Chapter 1:
Introduction to Environmental Policy

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Environmental Regulation: A Game-Theoretic Approach

Overview of Chapter 1

- **Section 1.1.** Environmental externalities and regulation.
- **Section 1.2.** Policy instruments for environmental regulation.
- **Section 1.3.** Market structure and strategic behavior.
- **Section 1.4.** A short history of environmental regulation.
- **Book Overview.** Roadmap for Chapters 2–8.

Section 1.1
Environmental Externalities and Regulation

The Climate Crisis – Key Facts

- Global CO_2 emissions from energy-related activities reached a record **37.8 gigatonnes (Gt)** in 2024.
  - Atmospheric CO_2 concentration: 422.5 parts per million.
  - Over 50% higher than pre-industrial levels; see IEA (2025).
- **2024 was the hottest year on record**:
  - Global surface temperatures: 1.29°C above 20th-century baseline.
  - 1.5°C threshold exceeded for a full calendar year for the first time; see NASA (2025).
- Consequences: extreme weather, rising sea levels, ecosystem disruption.

The Economic Toll

- Natural catastrophes in 2024 inflicted **$417 billion** in global economic losses.
  - Only $154 billion covered by insurance.
- In the United States alone:
  - 27 separate billion-dollar weather disasters in 2024.
  - $182.7 billion in damages; over 560 lives lost; see NOAA (2025).
  - Hurricanes, wildfires, floods, and severe storms.
- These figures underscore the urgency of effective environmental policy.
  - Policies must mitigate emissions *and* build resilience.
  - A strategic, economically grounded approach is required.

What Is an Externality?

- An **externality** occurs when the actions of one party impose costs (or benefits) on others that are not reflected in market prices.
- **Negative externalities** from pollution:
  - Air pollution from manufacturing.
  - Water contamination from agriculture.
  - Greenhouse gas emissions from transportation.
- **Market outcome**: Too much pollution is produced, and too few resources are allocated to mitigation.
  - Firms equate private marginal cost to marginal revenue.
  - Social marginal cost $=$ private marginal cost $+$ marginal external damage.
  - Equilibrium quantity $>$ socially optimal quantity.

Two Sources of Market Failure

- **Market failure 1: Negative externality.**
  - Pollution damages not internalized by the polluting firm.
  - Private optimum $\neq$ social optimum.
- **Market failure 2: Imperfect competition.**
  - Oligopolistic industries produce *less* output than the competitive benchmark.
  - Regulation interacts with market power: an emission fee may worsen the output distortion.
  - Subsidies may exacerbate overproduction under certain market structures.
- This book studies both market failures **simultaneously**:
  - Emission fees, permits, and abatement in imperfectly competitive markets.

Why Game Theory?

- In oligopolistic markets, firms **anticipate each other's reactions** and adjust behavior accordingly.
- Strategic interdependence affects:
  - Output levels and pricing.
  - Investment in abatement technology.
  - Responses to emission fees and permit systems.
- Game theory provides the tools to analyze these interactions:
  - **Nash Equilibrium**: each firm's strategy is a best response to others.
  - **Subgame Perfect Equilibrium**: backward induction removes non-credible threats.
- *Goal*: a unified framework for regulation in strategic environments.

Section 1.2
Policy Instruments for Environmental Regulation

Overview of Policy Instruments

- **Command-and-control (C&C) regulations**:
  - Technology standards and emission limits.
  - Direct mandates on what firms must do or use.
- **Market-based instruments**:
  - Emission taxes (Pigouvian fees): charge per unit of pollution.
  - Tradable permits (cap-and-trade): set a quantity ceiling.
  - Subsidies for clean technologies.
- **Information-based tools**:
  - Labeling and disclosure requirements.
- **Voluntary agreements**:
  - Firms commit to environmental targets in exchange for flexibility.

Emission Taxes (Pigouvian Fees)

- A tax t per unit of pollution sets the private cost of emitting equal to the social cost.
- **Optimal Pigouvian tax**: t^* = marginal external damage at the social optimum.
  - Firms reduce emissions until their marginal abatement cost = t^*.
  - Achieves the socially optimal emission level at least cost.
- **Strengths**:
  - Continuous incentive for abatement and innovation.
  - Revenue can fund environmental programs or reduce other taxes.
- **Weaknesses**:
  - Politically sensitive; uncertain abatement quantity.
  - Setting the right tax level requires information about damages.

Tradable Permits (Cap-and-Trade)

- Regulator sets a **cap** on total emissions and issues permits.
  - Firms buy and sell permits in a market.
  - Equilibrium permit price = marginal abatement cost across firms.
- **Strengths**:
  - Achieves cost-effective aggregate reductions (least-cost principle).
  - Certain quantity of aggregate emissions (unlike a tax).
- **Weaknesses**:
  - Requires robust market infrastructure and monitoring.
  - Permit price volatility can create investment uncertainty.
  - In oligopolistic markets: strategic permit trading may affect competitiveness.
- Studied extensively in Chapter 6 (tradable permits and market power).

Command-and-Control vs. Market-Based

- **C&C**: Technology standards, emission limits.
  - Easy to enforce and monitor.
  - May stifle innovation; firms meet the standard but no more.
  - Ignores heterogeneity in abatement costs.
- **Market-based**: Taxes and permits.
  - Achieve cost-efficiency by equating marginal abatement costs.
  - Encourage ongoing innovation beyond the standard.
- **Weitzman (1974) insight**: Under uncertainty, the instrument choice depends on the slope of marginal costs vs. marginal damages.
  - Steep marginal damage $\Rightarrow$ prefer quantity instrument (permits).
  - Steep marginal abatement cost $\Rightarrow$ prefer price instrument (tax).
- Analyzed rigorously in Chapters 3 and 4.

Equivalence and Non-Equivalence of Instruments

- Under **perfect competition** and certainty, an emission tax and a tradable permit system achieving the same aggregate quantity are equivalent.
  - The permit price equals the tax rate at equilibrium.
- Under **imperfect competition**, this equivalence breaks down:
  - Strategic output decisions interact differently with price vs. quantity instruments.
  - Chapter 2: Pigouvian tax vs. emission quota in a Cournot oligopoly.
  - Chapter 6: Tradable permits under market power.
- Asymmetric information also breaks equivalence:
  - Taxes fix price, permits fix quantity — but under uncertainty, the expected welfare differs.

Section 1.3
Market Structure and Strategic Behavior

Cournot Oligopoly

- n firms compete in quantities **simultaneously**.
  - Inverse demand: $p(Q) = a - bQ$, where $Q = \sum_{i=1}^{n} q_i$.
  - Firm i's profit: $\pi_i = p(Q) \cdot q_i - c \cdot q_i$.
- **Best response function (BRF)** of firm i:

$$q_i^*(q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}.$$

- **Symmetric Nash Equilibrium**: $q_i^* = \frac{a-c}{(n+1)b}$.
  - As $n \to \infty$, price $\to$ marginal cost (competitive outcome).
  - Monopoly when $n = 1$; duopoly when $n = 2$.

Stackelberg Model

- **Leader-follower** setting: firm 1 moves first, firm 2 responds.
- **Backward induction**:
  1. Firm 2 observes q_1 and best-responds: $q_2^*(q_1)$.
  2. Firm 1 anticipates $q_2^*(q_1)$ and maximizes its own profit.
- Equilibrium:

$$q_1^L = \frac{a-c}{2b}, \quad q_2^F = \frac{a-c}{4b}.$$

  - Leader produces more than under Cournot; follower less.
  - Total output $>$ Cournot: $Q^S = \frac{3(a-c)}{4b}$.
- Relevant for regulatory settings: regulator as a Stackelberg "leader."

Nash Equilibrium

- A **Nash Equilibrium (NE)** is a strategy profile $(s_1^*, \ldots, s_n^*)$ such that no player can improve by deviating unilaterally:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \text{ for all } s_i \text{ and all } i.$$

- **Existence**: Every finite game has at least one NE (possibly in mixed strategies).
- **Key property**: mutual best responses – no firm regrets its choice given the others' choices.
- In a *static game*, NE is the standard solution concept.
  - Cournot duopoly: intersection of BRFs.

Subgame Perfect Equilibrium (SPE)

- In **sequential games**, a Nash Equilibrium may rely on non-credible threats.
- A **Subgame Perfect Equilibrium** requires Nash Equilibrium behavior in *every* subgame.
  - Eliminates non-credible threats via backward induction.
- **Backward induction procedure**:
  1. Solve the last stage first.
  2. Given last-stage solutions, solve the second-to-last stage.
  3. Continue until the first stage.
- Example: emission fee game.
  - Stage 1: regulator sets fee t.
  - Stage 2: firms choose output q_i.
  - SPE: solve firm decisions first, then regulator's optimal fee.

Heterogeneous Firms

- In practice, firms differ in:
  - Production costs: $c_1 \neq c_2$.
  - Pollution intensity: e_i / q_i differs across firms.
  - Abatement costs: marginal abatement cost curves differ.
- **Implications for regulation**:
  - A uniform emission fee may impose disproportionate burdens on high-cost abaters.
  - Differentiated regulation can be welfare-improving but harder to implement.
  - Permit trading allows abatement to occur where it is cheapest.
- Studied in Chapter 3 (asymmetric firms and second-best regulation).

Uncertainty and Instrument Choice

- Regulators typically face uncertainty about:
  - Marginal abatement costs (private information of firms).
  - Marginal damage from pollution.
- **Weitzman (1974)**: Under uncertainty, the choice between taxes (price instruments) and permits (quantity instruments) depends on the relative slopes of marginal cost and marginal damage:
  - If marginal damage is steep $\Rightarrow$ quantity instrument preferable.
  - If marginal abatement cost is steep $\Rightarrow$ price instrument preferable.
- **Hybrid instruments** (price-quantity combinations) can outperform either pure instrument in some settings.
- This book analyzes these trade-offs in strategic market environments.

Abatement Investment

- Firms can invest in **abatement technology** to reduce emissions per unit of output.
  - Emission: $e_i = q_i - z_i$, where z_i is abatement.
  - Abatement cost: $C(z_i)$ – increasing and convex.
- Multi-stage game:
  1. Firms invest in abatement (z_i).
  2. Regulator sets emission fee (t).
  3. Firms choose output (q_i).
- **Strategic incentive**: firms may invest in abatement to influence the regulator's future fee.
  - More abatement $\Rightarrow$ lower fee expected $\Rightarrow$ higher profits.
- Analyzed in Chapters 5 and 7.

Mergers, Cross-Ownership, and Regulation

- **Mergers** reduce the number of firms, affecting both market power and aggregate emissions.
  - Less output: lower emissions (good for environment).
  - But also less competition: higher prices (bad for consumers).
- Antitrust authority weighs both effects when evaluating mergers.
- **Cross-ownership**: firm i holds a partial stake α in firm j.
  - Softens competition: each firm internalizes a share of rivals' profits.
  - Environmental implications: cross-ownership may raise or lower industry emissions.
- **Environmental R&D cartels**: firms cooperate on abatement investment.
  - Internalize abatement spillovers; may raise social welfare.
- Studied in Chapters 7 and 8.

Section 1.4
A Short History of Environmental Regulation

Origins: London Smog and Early US Regulation

- Industrial pollution and urban smog became visible public health threats in the mid-20th century.
- **The Great London Smog of 1952**:
  - Five days of severe air pollution killed an estimated 4,000–12,000 people.
  - Led to the UK Clean Air Act of 1956.
- **US Clean Air Act of 1970**:
  - Established the Environmental Protection Agency (EPA).
  - Mandated large reductions in automobile emissions by 1975.
  - Led to catalytic converters and unleaded gasoline.
  - Significantly reduced lead and CO levels; see EPA (2011).

The 1990 Clean Air Act Amendments

- Significantly expanded regulatory authority in the US.
- Introduced **market-based mechanisms**:
  - Tradable permits for SO_2 under the Acid Rain Program.
  - Reformulated fuels to reduce urban smog.
- **Outcomes by early 2000s**:
  - NOx and particulate matter (PM) emissions reduced by over 90% in some sectors.
  - The SO_2 cap-and-trade program became a flagship example of successful environmental market design.
- **Lesson**: Well-designed market-based instruments can achieve large environmental gains at low cost.

Sweden's Carbon Tax and Chile's Carbon Price

- **Sweden (1991)**:
  - First country to introduce a comprehensive carbon tax.
  - Exceeded €115 per ton of CO_2 by the 2020s.
  - Contributed to a 27% reduction in greenhouse gas emissions between 1990 and 2018 – while maintaining GDP growth.
- **Chile (2017)**:
  - Carbon tax targeting large thermal power plants.
  - Prompted a shift toward cleaner energy sources; see United Nations (2021).
- **Canada – Output-Based Pricing System (OBPS)**:
  - Sector-specific emission fees to incentivize low-carbon production.
  - Allows output-based credits for efficiency improvements.
- Analyzed in Chapter 2: how emission fees align private incentives with social goals.

Developing Countries: Colombia and South Africa

- Adoption of emission fees in developing countries has been more gradual but increasingly significant.
- **Colombia (2017)**:
  - Modest carbon tax covering fossil fuel use.
  - Revenue directed toward environmental programs.
- **South Africa (2019)**:
  - National carbon tax initially set at R120 ($\approx$ \$8) per ton of CO_2.
  - Exemptions for energy-intensive sectors to ease transition.
- **Challenges in developing countries**:
  - Limited administrative capacity.
  - Political resistance and equity concerns.
  - Nonetheless: important steps toward internalizing externalities; see World Bank (2025).

EU ETS and US Cap-and-Trade

- **EU Emissions Trading System (EU ETS)** – launched 2005:
  - World's largest carbon market, covering ∼40% of EU greenhouse gas emissions.
  - Phase I–IV improvements: tighter caps, auctioning, carbon price $>$ €60/ton by 2022.
- **US Regional Greenhouse Gas Initiative (RGGI)**:
  - Cap-and-trade program covering power sector in 12 Northeast states.
  - Demonstrated feasibility of state-level carbon pricing.
- **California Cap-and-Trade (2013)**:
  - Linked with Quebec; covers $\sim$ 85% of California GHG emissions.
  - Allowance prices in the \$25–35 range.
- Analyzed in Chapter 6: tradable permits in imperfectly competitive markets.

The Paris Agreement and Global Climate Governance

- **Paris Agreement (2015)**: 196 parties committed to limiting global warming to well below 2°C above pre-industrial levels.
  - Countries submit Nationally Determined Contributions (NDCs).
  - Voluntary pledges reviewed and ratcheted up every five years.
- **Economic challenges**:
  - Free-riding: countries benefit from others' abatement without contributing.
  - Verification and enforcement: no binding compliance mechanism.
  - Carbon leakage: production shifts to unregulated jurisdictions.
- **Market-based solutions**: Carbon border adjustments (EU CBAM), international carbon markets (Article 6 of Paris Agreement).
- These strategic dimensions are central to the models in this book.

The Coase Theorem

- **Coase (1960)**: In the absence of transaction costs and with well-defined property rights, private bargaining leads to an efficient outcome regardless of how rights are initially assigned.
- **Implication**: Government intervention may be unnecessary if:
  - Property rights are clearly defined.
  - Transaction costs are negligible.
  - Parties can negotiate freely.
- **Limitations in practice**:
  - Many polluters and victims $\Rightarrow$ high transaction costs.
  - Asymmetric information, hold-up problems.
  - Unclear property rights (who owns clean air?).
- **Empirical evidence**: Tradable permit systems partly reflect Coasian logic.
- Revisited in Chapter 2.

Book Overview
Roadmap for Chapters 2–8

Chapter 2: Regulating a Polluting Oligopoly

- Baseline model: n Cournot firms with emissions $e_i = q_i$.
- Regulator maximizes social welfare:
  $W = CS + PS + T - \frac{d}{2}E^2$, where $E = \sum e_i$.
- **Key results**:
  - Optimal emission fee and its relationship to market structure.
  - Comparison: fee vs. emission quota.
  - The Coase Theorem revisited in an oligopoly context.
- **Tools introduced**:
  - Best response functions (BRFs).
  - Two-stage Stackelberg game between regulator and firms.
  - Social marginal benefit / social marginal cost framework.

Chapters 3–4: Asymmetric Firms and Uncertainty

- **Chapter 3**: Regulating asymmetric Cournot oligopolists.
  - Firms differ in marginal costs or pollution intensity.
  - First-best vs. second-best (uniform fee) regulation.
  - Price vs. quantity instruments under asymmetric information.
- **Chapter 4**: Uncertainty and the Weitzman result.
  - Uncertainty about abatement costs and/or damages.
  - Expected welfare comparison: emission tax vs. quantity standard.
  - Extension to oligopolistic markets.
  - Hybrid instruments (taxes with a safety valve).

Chapters 5–6: Sequential Competition and Abatement Investment

- **Chapter 5**: Sequential competition and regulation.
  - Stackelberg (leader-follower) market structure.
  - How first-mover advantage interacts with emission fees.
  - Two-period model with pollution persistence; regulatory commitment.
- **Chapter 6**: Regulation when firms invest in abatement.
  - Firms choose output *and* abatement investment simultaneously.
  - Tax-saving effect (TSE) and emission-reduction effect (ERE).
  - Spillovers, free-riding, and environmental R&D joint ventures (ERJVs).

Chapters 7–8: Mergers and Managerial Incentives

- **Chapter 7**: Environmentally friendly mergers?
  - Effect of horizontal mergers on emissions and social welfare.
  - When do mergers reduce pollution? Antitrust vs. environmental goals.
  - Optimal emission fee before and after a merger.
- **Chapter 8**: Managerial incentives and environmental policy.
  - Owners delegate decisions to managers with profit-sales objectives.
  - How managerial contracts interact with emission regulation.
  - Implications for fee design when managers, not owners, choose output.
- **Unifying theme**: All chapters share the same baseline welfare function and oligopoly framework introduced in Chapter 2.

Analytical Framework and Notation

- **Baseline model** (used throughout):
  - n firms; inverse demand $p(Q) = 1 - Q$; marginal cost $c \in [0, 1]$.
  - Emissions: $e_i = q_i - z_i$ ($z_i \geq 0$: abatement).
  - Environmental damage: $ED = \frac{d}{2}E^2$, $d > 0$.
- **Social welfare**:

$$W = CS + PS + T - ED = \frac{Q^2}{2} + \sum_i \pi_i + tE - \frac{d}{2}E^2.$$

- **Key notation**:
  - t: emission fee per unit; $E = Q - Z$: aggregate net emissions.
  - $Z = \sum z_i$: aggregate abatement.
  - Superscripts: NE (Nash), SO (social optimum), NM (no merger), M (merger).

How Each Chapter Adds Complexity

- **Ch. 2** – Baseline: n symmetric Cournot firms, linear demand, per-unit emission fee.
- **Ch. 3** – Extension: asymmetric firms (different costs or pollution intensities); second-best uniform fees.
- **Ch. 4** – Extension: uncertainty about costs or damages; Weitzman price-vs.-quantity choice.
- **Ch. 5** – Extension: sequential (Stackelberg) competition; intertemporal pollution persistence.
- **Ch. 6** – Extension: endogenous abatement investment; spillovers; ERJVs.
- **Ch. 7** – Extension: horizontal mergers; antitrust-environment interaction.
- **Ch. 8** – Extension: managerial incentive contracts; owners delegate to profit-sales maximizing managers.

Prerequisites

- **Calculus**: Partial derivatives, constrained optimization, the envelope theorem.
- **Microeconomics**: Consumer surplus, producer surplus, welfare analysis; monopoly and oligopoly pricing.
- **Game theory** (covered in this chapter):
  - Normal-form and extensive-form games.
  - Nash Equilibrium and Subgame Perfect Equilibrium.
  - Backward induction; Stackelberg competition.
- **Not required**: Advanced econometrics, stochastic calculus, or industrial organization beyond intermediate microeconomics.
- Each chapter is self-contained and can be read independently after Chapter 2.

Chapter 1 – Key Takeaways

- **Environmental externalities** cause market failures: too much pollution, too little abatement.
- **Policy instruments** – taxes, permits, standards – differ in efficiency, enforceability, and political feasibility.
- **Market structure matters**: in oligopolies, regulation interacts with strategic behavior, making simple Pigouvian prescriptions incomplete.
- **Game theory** provides the tools to model these interactions: Nash Equilibrium, backward induction, and Subgame Perfect Equilibrium.
- **Historical evidence**: well-designed market-based instruments (US SO_2 trading, Sweden's carbon tax) can achieve large environmental gains at low cost.
- **This book's approach**: progressively add complexity to the baseline model, always solving for SPE via backward induction.

Thank you!

Chapter 2

Regulating a Polluting Oligopoly

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 2

- This chapter analyzes environmental regulation in oligopolistic industries using a game-theoretic framework.
- Sections:
  - 2.1 Introduction and guiding questions.
  - 2.2 Benchmark: unregulated oligopoly.
  - 2.3 Regulated oligopoly: the optimal emission fee.
  - 2.4 Marginal benefits and costs interpretation.
  - 2.5 Other regulations: quotas, tradable permits, Coase Theorem.
  - 2.6 Case studies.
  - 2.7 Which results depend on the functional forms?
  - 2.8 Summary.

Section 2.1

Introduction

The Central Problem

- Firms in oligopolistic industries generate *emissions* as a by-product of production $\Rightarrow$ negative externalities reduce social welfare.
- How should policymakers regulate pollution while accounting for *strategic* firm behavior?
- Chapter 2 provides a game-theoretic answer:
  - Section 2.2: derive the unregulated benchmark (Cournot equilibrium).
  - Section 2.3: introduce emission fees and find the socially optimal fee.
  - Sections 2.4–2.7: interpret results, compare other instruments, and check which results depend on the functional forms.
- Key insight: two market failures coexist in oligopoly, *market concentration* (underproduction) and *pollution* (overproduction). The optimal policy must balance both.

Two Competing Market Failures in Oligopoly

- **Market failure 1 – Pollution externality:** each firm ignores the damage its emissions impose on society $\Rightarrow$ firms produce *too much* output and generate excessive emissions.
  - Corrective instrument: **emission fee** (positive t).
- **Market failure 2 – Market concentration:** a duopoly produces *less* than the competitive level due to market power $\Rightarrow$ underproduction relative to the social optimum.
  - Corrective instrument: **output subsidy** (negative t).
- The optimal emission fee **balances** these two failures:

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

- When $d > \frac{1}{2}$: pollution failure dominates $\Rightarrow t^* > 0$.
- When $d < \frac{1}{2}$: concentration failure dominates $\Rightarrow t^* < 0$ (subsidy).

Guiding Questions

1. How does Cournot competition affect pollution and welfare in unregulated markets?
2. What are the marginal benefits and costs of optimal regulation?
3. What is the role of emission fees in aligning private and social incentives?
4. Are alternative policy instruments, like quotas and tradable permits, as effective as emission fees at addressing pollution?

Real-World Context: Pollution in Concentrated Industries

- In several polluting industries, a few firms account for most production: firms have market power, and their output pollutes.
- **Cement (Canada):** six companies run the country's 15 cement plants (section 2.6.1).
- **Steel, aluminum, cement (US):** the proposed Clean Competition Act would charge emissions above an industry benchmark (section 2.6.2).
- **Electricity (Chile, EU):** generators paid 94% of Chile's green taxes on stationary sources in 2017; the EU ETS caps their emissions (sections 2.6.4–2.6.5).
- Chapter 2 provides the baseline model that chapters 3–8 extend.

Section 2.2

Benchmark: Unregulated Oligopoly

Model Setup

- **Industry**: two firms competing à la Cournot (simultaneous output choices). (Later extended to n firms.)
- **Inverse demand**: $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- **Marginal production cost**: c, symmetric, with $1 > c \geq 0$.
- **Emissions**: every unit of output generates one unit of emissions, so $e_i = q_i$ and aggregate emissions $E = Q$.
- **Environmental damage**: $Env = \frac{d}{2}E^2$, where $d > \frac{1}{2}$ measures pollution severity.
  - Damage is increasing and convex in emissions: marginal damage dE rises with E.
- **Social welfare**: $W = CS + PS - Env$, where $CS = \frac{Q^2}{2}$ and $PS = \pi_1 + \pi_2$.

Firm 1's Profit-Maximization Problem

Firm 1 chooses q_1 to solve:

$$\max_{q_1 \geq 0} \pi_1 = (1 - q_1 - q_2)q_1 - cq_1$$

- First term: total revenue $p(Q) \cdot q_1 = (1 - q_1 - q_2)q_1$.
- Second term: total production cost cq_1.
- Firm 1 takes q_2 as *given* (cannot control rival's output).

Taking the first-order condition (FOC) with respect to q_1:

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - c = 0$$

Interpretation: firm 1 increases q_1 until marginal revenue $1 - 2q_1 - q_2$ equals marginal cost c.

Firm 1's Best Response Function (Section 2.2.1)

Rearranging the FOC yields firm 1's **best response function (BRF)**:

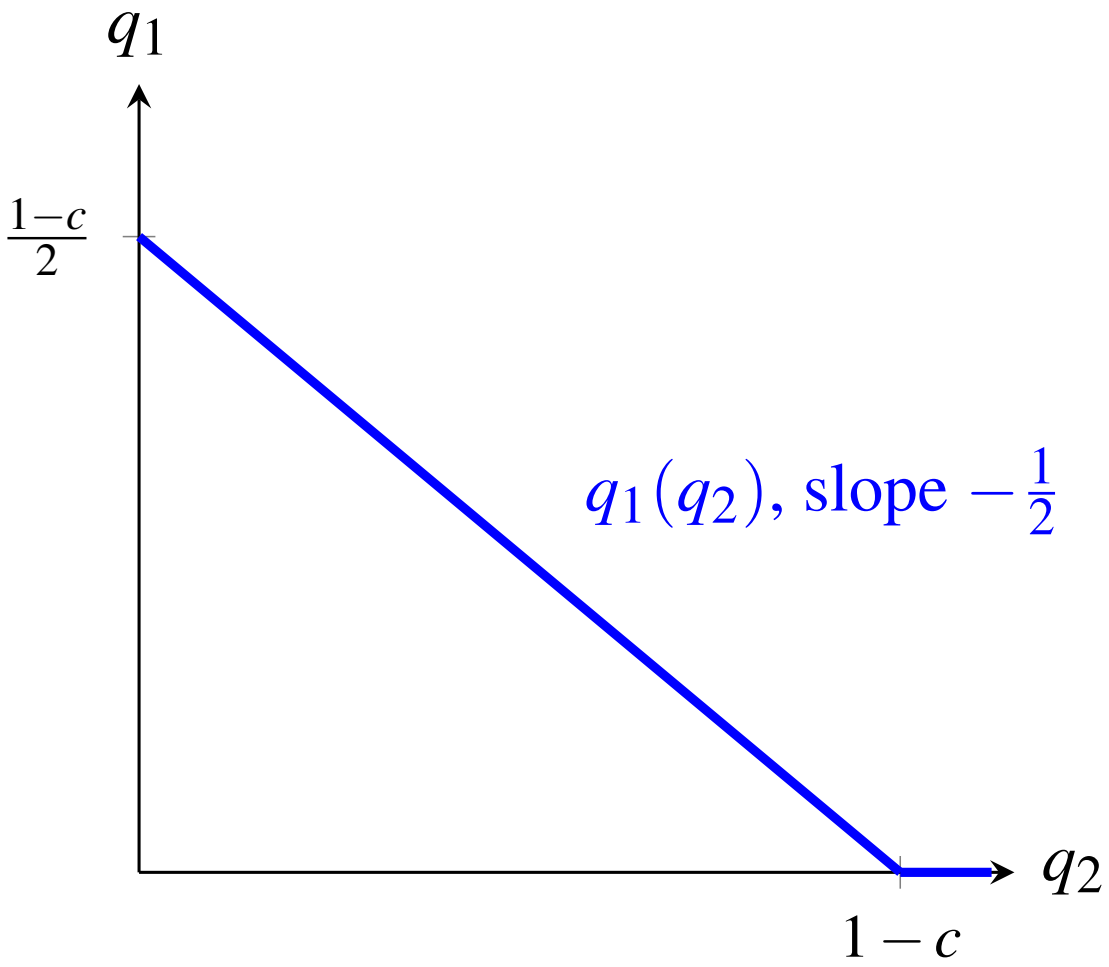
$$q_1(q_2) = \begin{cases} \frac{1-c}{2} - \frac{1}{2}q_2 & \text{if } q_2 \leq 1-c \\ 0 \quad \text{otherwise} \end{cases}$$

Key properties:

1. When $q_2 = 0$: firm 1 produces the *monopoly output* $\frac{1-c}{2}$.
2. As q_2 rises, q_1 *falls* (strategic substitutes), with slope $-\frac{1}{2}$.
3. When $q_2 = 1 - c$: firm 1 shuts down ($q_1 = 0$).

Interpretation: firm 1's optimal output is decreasing in its rival's output because rival production reduces the residual demand facing firm 1.

Figure 2.1 – Firm 1's Best Response Function



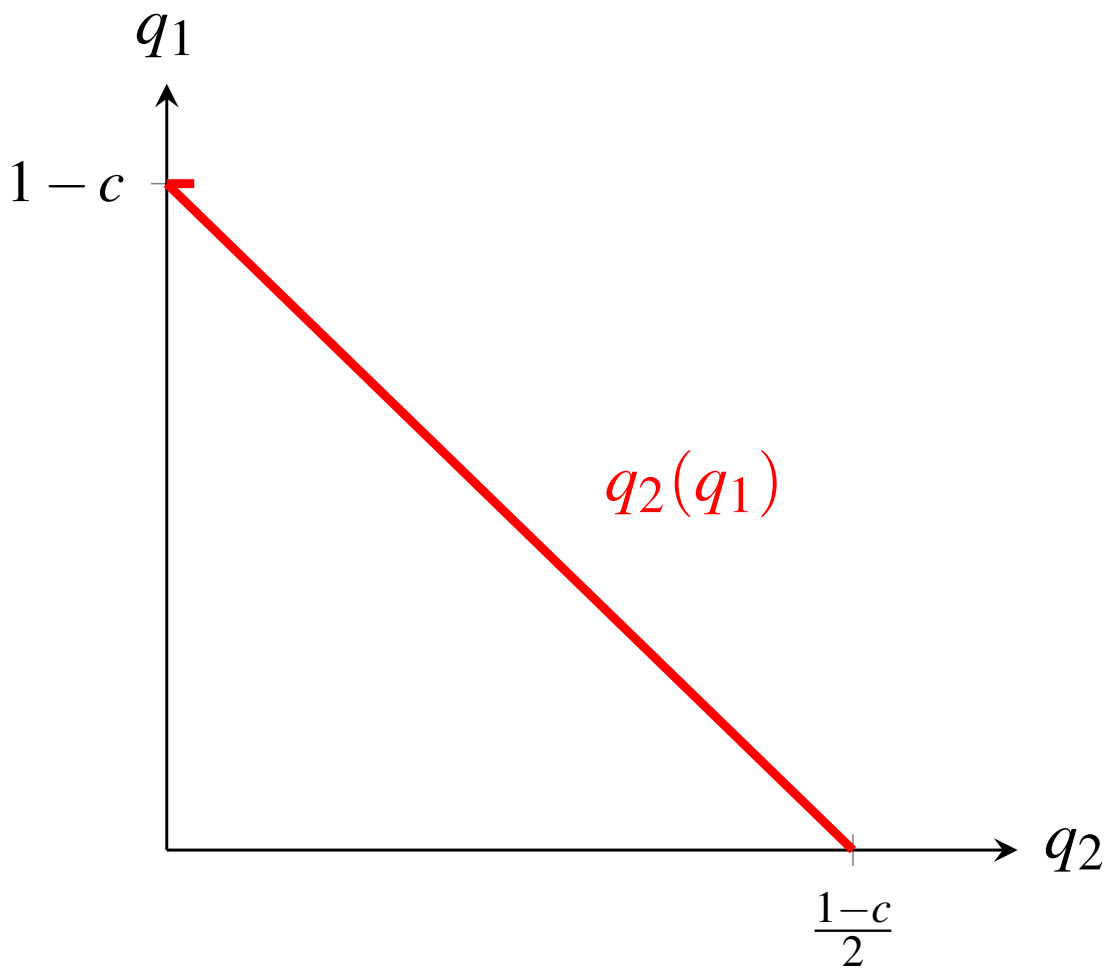
Firm 2's Best Response Function

By symmetry (same costs and demand), firm 2's BRF is:

$$q_2(q_1) = \begin{cases} \frac{1-c}{2} - \frac{1}{2}q_1 & \text{if } q_1 \leq 1-c \\ 0 \quad \text{otherwise} \end{cases}$$

- Figure 2.2 uses the *same axes* as Figure 2.1: q_2 on the horizontal axis, q_1 on the vertical axis.
- Its intercept, $q_2 = \frac{1-c}{2}$, lies on the horizontal axis, where $q_1 = 0$.
- q_2 falls as q_1 rises; firm 2 shuts down when q_1 reaches $1 - c$.

Figure 2.2 – Firm 2's Best Response Function



Finding Equilibrium Output (Section 2.2.2)

Two methods to find the Nash equilibrium:

Method 1: Invoke symmetry (same costs and demand):

- Impose $q_1 = q_2 = q$ in BRF_1: $q = \frac{1-c}{2} - \frac{1}{2}q$
- Solve: $\frac{3}{2}q = \frac{1-c}{2}$, so $q^{NR} = \dfrac{1-c}{3}$

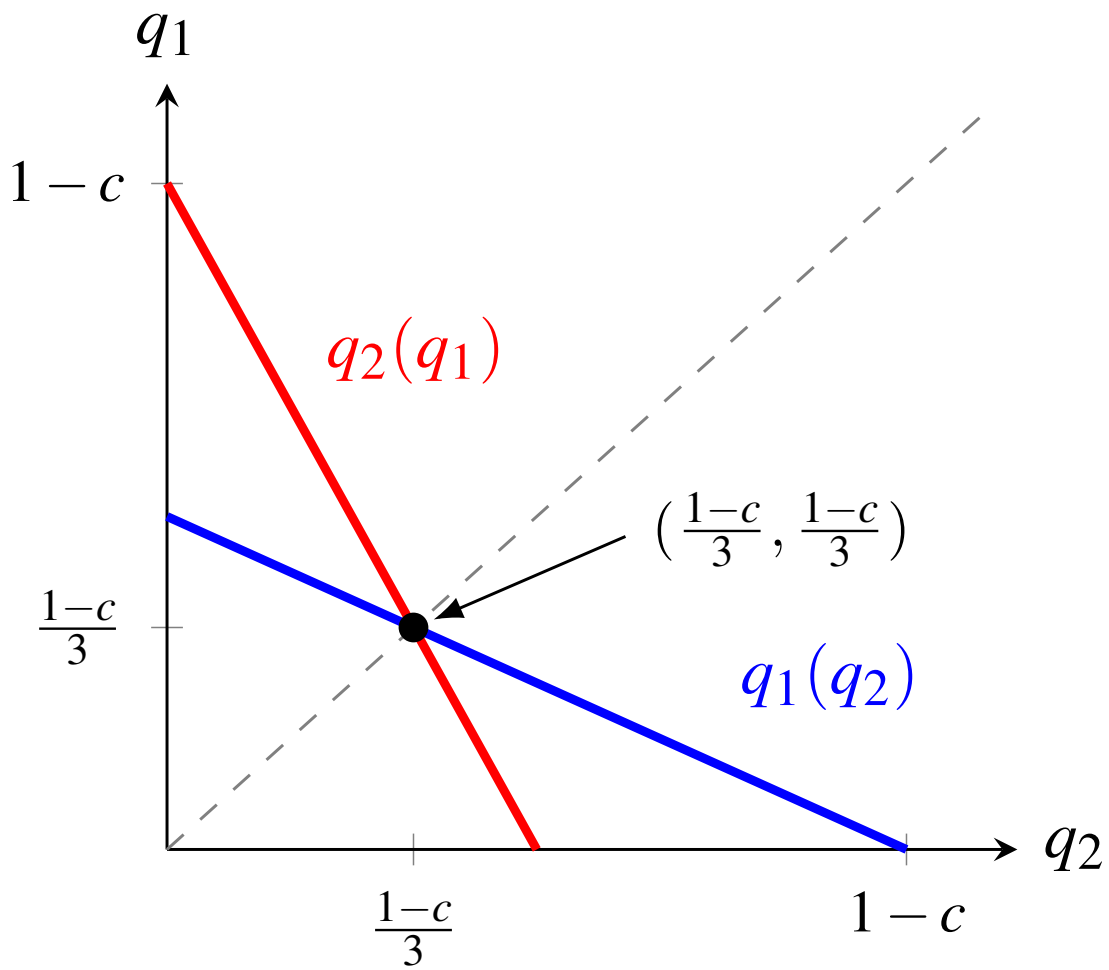
Method 2: Substitute one BRF into the other:

- Insert $q_2(q_1)$ into $q_1(q_2)$: $q_1 = \frac{1-c}{2} - \frac{1}{2}\left(\frac{1-c}{2} - \frac{1}{2}q_1\right)$
- Simplify: $q_1 - \frac{1}{4}q_1 = \frac{1-c}{2} - \frac{1-c}{4}$, yielding $q_1 = \frac{1-c}{3}$. $\Rightarrow$ same result.

Equilibrium outcomes (superscript NR = no regulation):

$$q_1^{NR} = q_2^{NR} = \frac{1-c}{3}, \quad Q^{NR} = \frac{2(1-c)}{3}, \quad p^{NR} = \frac{1+2c}{3}$$

Figure 2.3 – Equilibrium Output



Pollution and Welfare Without Regulation (Section 2.2.3)

Since $e_i = q_i$, aggregate emissions are $E^{NR} = Q^{NR} = \frac{2(1-c)}{3}$.

Welfare under no regulation:

$$\begin{aligned} W^{NR} &= CS^{NR} + PS^{NR} - Env^{NR} \\ &= \frac{(Q^{NR})^2}{2} + \left[(1 - Q^{NR})Q^{NR} - cQ^{NR}\right] - \frac{d}{2}(E^{NR})^2 \\ &= \frac{2(1-c)^2(2-d)}{9} \end{aligned}$$

Welfare Without Regulation: Observations

- W^{NR} is *decreasing* in pollution severity d.
- $W^{NR} < 0$ if $d > 2$ (environmental damage dominates all surplus).
- If $d > \frac{1}{2}$: firms ignore the externality $\Rightarrow$ socially *excessive* output and emissions.
- Regulation is needed to correct this market failure.

Extending to n Firms (Section 2.2.4)

With $n \geq 2$ symmetric firms, every firm i solves:

$$\max_{q_i \geq 0} \pi_i = (1 - q_i - Q_{-i})q_i - cq_i$$

where $Q_{-i} = \sum_{j \neq i} q_j$.

Invoking symmetry ($q_i = q$ for all i, so $Q_{-i} = (n-1)q$):

$$q^{NR}(n) = \frac{1-c}{n+1}, \qquad Q^{NR}(n) = \frac{n(1-c)}{n+1}$$

Key findings:

- *Individual* output *decreases* in n: each firm is smaller.
- *Aggregate* output and emissions *increase* in n: while each firm contributes less, the industry as a whole emits more.

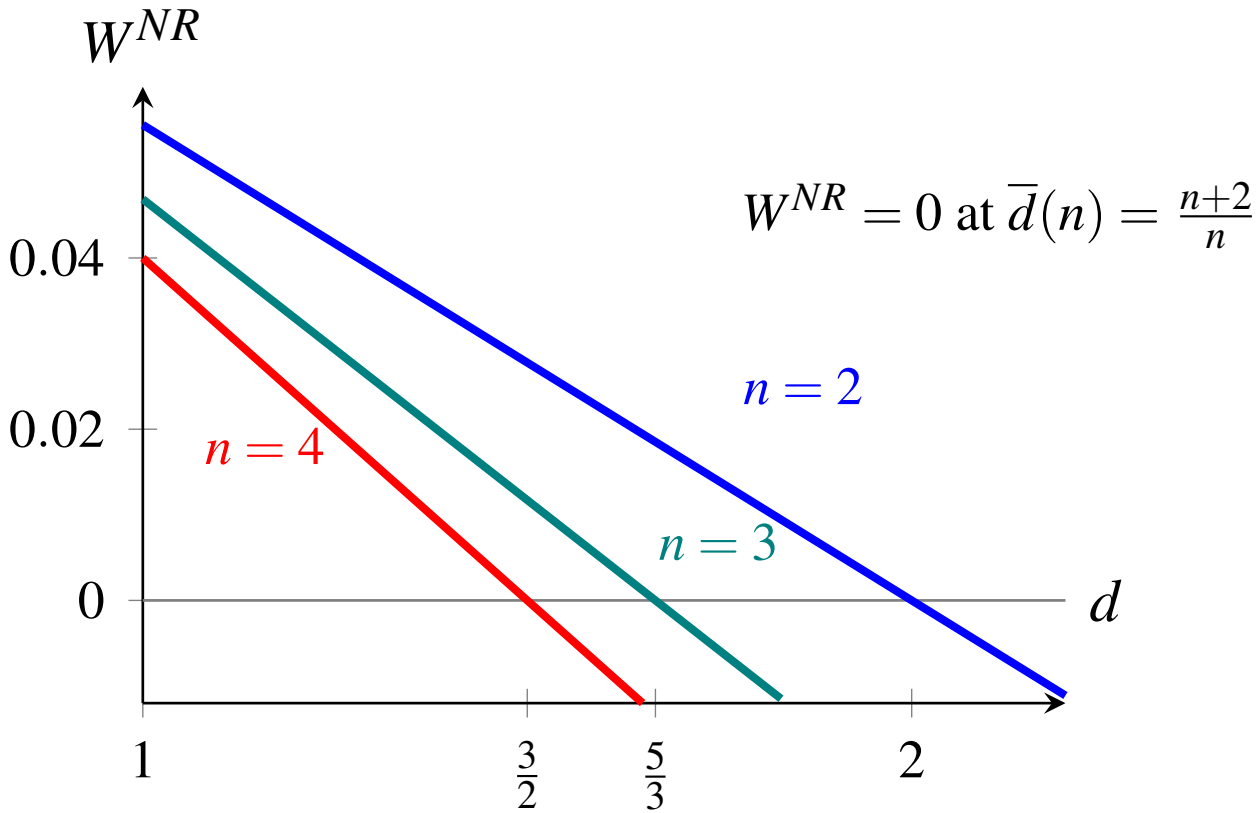
Welfare With n Firms: Conditions for Positive Welfare

- With n firms, social welfare under no regulation is:

$$W^{NR}(n) = \frac{n(1-c)^2[2+n(1-d)]}{2(n+1)^2}$$

- $W^{NR}(n) > 0$ if and only if $d < \overline{d}(n) \equiv \frac{n+2}{n}$; for instance, $\overline{d}(2) = 2$ and $\overline{d}(3) = \frac{5}{3} \approx 1.67$.
- $\frac{\partial \overline{d}(n)}{\partial n} = -\frac{2}{n^2} < 0$: more firms $\Rightarrow$ aggregate emissions and environmental damage rise, so welfare is more likely negative.
- **Policy implication**: competitive industries face a *more urgent* need for environmental regulation than concentrated ones, because competition raises aggregate pollution.

Figure 2.4 – Welfare $W^{NR}(n)$ as a Function of d



At $n = 2, 3, 4$ and $c = \frac{1}{2}$: every line crosses zero at $d = \overline{d}(n)$, which falls in n.

Section 2.3

Regulated Oligopoly

The Two-Stage Regulatory Game

The regulator (Environmental Agency, EA) introduces a per-unit emission fee t. The game has two stages:

1. **Stage 1 (EA)**: The EA sets emission fee t, paid by every firm per unit of emissions ($t < 0$ is a per-unit subsidy).
2. **Stage 2 (Firms)**: Each firm i observes t and chooses output q_i simultaneously (Cournot) to maximize profits.

- Solution concept: **Subgame Perfect Equilibrium (SPE)**, found by *backward induction* (section B.4 of the Game Theory Appendix).
- We solve Stage 2 first, obtaining $q(t)$; the EA anticipates $q(t)$ when setting the fee in Stage 1.

Stage 2: Firms' Output Decisions (Section 2.3.1)

Each firm i takes fee t as given and solves:

$$\max_{q_i \geq 0} \pi_i = (1 - q_i - q_j)q_i - (c + t)q_i$$

- The fee raises effective marginal cost from c to $c + t$.
- FOC: $1 - 2q_i - q_j = c + t$, so firm i is active if $q_j \leq 1 - c - t$.

Invoking symmetry ($q_i = q_j = q$):

$$q(t) = \frac{1 - c - t}{3}$$

Stage 2: Aggregate Output and Emissions

Aggregate output and emissions as a function of the fee:

$$Q(t) = \frac{2(1-c-t)}{3} = E(t)$$

- $Q(t)$ is *decreasing* in t.
- When $t = 0$: $Q(0) = Q^{NR} = \frac{2(1-c)}{3}$.
- When $t = 1 - c$: $Q = 0$ (firms shut down).

Stage 1: EA Sets the Optimal Fee (Direct Approach)

Anticipating $Q(t)$, the EA maximizes social welfare:

$$\max_t W = CS + PS + T - Env$$

where $T = tQ(t)$ is total tax revenue (returned to society as a lump-sum transfer). Note that $PS + T = (1 - Q)Q - cQ$, so:

$$W = \frac{Q(t)^2}{2} + \left[(1 - Q(t))Q(t) - cQ(t)\right] - \frac{d}{2}Q(t)^2$$

FOC with respect to t:

$$\frac{\partial W}{\partial t} = \frac{2(1-c)(2d-1) - 4t(1+d)}{9} = 0$$

Optimal emission fee:

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Interpreting the Optimal Fee

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)} \begin{cases} > 0 & \text{if } d > \frac{1}{2} \\ = 0 & \text{if } d = \frac{1}{2} \\ < 0 & \text{if } d < \frac{1}{2} \end{cases}$$

Two competing market failures in oligopoly:

1. **Pollution externality**: firms emit too much $\Rightarrow$ calls for a *fee* (positive t).
2. **Market concentration**: duopoly output is below the competitive level $\Rightarrow$ calls for a *subsidy* (negative t).

- $d > \frac{1}{2}$: pollution dominates; $d < \frac{1}{2}$: concentration dominates; $d = \frac{1}{2}$: the two failures offset each other.
- Throughout the book, we consider $d > \frac{1}{2}$, so $t^* > 0$.

No Pollution Damage: An Output Subsidy (Section 2.3.2)

- **No damage**, $d = 0$: only the second market failure, market concentration, arises.
- Duopoly output is socially *insufficient*, so the EA subsidizes output:

$$t^* = -\frac{1-c}{2} < 0$$

- Every firm receives a subsidy of $\frac{1-c}{2}$ dollars per unit, giving it incentives to increase production.
- The same argument applies with mild damage, $0 < d < \frac{1}{2}$.

Subsidies and Corner Solutions (Section 2.3.2)

- Throughout the book, the EA can set a *negative* fee, $t < 0$, which we interpret as a per-unit **subsidy**.
- If subsidies are *not available* (political or budgetary constraints), the EA chooses t subject to $t \geq 0$.
- Welfare (2.9) is concave in t, so the EA sets:

$$t = \begin{cases} 0 & \text{if } d < \frac{1}{2} \\ t^* & \text{otherwise} \end{cases}$$

- $t = 0$ is a **corner solution** (Appendix A.4). The same applies to every negative fee in later chapters.

Comparative Statics of t^*: Pollution Severity

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Effect of pollution severity d:

$$\frac{\partial t^*}{\partial d} = \frac{3(1-c)}{2(1+d)^2} > 0$$

More severe pollution $\Rightarrow$ *stricter* optimal fee.

Comparative Statics of t^*: Production Costs

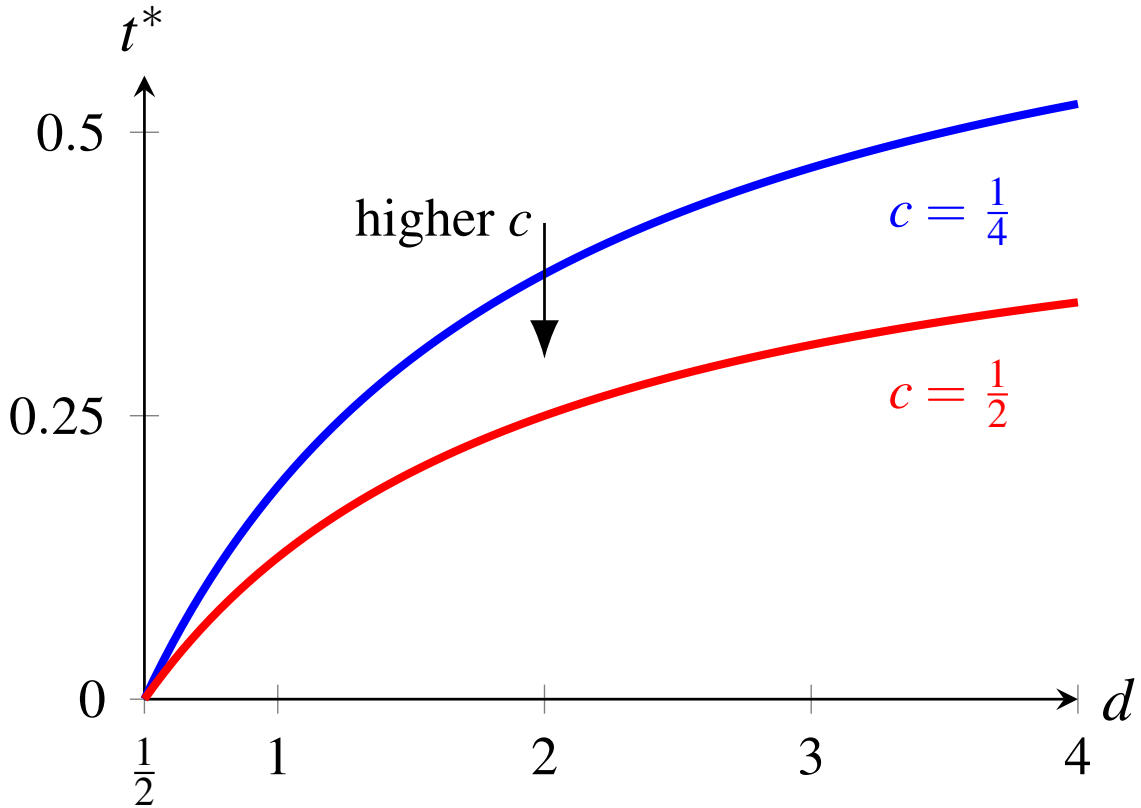
Effect of marginal cost c:

$$\frac{\partial t^*}{\partial c} = -\frac{2d-1}{2(1+d)} < 0 \quad (\text{since } d > \tfrac{1}{2})$$

Higher production costs $\Rightarrow$ EA anticipates less output and emissions $\Rightarrow$ *less stringent* fee.

Summary: the optimal fee is jointly shaped by the severity of the pollution externality *and* the degree of market inefficiency.

Figure 2.5 – Fee t^* Increases in d and Decreases in c



Evaluated at $c = \frac{1}{4}$ and $c = \frac{1}{2}$ (less efficient firms).

Equilibrium Output Under the Optimal Fee

Inserting t^* into $q(t) = \frac{1-c-t}{3}$:

$$q(t^*) = \frac{1-c-t^*}{3} = \frac{1-c}{2(1+d)}$$

Aggregate regulated output:

$$Q(t^*) = \frac{2(1-c-t^*)}{3} = \frac{1-c}{1+d}$$

Regulated vs. Unregulated Output

Comparison with unregulated output (since $d > \frac{1}{2} \Rightarrow 1 + d > \frac{3}{2}$):

$$Q(t^*) = \frac{1-c}{1+d} < \frac{2(1-c)}{3} = Q^{NR}$$

Conclusion: the emission fee reduces output (and emissions) below the unregulated level, moving the market toward the social optimum.

Indirect Approach: Finding Socially Optimal Output

Step 1: find Q^{SO} that maximizes welfare directly:

$$\max_{Q \geq 0} W = \frac{Q^2}{2} + \left[(1-Q)Q - cQ\right] - \frac{d}{2}Q^2$$

FOC: $Q + (1-2Q) - c - dQ = 0$, yielding:

$$Q^{SO} = \frac{1-c}{1+d}$$

Indirect Approach: Finding the Fee

Step 2: find fee t that induces $Q(t) = Q^{SO}$:

$$\frac{1-c}{1+d} = \frac{2(1-c-t)}{3} \quad \Rightarrow \quad t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Key result: both approaches yield the same t^*.

The indirect approach separates two sub-problems: (i) what is the socially optimal outcome? and (ii) what fee implements it?

Fee t^* Achieves the First Best (Section 2.3.2)

- At t^*, every firm produces $q(t^*) = \frac{1-c}{2(1+d)} > 0$: **both firms remain active**.
- Firms are symmetric with constant marginal costs $\Rightarrow$ the division of Q^{SO} between them does not affect total production costs.
- Therefore, t^* achieves the **first best**: firms produce Q^{SO} at the lowest possible cost.

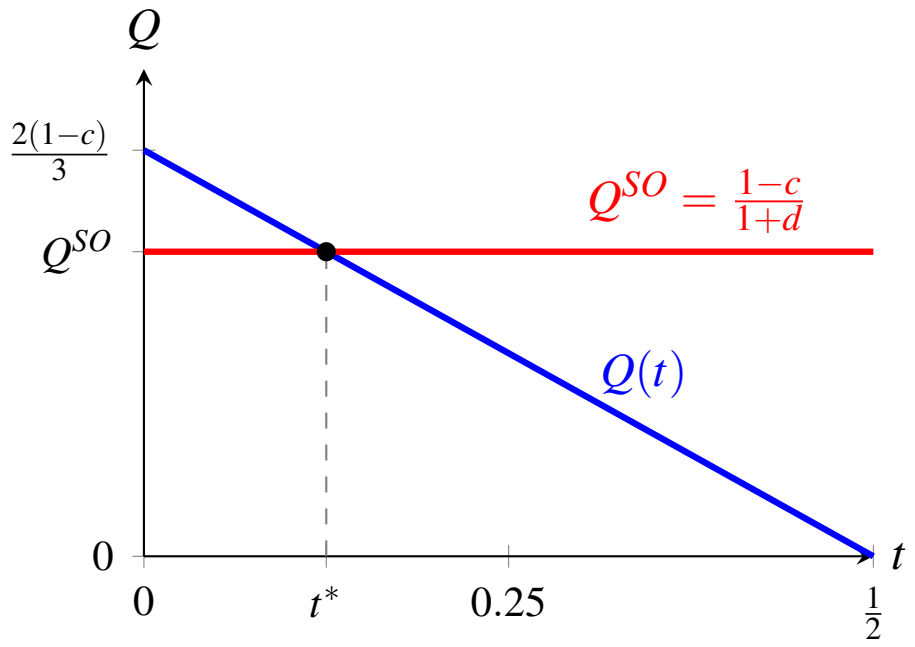
Two Checks After Every Fee

In later chapters, we check two properties after every fee:

1. Do **all firms remain active** at the fee?
2. Does a single fee achieve the **first best**, or only a **second best**?

In addition, when the fee is negative (a subsidy) and subsidies are not available, the EA sets $t = 0$ instead.

Figure 2.6 – Using $Q(t)$ and Q^{SO} to Find the Fee t^*



- At $c = \frac{1}{2}$, $d = 1$: $Q(t)$ falls from $\frac{1}{3}$ at $t = 0$ to zero at $t = 1 - c$; Q^{SO} does not depend on t.
- $t < t^*$: $Q(t) > Q^{SO}$, fee too lax; $t > t^*$: $Q(t) < Q^{SO}$, too stringent.

Welfare Comparison: Regulated vs. Unregulated

- **Welfare without regulation**:

$$W^{NR} = \frac{2(1-c)^2(2-d)}{9}$$

- **Welfare under the optimal fee**, where output is Q^{SO}:

$$W(t^*) = \frac{(1-c)^2}{2(1+d)}$$

- $W(t^*) \geq W^{NR}$, with equality only at $d = \frac{1}{2}$, where $t^* = 0$.

Where Does the Welfare Gain Come From?

- The fee moves aggregate output from Q^{NR} to Q^{SO}, where $SMB = SMC$.
- If $d > \frac{1}{2}$: lower output reduces environmental damage by more than it reduces consumer and producer surplus.
- If $d < \frac{1}{2}$: the subsidy raises output, increasing surplus by more than environmental damage.
- Fee revenue is a *transfer*: firms pay it, and the EA returns it to society.

Section 2.4

Marginal Benefits and Costs Interpretation

Deriving SMB and SMC

From the welfare-maximization problem:

$$\max_{Q \geq 0} W = \frac{Q^2}{2} + \left[(1-Q)Q - cQ\right] - \frac{d}{2}Q^2$$

FOC and rearrangement:

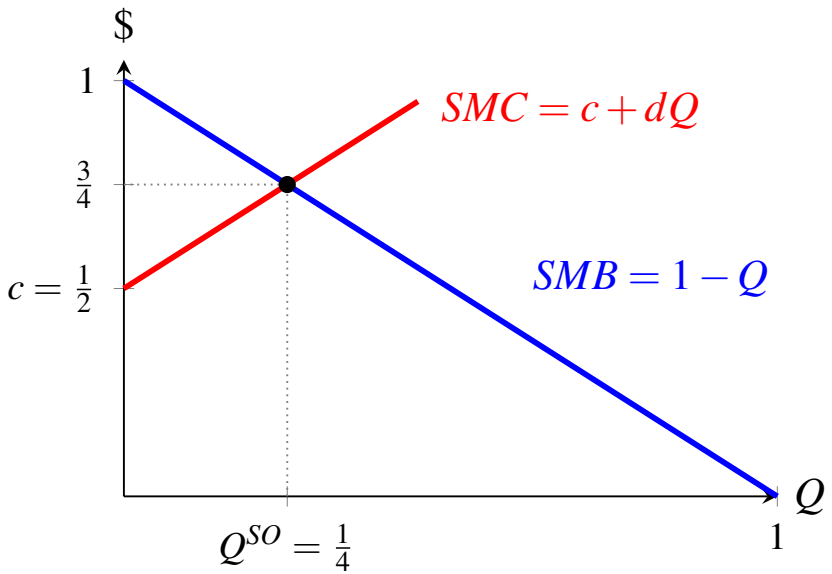
$$\underbrace{1-Q}_{SMB} = \underbrace{c+dQ}_{SMC}$$

Solving $SMB = SMC$ gives $Q^{SO} = \frac{1-c}{1+d}$, confirming the result from the Indirect Approach.

Interpreting SMB and SMC

- **Social Marginal Benefit**, $SMB = Q + (1 - 2Q) = 1 - Q$: the social gain from one more unit (higher consumer surplus and producers' marginal revenue).
- **Social Marginal Cost**, $SMC = PMC + MEC = c + dQ$: the social cost of one more unit, accounting for production and environmental damage.
  - $PMC = c$: private marginal cost, what the firm pays per unit.
  - $MEC = dQ$: marginal external cost, the environmental damage that the firm *ignores*, increasing in aggregate output.

Figure 2.7 – *SMB* and *SMC*



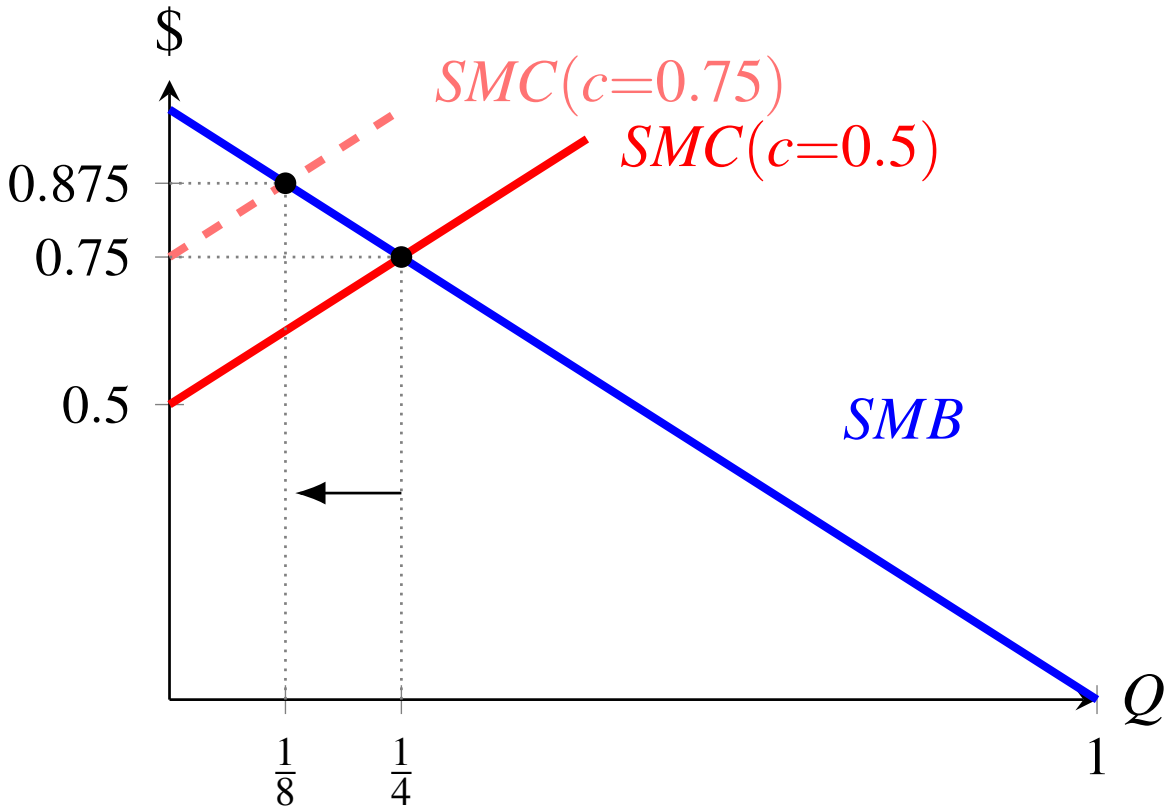
At $c = \frac{1}{2}$, $d = 1$, the curves cross at $Q^{SO} = \frac{1}{4}$.

- $Q > Q^{SO}$: $SMC > SMB$; *reducing* output raises welfare.
- $Q < Q^{SO}$: $SMC < SMB$; *increasing* output raises welfare.
- $Q = Q^{SO}$: $SMB = SMC$, the social optimum.

Comparative Statics: Effect of Higher Cost c

- $SMC = c + dQ$ shifts *upward* (parallel shift) when c rises.
- The *SMB–SMC* crossing point moves *leftward*: Q^{SO} decreases.
- Intuition: less efficient production $\Rightarrow$ it is socially optimal to produce fewer units.
- Formally: $\frac{\partial Q^{SO}}{\partial c} = -\frac{1}{1+d} < 0$.
- The optimal fee also decreases: $\frac{\partial t^*}{\partial c} = -\frac{2d-1}{2(1+d)} < 0$ since $d > \frac{1}{2}$.
- **Policy implication**: industries with higher production costs warrant *less stringent* emission fees, all else equal, because the market already produces closer to the social optimum.

Figure 2.8a – A Change in c

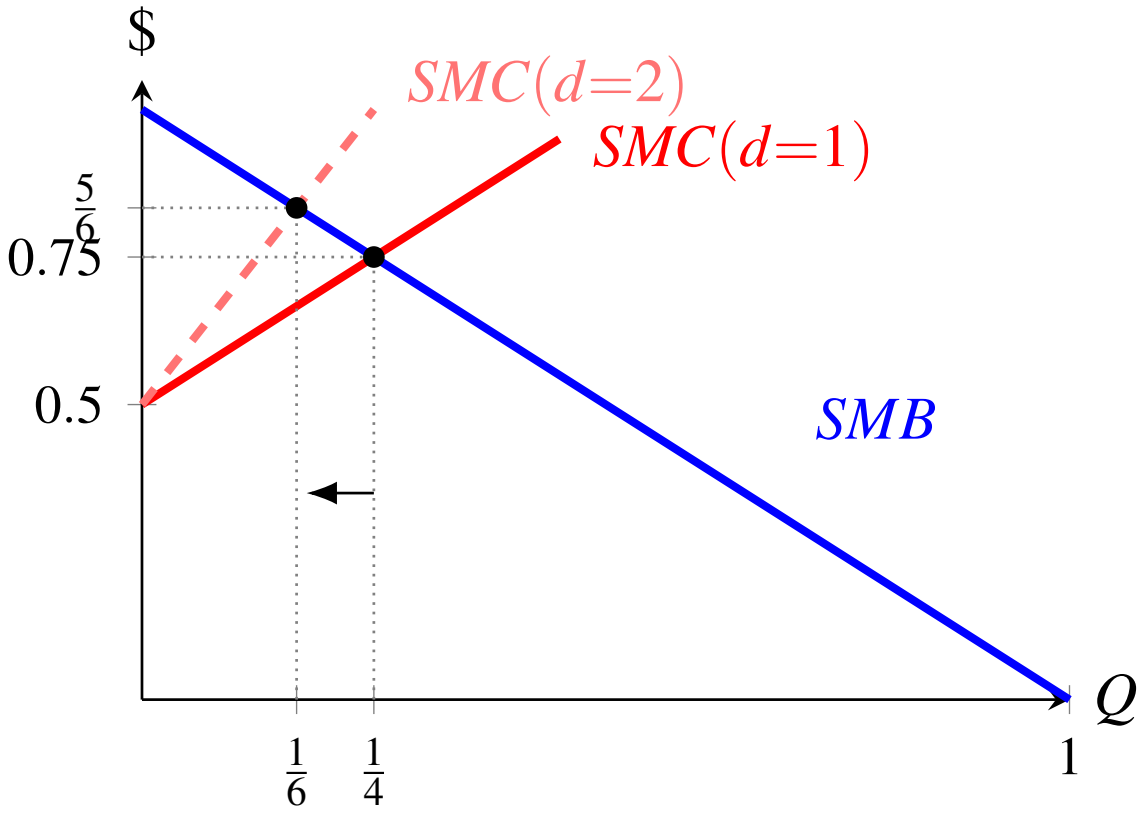


c rises from 0.50 to 0.75: SMC shifts up, and Q^{SO} falls from $\frac{1}{4}$ to $\frac{1}{8}$.

Comparative Statics: Effect of Higher Pollution Severity d

- $SMC = c + dQ$ becomes *steeper* (pivots upward) when d rises.
- The *SMB–SMC* crossing point moves *leftward*: Q^{SO} decreases.
  - Intuition: each unit causes more damage $\Rightarrow$ society wants less output.
  - Formally: $\frac{\partial Q^{SO}}{\partial d} = -\frac{1-c}{(1+d)^2} < 0.$
- The optimal fee increases: $\frac{\partial t^*}{\partial d} = \frac{3(1-c)}{2(1+d)^2} > 0.$
- **Policy implication**: more polluting industries (or industries with more severe health/environmental impacts) should face *stricter* emission fees.

Figure 2.8b – A Change in d



d rises from 1 to 2: SMC becomes steeper, and Q^{SO} falls from $\frac{1}{4}$ to $\frac{1}{6}$.

Pigouvian Tax vs. Oligopoly Fee

- **Pigouvian tax**: set t equal to the marginal external cost at the social optimum:

$$t^{Pigouvian} = d \cdot Q^{SO} = \frac{d(1-c)}{1+d}$$

- **Oligopoly fee** (Cournot duopoly):

$$t^{*} = \frac{(1-c)(2d-1)}{2(1+d)}$$

- **Comparison**: $t^{Pigouvian} - t^{*} = \frac{1-c}{2(1+d)} > 0$, which is firms' markup at Q^{SO} (section 2.7).
- **Intuitively**, the EA sets a *lower* fee than under competition to avoid over-restricting output in a concentrated market.

Worked Example: Computing the Optimal Fee

Parameters: $c = \frac{1}{2}$ and $d = 1$, as in Figures 2.6 and 2.7.

Step 1. Unregulated equilibrium:

$$q^{NR} = \frac{1 - \frac{1}{2}}{3} = \frac{1}{6}, \quad Q^{NR} = \frac{1}{3}, \quad p^{NR} = 1 - \frac{1}{3} = \frac{2}{3}$$

Step 2. Optimal fee:

$$t^* = \frac{\frac{1}{2}(2 \cdot 1 - 1)}{2(1+1)} = \frac{1}{8}$$

Worked Example: Regulated Output

Step 3. Regulated equilibrium:

$$q(t^*) = \frac{1 - \frac{1}{2} - \frac{1}{8}}{3} = \frac{1}{8}, \qquad Q(t^*) = \frac{1}{4} = Q^{SO}$$

- **Step 4.** At $Q^{NR} = \frac{1}{3}$, $SMC = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ exceeds $SMB = 1 - \frac{1}{3} = \frac{2}{3}$: unregulated output is socially excessive.
- The fee reduces output and emissions by 25%, from $\frac{1}{3}$ to $\frac{1}{4}$, reaching the social optimum.

Section 2.5

Other Regulations

Emission Quotas (Section 2.5.1)

Quota system: each firm receives an individual output cap $\overline{q}_i$ (non-tradable).

Two-stage game under quotas:

1. **Stage 1**: regulator sets $\overline{q}_i$ for firm i.
2. **Stage 2**: firm i solves:
   $\max_{0 \leq q_i \leq \overline{q}_i} \pi_i = (1 - q_i - q_j)q_i - cq_i$.

Emission Quotas (Section 2.5.1)

Stage 2: firm i's best response is $q_i(q_j) = \min\left\{\overline{q}_i, \frac{1-c-q_j}{2}\right\}$.
With a common quota $\overline{q} < \frac{1-c}{3}$, every firm produces at the quota.

Stage 1: regulator sets aggregate output equal to $Q^{SO} = \frac{1-c}{1+d}$, giving each firm:

$$\overline{q}_1 = \overline{q}_2 = \frac{Q^{SO}}{2} = \frac{1-c}{2(1+d)}$$

Since $d > \frac{1}{2}$, we have $\frac{1-c}{2(1+d)} < \frac{1-c}{3}$: the quota is *binding*.

Policy Equivalence: Fees vs. Quotas

Result: When the regulator is *fully informed*, emission fees and quotas are *equally effective*:

- Both instruments induce aggregate output $Q^{SO} = \frac{1-c}{1+d}$.
- Both achieve the social optimum.

Policy Equivalence: Fees vs. Quotas

When does equivalence break down?

- When the regulator is *uninformed* about some detail of the industry, such as demand or production costs.
- A fee chosen under wrong beliefs may induce $Q \neq Q^{SO}$; a quota fixes output but cannot adjust to the true conditions.
- **Weitzman (1974)**: under uncertainty, fees are preferred when marginal abatement costs are steeper than marginal damages; quotas otherwise. Chapter 4 studies this comparison.

Price vs. Quantity Instruments: The Weitzman Trade-off

- **Price instrument (emission fee)**: the regulator sets the per-unit cost of emissions; output adjusts.
- **Quantity instrument (quota)**: the regulator caps output; the price adjusts.
- **Weitzman's insight**: the relative slopes of *SMB* and *SMC* determine the preferred instrument:
  - Steep *SMB*, flat *SMC*: **fees** preferred.
  - Flat *SMB*, steep *SMC* (severe pollution or damage thresholds): **quotas** preferred.
- Chapter 4: with uncertain demand or costs, fees are preferred if $d < 2b$, where b is the slope of inverse demand.

Tradable Permits / Cap-and-Trade (Section 2.5.2)

Cap-and-trade: regulator sets an aggregate emissions cap, distributes permits as individual quotas, and *allows firms to trade* permits.

Key advantage over non-tradable quotas:

- Permits flow to firms with the highest abatement cost.
- Low-abatement-cost firms sell permits; high-abatement-cost firms buy.
- Result: *cost-effective* achievement of the aggregate target.

Tradable Permits / Cap-and-Trade (Section 2.5.2)

In equilibrium: the market permit price equals the marginal abatement cost, replicating the outcome under an optimal emission fee.

Real-world examples:

- EU Emissions Trading System (EU ETS, since 2005).
- California Cap-and-Invest Program (since 2013; renamed in 2025).
- Washington State Cap-and-Invest Program (since 2023).

Market Power in Permit Markets (Section 2.5.2)

- Tradable permits are efficient if firms take the permit price *as given*.
- A firm that is large relative to the permit market can influence this price (Hahn, 1984):
  - More permits than it needs: it sells too few, keeping the price high, as a monopolist would.
  - Fewer permits than it needs: it buys too few, keeping the price low, as a monopsonist would.
- Permits are allocated efficiently only if this firm receives the permits it would hold in a competitive permit market.

Permits as a Strategic Tool (Section 2.5.2)

- When firms also compete in the output market:
  - A dominant firm may buy *more* permits than it needs to raise its rivals' production costs (Misiolek and Elder, 1989).
  - In a Cournot duopoly, firms may use permits to drive their rival out of the market (von der Fehr, 1993).
- These strategic motives do not arise under emission fees, where firms cannot affect the stringency of the policy they face.
- For this reason, most chapters in the book focus on emission fees.

The Coase Theorem (Section 2.5.3)

Coase Theorem (Coase, 1960): if property rights are *clearly defined* and transaction costs are *negligible*, private bargaining achieves an efficient allocation, regardless of who holds the initial rights.

Application to pollution:

- A polluting firm and affected residents negotiate over emissions.
- Bargaining drives the outcome to $SMB = SMC$ (social optimum).

The Coase Theorem (Section 2.5.3)

If residents hold clean-air rights:

- The firm *pays* residents to accept more pollution, up to $SMB = SMC$.

If the firm holds pollution rights:

- Residents *pay* the firm to reduce pollution, until $SMB = SMC$.

Both scenarios reach the same efficient outcome: Q^{SO}.

Limits of the Coase Theorem

Why private bargaining often fails in practice:

- **High transaction costs**: legal fees, negotiation time, coordination among many affected parties.
- **Ill-defined property rights**: who owns clean air? Clean water?
- **Asymmetric information**: firms may conceal true emissions; residents may exaggerate harm.
- **Many affected parties**: emissions affect many people at the same time, creating free-rider problems.

Conclusion: regulation is needed when bargaining breaks down (evidence in Case Study 2.6.6).

Comparing the Three Instruments

	Fee	**Quota**	**Permits**
Achieves Q^{SO}?	Yes	Yes	Yes
Cost-effective?	Yes	If symmetric	Yes
Revenue?	Yes	No	If auctioned
Open to manipulation?	No	No	With market power
Allows flexibility?	Yes	No	Yes

- With full information and symmetric firms, all three instruments achieve the social optimum.
- Under uncertainty, the ranking depends on the slopes of *SMB* and *SMC* (chapter 4).

Section 2.6

Case Studies

Case Study 2.6.1: A Concentrated Industry

- Six companies operate Canada's 15 cement plants; the five members of the Cement Association of Canada operate 14 of them.
- Producers include Lafarge Canada (part of Amrize since 2025) and St. Marys Cement.

Source: Cement Association of Canada (2023).

Case Study 2.6.1: Cement Industry in Canada

- **Federal Output-Based Pricing System** (OBPS, since 2019): each facility's emissions limit equals a sector performance standard times its output.
- Above the limit: pay an excess emissions charge or remit compliance units. Below the limit: earn surplus credits to sell or bank.
- Ontario, where St. Marys Cement operates several plants, moved its facilities to its own Emissions Performance Standards program in 2022.
- In the 2010s, St. Marys hosted pilot projects that used kiln exhaust gas to grow algae.

Case Study 2.6.1: Connection to the Model

- At the margin, an output-based system charges a facility per tonne of emissions, as fee t in section 2.3.
- It collects less revenue, since emissions below the limit are not charged.
- The case illustrates the instrument; it does not show that the system caused firms' output or abatement decisions.
- Chapter 6 studies investment in abatement.

Case Study 2.6.2: The Clean Competition Act in the US

- Introduced in the Senate in June 2022; reintroduced in December 2023 and December 2025.
- Proposes a charge on the carbon content of energy-intensive goods, such as steel, aluminum, cement, and nitrogen fertilizer.
- Domestic producers pay only on emissions above their industry's average carbon intensity; imports face comparable charges.
- 2025 version: \$60 per tonne of CO_2 above the benchmark, rising each year.

Case Study 2.6.2: Connection to the Model

- The proposed charge is a per-unit fee, as t in section 2.3, applied only to emissions above a benchmark.
- The bill has not been enacted, so the case illustrates the design of the instrument rather than its effects.

Case Study 2.6.3: Sweden's Carbon Tax

- Introduced in 1991 at SEK 250 per tonne of CO_2; SEK 1,520 (about €134) in 2026, among the highest rates in the world.
- Falls mainly on fossil fuels used for transport and heating.
- Installations in the EU Emissions Trading System, such as cement plants, are exempt: they pay through EU allowances instead.
- Cementa, Sweden's only cement producer, planned to capture up to 1.8 million tonnes of CO_2 per year at Slite by 2030, but paused the project in 2025.

Case Study 2.6.3: Connection to the Model

- In section 2.3, a single fee applies to all emissions in the industry.
- A high headline tax rate need not reach the most emission-intensive industries, which in Sweden face the EU allowance price instead.
- For these firms, the relevant per-unit charge t is that allowance price.

Case Study 2.6.4: Chile's Electricity Generation

- Carbon tax from the 2014 tax reform; in effect since 2017 at US$5 per tonne of CO_2, on facilities of at least 50 MWt.
- In 2017, power generators paid 94% of the green taxes collected from stationary sources; CO_2 was 88% of that revenue.
- From 2023 (2020 reform): facilities emitting at least 25,000 tonnes of CO_2 or 100 tonnes of particulate matter per year.
- The rate remains far below the US$32.5 per tonne shadow price of carbon that the government used when introducing the tax.

Case Study 2.6.4: Connection to the Model

- The Chilean tax is a per-unit fee on emissions from large facilities, as t in section 2.3.
- Section 2.7: when firms exercise market power, the optimal fee lies *below* marginal damage.
- Assessing whether the Chilean rate is too low requires estimates of both marginal damage and firms' markups, so the case does not test this prediction.

Case Study 2.6.5: Emission Trading Systems

EU ETS (since 2005):

- Largest carbon market by trading value; China's national ETS (2021) is larger by covered emissions.
- Caps emissions from electricity, industry, aviation within Europe, and (since 2024) maritime transport.
- "Fit for 55": cap 62% below 2005 levels by 2030; ETS2 (buildings, road transport fuels) scheduled for 2028.
- Covered emissions were about 50% below 2005 levels in 2024, also due to other factors; Bayer and Aklin (2020): the ETS reduced total EU emissions by about 3.8% over 2008–2016.

Case Study 2.6.5: Emission Trading Systems

California Cap-and-Invest (since 2013):

- Linked with Québec since 2014; covers about 80% of the state's greenhouse gas emissions.
- Quarterly auctions, banking, offsets, and price controls; most auction revenue goes to the Greenhouse Gas Reduction Fund.
- In 2025, extended through 2045 and renamed Cap-and-Invest.

Case Study 2.6.5: Emission Trading Systems

Washington State Cap-and-Invest (since January 2023):

- Created by the Climate Commitment Act of 2021; covers about 70–75% of state emissions.
- Quarterly auctions, free allocation for emissions-intensive, trade-exposed facilities, and price containment tools.
- Voters rejected a repeal initiative in November 2024. In June 2026, Washington, California, and Québec agreed to link their markets.
- The program supports the state's legal goal of net-zero emissions by 2050.

Case Study 2.6.5: Connection to the Model

- These programs are tradable permit systems (section 2.5.2): the regulator sets an aggregate cap.
- The allowance price works as a per-unit emission cost common to all covered firms.
- Firms with market power in the permit market could distort this price.
- The decline in covered emissions is consistent with a binding cap, but it also reflects other factors.

Case Study 2.6.6: Testing Coase's Theorem

Laboratory experiments:

- Hoffman and Spitzer (1982): with known payoffs and a single property right holder, 89.5% of 114 bargaining decisions maximized joint payoffs.
- Success rates were lower under limited information and joint control; Hoffman and Spitzer (1986) extended the tests to larger groups.
- Survey by Deryugina et al. (2021): in the lab, efficient outcomes are common with complete information and low transaction costs.

Case Study 2.6.6: Testing Coase's Theorem

Field studies:

- Cheung (1973): pollination contracts between orchard owners and beekeepers in Washington State address the externality without regulation.
- Ellickson (1986): cattle trespass disputes in Shasta County, California, resolved by informal norms rather than legal entitlements.
- Deryugina et al. (2021): Coasian bargaining is limited in scope; most applications involve more than two parties, often a government or an environmental group.

Case Study 2.6.6: Connection to the Model

- These studies relate to section 2.5.3: bargaining achieves efficient outcomes when property rights are clearly defined and transaction costs are low.
- These conditions rarely hold when pollution affects many people, as in the industries that this book studies.

Section 2.7

Which Results Depend on the Functional Forms?

Our Functional Forms (Section 2.7)

Throughout the book, we consider:

- Linear inverse demand, $p(Q) = 1 - Q$, and constant marginal costs, c.
- One unit of emissions per unit of output, and quadratic damage, $Env = \frac{d}{2}Q^2$.
- Firms competing in quantities (Cournot).

These forms yield **closed-form** output levels, fees, and cutoffs, helping us interpret every result. *Which results rely on them?*

Table 2.1 – Functional Forms and the Generality of Our Results

Assumption	*What it provides*	*Results that rely on it*	*What holds more generally*
Linear demand	Closed-form output levels and fees	Closed-form cutoffs, such as $d > \frac{1}{2}$ (chapter 2) or $\frac{5}{7}$ (chapter 7)	The optimal fee equals marginal damage minus firms' markup; cutoffs change with demand curvature
Quadratic damage	Linear marginal damage, dQ	Closed-form fees; the sign of the fee depends on d alone	The fee is positive if and only if marginal damage exceeds firms' markup at Q^{SO}

Table 2.1 (continued)

Assumption	*What it provides*	*Results that rely on it*	*What holds more generally*
Constant and symmetric marginal costs	The division of output between firms does not affect total costs	A uniform fee achieves the first best (chapter 2, sections 8.2 and 8.6)	With asymmetric costs (sections 3.4 and 5.2) or economies of scale (section 3.5), a uniform fee is a second best; with convex costs it remains first best
Quantity competition	Outputs are strategic substitutes	The level of fees in chapters 3–8 (except section 3.8)	Under price competition, the fee is higher and closer to marginal damage (section 3.8)
One unit of emissions per unit of output	The fee works as an output tax	Chapters 2–5 (except sections 3.3 and 3.6), 7, and 8	With abatement, a single fee cannot correct both output and abatement, and becomes a second best (chapter 6)

One Result Holds More Generally (Section 2.7)

- Symmetric firms and a fee that induces $Q^{SO} \Rightarrow$ price equals the social marginal cost of output:

$$p(Q^{SO}) = c + MD(Q^{SO})$$

  where MD denotes marginal damage.
- Firms price above their after-tax cost, $p = c + t + m$, where $m \equiv p - c - t$ is their **markup**.
- Combining both expressions, the optimal fee is

$$t^* = MD(Q^{SO}) - m \tag{2.25}$$

Marginal Damage Minus Markup

- In words, the optimal fee equals marginal damage minus firms' markup, as in Barnett (1980).
- In our model, $MD(Q^{SO}) = dQ^{SO} = \frac{d(1-c)}{1+d}$ and $m = \frac{1-c}{2(1+d)}$.
- Subtracting, we obtain fee (2.10):

$$t^* = \frac{d(1-c)}{1+d} - \frac{1-c}{2(1+d)} = \frac{(1-c)(2d-1)}{2(1+d)}$$

Implications of the Rule

- When firms exercise market power, $m > 0$, the optimal fee lies *below* marginal damage.
- The fee becomes a **subsidy** when the markup exceeds marginal damage, as when $d < \frac{1}{2}$ in our model.
- Cutoffs such as $d > \frac{1}{2}$ depend on the functional forms; the rule holds more generally.
- Section 3.8 shows that the same rule applies when firms compete in prices.

Further Reading (Beyond the Book)

- Cement Association of Canada (2023) *Concrete Zero: Canada's Cement and Concrete Industry Action Plan to Net-Zero*.
  - Industry plan: cut emissions 40% by 2030 and reach net zero by 2050, relative to 2020; calls for a predictable carbon price.
- Enel Chile (2022) *Annual Report on Form 20-F, Fiscal Year 2021*, U.S. Securities and Exchange Commission.
  - Describes Chile's green tax (Law 20,780, US$5 per tonne of CO_2) and notes that increases in it could discourage thermal generation.

Section 2.8

Summary

Overview: Chapter 2 Results at a Glance

Variable	No regulation	With optimal fee
Per-firm output	$\frac{1-c}{3}$	$\frac{1-c}{2(1+d)}$
Aggregate output	$\frac{2(1-c)}{3}$	$\frac{1-c}{1+d}$
Aggregate emissions	$\frac{2(1-c)}{3}$	$\frac{1-c}{1+d}$
Emission fee	0	$\frac{(1-c)(2d-1)}{2(1+d)}$
Welfare	$\frac{2(1-c)^2(2-d)}{9}$	$\frac{(1-c)^2}{2(1+d)}$

- Since $d > \frac{1}{2}$: regulated output $<$ unregulated output, and regulated welfare $>$ unregulated welfare.
- The optimal fee lies below the Pigouvian level, $\frac{d(1-c)}{1+d}$, by firms' markup, $\frac{1-c}{2(1+d)}$.

Summary: Benchmark

Benchmark (Unregulated Cournot Oligopoly):

- Firms compete à la Cournot; each unit of output generates one unit of emissions.
- Equilibrium: $q^{NR} = \frac{1-c}{3}$, $Q^{NR} = \frac{2(1-c)}{3}$.
- When $d > \frac{1}{2}$, firms produce *more* than the social optimum: they ignore $MEC = dQ$.
- More firms $\Rightarrow$ higher aggregate emissions; welfare cutoff $\overline{d}(n) = \frac{n+2}{n}$ decreases in n.

Summary: Emission Fees

Optimal Emission Fee (Two-Stage SPE):

- $t^* = \frac{(1-c)(2d-1)}{2(1+d)} > 0$ iff $d > \frac{1}{2}$; a subsidy otherwise ($t = 0$ if subsidies are unavailable).
- Balances two market failures: pollution (overproduction) and market concentration (underproduction).
- Fee increases in d and decreases in c.
- Both firms remain active, and $Q(t^*) = Q^{SO} = \frac{1-c}{1+d}$: the fee achieves the first best.

Summary: SMB and SMC

SMB/SMC Framework:

- $SMB = 1 - Q$; $SMC = PMC + MEC = c + dQ$.
- Social optimum: $SMB = SMC \Rightarrow Q^{SO} = \frac{1-c}{1+d}$.
- Higher c or higher d shifts SMC upward or makes it steeper $\Rightarrow$ lower Q^{SO}.
- In oligopoly, $t^* < t^{Pigouvian}$: the fee equals marginal damage minus firms' markup.

Summary: Other Instruments

- **Emission quotas**: caps on output; achieve Q^{SO} when the regulator is fully informed. The quota binds iff $d > \frac{1}{2}$.
- **Tradable permits**: combine quantity certainty with cost-effectiveness; firms with market power in the permit market can manipulate the permit price or use permits to raise their rivals' costs.
- **Coase Theorem**: private bargaining achieves Q^{SO} if property rights are well defined and transaction costs are negligible; in practice, these conditions often fail.

Summary: Functional Forms

- Linear demand, quadratic damage, constant costs, and quantity competition yield closed-form results; cutoffs, such as $d > \frac{1}{2}$, depend on these forms.
- More generally, the optimal fee equals marginal damage minus firms' markup, so it lies below marginal damage when firms exercise market power.

Chapter 2 as Baseline: Chapters 3–5

- **Chapter 2** establishes the *baseline model*: Cournot duopoly and an emission fee; no abatement, no asymmetries.
- **Chapter 3**: more firms, pollution and cost asymmetries, nonlinear costs, product differentiation, cost externalities, and price competition.
- **Chapter 4**: regulation under uncertainty, mechanisms to infer emissions, and green certification.
- **Chapter 5**: sequential competition, two time periods, and regulatory commitment.

Chapter 2 as Baseline: Chapters 6–8

- **Chapter 6**: firms invest in abatement (non-cooperative, cooperative, and sequential decisions), and regulatory timing.
- **Chapter 7**: mergers and cartels in polluting industries.
- **Chapter 8**: managerial incentives, such as mixed duopolies, sales incentives, cross-ownership, and corporate social responsibility.

Key Takeaways and Policy Implications

1. **Two failures, one instrument**: the fee corrects pollution and market concentration.
2. **Fee**: below marginal damage by firms' markup; a subsidy when $d < \frac{1}{2}$.
3. **First best**: both firms remain active and produce Q^{SO}.
4. **Instruments**: fees, quotas, and permits are equivalent under full information; market power can distort permit markets.
5. **Functional forms**: cutoffs depend on them; "fee = marginal damage minus markup" holds more generally.

Thank You

Environmental Regulation: A Game-Theoretic Approach

Chapter 2: Regulating a Polluting Oligopoly

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 3

Environmental Regulation – Extensions

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Section 3.1
Introduction

Overview and Roadmap of Chapter 3 (1/2)

- Chapter 2 established the benchmark: two symmetric Cournot firms, linear costs, and a homogeneous product.
- Chapter 3 **relaxes one assumption at a time**, asking whether the fee becomes more or less stringent, and whether a *uniform* fee still achieves the first best.
- **3.2 More firms:** more aggregate output and emissions, requiring a more stringent fee.
- **3.3 Pollution asymmetries:** a uniform fee may be inefficient when firms generate different amounts of emissions.

Overview and Roadmap of Chapter 3 (2/2)

- **3.4 Cost asymmetries:** different production costs alter strategic behavior and the regulator's response.
- **3.5 Nonlinear costs:** cost curvature affects output and fee design.
- **3.6 Product differentiation:** softer competition raises emissions; green consumers reduce emissions.
- **3.7 Cost externalities:** one firm's output raises its rival's costs.
- **3.8 Price competition:** the fee becomes more stringent, and closer to marginal damage.
- **3.9–3.11:** case studies, related literature, and summary.

Guiding Questions for Chapter 3 (1/2)

1. How does increasing the number of firms affect aggregate emissions and optimal fees?
2. Why might uniform emission fees be inefficient under pollution asymmetries?
3. How do cost asymmetries influence strategic behavior and regulatory design?
4. What are the implications of nonlinear cost structures for emission fee policies?

Guiding Questions for Chapter 3 (2/2)

5. How does product differentiation affect competition and environmental outcomes?
6. How do environmentally aware consumers affect emissions and the emission fee?
7. What challenges arise when firms' outputs impose cost externalities on rivals?
8. How does the emission fee change when firms compete in prices rather than quantities?

Section 3.2
Allowing for More Firms

Setup: n Symmetric Cournot Firms

- Setting of Section 2.3, but with $n \geq 2$ symmetric firms, each with marginal cost c.
- **Inverse demand:** $p = 1 - Q$, where $Q = \sum_{i=1}^{n} q_i$. **Damage:** $Env = \frac{d}{2} Q^2$.
- **Two-stage game:** the EA sets fee t in the first stage; firms compete à la Cournot in the second stage.
- **Conjecture:** more firms produce more aggregate output and emissions, requiring a more stringent fee.

Section 3.2.1 – Second Stage (Output)

- Every firm i solves

$$\max_{q_i \geq 0} \quad \pi_i = (1 - q_i - Q_{-i})q_i - (c + t)q_i \tag{3.1}$$

- **Best response function:**

$$q_i(Q_{-i}) = \frac{1 - c - t}{2} - \frac{1}{2}Q_{-i} \quad \text{if } Q_{-i} \leq 1 - c - t$$

- In a symmetric equilibrium, $q_i = q$ for every firm, so $Q_{-i} = (n - 1)q$.

Section 3.2.1 – Equilibrium Output

- Inserting $Q_{-i} = (n-1)q$ and solving for q:

$$q(t) = \frac{1-c-t}{n+1}$$

- $q(t)$ decreases in the fee, t, and in the number of firms, n.
- **Aggregate output** increases in n:

$$Q(t) = nq(t) = \frac{n(1-c-t)}{n+1}, \qquad \frac{\partial Q(t)}{\partial n} = \frac{1-c-t}{(n+1)^2} > 0$$

Section 3.2.2 – First Stage: Direct Approach

- The EA maximizes $W = CS + PS + T - Env$ evaluated at $Q(t)$, as in problem (2.8); see (3.2)–(3.3).
- **First-order condition:**

$$\frac{\partial W}{\partial t} = \frac{n\left[(1-c)(nd-1) - nt(1+d)\right]}{(n+1)^2} = 0$$

- **Optimal emission fee:**

$$t^*(n) = \frac{(1-c)(dn-1)}{n(1+d)}$$

- At $n = 2$: $t^*(2) = \frac{(1-c)(2d-1)}{2(1+d)}$, as in Section 2.3.

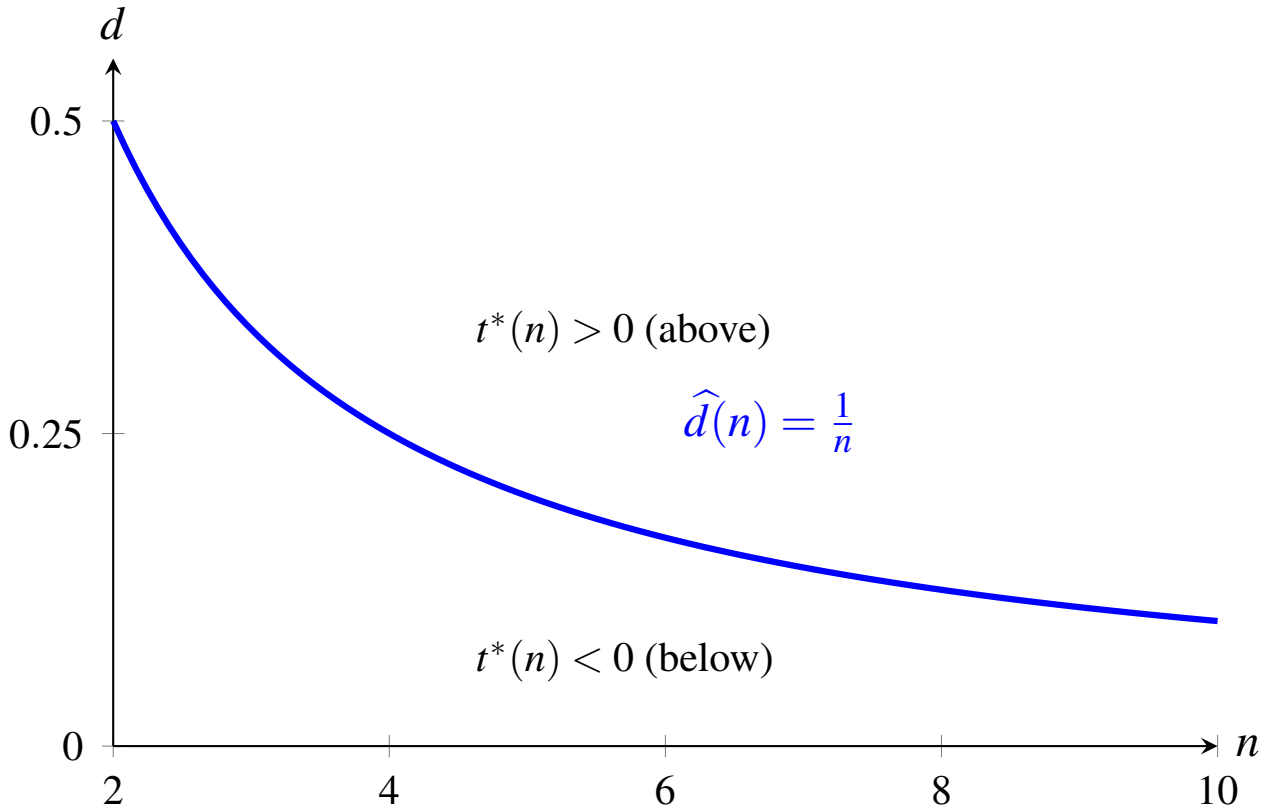
Sign of the Fee: Cutoff $\widehat{d}(n)$

- $t^*(n) > 0$ if and only if $dn > 1$, that is,

$$d > \widehat{d}(n) \equiv \frac{1}{n}$$

- At $n = 2$, $\widehat{d}(2) = \frac{1}{2}$, as in Section 2.3.
- With $n \geq 3$, the cutoff decreases in n: $\widehat{d}(3) = \frac{1}{3}$, $\widehat{d}(4) = \frac{1}{4}$, and so on.
- **Intuitively,** a more competitive market produces more output and emissions, so the fee is positive for a wider range of d.

Figure 3.1 – Cutoff $\widehat{d}(n) = \frac{1}{n}$



Comparative Statics of $t^*(n)$

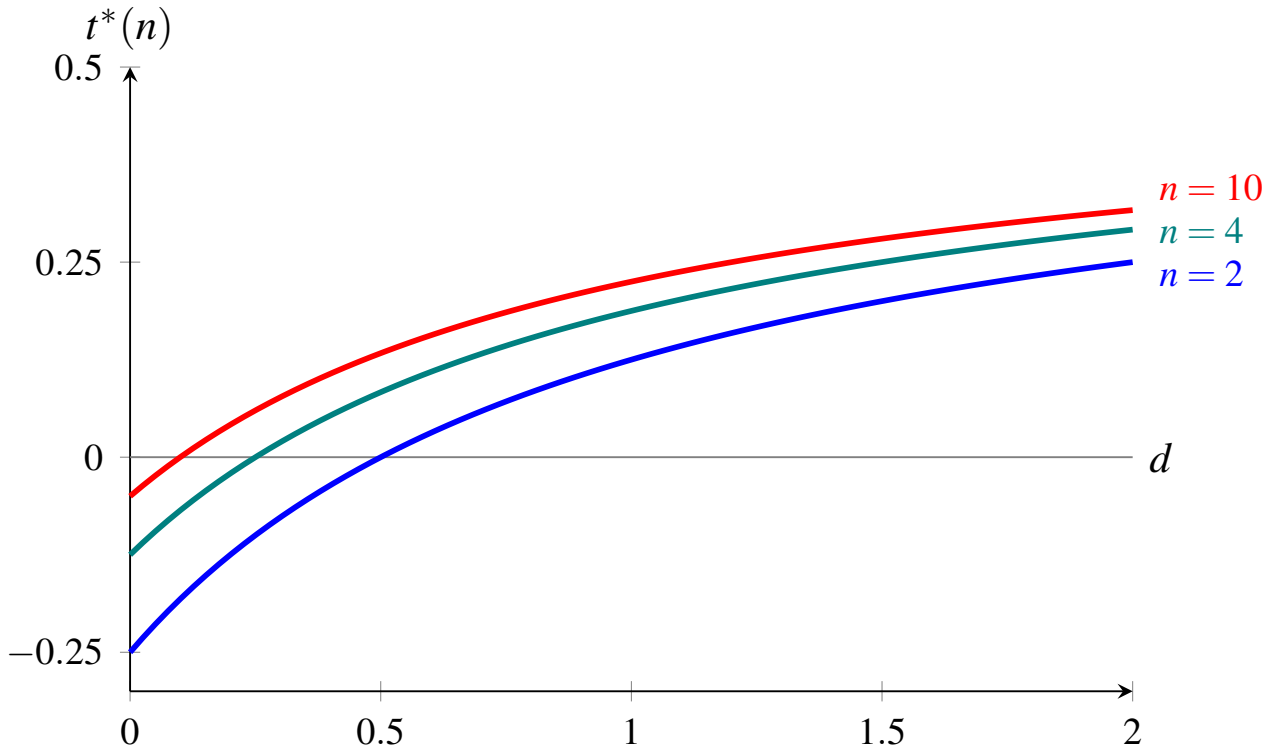
- The fee increases in the number of firms:

$$\frac{\partial t^*(n)}{\partial n} = \frac{1-c}{n^2(1+d)} > 0$$

- **Intuitively,** a more competitive industry produces more aggregate output and emissions, requiring a more stringent fee.
- The fee also increases in d and decreases in c:

$$\frac{\partial t^*(n)}{\partial d} = \frac{(1-c)(n+1)}{n(1+d)^2} > 0, \qquad \frac{\partial t^*(n)}{\partial c} = \frac{1-dn}{n(1+d)} < 0$$

Figure 3.2 – Fee $t^*(n)$ Increases in n



Fee $t^*(n)$ as a function of d, at $c = \frac{1}{2}$ and $n = 2$, 4, and 10.

Section 3.2.2 – First Stage: Indirect Approach

- **Step 1:** from Section 2.3.2, $Q^{SO} = \frac{1-c}{1+d}$, which does not depend on n: a "pollution target" for the EA.
- **Step 2:** set $Q^{SO} = Q(t)$,

$$\frac{1-c}{1+d} = \frac{n(1-c-t)}{n+1} \tag{3.4}$$

  which yields the same fee, $t^*(n)$.
- At $t^*(n)$, every firm produces $\frac{1-c}{n(1+d)} > 0$, so all firms remain active.
- With symmetric firms and constant marginal costs, $t^*(n)$ achieves the **first best**.

Section 3.2.3 – Regulating Market Entry

- Instead of a fee, the EA can provide **entry licenses** until $Q(0) = Q^{SO}$ (no fee, $t = 0$):

$$\frac{1-c}{1+d} = \frac{n(1-c)}{n+1} \implies n^{SO} = \frac{1}{d} \tag{3.5}$$

- n^{SO} decreases in d. As $d \to 0$, $n^{SO} \to +\infty$: free entry, close to perfect competition.
- When $d > 0$, n^{SO} is bounded: fixed entry costs or licenses limit the number of firms.

Entry Regulation Is a Coarse Tool

- The number of firms must be an integer, which only occurs at specific values of d: $d = \frac{1}{2}$ yields $n^{SO} = 2$; $d = 1$ yields $n^{SO} = 1$ (a monopoly).
- When $d > 1$, even a monopoly overproduces, since $\frac{1-c}{2} > \frac{1-c}{1+d}$: restricting entry cannot induce Q^{SO}.
- Entry restrictions can **complement** emission fees where fees are politically controversial, but they rarely replace them.

Section 3.3
Allowing for Pollution Asymmetries

Setup: Emission Intensity

- Builds on Akhundjanov and Muñoz-Garcia (2016a, 2016b).
- Thus far, every unit of output generated one unit of emissions, $E = Q$.
- If every unit generates λ units, $E = \lambda Q$, we redefine pollution severity as $d_R \equiv d\lambda^2$:

$$Env = \frac{d}{2}(\lambda Q)^2 = \frac{d_R}{2}Q^2$$

- A common intensity is just a normalization. The interesting case is **asymmetric** intensities.

Setup: Asymmetric Pollution Intensities

- Firm 1: $e_1 = q_1$. Firm 2: $e_2 = \theta q_2$, where $\theta \geq 0$.
- $\theta = 0$: firm 2 is clean; $\theta \in (0, 1)$: firm 2 is cleaner; $\theta = 1$: symmetric (Chapter 2); $\theta > 1$: firm 2 is more polluting. Example: U.S. coal plants emitted 2.31 lb of CO_2 per kWh in 2023, natural gas plants 0.96 lb (EIA), so $\theta \approx 2.4$.
- **Environmental damage:**

$$Env = \frac{d}{2}\left(q_1 + \theta q_2\right)^2$$

- **Question:** is a *uniform* fee still enough, or does the EA need *type-dependent* fees?

Section 3.3.1 – Second Stage (Output)

- For a given uniform fee t, outputs coincide with Section 2.3.1:

$$q(t) = \frac{1-c-t}{3}, \qquad Q(t) = \frac{2(1-c-t)}{3} \tag{3.6}$$

- For a given t, firms do not care about pollution intensities. The EA does.
- The EA charges t per unit of *output*, since it observes output but not emission intensity.
- A fee per unit of emissions would charge firm 2 θt per unit of output: a type-dependent fee (Section 3.3.3).

Section 3.3.2 – First Stage (Emission Fee)

- The EA solves problem (3.7), with the new damage term:

$$\max_{t} \quad W = CS + PS + T - \underbrace{\frac{d}{2}\left[q(t) + \theta q(t)\right]^2}_{\text{New } Env}$$

- **First-order condition:**

$$\frac{\partial W}{\partial t} = \frac{-2 - 4t + d(1-t)(1+\theta)^2 - c\left[d(1+\theta)^2 - 2\right]}{9} = 0 \tag{3.8}$$

Optimal Uniform Fee $t^*(\theta)$

- Solving (3.8) for t:

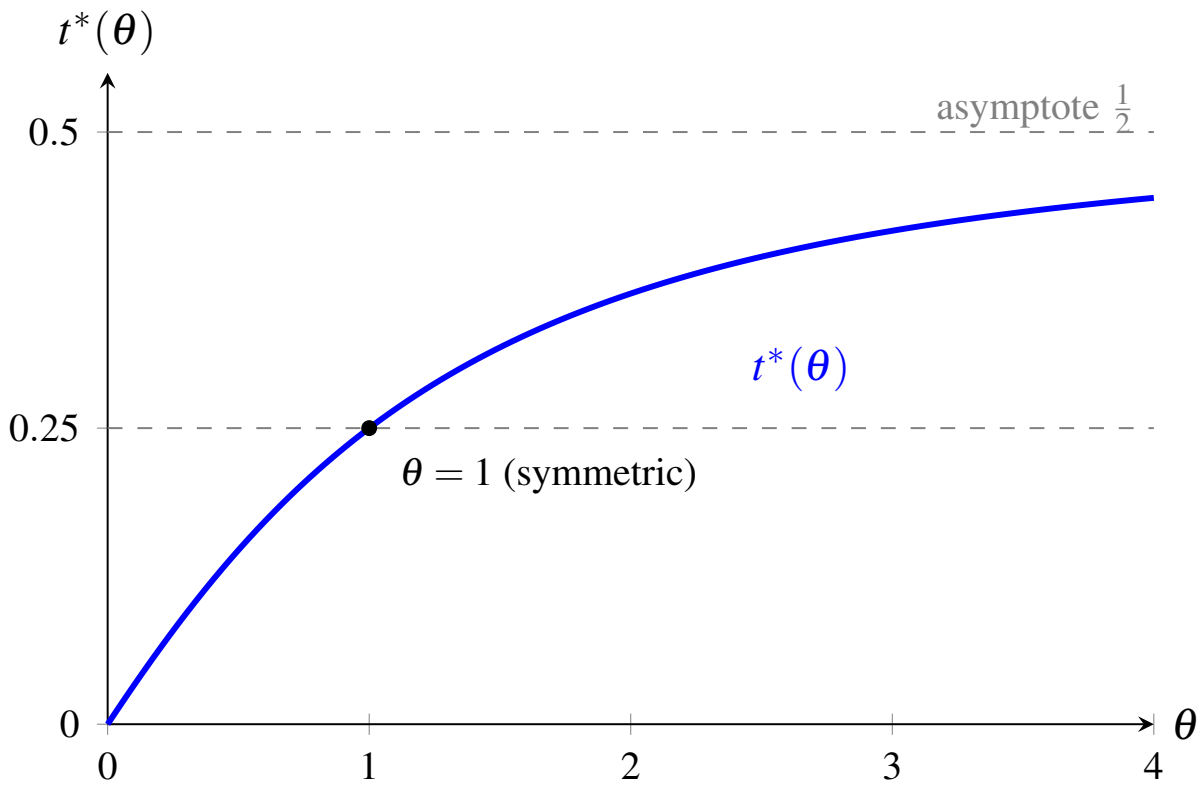
$$t^*(\theta) = \frac{(1-c)\left[d(1+\theta)^2 - 2\right]}{4 + d(1+\theta)^2} \tag{3.9}$$

- At $\theta = 1$: $t^*(1) = \frac{(1-c)(2d-1)}{2(1+d)}$, as in (2.10).
- The fee increases in firm 2's pollution intensity:

$$\frac{\partial t^*(\theta)}{\partial \theta} = \frac{12d(1-c)(1+\theta)}{\left[4 + d(1+\theta)^2\right]^2} > 0 \tag{3.10}$$

- **Intuitively,** firm 2 generates more emissions per unit of output, requiring a more stringent fee.

Figure 3.3 – Emission Fee $t^*(\theta)$



At $c = \frac{1}{2}$ and $d = 2$: $t^*(\theta) = \frac{(1+\theta)^2-1}{4+2(1+\theta)^2}$, with $t^*(1) = \frac{1}{4}$.

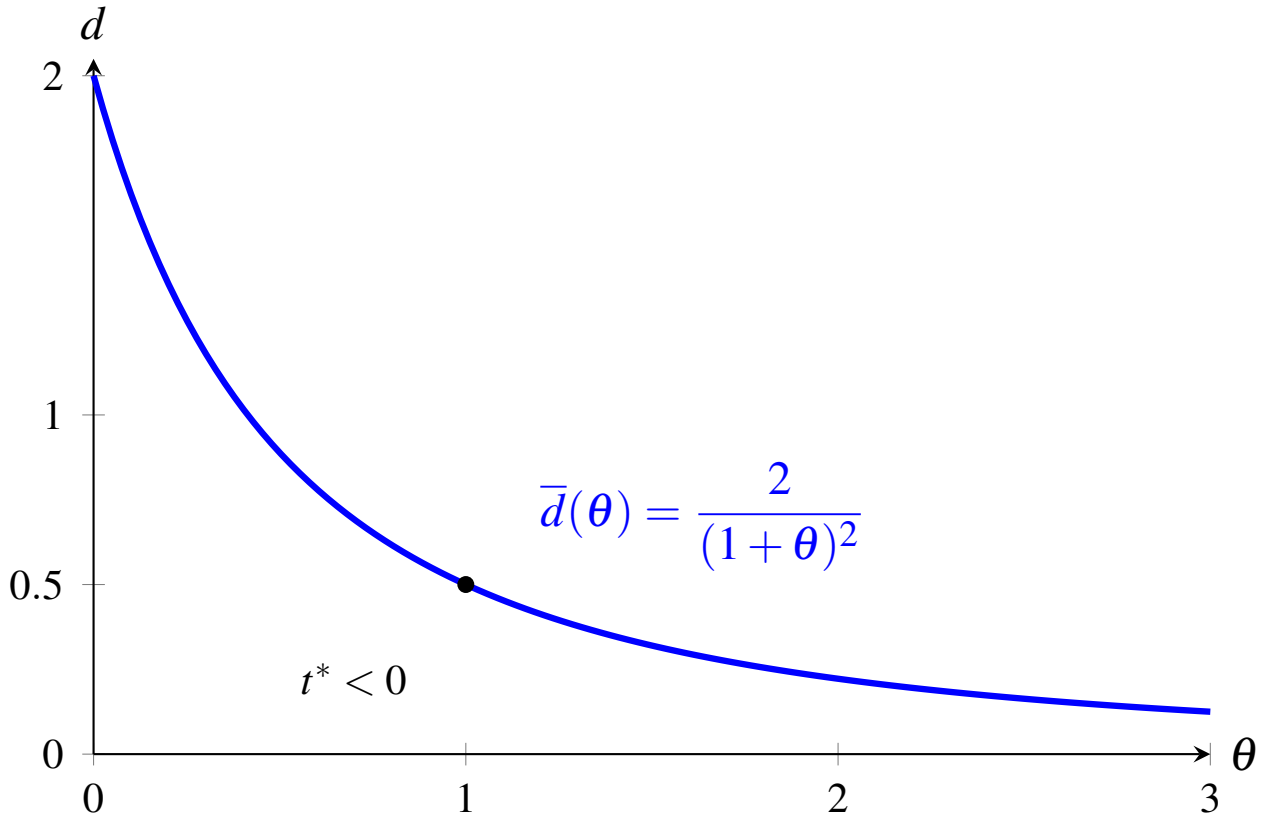
Sign of the Uniform Fee: Cutoff $\overline{d}(\theta)$

- $t^*(\theta) > 0$ if and only if

$$d > \overline{d}(\theta) \equiv \frac{2}{(1+\theta)^2}$$

  When $d < \overline{d}(\theta)$, the fee is negative (a subsidy).
- At $\theta = 1$: $\overline{d}(1) = \frac{1}{2}$, as in Section 2.3.2.
- A higher θ lowers the cutoff: the fee is positive for a wider range of d.

Figure 3.4 – Cutoff $\overline{d}(\theta)$



A Cleaner Firm 2 and the Uniform Fee

- When $\theta < 1$, $\overline{d}(\theta) > \frac{1}{2}$: the EA subsidizes production even when $\frac{1}{2} < d < \overline{d}(\theta)$.
- **Intuitively,** a cleaner firm 2 reduces emissions per unit of output, so underproduction from market power becomes the dominant market failure.
- At $t^*(\theta)$, every firm produces $\frac{2(1-c)}{4+d(1+\theta)^2} > 0$: both firms remain active.
- This uniform fee, however, only achieves a **second best** when $\theta \neq 1$.

Section 3.3.3 – Type-Dependent Fees

- A uniform fee tries to solve two problems with one tool: excessive pollution, and its distribution across firms.
- **Type-dependent fees:** one fee per firm, t_1 and t_2, tailored to each firm's characteristics.
- **Uniform fees:** the same fee on all firms in the industry.
- In practice, fees typically differ only when pollutants are chemically different.

Type-Dependent Fees: Second Stage

- Firm 1 solves $\max_{q_1 \geq 0}(1 - q_1 - q_2)q_1 - (c + t_1)q_1$ (3.11); firm 2 solves a symmetric problem with t_2.
- **Output functions:**

$$q_1(t_1, t_2) = \frac{1 - c - 2t_1 + t_2}{3}, \quad q_2(t_1, t_2) = \frac{1 - c - 2t_2 + t_1}{3} \tag{3.12}$$

- Each output decreases in its own fee, but increases in its rival's fee.
- When $t_1 = t_2 = t$, both become $\frac{1-c-t}{3}$, as above.

Type-Dependent Fees: First Stage

- The EA chooses (t_1, t_2) to maximize welfare (3.13), with damage $\frac{d}{2}\left[q_1 + \theta q_2\right]^2$.
- The first-order conditions yield $t_1 = \frac{(1-c)\theta}{1-\theta}$ and $t_2 = -\frac{1-c}{1-\theta}$.
- When firm 2 is cleaner, $\theta < 1$, these fees induce $q_1 = \frac{\theta(1-c)}{\theta-1} < 0$, which is **not feasible**.
- We then impose firm 1's constraint, $q_1 \geq 0$, as binding (Appendix A.4).

Why Firm 1 Should Not Produce

- Both firms face the same production cost.
- Shifting one unit of output from firm 1 to firm 2 leaves consumer surplus and production costs unaffected, but reduces emissions by $1 - \theta$ units.
- The EA, then, prefers that firm 2 produce **all** output.
- As a monopolist, firm 2 produces $\frac{1-c-t_2}{2}$. The EA seeks the output that maximizes $(1-c)q_2 - \frac{1}{2}q_2^2 - \frac{d}{2}(\theta q_2)^2$, that is, $\frac{1-c}{1+d\theta^2}$.

Optimal Type-Dependent Fees

- **Fees:**

$$t_2^*(\theta) = \frac{(1-c)(d\theta^2-1)}{1+d\theta^2} \quad \text{and} \quad t_1^*(\theta) \geq \frac{(1-c)d\theta^2}{1+d\theta^2} \quad (3.14)$$

- t_1^* keeps firm 1 out: its best response, $\frac{1-c-t_1-q_2}{2}$, is zero iff $t_1 \geq 1-c-q_2$.
- **Equilibrium outputs** (3.15)–(3.16):

$$q_1 = 0 \quad \text{and} \quad q_2 = \frac{1-c}{1+d\theta^2} > 0$$

Interpreting Type-Dependent Fees

- **Two tools, two goals:** t_1^* keeps the more polluting firm 1 out; t_2^* corrects the cleaner firm's output.
- t_2^* is a **subsidy** when $d\theta^2 < 1$: the EA mainly corrects the underproduction of a monopolist.
- If firm 2 is completely clean, $\theta = 0$: $t_2^*(0) = -(1-c)$, inducing the perfectly competitive output $1-c$.
- Firm 2's output decreases in c, d, and θ.

Type-Dependent versus Uniform Fees

- Type-dependent fees achieve the **first best**. The uniform fee keeps both firms active, producing the same output: a **second best**.
- **Example:** $c = \frac{1}{2}$, $d = 1$, and $\theta = \frac{1}{2}$. Welfare is 0.1 under type-dependent fees, but 0.08 under the uniform fee.
- If firm 2 is more polluting, $\theta > 1$, the argument is symmetric: $t_2 \geq \frac{(1-c)d}{1+d}$ keeps firm 2 out, and $t_1 = \frac{(1-c)(d-1)}{1+d}$ induces firm 1 to produce $\frac{1-c}{1+d}$ as a monopolist.

Section 3.4
Allowing for Cost Asymmetries

Setup: Cost Asymmetries

- Builds on Simpson (1995); see also Akhundjanov and Muñoz-Garcia (2016a).
- Duopoly of Section 2.3, but firm 1 has a cost advantage: $1 > c_2 \geq c_1 \geq 0$.
- Let $c_2 = c$ and $c_1 = \alpha c$, where $\alpha \in [0, 1]$ measures cost symmetry.
- $\alpha = 1$: symmetric firms; $\alpha \in (0, 1)$: firm 1 has a cost advantage; $\alpha = 0$: firm 1's cost is zero.

Section 3.4.1 – Second Stage: Best Responses

- Firm 1 solves (3.17), $\max_{q_1 \geq 0}(1 - q_1 - q_2)q_1 - (\alpha c + t)q_1$; firm 2 solves (3.18) with cost $c + t$.
- **Best response functions:**

$$q_1(q_2) = \frac{1 - \alpha c - t}{2} - \frac{1}{2}q_2,$$
$$q_2(q_1) = \frac{1 - c - t}{2} - \frac{1}{2}q_1.$$

- Firm 2's best response starts at a lower intercept: it produces less, for a given output of its rival.
- We cannot impose $q_1 = q_2$; we solve both functions simultaneously.

Section 3.4.1 – Equilibrium Outputs

- **Output functions:**

$$q_1(t) = \frac{1 - t - c(2\alpha - 1)}{3} \tag{3.21}$$

$$q_2(t) = \frac{1 - t - c(2 - \alpha)}{3} \tag{3.23}$$

- Both coincide with $\frac{1-c-t}{3}$ when $\alpha = 1$.
- A larger cost advantage (lower α) raises firm 1's output, $\frac{\partial q_1(t)}{\partial \alpha} = -\frac{2c}{3} < 0$, and lowers firm 2's.

Which Firms Are Active?

- Firm 1 is active iff $\alpha < \overline{\alpha}_1 \equiv \frac{1+c-t}{2c}$ (3.22), which holds for all $\alpha \in [0,1]$ since $\overline{\alpha}_1 > 1$ when $t < 1-c$.
- Firm 2 is active iff

$$\alpha > \overline{\alpha}_2 \equiv \frac{2c+t-1}{c} \tag{3.24}$$

- **Intermediate fee**, $1-2c < t < 1-c$: $\overline{\alpha}_2 > 0$, so firm 2 is active only if firms are sufficiently symmetric.
- **Lax fee**, $t < 1-2c$: $\overline{\alpha}_2 < 0$, so firm 2 is active for all $\alpha \in [0,1]$.

Output Differential and Aggregate Output

- **Output differential:**

$$q_1(t) - q_2(t) = c(1 - \alpha) \tag{3.25}$$

  zero when $\alpha = 1$, and increasing as α falls.

- **Aggregate output:**

$$Q(t) = \frac{2(1-t) - c(1+\alpha)}{3}, \qquad \frac{\partial Q(t)}{\partial \alpha} = -\frac{c}{3} < 0 \tag{3.26}$$

- As firms become more asymmetric, firm 2's output falls, but firm 1's output increases more: aggregate output increases.

Section 3.4.2 – First Best: Type-Dependent Fees

- With cost asymmetries, the direct and indirect approaches yield different fees (indirect approach: Appendix 3.a).
- The cheapest way to produce any aggregate output is to let the low-cost firm 1 produce all of it.
- **First best:** $t_1 = \frac{(1-\alpha c)(d-1)}{1+d}$ induces firm 1 to produce $\frac{1-\alpha c}{1+d}$ as a monopolist, while $t_2 \geq 1 - c - \frac{1-\alpha c}{1+d}$ keeps firm 2 out.
- If the EA must set the same fee on all firms, it uses a **uniform fee**: a second-best scenario.

Uniform Fee: The EA's Problem

- Anticipating $q_1(t)$ and $q_2(t)$, the EA solves (3.27), which simplifies to

$$\max_t \quad W = \frac{Q^2}{2} + [(1-Q)Q - \alpha c q_1 - c q_2] - \frac{d}{2}Q^2 \quad (3.28)$$

  evaluated at $q_1(t)$, $q_2(t)$, and $Q(t)$.

- **First-order condition:**

$$\frac{\partial W}{\partial t} = \frac{(2d-1)\left[2 - c(1+\alpha)\right] - 4t(1+d)}{9} = 0 \quad (3.29)$$

Uniform Fee $t^*(\alpha)$

- Solving (3.29) for t:

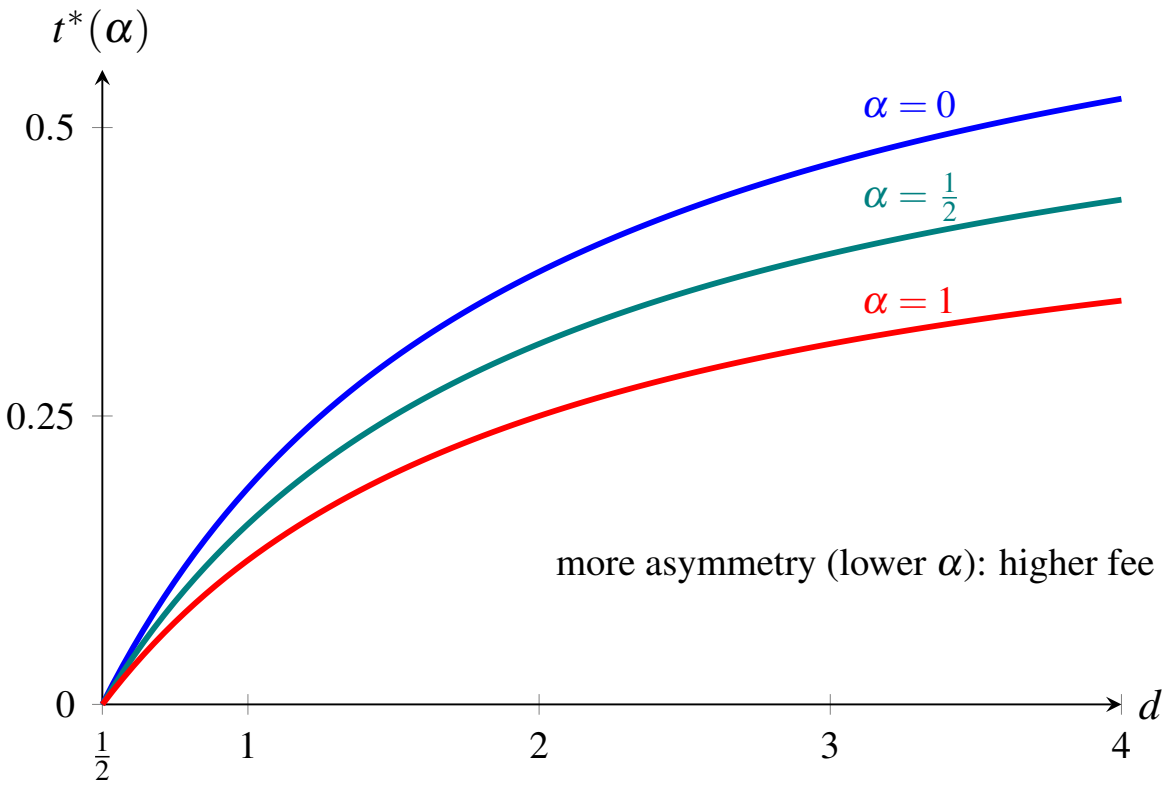
$$t^*(\alpha) = \frac{(2d-1)\left[2 - c(1+\alpha)\right]}{4(1+d)} \tag{3.30}$$

- At $\alpha = 1$: $t^*(1) = \frac{(2d-1)(1-c)}{2(1+d)}$, as in Section 2.3.2.
- More asymmetric firms (lower α) face a **more stringent** fee:

$$\frac{\partial t^*(\alpha)}{\partial \alpha} = -\frac{(2d-1)c}{4(1+d)} < 0 \tag{3.31}$$

- **Intuitively,** the EA anticipates more aggregate output and emissions.

Figure 3.5 – Emission Fee $t^*(\alpha)$



Fee $t^*(\alpha)$ as a function of d, at $c = \frac{1}{2}$, for $\alpha = 0$, $\frac{1}{2}$, and 1.

Equilibrium Outputs at $t^*(\alpha)$

- The fee is positive, since $d > \frac{1}{2}$ and $2 - c(1+\alpha) > 0$ for all $\alpha \in [0,1]$.
- **Firm 1**, active under all parameter conditions:

$$q_1(t^*(\alpha)) = \frac{2 + c\left[(1-3\alpha) + 2d(1-\alpha)\right]}{4(1+d)} \tag{3.32}$$

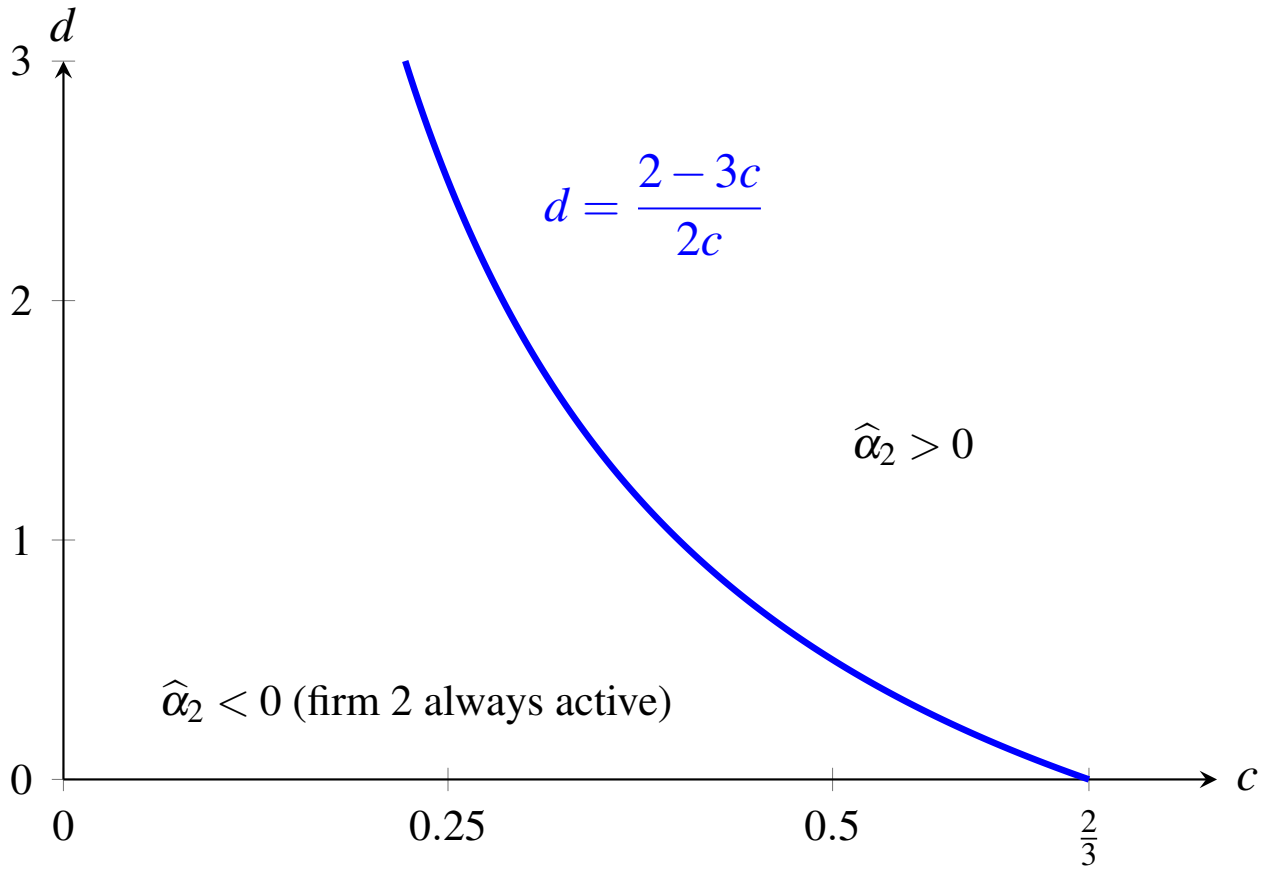
- **Firm 2**, increasing in α:

$$q_2(t^*(\alpha)) = \frac{2 - c\left[(3-\alpha) + 2d(1-\alpha)\right]}{4(1+d)} \tag{3.33}$$

When Is Firm 2 Active? Cutoff $\widehat{\alpha}_2$

- $q_2(t^*(\alpha)) > 0$ iff $\alpha > \widehat{\alpha}_2 \equiv \frac{(3+2d)c-2}{c(1+2d)}$, where $\widehat{\alpha}_2 > 0$ iff $d > \frac{2-3c}{2c}$.
- $d \leq \frac{2-3c}{2c}$: pollution is not severe, the fee is lax, and firm 2 is active for all $\alpha \in (0, 1]$.
- $d > \frac{2-3c}{2c}$: the fee is more stringent, and firm 2 is active only if $\alpha > \widehat{\alpha}_2$ (sufficiently symmetric firms).
- At $\alpha = 1$: $q_1 = q_2 = \frac{1-c}{2(1+d)}$ (3.34), as in Section 2.3.2.

Figure 3.6 – Cutoff $d = \frac{2-3c}{2c}$



When the High-Cost Firm Exits

- If $\alpha \leq \widehat{\alpha}_2$, fee $t^*(\alpha)$ would require $q_2 < 0$: we account for firm 2's exit (Appendix A.4).
- Let $\widetilde{\alpha}_2 \equiv \frac{c(1+d)-1}{cd}$, which lies below $\widehat{\alpha}_2$.
- **Case (i)**, $\widetilde{\alpha}_2 < \alpha \leq \widehat{\alpha}_2$: the EA sets the fee at which firm 2 just stops producing, $t = 1 - c(2-\alpha)$, negative if $\alpha < \frac{2c-1}{c}$.
- Firm 1 then produces $c(1-\alpha)$ as a monopolist, less than the first-best $\frac{1-\alpha c}{1+d}$. A laxer fee would bring firm 2 back.

When the High-Cost Firm Exits (cont.)

- **Case (ii)**, $\alpha \leq \widetilde{\alpha}_2$ (requires $d > \frac{1-c}{c}$): the EA sets $t = \frac{(1-\alpha c)(d-1)}{1+d}$, negative if $d < 1$.
- Firm 1 then produces the first-best output $\frac{1-\alpha c}{1+d}$ as a monopolist, and firm 2 stays out.
- In case (ii), the uniform fee coincides with the first-best fee on firm 1: it achieves the **first best**.
- In case (i), and when both firms are active, the uniform fee only achieves a **second best**.

Indirect Approach (Appendix 3.a)

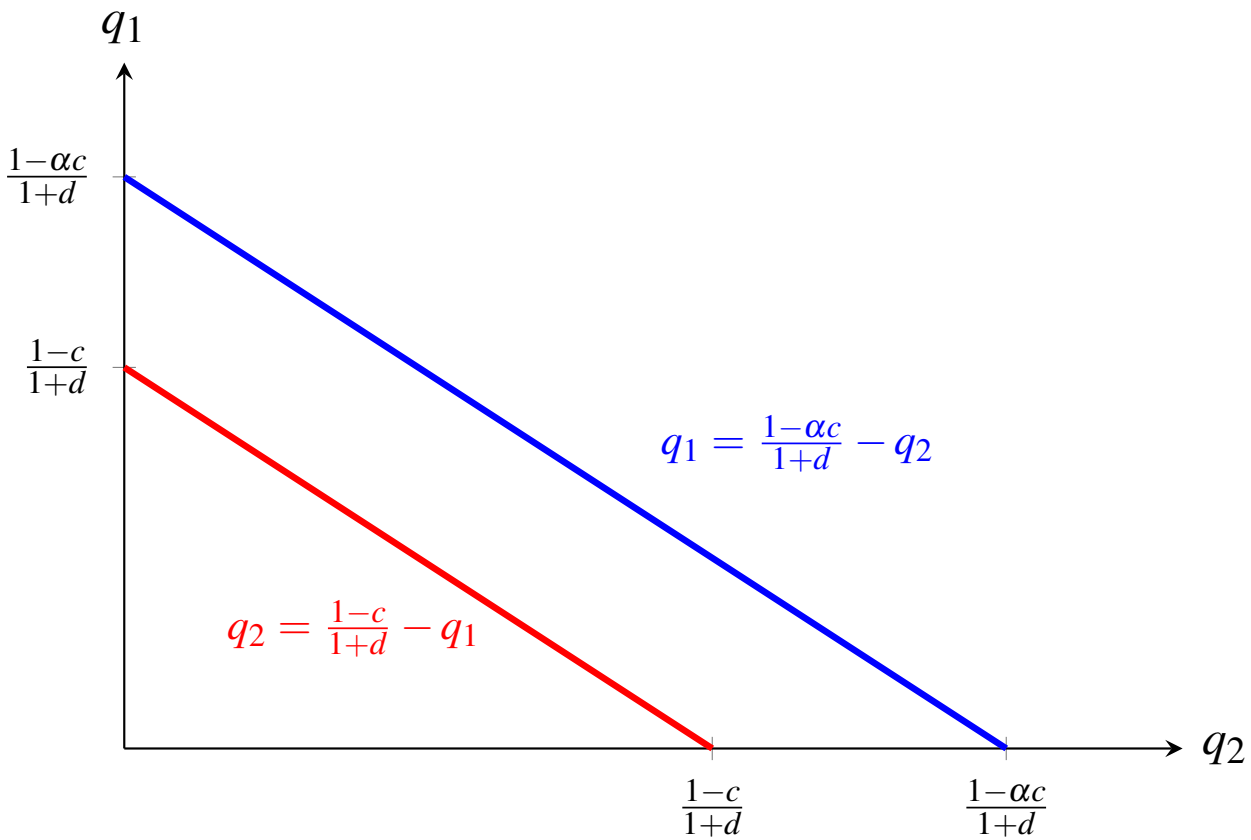
- **Step 1:** maximize W over (q_1, q_2). The first-order conditions, $1 - \alpha c = (1 + d)(q_1 + q_2)$ and $1 - c = (1 + d)(q_1 + q_2)$, cannot both hold when $\alpha < 1$.
- An additional unit from firm 1 raises welfare by more than one from firm 2: a corner solution,

$$q_1 = \frac{1 - \alpha c}{1 + d} \quad \text{but} \quad q_2 = 0 \tag{3.91}$$

- **Step 2:** setting $Q(t) = \frac{1-\alpha c}{1+d}$ yields

$$t^*_{Ind.}(\alpha) = \frac{2d - 1 - c[1 + d - \alpha(2 - d)]}{2(1 + d)} \tag{3.93}$$

Figure 3.A – Equations (3.90) for q_1 and q_2



Direct versus Indirect Approach

- The direct approach sets a more stringent fee:

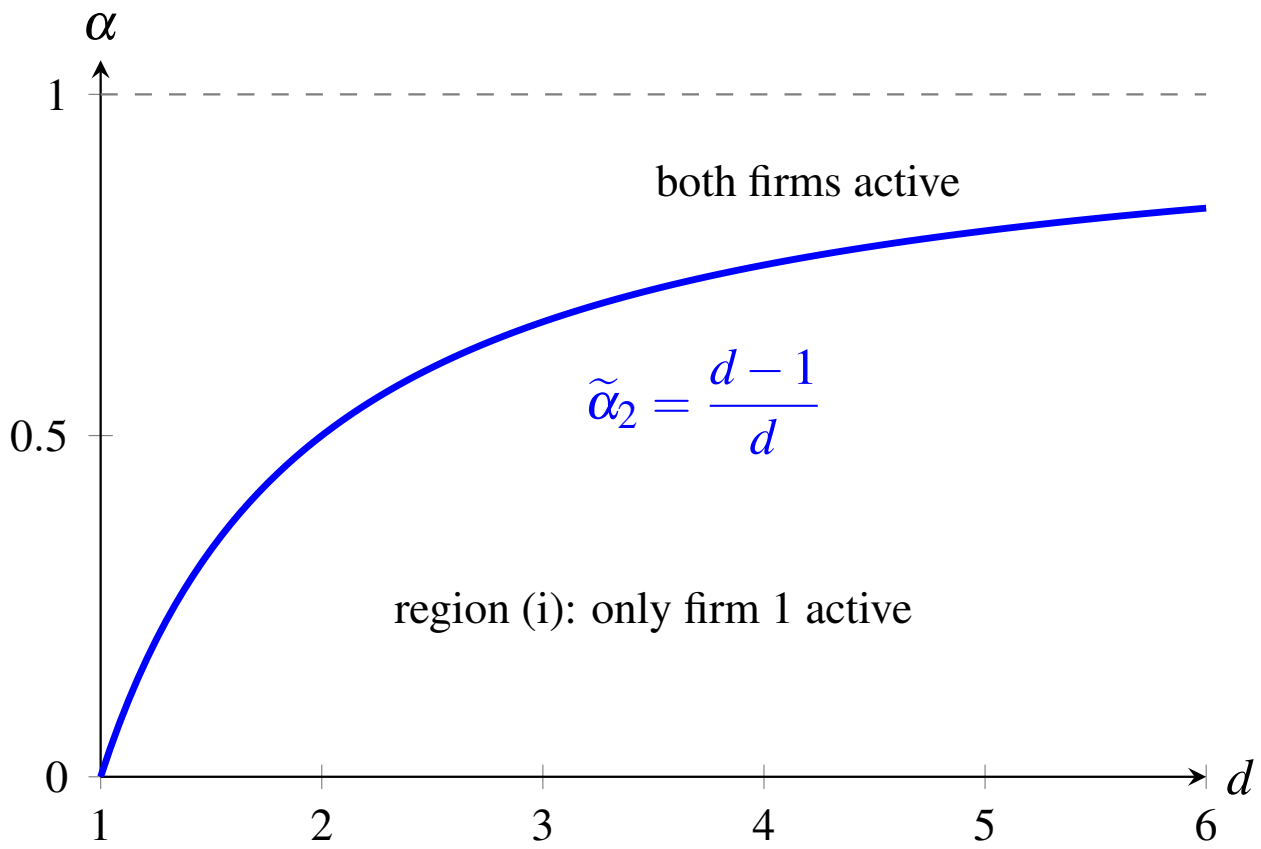
$$t^*(\alpha) - t^*_{Ind.}(\alpha) = \frac{3c(1-\alpha)}{4(1+d)} \tag{3.94}$$

- $t^*_{Ind.}(\alpha)$ induces the first-best aggregate output, but splits it between both firms when both are active.
- Hence, $t^*_{Ind.}(\alpha)$ is not the best uniform fee: welfare under $t^*(\alpha)$ is higher by $\frac{c^2(1-\alpha)^2}{8(1+d)}$.
- Achieving the first best requires type-dependent fees.

Indirect Approach: When Is Firm 2 Active?

- At $t^*_{Ind.}(\alpha)$, firm 1 is always active, and firm 2 is active iff $\alpha \geq \widetilde{\alpha}_2 \equiv \frac{c(1+d)-1}{cd}$.
- When $\alpha < \widetilde{\alpha}_2$, the uniform fee $\frac{(1-\alpha c)(d-1)}{1+d}$ induces firm 1 to produce the first-best output as a monopolist.
- At $c = \frac{1}{2}$, $\widetilde{\alpha}_2 = \frac{d-1}{d}$, which increases in d: more severe pollution makes it harder for firm 2 to remain active.

Figure 3.B – Cutoff $\widetilde{\alpha}_2$



Section 3.5
Allowing for Nonlinear Costs

Setup: Nonlinear Costs

- Duopoly of Section 2.3, but every firm's cost function is

$$C(q_i) = cq_i + \frac{\beta}{2}q_i^2 \tag{3.35}$$

  with marginal cost $c + \beta q_i$.
- $\beta = 0$: constant marginal costs; $\beta > 0$: diseconomies of scale; $\beta < 0$: economies of scale. We assume $\beta > -1$.
- **Conjecture:** more convex costs lower output and emissions, calling for a less stringent fee.

Section 3.5.1 – Second Stage: Best Response

- Firm i solves (3.36). **First-order condition:**

$$1 - 2q_i - q_j - c - \beta q_i - t = 0 \tag{3.37}$$

- **Best response function:**

$$q_i(q_j) = \frac{1 - c - t}{2 + \beta} - \frac{1}{2 + \beta} q_j$$

- With convex costs, $\beta > 0$, the best response shifts inward (lower intercept) and becomes flatter (slope $\frac{1}{2+\beta}$ closer to zero).

Section 3.5.1 – Equilibrium Output

- In a symmetric equilibrium, $q_i = q_j = q$:

$$q(t) = \frac{1 - c - t}{3 + \beta} \tag{3.39}$$

- $\beta = 0$ yields $q(t) = \frac{1-c-t}{3}$, as in (2.7).
- Convex costs: every firm produces fewer units. Concave costs: more units.
- Aggregate output, $Q(t) = \frac{2(1-c-t)}{3+\beta}$, exhibits the same comparative statics in β.

Section 3.5.2 – First Stage: Social Marginal Cost

- **Indirect approach:** find the output the EA seeks, then the fee that induces it.
- With two symmetric firms producing $q = \frac{Q}{2}$ each, the industry cost is $2\left[cq + \frac{\beta}{2}q^2\right] = cQ + \frac{\beta}{4}Q^2$, with marginal cost $c + \frac{\beta}{2}Q$.
- Maximizing W over Q (3.40)–(3.41) yields

$$\underbrace{1 - Q}_{SMB} = \underbrace{c + \left(d + \frac{\beta}{2}\right) Q}_{SMC} \tag{3.42}$$

- With $\beta > 0$, SMC is larger than with linear costs.

Socially Optimal Output $Q^{SO}(\beta)$

- Solving (3.42) for Q:

$$Q^{SO}(\beta) = \frac{1-c}{1+d+\frac{\beta}{2}} \tag{3.43}$$

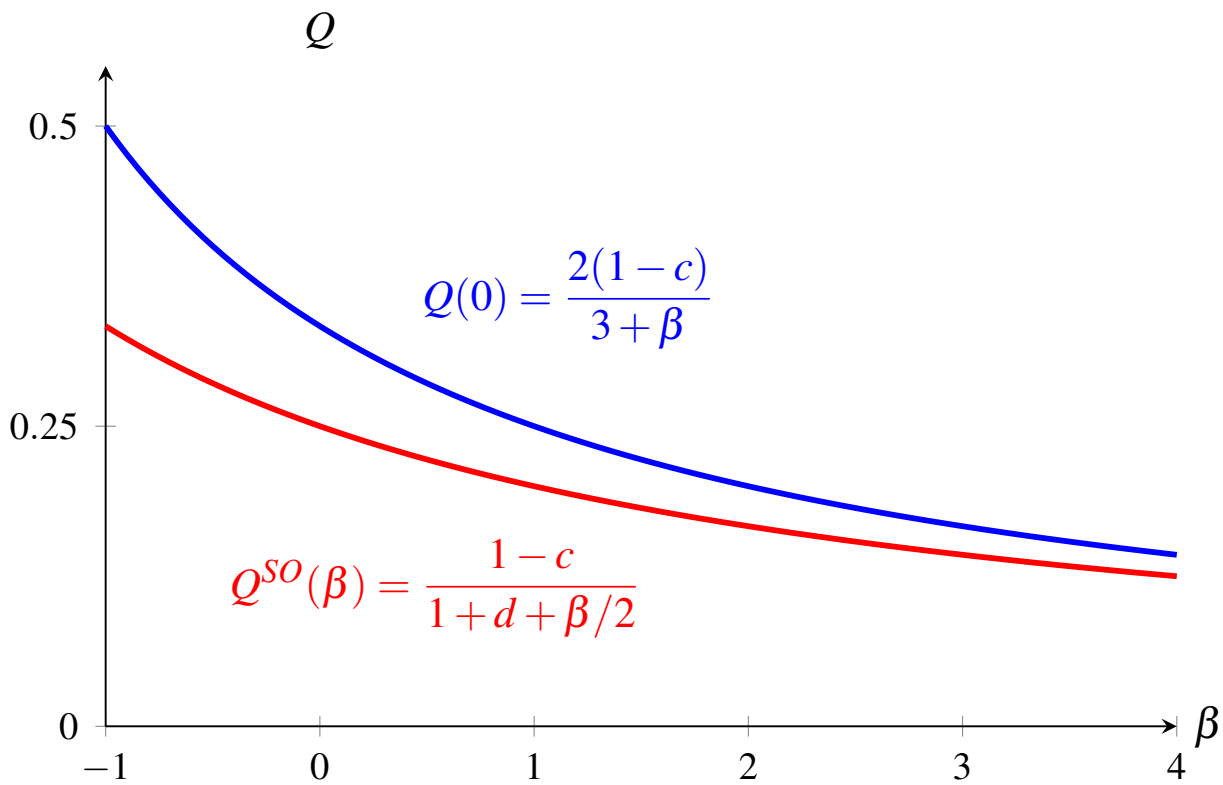
  which decreases in β, and equals $\frac{1-c}{1+d}$ when $\beta = 0$.

- Unregulated output, $Q(0) = \frac{2(1-c)}{3+\beta}$, is socially excessive iff

$$d > \frac{1}{2}, \quad \text{independent of } \beta \tag{3.44}$$

- β enters both denominators in the same way, $3+\beta$ and $2+2d+\beta$, so the comparison reduces to 3 versus $2+2d$.

Figure 3.7 – Market Output $Q(0)$ and $Q^{SO}(\beta)$



At $c = \frac{1}{2}$ and $d = 1$: both curves decrease in β, and $Q(0)$ lies above $Q^{SO}(\beta)$.

Optimal Fee $t^*(\beta)$

- Setting $Q^{SO}(\beta)$ equal to $Q(t)$:

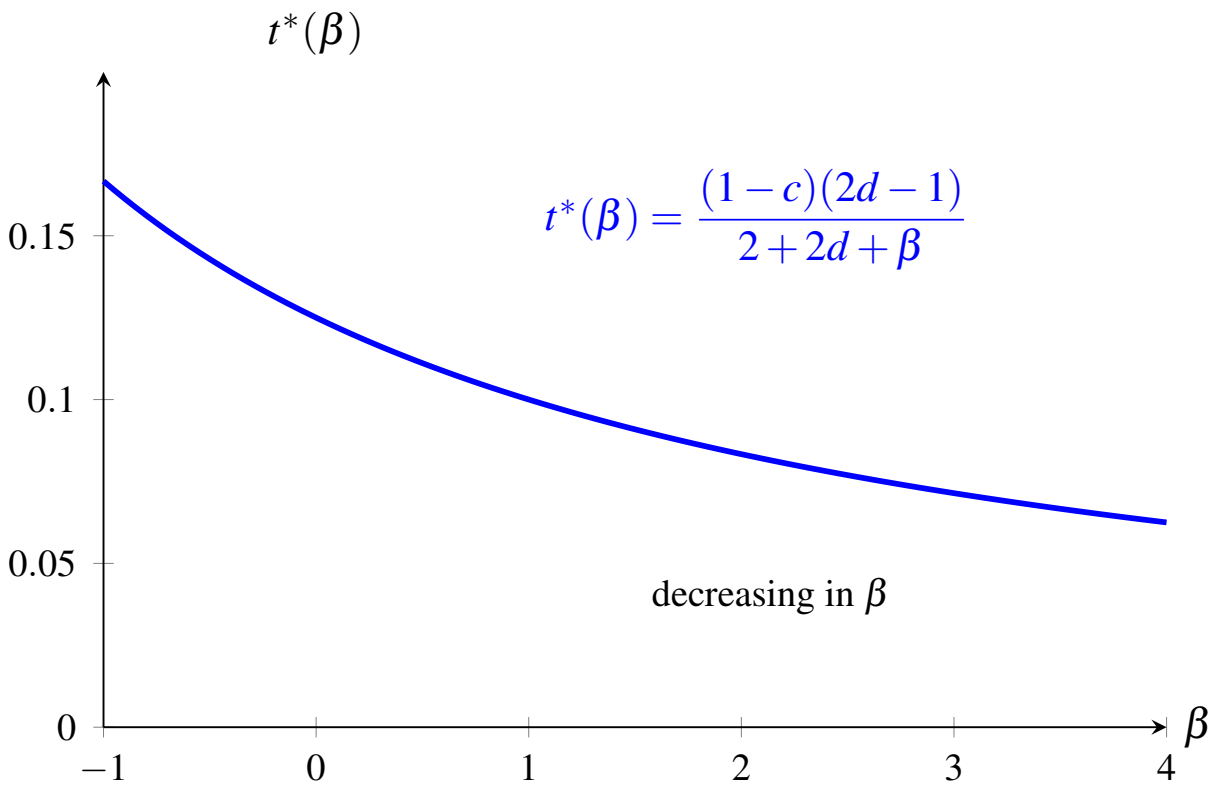
$$\frac{1-c}{1+d+\frac{\beta}{2}} = \frac{2(1-c-t)}{3+\beta} \tag{3.45}$$

- Solving for t:

$$t^*(\beta) = \frac{(1-c)(2d-1)}{2+2d+\beta} \tag{3.46}$$

- Positive iff $d > \frac{1}{2}$, for all admissible β: with diseconomies ($\beta > 0$) or economies of scale ($\beta < 0$).

Figure 3.8 – Fee $t^*(\beta)$



Fee $t^*(\beta)$ as a function of β, at $c = \frac{1}{2}$ and $d = 1$.

Economies of Scale: A Second-Best Fee

- With $\beta < 0$, the industry cost $cQ + \frac{\beta}{2}(q_1^2 + q_2^2)$ is minimized at $q_1 = Q$ or $q_2 = Q$: a planner would select

$$Q^{FB}(\beta) = \frac{1-c}{1+d+\beta} > Q^{SO}(\beta)$$

- A uniform fee cannot shift production to one firm, as both produce the same output in (3.39): $t^*(\beta)$ is the best uniform fee given that both remain active, and $Q^{SO}(\beta)$ the best output under this constraint.
- **Example:** $c = \frac{1}{2}$, $d = 1$, and $\beta = -\frac{1}{2}$. Welfare is $\frac{1}{12}$ under the single-firm allocation, and $\frac{1}{14}$ at $Q^{SO}(\beta)$: a loss of $\frac{1}{84}$.
- When $\beta \geq 0$, splitting output equally minimizes costs: $Q^{SO}(\beta)$ and $t^*(\beta)$ are the **first best**.

Section 3.5.3 – Role of Diseconomies of Scale

- The fee decreases in cost convexity:

$$\frac{\partial t^*(\beta)}{\partial \beta} = -\frac{(1-c)(2d-1)}{(2+2d+\beta)^2} < 0 \tag{3.47}$$

- A higher β lowers both $Q(0)$ and $Q^{SO}(\beta)$, but $Q(0)$ falls faster, shrinking the wedge:

$$Q(0) - Q^{SO}(\beta) = \frac{2(1-c)(2d-1)}{(3+\beta)(2+2d+\beta)} \tag{3.48}$$

- **Intuitively,** convex costs already do part of the regulator's job of curbing output, so a laxer fee suffices.

Section 3.6
Allowing for Product Differentiation

Setup: Differentiated Products

- Think of two soft drinks with different flavors, two car models, or two smartphones in different colors.
- **Inverse demand** (Singh and Vives, 1984):

$$p(q_i, q_j) = 1 - q_i - \gamma q_j, \qquad \gamma \in [0, 1] \tag{3.49}$$

- $\gamma = 1$: homogeneous products (Chapter 2); $\gamma \in (0, 1)$: horizontally differentiated; $\gamma = 0$: unrelated goods.
- **Conjecture:** more differentiation (lower γ) softens competition, raising output and emissions, so the EA sets a more stringent fee.

Section 3.6.1 – Second Stage: Best Response

- Firm i solves

$$\max_{q_i \geq 0} \quad \pi_i = (1 - q_i - \gamma q_j) q_i - (c + t) q_i \tag{3.50}$$

- **Best response function:**

$$q_i(q_j) = \frac{1 - c - t}{2} - \frac{\gamma}{2} q_j$$

- With $\gamma < 1$, the intercept is unaffected, but the function becomes flatter: more differentiated goods attenuate output competition.
- With $\gamma = 0$, $q_i = \frac{1-c-t}{2}$, the monopoly output.

Section 3.6.1 – Equilibrium Output

- In a symmetric equilibrium:

$$q(t) = \frac{1-c-t}{2+\gamma}, \qquad Q(t) = \frac{2(1-c-t)}{2+\gamma} \quad (3.52\text{–}3.53)$$

- $\gamma = 1$ yields $q(t) = \frac{1-c-t}{3}$, as in Chapter 2.
- More differentiated products (lower γ) raise individual and aggregate output.
- Aggregate output ranges from $\frac{2(1-c-t)}{3}$ when $\gamma = 1$ to $1-c-t$ when $\gamma = 0$.

Section 3.6.2 – First Stage: Welfare

- With horizontal differentiation, consumer surplus is

$$CS = \frac{q_i^2 + 2\gamma q_i q_j + q_j^2}{2},$$

  which becomes $\frac{Q^2}{2}$ when $\gamma = 1$ (3.55).
- With equal outputs, $CS = (1+\gamma)q^2$ (3.56).
- The EA solves

$$\max_t \quad W = (1+\gamma)q^2 + [(1-q-\gamma q)Q - cQ] - \frac{d}{2}Q^2 \tag{3.57}$$

  evaluated at $q(t)$ and $Q(t)$.

Optimal Fee $t^*(\gamma)$

- **First-order condition:**

$$\frac{\partial W}{\partial t} = \frac{2\left[c(1-2d) + 2d(1-t) - t(1+\gamma) - 1\right]}{(2+\gamma)^2} = 0 \tag{3.58}$$

- **Optimal fee:**

$$t^*(\gamma) = \frac{(1-c)(2d-1)}{2d+1+\gamma} \tag{3.59}$$

- At $\gamma = 1$, it coincides with (2.10). It is positive iff $d > \frac{1}{2}$, regardless of γ.

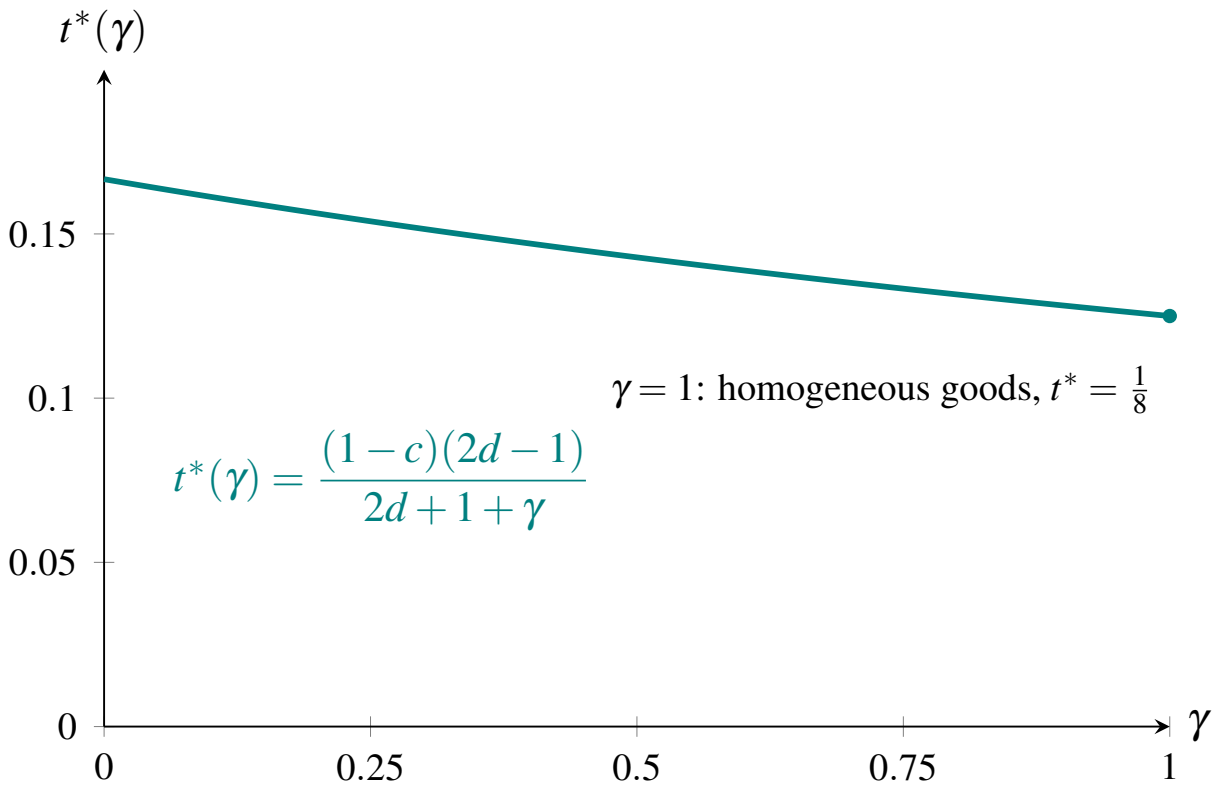
Comparative Statics of $t^*(\gamma)$

- The fee decreases in product homogeneity:

$$\frac{\partial t^*(\gamma)}{\partial \gamma} = -\frac{(1-c)(2d-1)}{(2d+1+\gamma)^2} < 0 \tag{3.60}$$

- **Intuitively,** more differentiated goods (lower γ) imply a larger aggregate output, requiring a more stringent fee.
- At $t^*(\gamma)$, every firm produces $\frac{1-c}{2d+1+\gamma} > 0$, which is the socially optimal output: $t^*(\gamma)$ achieves the **first best**.

Figure 3.9 – Fee $t^*(\gamma)$



At $c = \frac{1}{2}$ and $d = 1$: $t^*(\gamma) = \frac{1}{2(3+\gamma)}$, which decreases in γ.

Section 3.6.3 – Vertical Differentiation and Green Consumers

- **Horizontal** differentiation: consumers disagree about which product they prefer.
- **Vertical** differentiation: all consumers agree that one product is better, here in its environmental quality.
- Firm G sells a **green** good with no emissions; firm B sells a **brown** good, with one unit of emissions per unit of output.
- Builds on Mussa and Rosen (1978), applied to environmental quality by Cremer and Thisse (1999) and Moraga-González and Padrón-Fumero (2002).

Green Consumers: Preferences and Assumptions

- Consumers are uniformly distributed over $v \in [0, 1]$, and each buys at most one unit.
- **Utility:** $v - p_G$ from the green good, and $(1 - \kappa)v - p_B$ from the brown good, where $\kappa \in [0, 1)$ measures consumers' **environmental awareness**.
- $\kappa = 0$: homogeneous goods, as in Chapter 2. $\kappa > 0$: every consumer values the brown good less, by κv.
- Costs are zero, $c = 0$. Only the brown good pollutes: $Env = \frac{d}{2} q_B^2$, where $d > \frac{1}{2}$.
- The EA sets a fee t per unit of emissions, which only firm B pays; firms then observe t and compete à la Cournot.

Green Consumers: Demand

- Indifferent between green and brown: $v_{GB} = \frac{p_G - p_B}{\kappa}$.
  Indifferent between brown and not buying: $v_B = \frac{p_B}{1-\kappa}$.
- Demands: $q_G = 1 - v_{GB}$ and $q_B = v_{GB} - v_B$.
- **Inverse demands:**

$$p_G = 1 - q_G - (1-\kappa)q_B \quad \text{and} \quad p_B = (1-\kappa)(1 - q_G - q_B) \tag{3.61}$$

- With $\kappa > 0$, the brown good is cheaper:
  $p_G - p_B = \kappa(1 - q_G) > 0$.

Green Consumers: Second Stage

- **Profits:** $\pi_G = [1 - q_G - (1-\kappa)q_B]\, q_G$ and $\pi_B = [(1-\kappa)(1 - q_G - q_B) - t]\, q_B$.
- **Best response functions:**

$$q_G(q_B) = \frac{1}{2} - \frac{1-\kappa}{2} q_B \quad \text{and} \quad q_B(q_G) = \frac{1}{2} - \frac{t}{2(1-\kappa)} - \frac{1}{2} q_G \tag{3.62}$$

- Awareness flattens firm G's best response. When $t > 0$, it lowers the intercept of firm B's best response.

Green Consumers: Equilibrium Outputs

- Solving (3.62) simultaneously:

$$q_G(t) = \frac{1+\kappa+t}{3+\kappa} \quad \text{and} \quad q_B(t) = \frac{1-\kappa-2t}{(1-\kappa)(3+\kappa)} \qquad (3.63)$$

- Brown output is positive iff $t < \frac{1-\kappa}{2}$. A more stringent fee lowers brown output and raises green output.
- **Without regulation:** $q_G(0) = \frac{1+\kappa}{3+\kappa}$ and $q_B(0) = \frac{1}{3+\kappa}$, both $\frac{1}{3}$ when $\kappa = 0$, as in Chapter 2 with $c = 0$.
- Awareness raises green output and lowers brown output: green consumers reduce emissions even without regulation.

Green Consumers: First Stage

- **Consumer surplus** with vertically differentiated goods:

$$CS = \frac{q_G^2 + 2(1-\kappa)q_G q_B + (1-\kappa)q_B^2}{2} \tag{3.64}$$

- Welfare includes consumers' lower valuation of the brown good (Bansal and Gangopadhyay, 2003), which differs from the damage *Env*.
- **Optimal fee:**

$$t^*(\kappa) = \frac{(1-\kappa)\left[2d - (1-\kappa)^2\right]}{4d + (1+3\kappa)(1-\kappa)} \tag{3.66}$$

Comparing $t^*(\kappa)$ with Chapter 2

- With $c = 0$, fee (2.10) becomes $\frac{2d-1}{2(1+d)}$.
- At $\kappa = 0$, $t^*(0) = \frac{2d-1}{4d+1}$: also positive iff $d > \frac{1}{2}$, but lower, since $4d + 1 > 2(1 + d)$ iff $d > \frac{1}{2}$.
- **Intuitively,** only the brown firm pollutes here, whereas both firms pollute in Chapter 2.
- $t^*(\kappa) > 0$ for all $\kappa \in [0, 1)$, and it approaches zero as $\kappa \to 1$.

Outputs at $t^*(\kappa)$

- **Equilibrium outputs:**

$$q_B^* = \frac{1-\kappa}{4d + (1+3\kappa)(1-\kappa)} \quad \text{and}$$

$$q_G^* = \frac{2\left[d + \kappa(1-\kappa)\right]}{4d + (1+3\kappa)(1-\kappa)} \tag{3.67}$$

  Both are positive: both firms remain active.

- Brown output, and thus emissions, decreases in awareness:

$$\frac{\partial q_B^*}{\partial \kappa} = -\frac{4d + 3(1-\kappa)^2}{\left[4d + (1+3\kappa)(1-\kappa)\right]^2} < 0$$

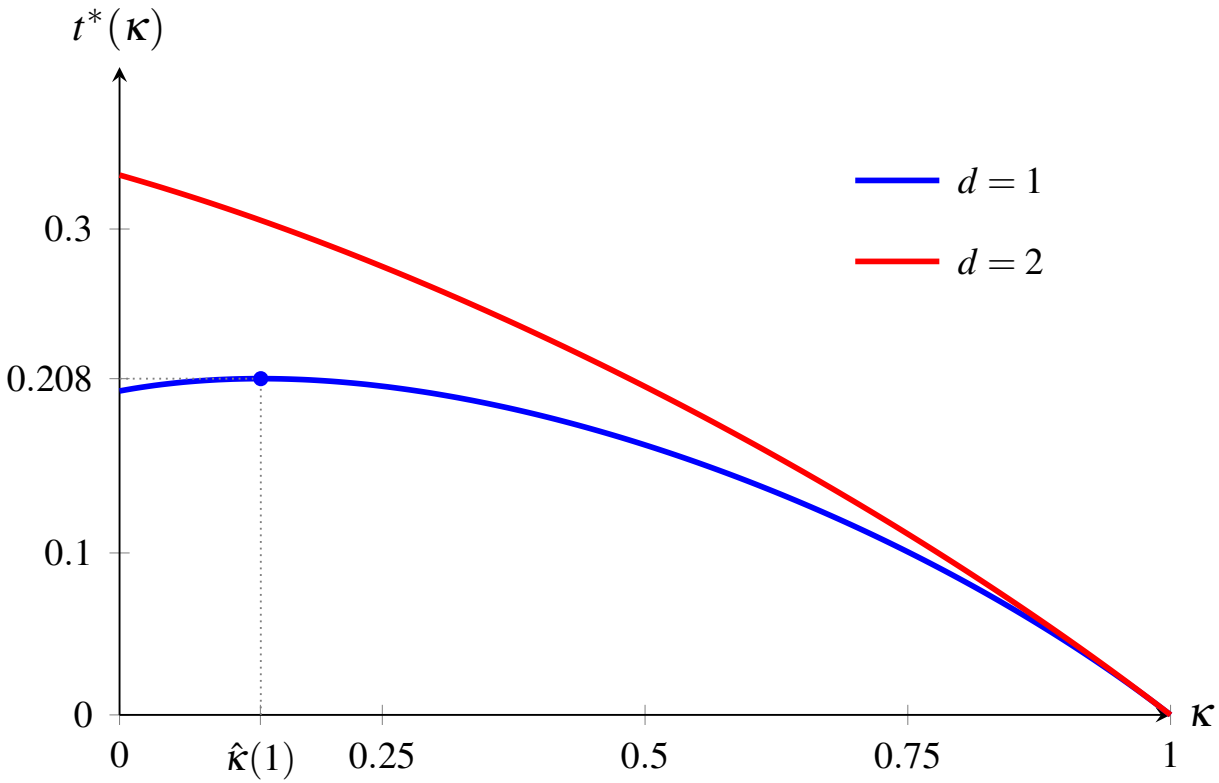
How Awareness Affects the Fee

- Differentiating $t^*(\kappa)$:

$$\frac{\partial t^*(\kappa)}{\partial \kappa} = -\frac{N(\kappa)}{\left[4d + (1+3\kappa)(1-\kappa)\right]^2} \tag{3.68}$$

  where $N(\kappa) \equiv 8d^2 - 6d(1-\kappa)^2 + 3(1-\kappa)^4 - 8(1-\kappa)^3$.
- $N(\kappa)$ increases in κ, starting at $N(0) = (2d+1)(4d-5)$.
- If $d \geq \frac{5}{4}$: $N(\kappa) > 0$, so the fee decreases in awareness.
- If $\frac{1}{2} < d < \frac{5}{4}$: the fee increases for $\kappa < \hat{\kappa}(d)$ and decreases otherwise, where $N(\hat{\kappa}(d)) = 0$.

Figure 3.10 – Fee $t^*(\kappa)$ with Green Consumers



With $d = 2$, the fee decreases in κ. With $d = 1$, it increases up to $\hat{\kappa}(1) \simeq 0.13$, and then decreases.

Two Effects of Environmental Awareness

- **Emission effect:** consumers shift from the brown to the green good, reducing emissions, so a less stringent fee suffices.
- **Competition effect:** the brown good becomes a weaker substitute, doing less to lower the green good's price; the EA has a weaker reason to protect brown output with a lenient fee.
- **Severe pollution:** the emission effect dominates, and the fee decreases in κ.
- **Mild pollution:** the EA cares more about the green firm's market power, and the competition effect dominates for low κ.

Green Consumers: A Second-Best Fee

- A planner would only produce the green good (valued more, same cost, no emissions), selling $q_G = 1$ at a price equal to marginal cost.
- A single fee on firm B cannot correct firm G's market power: $t^*(\kappa)$ is a **second best**.
- Driving firm B out, $t \geq \frac{1-\kappa}{2}$, leaves firm G as a monopolist producing $\frac{1}{2}$, with welfare $\frac{3}{8}$.
- $t^*(\kappa)$ yields a higher welfare, $W^* - \frac{3}{8} = \frac{(1-\kappa)^2}{8[4d+(1+3\kappa)(1-\kappa)]} > 0$: the brown firm's competition lowers the green good's price.
- Chapter 4 (section 4.8) considers consumers who cannot observe environmental quality.

Section 3.7
Allowing for Cost Externalities

Setup: Cost Externalities

- Each firm's cost depends on its rival's output: shared infrastructure, congestion, or resource competition.
- **Cost function:**

$$C(q_i, q_j) = cq_i + \phi q_i q_j, \qquad \phi \geq 0 \tag{3.69}$$

- $\phi = 0$: cost cq_i, as in Section 2.2. Marginal cost $c + \phi q_j$ increases in the rival's output.
- **At first glance,** a higher ϕ lowers output, calling for a laxer fee. While output indeed falls, the fee becomes more stringent when $d < 2$, and less stringent only when $d > 2$.

Section 3.7.1 – Second Stage: Best Response

- **First-order condition** of problem (3.70):

$$1 - 2q_i - q_j - c - \phi q_j - t = 0 \tag{3.71}$$

- **Best response function:**

$$q_i(q_j) = \frac{1 - c - t}{2} - \frac{1 + \phi}{2} q_j \tag{3.72}$$

- Same intercept, but a steeper slope: a higher q_j raises firm i's costs, so it produces fewer units.
- Cost externalities increase the strategic substitutability between firms' output.

Section 3.7.1 – Equilibrium Output

- In a symmetric equilibrium:

$$q(t) = \frac{1-c-t}{3+\phi}, \qquad Q(t) = \frac{2(1-c-t)}{3+\phi} \tag{3.74}$$

- Output decreases in cost externalities: $\frac{\partial q(t)}{\partial \phi} = -\frac{1-c-t}{(3+\phi)^2} < 0$.
- Output still decreases in the fee, $\frac{\partial q(t)}{\partial t} = -\frac{1}{3+\phi} < 0$, but less sensitively than when $\phi = 0$.
- The EA may then need a more stringent fee to achieve the same pollution reduction.

Section 3.7.2 – First Stage (Emission Fee)

- With two symmetric firms producing $q = \frac{Q}{2}$ each, the industry externality cost is $2\phi q^2 = \frac{\phi}{2}Q^2$.
- The EA solves (3.75), evaluated at $Q(t)$:

$$\max_t \quad W = \frac{Q^2}{2} + \left[(1-Q)Q - cQ - \frac{\phi}{2}Q^2 - tQ\right] + tQ - \frac{d}{2}Q^2$$

- Fee revenue, $tQ(t)$, cancels with firms' fee payments.

Optimal Fee $t^*(\phi)$

- **First-order condition:**

$$\frac{\partial W}{\partial t} = \frac{2(1-c)(2d-1+\phi) - 4(1+d+\phi)t}{(3+\phi)^2} = 0 \quad (3.76)$$

- Solving for t:

$$t^*(\phi) = \frac{(1-c)(2d-1+\phi)}{2(1+d+\phi)} \quad (3.77)$$

  which coincides with (2.10) when $\phi = 0$.

- Positive iff $d > \underline{d}(\phi) \equiv \frac{1-\phi}{2}$, which holds for all $d > \frac{1}{2}$: $t^*(\phi)$ is unambiguously positive.

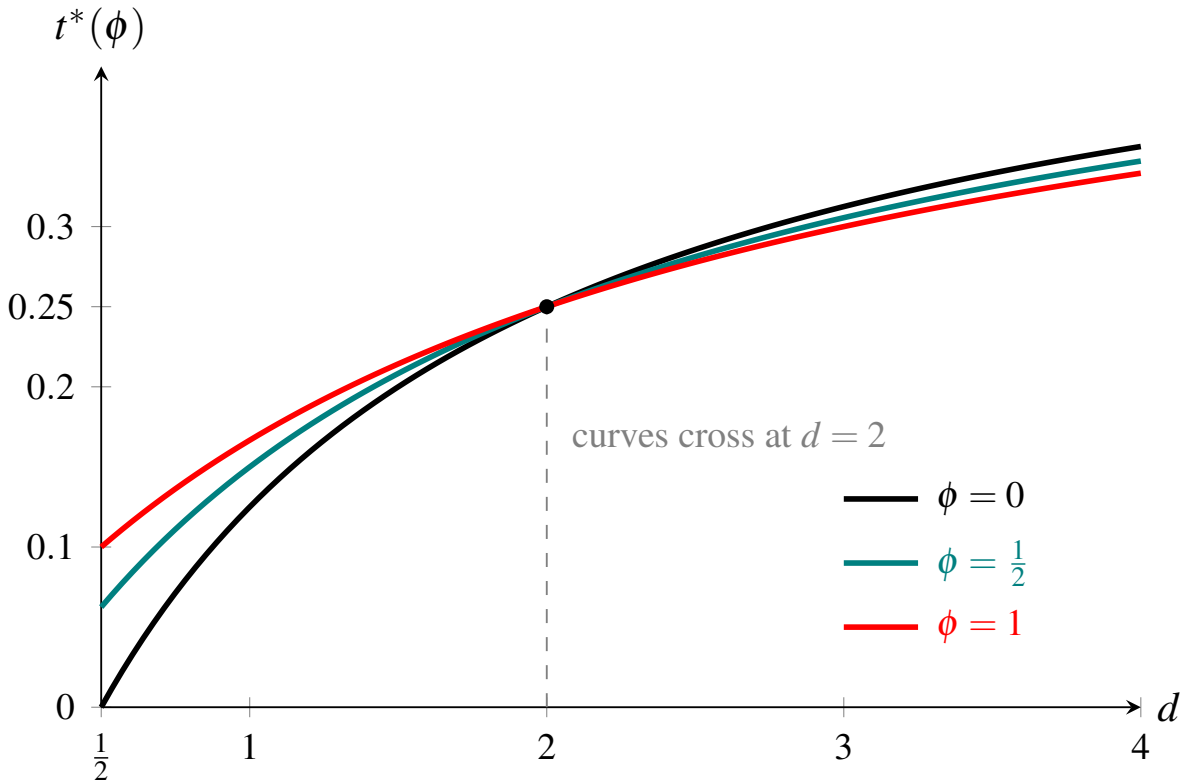
With or Without Cost Externalities?

- Decomposing the fee:

$$t^*(\phi) = t^*(0) + \frac{\phi(1-c)(2-d)}{2(1+d)(1+d+\phi)} \tag{3.78}$$

- $d < 2$: the last term is positive, so the fee is **more** stringent with cost externalities, $t^*(\phi) > t^*(0)$.
- $d > 2$: the last term is negative, so the fee is **less** stringent, $t^*(\phi) < t^*(0)$.
- Equivalently, $\frac{\partial t^*(\phi)}{\partial \phi} = \frac{(1-c)(2-d)}{2(1+d+\phi)^2}$, positive iff $d < 2$.

Figure 3.11 – Emission Fee $t^*(\phi)$



At $c = \frac{1}{2}$, for $\phi = 0$, $\frac{1}{2}$, and 1: the fee shifts up when $d < 2$, and down when $d > 2$.

Intuition: Two Output Reductions

- $SMC = c + (d + \phi)Q$, so the socially optimal output is $Q^{SO} = \frac{1-c}{1+d+\phi}$.
- A higher ϕ reduces both market and socially optimal output:

$$\frac{\partial Q(t)}{\partial \phi} = -\frac{2(1-c-t)}{(3+\phi)^2} < 0 \quad \text{and}$$

$$\frac{\partial Q^{SO}}{\partial \phi} = -\frac{1-c}{(1+d+\phi)^2} < 0 \tag{3.79}$$

- **Mild pollution**, $d < 2$: the reduction in Q^{SO} dominates, so the EA tightens the fee.
- **Severe pollution**, $d > 2$: the reduction in market output dominates, so a laxer fee suffices.

Cost Externalities: A Second-Best Fee

- At $t^*(\phi)$, every firm produces $\frac{1-c}{2(1+d+\phi)} > 0$: both firms remain active.
- The total externality cost, $2\phi q_1 q_2$, is largest when output is divided equally between firms.
- A planner would concentrate all output in one firm, producing $\frac{1-c}{1+d}$: type-dependent fees $t_i = \frac{(1-c)(d-1)}{1+d}$ and $t_j \geq \frac{(1-c)(d-\phi)}{1+d}$ achieve it.
- The uniform fee $t^*(\phi)$ only achieves a **second best** when $\phi > 0$.

Section 3.8
Allowing for Price Competition

Setup: Price Competition

- In retail electricity, gasoline stations, or consumer goods, firms set prices rather than output levels.
- Inverse demand of Section 3.6, $p(q_i, q_j) = 1 - q_i - \gamma q_j$, now with $\gamma \in [0, 1)$.
- **Direct demand:**

$$q_i(p_i, p_j) = \frac{1 - \gamma - p_i + \gamma p_j}{1 - \gamma^2} \tag{3.80}$$

- Lange and Requate (1999) study emission taxes with price-setting firms in a more general setting.

Section 3.8.1 – Second Stage (Prices)

- Every firm i solves

$$\max_{p_i \geq 0} \quad \pi_i = (p_i - c - t)\frac{1 - \gamma - p_i + \gamma p_j}{1 - \gamma^2} \tag{3.81}$$

- **Best response function:**

$$p_i(p_j) = \frac{1 - \gamma + c + t}{2} + \frac{\gamma}{2} p_j \tag{3.82}$$

- Prices are **strategic complements**: when firm j raises its price, firm i raises its own.
- A higher fee shifts this function upward: firms pass part of the fee onto prices.

Equilibrium Price and Output

- In a symmetric equilibrium:

$$p(t) = \frac{1-\gamma+c+t}{2-\gamma} \quad \text{and} \quad q(t) = \frac{1-c-t}{(2-\gamma)(1+\gamma)} \tag{3.83}$$

- Compared with quantity competition, $q^C(t) = \frac{1-c-t}{2+\gamma}$:

$$q(t) - q^C(t) = \frac{\gamma^2(1-c-t)}{(2-\gamma)(1+\gamma)(2+\gamma)} \geq 0 \tag{3.84}$$

- For a given fee, firms produce more under price competition (Singh and Vives, 1984); both coincide at $\gamma = 0$.

Section 3.8.2 – First Stage: Fee $t^B(\gamma)$

- **Emission fee under Bertrand competition:**

$$t^B(\gamma) = \frac{(1-c)(2d+\gamma^2-1)}{2d+1+\gamma} \tag{3.85}$$

- Every firm produces $\frac{1-c}{2d+1+\gamma} > 0$, as under quantity competition at $t^*(\gamma)$: both fees achieve the **first best**.
- Comparing both fees:

$$t^B(\gamma) - t^*(\gamma) = \frac{\gamma^2(1-c)}{2d+1+\gamma} \geq 0 \tag{3.86}$$

Why Price Competition Calls for a Stricter Fee

- Price competition leads firms to produce more units for a given fee, (3.84), so the EA needs a more stringent fee to induce the same output.
- Both fees coincide when goods are unrelated, $\gamma = 0$: firms behave as monopolists under both forms of competition.
- As $\gamma \to 1$, price approaches $c + t$, and

$$t^B(\gamma) \to \frac{d(1-c)}{1+d} = dQ^{SO},$$

the Pigouvian fee under perfect competition.

Sign and Comparative Statics of $t^B(\gamma)$

- $t^B(\gamma) > 0$ iff $d > \frac{1-\gamma^2}{2}$, less demanding than $d > \frac{1}{2}$: price competition already curbs firms' market power.
- The fee increases in pollution severity:

$$\frac{\partial t^B(\gamma)}{\partial d} = \frac{2(1-c)(2-\gamma)(1+\gamma)}{(2d+1+\gamma)^2} > 0$$

- It decreases in production costs when positive: $\frac{\partial t^B(\gamma)}{\partial c} = -\frac{2d+\gamma^2-1}{2d+1+\gamma}$.

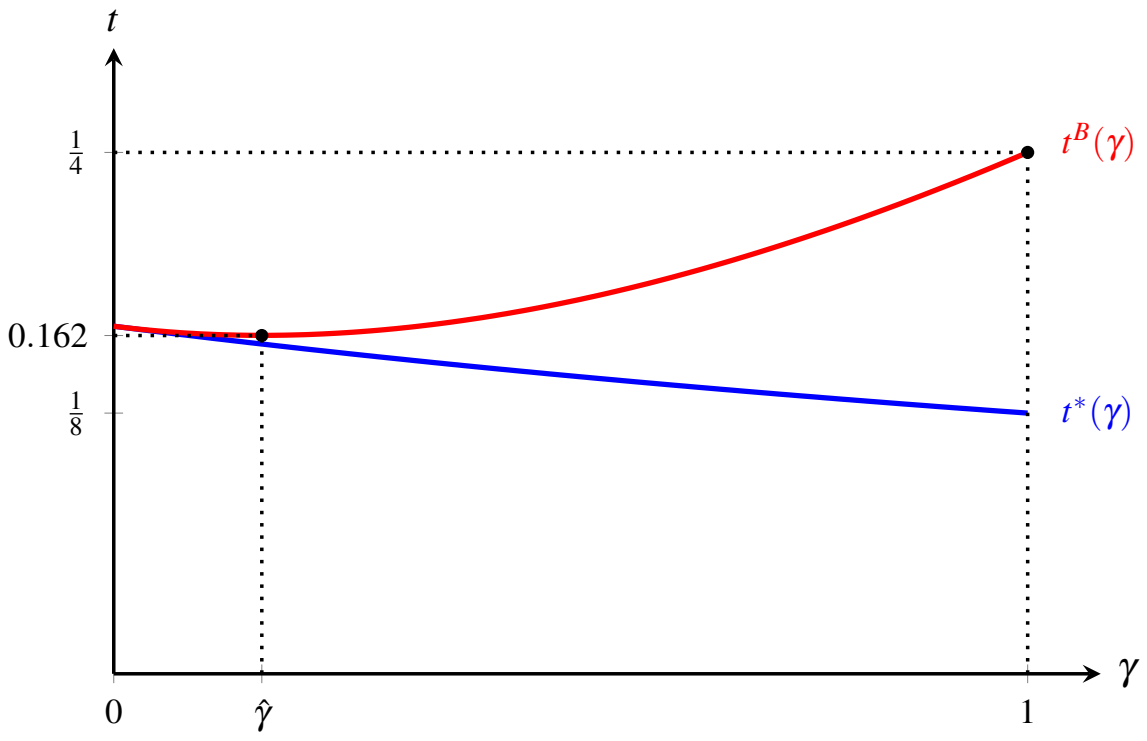
Fee $t^B(\gamma)$ Is U-Shaped in γ

- Differentiating:

$$\frac{\partial t^B(\gamma)}{\partial \gamma} = \frac{(1-c)\left[(1+\gamma)^2 - 2d(1-2\gamma)\right]}{(2d+1+\gamma)^2} \tag{3.87}$$

  negative for $\gamma < \hat{\gamma} \equiv \sqrt{4d^2+6d} - (1+2d)$, and positive otherwise.
- More homogeneous goods (higher γ) reduce Q^{SO}, calling for a less stringent fee.
- They also intensify price competition, reducing markups, which calls for a more stringent fee.
- The first effect dominates when goods are very differentiated; the second dominates otherwise.

Figure 3.12 – Price versus Quantity Competition



At $c = \frac{1}{2}$ and $d = 1$: $t^*(\gamma)$ falls from $\frac{1}{6}$ to $\frac{1}{8}$; $t^B(\gamma)$ is lowest at $\hat{\gamma} = \sqrt{10} - 3 \simeq 0.16$, then rises toward $\frac{1}{4}$.

Fees and Markups

- Markup at the socially optimal output, $p - c - t$: $\frac{(1-\gamma^2)(1-c)}{2d+1+\gamma}$ under price competition, and $\frac{1-c}{2d+1+\gamma}$ under quantity competition.
- In both cases, the fee equals marginal damage minus the markup:

$$t = dQ^{SO} - (p - c - t), \qquad Q^{SO} = \frac{2(1-c)}{2d+1+\gamma} \tag{3.88}$$

- The EA starts from the Pigouvian fee and reduces it by the markup that firms already charge, since this markup also reduces output.
- Price competition yields a smaller markup, and thus a fee closer to marginal damage (Section 2.7 extends this rule).

Table 3.1 – Summary of Comparative Statics (1/2)

Extension	$q(t)$	$Q(t)$	Q^{SO}	t
3.2: More firms (higher n)	$-$	$+$	$=$	$+$
3.3: Pollution asymmetry (lower θ)	$=$	$=$	$+$	$-$
3.4: Cost asymmetry (lower α)	$+$ for q_1, $-$ for q_2	$+$	$+$	$+$
3.5: Cost convexity (higher β)	$-$	$-$	$-$	$-$

- $+$ $(-)$: the parameter raises (lowers) the variable; $=$: no effect.
- **Higher β:** output falls by more than Q^{SO}: a less stringent fee.
- Column Q^{SO}: first-best output in 3.3–3.4; in 3.7, the best output when both firms produce the same amount.

Table 3.1 – Summary of Comparative Statics (2/2)

Extension	$q(t)$	$Q(t)$	Q^{SO}	t
3.6: Product differentiation (lower γ)	$+$	$+$	$+$	$+$
3.6.3: Green consumers (higher κ)	$+$ for q_G, $-$ for q_B	$+/-$	$=$	$-$ if $d \geq \frac{5}{4}$; $+$ then $-$ otherwise
3.7: Cost externalities (higher ϕ)	$-$	$-$	$-$	$+$ iff $d < 2$
3.8: Price competition (lower γ)	$+$ iff $\gamma < \frac{1}{2}$	$+$ iff $\gamma < \frac{1}{2}$	$+$	$+$ iff $\gamma < \hat{\gamma}$

- **Lower γ:** output rises by more than Q^{SO}: a stricter fee.

Section 3.9
Case Studies

Case Study 3.9.1 – California Carbon Pricing

- Cap-and-trade (Cap-and-Invest since 2025) covers large industrial facilities (cement plants, petroleum refineries), electricity, and transportation fuels.
- One allowance per tonne of CO_2-equivalent: the allowance price works as a per-unit emission charge. A small per-tonne AB 32 Cost of Implementation Fee funds the state's climate programs.
- CalPortland converted its Mojave cement plant to portland-limestone cement in 2022, which, it reports, cuts CO_2 per tonne by about 10%.
- **Model:** the allowance price is the common fee t faced by firms with different technologies (Sections 3.3–3.4); the case does not show that the price caused their choices.

Case Study 3.9.2 – Firm Entry in Poland's Power Generation

- Entry into a market dominated by a large coal-based incumbent, PGE: more than 20 GW of solar capacity added between 2020 and 2025; coal's share of generation fell to about 51% in 2025.
- Plants pay for emissions through the EU Emissions Trading System: the price per tonne is set at the EU level.
- **Model:** in Section 3.2, more firms raise aggregate output for a given fee, and $t^*(n)$ becomes more stringent.
- The EU price cannot respond to entry in one national market: the case illustrates a fee that cannot adjust to the number of firms, rather than evidence for Section 3.2.

Case Study 3.9.3 – Pollution Asymmetries in Spain's Cement Industry

- Plants differ in emission intensity, partly because of their fuel mix. Cementos Molins: alternative fuels, such as solid recovered fuel replacing part of the petroleum coke, supplied 25% of its kilns' thermal energy in 2025, and more than 55% at its Sant Vicenç dels Horts plant.
- Plants pay the same EU allowance price per tonne; Spain has no national carbon tax on cement.
- **Model:** a uniform fee per unit of output is a second best (Section 3.3). A charge per tonne costs a plant more per unit of output the higher its intensity, moving toward the type-dependent fees of Section 3.3.3. The case does not test the model.

Case Study 3.9.4 – Cost Asymmetries in Sweden's Industrial Sector

- Sweden's carbon tax excludes EU ETS installations: steel and cement producers face the EU allowance price.
- SSAB is replacing blast furnaces with electric arc furnaces: Oxelösund (SEK 6.2 billion, start-up planned for 2027), and public funding for its Luleå plant.
- Heidelberg Materials Cement Sverige (formerly Cementa) paused CO_2 capture at Slite in November 2025, after the Swedish Energy Agency rejected public co-funding.
- **Model:** in Section 3.4, a uniform fee can drive the high-cost firm out. Here, firms differ in *abatement* costs (Chapter 6); the case does not test the model.

Section 3.10
Related Literature

3.10.1 – Emission Fees in Markets with Several Firms

- **Desjardins and Sinclair-Desgagné (2025):** internal carbon pricing within multidivisional firms.
- **Aldy and Stavins (2012):** theory and experience of carbon taxes and cap-and-trade systems.
- **Requate and Unold (2003):** incentives that taxes, permits, and standards give firms to adopt advanced abatement technology.
- **Requate (2006):** survey of environmental policy under imperfect competition.

3.10.2–3.10.3 – Asymmetric Intensity and Convex Costs

- **Ino and Matsumura (2021):** emission intensity-based pricing that achieves the first best under imperfect competition.
- **Requate (2005):** incentives to develop and adopt abatement technology.
- **Requate (2006):** with different firms, a uniform tax leaves production inefficient; adding an output subsidy only achieves a second best.
- **Long and Soubeyran (2005):** firm-specific taxes favor efficient firms and raise industry concentration.

3.10.4 – Green Consumers and Vertical Differentiation

- **Cremer and Thisse (1999):** a tax on polluting products can raise the number of firms and welfare.
- **Moraga-González and Padrón-Fumero (2002):** an emission standard can raise emissions and reduce welfare.
- **Bansal and Gangopadhyay (2003):** a discriminatory subsidy cuts pollution; a discriminatory tax may raise it.
- **Eriksson (2004):** a green premium has only a modest effect.
- **Birg and Voßwinkel (2021):** emission taxes affect relocation and competition.
- **Lambertini (2013):** survey of environmental quality competition.

Further Reading (Beyond the Book) (1/2)

- **Buchanan (1969)**, "External Diseconomies, Corrective Taxes, and Market Structure," *American Economic Review*: under monopoly, a tax equal to marginal damage can reduce welfare, since output is already too low.
- **Ryan (2012)**, "The Costs of Environmental Regulation in a Concentrated Industry," *Econometrica*: the 1990 Clean Air Act Amendments raised sunk entry costs in U.S. cement.
- **Gordon (1954)**, "The Economic Theory of a Common-Property Resource: The Fishery," *Journal of Political Economy*: open access dissipates the rent (Section 3.7).

Further Reading (Beyond the Book) (2/2)

- **Hardin (1968)**, "The Tragedy of the Commons," *Science*: each user of a shared resource bears only part of the cost of using it more, so the resource is overused.
- **Greenstone (2002)**, "The Impacts of Environmental Regulations on Industrial Activity," *Journal of Political Economy*: nonattainment status under the Clean Air Act cost polluting industries about 590,000 jobs in 1972–1987.
- **Fowlie, Holland, and Mansur (2012)**, "What Do Emissions Markets Deliver and to Whom?" *American Economic Review*: Southern California's NO_x market cut emissions by about 20% relative to comparable plants.

Section 3.11
Summary

Summary (1/3)

- **More firms:** more aggregate output and emissions, requiring a more stringent fee.
- **Pollution asymmetries:**
  - A uniform fee increases in firm 2's pollution intensity, but only achieves a second best when intensities differ.
  - Type-dependent fees keep the more polluting firm out and correct the cleaner firm's output, possibly with a subsidy.
- **Cost asymmetries:** high-cost firms produce fewer units; the uniform fee becomes more stringent as asymmetries increase, and with large asymmetries it drives the high-cost firm out.

Summary (2/3)

- **Nonlinear costs:** more convex costs lower output and emissions, requiring a less stringent fee. With economies of scale, the fee is a second best.
- **Product differentiation:** more differentiated products raise output and emissions, requiring a more stringent fee.
- **Green consumers:**
  - Aware consumers shift purchases from the brown to the green good, reducing emissions even without regulation.
  - The fee decreases in awareness when $d \geq \frac{5}{4}$; otherwise, it first increases and then decreases. It is a second best.

Summary (3/3)

- **Cost externalities:** the fee becomes more stringent when $d < 2$, and less stringent otherwise; a uniform fee is a second best.
- **Price competition:** firms produce more for a given fee, so the EA sets a more stringent fee, closer to marginal damage.
- **Functional forms:** each extension keeps linear demand and quadratic damage. Cutoffs depend on these forms, while the rule "fee = marginal damage − markup" holds more generally (Section 2.7).

Looking Ahead: Chapters 4 to 8

- Chapter 3 extended the two-stage regulatory game to richer market structures.
- The remaining chapters build on this framework:
  - **Chapter 4:** Regulation under Uncertainty.
  - **Chapter 5:** Sequential Competition and Regulation.
  - **Chapter 6:** Regulation When Firms Invest in Abatement.
  - **Chapter 7:** Environmentally Friendly Mergers?
  - **Chapter 8:** Managerial Incentives.

Thank You

Chapter 3: Environmental Regulation – Extensions

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 4

Regulation under Uncertainty

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 4: Outline

- **4.1** Introduction
- **4.2** Allowing for Uncertain Environmental Damages
- **4.3** Allowing for Uncertain Demand
- **4.4** Allowing for Uncertain Production Costs
- **4.5** Using Mechanisms to Infer Emissions
- **4.6** Case Studies on Uninformed Regulation
- **4.7** Related Literature on Uninformed Regulation
- **4.8** Green Certification and Greenwashing
- **4.9** Related Literature on Green Certification
- **4.10** Case Studies on Green Certification
- **4.11** Summary

Section 4.1
Introduction

Motivation: Why Uncertainty Matters

- Previous chapters assume that the regulator knows pollution severity, market demand, and firms' production costs.
- In practice, regulators often lack reliable data on these parameters.
- The environmental agency (EA) then sets policy based on *expected* values rather than realized ones.
- Which instrument, emission fees or quotas, generates the smaller welfare loss?

Guiding Questions

1. What types of uncertainty do regulators face in environmental policy?
2. How does uncertainty in pollution severity affect the choice of policy instruments?
3. Under what conditions are emission fees preferred over quotas?
4. How can mechanism design help regulators elicit truthful information from firms?
5. What are the risks and strategic implications of greenwashing?
6. How do signaling models explain the credibility of green certification?

Chapter Roadmap

- **Sections 4.2–4.4**: the EA does not observe pollution severity, demand, or production costs. We compare fees and quotas in each case.
- **Section 4.5**: a circular monitoring mechanism elicits firms' emissions.
- **Section 4.6**: case studies from Sweden, Turkey, Côte d'Ivoire, and Mexico.
- **Section 4.8**: green certification and greenwashing as a signaling game.

Section 4.2
Uncertain Environmental Damages

Building on Weitzman (1974)

- Sections 4.2–4.4 adapt the comparison between price and quantity instruments of **Weitzman (1974)**.
- Setting: a polluting Cournot duopoly where the regulator does not observe pollution severity, demand, or production costs.
- Why might the EA not observe pollution severity d? Firms know their production technology better than the regulator.
- Random events also matter: a hot and sunny day may increase the accumulation of air pollutants relative to a cold rainy day.

4.2.1 Modeling Uncertain Pollution

- **Binary types**:

$$d = \begin{cases} d_H & \text{with probability } p \\ d_L & \text{with probability } 1-p \end{cases}$$

  where $d_H > d_L$ and $p \in (0,1)$.
- **More types**: d takes $k \geq 2$ values $d_1 < d_2 < ... < d_k$ with probabilities $p_1, p_2, ..., p_k \in (0,1)$, where $\sum_{i=1}^{k} p_i = 1$.
- **Continuous types**: $d \in [a,b]$, where $b > a$ (e.g., $d \in [2,4]$), with cumulative distribution function $F(x) = \Pr\{d \leq x\}$. If $d \sim U[0,1]$, then $F(x) = x$.

4.2.2 Regulation under Uncertain Pollution: Timing

- Same setting as in section 2.3:
  - **Stage 1**: the EA sets a per-unit emission fee t.
  - **Stage 2**: every firm observes t and chooses its output $q_i(t)$.
- **New element**: the EA sets t based on the expected pollution severity, $E[d]$, not on the realized d.
- Second stage, as in section 2.3.1:

$$q_i(t) = \frac{1 - (c + t)}{3}$$

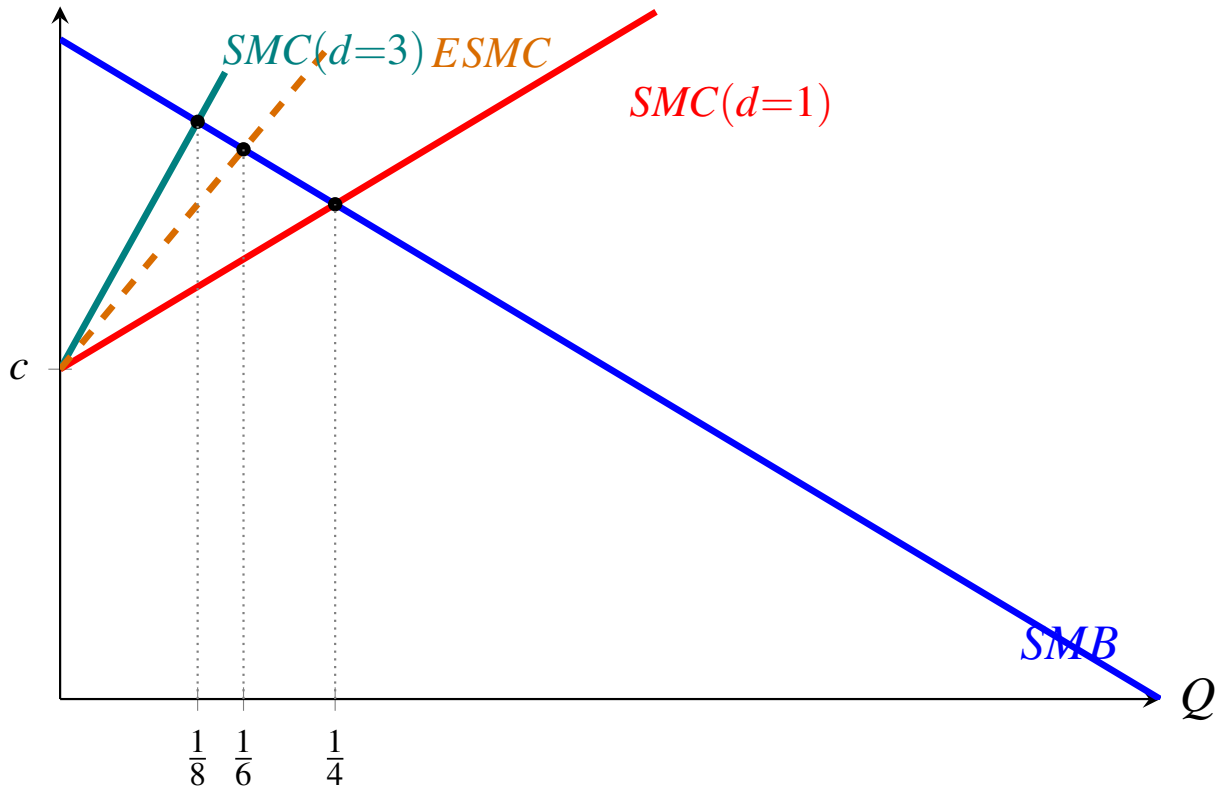
4.2.2 The EA's Problem

- In the first stage, the EA solves

$$\max_t \; W = \frac{(Q(t))^2}{2} + [(1 - Q(t))Q(t) - cQ(t)] - \frac{E[d]}{2}[Q(t)]^2 \tag{4.1}$$

- Fee revenue, $tQ(t)$, cancels with firms' fee payments. Consumer and producer surplus are observable; only the environmental damage is uncertain.
- The EA seeks an aggregate output where $SMB = 1 - Q$, unaffected by uncertainty, equals the *expected* social marginal cost, $ESMC = c + E[d]Q$ (Figure 4.1).

Figure 4.1: *SMB*, *SMC*, and *ESMC*



Parameters: $c = \frac{1}{2}$; $d = 1$ or $d = 3$, equally likely;
$ESMC = \frac{1}{2} + 2Q$.

4.2.2 The Fee under Uncertainty

- Differentiating (4.1) with respect to t and solving for t:

$$t^U = \frac{(1-c)(2E[d]-1)}{2(1+E[d])} \tag{4.2}$$

- Same as the fee under certainty, $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$, but evaluated at $E[d]$ instead of d.
- t^U is a fee if $E[d] > \frac{1}{2}$, and a subsidy otherwise. Every firm produces $\frac{1-c}{2(1+E[d])} > 0$ at this fee.
- t^U maximizes *expected* welfare, but it is a **second-best** policy: unless $d = E[d]$, the EA would set a different fee if it observed d.

4.2.3 Fee Comparison

- The **fee inefficiency** from the EA's lack of information is

$$t^U - t^* = \frac{3(1-c)(E[d]-d)}{2(1+d)(1+E[d])} \tag{4.3}$$

- $E[d] > d$: the EA **overtaxes** firms, $t^U > t^*$, inducing a socially insufficient output.
- $E[d] < d$: the EA **undertaxes** firms, $t^U < t^*$, inducing a socially excessive output.
- Only when $E[d] = d$ do both fees coincide.

4.2.3 Welfare Comparison

- Informed EA, setting t^*: $W^* = \frac{(1-c)^2}{2(1+d)}$. Uninformed EA, setting t^U:

$$W^U = \frac{(1-c)^2(1-d+2E[d])}{2(1+E[d])^2} \tag{4.4}$$

If $E[d] = d$, then $W^U = W^*$: uncertainty is welfare inconsequential.

- When $E[d] \neq d$, fee t^U generates a welfare loss

$$WL_t = W^* - W^U = \frac{(1-c)^2(E[d]-d)^2}{2(1+d)(1+E[d])^2} \tag{4.5}$$

which is unambiguously positive: the EA induces a socially insufficient or excessive output.

4.2.3 Comparative Statics of WL_t (1/2)

- WL_t is unambiguously decreasing in c:

$$\frac{\partial WL_t}{\partial c} = -\frac{(1-c)(E[d]-d)^2}{(1+d)(1+E[d])^2} < 0$$

- WL_t is increasing in $E[d]$ if

$$\frac{\partial WL_t}{\partial E[d]} = \frac{(1-c)^2(E[d]-d)}{(1+E[d])^3} > 0,$$

which holds if the EA overestimates pollution severity, $E[d] > d$.

4.2.3 Comparative Statics of WL_t (2/2)

- Effect of the true severity:

$$\frac{\partial WL_t}{\partial d} = -\frac{(1-c)^2(2+d+E[d])(E[d]-d)}{2(1+d)^2(1+E[d])^2}$$

- If $E[d] > d$: WL_t increases in $E[d]$ and decreases in d.
- If $E[d] < d$: WL_t decreases in $E[d]$ and increases in d.
- Intuitively, the loss grows when $E[d]$ moves further away from d.

4.2.4 What If the EA Regulates Quantities?

- The EA, still uncertain about d, sets a maximum output per firm. It induces the aggregate output where $SMB = ESMC$:

$$1 - Q = c + E[d]Q \tag{4.6}$$

so $Q^{SO,U} = \frac{1-c}{1+E[d]}$ and every firm produces $q^{SO,U} = \frac{1-c}{2(1+E[d])}$.

- Relative to $Q^{SO} = \frac{1-c}{1+d}$ under complete information:

$$Q^{SO} - Q^{SO,U} = \frac{(1-c)(E[d]-d)}{(1+d)(1+E[d])} \tag{4.7}$$

- $E[d] > d$: socially insufficient output, $Q^{SO} > Q^{SO,U}$ ($SMB > SMC$). $E[d] < d$: socially excessive output, $Q^{SO} < Q^{SO,U}$ ($SMB < SMC$).

4.2.4 Fees and Quotas Yield the Same Welfare Loss

- W^* and W^U coincide with those under fees, so

$$WL_Q = W^* - W^U = \frac{(1-c)^2(E[d]-d)^2}{2(1+d)(1+E[d])^2} = WL_t,$$

  with the same comparative statics.
- When the EA is uncertain only about pollution damage, neither instrument is preferable to the other.
- The equivalence also holds with nonlinear demand, costs, or damages: firms' output does not depend on the realized d, so a fee and a quota can induce the same output.
- With uncertainty about the social marginal benefit (e.g., demand), we can rank the two instruments.

Section 4.3
Uncertain Demand

4.3 Setup: Uncertain Demand

- The EA does not observe market demand:

$$\begin{aligned} p_H(Q) &= 1 - bQ \text{ with probability } p, \text{ and} \\ p_L(Q) &= a - bQ \text{ with probability } 1 - p \end{aligned}$$

- $b > 0$ is the common slope, and $a \in [c, 1]$ measures demand symmetry: $a = 1$ means identical demands.
- Firms observe the demand realization before choosing output; the EA does not observe it when setting its fee.
- A higher (lower) demand induces more (less) output and emissions, requiring a more (less) stringent fee.
- Unlike previous chapters ($b = 1$), we keep $b > 0$ generic: comparing the slope of *SMB* with that of *SMC* helps us rank fees and quotas.

4.3.1 Second Stage (Output)

- As in section 2.3.1, every firm produces

$$q_H(t) = \frac{1-(c+t)}{3b} \quad \text{and} \quad q_L(t) = \frac{a-(c+t)}{3b}$$

  under high and low demand, respectively.
- Output is higher under high demand, $q_H(t) > q_L(t)$.
- Aggregate output:

$$Q_H(t) = \frac{2(1-(c+t))}{3b} \quad \text{and} \quad Q_L(t) = \frac{2(a-(c+t))}{3b}$$

4.3.2 The EA's Problem

- In the first stage, the EA anticipates $Q_H(t)$ and $Q_L(t)$ and solves

$$\max_t \; W = \; p\left[\frac{bQ_H^2}{2} + (1 - bQ_H)Q_H - cQ_H - \frac{d}{2}Q_H^2\right] + (1-p)\left[\frac{bQ_L^2}{2} + (a - bQ_L)Q_L - cQ_L - \frac{d}{2}Q_L^2\right] \tag{4.8}$$

- Consumer surplus is $CS_H = \frac{bQ_H(t)^2}{2}$, and similarly under low demand; fee payments and fee revenue cancel.
- Damage d and costs are observable, but demand is not.

4.3.2 The Fee under Uncertain Demand

- Problem (4.8) is *not* equivalent to evaluating welfare at expected output, $E[Q(t)] = pQ_H(t) + (1-p)Q_L(t)$, and at the expected demand intercept, $E[a] = p + (1-p)a$, problem (4.9): consumer surplus and damages are nonlinear in output.
- Inserting $Q_H(t)$ and $Q_L(t)$ into (4.8), and differentiating with respect to t:

$$\frac{2[E[a]-c](2d-b) - 4(b+d)t}{9b^2} = 0 \tag{4.10}$$

- Solving for t:

$$t^{UD} = \frac{[E[a]-c](2d-b)}{2(b+d)} \tag{4.11}$$

4.3.2 When Is t^{UD} a Fee?

- $t^{UD} > 0$ if $E[a] > c$, which holds since 1 and a are weakly above c (one strictly), and if $d > \frac{b}{2}$ ($d > \frac{1}{2}$ when $b = 1$). Otherwise, it is a subsidy, as in chapter 2. A steeper demand makes $d > \frac{b}{2}$ more demanding.
- At t^{UD}, output is positive under high demand. Under low demand, every firm produces

$$q_L(t^{UD}) = \frac{3b(a-c) - p(1-a)(2d-b)}{6b(b+d)}$$

positive if and only if $a > \frac{3bc+p(2d-b)}{3b+p(2d-b)}$: the low demand is not too weak. Otherwise, the fee, set for an average demand, exceeds firms' margin. We assume that this condition holds.

4.3.2 Comparative Statics of t^{UD}

- Expected demand and production costs:

$$\frac{\partial t^{UD}}{\partial E[a]} = \frac{2d-b}{2(b+d)} > 0 \quad \text{and} \quad \frac{\partial t^{UD}}{\partial c} = -\frac{2d-b}{2(b+d)} < 0$$

  A higher p or a raises $E[a]$: the EA anticipates more output and emissions, setting a more stringent fee. A higher c has the opposite effect.

- Pollution severity and demand slope (for all $d > 0$):

$$\frac{\partial t^{UD}}{\partial d} = \frac{3b[E[a]-c]}{2(b+d)^2} > 0 \quad \text{and} \quad \frac{\partial t^{UD}}{\partial b} = -\frac{3d[E[a]-c]}{2(b+d)^2} < 0$$

  A steeper demand means less output and emissions, and a less stringent fee.

4.3.3 Fees When Demand Is Known

- If the EA knows that demand is high, $p = 1$:

$$t_H = \frac{(1-c)(2d-b)}{2(b+d)} \tag{4.12}$$

  which coincides with the fee in section 2.3.2 when $b = 1$.

- If the EA knows that demand is low, $p = 0$:

$$t_L = \frac{(a-c)(2d-b)}{2(b+d)}$$

4.3.3 Fee Differentials

- High demand:

$$t_H - t^{UD} = \frac{(1-p)(1-a)(2d-b)}{2(b+d)} \tag{4.13}$$

  which is positive when $d > \frac{b}{2}$.

- Low demand:

$$t_L - t^{UD} = -\frac{p(1-a)(2d-b)}{2(b+d)} \tag{4.14}$$

  which is negative when $d > \frac{b}{2}$.

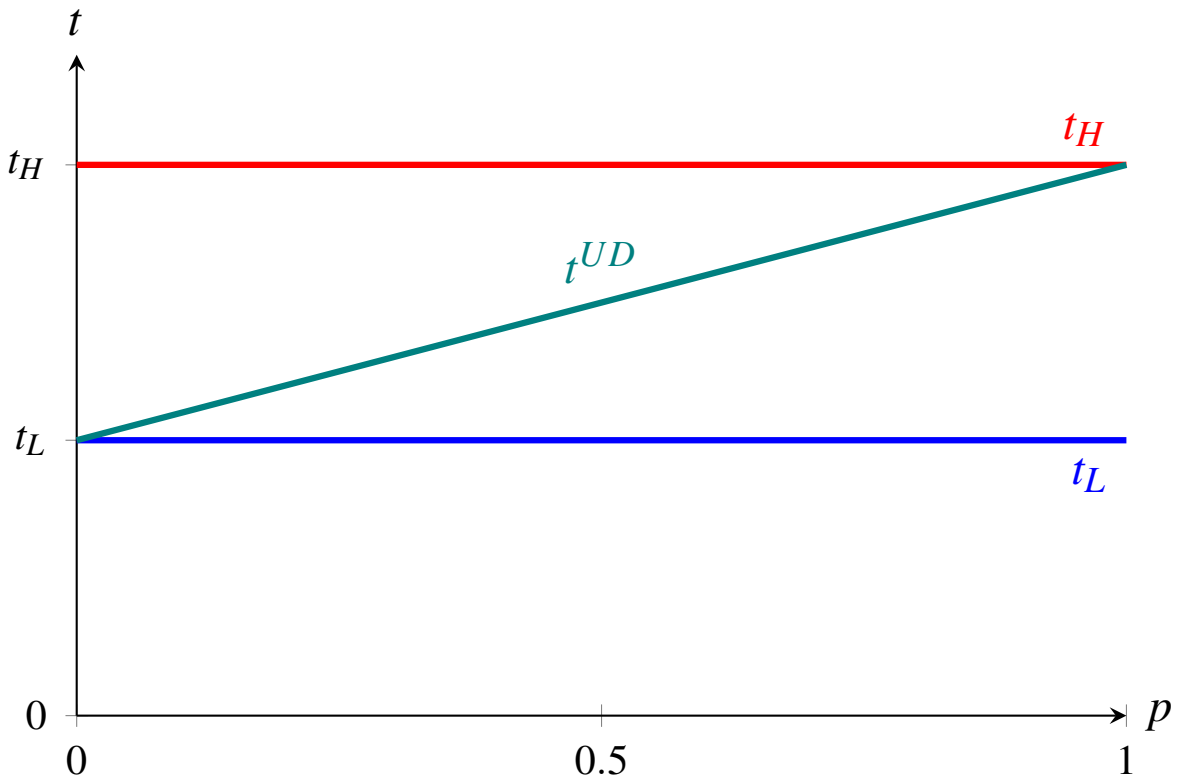
4.3.3 Fee Ranking

- Combining (4.13) and (4.14):

$$t_L < t^{UD} < t_H \tag{4.15}$$

  for all $d > \frac{b}{2}$.
- When $d < \frac{b}{2}$, all three fees are subsidies and the ranking reverses.
- Uncertainty leads the EA to set an intermediate fee: not as stringent as when demand is high, and not as lax as when demand is low.

Figure 4.2: Fees t_H, t_L, and t^{UD}



Parameters: $c = \frac{1}{2}$, $d = 2$, $b = 1$, and $a = \frac{3}{4}$. $t^{UD} = t_L$ at $p = 0$ and $t^{UD} = t_H$ at $p = 1$; for $p \in (0, 1)$, $t_L < t^{UD} < t_H$, and t^{UD} approaches t_H as p increases.

4.3.3 Welfare Comparison: Informed EA

- An informed EA sets t_H when demand is high, yielding welfare $\frac{(1-c)^2}{2(b+d)}$; and t_L when demand is low, yielding $\frac{(a-c)^2}{2(b+d)}$.
- Expected welfare under certainty:

$$\overline{W} = \frac{p(1-c)^2 + (1-p)(a-c)^2}{2(b+d)} \tag{4.16}$$

- With $b = 1$ and $p = 1$, it becomes $\frac{(1-c)^2}{2(1+d)}$, as in section 4.2.3.

4.3.3 Welfare Loss from Fees

- An uninformed EA sets t^{UD} in both states:

$$W^{UD} = \overline{W} - \frac{p(1-p)(1-a)^2(2d-b)^2}{18b^2(b+d)} \tag{4.17}$$

- Welfare loss due to regulatory uncertainty:

$$WL_t = \overline{W} - W^{UD} = \frac{p(1-p)(1-a)^2(2d-b)^2}{18b^2(b+d)} \tag{4.18}$$

- $WL_t = 0$ when the EA faces no uncertainty ($p = 0$ or $p = 1$), when both demands coincide ($a = 1$), or when $d = \frac{b}{2}$, where the fee is zero in both states. Otherwise, $WL_t > 0$, and it increases in $1 - a$.
- t^{UD} is a **second-best** policy: the first best would require a different fee in each demand state.

4.3.4 What If the EA Regulates Quantities?

- The EA chooses a single aggregate output Q for both demand states, problem (4.19), which simplifies to

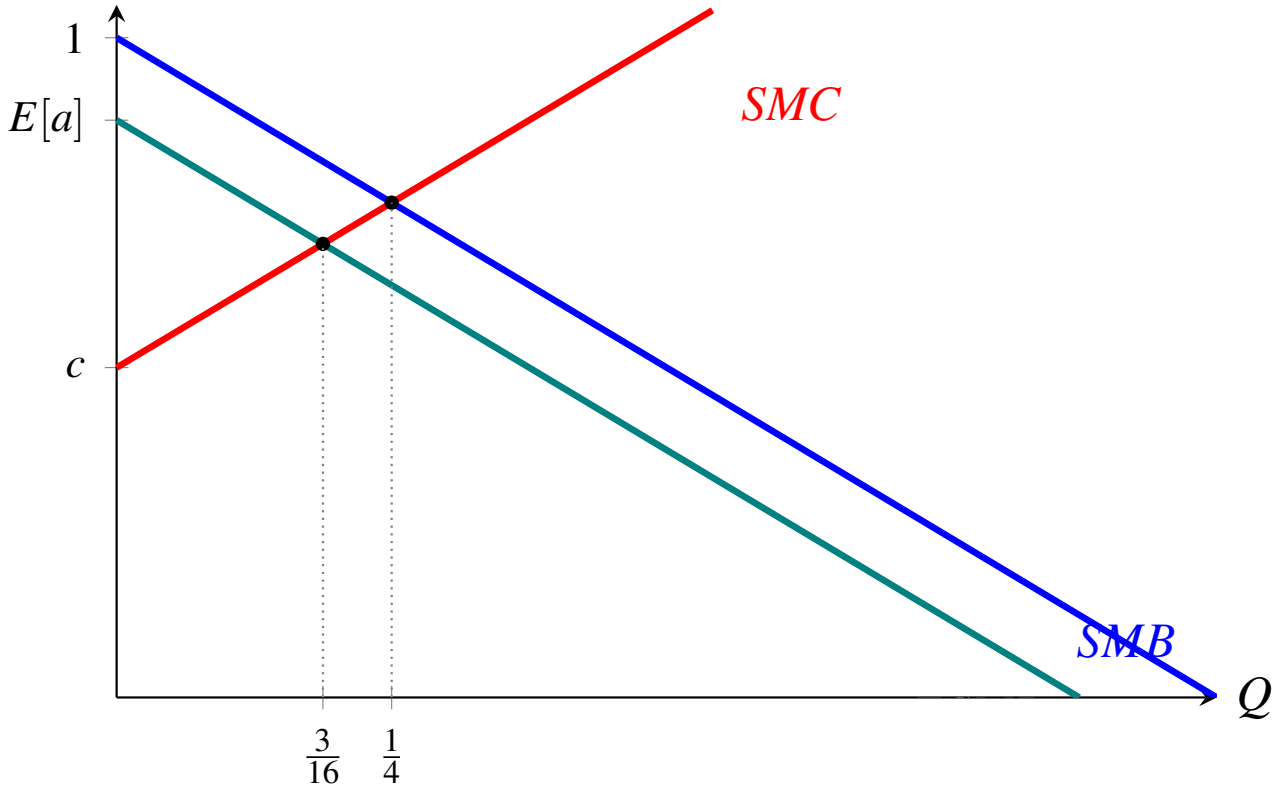
$$\max_{Q \geq 0} \; W = \frac{bQ^2}{2} + [(E[a] - bQ)Q - cQ] - \frac{d}{2}Q^2 \qquad (4.20)$$

- Differentiating with respect to Q:

$$\underbrace{E[a] - bQ}_{ESMB} \; = \; \underbrace{c + dQ}_{SMC} \qquad (4.21)$$

- SMC is unaffected; $ESMB$ has intercept $E[a]$ and slope $-b$ (it becomes $SMB = 1 - bQ$ if $p = 1$), so it lies below SMB, with the same slope.

Figure 4.3: *SMB*, *SMC*, and *ESMB*



Parameters: $b = d = 1$ and $c = \frac{1}{2}$, as in chapter 2. Since *ESMB* lies below *SMB*, $Q^{SO,UD} < Q^{SO}$; a higher p shifts *ESMB* upward, closer to *SMB*.

4.3.4 Socially Optimal Output

- Solving (4.21) for Q:

$$Q^{SO,UD} = \frac{E[a] - c}{b + d} \tag{4.22}$$

- If $p = 1$: $Q^{SO} = \frac{1-c}{b+d}$; with $b = 1$, $Q^{SO} = \frac{1-c}{1+d}$, as under certainty.
- Output inefficiency:

$$Q^{SO} - Q^{SO,UD} = \frac{(1 - a)(1 - p)}{b + d} \tag{4.23}$$

which is positive when $a < 1$ and $p < 1$.

4.3.4 Welfare Loss from Quotas

- **Fees**: $t^{UD} < t_H$, so output and pollution are socially excessive when demand turns out to be high. **Quotas**: *less* output than when demand is known to be high, since $ESMB < SMB$.
- Under uncertainty, $W_Q^{UD} = \frac{(E[a]-c)^2}{2(b+d)}$. Relative to $\overline{W}$ in (4.16),

$$WL_Q = \overline{W} - W_Q^{UD} = \frac{p(1-p)(1-a)^2}{2(b+d)} \tag{4.24}$$

  which differs from WL_t in (4.18).
- We assume the quota binds in both states, $\frac{2(a-c)}{3b} \geq \frac{E[a]-c}{b+d}$.

4.3.5 Welfare Loss Comparison

- Both policies are suboptimal. Which one is "less bad"?
- Difference in welfare losses:

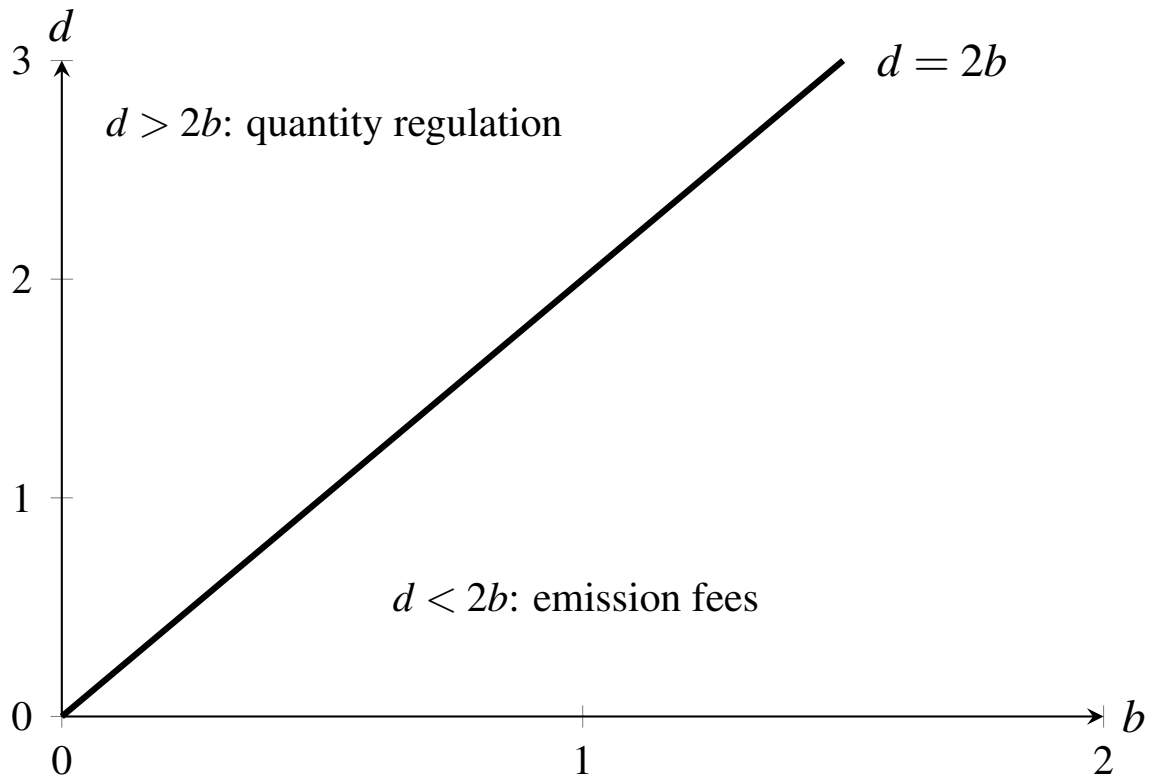
$$\Delta WL = WL_t - WL_Q = \frac{2p(1-a)^2(1-p)(d-2b)}{9b^2} \quad (4.25)$$

- $\Delta WL > 0$ if and only if

$$d > 2b \quad (4.26)$$

which is more demanding than $d > \frac{b}{2}$, since $2b > \frac{b}{2}$.

Figure 4.4: Welfare Ranking of Policy Tools



$d > 2b$ (high d or low b): $WL_t > WL_Q$, so quantity regulation is preferred. $d < 2b$ (low d or high b): emission fees are preferred.

Table 4.1: Welfare Losses ($\times$1,000)

Parameters: $c = p = \frac{1}{2}$ and $a = \frac{4}{5}$.

d	b	WL_t	WL_Q	ΔWL	*Preferred policy*
2	1	1.67	1.67	0	Indifferent
2	2	0.14	1.25	-1.11	Emission fees
3	1	3.47	1.25	2.22	Quantity regulation
3	2	0.44	1.00	-0.56	Emission fees
3	3	0.09	0.83	-0.74	Emission fees
4	1	5.44	1.00	4.44	Quantity regulation
4	2	0.83	0.83	0	Indifferent
4	3	0.22	0.71	-0.49	Emission fees

$d = 2b$: indifferent; $d < 2b$: emission fees; $d > 2b$: quantity regulation.

4.3.5 Comparative Statics of ΔWL

- Pollution severity and demand slope:

$$\frac{\partial \Delta WL}{\partial d} = \frac{2p(1-a)^2(1-p)}{9b^2} > 0$$
$$\frac{\partial \Delta WL}{\partial b} = -\frac{4p(1-a)^2(1-p)(d-b)}{9b^3}$$

  where the latter is negative if and only if $d > b$.

- Three regions:
  - (i) $d < b$: fees preferred, and ΔWL increasing in b.
  - (ii) $2b \geq d > b$: fees preferred, and ΔWL decreasing in b.
  - (iii) $d > 2b$: quotas preferred, and ΔWL decreasing in b.

4.3.5 Connection to Weitzman (1974)

- This result is in line with Weitzman (1974).
- The cutoff is $d = 2b$ rather than $d = b$: under a fee, duopolists adjust aggregate output by only $\frac{2}{3b}$ per unit change in the demand intercept, rather than by $\frac{1}{b}$ as competitive firms do.
- If *SMB* is relatively steep compared to *SMC*, price instruments (e.g., taxes) generate smaller welfare losses.
- If *SMB* is relatively flat compared to *SMC*, quantity instruments are more effective.

Section 4.4
Uncertain Production Costs

4.4 Setup: Uncertain Costs

- The EA does not observe firms' marginal cost:

$$\text{marginal cost} = \begin{cases} c & \text{with probability } p \\ \alpha c & \text{with probability } 1-p \end{cases}$$

- $\alpha \in [0,1]$ measures similarity: $\alpha = 1$ means both costs coincide; $\alpha = 0$ means the low cost is zero.
- Both firms use the same technology and have the same marginal cost.
- A lower cost means more output and emissions, requiring a more stringent fee.

4.4.1 Second Stage (Output)

- As in section 2.3.1, every firm produces

$$q_H(t) = \frac{1-(c+t)}{3b} \quad \text{and} \quad q_L(t) = \frac{1-(\alpha c+t)}{3b}$$

  under high and low costs, respectively.
- Output is higher when costs are low, $q_L(t) > q_H(t)$.
- Aggregate output:

$$Q_H(t) = \frac{2(1-(c+t))}{3b} \quad \text{and} \quad Q_L(t) = \frac{2(1-(\alpha c+t))}{3b}$$

4.4.2 Regulation under Uncertain Costs

- The EA anticipates $Q_H(t)$ and $Q_L(t)$ and maximizes expected welfare, problem (4.27).
- Differentiating with respect to t:

$$\frac{2[1-E[c]](2d-b)-4(b+d)t}{9b^2}=0 \qquad (4.28)$$

  where $E[c] \equiv pc+(1-p)\alpha c$ is the expected marginal cost.
- Solving for t:

$$t^{UC}=\frac{[1-E[c]](2d-b)}{2(b+d)} \qquad (4.29)$$

4.4.2 Is Output Positive under High Costs?

- $t^{UC} > 0$ if $1 > E[c]$, which holds since $c < 1$ and $\alpha c \leq c$, and if $d > \frac{b}{2}$.
- Under high costs, every firm produces $\frac{1-c-t^{UC}}{3b}$, which is positive if and only if $(1 - E[c])(2d - b) < 2(b + d)(1 - c)$.
- This can fail when pollution is severe and high costs are unlikely (low p) or much higher than low costs (low α). We assume that it holds.
- As t^{UD}, fee t^{UC} is a second-best policy.

4.4.2 Comparative Statics of t^{UC}

- More severe pollution and steeper demand:

$$\frac{\partial t^{UC}}{\partial d} = \frac{3b[1 - E[c]]}{2(b+d)^2} > 0 \quad \text{and} \quad \frac{\partial t^{UC}}{\partial b} = -\frac{3d[1 - E[c]]}{2(b+d)^2} < 0$$

- Higher expected cost:

$$\frac{\partial t^{UC}}{\partial E[c]} = -\frac{2d - b}{2(b+d)} < 0$$

- $E[c]$ increases when high costs are more likely (higher p) or costs are more symmetric (higher α). The intuition is analogous to that in section 4.3.2.

4.4.3 Fee Differentials

- Fee if the EA knows costs are high: $t_H = \frac{(1-c)(2d-b)}{2(b+d)}$, the fee in chapter 2 when $b = 1$.
- High costs:

$$t_H - t^{UC} = -\frac{c(1-p)(1-\alpha)(2d-b)}{2(b+d)} \tag{4.30}$$

- Low costs:

$$t_L - t^{UC} = \frac{cp(1-\alpha)(2d-b)}{2(b+d)} \tag{4.31}$$

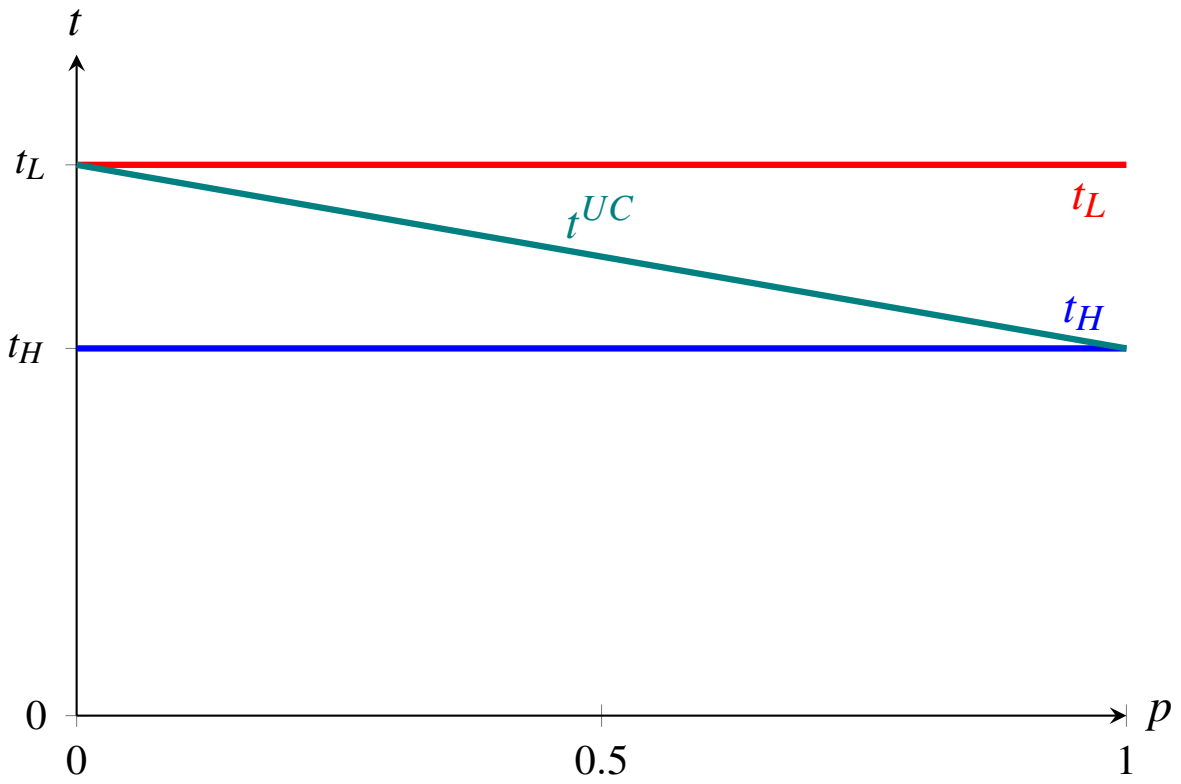
4.4.3 Fee Ranking

- Combining (4.30) and (4.31):

$$t_H < t^{UC} < t_L \tag{4.32}$$

  for all $d > \frac{b}{2}$. When $d < \frac{b}{2}$, all three fees are subsidies and the ranking reverses.
- Intermediate fee: not as stringent as when costs are low, and not as lax as when costs are high.
- Low costs, like high demand, mean more output and emissions, so t_L is now the most stringent fee.

Figure 4.5: Fees t_L, t_H, and t^{UC}



Parameters: $c = \alpha = \frac{1}{2}$, $b = 1$, and $d = 2$. $t^{UC} = t_L$ at $p = 0$ and $t^{UC} = t_H$ at $p = 1$; for $p \in (0, 1)$, $t_H < t^{UC} < t_L$, and t^{UC} approaches t_H as p increases.

4.4.3 Welfare Comparison

- A fully informed EA sets t_H or t_L, achieving

$$\overline{W} = pW_H + (1-p)W_L = \frac{p(1-c)^2 + (1-p)(1-\alpha c)^2}{2(b+d)} \tag{4.33}$$

- The uninformed EA sets the single fee t^{UC}:

$$WL_t = \overline{W} - E[W(t^{UC})] = \frac{p(1-p)c^2(1-\alpha)^2(2d-b)^2}{18b^2(b+d)} \tag{4.34}$$

which is unambiguously positive.

4.4.4 What If the EA Regulates Quantities?

- The EA chooses a single aggregate output Q, problem (4.35), which simplifies to

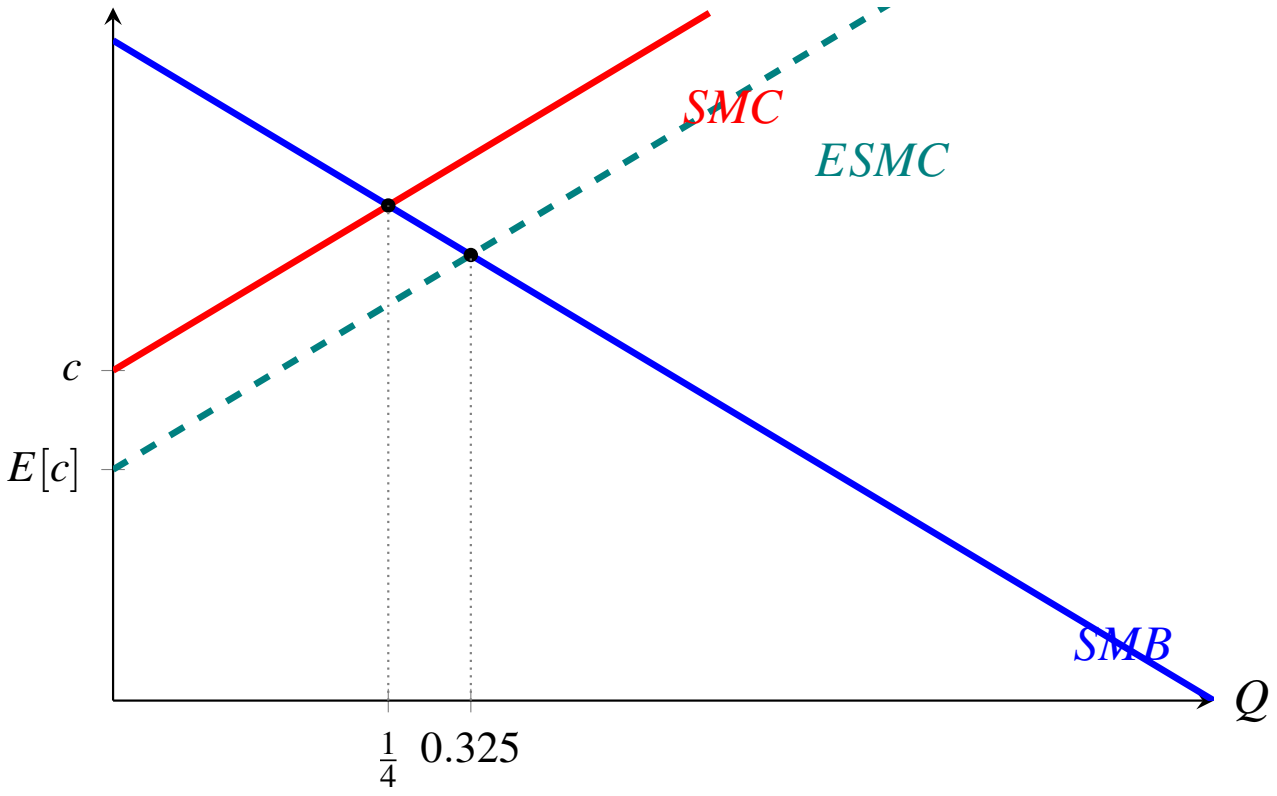
$$\max_{Q \geq 0} \; W = \frac{bQ^2}{2} + [(1 - bQ)Q - E[c]Q] - \frac{d}{2}Q^2 \qquad (4.36)$$

- Differentiating with respect to Q:

$$\underbrace{1 - bQ}_{SMB} \; = \; \underbrace{E[c] + dQ}_{ESMC} \qquad (4.37)$$

- *SMB* is unaffected, but the intercept of *ESMC* shifts down ($E[c] < c$ for $p < 1$ and $\alpha < 1$), so $Q^{SO,UC} > Q^{SO}$ (Figure 4.6).

Figure 4.6: *SMB*, *SMC*, and *ESMC* (Uncertain Costs)



4.4.4 Socially Optimal Output

- Solving (4.37) for Q:

$$Q^{SO,UC} = \frac{1 - E[c]}{b + d} \tag{4.38}$$

- Output inefficiency:

$$Q^{SO} - Q^{SO,UC} = \frac{E[c] - c}{b + d} \tag{4.39}$$

  which is negative since $E[c] < c$.
- Aggregate output is higher with than without cost uncertainty.

4.4.4 Welfare Loss from Quotas

- Against the same fully informed benchmark $\overline{W}$:

$$WL_Q = \overline{W} - E[W_Q] = \frac{p(1-p)c^2(1-\alpha)^2}{2(b+d)} \tag{4.40}$$

  which differs from WL_t in (4.34).
- We assume the quota binds in both cost states: $\frac{2(1-c)}{3b} \geq \frac{1-E[c]}{b+d}$.

4.4.5 Welfare Loss Comparison

- Difference in welfare losses:

$$\Delta WL = WL_t - WL_Q = \frac{2pc^2(1-\alpha)^2(1-p)(d-2b)}{9b^2} \tag{4.41}$$

- $\Delta WL > 0$ if and only if

$$d > 2b \tag{4.42}$$

- Same condition as under demand uncertainty, in line with Weitzman's (1974) results: fees if $d < 2b$, quantity regulation if $d > 2b$.

Section 4.5
Using Mechanisms to Infer Emissions

4.5 Can the EA Learn Firms' Emissions?

- Firms may want to understate their emissions to lower their tax burden.
- Duggan and Roberts (2002): the EA can design an incentive scheme for firms to truthfully report emissions, and then set the optimal fee.
- This is a **direct revelation mechanism**: the uninformed EA designs the rules of a game to induce informed firms to report truthfully.
- For an introduction to mechanism design, see Mas-Colell et al. (1995) and Börgers (2015).

4.5 Setup

- Two polluting firms; profit $\pi_i(q_i) = \ln q_i$, increasing in output at a decreasing rate.
- Every unit of output generates one unit of pollutants, with damage

$$Env = \frac{d_1}{2} q_1^2 + \frac{d_2}{2} q_2^2 \tag{4.43}$$

- No interaction effects: $q_1^2 + q_2^2$ rather than $(q_1 + q_2)^2$. With $q_1 = 3$ and $q_2 = 4$: 25 rather than 49.
- Welfare: $W = PS - Env$, abstracting from consumer surplus.

4.5.1 Complete Information: Firms' Output

- Benchmark: the EA observes pollution and sets a fee t_i on firm i (as in section 3.3, with $d_1 \neq d_2$).
- Second stage:

$$\max_{q_i \geq 0} \ln q_i - t_i q_i \tag{4.44}$$

- First-order condition $\frac{1}{q_i} - t_i = 0$ yields

$$q_i(t_i) = \frac{1}{t_i} \tag{4.46}$$

which is decreasing in the fee t_i.

4.5.1 Complete Information: Social Optimum

- Fees are transfers, so welfare is $\ln q_1 + \ln q_2 - \frac{d_1}{2} q_1^2 - \frac{d_2}{2} q_2^2$.
- Differentiating with respect to q_i:

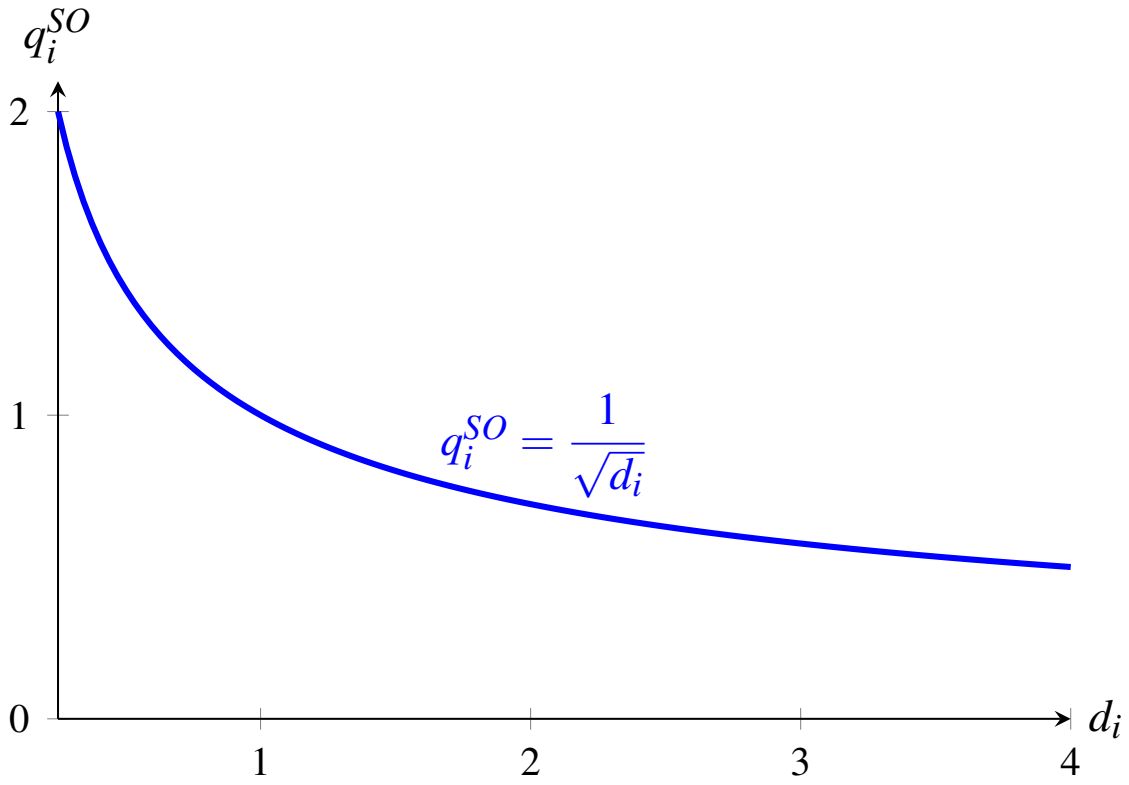
$$\frac{1}{q_i} = d_i q_i \quad (SMB = SMC) \tag{4.47}$$

- Socially optimal pollution:

$$q_i^{SO} = \frac{1}{\sqrt{d_i}} \tag{4.48}$$

which is decreasing in d_i (Figure 4.7).

Figure 4.7: Optimal Emissions, Complete Information



4.5.1 The Optimal Fee

- Setting $q_i(t_i) = q_i^{SO}$:

$$\frac{1}{t_i} = \frac{1}{\sqrt{d_i}} \tag{4.49}$$

- Emission fee:

$$t_i = \sqrt{d_i} \tag{4.50}$$

- Intuitively, the fee equals the marginal damage of firm i's emissions at the socially optimal output, $d_i q_i^{SO} = \sqrt{d_i}$.

4.5.2 Incomplete Information: Circular Monitoring

- The EA cannot observe emissions, but every firm observes its own and its rival's. Firm 1 reports firm 2's emissions, and vice versa. Fees:

$$t_1 = \frac{\partial Env(\overline{q}_1, q_2)}{\partial q_1} \quad \text{and} \quad t_2 = \frac{\partial Env(\overline{q}_2, q_1)}{\partial q_2} \tag{4.51}$$

  where $\overline{q}_i$ is firm i's emissions, as reported by its rival.
- Penalty $(\overline{q}_j - q_j)^2$ for misreporting (Varian 1994).
- Misreporting does not affect firm i's own fee but can lead to the penalty, so every firm i truthfully reports its rival's emissions, q_j.

4.5.2 Equilibrium Output

- Every firm i solves

$$\max_{q_i \geq 0} \ \pi_i(q_i) - t_i q_i - (\overline{q}_j - q_j)^2 \tag{4.52}$$

- With truthful reports, $\overline{q}_i = q_i$, the fee is $t_i = d_i q_i$, so

$$\max_{q_i \geq 0} \ \ln q_i - d_i q_i^2 \tag{4.54}$$

- First-order condition $\frac{1}{q_i} = 2 d_i q_i$ yields

$$q_i^* = \frac{1}{\sqrt{2 d_i}} \tag{4.56}$$

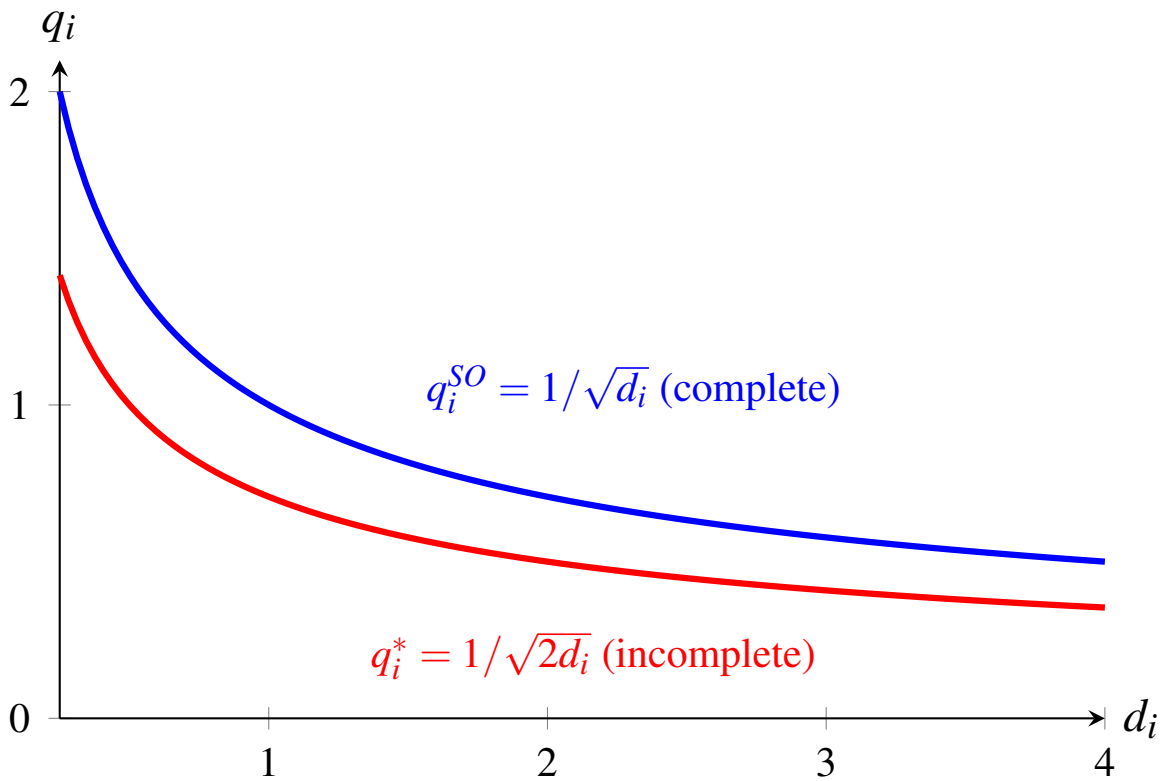
4.5.2 Output Inefficiency

- q_i^* is decreasing in d_i, and lower than $q_i^{SO} = \frac{1}{\sqrt{d_i}}$.
- Output difference:

$$OD_i(d_i) = \frac{1}{\sqrt{d_i}} - \frac{1}{\sqrt{2d_i}} = \frac{\sqrt{2}-1}{\sqrt{2d_i}} \simeq \frac{0.29}{\sqrt{d_i}} \tag{4.57}$$

- OD_i decreases in d_i: the inefficiency shrinks when pollution becomes more severe.

Figure 4.8: Emissions, Complete vs. Incomplete Information



4.5.2 Where Does the Inefficiency Come From?

- *Not* from the EA's lack of information: firms report truthfully, so the EA learns every firm's emissions, as in section 4.5.1.
- It comes from the **design of the fee**: $t_i = d_i \overline{q}_i$ depends on firm i's own emissions, which its rival reports truthfully.
- One more unit of output also increases the fee on all of firm i's units: its effective marginal charge is $2d_i q_i$, twice the marginal damage $d_i q_i$.
- As a result, every firm produces too little.

4.5.2 Restoring the First Best

- The EA charges half of this fee, $\frac{d_i}{2}\overline{q}_i$, per unit. Firm i then solves $\max_{q_i \geq 0} \ln q_i - \frac{d_i}{2} q_i^2$, which yields $q_i^{SO} = \frac{1}{\sqrt{d_i}}$.
- Circular monitoring elicits truthful information, and an appropriately designed fee uses this information to implement the first best.
- **Tax burden**: under fee $t_i = d_i \overline{q}_i$, the fee on each firm is $t_i = \frac{\sqrt{d_i}}{\sqrt{2}}$, so

$$t_i q_i = \frac{\sqrt{d_i}}{\sqrt{2}} \frac{1}{\sqrt{2 d_i}} = \frac{1}{2} \tag{4.58}$$

Section 4.6
Case Studies on Uninformed Regulation

Sweden: Funding an Unproven Technology

- In October 2024, the Swedish Energy Agency granted SaltX Technology SEK 12.5 million under its pilot and demonstration program, to test electric calcination for cement production at industrial scale.
- The agency based its support on the potential for large emission reductions, but the technology was not yet in commercial use, so its long-run costs and emissions could only be projected (SaltX Technology 2024; Swedish Energy Agency 2025).
- *Connection to the model*: as in sections 4.2–4.5, the regulator sets policy without observing damages, costs, or emissions; this grant was made under uncertain costs, as in section 4.4.

Turkey and Côte d'Ivoire: Collecting Information

- **Turkey**: the Public Oversight, Accounting and Auditing Standards Authority made sustainability reporting mandatory for large companies from 2024, requiring information that authorities did not previously collect (Public Oversight, Accounting and Auditing Standards Authority 2023).
- **Côte d'Ivoire**: in July 2025, the government launched a sustainability-linked financing framework that ties borrowing costs to verifiable targets, such as non-hydro renewable capacity and forest cover (World Bank 2025b).
- *Connection to the model*: the Turkish requirement relates to section 4.5, where the regulator needs information about firms' emissions; the Ivorian framework ties payments to verifiable outcomes.

Mexico: Voluntary Agreements with Tanneries

- During the 1980s and 1990s, environmental authorities negotiated four voluntary agreements with leather tanneries in León, Guanajuato.
- Authorities lacked the legal, institutional, and monitoring capacity to enforce conventional standards. The agreements produced few concrete results (Blackman and Sisto 2006).
- Marcus et al. (2011) discuss how firms can respond when environmental regulation is uncertain.
- *Connection to the model*: the agreements relate to section 4.5. The cases do not test the models' predictions about fees and quotas.

Section 4.7
Related Literature on Uninformed Regulation

4.7 Prices versus Quantities and Mechanism Design

- **Weitzman (1974)**: fees preferred when marginal abatement costs are steeper than marginal damages.
- Kolstad (1986): incentives vs. command-and-control, empirically.
- Fishelson (1976), Kolstad (1996), Requate (2005): instrument design.
- Stranlund and Ben-Haim (2008): Knightian uncertainty.
- Duggan and Roberts (2002): direct revelation mechanisms.
- Kwerel (1977): licensing with subsidies; Montero (2008): permit auctions.
- Carson and LaRiviere (2018): unknown emissions-to-damage link.

4.7 Firm Behavior and Market Structure

- Desjardins and Sinclair-Desgagné (2025): internal carbon pricing.
- Aldy and Stavins (2012): carbon taxes and cap-and-trade.
- Wang et al. (2021): principal-agent pollution cost control.
- Ino and Matsumura (2021): emission intensity-based pricing.
- Birg and Voßwinkel (2017), Lange and Requate (1999): product differentiation and market structure.
- Absi et al. (2013): production planning under emission constraints.
- Farzin and Kort (2000): regulatory uncertainty and abatement investment.

Further Reading (Beyond the Book)

- **Pizer (2002)**, "Combining Price and Quantity Controls to Mitigate Global Climate Change," *J. Public Econ.*: simulated expected gains from the optimal price policy are five times those of the quantity policy.
- **Davis (2008)**, "The Effect of Driving Restrictions on Air Quality in Mexico City," *JPE*: no evidence that *Hoy No Circula* (1989) improved air quality; the fleet grew and shifted toward high-emitting vehicles.
- **Spence (1973)**, "Job Market Signaling," *QJE*: a costly signal separates types when its cost differs across types, the logic of certification in section 4.8.
- **Eichholtz, Kok, and Quigley (2010)**, "Doing Well by Doing Good? Green Office Buildings," *AER*: Energy Star or LEED offices rent for about 3% more, sell for about 16% more.

Section 4.8
Green Certification and Greenwashing

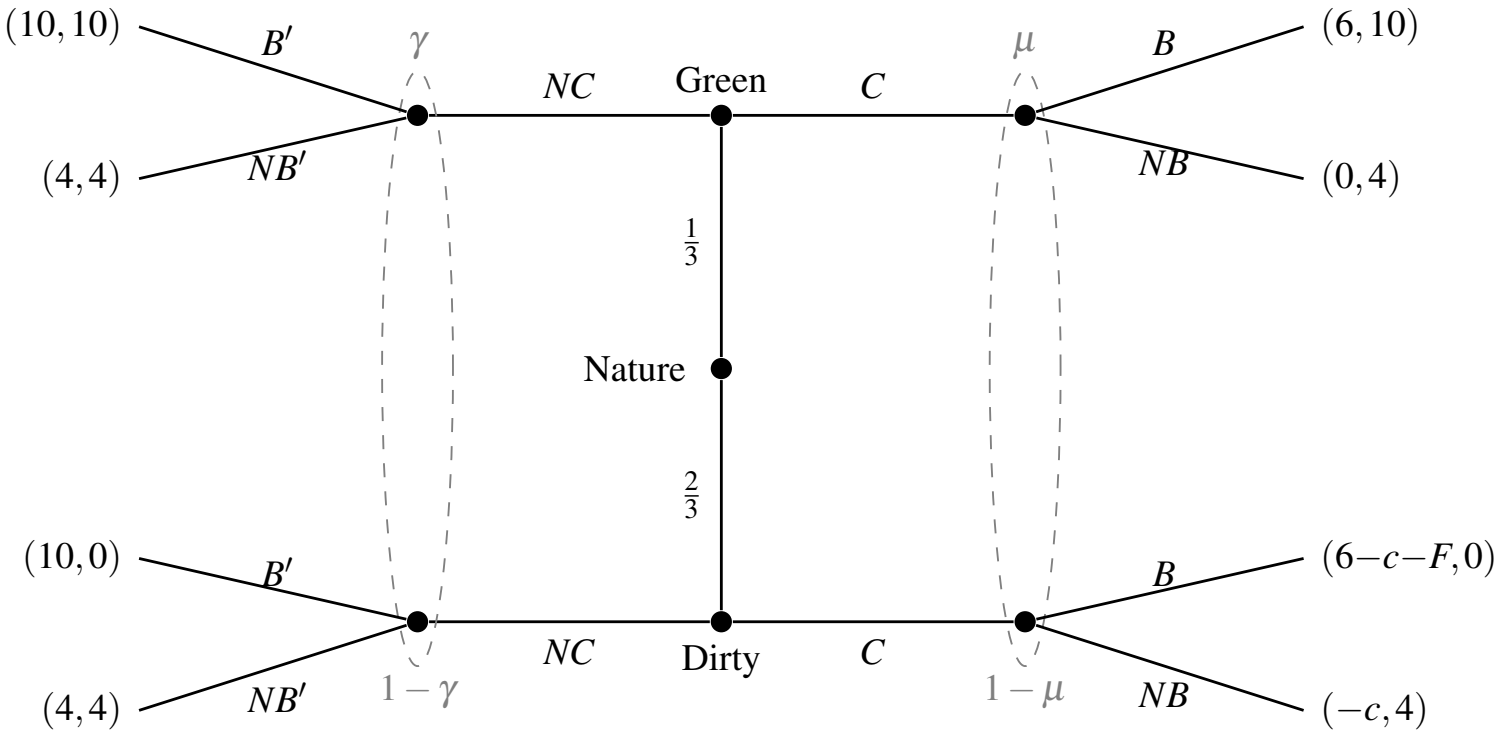
4.8 Green Certification and Greenwashing

- **Green certification**: third-party verification that a building or product meets an environmental standard, e.g., Energy Star (appliances), USDA Organic (food), the Global Organic Textile Standard (textiles), and the Forest Stewardship Council (wood products).
- Do firms certify to credibly signal environmental quality, or to obscure it, particularly when all firms in an industry certify?
- **Greenwashing**: firms seek reputational gains without significant changes to their production practices.
- Models of green certification often use signaling games under asymmetric information; Garrido et al. (2020) let firms choose between certified and uncertified claims, and certification can be mandatory.

4.8.1 Modeling Certification as a Signal

- We focus on **pure-strategy PBEs**.
- The firm privately observes whether its product is green (G), with probability $\frac{1}{3}$, or dirty (D), with probability $\frac{2}{3}$.
- The firm certifies its product (C) or not (NC).
- The consumer observes this message, updates her beliefs, and buys (B) or does not buy (NB).

Figure 4.9: Certification as a Signal



4.8.1 Consumer Beliefs

- After observing certification, by Bayes' rule:

$$\mu = \Pr\{G|C\} = \frac{\Pr(G) \times \Pr\{C|G\}}{\Pr(C)} \tag{4.59}$$

  Among all certified products, what share comes from green firms?

- $\mu > \frac{1}{3}$: certification makes the consumer more confident that the product is green; $\mu < \frac{1}{3}$: the opposite.
- After observing no certification:

$$\gamma = \Pr\{G|NC\} \quad \text{and} \quad 1 - \gamma = \Pr\{D|NC\} \tag{4.60}$$

4.8.1 Firm Payoffs

- The green firm's certification cost is 4: $10 - 6 = 4$ if the consumer buys, and $4 - 0 = 4$ if she does not.
- For the dirty firm, conditional on a purchase:

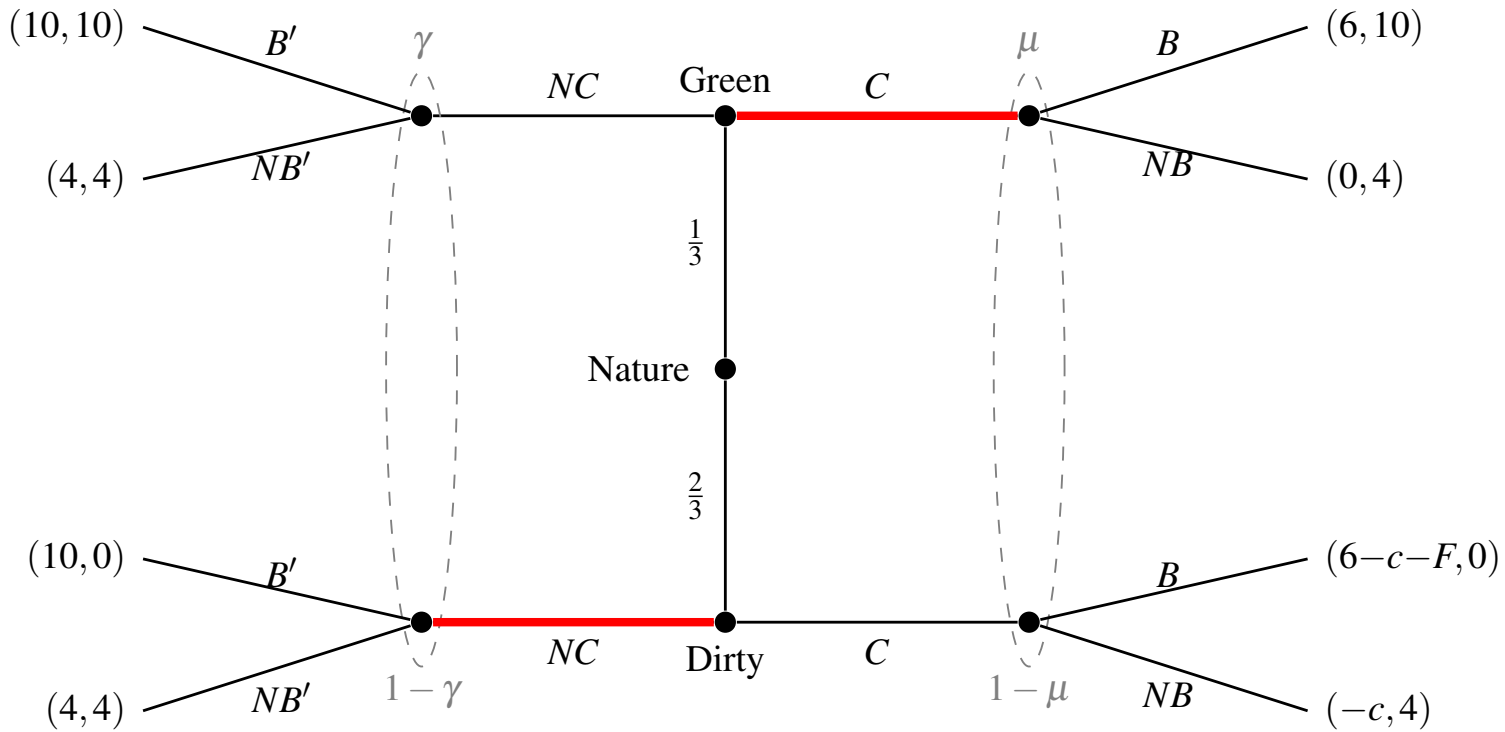
$$10 - (6 - c - F) = 4 + c + F \tag{4.61}$$

- $c \geq 0$: the dirty firm's additional certification cost relative to the green firm.
- $F \geq 0$: the expected fine if consumers purchase a good with a false certification.

4.8.1 Interpreting c and F; Consumer Payoffs

- $c = F = 0$: both firm types earn the same payoffs (6 or 0 if certified; 10 or 4 if not). $F = 0$ and $c > 0$: the dirty firm faces a longer and more costly certification process. $c = 0$ and $F > 0$: consumer organizations may take the dirty firm to court after its certified product is found to be dirty.
- If the dirty firm does not certify, its payoffs are unaffected by c or F.
- **Consumer**: buying yields 10 if the product is green and 0 if it is dirty; not buying yields 4. Certification has no intrinsic value: it only serves as a signal.
- **Roadmap**: separating PBE $C^G NC^D$, then pooling PBEs.

Figure 4.10: Separating Strategy Profile $C^G NC^D$

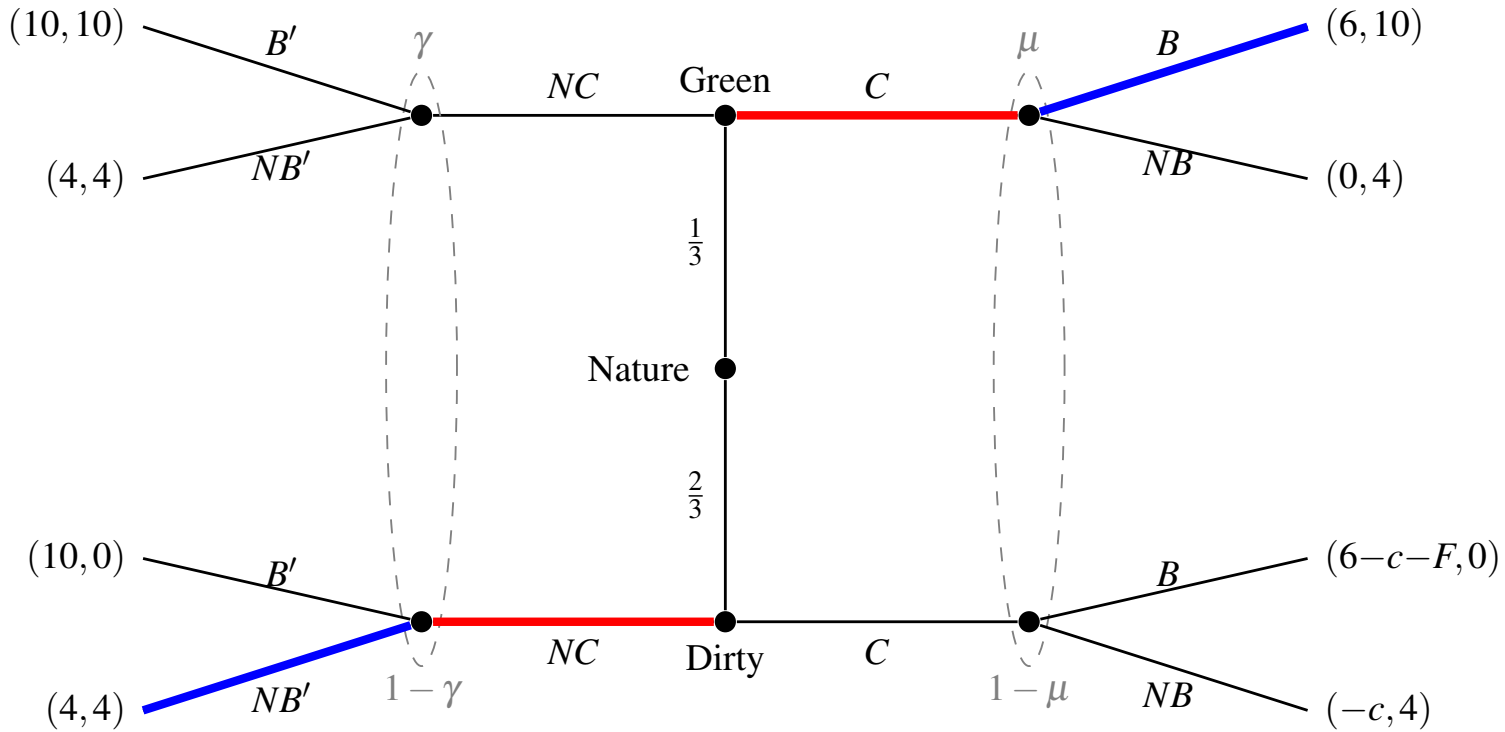


Only the green firm certifies its product.

4.8.2 Steps 1–2: Beliefs and Responses

- **Step 1**: $\mu = 1$, since a certified product can only come from the green firm; and $\gamma = 0$, since only the dirty firm does not certify.
- **Step 2**:
  - After a certified product, the consumer buys, B, since $10 > 4$.
  - After an uncertified product, she does not buy, NB', since $4 > 0$.

Figure 4.11: Consumer Responses in $C^G NC^D$



4.8.2 Steps 3–4: Optimal Messages

- **Green firm**: earns 6 from C^G; deviating to NC^G yields 4. No incentive to deviate.
- **Dirty firm**: earns 4 from NC^D; deviating to C^D fools the consumer and yields $6 - c - F$. It does not deviate if $4 \geq 6 - c - F$:

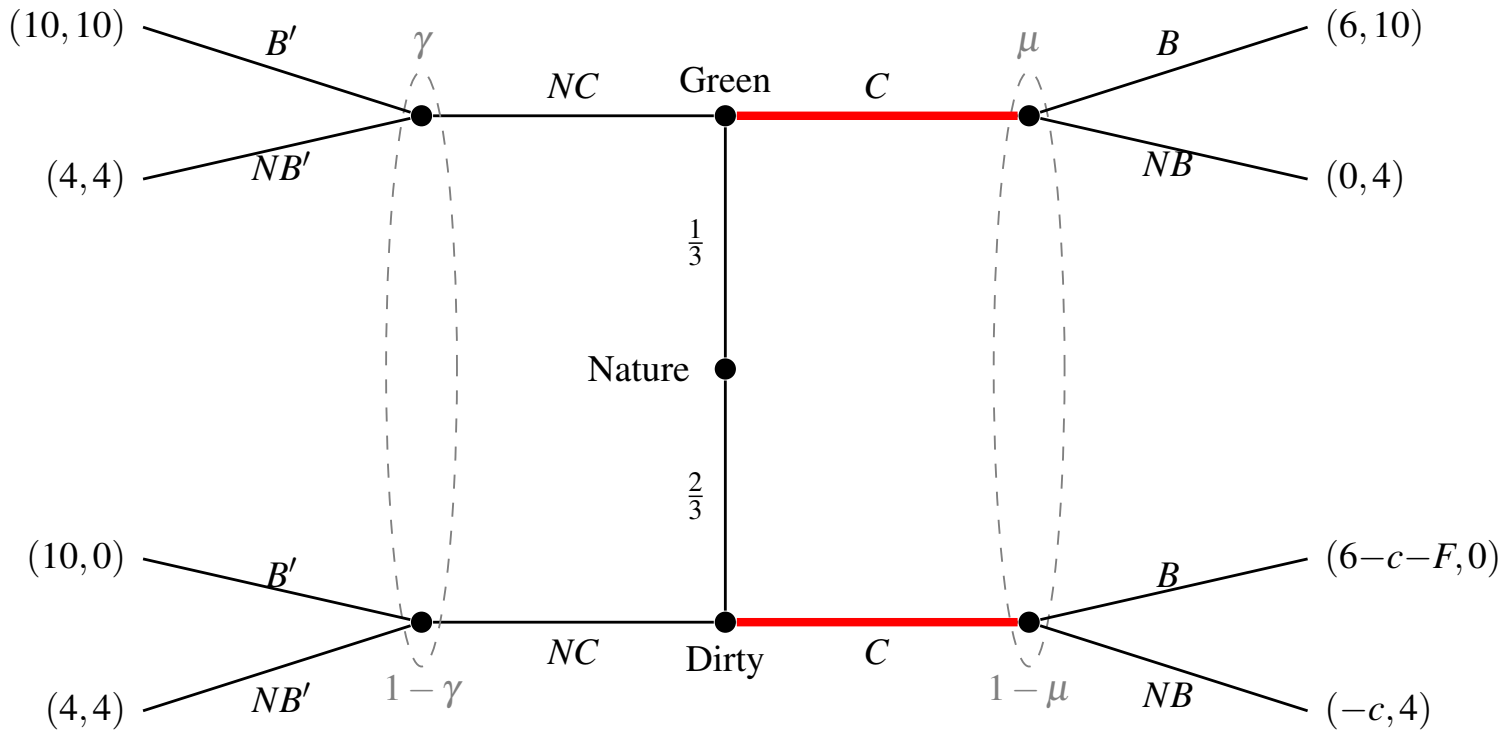
$$c + F \geq 2 \tag{4.62}$$

- $c + F$ is the cost of greenwashing before (c) and after (F) the purchase. If $c = F = 3$, the dirty firm does not deviate; if $c = 1$ and $F = 0$, it certifies.
- **Step 4**: $C^G NC^D$ is a PBE if and only if $c + F \geq 2$.

Complete Information and the Other Separating Profile

- **Complete information**: the green firm chooses NC^G, and the consumer buys only from it. Payoffs: $(10, 10)$ for the green firm and $(4, 4)$ for the dirty firm, versus $(6, 10)$ and $(4, 4)$ in the separating PBE. Signaling is costly: $10 - 6 = 4$.
- $NC^G C^D$: beliefs $\mu = 0$ and $\gamma = 1$; responses NB after a certified product and B' after an uncertified one.
- The green firm earns 10 from NC^G (deviating yields 0), but the dirty firm earns $-c$ from C^D and 10 from NC^D, for all parameter values: $NC^G C^D$ cannot be supported as a PBE.

Figure 4.12: Pooling Strategy Profile $C^G C^D$



Section 4.8.3: both firm types certify their products.

4.8.3 Step 1: Consumer Beliefs

- On the equilibrium path, by Bayes' rule:

$$\mu = \frac{\frac{1}{3} \times 1}{\left(\frac{1}{3} \times 1\right) + \left(\frac{2}{3} \times 1\right)} = \frac{1}{3} \tag{4.63}$$

  Certification provides no information: beliefs equal the prior.

- Off the equilibrium path, Bayes' rule has no bite:

$$\gamma = \frac{\frac{1}{3} \times 0}{\left(\frac{1}{3} \times 0\right) + \left(\frac{2}{3} \times 0\right)} = \frac{0}{0} \tag{4.64}$$

  so $\gamma \in [0, 1]$ is unrestricted.

4.8.3 Step 2: Consumer Responses

- After a certified product, she buys if

$$\frac{1}{3}10 + \frac{2}{3}0 > \frac{1}{3}4 + \frac{2}{3}4 \tag{4.65}$$

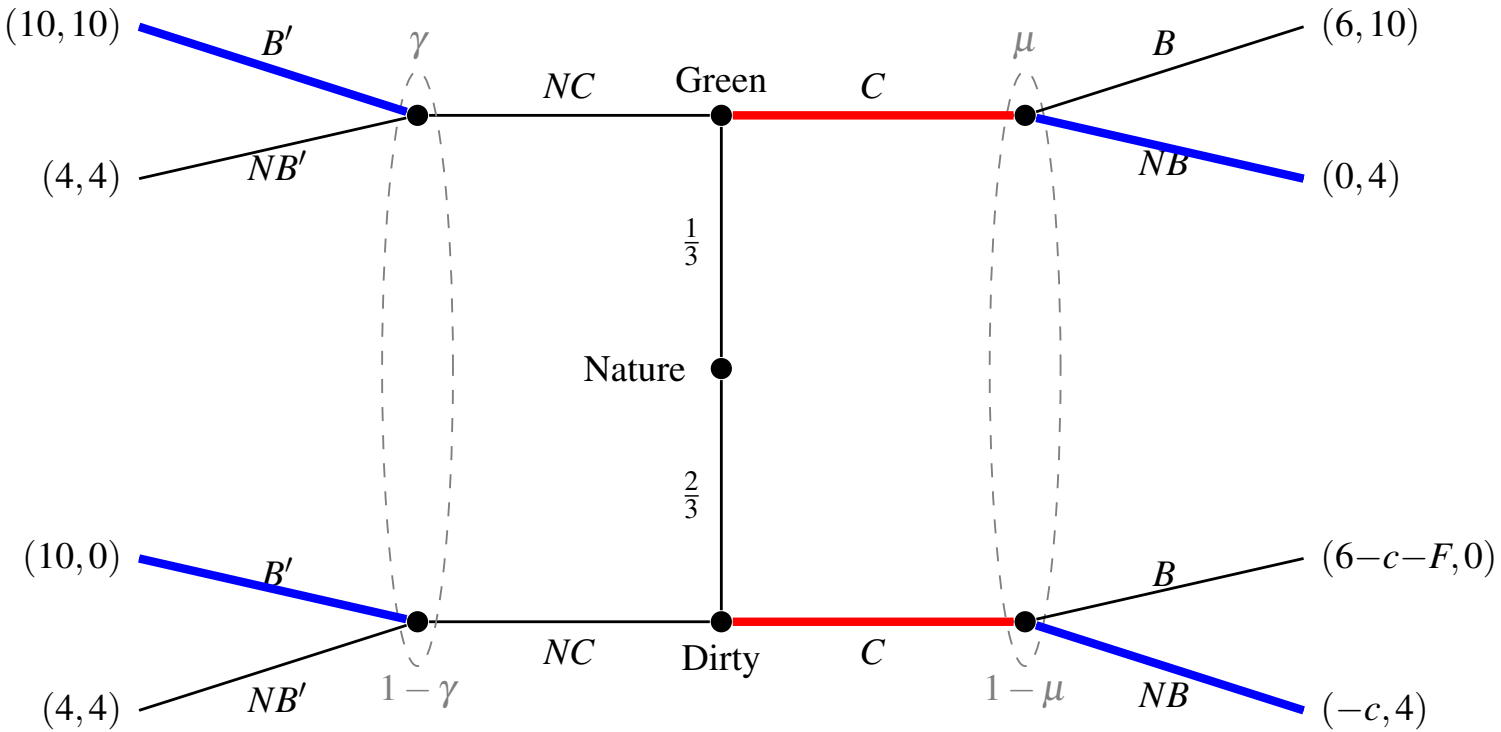
  that is, $\frac{10}{3} \simeq 3.33 > 4$, which does not hold: she responds with *NB*. The firm *conceals* its type.

- After an uncertified product, she buys if

$$\gamma 10 + (1-\gamma)0 > \gamma 4 + (1-\gamma)4 \tag{4.66}$$

  that is, $\gamma > \frac{2}{5}$.

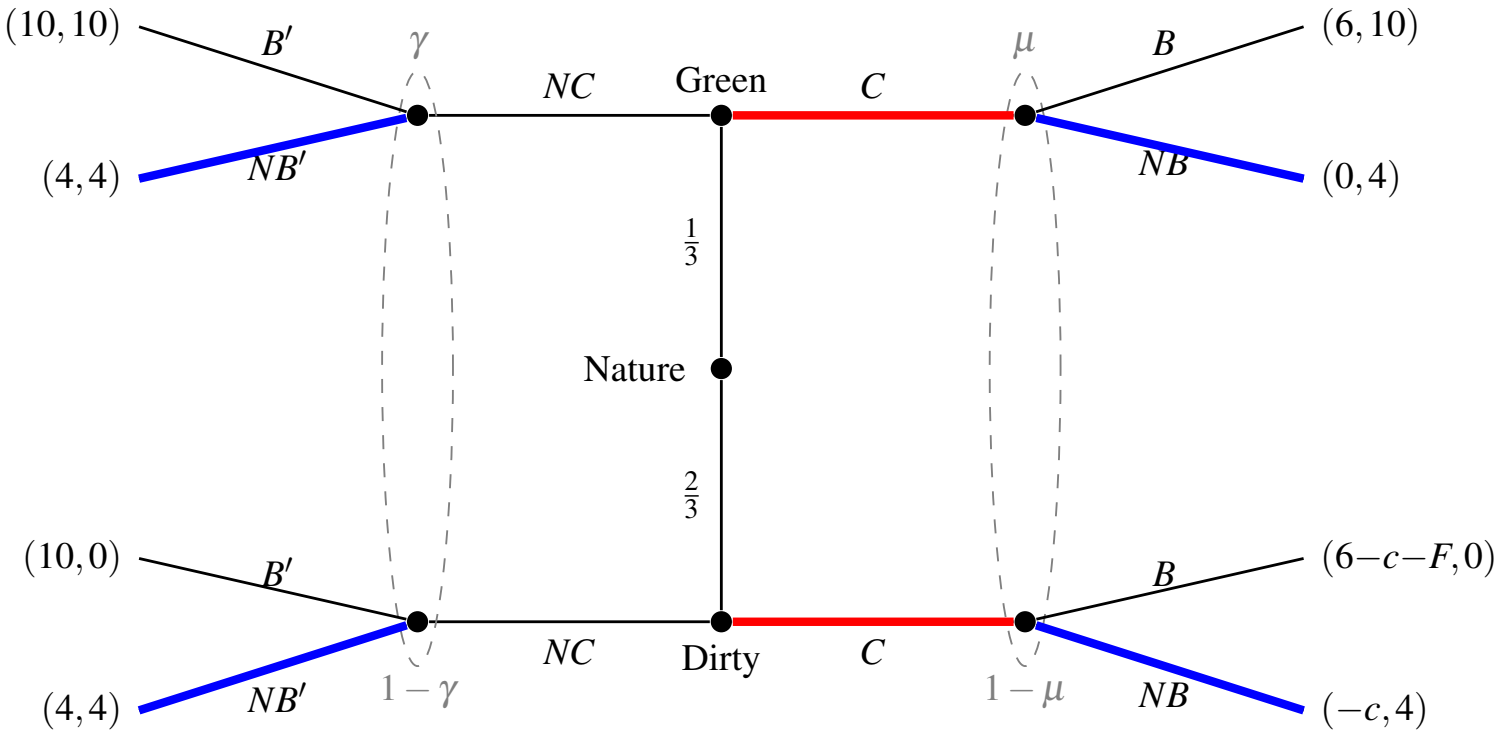
Figure 4.13: $C^G C^D$, Responses if $\gamma > \frac{2}{5}$



4.8.3 Step 3: Optimal Messages if $\gamma > \frac{2}{5}$

- **Green firm**: earns 0 from C^G, since its product is not purchased. Deviating to NC^G triggers B' and yields 10.
- **Dirty firm**: earns $-c$ from C^D, and 10 by deviating.
- At least one firm type has incentives to deviate, so $C^G C^D$ cannot be sustained as a PBE.

Figure 4.14: $C^G C^D$, Responses if $\gamma \leq \frac{2}{5}$



4.8.3 Step 3: Optimal Messages if $\gamma \leq \frac{2}{5}$

- **Green firm**: deviates from C^G to NC^G since $4 > 0$. Its product is not purchased either way, so it saves on certification costs.
- **Dirty firm**: deviates from certifying, $-c$, to not certifying, 4.
- **Step 4**: firms have incentives to deviate for all values of γ, so $C^G C^D$ cannot be sustained as a pooling PBE.

4.8.3 Pooling Profile $NC^G NC^D$

- Beliefs: $\gamma = \frac{1}{3}$ on the equilibrium path, and $\mu \in [0, 1]$ off the equilibrium path.
- Responses: NB' after an uncertified product; after a certified one, B if $\mu > \frac{2}{5}$ and NB otherwise.
- If $\mu > \frac{2}{5}$: the green firm deviates, since $6 > 4$, so $NC^G NC^D$ is not a PBE.
- If $\mu \leq \frac{2}{5}$: neither type deviates ($4 > 0$ and $4 > -c$), so $NC^G NC^D$ can be sustained as a PBE.

4.8.3 Applying the Intuitive Criterion

- $NC^G NC^D$ requires $\mu \leq \frac{2}{5}$: a certified product is likely dirty. These beliefs may be insensible if only the green firm can benefit from certifying. We use Cho and Kreps' (1987) Intuitive Criterion.
- Upon observing a certified product, which type could benefit?
- **Green firm**: from 4 to 6 if the consumer responds with B.
- **Dirty firm**: from 4 to $6 - c - F$ if she responds with B. The deviation is not equilibrium dominated if $6 - c - F \geq 4$, that is, $2 \geq c + F$.

4.8.3 Two Cases

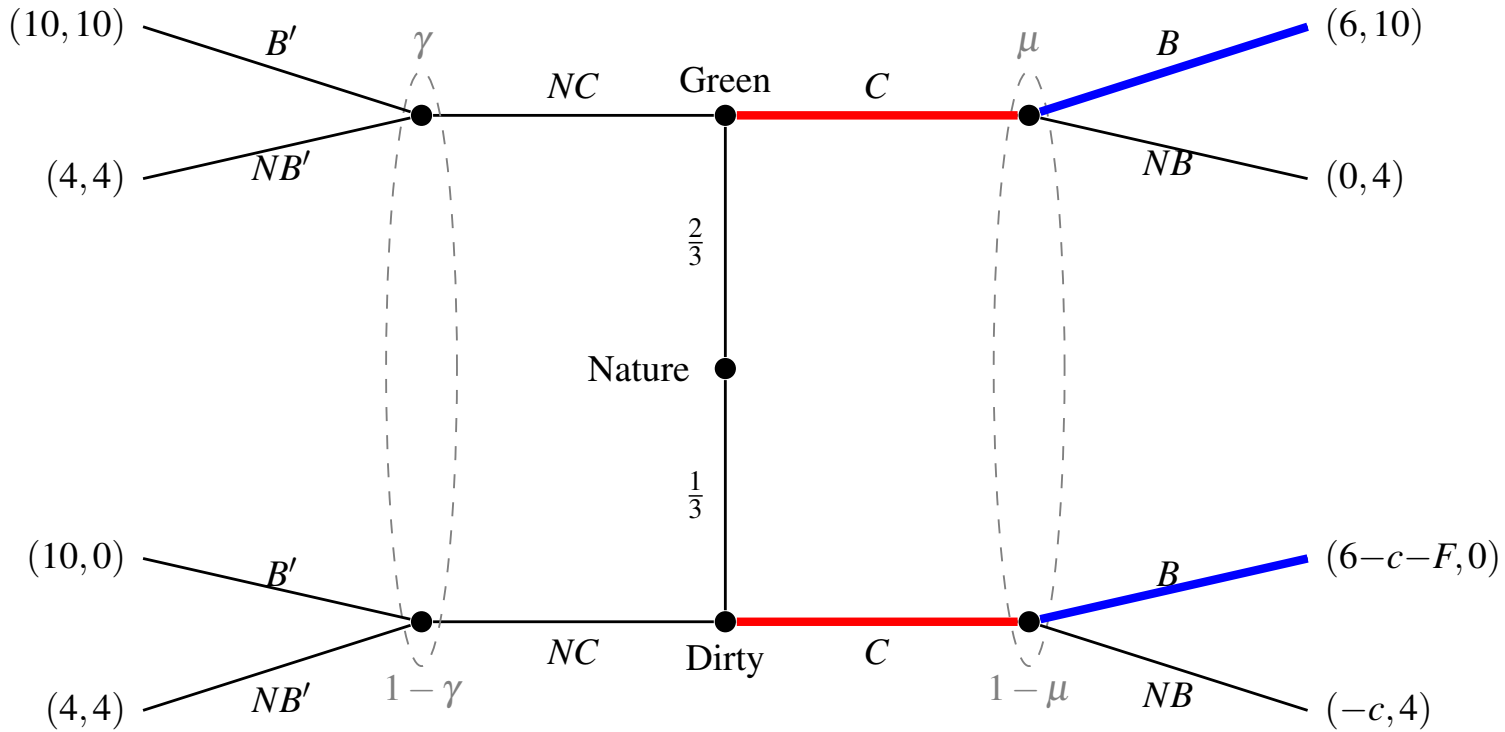
1. $2 \geq c + F$: both types could benefit from certifying, so consumers cannot update their off-the-equilibrium beliefs. $NC^G NC^D$ **survives** the Intuitive Criterion.
2. $2 < c + F$: only the green firm can benefit from certifying. Consumers update to $\mu = 1$ and buy, so the green firm deviates to C^G. $NC^G NC^D$ **does not survive**.

Summary: if $c + F > 2$, only the separating PBE $C^G NC^D$ survives; if $c + F < 2$, the unique pure-strategy PBE is $NC^G NC^D$; if $c + F = 2$, both survive.

4.8.4 What If Green Products Are More Likely?

- So far, the prior probability of a green firm is $\frac{1}{3}$ and that of a dirty firm is $\frac{2}{3}$.
- These priors played no role in the separating PBEs.
- But they explain why consumers do not buy after a pooling message; see (4.65).
- Now the green firm is more likely: probability $\frac{2}{3}$, while the dirty firm has probability $\frac{1}{3}$ (Figure 4.15).

Figure 4.15: $C^G C^D$ with More Likely Green Firm



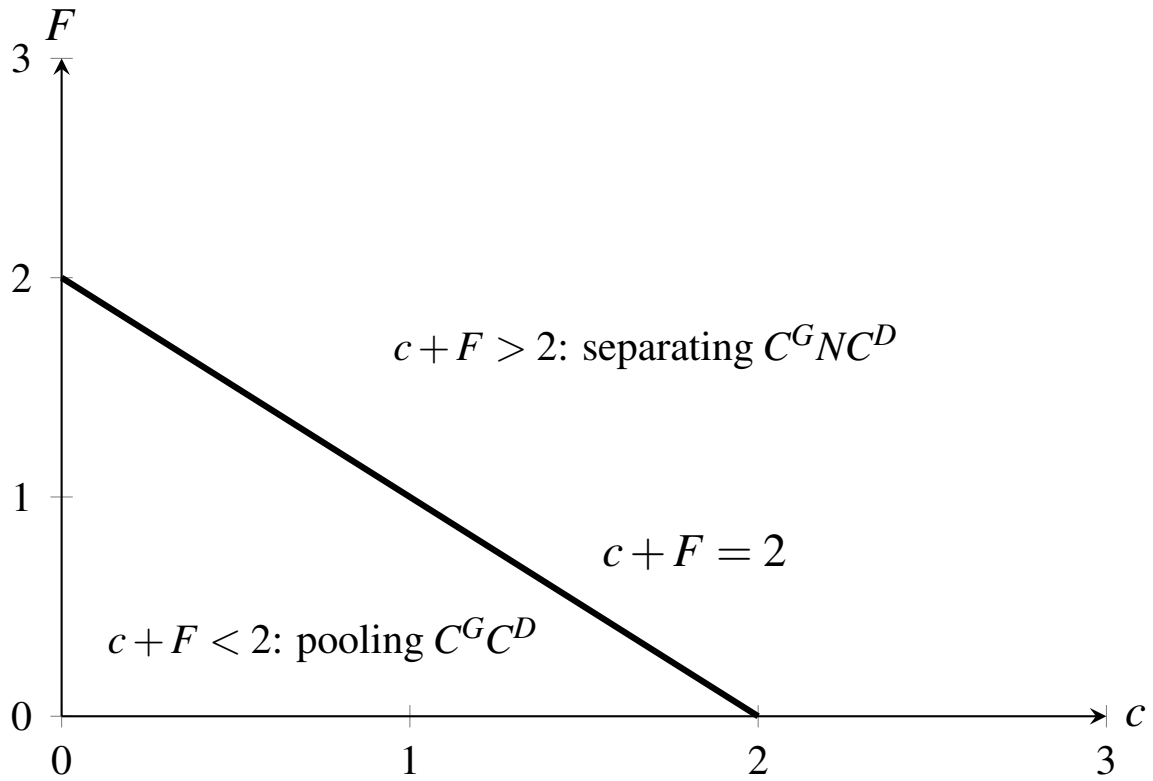
4.8.4 Pooling Profile $C^G C^D$

- After a certified product, the consumer buys, B, since

$$\frac{2}{3}10 + \frac{1}{3}0 > \frac{2}{3}4 + \frac{1}{3}4 \tag{4.67}$$

  that is, $\frac{20}{3} \simeq 6.67 > 4$. After an uncertified product, she buys if $\gamma > \frac{2}{5}$, as in (4.66).
- $\gamma > \frac{2}{5}$: she responds with B', so the green firm deviates to NC^G, saving the certification costs.
- $\gamma \leq \frac{2}{5}$: the green firm does not deviate ($6 > 4$), but the dirty firm deviates to NC^D if $4 > 6 - c - F$, that is, $c + F > 2$.

Figure 4.16: Equilibria and Greenwashing Costs



Profiles where the green firm certifies. $c+F>2$: only $C^G NC^D$ can be sustained. $c+F<2$: only $C^G C^D$, so certification conveys no information. $c+F=2$: both.

4.8.4 The No-Certification Pooling PBE

- Unlike in section 4.8.3, $NC^G NC^D$ survives the Intuitive Criterion for all values of c and F.
- Consumers buy uncertified products, since $\gamma = \frac{2}{3} > \frac{2}{5}$, so both firm types earn 10, more than any payoff from certifying.
- No firm type can increase its payoff by deviating to certification.
- Intuitively, when green products are common, consumers buy without a certificate, and firms save on certification costs.

Section 4.9
Related Literature on Green Certification

4.9.1 Certification, Signaling, and Greenwashing

- Mahenc (2008): price as a signal of environmental performance.
- Mahenc (2017): a strategic certifier, honest only if it cares more about welfare than profit.
- Walter and Chang (2017): green clubs; subsidy plus certification tax.
- Wittreich (2025): truth-revealing certification improves welfare.
- Baksi and Bose (2007): penalties for misrepresentation.
- Heyes and Martin (2017): competing NGO labels.
- Ibanez and Grolleau (2008): ecolabeling and pollution.
- Daughety and Reinganum (2008): disclosure vs. price signaling.

Table 4.2: Signaling Models in Green Certification

		Audiences	
		One audience	Multiple audiences
Signal origin	One signal, one sender	Mahenc (2008); Wittreich (2025)	Baksi & Bose (2007)
	Two signals, same sender	Daughety & Reinganum (2008)	
	Two signals, different senders	Mahenc (2017)	

Section 4.8 assumes a binary decision, one signal, and one audience; the literature varies the number of signals, the senders, and the audiences.

4.9.2 Empirical Studies on Certification

- Potter et al. (2021) and Taillie et al. (2024): eco-labels increase consumer preference for sustainable products, especially when label clarity is high.
- Witek and Kuźniar (2021): among Polish consumers, gender, age, education, financial situation, and number of children affect green purchases.
- Nie and Wang (2025): certified firms in China's Green Factory program experience productivity gains.
- Zubeltzu-Jaka et al. (2024): certification is positively associated with financial performance, though causality may be bidirectional.

Section 4.10
Case Studies on Green Certification

4.10.1 Volkswagen's Dieselgate

- In September 2015, the U.S. Environmental Protection Agency found that Volkswagen diesel cars carried a "defeat device" that reduced nitrogen oxide emissions only during official tests.
- Volkswagen acknowledged that about 11 million vehicles worldwide had the affected engines.
- The company had advertised these cars as "clean diesel"; the U.S. Federal Trade Commission sued it in 2016 for deceptive advertising.

Dieselgate: Penalties and Connection to the Model

- 2017: Volkswagen pleaded guilty in the United States and agreed to a $2.8 billion criminal fine, plus $1.5 billion in civil penalties. 2019: an Australian court imposed a penalty of A$125 million.
- By 2020, fines, settlements, vehicle buybacks, and legal costs from the scandal exceeded 31 billion euros.
- *Connection to the model*: a dirty firm mimics the green firm when $c + F < 2$, as "clean diesel" marketing did. The penalties correspond to F, which supports separation when $c + F \geq 2$. The case does not show that larger fines would have prevented the deception.

4.10.2 Retail and Consumer Products

- April 2022: the U.S. Department of Justice, on behalf of the Federal Trade Commission, filed complaints against Kohl's and Walmart for marketing rayon textiles as "bamboo." Converting bamboo into rayon uses toxic chemicals and releases hazardous pollutants. Civil penalties: $2.5 million (Kohl's) and $3 million (Walmart).
- January 2022: Keurig Canada agreed to pay a C$3 million penalty over claims that its K-Cup pods were recyclable; outside British Columbia and Quebec, they were not widely accepted by municipal recycling programs.
- *Connection to the model*: consumers cannot verify these claims before purchasing; the penalties correspond to F. The cases do not measure their effect on firms' behavior.

Rules on Green Claims and Labels

- **US**: the FTC Green Guides (16 CFR Part 260) advise against unqualified general claims, such as "eco-friendly," which are hard to substantiate.
- **Food**: "organic" sellers with over $5,000 in annual organic sales need a USDA-accredited certifier; FDA has no formal definition of "natural."
- **EU**: Directive 2024/825, applied from September 2026, bans generic claims and labels not based on a certification scheme or set by public authorities.
- *Connection to the model*: as in section 4.8, consumers cannot verify these credence attributes (Darby and Karni 1973); rules on claims can raise the cost of mimicking, $c + F$.

4.10.3 Financial Sector

- September 2023: the U.S. Securities and Exchange Commission ordered DWS Investment Management Americas, part of Deutsche Bank's asset manager DWS Group, to pay $19 million for misstatements about its ESG investment process.
- From August 2018 to late 2021, DWS presented itself as a leader in ESG investing but did not adequately implement its own ESG integration policy. April 2025: prosecutors in Frankfurt fined DWS 25 million euros for misleading statements about its ESG capabilities.
- *Connection to the model*: investors cannot observe whether a fund follows its ESG policy; penalties raise the cost of misleading claims, as F does.

Section 4.11
Summary

Summary: Uncertain Damages

- The EA may not observe pollution severity d, due to technological gaps or random environmental factors.
- Emission fees are set on expected damages: overestimating d leads to excessive fees and underproduction; underestimating it, to lax fees and overproduction.
- Welfare losses arise from the gap between expected and actual damages.
- Fees and quotas yield equivalent inefficiencies when uncertainty is only in pollution damages.

Summary: Uncertain Demand and Costs

- Demand uncertainty shifts the *SMB* curve downward; cost uncertainty shifts the *SMC* curve downward.
- In both cases, the EA sets an intermediate fee, and welfare losses differ between price and quantity instruments:

Uncertainty about	**Fees preferred**	**Quotas preferred**
Damages (d)	Same welfare loss	
Demand	$d < 2b$	$d > 2b$
Production costs	$d < 2b$	$d > 2b$

- With nonlinear curves, the ranking depends on their relative slopes near the optimum, as in Weitzman (1974).

Summary: Mechanisms to Infer Emissions

- The EA can use incentive-compatible mechanisms to elicit truthful emission reports.
- Circular monitoring: firms report each other's emissions, and penalties for misreporting ensure truthful behavior.
- A fee equal to marginal damage at the reported emissions induces too little output, since every firm anticipates that its output increases its own fee.
- Charging half of this fee restores the first best.

Summary: Green Certification

- Firms use certification to convey or conceal environmental quality in markets with asymmetric information.
- The credibility of certification depends on the cost of signaling and the risk of greenwashing, $c + F$.
- When green products are unlikely: if $c + F > 2$, only the separating equilibrium survives the Intuitive Criterion; if $c + F < 2$, only the pooling equilibrium where no firm certifies survives.
- When green products are likely, the no-certification pooling PBE survives for all c and F.

Summary: Key Equations (1/2)

- Fee under uncertain damages:

$$t^U = \frac{(1-c)(2E[d]-1)}{2(1+E[d])} \tag{4.2}$$

- Fee under uncertain demand:

$$t^{UD} = \frac{[E[a]-c](2d-b)}{2(b+d)} \tag{4.11}$$

Summary: Key Equations (2/2)

- Fee under uncertain costs:

$$t^{UC} = \frac{[1 - E[c]](2d - b)}{2(b + d)} \tag{4.29}$$

- Fees versus quotas under uncertain demand:

$$\Delta WL = \frac{2p(1-a)^2(1-p)(d-2b)}{9b^2} \tag{4.25}$$

which is positive if and only if $d > 2b$.

Looking Ahead

- Chapter 4 examined how regulation performs when the EA lacks information about damages, demand, costs, or emissions.
- **Chapter 5**: sequential competition and regulation.
- Building on the Cournot framework of chapter 2, a leader chooses its output first, and a follower responds after observing it (Stackelberg competition).
- How does this sequential structure affect market outcomes and the design of optimal environmental policy?

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 4: Regulation under Uncertainty

Chapter 5

Sequential Competition and Regulation

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 5

- 5.1 Introduction and guiding questions.
- 5.2 Allowing for sequential competition: Stackelberg model without and with regulation; fee comparison; output advantage.
- 5.3 Allowing for two time periods: pollution persistence.
- 5.4 Regulatory commitment.
- 5.5–5.7 Case studies, related literature, and summary.

Section 5.1

Introduction

Why Sequential Competition?

- Chapter 2 assumed *simultaneous* Cournot competition: firms choose output without observing each other's decision.
- In this chapter, the leader chooses its output first; the follower responds after observing the leader's output.
- This sequential structure introduces strategic asymmetries that affect both market outcomes and the design of emission fees.

Roadmap of the Chapter

- **Benchmark**: unregulated Stackelberg duopoly; the leader's output advantage.
- **Regulation**: emission fees must be *more stringent* under sequential than under simultaneous competition.
- **Two periods**: pollution persists across periods; how fees in each period respond.
- **Commitment**: fee paths differ with and without commitment, but total welfare is unchanged.

Real-World Motivation

- **World crude oil**: before 2008, prices and quantities are closest to a Stackelberg outcome with Saudi Arabia as the leader (Huppmann and Holz, 2012).
- **Electricity (PJM)**: a leader-follower model where the largest generator chooses first (Chen et al., 2006).
- **Generation capacity**: a large incumbent can commit to capacity before its rivals.
- **Key insight**: when firms move sequentially, unregulated aggregate output (and pollution) is higher than under Cournot.

Guiding Questions

1. How does Stackelberg competition differ from Cournot in terms of strategic behavior?
2. What is the impact of sequential decision-making on emissions and welfare?
3. Why might emission fees need to be more stringent under sequential competition?
4. How does pollution persistence across time periods affect regulatory design?
5. What are the trade-offs between regulatory commitment and flexibility?
6. How do dynamic models enhance our understanding of intertemporal regulation?

Section 5.2

Allowing for Sequential Competition

Setup: Stackelberg Duopoly

- Following Choi et al. (2024): the leader (firm 1) chooses output first; the follower (firm 2) responds.
- Inverse demand $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- Marginal costs: the leader's is normalized to zero, $c_1 = 0$; the follower's is $c_2 = c \geq 0$.
- Then c measures the leader's cost advantage relative to the follower (or cost efficiency).

Setup: The Cost Advantage c

- When $c = 0$: firms are cost symmetric, $c_2 = c_1$; the leader benefits only from moving first.
- When $c > 0$: the leader enjoys a cost advantage, $c_2 > c_1$.
- Assumption: $c < \frac{1}{3}$, so firms are not extremely asymmetric. This avoids corner solutions where only one firm remains active.

Section 5.2.1 – No Regulation: Timing

- **Two-stage game (no regulation):**
  1. **Stage 1**: the leader (firm 1) chooses $q_1 \geq 0$.
  2. **Stage 2**: the follower (firm 2) observes q_1 and chooses $q_2 \geq 0$.
- Solution concept: **subgame perfect equilibrium** (SPE), found by **backward induction**.
- The leader can *commit* to its output: this is the source of the first-mover advantage.

Stage 2 (No Regulation): Follower's Problem

- The follower takes the leader's output q_1 as given and maximizes profit:

$$\max_{q_2 \geq 0} \pi_2 = (1 - q_1 - q_2)q_2 - cq_2. \tag{5.1}$$

- First-order condition: $1 - q_1 - 2q_2 - c = 0$.
- Solving for q_2, the follower's best response function is

$$q_2(q_1) = \frac{1-c}{2} - \frac{1}{2}q_1. \tag{5.2}$$

Stage 2 (No Regulation): Interpreting the Best Response

- **Vertical intercept** $\frac{1-c}{2}$: shifts downward when the follower suffers a larger cost disadvantage (higher c).
- **Slope** $-\frac{1}{2}$: when the leader increases q_1, the follower decreases q_2.
- Outputs are *strategic substitutes*, as in the Cournot models of previous chapters.
- Difference from Cournot: the follower *observes* q_1 before choosing q_2, and the leader anticipates this response.

Stage 1 (No Regulation): Leader's Problem

- The leader anticipates the follower's best response:

$$\max_{q_1 \geq 0} \pi_1 = \big(1 - q_1 - q_2(q_1)\big) q_1. \tag{5.3}$$

- Inserting $q_2(q_1) = \frac{1-c}{2} - \frac{1}{2} q_1$:

$$\max_{q_1 \geq 0} \pi_1 = \left(1 - q_1 - \left(\frac{1-c}{2} - \frac{1}{2} q_1\right)\right) q_1. \tag{5.4}$$

- The leader's problem now depends only on its own output q_1.

Stage 1 (No Regulation): Leader's Output

- Problem (5.4) simplifies to

$$\max_{q_1 \geq 0} \pi_1 = \left(\frac{1+c}{2} - \frac{1}{2} q_1 \right) q_1. \tag{5.5}$$

- First-order condition: $\frac{1+c}{2} - q_1 = 0$, so the leader produces

$$q_1^{NR} = \frac{1+c}{2}, \tag{5.6}$$

where NR stands for no regulation.

Stage 1 (No Regulation): Follower's Output

- Inserting q_1^{NR} into the follower's best response:

$$q_2^{NR} = \frac{1-c}{2} - \frac{1}{2} \cdot \frac{1+c}{2} = \frac{1-3c}{4}.$$

- $q_2^{NR} > 0$ if $c < \frac{1}{3}$, which holds by assumption.

Leader's Output Advantage (No Regulation)

- Output advantage:

$$\Delta q^{NR} \equiv q_1^{NR} - q_2^{NR} = \frac{1+c}{2} - \frac{1-3c}{4} = \frac{1+5c}{4}. \quad (5.7)$$

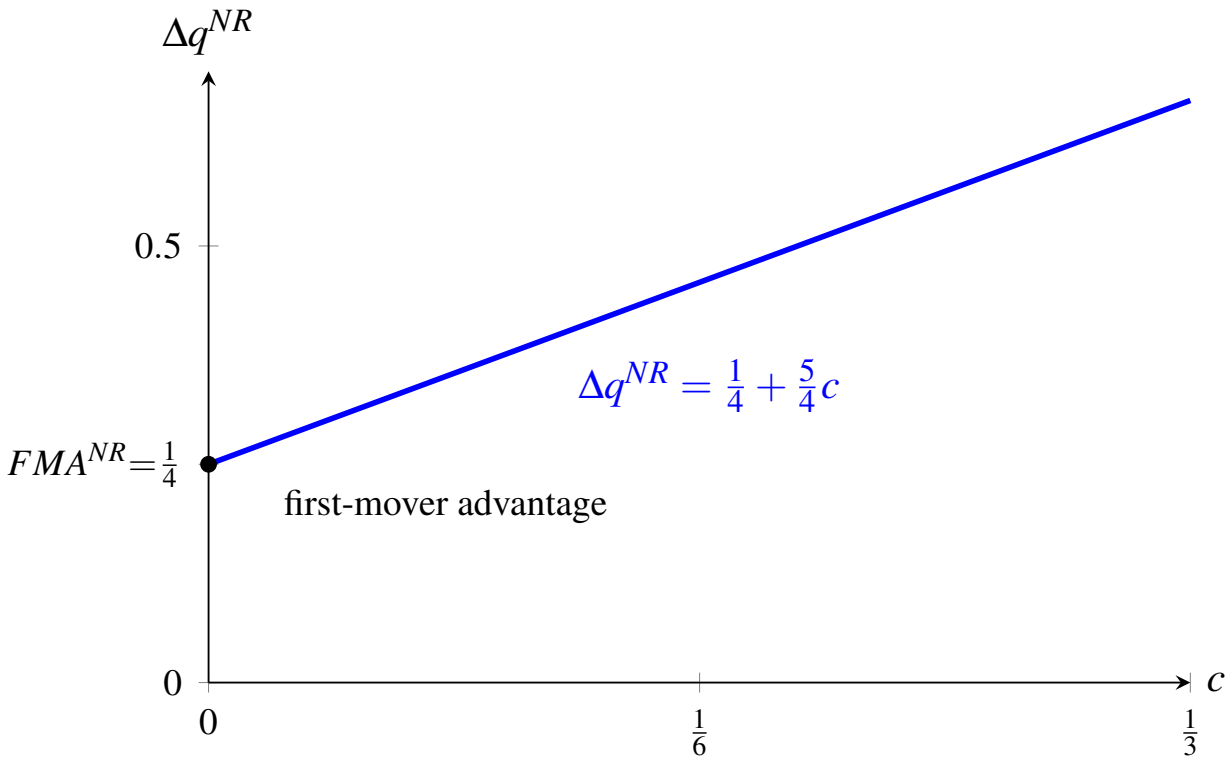
- Separating it into a first-mover advantage and a cost advantage:

$$\Delta q^{NR} = \underbrace{\frac{1}{4}}_{FMA^{NR}} + \underbrace{\frac{5}{4}c}_{CA^{NR}}. \quad (5.8)$$

Decomposing the Output Advantage

- $FMA^{NR} = \frac{1}{4}$: **first-mover advantage**. Positive even with symmetric costs ($c = 0$).
- $CA^{NR} = \frac{5}{4}c$: **cost advantage**. Zero when $c = 0$; increasing in c.
- The leader always benefits from a first-mover advantage. If costs are asymmetric, it benefits from both advantages.
- Question: does the EA, anticipating FMA^{NR} and CA^{NR}, set fees that ameliorate the leader's output advantage?

Figure 5.1 – Output Advantage Without Regulation



- Vertical intercept $FMA^{NR} = \frac{1}{4}$; slope $\frac{5}{4}$.

Section 5.2.2

Sequential Competition – Regulation

Regulated Stackelberg: Three-Stage Game

1. **Stage 1**: the regulator sets a per-unit emission fee t affecting both firms.
2. **Stage 2**: observing t, the leader chooses output q_1.
3. **Stage 3**: observing t and q_1, the follower responds choosing output q_2.

We solve the game by backward induction, starting from stage 3.

Regulated Stackelberg: Social Welfare

- Social welfare:

$$W = CS + PS + T - Env \tag{5.9}$$

  where $CS = \frac{Q^2}{2}$, $PS = \pi_1 + \pi_2$, $T = tQ$, and $Env = \frac{d}{2}Q^2$.
- Pollution severity: $d > \frac{1}{2}$, which guarantees positive emission fees under both simultaneous and sequential competition.
- No payoff discounting.

Stage 3 (With Regulation): Follower's Problem

- The follower pays fee t on each unit:

$$\max_{q_2 \geq 0} \pi_2 = (1 - q_1 - q_2)q_2 - (c + t)q_2. \tag{5.10}$$

- Best response function:

$$q_2(q_1, t) = \frac{1 - (c + t)}{2} - \frac{1}{2}q_1. \tag{5.11}$$

- Analogous to (5.2), but now shifts downward in both the follower's cost disadvantage, c, and the fee, t.

Stage 2 (With Regulation): Leader's Problem

- The leader anticipates $q_2(q_1, t)$:

$$\max_{q_1 \geq 0} \pi_1 = \big(1 - q_1 - q_2(q_1, t)\big) q_1 - t q_1. \tag{5.12}$$

- Inserting $q_2(q_1, t) = \frac{1-(c+t)}{2} - \frac{1}{2} q_1$, the problem simplifies to

$$\max_{q_1 \geq 0} \pi_1 = (1 - q_1 + c - t) \frac{q_1}{2}. \tag{5.13}$$

Stage 2 (With Regulation): Leader's Output

- First-order condition: $\frac{1+c-t-2q_1}{2} = 0$, so

$$q_1(t) = \frac{1+c-t}{2}. \tag{5.14}$$

- Positive if and only if $t < 1 + c$: the fee cannot be excessive, otherwise not even the leader would be active.
- $q_1(t)$ increases in the cost advantage c and decreases in the fee t.

Stage 2 (With Regulation): Follower's Output

- Inserting $q_1(t)$ into the follower's best response (5.11):

$$q_2(t) = \frac{1 - 3c - t}{4}, \tag{5.15}$$

  which decreases in both c and t.
- Positive if and only if $t < 1 - 3c$.
- Since $1 - 3c < 1 + c$, this condition is more demanding than the leader's; and $1 - 3c > 0$ because $c < \frac{1}{3}$.

Differential Effect of the Fee on Outputs

- From (5.14) and (5.15):

$$\frac{\partial q_1(t)}{\partial t} = -\frac{1}{2} < -\frac{1}{4} = \frac{\partial q_2(t)}{\partial t}. \tag{5.16}$$

- The leader's output is more responsive to the fee than the follower's.
- Intuitively, a unit increase in t lowers the follower's output by $\frac{1}{2}$ directly, but the leader's lower output raises it by $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ (strategic substitutes).

The Fee Reduces the Leader's Output Advantage

- The fee reduces the leader's output more than the follower's:

$$q_1^{NR} - q_1(t) > q_2^{NR} - q_2(t) \tag{5.17}$$

since the difference between both sides is $\frac{t}{2} - \frac{t}{4} = \frac{t}{4} > 0$ for every $t > 0$.

- Rearranging (5.17):

$$\Delta q^{NR} \equiv q_1^{NR} - q_2^{NR} > q_1(t) - q_2(t) \equiv \Delta q^{R}. \tag{5.18}$$

Stage 1 (With Regulation): The Regulator's Problem

- The regulator anticipates $q_1(t)$ and $q_2(t)$ and solves

$$\max_t W = CS + PS + T - Env \tag{5.19}$$

- Aggregate output: $Q(t) = \frac{3-c-3t}{4}$.
- Consumer surplus: $CS = \frac{(3-c-3t)^2}{32}$.

Stage 1 (With Regulation): Welfare Components

- Profits:

$$\pi_1 = \frac{(1+c-t)^2}{8} \quad \text{and} \quad \pi_2 = \frac{(1-3c-t)^2}{16},$$

  and $PS = \pi_1 + \pi_2$.
- Tax revenue: $T = tQ(t)$.
- Environmental damage: $Env = \frac{d(3-c-3t)^2}{32}$.

Stage 1: Deriving the Emission Fee

- Differentiating W with respect to t:

$$\frac{\partial W}{\partial t} = -\frac{3(1+cd) - c - 9d(1-t) + 9t}{16} = 0. \qquad (5.20)$$

- Solving for t, the fee under sequential competition is

$$t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}. \qquad (5.21)$$

- This fee assumes that the follower is active at t^{Seq}, which requires $c < c_A$ (Section 5.2.4).

Properties of the Fee t^{Seq}

- **Sign**: $t^{Seq} > 0$ if $c < 3$ and $d > \frac{1}{3}$, which holds since $c < \frac{1}{3}$ and $d > \frac{1}{2}$.
- **More severe pollution**: t^{Seq} becomes more stringent,

$$\frac{\partial t^{Seq}}{\partial d} = \frac{4(3-c)}{9(1+d)^2} > 0.$$

- **More cost asymmetry**: t^{Seq} becomes less stringent,

$$\frac{\partial t^{Seq}}{\partial c} = \frac{1-3d}{9(1+d)} < 0 \quad \text{since } d > \frac{1}{2}.$$

Section 5.2.3

Fee Comparison

Benchmark: Cournot Fee with the Same Firms

- Benchmark: simultaneous (Cournot) competition between the *same* firms, with marginal costs 0 and c.
- The uniform fee found in Section 3.4.2 when firm 1's cost is zero ($\alpha = 0$) is

$$t^C = \frac{(2-c)(2d-1)}{4(1+d)}.$$

- Question: is t^{Seq} more or less stringent than t^C?

Fee Differential: $t^{Seq} - t^{C}$

- Using a common denominator:

$$t^{Seq} - t^{C} = \frac{4(3-c)(3d-1) - 9(2-c)(2d-1)}{36(1+d)}.$$

- Expanding each term in the numerator:

$$\begin{aligned} 4(3-c)(3d-1) &= 36d - 12 - 12cd + 4c, \\ 9(2-c)(2d-1) &= 36d - 18 - 18cd + 9c. \end{aligned}$$

Fee Differential: Result

- Subtracting both terms, the numerator becomes $6 - 5c + 6cd$.
- Therefore, the fee differential is

$$\Delta t = t^{Seq} - t^{C} = \frac{6 - 5c + 6cd}{36(1 + d)}. \tag{5.22}$$

The Sequential Fee Is More Stringent

- $\Delta t > 0$ unambiguously, since $c < \frac{1}{3}$ implies $6 - 5c > 0$.
- Hence $t^{Seq} > t^{C}$: the emission fee is more stringent when firms compete sequentially than simultaneously.
- Intuitively, without regulation aggregate output is higher under Stackelberg:

$$Q^{NR} = \frac{3-c}{4} > \frac{2-c}{3} = Q^{NR,C}.$$

- Expecting more output and pollution, the regulator sets a more stringent fee.

Comparative Statics of Δt in c

- The fee differential widens as firms become more asymmetric if

$$\frac{\partial \Delta t}{\partial c} = \frac{6d - 5}{36(1 + d)} > 0, \tag{5.23}$$

  which holds if and only if $d > \frac{5}{6} \simeq 0.83$.

- Related to the difference in unregulated output:

$$\Delta Q^{NR} \equiv Q^{NR} - Q^{NR,C} = \frac{3 - c}{4} - \frac{2 - c}{3} = \frac{1 + c}{12}, \tag{5.24}$$

  which is increasing in c.

Comparative Statics of Δt in c: Intuition

- Both unregulated outputs decline when c increases, but the decline is larger under Cournot competition.
- When pollution is severe, $d > \frac{5}{6}$, the EA responds to this larger output gap.
- Then the fee under Cournot competition decreases faster in c than that under Stackelberg competition, widening Δt.

Comparative Statics of Δt in d

- The fee differential narrows as pollution becomes more severe:

$$\frac{\partial \Delta t}{\partial d} = \frac{11c - 6}{36(1+d)^2} < 0$$

  for all admissible costs, $c < \frac{1}{3}$.
- Both fees become more stringent as d rises, but t^{Seq} increases by less than t^{C}.
- As d grows, the gap between both fees approaches $\frac{c}{6}$.

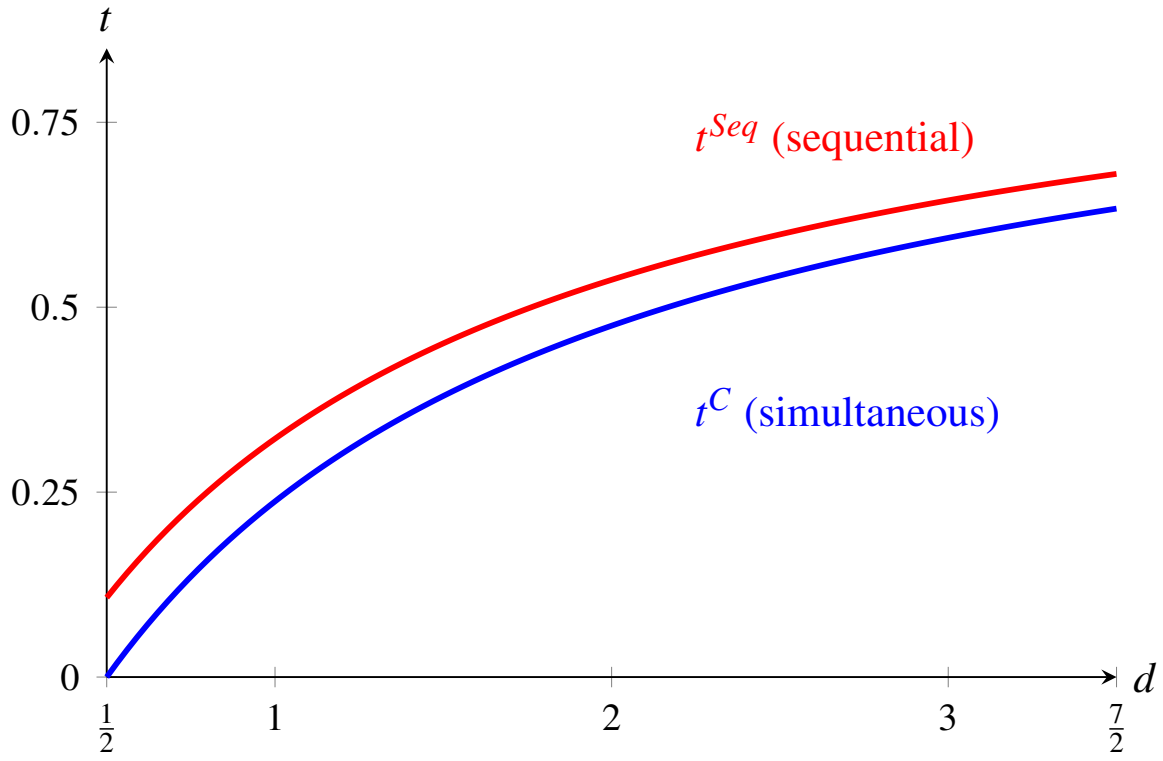
Figure 5.2: Fees at $c = \frac{1}{10}$

- Evaluating both fees at $c = \frac{1}{10}$:

$$t^{Seq} = \frac{29(3d-1)}{90(1+d)} \quad \text{and} \quad t^{C} = \frac{19(2d-1)}{40(1+d)}.$$

- Fee differential: $\Delta t = \frac{55+6d}{360(1+d)} > 0$.
- At this cost difference, the follower is active at t^{Seq} for all $d < \frac{23}{6}$; the figure considers d up to $\frac{7}{2}$.

Figure 5.2 – Emission Fees with Simultaneous and Sequential Competition



- $t^{Seq} > t^{C}$ for all pollution severities in the figure.

Sections 5.2.4–5.2.5

Equilibrium Output and Output Advantage with Regulation

Section 5.2.4 – Leader's Output at t^{Seq}

- Inserting t^{Seq} into $q_1(t)$:

$$q_1(t^{Seq}) = \frac{2\left[3 + c(2 + 3d)\right]}{9(1 + d)}, \tag{5.25}$$

  which is unambiguously positive.
- It decreases when pollution is more severe,

$$\frac{\partial q_1(t^{Seq})}{\partial d} = -\frac{2(3 - c)}{9(1 + d)^2} < 0,$$

- and increases with the leader's cost advantage,

$$\frac{\partial q_1(t^{Seq})}{\partial c} = \frac{2(2 + 3d)}{9(1 + d)} > 0.$$

Section 5.2.4 – Follower's Output at t^{Seq}

- Inserting t^{Seq} into $q_2(t)$:

$$q_2(t^{Seq}) = \frac{3 - c(7 + 6d)}{9(1 + d)}, \tag{5.26}$$

  which is positive if and only if $c < c_A \equiv \frac{3}{7+6d}$.
- Cutoff c_A originates at $\frac{3}{10}$ when $d = \frac{1}{2}$ and decreases in d, converging to zero.
- Since $c_A < \frac{1}{3}$ for all $d > \frac{1}{2}$, condition $c < c_A$ is more demanding than $c < \frac{1}{3}$.

Section 5.2.4 – Both Firms Active

- We assume $c < c_A$ in the rest of the section, so both firms are active at fee t^{Seq}.
- The follower's output decreases when pollution is more severe,

$$\frac{\partial q_2(t^{Seq})}{\partial d} = -\frac{3-c}{9(1+d)^2} < 0,$$

- and when its cost disadvantage grows,

$$\frac{\partial q_2(t^{Seq})}{\partial c} = -\frac{7+6d}{9(1+d)} < 0.$$

The Fee t^{Seq} Is a Second Best When $c > 0$

- The leader produces at zero cost, so a planner would assign all output to the leader.
- A uniform fee, however, keeps the high-cost follower active.
- Therefore, fee t^{Seq} only achieves a second best when $c > 0$.

Section 5.2.5 – Output Advantage with Regulation

- Evaluating $\Delta q^R = q_1(t^{Seq}) - q_2(t^{Seq})$:

$$\begin{aligned} \Delta q^R &= \frac{2\left[3 + c(2 + 3d)\right]}{9(1+d)} - \frac{3 - c(7 + 6d)}{9(1+d)} \\ &= \frac{3 + c(11 + 12d)}{9(1+d)}. \end{aligned} \tag{5.27}$$

Section 5.2.5 – First-Mover and Cost Advantages

- Separating first-mover and cost advantages:

$$\Delta q^R = \underbrace{\frac{1}{3(1+d)}}_{FMA^R} + \underbrace{\frac{11+12d}{9(1+d)}c}_{CA^R}. \tag{5.28}$$

- Both terms are positive; the expression holds for $c < c_A$.

Regulation and Each Type of Advantage

- More severe pollution: FMA^R decreases, while CA^R increases, $\frac{\partial CA^R}{\partial d} = \frac{c}{9(1+d)^2} > 0$.
- The fee **reduces** the first-mover advantage:

$$FMA^{NR} = \frac{1}{4} > \frac{1}{3(1+d)} = FMA^R \quad \text{iff } d > \frac{1}{3}. \tag{5.29}$$

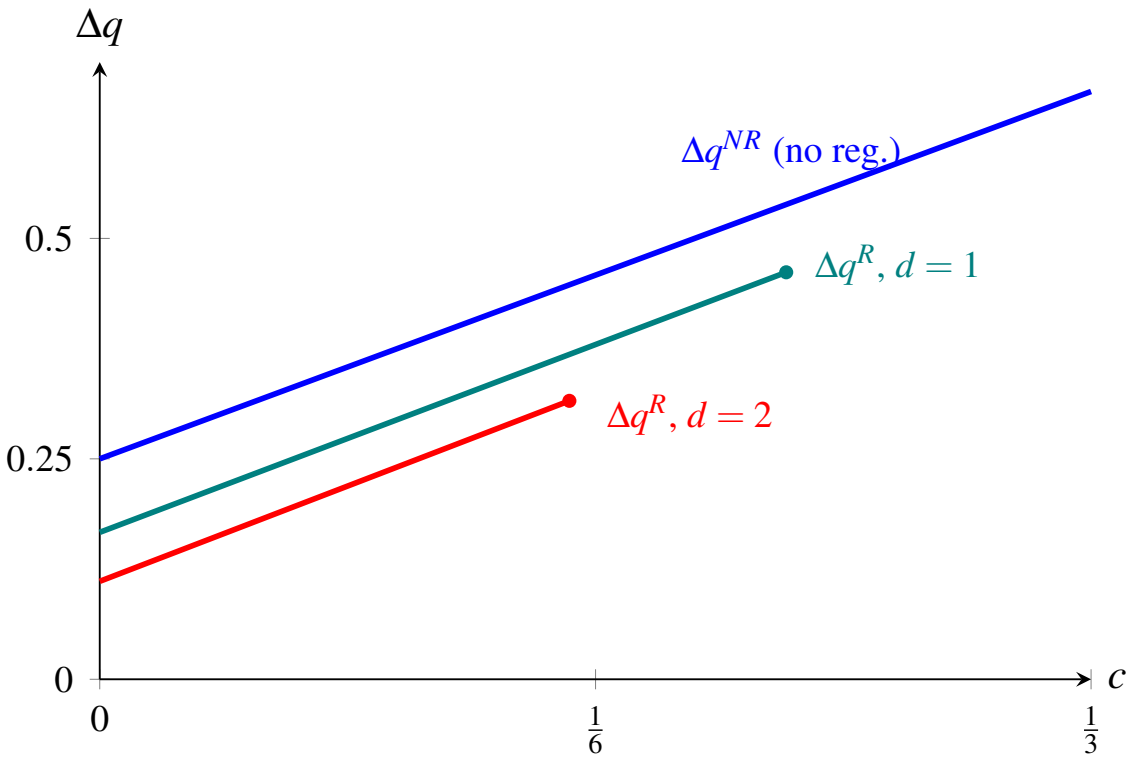
- The fee **increases** the cost advantage:

$$CA^R = \frac{11+12d}{9(1+d)}c > \frac{5c}{4} = CA^{NR} \quad \text{iff } d > \frac{1}{3}. \tag{5.30}$$

Comparing the Cost Advantages

- Both conditions in (5.29) and (5.30) hold since $d > \frac{1}{2}$ by assumption.
- In addition, $\frac{11+12d}{9(1+d)}c > c$ and $\frac{5c}{4} > c$ when $c > 0$.
- Therefore, $CA^R > CA^{NR} > c$.

Figure 5.3 – Output Advantage with and without Regulation



- Δq^R at $d = 1$ and $d = 2$, each drawn up to its cutoff c_A; both lie below Δq^{NR}.

Net Effect: Regulation Reduces the Output Advantage

- Comparing the output advantage with and without regulation:

$$\Delta q^{NR} - \Delta q^{R} = \frac{(3-c)(3d-1)}{36(1+d)}, \tag{5.31}$$

  or, more compactly,

$$\Delta q^{NR} - \Delta q^{R} = \frac{t^{Seq}}{4}. \tag{5.32}$$

- Same comparative statics as t^{Seq}: increases in d, decreases in c.

Recap: Section 5.2

- **No regulation**: $q_1^{NR} = \frac{1+c}{2}$, $q_2^{NR} = \frac{1-3c}{4}$; $\Delta q^{NR} = \frac{1}{4} + \frac{5}{4}c$.
- **With regulation**: $t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}$, increasing in d and decreasing in c.
- **Fee comparison**: $t^{Seq} > t^{C}$, since unregulated output is higher under Stackelberg.
- **Output advantage**: the fee reduces it by $\frac{t^{Seq}}{4}$; it lowers *FMA* but raises *CA*.
- **Second best**: with $c > 0$, a uniform fee keeps the high-cost follower active.

Section 5.3

Allowing for Two Time Periods

Motivation: Why a Two-Period Model?

- Many pollutants accumulate over time: part of first-period emissions persists into the second period.
- Then output decisions in one period affect environmental damage in the other.
- We can no longer treat the choice of emission fees in each period as two independent problems.

Two Periods: Timing

- **First period:**
  1. The regulator sets fee t_1, which only applies to first-period emissions.
  2. Every firm observes t_1 and simultaneously chooses its first-period output, x_i.
- **Second period:**
  3. The regulator sets fee t_2, which only applies to second-period emissions.
  4. Every firm observes t_2 and simultaneously chooses its second-period output, q_i.

Two Periods: Demand and Costs

- First period: $p(X) = 1 - X$, where $X = x_1 + x_2$.
- Second period: $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- Symmetric marginal cost, $c_i = c$, in both periods.
- In this section, c denotes the common marginal cost, not the cost advantage of Section 5.2.

Two Periods: Welfare in Each Period

- First period:

$$W_1 = CS_1 + PS_1 + T_1 - Env_1 \tag{5.33}$$

with $CS_1 = \frac{X^2}{2}$, $PS_1 = (1 - X)X - (c + t_1)X$, $T_1 = t_1 X$, and $Env_1 = \frac{d}{2}X^2$.

- Second period:

$$W_2 = CS_2 + PS_2 + T_2 - Env_2 \tag{5.34}$$

with $Env_2 = \frac{d}{2}E^2 = \frac{d}{2}(Q + \lambda X)^2$.

Pollution Persistence λ

- Second-period emissions: $E = Q + \lambda X$, where $\lambda \in [0, 1]$ denotes pollution persistence across periods.
- $\lambda = 0$: first-period emissions fully dissipate, $E = Q$; two independent problems, as in Section 2.3.
- $\lambda > 0$: part of first-period emissions persists, so

$$Env_2 = \frac{d}{2}(Q + \lambda X)^2 > \frac{d}{2}Q^2. \tag{5.35}$$

- $\lambda = 1$: all first-period emissions persist, $E = Q + X$.

Section 5.3.1 – Fourth Stage: Second-Period Output

- Taking t_2 as given, every firm solves

$$\max_{q_i \geq 0} \pi_2 = (1 - q_i - q_j)q_i - (c + t_2)q_i. \tag{5.36}$$

- Best response: $q_i(q_j) = \frac{1-(c+t_2)}{2} - \frac{1}{2}q_j$ if $q_j \leq 1 - c - t_2$, and zero otherwise. (5.37)
- Symmetric equilibrium:

$$q(t_2) = \frac{1 - c - t_2}{3}, \qquad Q(t_2) = \frac{2(1 - c - t_2)}{3}. \tag{5.38}$$

Section 5.3.1 – Third Stage: Second-Period Fee

- Observing λX, the regulator solves
$$\max_{t_2} W_2 = CS_2 + PS_2 + T_2 - Env_2 \text{ at } Q(t_2). \tag{5.39}$$
- Differentiating and solving for t_2:
$$t_2(X) = \underbrace{\frac{(1-c)(2d-1)}{2(1+d)}}_{t^*} + \frac{3d}{2(1+d)}\lambda X. \tag{5.40}$$
- $\lambda = 0$: $t_2(X) = t^*$, unaffected by X.
- $\lambda > 0$: more stringent, to account for the persisting emissions λX.

Second Period at Fee $t_2(X)$

- Every firm produces $q(t_2(X)) = \frac{1-c-d\lambda X}{2(1+d)}$, earning

$$\pi_2(t_2(X)) = \frac{(1-c-d\lambda X)^2}{4(1+d)^2}, \tag{5.41}$$

  which decreases in first-period output X.
- Second-period welfare:

$$W_2 = \frac{(1-c)^2}{2(1+d)} - \frac{d[2(1-c)+\lambda X]}{2(1+d)}\lambda X. \tag{5.42}$$

- First term: welfare when $\lambda = 0$. Second term: welfare loss from persistence.

Section 5.3.2 – Second Stage: First-Period Output

- Every firm anticipates $t_2(X)$ and its second-period profit:

$$\max_{x_i \geq 0} \underbrace{(1 - x_i - x_j)x_i - (c + t_1)x_i}_{\pi_1} + \pi_2(t_2(X)). \qquad (5.43)$$

- **Trade-off**: a larger x_i raises current profit, but more pollution persists and $t_2(X)$ becomes more stringent.
- A higher λ worsens this trade-off, giving firms more incentives to reduce first-period output.

Second Stage: Best Response

- Differentiating with respect to x_i:

$$x_i(x_j) = \frac{2(1-c-t_1)(1+d)^2-(1-c)d\lambda}{4(1+d)^2-d^2\lambda^2} - \frac{2+d[4+d(2-\lambda^2)]}{4(1+d)^2-d^2\lambda^2}x_j. \quad (5.44)$$

- $\lambda = 0$: $x_i(x_j) = \frac{1-c-t_1}{2} - \frac{1}{2}x_j$, as in the fourth stage.
- A small increase in λ from $\lambda = 0$ shifts this best response downward.

Second Stage: First-Period Output Function

- Invoking symmetry, $x_i = x_j = x$:

$$x(t_1) = \frac{2(1-c-t_1)(1+d)^2 - (1-c)d\lambda}{6(1+d)^2 - 2d^2\lambda^2}. \qquad (5.45)$$

- $\lambda = 0$: $x(t_1) = \frac{1-c-t_1}{3}$, symmetric with second-period output.

First Stage: First-Period Fee

- The regulator anticipates $x(t_1)$, $t_2(X)$, and $q(t_2(X))$, and solves

$$\max_{t_1} CS_1 + PS_1 + T_1 - Env_1 + W_2 \tag{5.46}$$

  evaluated at $X(t_1)$.
- A more stringent t_1 curbs current and future pollution, but lowers first-period welfare below its static maximum when $t_1 > t^*$.

First Stage: The Fee t_1

- Differentiating with respect to t_1 and solving:

$$t_1 = t^* + \frac{(1-c)d\left[2+3\lambda+d(2-\lambda)(1+\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}\lambda. \quad (5.47)$$

- Here $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ is the fee in (5.40) when $\lambda = 0$.
- $\lambda = 0$: $t_1 = t^*$; no intertemporal effects.
- $\lambda > 0$: the second term is positive, so $t_1 > t^*$.

Two Welfare Effects of a Stringent t_1

- **Positive**: it reduces future environmental damage.
- **Negative**: current welfare is lower than under fee t^*.
- Overall, the sum of welfare across periods is maximized with $t_1 > t^*$, and $t_2 > t^*$ as well.

The Second-Period Fee in Equilibrium

- First-period aggregate output at fee t_1:

$$X(t_1) = \frac{(1-c)[1+d(1-\lambda)]}{1+d(2+d+\lambda^2)}. \tag{5.48}$$

- Inserting $X(t_1)$ into $t_2(X)$ (see (5.49)):

$$t_2 = t^* + \frac{3(1-c)d\,[1+d(1-\lambda)]}{2(1+d)\,\left[1+d(2+d+\lambda^2)\right]}\lambda. \tag{5.50}$$

- When $\lambda > 0$, $t_2 > t^*$: both fees are more stringent than without intertemporal effects.

Positive Fees and the First Best

- Both fees are positive for all $d > \frac{1}{2}$, and can remain positive when pollution is milder.
- Example: $t_2 > 0$ iff $2d^2 + (1 + 3\lambda - \lambda^2)d - 1 > 0$, which holds for all $d > \frac{1}{2}$ when $\lambda = 0$, and for all $d > 0.28$ when $\lambda = 1$.
- The EA has two instruments, t_1 and t_2, to induce two output levels: fees t_1 and t_2 achieve the **first best**.

Section 5.3.3 – Fee Comparison across Periods

- Figure 5.4 evaluates t_1 and t_2 at $c = \frac{1}{2}$ and $d = 2$, as functions of λ; t^* is unaffected by λ.
- At $\lambda = 0$, all three fees coincide at $\frac{1}{4}$.
- Low λ: t_2 increases faster than t_1, so $t_2 > t_1 > t^*$.
- High λ: t_2 flattens and decreases, so $t_1 > t_2 > t^*$.

The Cutoff $\overline{\lambda}$

- $t_2 > t_1$ if and only if

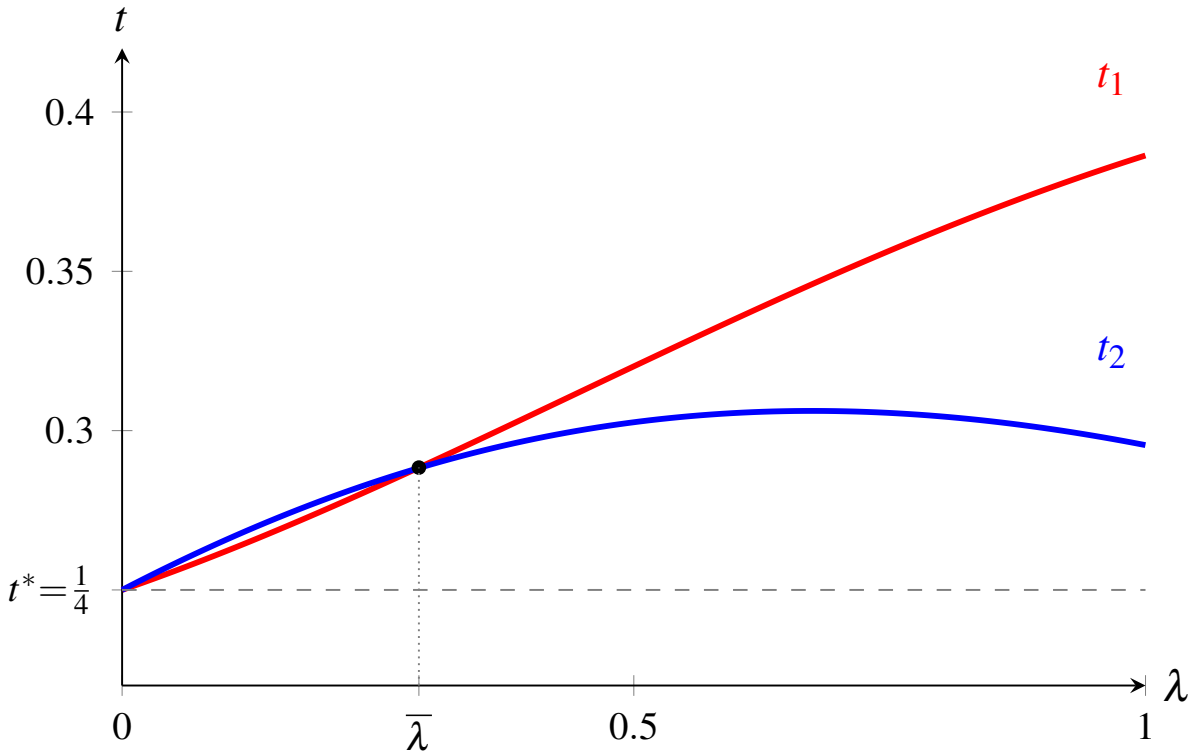
$$\lambda < \overline{\lambda} \equiv \frac{3 + 4d - [9 + 4d(5 + 3d)]^{1/2}}{2d}.$$

- At $d = 2$, $\overline{\lambda} = 0.29$, marked on the horizontal axis of Figure 5.4.
- $\overline{\lambda}$ decreases in d:

$$\frac{\partial \overline{\lambda}}{\partial d} = \frac{9 + 10d - 3[9 + 4d(5 + 3d)]^{1/2}}{2d^2[9 + 4d(5 + 3d)]^{1/2}} < 0. \qquad (5.51)$$

- More severe pollution expands the region where $t_1 > t_2$.

Figure 5.4 – Fees t_1 and t_2 as a Function of λ



- Both fees are almost identical when λ is low.

Section 5.3.4 – First-Period Welfare

- Evaluated at equilibrium fees and output:

$$W_1 = \frac{(1-c)^2}{2(1+d)} - \frac{(1-c)^2 d^2 \lambda (1+d+\lambda)^2}{2(1+d)[1+d(2+d+\lambda^2)]^2}\lambda. \quad (5.52)$$

- The first term, $\frac{(1-c)^2}{2(1+d)}$, is welfare when pollution is non-persistent, $\lambda = 0$.

Section 5.3.4 – Second-Period Welfare

- Using the same format:

$$
\begin{aligned}
W_2 \quad = \quad & \frac{(1-c)^2}{2(1+d)} \\
& - \frac{(1-c)^2 d\lambda\left[1+d(1-\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]^2} \\
& \times \left[2+4d+2d^2+\lambda+d\lambda+d\lambda^2\right].
\end{aligned}
\tag{5.53}
$$

- Both W_1 and W_2 originate at $\frac{(1-c)^2}{2(1+d)}$ when $\lambda = 0$.

Section 5.3.4 – Total Welfare

- Total welfare, $W = W_1 + W_2$:

$$W = \frac{(1-c)^2}{1+d} - \frac{(1-c)^2 d\lambda\left[2+\lambda+d(2-\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}. \quad (5.54)$$

- The last term is positive, so $W < \frac{(1-c)^2}{1+d}$ when $\lambda > 0$.
- Every unit of first-period emissions generates more damage, and the regulator sets more stringent fees in both periods.

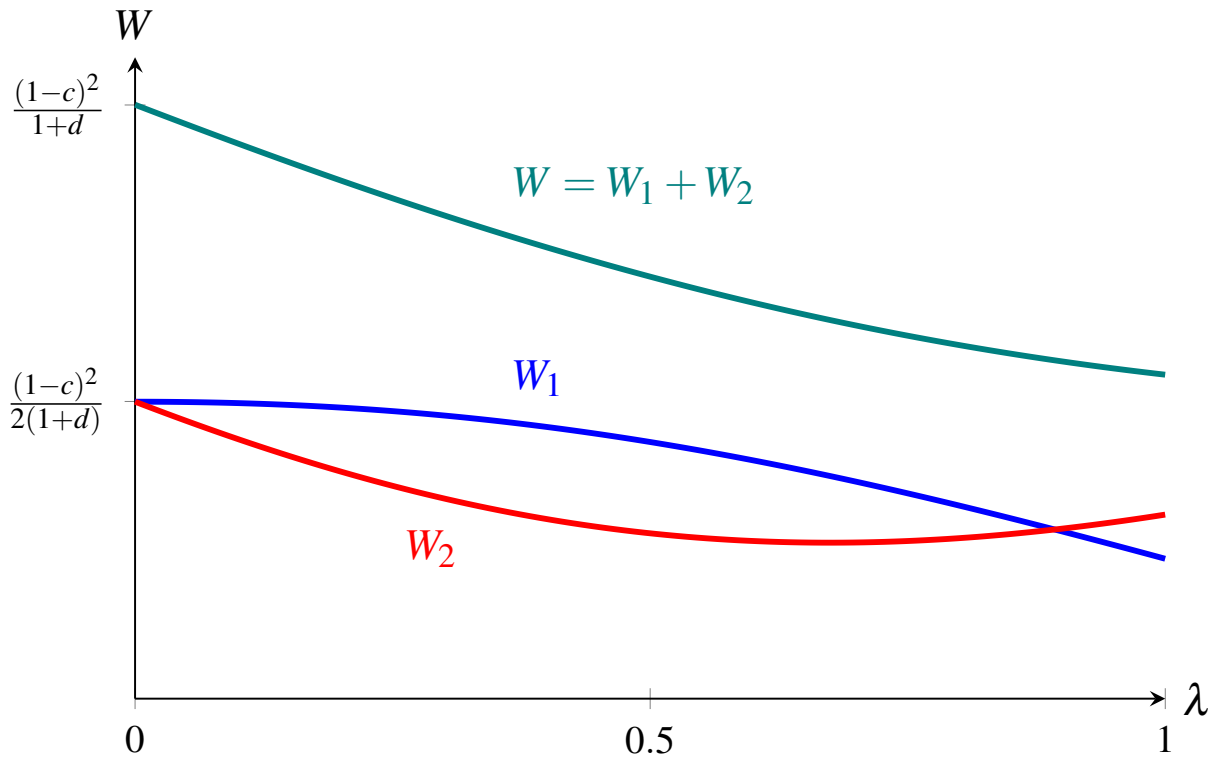
Total Welfare Decreases in λ

- Differentiating (5.54) with respect to λ:

$$\frac{\partial W}{\partial \lambda} = -\frac{(1-c)^2 d(1+d+\lambda)[1+d(1-\lambda)]}{[1+d(2+d+\lambda^2)]^2} < 0.$$

- More persistent pollution decreases total welfare, even though both fees adjust optimally.

Figure 5.5 – Equilibrium Welfare in Each Period



- W_1 decreases in λ; W_2 decreases, then increases ($\lambda \simeq 0.67$).
- At $c = \frac{1}{2}$ and $d = 2$, $W_2 < W_1$ for all $\lambda < 0.89$.

Recap: Section 5.3 Key Results

- $\lambda = 0$: both periods are independent and $t_1 = t_2 = t^*$.
- $\lambda > 0$: both fees exceed t^*; $t_2(X)$ responds to the persisting emissions λX.
- **Fee ranking**: $t_2 > t_1$ if $\lambda < \overline{\lambda}$; $t_1 > t_2$ otherwise.
- **Welfare**: total welfare decreases in λ; fees t_1 and t_2 achieve the first best.

Section 5.4

Regulatory Commitment

Time-Consistent Policy vs. Commitment

- In Section 5.3, the regulator chooses t_2 after observing first-period output and the emissions that persist, λX.
- This ability to revise t_2 at the beginning of the second period is known as *time-consistent policy* (Petrakis and Xepapadeas, 1999; Poyago-Theotoky, 2007).
- Alternatively, the regulator could **commit** to the fee pair (t_1, t_2) in the first stage, without revising it later.

Commitment: Game Structure

1. **Stage 1**: the regulator commits to the fee pair (t_1, t_2).
2. **Stage 2**: every firm i chooses its first-period output, x_i.
3. **Stage 3**: every firm i chooses its second-period output, q_i.

Stage 3 coincides with stage 4 in Section 5.3.1, so $Q(t_2) = \frac{2(1-c-t_2)}{3}$.

Commitment: First-Period Output

- In stage 2, every firm solves

$$\max_{x_i \geq 0} \underbrace{(1 - x_i - x_j)x_i - (c + t_1)x_i}_{\pi_1} + \pi_2(t_2), \tag{5.55}$$

  where $\pi_2(t_2) = \frac{(1-c-t_2)^2}{9}$ since t_2 was fixed in stage 1.
- Then $\pi_2(t_2)$ is a constant, not affecting first-period output.
- In Section 5.3.2, in contrast, the firm anticipates $t_2(X)$ for each value of X.

Commitment: First-Period Output Function

- Best response: $x_i(x_j) = \frac{1-(c+t_1)}{2} - \frac{1}{2}x_j$ if $x_j \leq 1 - c - t_1$, and zero otherwise. (5.56)
- Invoking symmetry:

$$x^{Com}(t_1) = \frac{1 - c - t_1}{3}, \tag{5.57}$$

  so $X^{Com}(t_1) = \frac{2(1-c-t_1)}{3}$.
- Firms anticipate that their first-period output does not affect the second-period fee.

Commitment: The Regulator's Problem

- In stage 1, the regulator solves $\max_{t_1, t_2} W_1 + W_2$, where

$$\begin{aligned} W_1 &= \frac{X^2}{2} + (1 - X)X - cX - \frac{d}{2}X^2, \\ W_2 &= \frac{Q^2}{2} + (1 - Q)Q - cQ - \frac{d}{2}(Q + \lambda X)^2. \end{aligned}$$

- Evaluated at $X(t_1) = \frac{2(1-c-t_1)}{3}$ and $Q(t_2) = \frac{2(1-c-t_2)}{3}$.

Equilibrium Fees under Commitment

- Differentiating with respect to t_1 and t_2:

$$t_1^{Com} = t^* + \frac{3d(1-c)(1+d+\lambda)}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}\lambda, \quad (5.58)$$

$$t_2^{Com} = t^* + \frac{3d(1-c)[1+d(1-\lambda)]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}\lambda, \quad (5.59)$$

  where $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$.

- Superscript *Com* denotes commitment.

Fee Ranking under Commitment

- The difference between the second terms in (5.58) and (5.59) is

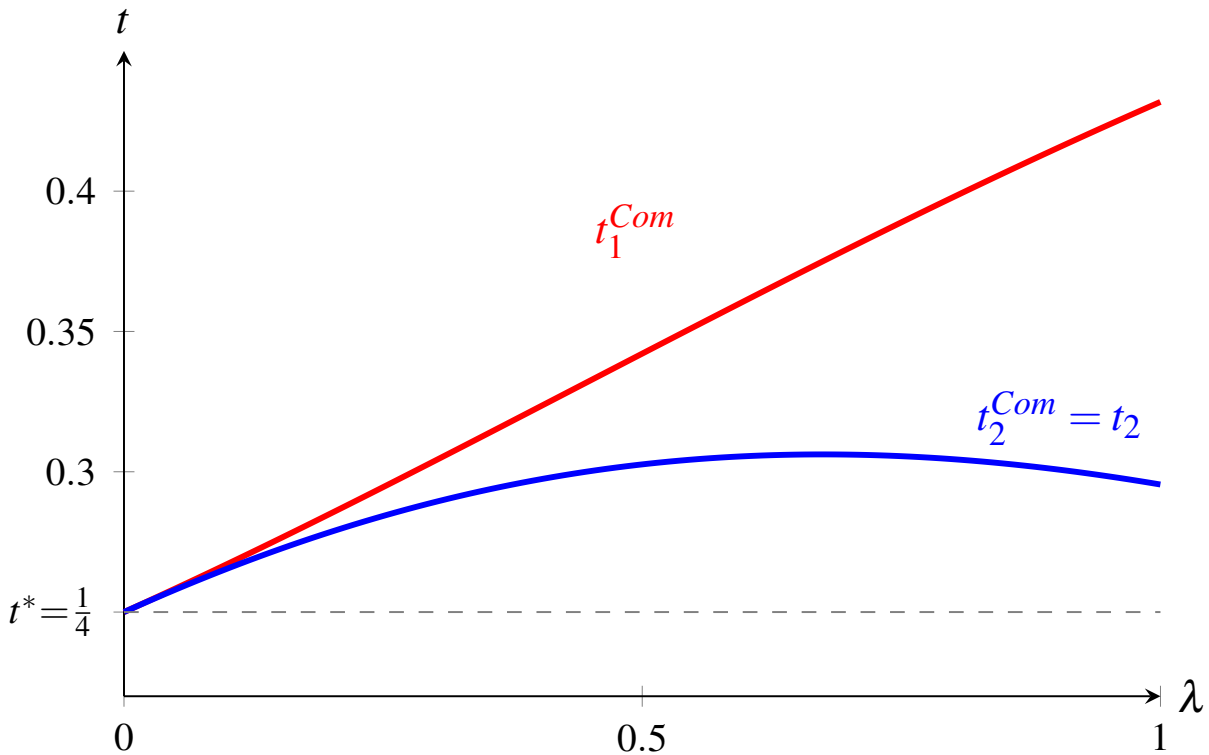
$$\begin{aligned} & \frac{3d(1-c)(1+d+\lambda)}{2(1+d)\left[1+d(2+d+\lambda^2)\right]} \\ & -\frac{3d(1-c)[1+d(1-\lambda)]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]} \\ = \quad & \frac{3d\lambda(1-c)}{2\left[1+d(2+d+\lambda^2)\right]} > 0. \end{aligned} \tag{5.60}$$

- Unambiguous fee ranking: $t_1^{Com} \geq t_2^{Com} \geq t^*$. (5.61)

Fees under Commitment and λ

- $\lambda = 0$: all fees coincide, $t_1^{Com} = t_2^{Com} = t^*$.
- $\lambda > 0$: every unit of emissions generates more damage, so both fees become more stringent.
- t_1^{Com} increases in λ; t_2^{Com} increases in λ when pollution is not very persistent but decreases otherwise.

Figure 5.6 – Emission Fees with Commitment



- Same parameters as Figure 5.4; t_1 appears in Figure 5.4.

Section 5.4.1 – Fees with and without Commitment

- First-period fees:

$$t_1^{Com} - t_1 = \frac{d\lambda(1-c)\left[1+d(1-\lambda(1-\lambda))\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]} > 0. \quad (5.62)$$

- Positive since $1-\lambda(1-\lambda) \geq \frac{3}{4}$ for all $\lambda \in [0,1]$, with its minimum at $\lambda = \frac{1}{2}$.
- Second-period fees coincide: $t_2^{Com} = t_2$.

Why Is t_1^{Com} More Stringent?

- Under commitment, firms no longer anticipate that a larger first-period output raises the second-period fee.
- Then they produce more at any given t_1.
- The regulator offsets this with a more stringent first-period fee, $t_1^{Com} > t_1$.

Section 5.4.2 – Welfare with and without Commitment

- First-period welfare with commitment coincides with (5.52):

$$W_1^{Com} = W_1. \tag{5.63}$$

- Second-period welfare with commitment coincides with (5.53):

$$W_2^{Com} = W_2. \tag{5.64}$$

- Therefore, total welfare $W^{Com} = W_1^{Com} + W_2^{Com}$ is the same as under no commitment.

Why Is Commitment Inconsequential for Welfare?

- At first glance, commitment seems "bad news": the regulator cannot revise t_2.
- But the regulator anticipates this when choosing t_1^{Com}, offsetting firms' larger first-period output.
- Both regimes induce the same output levels, and welfare, across periods.
- These output levels coincide with those a social planner would choose: the first best, with and without commitment.

Welfare Equivalence Requires Optimal Fees

- The result $W^{Com} = W$ compares both regimes when the regulator chooses its fees optimally.
- With commitment, welfare is strictly concave in (t_1, t_2) and reaches the first best only at (t_1^{Com}, t_2^{Com}).
- Committing to any other fee pair, such as $t_1 = t_2 = t^*$ when $\lambda > 0$, yields lower welfare than the regulator obtains without commitment.

Recap: Section 5.4 Key Results

- **Commitment**: the regulator sets (t_1, t_2) in advance; firms take t_2 as given.
- **Fee ranking**: $t_1^{Com} \geq t_2^{Com} \geq t^*$ for all λ; without commitment, the ranking depends on λ.
- **Fee comparison**: $t_1^{Com} > t_1$ and $t_2^{Com} = t_2$.
- **Welfare**: $W^{Com} = W$; the regulator achieves the first best in both regimes.

Section 5.5

Case Studies

Case Study 5.5.1: World Crude Oil Market

- Huppmann and Holz (2012) compare Cournot, Stackelberg, cartel, and competitive numerical models of this market.
- Before 2008: prices and quantities closest to a Stackelberg outcome, with Saudi Arabia leading a non-cooperative OPEC oligopoly and a competitive fringe; in 2008 and 2009, closer to the competitive benchmark.
- Norway has taxed CO_2 emissions from petroleum activities on its continental shelf since 1991, in addition to the costs of EU ETS allowances.
- *Connection to the model*: the leader's output shifts the followers' residual demand; a fee raises every producer's marginal cost. The evidence comes from a calibrated model, not a direct test of the optimal fee.

Case Study 5.5.2: PJM Electricity Market

- NO_x emissions from power plants were capped under allowance trading programs (1999–2002 and 2003–2008).
- Chen et al. (2006) calibrate a leader-follower model: the largest generator chooses first, medium-sized firms respond as Cournot followers, and small firms act as a competitive fringe.
- In their simulations, the leader withholds NO_x allowances, raising the allowance price and rivals' compliance costs; electricity prices barely change.
- *Connection to the model*: numerical results relying on tradable allowances (Section 2.5.2). Under a fixed fee, as in Section 5.2, the leader could not affect its rivals' compliance cost per unit of emissions.

Case Study 5.5.3: Spanish Electricity Generation

- Wholesale electricity is traded in a daily auction run by OMIE; generators submit their offers by the same deadline.
- Power plants do not pay a national carbon tax; they must hold EU ETS allowances, so allowance prices act as a per-unit emission cost.
- Fabra and Reguant (2014), with plant-level data: emission costs were almost fully passed through to electricity prices.
- *Connection to the model*: with simultaneous offers, the timing of Section 5.2 is an interpretation, most useful for longer-run choices such as capacity investment. The pass-through evidence does not test sequential vs. simultaneous competition.

Additional Example: Aviation in the EU ETS

- Since 2012, the EU ETS has covered aviation; its scope has been temporarily limited to flights within the European Economic Area.
- Free allocation of allowances to airlines is being phased out, with full auctioning from 2026.
- In June 2024, the Lufthansa Group announced a surcharge of 1 to 72 euros per ticket, for departures from January 2025, to cover the EU's sustainable-fuel blending quota, EU ETS changes, and other environmental costs.
- *Connection to the model*: the allowance price raises every airline's marginal cost, as fee t does. Viewing a hub carrier as a leader is an interpretation, not a documented feature of this market.

Section 5.6

Related Literature

Related Literature

- **Espinola-Arredondo et al. (2019)**: an incumbent decides whether to adopt green technology; then an entrant decides whether to enter and invest. Committing to fees before firms act lets the regulator induce competition, deter entry, or encourage green investment, which can yield higher welfare than reactive regulation.
- **Choi et al. (2024)**: separating first-mover and cost advantages, fees under Stackelberg competition reduce the leader's dominance, particularly when pollution is severe. Assuming simultaneous competition when firms compete sequentially leads to inefficient fees and welfare losses. Their model also includes firm-specific fees and abatement investment.

Further Reading (Beyond the Book) I

- **Stackelberg (1934)**, *Marktform und Gleichgewicht*, Springer: introduces the leader-follower duopoly model.
- **Ulph (1996)**, "Environmental Policy and International Trade when Governments and Producers Act Strategically," *Journal of Environmental Economics and Management*: strategic R&D by producers weakens governments' incentive to distort policy.
- **Kydland and Prescott (1977)**, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*: discretionary policy does not maximize the social objective.

Further Reading (Beyond the Book) II

- **Biglaiser, Horowitz, and Quiggin (1995)**, "Dynamic Pollution Regulation," *Journal of Regulatory Economics*: emission taxes achieve the first best and are time consistent.
- **Benchekroun and Long (2002)**, "On the Multiplicity of Efficiency-Inducing Tax Rules," *Economics Letters*: with a pollution stock, many tax rules induce the optimal output path.
- **Requate (2005)**, "Dynamic Incentives by Environmental Policy Instruments—A Survey," *Ecological Economics*: incentives to develop and adopt abatement technology, including policy timing and commitment.

Section 5.7

Summary

Summary I: Sequential Competition

- In Stackelberg settings, the leader moves first, influencing the follower's output decision.
- The leader benefits from a first-mover advantage and, when costs differ, a cost advantage: it produces more than the follower.
- Unregulated aggregate output is higher under Stackelberg than under Cournot competition.

Summary II: Emission Fees and Output Advantage

- The fee is more stringent under Stackelberg than under Cournot competition, $t^{Seq} > t^{C}$.
- The fee becomes more stringent with more severe pollution and less stringent with more cost asymmetry.
- Fees reduce the leader's output advantage by $\frac{t^{Seq}}{4}$, lowering the leader's output more than the follower's.

Summary III: Two Periods and Commitment

- With persistent pollution, both fees exceed t^*; total welfare decreases in λ.
- Without commitment, the regulator adjusts t_2 to realized first-period emissions.
- With commitment, t_1 is higher, offsetting firms' weaker incentive to restrain first-period output.
- Total welfare is the same with and without commitment.

Summary IV: Functional Forms

- Closed-form fees rely on linear demand and constant marginal costs.
- When firms' costs differ, a single (uniform) fee only achieves a second best.

Thank You

Questions and Discussion

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 6

Regulation When Firms Invest in Abatement

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 6

- This chapter extends the baseline model by allowing firms to invest in abatement technologies that reduce emissions.
- Sections:
  - 6.1 Introduction. 6.2 Modeling assumptions.
  - 6.3 Non-cooperative abatement: no spillovers and spillovers.
  - 6.4 Cooperative abatement decisions (ERCs and ERJVs).
  - 6.5 Sequential abatement decisions (no spillovers).
  - 6.6 Different regulatory timing: the regulator moves first.
  - 6.7 Other extensions. 6.8 Case studies. 6.9 Summary.

Section 6.1

Introduction

What this chapter studies

- Previous chapters examined how firms adjust *output* in response to emission fees, taking abatement technology as fixed.
- This chapter introduces a richer environment: firms choose both their output levels **and** how much to invest in abatement technologies.
- Examples of abatement investment: smokestack scrubbers, wastewater treatment systems, advanced filtration, carbon capture equipment.
- Key question: how do abatement investments interact with the regulator's emission fee? Does one "crowd out" the other?
- The baseline model from Chapter 2 is recovered as a special case when abatement is prohibitively expensive ($\gamma \to +\infty$).

Two new strategic layers

- **Firm–regulator interaction:** each firm's abatement investment lowers net emissions, inducing the regulator to set a *less stringent* fee: the **tax-saving externality (TSE)**.
- **Firm–firm interaction:** when abatement generates spillovers, one firm's investment also reduces its rival's net emissions: the **emission-reduction externality (ERE)**.
- These two externalities create **free-riding incentives** and determine whether firms should coordinate their abatement choices.
- This chapter systematically studies TSE and ERE under non-cooperative and cooperative abatement.

Chapter 6 in Context: Extensions of Chapter 2

- **Chapter 2:** Cournot duopoly + emission fee; no abatement investment by firms.
- **Chapter 6 adds:**
  - Abatement investment stage (before fee-setting).
  - Spillovers in abatement (β).
  - Cooperative abatement (ERC/ERJV).
  - Sequential timing in abatement.
  - Alternative regulatory timing (fee before abatement).
- **What changes:**
  - The optimal emission fee is lower (abatement partially corrects externality).
  - Firms' strategies now involve both output and abatement.
  - Free-riding incentives are richer: TSE and ERE.
- Chapter 6 provides a more complete picture of how firms respond to environmental regulation in the presence of clean technology investment.

Guiding Questions

1. What motivates firms to invest in abatement technologies under regulation?
2. How do spillovers in abatement affect strategic and cooperative behavior?
3. What are the welfare implications of environmental research joint ventures (ERJVs)?
4. How does the timing of abatement decisions influence equilibrium outcomes?
5. What is the role of regulatory commitment in shaping investment strategies?
6. How do different regulatory designs affect incentives for green innovation?

Real-World Examples of Abatement Investment

- **Smokestack scrubbers:** power plants install flue-gas desulfurization systems to capture SO_2 before it is emitted.
- **Wastewater treatment:** chemical plants invest in treatment facilities to reduce toxic discharges; the cost is sunk once installed.
- **Advanced filtration:** industrial plants install filtration systems that capture emissions before they leave the plant.
- **In the model:** the regulator observes total industry abatement and then sets the emission fee. This interaction is the core mechanism of Chapter 6.

Section 6.2

Modeling Assumptions

Basic Setup

- Duopoly with inverse demand $p(Q) = 1 - Q$ and symmetric marginal cost c, where $1 > c \geq 0$.
- Firms can invest in **abatement technologies** $z_i \geq 0$ to reduce their net emissions.
- Without abatement: $e_i = q_i$ (emissions equal output).
- With abatement and **spillover parameter** $\beta \in [0, 1]$:

$$e_i = q_i - z_i - \beta z_j$$

- Three cases for β:
  - $\beta = 0$: no spillovers – firm i's emissions reduced only by its own investment.
  - $\beta \in (0, 1)$: partial spillovers – firm i benefits from a share β of rival's abatement.
  - $\beta = 1$: full spillovers – $e_i = q_i - z_i - z_j$.

Abatement Cost and Investment Efficiency

- Total cost of investing in abatement:

$$C(z_i) = \frac{\gamma(z_i)^2}{2}$$

  increasing and *convex* in abatement z_i.
- Parameter $\gamma > 0$ denotes **investment efficiency**:
  - Lower $\gamma \Rightarrow$ higher efficiency (abatement cheaper).
  - $\gamma \to 0$: abatement is essentially free.
  - $\gamma \to +\infty$: abatement is prohibitively costly; $z_i = 0$ and results collapse to Chapter 2.
- Marginal cost of abatement: γz_i – positive and increasing in z_i.
- Firm pays emission fee $te_i = t(q_i - z_i - \beta z_j)$ in stage 2.

Three-Stage Game (Poyago-Theotoky, 2007)

- **Stage 1 (Abatement):** Every firm i independently and simultaneously invests in abatement $z_i \geq 0$.
- **Stage 2 (Fee-setting):** The regulator observes investment decisions $z = (z_1, z_2)$ and sets per-unit emission fee t (a subsidy if $t < 0$).
- **Stage 3 (Output):** Every firm i observes z and t, then chooses output q_i (Cournot competition).
- Key feature: firms invest in abatement *before* the regulator sets the fee – strategic anticipation of regulatory response.
- Stages 2–3 coincide with Chapter 2; Stage 1 is new.
- Section 6.6 reverses this timing: regulator moves first.

Welfare Function with Abatement

- Welfare function retains the same structure as in previous chapters:
$$W = CS + PS + T - Env$$
- Aggregate abatement: $Z = z_1 + z_2$; aggregate emissions: $E = Q - Z$.
- Tax revenue: $T = tE = t(Q - Z)$.
- Environmental damage:
$$Env = \frac{d}{2}E^2 = \frac{d}{2}(Q - Z)^2$$
where $d > \frac{1}{2}$ measures **pollution severity**.
- Two channels through which abatement improves welfare:
  - *Direct*: reduces environmental damage for given output.
  - *Indirect*: lowers fee stringency, boosting output, CS and PS.

Backward Induction: Solving the Three-Stage Game

- We solve the game by **backward induction**: start from Stage 3 and work back to Stage 1.
- **Stage 3** $\rightarrow$ output as a function of (t, z_i, z_j).
- **Stage 2** $\rightarrow$ optimal fee as a function of (z_i, z_j).
- **Stage 1** $\rightarrow$ optimal abatement given anticipated fee and output.
- This approach ensures *subgame perfect* equilibrium: each firm's strategy is optimal at every stage, given the strategies of all other players.
- Key insight: firms in Stage 1 anticipate how their abatement affects the regulator's fee in Stage 2, which in turn affects output in Stage 3. This is the strategic "abatement for tax relief" mechanism.

Welfare Components with Abatement

- Welfare $W = CS + PS + T - Env$ with abatement:
  - $CS = \frac{Q^2}{2}$ with $Q = Q(t)$.
  - $PS = \sum_i \pi_i = \sum_i \left[q^2 + tz_i - \frac{\gamma z_i^2}{2} \right]$.
  - $T = tE = t(Q - Z)$ – tax revenue net of abatement.
  - $Env = \frac{d}{2}(Q - Z)^2$ – environmental damage on net emissions.
- Note: abatement costs $\frac{\gamma}{2}(z_i^2 + z_j^2)$ are *subtracted* from profits and thus from welfare – abatement is costly but improves the environment.
- Regulator maximizes W choosing t in Stage 2, taking $Z = (z_1 + z_2)$ as given (already chosen in Stage 1).
- The regulator does *not* control abatement directly; only the emission fee.

Section 6.3

Non-Cooperative Abatement

Stage 3: Output Competition (No Spillovers, $\beta = 0$)

- Each firm i chooses q_i to maximize:

$$(1 - q_i - q_j)q_i - cq_i - t(q_i - z_i)$$

- First-order condition yields best response: $q_i(q_j) = \frac{1-c-t}{2} - \frac{1}{2}q_j$.
- Symmetric equilibrium: each firm's output $q = \frac{1-c-t}{3}$, aggregate output $Q(t) = \frac{2(1-c-t)}{3}$.
- Equilibrium profit:

$$\pi_i = \frac{(1-c-t)^2}{9} + tz_i = q^2 + tz_i$$

- Profit π_i increases in abatement z_i because abatement lowers the fee the firm effectively pays.

Stage 2: Optimal Emission Fee

- Regulator sets t to maximize $W = CS + PS + T - Env$, anticipating equilibrium output $Q(t) = \frac{2(1-c-t)}{3}$.
- First-order condition (differentiating W w.r.t. t) yields fee (6.9):

$$t(Z) = \frac{(2d-1)(1-c) - 3dZ}{2(1+d)}$$

- Fee properties:
  - *Increases* with pollution severity d: $\frac{\partial t(Z)}{\partial d} > 0$ when $Z < 1 - c$.
  - *Decreases* with marginal cost c: $\frac{\partial t(Z)}{\partial c} < 0$.
  - **Decreases with aggregate abatement** Z: $\frac{\partial t(Z)}{\partial z_i} = -\frac{3d}{2(1+d)} < 0$.
- Key insight: when either firm invests more in abatement, the regulator anticipates fewer net emissions and *reduces* the fee.

Result: Abatement Reduces the Emission Fee

- **Result (6.10).** Without spillovers ($\beta = 0$), the fee $t(Z)$ satisfies
$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d}{2(1+d)} < 0$$
- That is, **any increase in abatement, by either firm, reduces the emission fee**.
- **Intuition:** higher abatement $\Rightarrow$ lower net emissions $\Rightarrow$ less need for a stringent fee to correct the environmental externality.

Free-Riding through a Less Stringent Fee

- The effect in (6.10) is *symmetric* across firms.
- This symmetry allows firm i to **free-ride** on its rival's investment z_j:
  - Firm j invests more in abatement.
  - The regulator reduces the emission fee for every firm.
  - Firm i benefits from the lower fee *without* investing.
- Section 6.3.2 calls this the **tax-saving externality (TSE)**; it arises *with and without* spillovers.

Stage 1: Abatement Best Response Function

- Each firm i chooses z_i to maximize profit, anticipating equilibrium fee $t(Z)$:

$$\max_{z_i \geq 0} \pi_i = q^2 + t(Z)z_i - \frac{\gamma(z_i)^2}{2}$$

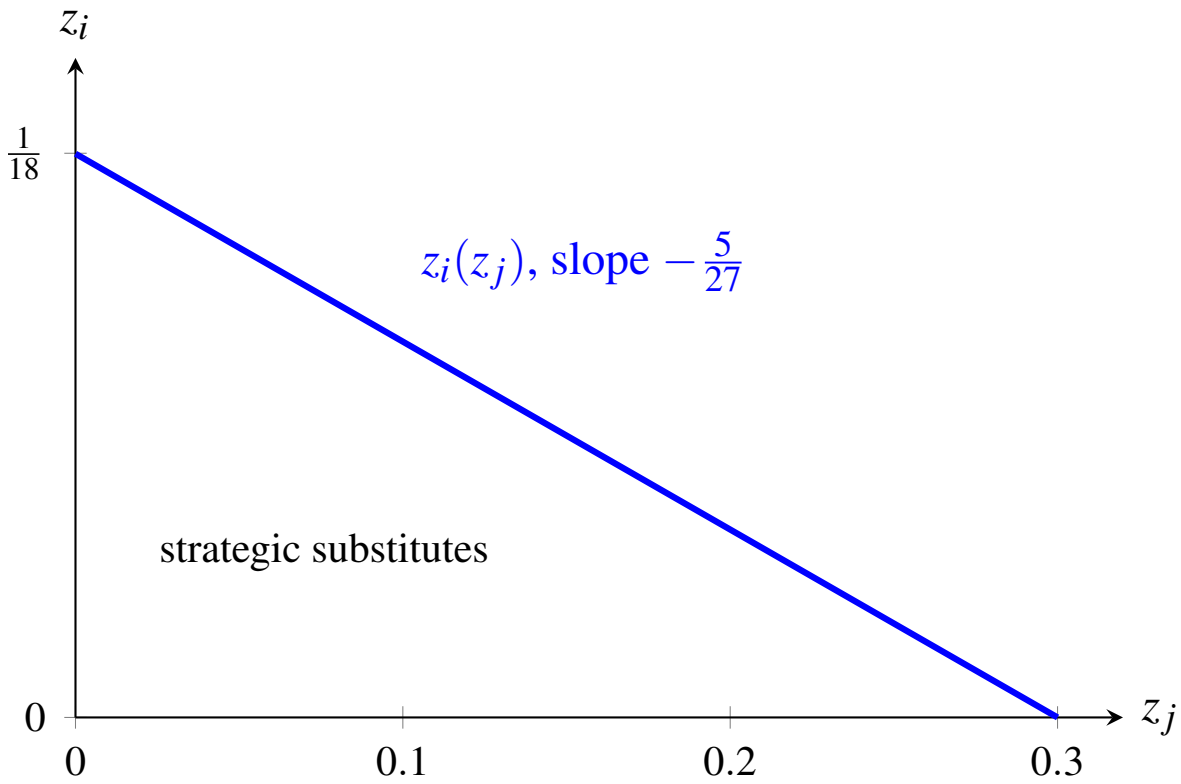
- First-order condition yields best response function (6.12):

$$z_i(z_j) = \underbrace{\frac{[2d(1+d)-1](1-c)}{B}}_{\text{vertical intercept}>0} - \underbrace{\frac{d(3+2d)}{B}}_{|\text{slope}|} z_j$$

  where $B \equiv d(6+5d) + 2\gamma(1+d)^2$.

- **Negative slope**: abatement levels are *strategic substitutes*.

Figure 6.1 – Best Response Function $z_i(z_j)$



- Evaluated at $c = \frac{1}{2}$, $d = 1$, and $\gamma = 2$: $z_i(z_j) = \frac{1}{18} - \frac{5}{27} z_j$.

Intuition: Strategic Substitutability in Abatement

- **Why are abatement decisions strategic substitutes?**
- When firm j invests more: $z_j \uparrow$
  1. Total abatement $Z = z_i + z_j$ increases.
  2. Regulator anticipates lower net emissions $\Rightarrow$ reduces fee $t(Z)$.
  3. Lower fee reduces marginal benefit of firm i's abatement (tax saved per unit of abatement falls).
  4. Firm i responds by investing less: $z_i \downarrow$.
- The negative slope of $z_i(z_j)$ is the graphical representation of this substitutability.

Pollution Severity and the Best Response: Intercept

- Let $z_i^{Vert.}$ and z_i^{Slope} denote the first and second terms in best response function (6.12).
- **Vertical intercept**: $\frac{\partial z_i^{Vert.}}{\partial d} > 0$, see (6.13). The best response shifts *upward* when pollution is more severe.
- Intuition: higher $d \Rightarrow$ firms anticipate a more stringent fee $\Rightarrow$ stronger incentive to invest in abatement.

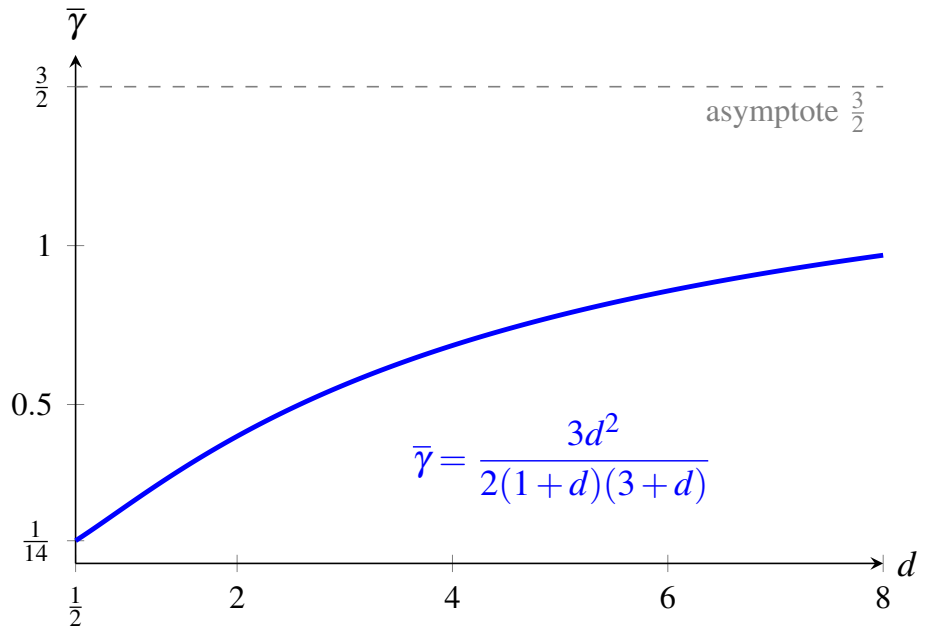
Pollution Severity and the Best Response: Slope

- **Slope**: from (6.14) and (6.15),

$$\frac{\partial z_i^{Slope}}{\partial d} = \frac{3d^2 - 2\gamma(1+d)(3+d)}{\left[2\gamma(1+d)^2 + d(6+5d)\right]^2}$$

  which is negative if and only if $\gamma > \overline{\gamma} \equiv \frac{3d^2}{2(1+d)(3+d)}$.
- When abatement is relatively costly, $\gamma > \overline{\gamma}$, the best response becomes *steeper* as d increases: stronger free-riding incentives.
- Reason: the fee responds more strongly to abatement when pollution is more severe, since $\left|\frac{\partial t(Z)}{\partial z_j}\right| = \frac{3d}{2(1+d)}$ increases in d.

Figure 6.2 – Cutoff $\overline{\gamma}$ as a Function of d



- $\overline{\gamma}$ originates at $\gamma = \frac{1}{14}$ when $d = \frac{1}{2}$, increases in d, and approaches $\gamma = \frac{3}{2}$ when $d \to +\infty$.
- Hence, when abatement is relatively costly, $\gamma > \frac{3}{2}$, condition $\gamma > \overline{\gamma}$ holds for *all* values of d.

Equilibrium Abatement (No Spillovers)

- In symmetric equilibrium ($z_i = z_j$), equilibrium abatement is (6.16):

$$z_i^* = \frac{(1-c)[2d(1+d)-1]}{2\gamma(1+d)^2 + d(9+7d)} > 0$$

  (positive since $d > \frac{1}{2}$ by assumption).
- **Comparative statics:**
  - $\frac{\partial z_i^*}{\partial d} > 0$: higher pollution severity $\Rightarrow$ more abatement investment.
  - $\frac{\partial z_i^*}{\partial \gamma} < 0$: more expensive abatement $\Rightarrow$ less investment.
  - $\frac{\partial z_i^*}{\partial c} < 0$: higher marginal cost $\Rightarrow$ less output, fewer emissions, less stringent fee, less investment.

Result: Equilibrium Abatement Increases in d

- **Result (6.17).** $\frac{\partial z_i^*}{\partial d} > 0$, $\frac{\partial z_i^*}{\partial \gamma} < 0$, and $\frac{\partial z_i^*}{\partial c} < 0$.
- **Higher d:** firms anticipate a more stringent fee, which increases the tax savings from every unit of abatement.
- **Higher γ:** abatement is more expensive, so firms invest less.
- **Higher c:** less output $\Rightarrow$ fewer emissions $\Rightarrow$ less stringent fee $\Rightarrow$ weaker need to invest in abatement.

Figure 6.3a – Equilibrium Abatement Decreases in γ

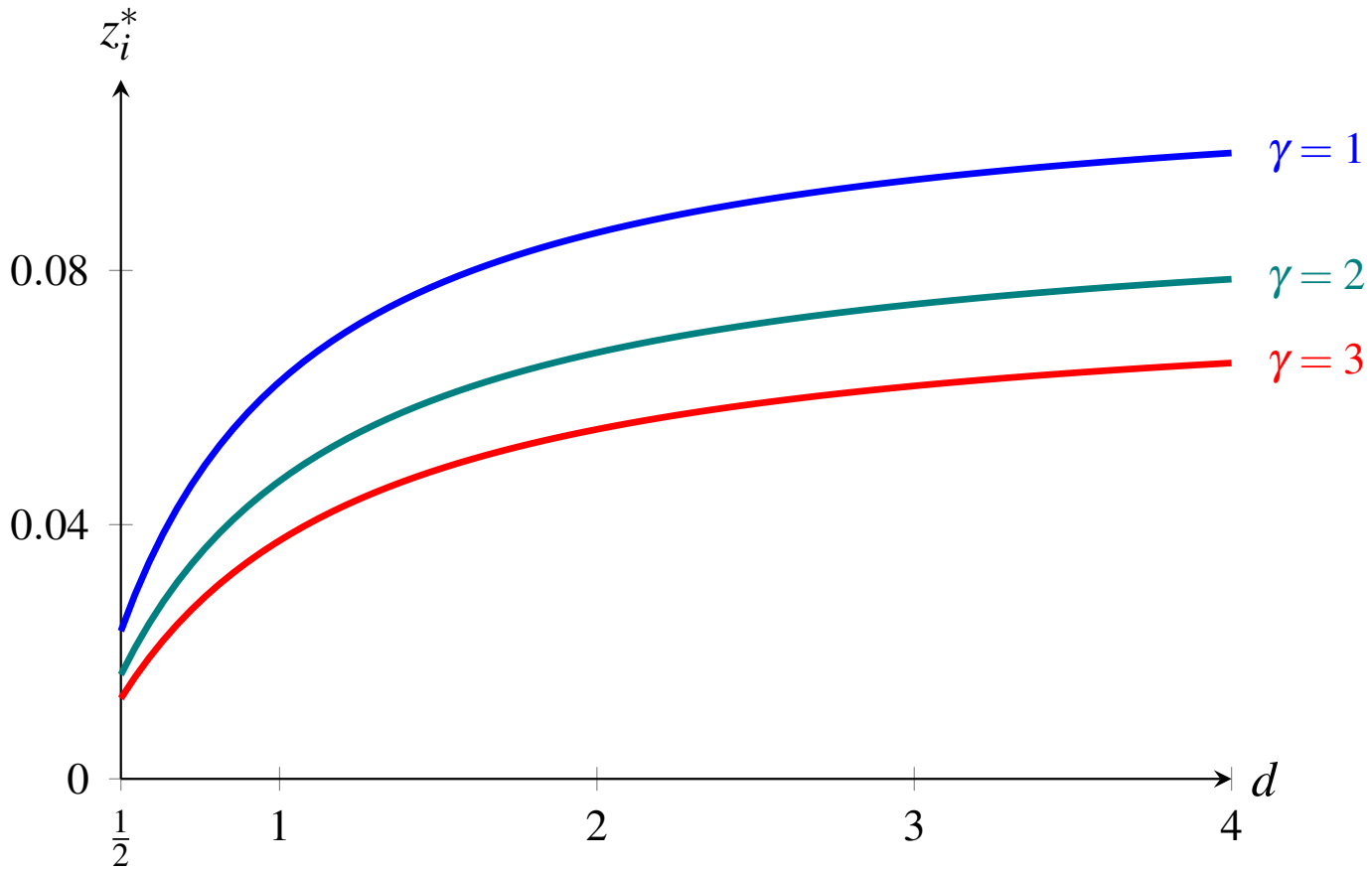
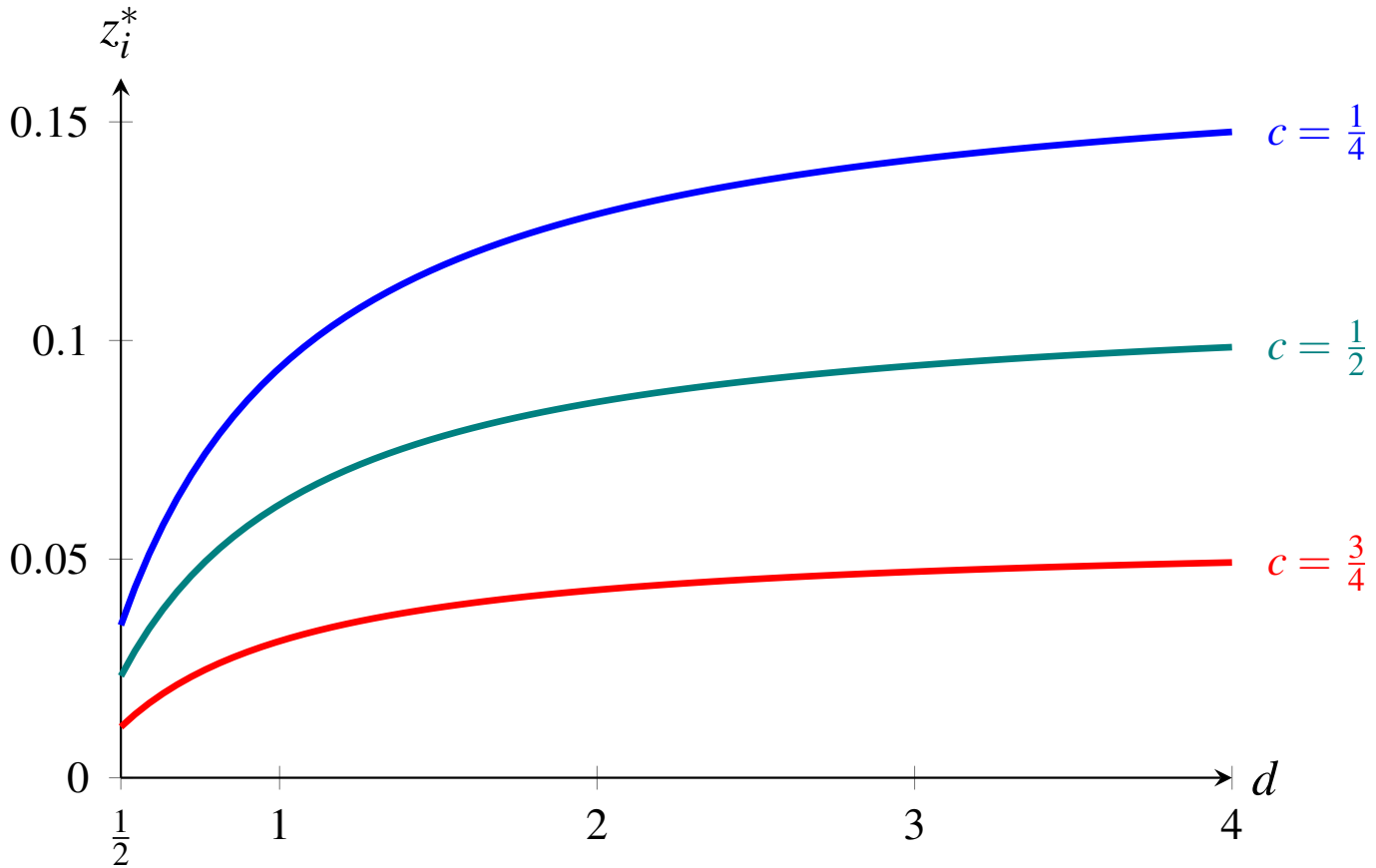


Figure 6.3b – Equilibrium Abatement Decreases in c



Equilibrium Emission Fee (No Spillovers)

- Substituting $Z^* = 2z_i^*$ into $t(Z)$, equilibrium fee (6.18):

$$t(Z^*) = \frac{(1-c)[d(2\gamma - 3 + 2d(1+2\gamma)) - 2\gamma]}{4\gamma(1+d)^2 + 2d(9+7d)}$$

- Fee is **positive** if and only if (6.19):

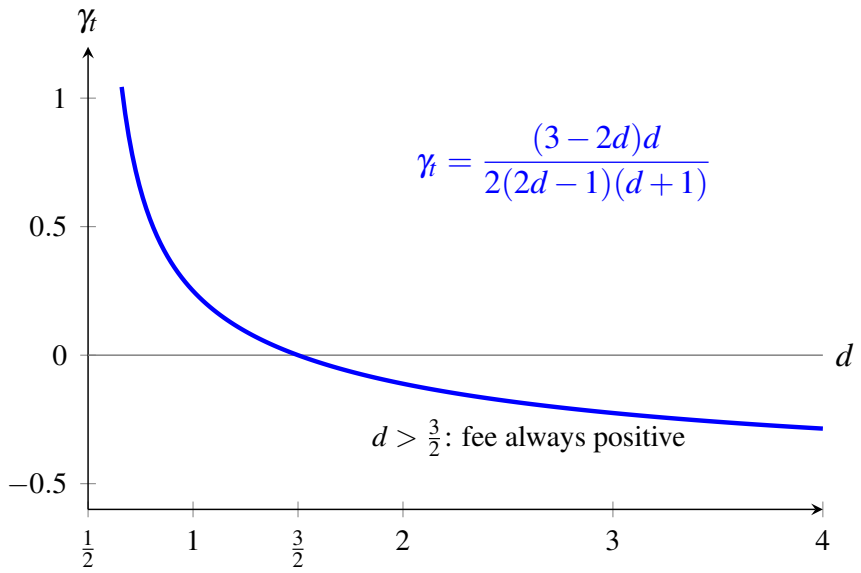
$$\gamma > \gamma_t \equiv \frac{(3-2d)d}{2(2d^2+d-1)}$$

- When $d > \frac{3}{2}$: cutoff $\gamma_t < 0$, so fee is always positive.
- When $d < \frac{3}{2}$: abatement must be sufficiently costly ($\gamma > \gamma_t$) for a positive fee.
- When $\gamma \leq \gamma_t$: firms invest heavily in abatement $\Rightarrow$ per-unit *subsidy* is socially optimal (not a fee).

Intuition: Fee vs. Subsidy

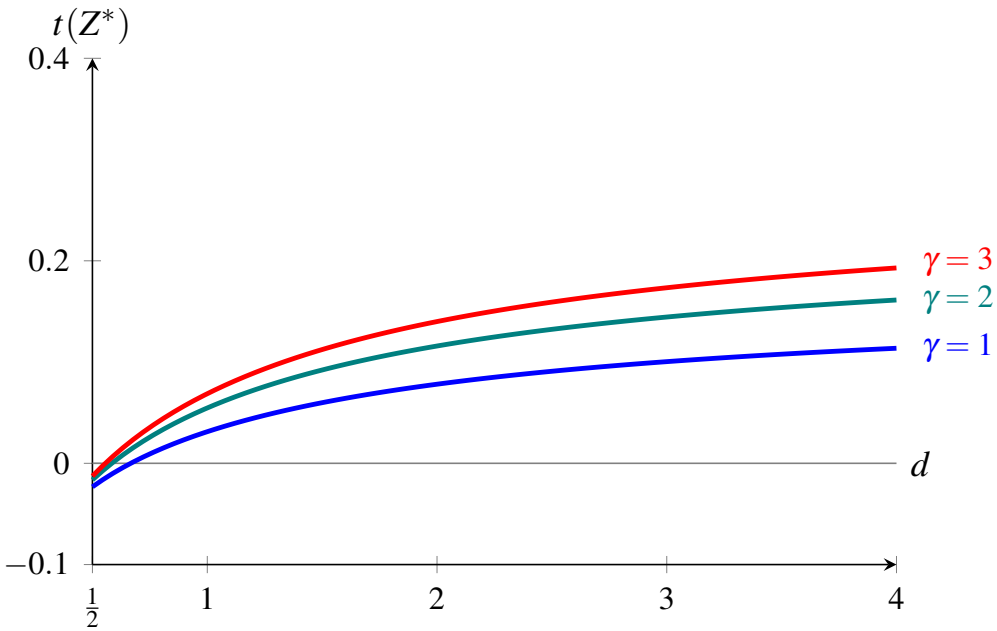
- **Why can the optimal instrument be a subsidy?**
  - When abatement is cheap (γ small), firms invest heavily, reducing net emissions substantially.
  - When pollution is also mild ($d < \frac{3}{2}$), environmental externality is small relative to the oligopoly distortion (underproduction).
  - Boosting output via a *subsidy* can dominate using an emission *fee*.
- **Key contrast with Chapter 2 (no abatement):**
  - Without abatement: the fee is always positive (emission fee needed to correct the externality).
  - With abatement: abatement itself partially corrects the externality, so the optimal fee can be zero or negative.

Figure 6.4 – Cutoff γ_t as a Function of d



- γ_t originates at a vertical asymptote at $d = \frac{1}{2}$, decreases in d, crosses the horizontal axis at $d = \frac{3}{2}$, and converges to $\gamma = -\frac{1}{2}$ when $d \to +\infty$.
- $d > \frac{3}{2}$: the fee is positive for every γ. $d < \frac{3}{2}$: the fee is positive only if $\gamma > \gamma_t > 0$.

Figure 6.5a – Emission Fee Increases in d and γ



- At $c = \frac{1}{2}$, the fee can be negative when pollution is not severe (low d), but this portion shrinks as γ increases.
- Higher $\gamma \Rightarrow$ less abatement $\Rightarrow$ more emissions per unit of output $\Rightarrow$ more stringent fee.

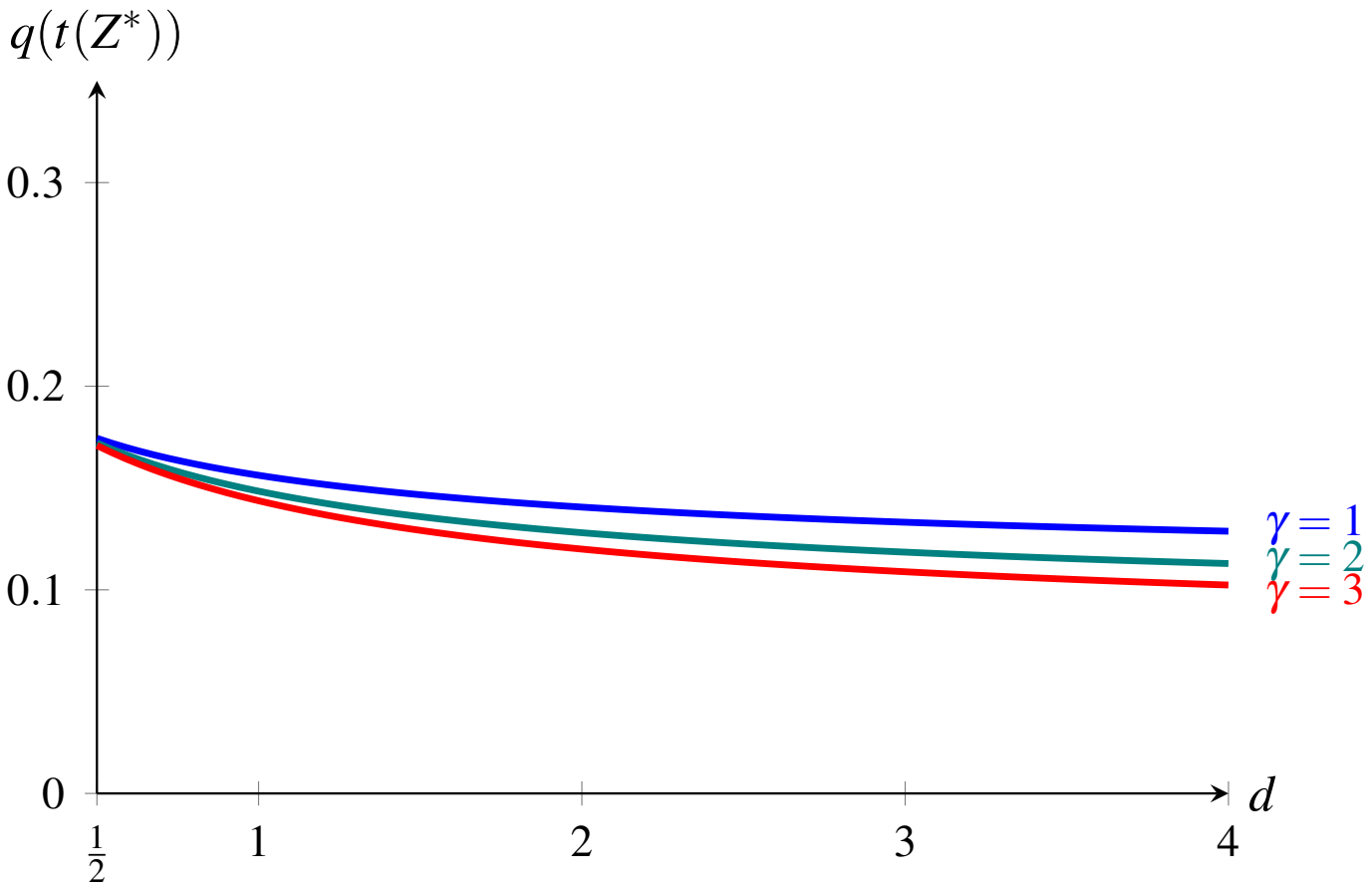
Equilibrium Output (No Spillovers)

- Substituting $t(Z^*)$ into $q(t) = \frac{1-c-t}{3}$ yields (6.20):

$$q(t(Z^*)) = \frac{(1-c)[d(7+4d)+2\gamma(1+d)]}{4\gamma(1+d)^2+2d(9+7d)} > 0$$

- Output is *lower* when pollution is more severe (higher d) or abatement is more expensive (higher γ): the regulator sets a more stringent fee in both cases.
- Comparison with Chapter 2: when $\gamma \to +\infty$, abatement investment is zero and equilibrium output converges to the Chapter 2 result.
- When γ is finite, abatement lowers net emissions $\Rightarrow$ regulator reduces fee $\Rightarrow$ output is *higher* than in Chapter 2.

Figure 6.5b – Equilibrium Output Decreases in d and γ



Net Emissions and the First Best (Section 6.3.1)

- At this equilibrium, every firm's abatement, output, and net emissions are positive:

$$q - z_i^* = \frac{(1-c)(2d\gamma + 3d + 2\gamma + 2)}{4\gamma(1+d)^2 + 2d(9+7d)} > 0$$

- Given aggregate abatement, fee $t(Z)$ induces the socially optimal output.
- But firms invest *less* than the socially optimal level,

$$z^{FB} = \frac{d(1-c)}{2d + \gamma(1+d)},$$

for all admissible parameters.

Why the Fee Is Only a Second Best

- Intuitively, a larger abatement leads the regulator to set a less stringent fee, see (6.10).
- This reduces every firm's return from abating.
- A single fee cannot correct both output and abatement decisions: the emission fee only achieves a **second best**.
- This result also applies to the extensions in the rest of the chapter.

Section 6.3.2 – Spillovers in Abatement ($\beta > 0$)

- With spillovers, firm i's emissions become:

$$e_i = q_i - z_i - \beta z_j$$

- Sources of spillovers: non-patentable innovations, worker mobility (know-how transfer), geographical proximity (measurement errors).
- **Symmetric spillovers**: model assumes firm i equally benefits from any rival's investment (parameter β is identical for all pairs).
- **Asymmetric spillovers** (richer model): $e_1 = q_1 - z_1 - \beta_1 z_2$ and $e_2 = q_2 - z_2 - \beta_2 z_1$ with $\beta_1 \neq \beta_2$.
- We focus on symmetric case; equilibrium output in Stage 3 is unchanged: $q = \frac{1-c-t}{3}$ (spillovers affect output only indirectly via the fee).

Asymmetric Spillovers (Beyond the Book)

- Firms may absorb spillovers differently, for instance because they differ in *absorptive capacity*, their ability to learn from outside knowledge (Cohen & Levinthal, 1989).
- Solving the model of Section 6.3.2 with emissions (6.22) and $\beta_1 > \beta_2$, firm 1 invests less in an interior equilibrium:

$$z_1^* < z_2^* \text{ for all admissible parameters.}$$

- Intuitively, firm 1 receives a larger share of its rival's abatement for free, which weakens its own incentive to invest: it free-rides more.

Stage 2: Optimal Fee with Spillovers

- Regulator's optimal fee with spillovers (6.24):

$$t(Z) = \frac{(2d-1)(1-c) - 3d(1+\beta)Z}{2(1+d)}$$

- Coincides with equation (6.9) when $\beta = 0$.
- Fee sensitivity to abatement now *amplified* by spillovers (6.25):

$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d(1+\beta)}{2(1+d)} < 0$$

- Its magnitude increases in β: a given investment reduces emissions for both firms more effectively, so stronger spillovers reduce the need for high emission fees.
- Equilibrium output: $q = \frac{1-c+d(1+\beta)Z}{2(1+d)}$, increasing in aggregate abatement Z.

Stage 1: Best Response with Spillovers

- Differentiating profit (6.26) with respect to z_i yields (6.27):

$$z_i(z_j) = \frac{A}{B} - \frac{d(3+2d)(1+\beta)^2}{B} z_j$$

  where $A \equiv [d(2+\beta+2d)-1](1-c)$ and $B \equiv d(1+\beta)[6+(5-\beta)d] + 2\gamma(1+d)^2$.

- Coincides with (6.12) when $\beta = 0$.
- Still negatively sloped (strategic substitutes), but the slope magnitude increases with β: spillovers amplify each firm's incentive to free-ride.

TSE and ERE: Two Externalities from Rivals' Abatement

- Firm j's investment z_j generates two externalities for firm i:
  1. **Tax-saving externality (TSE):** a reduction in the emission fee t.
  2. **Emission-reduction externality (ERE):** a decrease in firm i's emissions due to spillovers.
- The TSE arises *regardless* of spillovers ($\beta \geq 0$).
- The ERE arises *only when* spillovers are present, $\beta > 0$.

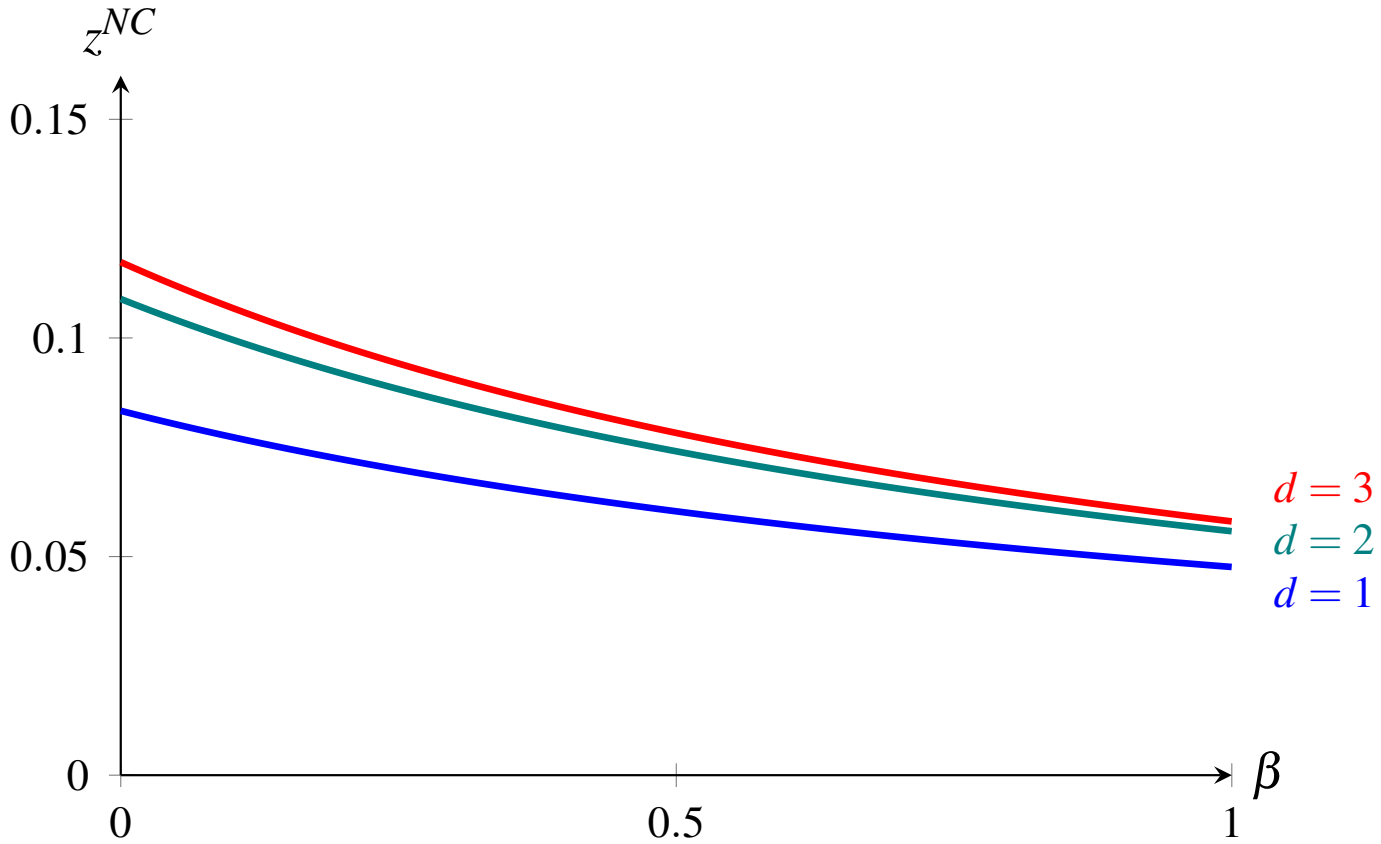
Equilibrium Abatement with Spillovers (z^{NC})

- In symmetric equilibrium, non-cooperative abatement is (6.28):

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$$

- Superscript NC = non-cooperative (firms do not coordinate).
- **Spillovers:** stronger spillovers expand the ERE, so every firm invests less, unless abatement is very costly.
- **Pollution and costs:** higher d shifts z^{NC} up; higher c shifts it down.

Figure 6.6 – Abatement z^{NC} as a Function of β



When Do Spillovers Reduce Abatement?

- z^{NC} decreases in β when abatement is not too costly:

$$2\gamma(1+d)^2 < 16d^3 + 33d^2 + 7d - 12 + 2\beta(2d^3 + 8d^2 + 5d - 3) + \beta^2 d(d+3)$$

- Figure 6.6 ($c = \frac{1}{2}$, $\gamma = \frac{1}{4}$, and $d = 1, 2, 3$) satisfies this condition.
- Otherwise, when abatement is very costly, z^{NC} increases in β.
- Higher $d \Rightarrow$ more stringent fee $\Rightarrow$ z^{NC} shifts upward in the figure.

When Does a Subsidy Replace the Fee? (Spillovers)

- Inserting z^{NC} into fee (6.24), the fee is a **subsidy** ($t < 0$) only when

$$d < \frac{3}{2} \quad \text{and} \quad \gamma < (1+\beta)^2 \gamma_t,$$

  where γ_t is the cutoff in (6.19).
- Mild pollution and inexpensive abatement: the oligopoly's underproduction then dominates the environmental externality.
- As in Section 6.3.1, abatement, output, and net emissions are positive.
- Firms invest less than the socially optimal level, so the fee only achieves a second best.

Section 6.4

Cooperative Abatement Decisions

Motivation for Cooperation in Abatement

- Every firm's investment gives rise to two externalities on its rival: the TSE and the ERE.
- **Environmental research cartel (ERC):** firms choose z_i and z_j to maximize *joint* profit, $\Pi_i + \Pi_j$, internalizing both externalities.
- **Environmental research joint venture (ERJV):** an ERC where firms also fully share information, $\beta = 1$, avoiding duplication in R&D.
- Stages 2–3 are unaffected by cooperation: equilibrium output and fee formula remain the same.

Cooperative Abatement: ERC Equilibrium

- Firms jointly choose (z_i, z_j) to maximize $\Pi_i + \Pi_j$, see (6.29).
- ERC equilibrium abatement (6.30):

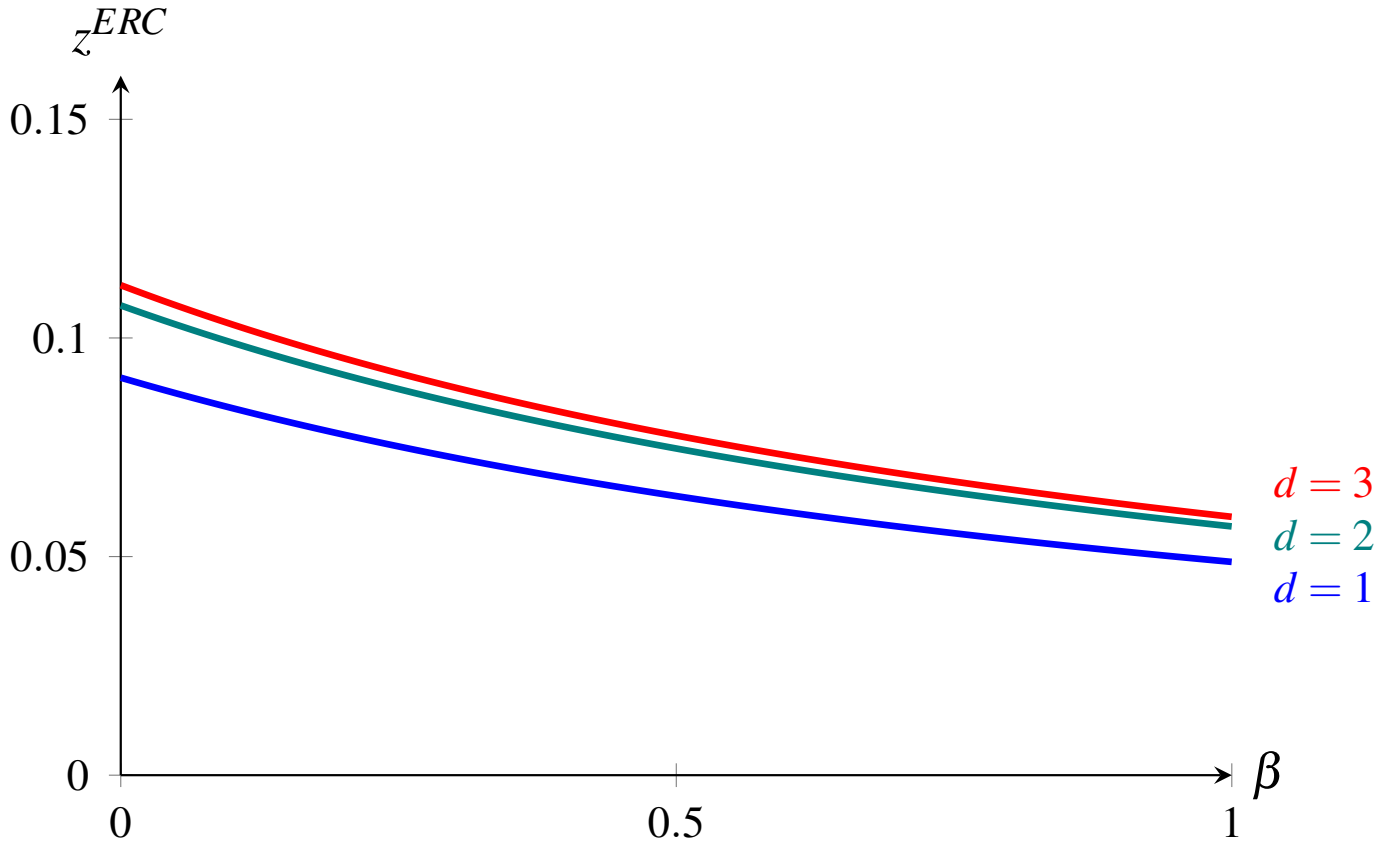
$$z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2 + 2\gamma(1+d)^2} > 0$$

  (positive since $d > \frac{1}{2}$).
- Figures 6.6 and 6.7 suggest that $z^{ERC} > z^{NC}$ under most, but not all, parameter conditions; Section 6.4.1 ranks them.

ERC Joint Profit Maximization: First-Order Conditions

- Relative to the NC problem, the ERC's first-order condition adds the effect of z_i on its rival's profit:
  1. **Through the fee (TSE):** without spillovers, this effect is $\frac{d(2q-3z_i)}{2(1+d)}$, which can be positive or negative.
  2. **Through spillovers (ERE, if $\beta > 0$):** lower emissions for the rival, a positive effect.
- The ERC invests more than under NC when the combined externality is positive, and less when it is negative.

Figure 6.7 – z^{ERC} as a Function of β



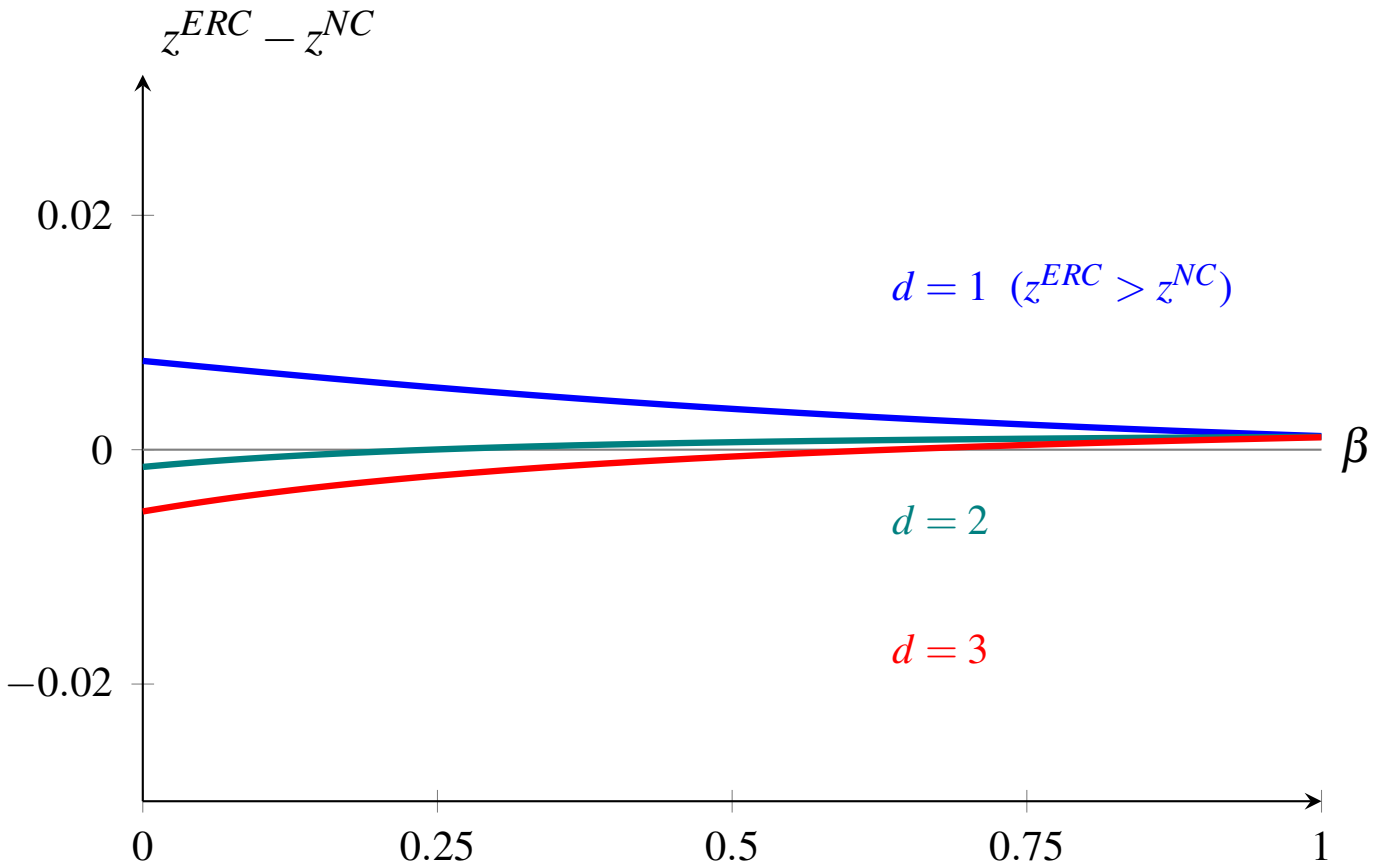
Investment Comparison: z^{ERC} vs. z^{NC} – No Spillovers

- When $\beta = 0$, only the TSE is present:

$$z^{ERC} > z^{NC} \quad \text{if and only if} \quad d < \frac{3}{2} + \gamma$$

- Small damages (e.g., $d = 1$ in Figure 6.8): $z^{ERC} > z^{NC}$. Larger damages (e.g., $d = 3$): the ranking reverses.
- Firm j's investment lowers the fee, which affects firm i's profit in two ways:
  - Lower fee on firm i's output (positive effect).
  - Lower fee savings from firm i's own abatement (negative effect).

Figure 6.8 – $z^{ERC} - z^{NC}$ as a Function of β



Intuition: Why the Ranking Reverses ($\beta = 0$)

- The effect of z_j on firm i's profit is $\frac{d(2q-3z_i)}{2(1+d)}$: positive when firm i's output is large relative to its abatement.
- **Small d:** fees are less stringent and firms invest little $\Rightarrow$ positive externality $\Rightarrow$ the ERC raises investment.
- **Large d:** fees are more stringent, firms produce less and invest more $\Rightarrow$ the negative effect dominates $\Rightarrow$ the ERC lowers investment.

Investment Comparison: With Spillovers ($\beta > 0$)

- The ERC internalizes the TSE (positive or negative) and the ERE (unambiguously positive).
- When β is sufficiently high, the ERE dominates: all curves in Figure 6.8 lie in the positive quadrant, $z^{ERC} > z^{NC}$.
- In Figure 6.8, where $\gamma = \frac{1}{4}$, this occurs for all $\beta > 0.24$ when $d = 2$, and for all $\beta > 0.63$ when $d = 3$.
- ERJV ($\beta = 1$): $z^{ERC} - z^{NC}$ is positive and barely affected by d (0.00116, 0.00110, and 0.00106 when $d = 1, 2, 3$).

Result: Investment Ranking under ERJV ($\beta = 1$)

- **Result.** When $\beta = 1$ (firms fully share their innovation): $z^{ERC} > z^{NC}$.
- **Intuition:**
  - With $\beta = 1$, the ERE is maximal: each unit of the rival's investment fully reduces firm i's emissions.
  - This positive externality dominates the TSE, whatever its sign.
- **Implication:** when innovation is fully shared, coordination always raises abatement.

ERC: Fee and First Best

- Under the ERC, abatement, output, and net emissions are positive; the fee is a subsidy only when $d < \frac{3}{2}$ and $\gamma < 2(1+\beta)^2\gamma_t$.
- Socially optimal abatement:

$$z^{FB} = \frac{d(1+\beta)(1-c)}{2d(1+\beta)^2 + \gamma(1+d)}$$

- The ERC invests too little when $\gamma(d-1) < 2d(1+\beta)^2(2d+1)$, and too much otherwise (severe pollution, $d > 1$, and costly abatement).
- The emission fee remains a second-best policy under the ERC.

Alternative Cooperative Regimes (Ouchida & Goto, 2016)

- Four institutional regimes (two binary policy choices):

	ERJVs banned ($\beta \neq 1$)	*ERJVs allowed* ($\beta = 1$)
Coord. banned	(i) R&D comp.	(iii) ERJV comp.
Coord. allowed	(ii) ERC	(iv) ERJV cartel

- **Model (i):** environmental R&D competition, generic β, as in Section 6.3.
- **Model (iv):** ERJV cartelization – ERC with $\beta = 1$.
- Model (ii) is the ERC of Section 6.4; model (iii) sets $\beta = 1$ with independent R&D choices.

Welfare Ranking of Cooperative Regimes

- Ouchida and Goto (2016): the welfare ranking depends on pollution severity and the cost of R&D.
- **Slight environmental damage:** ERJV cartelization (model (iv)) is socially preferred.
- **Severe damage and low R&D costs:** environmental R&D competition (model (i)) is socially superior to the other three regimes.
- **Severe damage and very high R&D costs:** ERJV cartelization is again socially preferred.

Section 6.5

Sequential Abatement Decisions

Motivation: Why Sequential Abatement?

- Baseline model: *simultaneous* abatement decisions.
- Small-scale projects (end-of-pipe scrubbers, minor filtration upgrades) are typically implemented simultaneously across firms.
- Larger projects (new machinery, retrofitting of facilities, post-disaster clean-up) unfold over long periods: firms may observe rivals' abatement before committing to their own.
- Strandholm et al. (2023) analyze this setting, abstracting from spillovers ($\beta = 0$) to isolate the role of timing.

Example: Intel and AMD

- May 2020: Intel commits to reduce its absolute operational greenhouse gas emissions by 10% by 2030.
- The following year, AMD, a rival in the semiconductor industry, responds with its own goals, including a 50% reduction in its absolute operational emissions by 2030.
- Similar patterns of staggered investment arise in steel manufacturing, where large-scale abatement projects are common.
- Model: a leader invests first; the follower observes its investment and responds.

Stages 2–3: Unaffected by Timing

- Equilibrium output in Stage 3 (6.31), unchanged:

$$q(t) = \frac{1 - c - t}{3}$$

- Regulator's fee in Stage 2 (6.32), the same as (6.9):

$$t(Z) = \frac{(2d - 1)(1 - c) - 3dZ}{2(1 + d)}$$

- Hence, $q(t(Z)) = \frac{1-c+dZ}{2(1+d)}$, increasing in aggregate abatement Z.
- The main divergence arises in **Stage 1**.

Follower's Best Response in Sequential Abatement

- Follower (firm 2) observes z_1 and chooses z_2 to maximize (6.34):

$$\max_{z_2 \geq 0} \pi_2 = \frac{[1 - c - t(Z)]^2}{9} + t(Z)z_2 - \frac{\gamma}{2}(z_2)^2$$

- Follower's best response (6.35), the same as (6.12):

$$z_2(z_1) = \frac{[2d(1+d) - 1](1-c) - d(3+2d)z_1}{d(6+5d) + 2\gamma(1+d)^2}$$

- Negative slope: strategic substitutes. The leader internalizes this reaction when choosing z_1^{Seq}.

Intuition: Leader's Investment Decision

- The leader solves $\max_{z_1 \geq 0} \ \pi_1(z_1, z_2(z_1))$.
- It anticipates that the follower's abatement *decreases* in its own, since abatement decisions are strategic substitutes.
- Should the leader push the follower's abatement down? It depends on whether a rival's abatement hurts or benefits each firm.
- From Section 6.4.1: every firm benefits from its rival's abatement when $d < \frac{3}{2} + \gamma$, that is, when $\gamma > \frac{2d-3}{2}$.

Free-Riding by the Leader

- The leader invests less than the follower, $z_1^{Seq} < z_2^{Seq}$, if and only if (6.36):

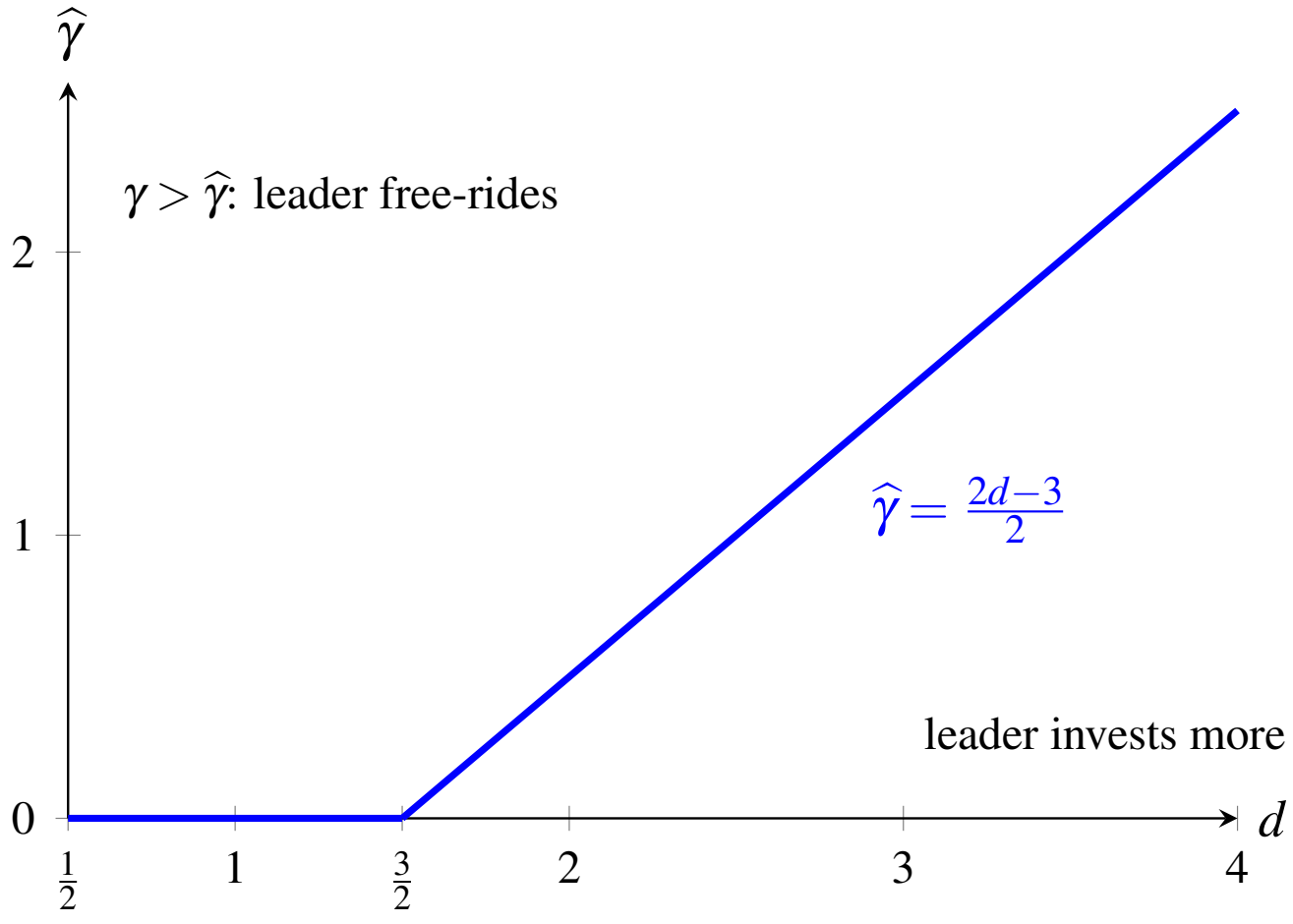
$$\gamma > \widehat{\gamma} \equiv \max\left\{0, \frac{2d-3}{2}\right\}$$

- $\gamma < \widehat{\gamma}$: every firm is hurt by its rival's abatement, so the leader invests more to induce the follower to invest less: $z_1^{Seq} > z_2^{Seq}$.
- $\gamma > \widehat{\gamma}$: every firm benefits from its rival's abatement, so the leader **free-rides**: $z_1^{Seq} < z_2^{Seq}$; the follower responds by investing more.

Cutoff $\widehat{\gamma}$: When Does the Leader Free-Ride?

- Term $\frac{2d-3}{2}$ originates at -1 when $d = \frac{1}{2}$, increases in d, and crosses the horizontal axis at $d = \frac{3}{2}$.
- $d \leq \frac{3}{2}$: $\widehat{\gamma} = 0$, so $\gamma > \widehat{\gamma}$ holds and the leader free-rides.
- $d > \frac{3}{2}$: $\widehat{\gamma} > 0$; the leader free-rides only if abatement costs are sufficiently high, $\gamma > \widehat{\gamma} > 0$.
- Analogy: sequential public good games, where early contributors reduce their effort, expecting later contributors to increase theirs.

Figure 6.9 – Leader's Free-Riding Incentives in Abatement



Role of Pollution Severity in Sequential Abatement

- Higher pollution severity (higher d) $\Rightarrow$ more stringent fee $\Rightarrow$ higher marginal benefit of abatement investment.
- As d increases, the threshold $\hat{\gamma}$ rises: the leader must bear higher costs before free-riding becomes optimal.
- Therefore, **severe pollution reduces free-riding incentives** for the leader.
- Conversely, when pollution is mild, regulatory response is weak $\Rightarrow$ benefit of abatement is low $\Rightarrow$ leader more likely to rely on the follower's investment.
- This underscores the importance of environmental context for determining strategic behavior in sequential investment settings.

Sequential vs. Simultaneous Abatement: Comparison

- Let $z^{NC} = z_i^*$ in (6.16), i.e., (6.28) at $\beta = 0$. Then

$$z_1^{Seq} > z^{NC} > z_2^{Seq} \quad \text{if and only if} \quad \gamma < \widehat{\gamma},$$

  while the ranking reverses when $\gamma > \widehat{\gamma}$.
- Strategic substitutes: the leader and the follower move in *opposite* directions relative to the simultaneous setting.

Aggregate Abatement under Sequential Timing

- $\gamma < \widehat{\gamma}$: the leader invests more to reduce the follower's abatement; the follower invests less.
- The leader's increase dominates, so aggregate abatement is higher: $z_1^{Seq} + z_2^{Seq} > 2z^{NC}$.
- $\gamma > \widehat{\gamma}$: the opposite occurs. The leader free-rides on the follower, and aggregate abatement is lower than under simultaneous investment.

Net Emissions and the Fee under Sequential Abatement

- Both firms' outputs are positive, and so are their net emissions, except for the leader when pollution is extremely severe, $d > \frac{33+3\sqrt{185}}{8} \simeq 9.23$, and abatement costs lie below a cutoff that increases in d ($\gamma < 0.04$ when $d = 10$, and $\gamma < 0.44$ when $d = 20$).
- The equilibrium fee can only be a subsidy when pollution is mild, $d < \frac{3}{2}$.
- As in Section 6.3, the fee is a **second-best** policy: firms' abatement levels differ from the socially optimal level. Even when they coincide, $\gamma = \widehat{\gamma}$, they equal z^{NC}, which lies below the first best.

Dominant Strategies under Pre-Commitment Regulation

- When the regulator sets the fee *before* firms invest in abatement (regulatory commitment), strategic interaction in abatement *disappears*.
- With a fixed fee t, each firm i chooses z_i to solve:

$$\max_{z_i \geq 0} \pi_i = q^2 + tz_i - \frac{\gamma}{2}(z_i)^2$$

- First-order condition: $t = \gamma z_i$, yielding (6.37):

$$z_i(t) = \frac{t}{\gamma}$$

- This optimal abatement is *independent* of rival's investment $z_j \Rightarrow$ abatement is a **strictly dominant strategy**.
- Sequential vs. simultaneous timing no longer matters when the fee is pre-committed.

Section 6.6

Different Regulatory Timing

Regulatory Commitment: Motivation

- Lambertini et al. (2017) consider a reversed timing:
  1. **Stage 1**: Regulator commits to an emission fee t.
  2. **Stage 2**: Firms independently choose abatement investments z_i.
  3. **Stage 3**: Firms compete à la Cournot in output markets.
- **Relevant when:** firms can easily revise abatement (scrubbers, pipe filters), but regulatory parameters are fixed by legislative or institutional constraints.
- **Contrast with Poyago-Theotoky (2007):** appropriate when investments are large-scale and irreversible (e.g., factory retrofits), while regulators can adjust policy in response to observed behavior.
- Lambertini et al. also allow for $n \geq 2$ firms to study how market structure affects aggregate abatement.

Stage 3: Output with n Firms

- Equilibrium output per firm:

$$q_i(t) = \frac{1 - c - t}{n + 1}$$

  decreasing in the fee t, the number of firms n, and the marginal cost c.

- Aggregate output $Q(t) = \frac{n(1-c-t)}{n+1}$ increases in n, approaching $1 - c - t$ as $n \to \infty$ (perfect competition).

Stage 2: Dominant-Strategy Abatement

- With a given fee t, the first-order condition is $t = \gamma z_i$, yielding (6.41):

$$z_i(t) = \frac{t}{\gamma}$$

  a *strictly dominant strategy* (independent of rivals' choices).
- Aggregate abatement (6.42): $Z(t) = \frac{nt}{\gamma}$, increasing in n and t, and decreasing in γ.
- If the fee were exogenous, this would be an **Arrowian result**: more competition increases aggregate R&D.

Section 6.6 – Equilibrium Emissions under Regulatory Commitment

- Each firm's net emissions under regulatory commitment, with n firms (6.43):

$$e_i(t) = \frac{1-c-t}{n+1} - \frac{t[1+\beta(n-1)]}{\gamma}$$

- First term: output $q_i(t)$. Second term: abatement by firm i plus spillovers from its $n-1$ rivals.
- Equilibrium profits (6.44):

$$\pi_i(t) = \left(\frac{1-c-t}{n+1}\right)^2 + \frac{t^2[1+2\beta(n-1)]}{2\gamma}$$

Stage 1: The Regulator's Welfare Function

- With $d = 2$, the regulator maximizes welfare including the fee revenue, $tE = nte_i(t)$, returned to society as a lump-sum transfer (6.45):

$$W(t) = \frac{1}{2}[Q(t)]^2 + n\pi_i(t) + nte_i(t) - n^2[e_i(t)]^2$$

- Lambertini et al. (2017) abstract from the fee revenue. Including it, as in the rest of the book, changes the closed forms but not the qualitative results.
- It also guarantees that abatement never exceeds output at the optimal fee.

Stage 1: Regulator Sets the Optimal Fee

- Differentiating $W(t)$ with respect to t yields (6.47):

$$t(n) = \frac{\gamma\left[2n(n+1)K + \gamma(2n-1)\right](1-c)}{D}$$

where $K \equiv 1 + \beta(n-1)$ and

$$D \equiv 2n(n+1)^2K^2 + \gamma(n+1)\left[5n+1+4\beta n(n-1)\right] + 3\gamma^2 n$$

- Equilibrium abatement: $z_i(n) = \frac{t(n)}{\gamma}$.

Properties of the Fee under Regulatory Commitment

- The fee $t(n)$ is positive for all admissible parameters: the regulator never subsidizes production in this setting.
- Every firm's emissions are positive,

$$e_i(t(n)) = \frac{\gamma(1-c)\left[n+2+\gamma+\beta(n-1)\right]}{D} > 0,$$

  so abatement never exceeds output.
- A single fee cannot correct both output and abatement decisions: a second best.
- From now on, $c = 0$ and $\gamma = 5$ to focus on n and β.

Market Concentration and Abatement – No Spillovers ($\beta = 0$)

- With $\beta = 0$, $c = 0$, $\gamma = 5$, aggregate abatement is (6.49):

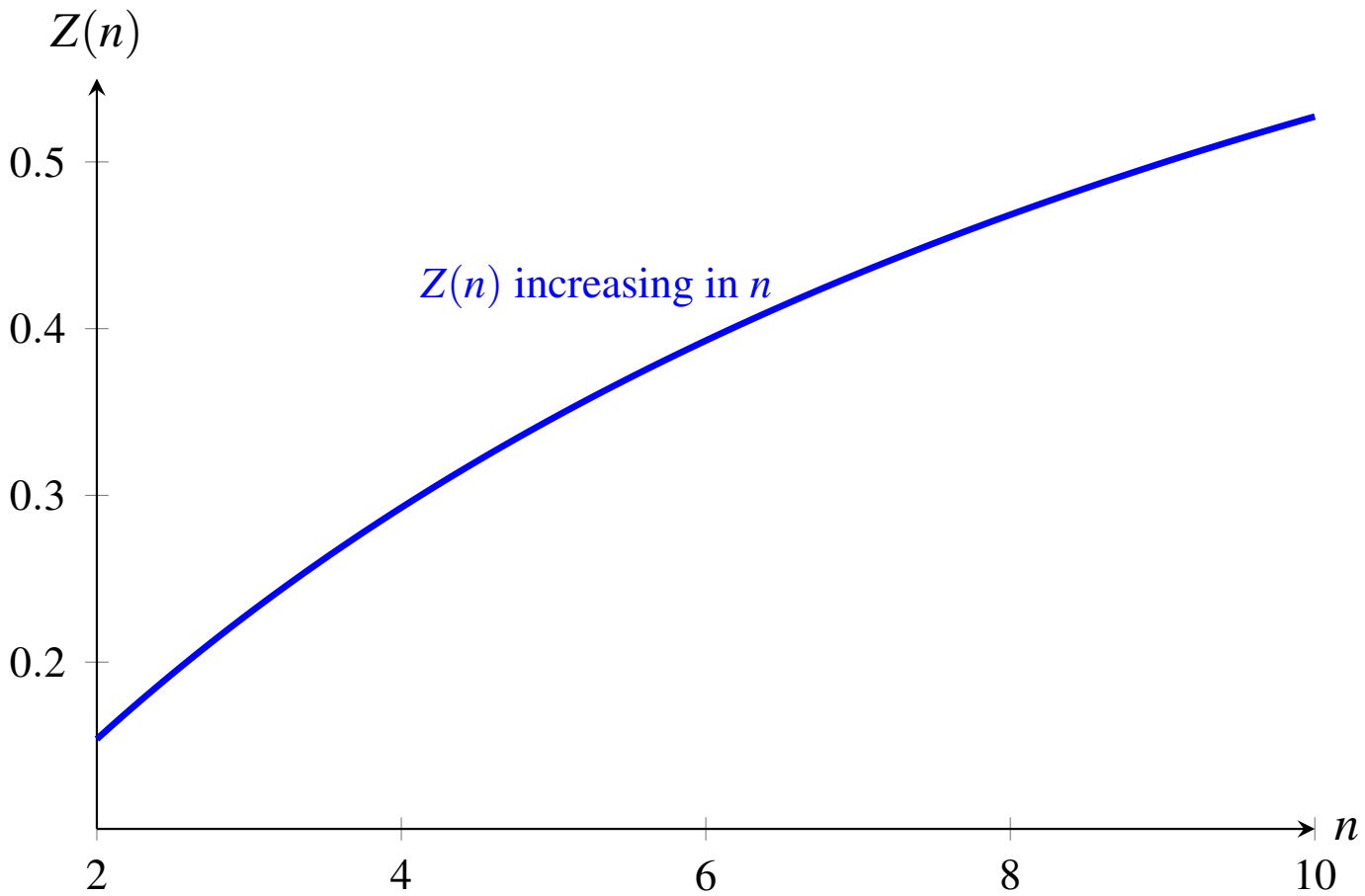
$$Z(n) = \frac{n(2n^2 + 12n - 5)}{2n^3 + 29n^2 + 107n + 5}$$

- From (6.50), $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$: aggregate abatement **increases monotonically** with the number of firms.
- Consistent with the **Arrowian prediction**: more competition encourages more innovation when spillovers are absent.

Why Abatement Increases in n ($\beta = 0$)

- No spillovers: the ERE is absent.
- Fee set before firms invest: the TSE is absent too.
- Every firm's abatement, $z_i(n) = \frac{t(n)}{\gamma}$, slightly decreases in n, but more firms invest, so aggregate abatement increases.
- Contrast: in Poyago-Theotoky (2007), the regulator responds to observed abatement, creating an incentive to invest strategically.

Figure 6.10 – Aggregate Abatement when $\beta = 0$



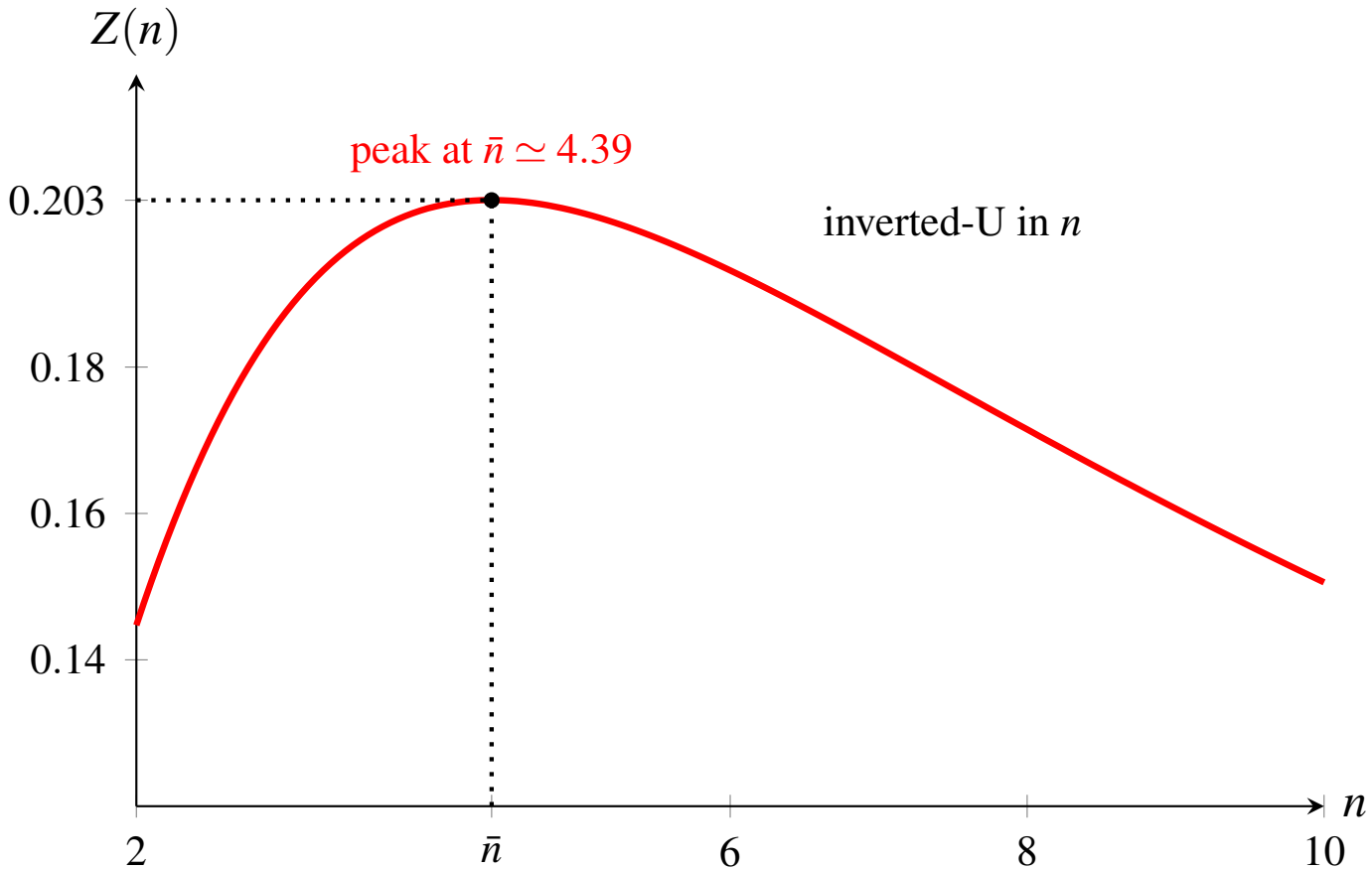
Market Concentration and Abatement – With Spillovers ($\beta = \frac{1}{2}$)

- With $\beta = \frac{1}{2}$, aggregate abatement is (6.51):

$$Z(n) = \frac{2n(n^3 + 2n^2 + 11n - 5)}{n^5 + 4n^4 + 26n^3 + 54n^2 + 191n + 10}$$

- From (6.52), $\frac{\partial Z(n)}{\partial n} > 0$ if and only if $n < \overline{n} \simeq 4.39$.
- **Inverted U-shape**: aggregate abatement increases as the industry grows from two to four firms, and decreases with every additional firm thereafter.

Figure 6.11 – Aggregate Abatement when $\beta = \frac{1}{2}$



Why Abatement Eventually Falls in n ($\beta = \frac{1}{2}$)

- Every firm's abatement is $z_i = \frac{t(n)}{\gamma}$: firms do *not* free-ride on each other's abatement in this setting.
- With spillovers, every unit of abatement reduces the emissions of $1 + \beta(n-1)$ firms.
- Hence, the regulator's fee $t(n)$, and thus individual abatement, falls quickly as n grows, eventually reducing aggregate abatement.

Result: Arrowian Result and Its Limits

- **Arrowian result (no spillovers):** $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$ when $\beta = 0$.
- **Spillovers break the Arrowian result:** with $\beta = \frac{1}{2}$, $Z(n)$ peaks at four firms ($\overline{n} \simeq 4.39$).
- The decline comes from a fee that falls quickly in n, not from free-riding.
- **Policy implication:** with spillovers, promoting competition does not always raise green investment.

Comparing Poyago-Theotoky vs. Lambertini: TSE Absent

- **Poyago-Theotoky (2007):** the regulator *responds* to observed abatement, so investing more leads to a less stringent fee (TSE present).
- **Lambertini et al. (2017):** the fee is set in the first stage, so abatement does not affect its stringency (TSE absent).
- The ERE also arises when $\beta > 0$, suggesting that an ERC could help internalize it.
- Because the TSE is absent, the case for an ERC is weaker than when firms invest before the regulator sets the fee.

Poyago-Theotoky (2007) vs. Lambertini et al. (2017): A Comparison

- **Poyago-Theotoky (2007):**
  - Timing: Abatement $\rightarrow$ Fee $\rightarrow$ Output.
  - Fee responds to observed abatement (TSE present).
  - Strategic abatement decisions: firms anticipate regulatory response.
  - Best for: irreversible capital investments; flexible regulation.
- **Lambertini et al. (2017):**
  - Timing: Fee $\rightarrow$ Abatement $\rightarrow$ Output.
  - Fee is pre-committed (TSE absent).
  - Abatement is a dominant strategy: $z_i = t/\gamma$.
  - Best for: modular/reversible abatement; rigid legislative fee.

Section 6.7

Other Extensions

Extension 1: Endogenous Entry (Katsoulacos & Xepapadeas, 1995)

- While most models assume an exogenous number of firms n, some settings feature **endogenous entry**.
- Framework: regulator sets fee first; firms decide whether to enter; then choose output and abatement.
- Key finding: equilibrium emission fee may *exceed* marginal environmental damage – **"overinternalization"** of externalities.
- This overinternalization can lead to excessive entry and inefficiencies.
- Optimal two-instrument policy:
  - An **entry fee** to limit socially excessive entry.
  - An **emission fee set below marginal damages** to avoid overinternalization.
- This combination yields higher welfare than a single emission fee alone.

Extension 2: Subsidies and Regulatory Design

- **Ouchida & Goto (2014)** extend Poyago-Theotoky (2007): subsidies can raise welfare when pollution is mild (low d), since underproduction may outweigh the externality, so it is optimal to subsidize output ($t < 0$).
- Mild pollution and costly R&D (large γ): firms invest little, and the subsidy may lead to higher emissions than under laissez-faire, but regulation can still improve welfare. Inexpensive R&D (low γ): firms invest heavily, so emissions are lower than when unregulated.
- **Buccella et al. (2025):** in a polluting managerial duopoly, a subsidy for end-of-pipe abatement always induces more abatement than an emission tax; the tax reduces output and, when the abatement technology is efficient enough, yields the lowest environmental damage.

Extension 3: Patenting vs. Licensing Green Technologies

- **Biglaiser & Horowitz (1995):** four-stage model – R&D investment, fee determination, technology adoption (patent or license), output choice.
- Firms choose between *exclusive use* of an innovation (patent) and *licensing* it to competitors.
- Trade-off: exclusive use provides competitive advantage; licensing generates revenue and may raise rivals' abatement.
- **Denicolo (1999):** single innovating firm decides whether to retain or license technology.
  - Compares two regulatory regimes: fee set before vs. after R&D.
  - Timing of fee-setting significantly affects licensing incentives and welfare.
- **Key insight**: the timing of regulatory intervention shapes both the direction and magnitude of technology diffusion.

Extension 4: Product Differentiation

- **Poyago-Theotoky & Teerasuwannajak (2002):** compare a pre-commitment fee (set before abatement) and a time-consistent fee (set afterward).
- Highly differentiated products: the time-consistent fee is less stringent than the pre-commitment fee. More homogeneous products: the ranking reverses, especially when R&D is efficient and pollution severe.
- Abatement is higher under time-consistent regulation when differentiation is strong, but may be lower when R&D costs are minimal.
- Pre-commitment regulation yields higher welfare with homogeneous products.

Extension 5: Mixed Oligopolies

- A public firm maximizes a weighted combination of profits and social welfare, including environmental damages, partially internalizing externalities.
- **Bárcena-Ruiz & Garzón (2006):** privatization can be welfare-reducing when emission taxes are present, as it exacerbates underproduction.
- **Haruna & Goel (2019):** the public firm's presence can instead reduce output and emissions, potentially leading to emission subsidies when pollution is mild.
- **Pal & Saha (2014):** first-best outcomes through abatement subsidies and output taxes, without privatization.

Further Extension (Beyond the Book): Stock Pollution

- **Stock pollution:** emissions accumulate, so abatement today also lowers future damage.
- **Benchekroun & Long (1998):** Cournot firms add to a pollution stock; a tax that depends on the current stock induces the socially optimal output path, and can be a subsidy when the stock is low.
- **Ulph & Ulph (2013):** a government that cannot commit to future carbon taxes lowers firms' return from investing in cleaner technology today, so R&D subsidies may be needed.
- **Golombek, Greaker & Hoel (2010):** with an optimal R&D subsidy today, lack of commitment creates no time-inconsistency problem.

Further Reading (Beyond the Book)

- Benchekroun, H., and N.V. Long (1998) "Efficiency Inducing Taxation for Polluting Oligopolists," Journal of Public Economics, 70(2), pp. 325–342.
- Cohen, W.M., and D.A. Levinthal (1989) "Innovation and Learning: The Two Faces of R&D," Economic Journal, 99(397), pp. 569–596.
- Golombek, R., M. Greaker, and M. Hoel (2010) "Carbon Taxes and Innovation without Commitment," B.E. Journal of Economic Analysis & Policy, 10(1).
- Ulph, A., and D. Ulph (2013) "Optimal Climate Change Policies When Governments Cannot Commit," Environmental and Resource Economics, 56(2), pp. 161–176.

Section 6.8

Case Studies

Case Study 1: Simultaneous Abatement – HP and Dell

- HP and Dell, two rivals in personal computers, have made similar environmental investments.
- **HP:** has run Planet Partners, a take-back program for print cartridges, since 1991; in September 2008, it became the first IT company to publish the greenhouse gas emissions of its largest suppliers.
- **Dell:** in May 2014, announced the first computer in the industry built with certified closed-loop recycled plastic, recovered from used electronics.
- **Connection to the model:** these cases fit Section 6.3: rivals choose abatement simultaneously, anticipating the regulator's response. They do not show why the firms acted as they did, or whether their investments were strategic substitutes, as the model predicts.

Case Study 2: ERJVs – Canada's Oil Sands Innovation Alliance

- March 2012: the chief executives of 12 oil sands producers, including Suncor Energy, Cenovus Energy, and Canadian Natural Resources, signed the charter of COSIA.
- Members share environmental technologies and fund joint research in four areas: greenhouse gases, land, tailings, and water.
- By late 2013, they had shared more than 560 technologies that had cost over C$900 million to develop.
- June 2022: COSIA became a division of the Pathways Alliance, renamed the Oil Sands Alliance in early 2026.

Case Study 2: Connection to the Model

- COSIA resembles the ERJVs of Section 6.4: firms coordinate their abatement R&D and share the resulting knowledge.
- Sharing knowledge corresponds to a high spillover parameter β.
- The case illustrates this form of coordination, but it does not measure whether members invest more than they would on their own, as the comparison in Section 6.4.1 examines.

Case Study 3: Sequential Abatement – Manufacturing

- **Brown et al. (2022):** about 33,500 firm-year observations on manufacturing firms in 18 OECD countries, 1990–2012.
- Higher national taxes on sulfur oxide emissions increase R&D spending; the increase comes entirely from high-pollution firms, especially in cement and mineral products, where outside knowledge is easier to acquire.
- Patenting did not generally rise: firms used R&D to absorb and adopt cleaner methods developed elsewhere.
- The study does not measure emissions directly.

Case Study 3 (cont.): U.S. Oil

- In the U.S. oil industry, two of the largest firms have made large low-carbon investments.
- **Chevron:** in June 2022, completed its \$3.15 billion acquisition of Renewable Energy Group, a biofuels producer.
- **ExxonMobil:** in November 2023, completed its \$4.9 billion acquisition of Denbury, which gave it the largest owned and operated CO_2 pipeline network in the United States.

Case Study 3: Connection to the Model

- Section 6.5 would treat large firms such as these as potential leaders.
- In that model, followers respond to a larger leader investment by investing less, since abatement decisions are strategic substitutes.
- Whether smaller firms have responded in this way is an open empirical question.
- The evidence in Brown et al. (2022) relates to how emission fees affect abatement R&D, as in Section 6.3, rather than to the timing of investment.

Additional Example: BMW and EU CO_2 Rules

- **EU Regulation 2019/631:** a carmaker whose average CO_2 emissions exceed its target pays €95 per g/km of excess for each new car it registers.
- **BMW:** in February 2023, announced an €800 million investment in its San Luis Potosí plant (Mexico) to build fully electric Neue Klasse models and their batteries from 2027.
- **Connection to the model:** the premium works like a per-unit emission fee t, and electric models lower emissions per car, as abatement z_i does in Section 6.3.
- The example does not show whether the premium drove BMW's investment, or how its rivals responded.

Section 6.9

Summary

Summary: Modeling Framework

- Chapter 6 introduces a **three-stage game**: firms invest in abatement (Stage 1), regulator sets emission fee (Stage 2), firms choose output (Stage 3).
- Abatement reduces net emissions and influences regulatory behavior, creating strategic incentives for investment.
- Two key parameters:
  - γ: investment efficiency (lower $\gamma \Rightarrow$ more abatement).
  - β: spillover strength (higher $\beta \Rightarrow$ more free-riding).
- Chapter 2 is a special case of this model when $\gamma \to +\infty$ (no abatement).
- Solving by backward induction yields equilibrium abatement, fee, output, and welfare.

Summary: Non-Cooperative Abatement

- Firms invest independently, anticipating that higher investment leads to lower emission fees.
- Best response functions are *downward-sloping*: abatement levels are **strategic substitutes**.
- Equilibrium abatement *increases* with pollution severity d, and *decreases* with abatement cost γ and production cost c.
- Firms invest less in abatement than the socially optimal level, so a single emission fee only achieves a **second best**.

Summary: Allowing for Spillovers

- Firms benefit from rivals' abatement efforts through spillovers.
- Spillovers amplify free-riding incentives and reduce the need for stringent emission fees.
- Equilibrium abatement increases with pollution severity, and decreases with the strength of spillovers unless abatement is very costly.
- The fee can be a subsidy only when pollution is mild and abatement is inexpensive.

Summary: Cooperative Abatement and ERJVs

- Firms coordinate abatement via an ERC (generic β) or an ERJV ($\beta = 1$, full information sharing).
- Coordination internalizes the externalities from spillovers (ERE) and tax savings (TSE), raising investment when spillovers are strong or pollution is mild ($d < \frac{3}{2} + \gamma$ when $\beta = 0$).
- With an ERJV ($\beta = 1$), $z^{ERC} > z^{NC}$.
- Welfare (Ouchida and Goto, 2016) depends on pollution severity and R&D costs.

Summary: Sequential and Regulatory-Commitment Models

- **Sequential abatement** (Strandholm et al., 2023; no spillovers): leader and follower move in opposite directions.
  - $\gamma < \widehat{\gamma}$: the leader invests more, the follower less; aggregate abatement rises.
  - $\gamma > \widehat{\gamma}$: the leader free-rides on the follower; aggregate abatement falls.
- **Regulatory commitment** (Lambertini et al., 2017): abatement $\frac{t}{\gamma}$ is a dominant strategy and the TSE disappears.
- Aggregate abatement increases in n without spillovers, but is inverted-U shaped with spillovers, peaking at four firms when $\beta = \frac{1}{2}$.

Summary: Functional Forms

- Closed-form results rely on quadratic abatement costs and quadratic environmental damage.
- The reason why a single fee only achieves a second best, one instrument for two decisions, applies more generally.

Key Formula Reference

- $e_i = q_i - z_i - \beta z_j$, $C(z_i) = \frac{\gamma z_i^2}{2}$, $q = \frac{1-c-t}{3}$, and

$$t(Z) = \frac{(2d-1)(1-c) - 3d(1+\beta)Z}{2(1+d)}$$

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)] + 2\gamma(1+d)^2}$$

$$z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2 + 2\gamma(1+d)^2}$$

- First best ($\beta = 0$): $z^{FB} = \frac{d(1-c)}{2d+\gamma(1+d)}$. Leader free-rides iff $\gamma > \widehat{\gamma} = \max\left\{0, \frac{2d-3}{2}\right\}$.
- Commitment: $z_i(t) = \frac{t}{\gamma}$ and $Z(t) = \frac{nt}{\gamma}$.

Looking Back: Chapter 6 vs. Chapter 2

- **Special case:** as $\gamma \to +\infty$, firms do not abate and all results reduce to Chapter 2, where $t^{Ch2} = \frac{(2d-1)(1-c)}{2(1+d)}$.
- **Finite** γ (Sections 6.3–6.4, any β): the fee is lower, output is higher, and net emissions are lower than in Chapter 2.
- **Welfare** is also higher than in Chapter 2, both with non-cooperative abatement and under the ERC, for all admissible parameters.
- Intuitively, the gains from abatement (lower damage and a less stringent fee, which raises output) exceed its cost.

Looking Ahead: Chapters 7 and 8

- Chapter 6 showed that abatement investment is a strategic tool: firms anticipate how their abatement changes the fee (TSE) and benefit from rivals' abatement (ERE).
- **Chapter 7:** environmentally friendly mergers, where changes in market structure affect output, emissions, and the fee.
- **Chapter 8:** managerial incentives, where firms pursue objectives beyond profit: public ownership, sales-based incentives, cross-ownership, and corporate social responsibility.

Chapter 7

Environmentally Friendly Mergers?

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 7

- **Section 7.1.** Introduction and guiding questions.
- **Section 7.2.** A merger-to-monopoly.
- **Section 7.3.** Allowing for cost-reducing mergers.
- **Section 7.4.** Allowing for mergers with non-linear costs.
- **Section 7.5.** Allowing for mergers between several firms.
- **Section 7.6.** Allowing for environmental regulation.
- **Section 7.7.** Case studies.
- **Section 7.8.** Merger review in practice.
- **Section 7.9.** Related literature.
- **Section 7.10.** Cartels in polluting industries.
- **Section 7.11.** Summary.

Section 7.1

Introduction

Mergers and Environmental Criteria

- Competition authorities face growing pressure to consider **pollution** in merger review: the European Commission, the UK's Competition and Markets Authority, and the Japan Fair Trade Commission.
- In contrast, the **2023 U.S. Merger Guidelines** do not include environmental considerations as a separate criterion.
- Recent European Commission decisions: *Norsk Hydro/Alumetal* (cleared unconditionally, May 4, 2023) and *Sika/MBCC* (cleared February 8, 2023, subject to a divestiture).
- In both cases, environmental aspects entered through **competition in green products**, not as a separate criterion.

The 2026 Draft EU Merger Guidelines

- On **April 30, 2026**, the European Commission published for consultation a **draft set of new Merger Guidelines**, replacing the horizontal (2004) and non-horizontal (2008) guidelines.
- Sustainability is a **non-price parameter** of competition (with privacy and resilience); efficiencies may include **dynamic** sustainability benefits.
- Consultation closed June 26, 2026; adoption is expected in the fourth quarter of 2026.
- If adopted, environmental effects would become an **explicit criterion**: the one our model assumes when the competition authority (CA) uses **total welfare, including environmental damage** (Sections 7.2–7.5).

Agreements, Mergers, and the UK Exception

- Agreements and mergers follow **separate tracks**. Sustainability guidance mostly concerns **agreements**: EC Horizontal Cooperation Guidelines (June 2023), a Dutch policy rule (October 2023), and the UK Green Agreements Guidance (October 2023), which does **not** extend to mergers.
- Merger review has moved more slowly. Exception: the UK **Merger Assessment Guidelines** (March 2021; efficiencies revised September 2026) count sustainability and the low-carbon transition among **customer benefits** that can offset a loss of competition.
- Until the European draft is adopted, environmental benefits enter mainly through the **efficiency defense** (Section 7.8.4).

Why a Merger Can Be Environmentally Friendly

- A merger reduces aggregate output: **consumer surplus** falls and **producer surplus** rises.
- Without environmental considerations, the consumer surplus loss dominates, and the CA blocks the merger.
- Lower output also means **fewer emissions**: a new (positive) welfare effect.
- The merger can then **improve welfare** if pollution is relatively severe, $d > \frac{5}{7}$ (Section 7.2).

Preview of Results

- **Section 7.3:** large cost reductions can raise consumer surplus; cost reductions make approval easier.
- **Section 7.4:** with convex costs, the merger becomes slightly more likely to reduce welfare.
- **Section 7.5:** larger mergers are more likely to reduce welfare, and are profitable only with a large market share (the "80% rule").
- **Section 7.6:** with an optimal emission fee, the CA's decision becomes **inconsequential**.
- **Section 7.10:** with competition policy, emission fees **hinder collusion**.

Guiding Questions

1. How can mergers affect pollution levels and overall welfare in oligopolistic industries?
2. Why might competition authorities consider environmental criteria in merger review?
3. Under what conditions can a merger reduce emissions enough to offset market power?
4. How do cost-reducing synergies influence the welfare implications of mergers?
5. What role do convex production costs play in shaping merger incentives?
6. How does environmental regulation interact with antitrust decisions?

Section 7.2

A Merger-to-Monopoly

Model Setup

- Two firms facing inverse demand $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- Symmetric marginal cost c, where $1 > c > 0$.
- **Environmental damage:** $Env = \frac{d}{2}Q^2$, where $d > 0$ measures the **severity** of environmental damage (as in Chapter 2).
- **Social welfare:**

$$W = CS + PS - Env$$

Time Structure

1. *Stage 1:* every firm chooses whether to submit a merger request to the CA.
2. *Stage 2:* if both firms submit, the CA approves or blocks, using either the **CS criterion** (as the U.S. Federal Trade Commission) or the **W criterion** (as in some European Commission cases).
3. *Stage 3:* if approved, firms choose output to maximize **joint profits**; otherwise, they compete à la Cournot.

- We solve the game by **backward induction**.

No Merger: Cournot Duopoly

- **Firm i's problem:**

$$\max_{q_i \geq 0} (1 - q_i - q_j) q_i - c q_i \tag{7.1}$$

- **First-order condition:** $1 - 2q_i - q_j - c = 0$.
- **Best response function:**

$$q_i(q_j) = \frac{1 - c}{2} - \frac{1}{2} q_j \tag{7.2}$$

No Merger: Cournot Equilibrium

- Symmetric equilibrium, $q_i = q_j$:

$$q_i^{NM} = \frac{1-c}{3} \tag{7.3}$$

- **Aggregate output:** $Q^{NM} = \frac{2(1-c)}{3}$ (7.4)
- **Profits:**

$$\pi_i^{NM} = \frac{(1-c)^2}{9} \tag{7.5}$$

No Merger: Welfare

- $CS^{NM} = \frac{(Q^{NM})^2}{2} = \frac{2(1-c)^2}{9}$ and $PS^{NM} = \pi_1^{NM} + \pi_2^{NM} = \frac{2(1-c)^2}{9}$.
- $Env^{NM} = \frac{d}{2}(Q^{NM})^2 = \frac{2d(1-c)^2}{9}$.
- **Welfare:**

$$\begin{aligned} W^{NM} &= CS^{NM} + PS^{NM} - Env^{NM} \\ &= \frac{2(1-c)^2(2-d)}{9} \end{aligned} \tag{7.6}$$

- Decreasing in pollution severity d; positive if $d < 2$, and in this case decreasing in c.

Merger: Joint Profit Maximization

- The merged entity maximizes joint profits $\pi_1 + \pi_2$, (7.7)–(7.8), which reduces to

$$\max_{Q \geq 0} (1 - Q)Q - cQ \tag{7.9}$$

- First-order condition $1 - 2Q - c = 0$ yields $Q^M = \frac{1-c}{2}$.
- Firms are cost symmetric, so they evenly divide output:

$$q_1^M = q_2^M = \frac{Q^M}{2} = \frac{1-c}{4} \tag{7.10}$$

Merger: Welfare

- $CS^M = \frac{(1-c)^2}{8}$, $\pi_i^M = \frac{(1-c)^2}{8}$, $PS^M = \frac{(1-c)^2}{4}$, $Env^M = \frac{d(1-c)^2}{8}$.
- **Welfare:**

$$W^M = \frac{(1-c)^2(3-d)}{8} \tag{7.11}$$

- Decreasing in d, and in c when welfare is positive.
- Positive if $d < 3$, a less demanding condition than $d < 2$: lower output yields fewer emissions.

Table 7.1: Welfare Effects of the Merger

	No merger	Merger	Welfare effect
Q	$\frac{2(1-c)}{3}$	$\frac{1-c}{2}$	$-$
CS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c)^2}{8}$	$-$
PS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c)^2}{4}$	$+$
Env	$\frac{2d(1-c)^2}{9}$	$\frac{d(1-c)^2}{8}$	$+$
W	$\frac{2(1-c)^2(2-d)}{9}$	$\frac{(1-c)^2(3-d)}{8}$	$+$ iff $d > \frac{5}{7}$

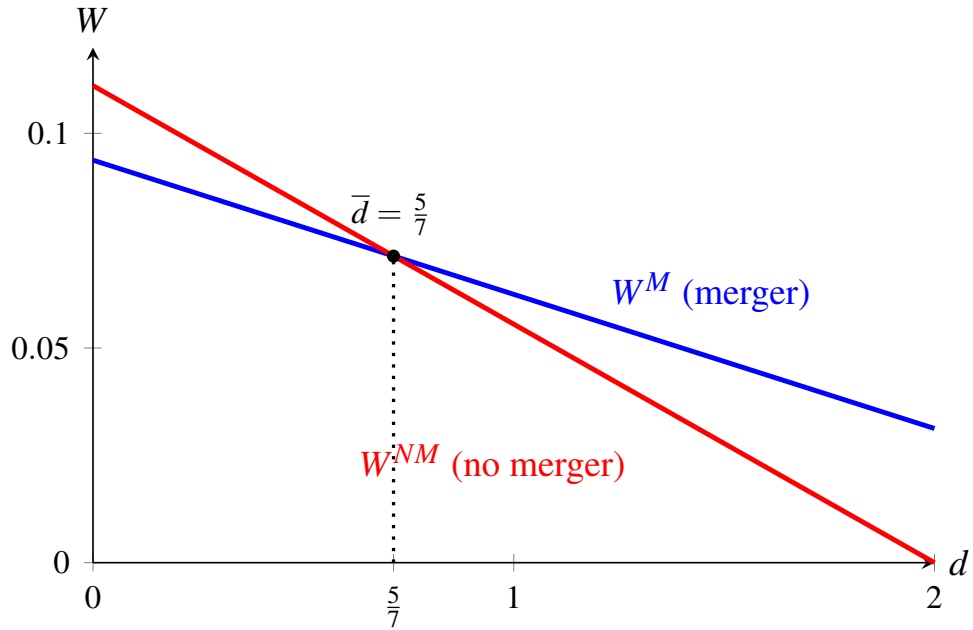
Welfare Comparison

- **Two positive effects** (higher PS, lower Env) and **one negative effect** (lower CS). The welfare gain from the merger is

$$\begin{aligned} \Delta W &= \frac{(1-c)^2(3-d)}{8} - \frac{2(1-c)^2(2-d)}{9} \\ &= \frac{(1-c)^2(7d-5)}{72} \end{aligned} \tag{7.12}$$

- $\Delta W > 0$ if and only if $d > \overline{d} \equiv \frac{5}{7} \simeq 0.71$.
- Special case $d = 0$ (no damage): $\Delta W = -\frac{5(1-c)^2}{72} < 0$; the consumer surplus loss dominates, and the CA blocks every merger request.

Figure 7.1: Welfare With and Without the Merger



- At $c = \frac{1}{2}$: $W^M = \frac{3-d}{32}$ and $W^{NM} = \frac{2-d}{18}$ both decrease in d and cross at $\overline{d} = \frac{5}{7}$.
- **Left**, $d < \frac{5}{7}$: the CA blocks the merger. **Right**, $d > \frac{5}{7}$: the emission reduction and the profit gain make it welfare enhancing.

Stage 2: CS and W Criteria

- **CS criterion:** approve if and only if $CS^M > CS^{NM}$, which never holds:

$$CS^M = \frac{(1-c)^2}{8} < \frac{2(1-c)^2}{9} = CS^{NM}$$

  The CA blocks the merger.

- **W criterion:** approve if and only if $W^M > W^{NM}$, that is,

$$d > \frac{5}{7}$$

Stage 1: Merger Request and Equilibrium

- **CS criterion:** firms anticipate a block for all admissible parameters, so they do not submit a request.
- **W criterion:** the merger is approved if $d > \frac{5}{7}$, and firms submit because $\pi_i^M > \pi_i^{NM}$, that is, $\frac{(1-c)^2}{8} > \frac{(1-c)^2}{9}$.
- **Equilibrium:** if $d > \frac{5}{7}$, firms submit, the CA approves, and $q_1^M = q_2^M = \frac{1-c}{4}$. Otherwise, firms compete à la Cournot, $q_1^{NM} = q_2^{NM} = \frac{1-c}{3}$.
- The profit comparison matters in Section 7.5, where a merger between k of n firms need not be profitable.

Section 7.3

Allowing for Cost-Reducing Mergers

Cost-Reducing Mergers: Setup

- Following Nocke and Whinston (2022), the merger lowers marginal cost from c to $c - x$, where $0 < x \leq c$.
- x is the merger's **cost-reduction effect** (better management practices, avoiding duplicated costs).
- Without the merger, marginal cost stays at c: the results of Section 7.2 apply.
- With the merger, aggregate output becomes

$$Q^M = \frac{1 - c + x}{2}$$

Table 7.2: Welfare Effects with Cost Reductions

	No merger	Merger	Welfare effect
Q	$\frac{2(1-c)}{3}$	$\frac{1-c+x}{2}$	$+$ iff $x > x_{CS}$
CS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c+x)^2}{8}$	$+$ iff $x > x_{CS}$
PS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c+x)^2}{4}$	$+$
Env	$\frac{2d(1-c)^2}{9}$	$\frac{d(1-c+x)^2}{8}$	$+$ iff $x < x_{CS}$
W	$\frac{2(1-c)^2(2-d)}{9}$	$\frac{(1-c+x)^2(3-d)}{8}$	see (7.16)

Consumer Surplus Comparison

- The merger increases consumer surplus if

$$\Delta CS(x) = \frac{(1-c+x)^2}{8} - \frac{2(1-c)^2}{9} > 0$$

  which holds if and only if $x > x_{CS} \equiv \frac{1-c}{3}$.
- **Small cost reductions**, $x < x_{CS}$ (including $x = 0$): output falls, as in Section 7.2.
- **Large cost reductions**, $x > x_{CS}$: lower costs offset the market power effect; output rises, price falls, and consumer surplus rises.

Producer Surplus and Environmental Damage

- Producer surplus unambiguously increases:

$$\Delta PS(x) = \frac{9(1-c+x)^2 - 8(1-c)^2}{36} > 0 \qquad (7.13)$$

- Environmental damage increases if

$$\Delta Env(x) = \frac{d(1-c+x)^2}{8} - \frac{2d(1-c)^2}{9} > 0 \qquad (7.14)$$

  which holds if and only if $x > x_{CS}$.

- Large cost reductions raise output and emissions; small ones lower both.

Welfare Comparison with Cost Reductions

- Welfare increases after the merger if and only if $\Delta W(x) > 0$, where

$$\begin{aligned}\Delta W(x) &= W^M - W^{NM} \\ &= \frac{(1-c+x)^2(3-d)}{8} - \frac{2(1-c)^2(2-d)}{9} \qquad (7.15)\end{aligned}$$

- The coefficient on d in $\Delta W(x)$ is

$$\frac{(1-c-3x)(7-7c+3x)}{72}$$

which is positive if and only if $x < x_{CS}$.

The Welfare Cutoff $\overline{d}(x)$

- Solving for d, the merger is welfare improving if and only if

$$\begin{cases} d > \overline{d}(x) & \text{if } x < x_{CS}, \\ \text{for all } d & \text{if } x = x_{CS}, \text{ and} \\ d < \overline{d}(x) & \text{if } x > x_{CS}, \end{cases} \tag{7.16}$$

- where

$$\overline{d}(x) \equiv \frac{5}{7} - \frac{144x(2-2c+x)}{7(7-7c+3x)(1-c-3x)}$$

Intuition for Condition (7.16)

- **Small cost reductions**, $x < x_{CS}$: the merger still reduces output and emissions, so it is welfare improving when pollution is severe enough, $d > \overline{d}(x)$.
- **Large cost reductions**, $x > x_{CS}$: the merger increases output and emissions, so it is welfare improving when pollution is not too severe, $d < \overline{d}(x)$.
- $x = x_{CS}$: output and emissions are unchanged; only the cost savings remain, raising welfare for all d.

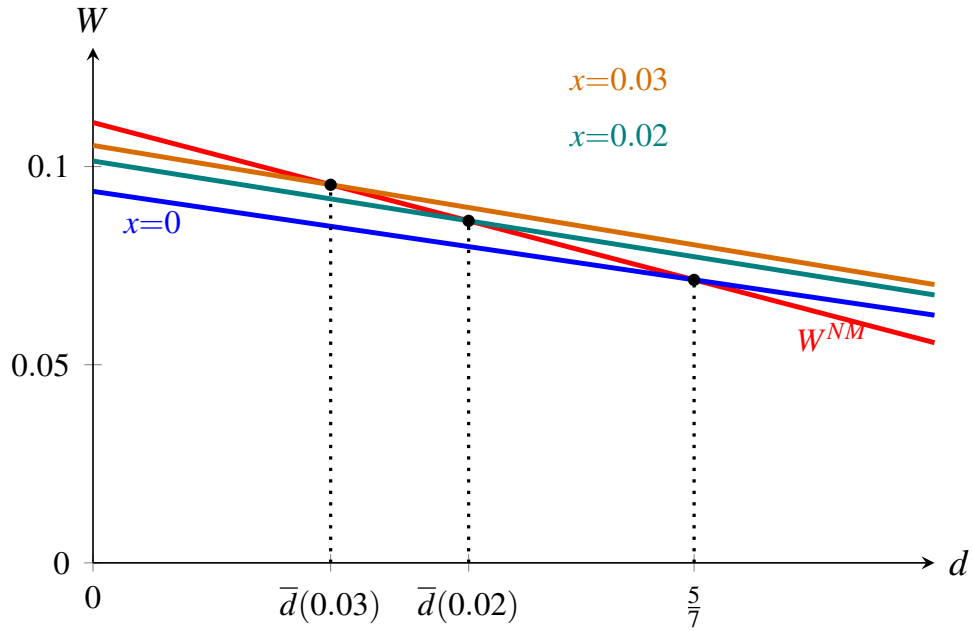
Small Cost Reductions Lower the Cutoff

- Without cost reductions, $\overline{d}(0) = \frac{5}{7}$, as in Section 7.2.
- For $0 < x < x_{CS}$, cutoff $\overline{d}(x)$ decreases in x, (7.17), so

$$\overline{d}(x) < \overline{d}(0) \quad \text{for all } 0 < x < x_{CS} \tag{7.18}$$

- *Intuitively*, cost savings raise producer surplus and reduce the output loss: the merger is welfare improving at lower pollution severities.
- The differential $\overline{d}(0) - \overline{d}(x)$ increases in x and in c, (7.19).

Figure 7.2: Welfare and the Cost-Reduction Effect x



- At $c = \frac{1}{2}$: W^M shifts upward as x increases (W^{NM} is unaffected), so the crossing point moves left.
- $\overline{d}(0) = \frac{5}{7} \simeq 0.71$, $\overline{d}(0.02) = 0.446$, $\overline{d}(0.03) = 0.282$; all these x lie below $x_{CS} = \frac{1}{6}$.

When Is the Merger Welfare Improving for All d?

- If x increases enough, cutoff $\overline{d}(x)$ becomes negative: $\overline{d}(x) < 0$ for all $(1-c)\left(\frac{4}{3}\sqrt{\frac{2}{3}}-1\right) < x < x_{CS}$.
- At $c = \frac{1}{2}$ (Figure 7.2), this range is $0.044 < x < \frac{1}{6}$.
- Together with $x = x_{CS}$: the merger is welfare improving regardless of pollution severity when $(1-c)\left(\frac{4}{3}\sqrt{\frac{2}{3}}-1\right) < x \leq x_{CS}$.
- For $x > x_{CS}$, $\overline{d}(x) > \frac{5}{7}$, and the merger is welfare improving only if $d < \overline{d}(x)$.

Second and First Stages (Sections 7.3.2–7.3.3)

- **CS criterion:** the CA approves the merger if and only if cost reductions are large, $x > x_{CS}$.
- **W criterion:** the CA approves under (7.16): if $d > \overline{d}(x)$ when $x < x_{CS}$; for all d when $x = x_{CS}$; and if $d < \overline{d}(x)$ when $x > x_{CS}$.
- The merger is profitable, and more so when x increases: firms submit a request whenever they anticipate approval.

Section 7.4

Allowing for Mergers with Non-Linear Costs

Convex Costs: Setup

- Every firm operates one plant with total cost

$$TC(q_i) = cq_i + \frac{h}{2}(q_i)^2 \tag{7.20}$$

  and marginal cost $MC(q_i) = c + hq_i$.
- $h = 0$: constant marginal cost c. $h > 0$: increasing marginal cost; h captures **cost convexities** (diseconomies of scale).
- The merged firm keeps **both plants open**, splitting production between them, as in Perry and Porter (1985).
- Cost convexities **reduce** firms' incentives to merge: increasing marginal costs already restrain output before the merger.

No Merger: Best Response with Convex Costs

- Every firm i solves

$$\max_{q_i \geq 0} (1 - q_i - q_j)q_i - cq_i - \frac{h}{2}(q_i)^2 \tag{7.21}$$

- First-order condition $1 - 2q_i - q_j - c - hq_i = 0$ yields

$$q_i(q_j) = \frac{1-c}{2+h} - \frac{1}{2+h}q_j \tag{7.22}$$

- $h > 0$: lower vertical intercept and flatter slope. Diseconomies of scale "soften" output competition.

No Merger: Equilibrium

- Symmetric equilibrium, (7.23)–(7.24):

$$q_i^{NM}(h) = \frac{1-c}{3+h} \qquad \text{and} \qquad Q^{NM}(h) = \frac{2(1-c)}{3+h}$$

  both decreasing in h.
- $CS^{NM}(h) = \frac{2(1-c)^2}{(3+h)^2}$.
- **Profits:**

$$\pi_i^{NM}(h) = \frac{(1-c)^2(2+h)}{2(3+h)^2} \tag{7.25}$$

  which collapses to $\frac{(1-c)^2}{9}$ when $h = 0$.

No Merger: Welfare

- $Env^{NM}(h) = \frac{2d(1-c)^2}{(3+h)^2}$ (7.26)
- **Welfare:**

$$W^{NM}(h) = \frac{(1-c)^2(4+h-2d)}{(3+h)^2} \tag{7.27}$$

- Decreasing in d, and in c when positive, $d < \frac{4+h}{2}$.
- Decreasing in h if and only if $d < \frac{5+h}{4}$: the output reduction from convexities is "good news" only when pollution is particularly severe.

Merger: Splitting Output Between Plants

- The merged entity chooses the output of each plant:

$$\max_{q_1,q_2\geq 0} (1-q_1-q_2)(q_1+q_2) - \left[cq_1+\frac{h}{2}(q_1)^2\right] - \left[cq_2+\frac{h}{2}(q_2)^2\right] \qquad (7.28)$$

- Equal split, $q_1 = q_2 = \frac{Q}{2}$, minimizes total cost, which becomes $cQ + \frac{h}{4}Q^2$.
- First-order condition $1 - 2Q - c - \frac{h}{2}Q = 0$ yields

$$Q^M(h) = \frac{2(1-c)}{4+h} \qquad \text{and} \qquad q_i^M(h) = \frac{1-c}{4+h}$$

The Merger Reduces Output by Less

- Aggregate output falls after the merger:

$$\Delta Q(h) = Q^{NM}(h) - Q^{M}(h) = \frac{2(1-c)}{(3+h)(4+h)} > 0 \quad (7.29)$$

- With linear costs, $\Delta Q(0) = \frac{1-c}{6}$.
- $\Delta Q(h)$ decreases in h: with more convex costs, every firm already restrains its output before the merger, leaving less room for the merger to reduce output.

Cost Convexities Reduce Merger Incentives

- $\pi_i^M(h) = \frac{(1-c)^2}{2(4+h)}$ (7.31), so $PS^M(h) = \frac{(1-c)^2}{4+h}$.
- The merger is profitable:

$$PS^M(h) - PS^{NM}(h) = \frac{(1-c)^2}{(3+h)^2(4+h)} > 0$$

- But this profit gain decreases in h, and so does the relative gain, $\frac{PS^M(h)-PS^{NM}(h)}{PS^{NM}(h)} = \frac{1}{(2+h)(4+h)}$.

Merger: Welfare

- $CS^M(h) = \frac{2(1-c)^2}{(4+h)^2}$ (7.30) and $Env^M(h) = \frac{2d(1-c)^2}{(4+h)^2}$ (7.32).
- **Welfare:**

$$W^M(h) = \frac{(1-c)^2(6+h-2d)}{(4+h)^2} \tag{7.33}$$

- Collapses to $W^M(0) = \frac{(1-c)^2(3-d)}{8}$ when costs are linear.

Table 7.3: Welfare Effects with Cost Convexities

	No merger	Merger	Welfare effect
Q	$\frac{2(1-c)}{3+h}$	$\frac{2(1-c)}{4+h}$	$-$
CS	$\frac{2(1-c)^2}{(3+h)^2}$	$\frac{2(1-c)^2}{(4+h)^2}$	$-$
PS	$\frac{(1-c)^2(2+h)}{(3+h)^2}$	$\frac{(1-c)^2}{4+h}$	$+$
Env	$\frac{2d(1-c)^2}{(3+h)^2}$	$\frac{2d(1-c)^2}{(4+h)^2}$	$+$
W	$\frac{(1-c)^2(4+h-2d)}{(3+h)^2}$	$\frac{(1-c)^2(6+h-2d)}{(4+h)^2}$	$+$ iff $d > \overline{d}(h)$

Welfare Comparison with Convex Costs

- $\Delta W(h) = W^M(h) - W^{NM}(h) > 0$, (7.34), if and only if

$$d > \overline{d}(h) \equiv \frac{10 + 3h}{2(7 + 2h)}$$

- Linear costs: $\overline{d}(0) = \frac{5}{7}$, as in Section 7.2.
- The cutoff increases in h:

$$\frac{\partial \overline{d}(h)}{\partial h} = \frac{1}{2(7 + 2h)^2} > 0 \qquad (7.35)$$

- But it remains below $\frac{3}{4}$, since $\frac{3}{4} - \overline{d}(h) = \frac{1}{4(7+2h)} > 0$.

Why the Cutoff Rises

- Since $Env = d \times CS$, the merger reduces environmental damage by d times the consumer surplus loss.
- Welfare change $=$ profit gain $+ (d-1)\times$ consumer surplus loss. So the merger is welfare improving iff $d > 1 - \frac{\text{profit gain}}{\text{CS loss}} = \overline{d}(h)$.
- As h rises, the profit gain decreases faster than the consumer surplus loss: the CA requires more severe pollution.
- The effect is small: $\overline{d}(h)$ only rises from $\frac{5}{7} \simeq 0.71$ to at most $\frac{3}{4}$.

Second and First Stages (Sections 7.4.2–7.4.3)

- **CS criterion:** output unambiguously decreases,

$$\Delta Q(h) = \frac{2(1-c)}{(3+h)(4+h)} \tag{7.36}$$

  so consumer surplus falls and the CA blocks; firms do not submit a request.
- **W criterion:** the CA approves if and only if $d > \overline{d}(h)$; only then do firms submit a request.

Section 7.5

Allowing for Mergers Between Several Firms

Several Firms: Setup

- Linear costs and no cost reduction, as in Section 7.2; now $n \geq 2$ firms, k of which merge, $n \geq k \geq 2$.
- No merger: n firms compete à la Cournot. Merger: the merged entity plus $n - k$ **outsiders**, $(n - k) + 1$ firms.
- Special cases: merger-to-monopoly, $n = k$; a two-firm merger, $n > k = 2$; and $n > k > 2$.
- Parameter of interest: the merging entity's market share, $\frac{k}{n}$.

No Merger: Cournot with n Firms

- Every firm i solves $\max_{q_i \geq 0} (1 - q_i - Q_{-i})q_i - cq_i$, (7.37), yielding

$$q_i(Q_{-i}) = \frac{1-c}{2} - \frac{1}{2}Q_{-i} \tag{7.38}$$

- Symmetry, $Q_{-i} = (n-1)q_i$, gives (7.39) and

$$q_i^{NM}(n) = \frac{1-c}{n+1} \tag{7.40}$$

- $Q^{NM}(n) = \frac{n(1-c)}{n+1}$ and $\pi_i^{NM}(n) = \frac{(1-c)^2}{(n+1)^2}$ (7.41).

No Merger: Welfare

- $CS^{NM}(n) = \frac{n^2(1-c)^2}{2(n+1)^2}$ and $PS^{NM}(n) = \frac{n(1-c)^2}{(n+1)^2}$.
- $Env^{NM}(n) = \frac{n^2 d(1-c)^2}{2(n+1)^2}$ (7.42)
- **Welfare:**

$$W^{NM}(n) = \frac{n(1-c)^2\left[2+n(1-d)\right]}{2(n+1)^2} \tag{7.43}$$

which reduces to $\frac{2(1-c)^2(2-d)}{9}$ when $n = 2$.

Merger: Merged Entity and Outsiders

- **Merged entity:**

$$\max_{q \geq 0} (1 - q - Q_O)q - cq \tag{7.44}$$

where Q_O is the aggregate output of all $n - k$ outsiders. Best response: $q(Q_O) = \frac{1-c}{2} - \frac{1}{2}Q_O$.

- **Every outsider** (7.45) has best response

$$q_i(q, q_{-O}) = \frac{1-c}{2} - \frac{1}{2}(q + q_{-O})$$

where q_{-O} is the output of the other outsiders.

Merger: Equilibrium Output

- In a symmetric equilibrium, every outsider produces q_O, and so does the merged entity, $q = q_O$ (no synergies). Hence $Q_O = (n-k)q_O$.
- Inserting Q_O into the merged entity's best response:

$$q_O = \frac{1-c}{2} - \frac{1}{2}(n-k)q_O \tag{7.46}$$

  so that $q_i^M(k,n) = \frac{1-c}{n-k+2}$.
- **Aggregate output:**

$$Q^M(k,n) = \frac{(n-k+1)(1-c)}{n-k+2} \tag{7.47}$$

Merger: Profits and Welfare

- $\pi_i^M(k,n) = \frac{(1-c)^2}{(n-k+2)^2}$ (7.48) and
  $PS^M(k,n) = \frac{(n-k+1)(1-c)^2}{(n-k+2)^2}$ (7.49).
- $Env^M(k,n) = \frac{d(n-k+1)^2(1-c)^2}{2(n-k+2)^2}$ (7.50)
- **Welfare:**

$$W^M(k,n) = \frac{(n-k+1)(1-c)^2\left[3+n-k-d(n-k+1)\right]}{2(n-k+2)^2} \tag{7.51}$$

which collapses to $\frac{(1-c)^2(3-d)}{8}$ when $n = k = 2$.

The Merger Reduces Output

- Aggregate output decreases after the merger:

$$Q^{NM}(n) - Q^{M}(k,n) = \frac{(1-c)(k-1)}{(n-k+2)(n+1)} > 0 \qquad (7.52)$$

- Consumer surplus decreases, but environmental damage is also lower.

Is the Merger Profitable?

- The merger between k firms is profitable if and only if $\pi_i^M(k,n) \geq k\pi_i^{NM}(n)$:

$$\left(\frac{1-c}{n-k+2}\right)^2 \geq k\left(\frac{1-c}{n+1}\right)^2 \tag{7.53}$$

- Simplifying, $(n+1)^2 \geq k(n-k+2)^2$, (7.54), or

$$(k-1)\left[-k^2+(2n+3)k-(n+1)^2\right] \geq 0 \tag{7.55}$$

The Profitability Cutoff $\widehat{k}(n)$

- Since $k \geq 2$, solving for k in the second term:

$$k \geq \frac{2n + 3 - \sqrt{4n + 5}}{2} \tag{7.56}$$

- Dividing both sides by n:

$$\frac{k}{n} \geq \frac{2n + 3 - \sqrt{4n + 5}}{2n} \equiv \widehat{k}(n) \tag{7.57}$$

- The merger must cover a large market share: $\frac{k}{n}$ must be at least $\widehat{k}(n)$, which never falls below 80%.

The 80% Rule

- $\widehat{k}(2) = 0.849$, $\widehat{k}(3) = 0.813$, $\widehat{k}(4) = 0.802$, and the minimum, $\widehat{k}(5) = 0.8$.
- For larger industries the cutoff increases again: $\widehat{k}(10) = 0.815$ and $\widehat{k}(20) = 0.845$, approaching 1 as n grows.
- For the merger to be profitable, at least 80% of the firms must merge: the "80% rule," after Salant et al. (1983).

Why the Merger Needs a Large Market Share

- **Positive effect:** merging firms coordinate and reduce output, which raises the market price and their margins.
- **Negative effect:** outsiders respond by *increasing* output (best responses are negatively sloped), which offsets part of the output reduction and limits the price increase.
- The positive effect dominates only when $\frac{k}{n}$ is large (k high, n low, or both).

Welfare Comparison with Several Firms

- $W^M(k,n) \geq W^{NM}(n)$ if and only if

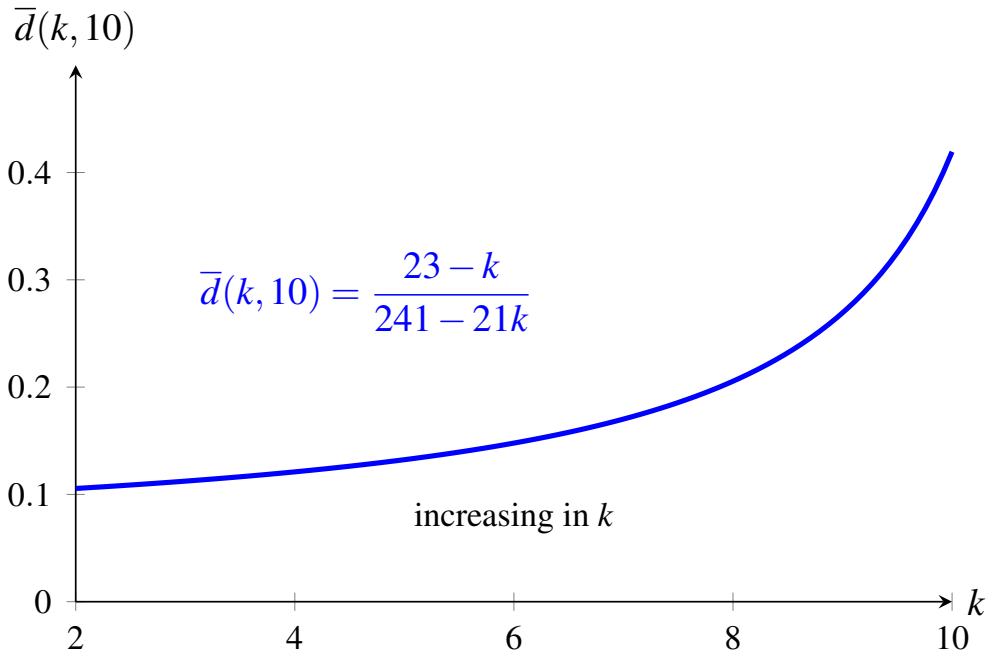
$$d > \overline{d}(k,n) \equiv \frac{2n-k+3}{2n(n-k+2)-k+1} \tag{7.58}$$

  with $\overline{d}(2,2) = \frac{5}{7}$, as in Section 7.2.
- The cutoff increases in the number of merging firms:

$$\frac{\partial \overline{d}(k,n)}{\partial k} = \frac{2(n+1)^2}{[2n(n-k+2)-k+1]^2} > 0 \tag{7.59}$$

- A larger merger reduces output more, so it requires more severe pollution to be welfare improving.

Figure 7.3: Cutoff $\overline{d}(k,n)$ Increases in k



- Evaluated at $n = 10$: $\overline{d}(k,10) = \frac{23-k}{241-21k}$.
- Increasing in k, since $\frac{\partial \overline{d}(k,10)}{\partial k} = \frac{242}{(241-21k)^2} > 0$.

Table 7.4: Welfare Effects of Larger Mergers (I)

	No merger	Merger	Welfare effect
Q	$\frac{n(1-c)}{n+1}$	$\frac{(n-k+1)(1-c)}{n-k+2}$	$-$
CS	$\frac{n^2(1-c)^2}{2(n+1)^2}$	$\frac{(n-k+1)^2(1-c)^2}{2(n-k+2)^2}$	$-$
PS	$\frac{n(1-c)^2}{(n+1)^2}$	$\frac{(n-k+1)(1-c)^2}{(n-k+2)^2}$	$+$

Table 7.4: Welfare Effects of Larger Mergers (II)

	No merger	Merger
Env	$\frac{n^2 d(1-c)^2}{2(n+1)^2}$	$\frac{d(n-k+1)^2(1-c)^2}{2(n-k+2)^2}$
W	$\frac{n(1-c)^2[2+n(1-d)]}{2(n+1)^2}$	$\frac{(n-k+1)(1-c)^2[3+n-k-d(n-k+1)]}{2(n-k+2)^2}$

- Welfare effect of the merger: Env, $+$; W, $+$ if and only if $d > \overline{d}(k, n)$.

Second and First Stages (Sections 7.5.2–7.5.3)

- **CS criterion:** output decreases, so the CA blocks the merger, and firms do not submit a request.
- **W criterion:** the CA approves if and only if $d > \overline{d}(k, n)$.
- The merger must also be profitable, $\frac{k}{n} \geq \widehat{k}(n)$: industry producer surplus rises, but outsiders capture part of the increase.
- When both conditions hold, firms submit a request and the CA approves it; otherwise, firms compete à la Cournot.

Section 7.6

Allowing for Environmental Regulation

Adding an Environmental Agency

- Builds on Choi et al. (2022), who also allow for cost asymmetries, product differentiation, and different pollution intensities.
- An environmental agency (EA), such as the U.S. Environmental Protection Agency, sets a per-unit **emission fee** t (a subsidy if $t < 0$).

1. *Stage 1:* firms decide whether to submit a merger request.
2. *Stage 2:* if a request is submitted, the CA approves or blocks it.
3. *Stage 3:* the EA sets the emission fee t.
4. *Stage 4:* firms compete à la Cournot if no merger occurs, or coordinate their output otherwise.

Stages 4 and 3: Output and Q^{SO}

- **Stage 4:** output as in Section 7.2.1, at cost $c + t$. No merger: $q_i^{NM}(t) = \frac{1-c-t}{3}$, $Q^{NM}(t) = \frac{2(1-c-t)}{3}$. Merger: $q_i^{M}(t) = \frac{1-c-t}{4}$, $Q^{M}(t) = \frac{1-c-t}{2}$.
- **Stage 3:** the EA induces the output that solves

$$\max_{Q \geq 0} \; W = CS + PS - Env \tag{7.60}$$

  where $CS = \frac{Q^2}{2}$, $PS = (1 - Q)Q - cQ$, and $Env = \frac{d}{2}Q^2$.
- First-order condition $Q + 1 - 2Q - c - dQ = 0$, (7.61), yields

$$Q^{SO} = \frac{1 - c}{1 + d} \tag{7.62}$$

Emission Fee Without the Merger

- The EA sets t such that $Q^{NM}(t) = Q^{SO}$, that is, $\frac{2(1-c-t)}{3} = \frac{1-c}{1+d}$, (7.63), yielding

$$t^{NM} = \frac{(1-c)(2d-1)}{2(1+d)} \tag{7.64}$$

- Positive for all $d > \frac{1}{2}$, and a subsidy otherwise.
- Increasing in d, $\frac{\partial t^{NM}}{\partial d} = \frac{3(1-c)}{2(1+d)^2} > 0$; decreasing in c for all $d > \frac{1}{2}$.
- Every firm produces $\frac{1-c}{2(1+d)}$: the fee achieves the first best.

Emission Fee After the Merger

- The EA sets t such that $Q^M(t) = Q^{SO}$, that is, $\frac{1-c-t}{2} = \frac{1-c}{1+d}$, (7.65), yielding

$$t^M = \frac{(1-c)(d-1)}{1+d} \tag{7.66}$$

- Positive for all $d > 1$, and a subsidy otherwise; increasing in d, and decreasing in c for all $d > 1$.
- The merged firm exercises more market power, so the EA subsidizes it for a larger range of pollution severities: $d < 1$ rather than $d < \frac{1}{2}$.
- Fee t^M also achieves the first best.

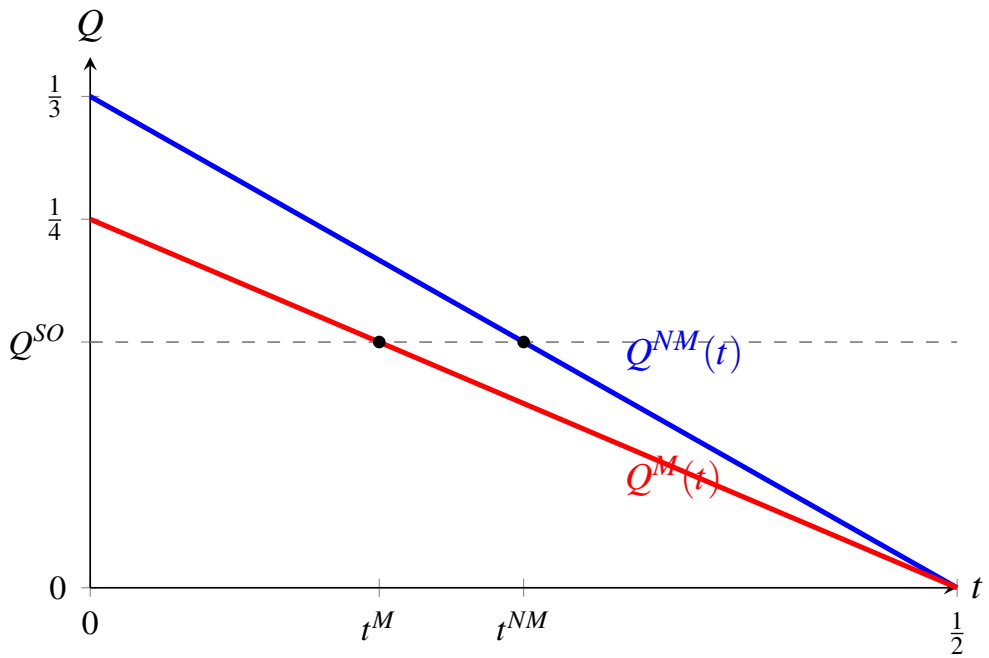
Comparing the Two Fees

- The fee is less stringent after the merger:

$$t^{NM} - t^{M} = \frac{1-c}{2(1+d)} > 0 \qquad (7.67)$$

- With fewer firms, the EA anticipates less aggregate output, $Q^{M}(t) < Q^{NM}(t)$.
- It sets a less stringent fee to induce the same aggregate output as before the merger.

Figure 7.4: Emission Fees t^M and t^{NM}



- At $c = \frac{1}{2}$ and $d = 2$: $Q^{NM}(t) = \frac{1-2t}{3}$, $Q^{M}(t) = \frac{1-2t}{4}$, and $Q^{SO} = \frac{1}{6} \simeq 0.17$.
- To induce Q^{SO}: $t^{NM} = \frac{1}{4}$ before the merger, but only $t^{M} = \frac{1}{6} \simeq 0.17$ after it.

Stage 2: The CA's Decision Is Inconsequential

- If the CA blocks the merger, the EA sets t^{NM}, inducing $Q^{NM}(t^{NM}) = Q^{SO}$; if it approves, the EA sets t^{M}, inducing $Q^{M}(t^{M}) = Q^{SO}$.
- Consumer surplus, environmental damage, and welfare are unchanged by the CA's decision, under both criteria.
- Profits **do** change: every firm's profit net of the (less stringent) fee increases from $\frac{(1-c)^2}{4(1+d)^2}$ to $\frac{(1-c)^2}{2(1+d)^2}$: it doubles.
- Fee collection decreases by the same amount, leaving welfare unaffected.

What If Subsidies Are Unavailable?

- The EA then sets $t = 0$ whenever the optimal fee is negative (Chapter 2), and the CA's decision is inconsequential only when $d \geq 1$.
- $\frac{1}{2} \leq d < 1$: the EA sets t^{NM} without the merger but $t = 0$ after it, so the merged firm produces $\frac{1-c}{2} < Q^{SO}$.
- $d < \frac{1}{2}$: the EA sets $t = 0$ in both cases, as in Section 7.2.
- In both cases, the merger reduces output, consumer surplus, and welfare, and the CA blocks it under both criteria.

Stage 1: Firms' Merger Request (Section 7.6.4)

- Firms anticipate that the CA is **indifferent** between approving and blocking: both yield the same consumer surplus and welfare.
- Assuming the CA approves when indifferent, firms submit the request, since the merger doubles their profits net of the fee.
- The CA approves it in equilibrium.

Section 7.7

Case Studies

Case Study 1: Norsk Hydro and Alumetal

- September 1, 2022: Norsk Hydro notified the European Commission of its proposed acquisition of **Alumetal**, a Polish producer of aluminum foundry alloys made from **recycled scrap**.
- In-depth investigation: concern that Hydro would eliminate a growing supplier of recycled alloys to automotive customers.
- May 4, 2023: cleared **unconditionally**. Combined shares were moderate, several sizeable rivals (including green producers) remained, and the two firms were not close competitors.
- Recycling aluminum requires about 5% of the energy needed for primary production.

Norsk Hydro and Alumetal: Connection to the Model

- A merger with little effect on prices, given the remaining rivals.
- Potential environmental gains if the merged firm expands its use of recycled inputs, lowering **emissions per unit** of output.
- Our model does not include this effect: the cost-reduction effect x of Section 7.3 lowers marginal costs, not emissions per unit.
- The Commission assessed competition only, and did not estimate effects on welfare or emissions.

Case Study 2: Sika and MBCC

- November 2021: Sika agreed to acquire **MBCC Group**, a supplier of construction chemicals.
- European Commission: very large shares in EEA markets for chemical admixtures, few rivals, and high barriers to entry.
- February 8, 2023: cleared on condition that Sika **divest** MBCC's chemical admixture business in the EEA, Australia, Canada, New Zealand, Switzerland, the UK, and the US.
- The Commission described admixtures as essential to the EU's green transition.

Sika and MBCC: Remedy and Connection to the Model

- The UK's Competition and Markets Authority, after an in-depth investigation, accepted a similar remedy on February 21, 2023.
- Sika sold the divested business to **Cinven** and completed the acquisition on May 2, 2023.
- **Model:** without remedies, such a merger would reduce output, lowering emissions but also consumer surplus (Section 7.2); the divestiture keeps rivalry in place.
- Neither authority measured effects on emissions.

Case Study 3: Summit Materials and Argos USA

- September 7, 2023: Cementos Argos agreed to combine its U.S. operations, **Argos USA**, with **Summit Materials** (about $3.2 billion), creating what the companies described as the fourth-largest cement platform in the United States.
- Hart-Scott-Rodino filing with the U.S. antitrust agencies; the waiting period expired on October 23, 2023.
- Summit's shareholders approved the deal on January 11, 2024, and it closed the following day. In February 2025, Summit was itself acquired by Quikrete.
- Cement is costly to transport, so price and emission effects depend on **regional markets**.

Summit and Argos: Connection to the Model

- Section 7.2: a merger raises welfare if the emission reduction from lower output outweighs the consumer surplus loss, which requires sufficiently severe pollution.
- Cement production accounts for around **8 percent** of global CO_2 emissions (Lehne and Preston, 2018).
- In a cement merger, consolidating production in cleaner plants could also lower **emissions per unit** of output, an effect that our model does not include.
- We know of no public evidence measuring either effect for this transaction.

Section 7.8

Merger Review in Practice

From Models to Decisions

- Our models ask whether a merger raises **total welfare**.
- A competition authority answers a narrower question, **in advance**: will *this* transaction harm the customers of *this* market?
- Between 2013 and 2023, the European Commission examined roughly **3,900 mergers** and prohibited **10** of them.
- How does an authority sort the 10 from the rest? Define the market, measure concentration, identify the theory of harm, weigh efficiencies, choose a remedy.

7.8.1 Defining the Relevant Market

- Sections 7.2–7.6 write a single inverse demand function, which **presumes the market is already defined**.
- **Hypothetical monopolist (SSNIP) test:** start from a narrow candidate market (e.g., premium bottled water sold in Denmark). Could a single owner profitably raise price by 5 to 10 percent?
- If enough buyers switch away (to tap water, soft drinks), the market is **too narrow**: add the next closest substitute and repeat.
- The smallest set of products for which the answer is yes is the relevant market.

7.8.1 Product and Geographic Markets

- **Product market:** goods that buyers regard as substitutes. **Geographic market:** the area with similar competitive conditions.
- Cement is heavy and costly to transport, so a plant competes within roughly 200 km: Summit Materials and Argos USA (Section 7.7.3) is best read region by region.
- Market definition determines market shares, and shares drive everything that follows.
- The European Commission revised its market definition notice in **February 2024** (replacing the 1997 notice): more weight on quality, innovation, and sustainability, and adapted to zero-price digital markets.

7.8.2 Measuring Concentration: The HHI

- **Herfindahl-Hirschman Index:** the sum of squared market shares, in percentage points.

	Shares before	Shares after
Firm 1	40	40
Firm 2	30	30
Firms 3 and 4	20 and 10	30
HHI	3,000	3,400

Table 7.5. HHI before and after the merger.

- $40^2 + 30^2 + 20^2 + 10^2 = 3{,}000$ before;
  $40^2 + 30^2 + 30^2 = 3{,}400$ after: an increase of 400 points.

7.8.2 A Shortcut and a Benchmark

- **Shortcut.** When firms with shares s_1 and s_2 merge, the index rises by

$$(s_1 + s_2)^2 - s_1^2 - s_2^2 = 2s_1 s_2$$

  here $2 \times 20 \times 10 = 400$: the competition the two firms no longer exert on each other.
- **Link to Section 7.5:** a higher $\frac{k}{n}$ reduces output more and requires more severe pollution to raise welfare; the HHI captures this **from the outside**, without knowing the firms' cost functions.
- **Benchmark.** With n equal firms, the HHI is $\frac{10{,}000}{n}$: 2,000 with five firms, 1,000 with 10.

7.8.2 Thresholds

- **2023 U.S. Merger Guidelines** (Department of Justice and Federal Trade Commission, December 2023): highly concentrated if HHI $>$ 1,800; an increase of more than 100 points is significant.
- Both conditions together: the merger is **presumed** to lessen competition substantially, and the burden shifts to the parties.
- The 2010 guidelines used 2,500 and 200: the 2023 revision **lowered both thresholds**.
- Our example (HHI of 3,400, increase of 400 points) is presumed anticompetitive under either standard.

7.8.3 Unilateral and Coordinated Effects

- **Unilateral effects:** the merged firm *alone* finds it profitable to raise its price, because lost sales now partly return to it. No agreement with rivals is required: the mechanism of Section 7.2.
- **Coordinated effects:** the merger makes collusion among the *remaining* firms easier to sustain. With fewer firms, deviations are easier to detect and punishments easier to coordinate (Section 7.10 develops the repeated-game logic).
- The two theories are **complements**: a merger may be challenged on either ground, and real decisions address both.

7.8.4 Weighing Cost Savings Against Market Power

- **Williamson (1968):** deadweight loss from a higher price vs. cost saving on every unit still produced.
- Example: price rises from 10 to 12, output falls from 100 to 90, and marginal cost falls from 10 to 8.

Deadweight loss, $\frac{1}{2} \times (12 - 10) \times (100 - 90)$	10
Cost saving, $(10 - 8) \times 90$	180
Net change in total welfare, $180 - 10$	$+170$

Table 7.6. Deadweight loss and cost savings from the merger.

7.8.4 Why Cost Savings Weigh So Much

- A cost reduction of two dollars per unit outweighs a price increase of two dollars per unit by a wide margin.
- *Intuitively*, the cost saving is a **rectangle** over every unit still produced, whereas the deadweight loss is a **triangle** over the units lost.
- Even a modest efficiency can offset a substantial exercise of market power.

7.8.4 Two Qualifications

- **Merger-specific and verifiable** (Farrell and Shapiro, 2001): efficiencies the firms could achieve separately do not count, and the authority must be able to verify them.
- **A different criterion.** Our welfare function adds *CS* and *PS* and subtracts environmental damage; the European Commission and the U.S. agencies apply a **consumer welfare standard**.
- In Table 7.6, consumers pay 12 instead of 10: consumer surplus falls by 190, while profits rise by 360. Under that standard, the merger would still be challenged.
- This gap explains many cases in which an economist judges a merger beneficial and an authority blocks it.

7.8.5 Remedies

- Most problems are resolved by **changing the transaction**, not by prohibition.
- **Structural remedies:** sell assets (a plant, a brand, a subsidiary) to a buyer who will compete. **Behavioral remedies:** keep the structure, constrain conduct (supply rivals, license technology, no bundling).
- Authorities **prefer structural** remedies: a divestiture needs no continuing supervision, whereas a behavioral commitment must be monitored for years.
- Often an **up-front buyer** condition: no closing until a suitable purchaser is approved.

7.8.5 Remedies in Practice

- **Sika/MBCC** (Section 7.7.2): a structural remedy. Sika divested MBCC's admixtures business to Cinven, and the deal proceeded.
- **Booking/eTraveli (2023):** Booking Holdings offered a choice screen displaying the offers of rival hotel booking sites.
- The European Commission rejected it: run by Kayak, a Booking subsidiary (not independent); too narrow in scope; ranking algorithm not monitorable. Merger **prohibited**, September 2023.
- A remedy is only useful if someone outside the firm can verify that it is being honored.

7.8.6 Mergers That Remove a Future Competitor

- Some cases involve buying a firm that is *not yet* a competitor. **Cunningham et al. (2021):** 5.3 to 7.4 percent of pharmaceutical acquisitions were **killer acquisitions** (project discontinued), clustering just below the review threshold.
- A polluting incumbent may buy a firm developing a **cleaner substitute** and shelve it: the loss is the **abatement that never takes place**.
- Section 7.6's result fails here: no emission fee can deliver abatement from a technology withdrawn from development.

7.8.6 Reviewing Below-Threshold Mergers

- **Illumina/GRAIL:** GRAIL had almost no turnover, and the deal met no European notification threshold.
- The European Commission accepted referrals from national authorities, prohibited the merger (September 2022), and fined Illumina €432 million for completing it before the review concluded.
- September 2024: the Court of Justice of the European Union held that the Commission had not been entitled to accept the referrals; the enforcement decisions fell away.
- Several countries now have **call-in powers** for below-threshold mergers: Denmark's took effect on July 1, 2024, and was first used in August 2025.

Section 7.9

Related Literature

Emission Taxes and Mergers: Theory

- **Gautier and Fikru (2024):** incumbents acquire new entrants, which changes the second-best emission tax; acquisitions can justify taxes that exceed marginal damages.
- **Katsoulacos and Xepapadeas (1995)** and **Requate (1997):** with endogenous entry, emission taxes both correct the externality and deter excessive entry.
- **Fikru and Gautier (2017):** horizontal mergers and environmental taxation in oligopoly markets with product differentiation.
- **Creti and Sanin (2017):** mergers under emissions trading; the permit market reshapes the profitability and welfare effects of consolidation.

Extensions and Empirical Evidence

- **Espinosa-Ramírez and Kayalica (2007):** welfare effects of mergers under pollution quotas and emission taxes; the environmental externalities of mergers can justify stricter regulatory responses.
- **Raghavendra et al. (2022):** carbon taxes and environmental regulation reduce inbound cross-border M&As in India, supporting the pollution haven hypothesis.
- Institutional quality, particularly corruption control, strengthens this effect.

Further Reading (Beyond the Book)

- **Farrell and Shapiro (1990):** "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 80(1), pp. 107–126.
  - In Cournot oligopoly, a merger without synergies raises the price; they find conditions under which a profitable merger also raises welfare.
- **Fikru and Gautier (2016):** "Mergers in Cournot Markets with Environmental Externality and Product Differentiation," *Resource and Energy Economics*, 45, pp. 65–79.
  - A merger becomes more attractive as products become more differentiated, and the optimal emission tax can be lower after the merger.

Section 7.10

Cartels in Polluting Industries

Cartels: Setup

- Firms coordinate output **without** the CA's approval, as in a cartel: they identify the joint-profit-maximizing output, and then choose output independently in an **infinitely repeated game**.
- The cartel is stable if no firm has incentives to unilaterally deviate.
- First, the CA does **not** monitor collusion; then, it imposes a fine $F > 0$ if it catches firms colluding.
- Goals: (i) how the CA deters collusion; (ii) how the emission fee t affects incentives to collude.

Cartels in Regulated Industries (I)

- **Trucks:** the European Commission fined Daimler, Volvo/Renault, Iveco, and DAF €2.93 billion in 2016 for collusion between 1997 and 2011 (MAN received immunity; Scania, which refused to settle, was fined €880.5 million in 2017).
- These firms are also regulated under EU standards for CO_2 and NO_X emissions from heavy-duty vehicles.
- **Steel abrasives:** fined €30.7 million in 2014 for collusion during 2003–2010 (Winoa, Metalltechnik Schmidt, and Eisenwerk Würth fined; Ervin received immunity), while subject to the EU Industrial Emissions Directive.

Cartels in Regulated Industries (II)

- **Auto parts:** the U.S. Department of Justice fined manufacturers more than $2 billion for collusion during the 2000s; these firms operate under the Resource Conservation and Recovery Act (RCRA).
- **Freight forwarding:** Schenker AG, Panalpina, Kuehne + Nagel, and others were fined $50.27 million in 2010 for collusion between 2002 and 2007; the air and road transport they organize is a large source of CO_2 emissions.

Competition and Collusion

- **Competition:** every firm solves

$$\max_{q_i \geq 0} \ \pi_i(t) = (1 - q_i - q_j)q_i - (c + t)q_i \qquad (7.68)$$

  with $0 \leq t < 1 - c$. Then $q_i^C(t) = \frac{1-c-t}{3}$ and $\pi^C(t) = \frac{(1-c-t)^2}{9}$.
- **Collusion:** firms maximize joint profits $\pi_i(t) + \pi_j(t)$, (7.69), so $q_i^M(t) = \frac{1-c-t}{4}$ and $\pi^M(t) = \frac{(1-c-t)^2}{8}$.
- Both profits decrease in t, and $\pi^M(t) > \pi^C(t)$.

Deviation

- If firm j keeps $q_j^M(t)$, firm i's best deviation is

$$q_i^D(t) = \frac{1-c-t}{2} - \frac{1}{2}\frac{1-c-t}{4} = \frac{3(1-c-t)}{8} \tag{7.70}$$

  earning $\pi^D(t) = \frac{9(1-c-t)^2}{64}$.
- Ranking of short-run profits:

$$\pi^D(t) \geq \pi^M(t) \geq \pi^C(t) \tag{7.71}$$

Grim-Trigger Strategy

1. In the first period, every firm i chooses the collusive output $q_i^M(t)$.
2. In every later period, if every firm chose $q_i^M(t)$ in all previous periods, firm i chooses $q_i^M(t)$. Otherwise, firm i reverts to the Cournot output $q_i^C(t)$ thereafter.

- A deviation "triggers" an infinite punishment phase. (Infinitely repeated games: Appendix B.7.)

When Is Collusion Sustainable?

- Collusion is sustained if

$$\frac{1}{1-\delta}\pi^M(t) \geq \pi^D(t) + \frac{\delta}{1-\delta}\pi^C(t) \qquad (7.72\text{–}7.73)$$

  where $\delta \in (0,1)$ is the discount factor.

- Equivalently:

$$\underbrace{\pi^D(t) - \pi^M(t)}_{\text{gain from cheating}} \leq \frac{\delta}{1-\delta}\underbrace{\left[\pi^M(t) - \pi^C(t)\right]}_{\text{loss from punishment}} \qquad (7.74)$$

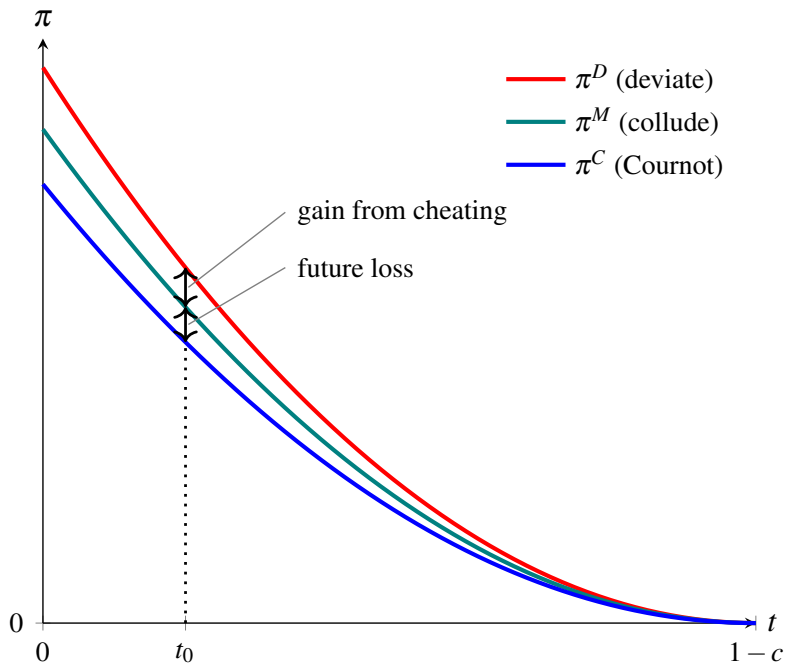
The Minimal Discount Factor

- Solving for δ:

$$\delta \geq \overline{\delta} \equiv \frac{\pi^D(t) - \pi^M(t)}{\pi^D(t) - \pi^C(t)} \tag{7.75}$$

- A larger gain from cheating, or a smaller per-period loss from punishment, raises $\overline{\delta}$.
- Higher $\overline{\delta}$: collusion is **hindered**. Lower $\overline{\delta}$: collusion is **facilitated**.

Figure 7.5: Payoff Gains and Losses from Cheating



- At $c = \frac{1}{2}$ and a given fee t_0: the upper arrow, $\pi^D(t_0) - \pi^M(t_0)$, is the gain from cheating.
- The lower arrow, $\pi^M(t_0) - \pi^C(t_0)$, is the future loss from cheating (return to Cournot).

Emission Fees Do Not Affect Collusion

- Inserting the profits into (7.75) and factoring out $(1-c-t)^2$:

$$\overline{\delta} = \frac{\frac{9}{64} - \frac{1}{8}}{\frac{9}{64} - \frac{1}{9}} = \frac{\frac{1}{64}}{\frac{17}{576}} = \frac{9}{17} \qquad (7.76\text{–}7.78)$$

- The same cutoff as in standard Cournot models without environmental regulation.
- The fee raises costs from c to $c+t$ under competition, collusion, and deviation alike, so it does not affect collusive incentives.

Competition Policy: Setup

- Following Aubert et al. (2006): collusion requires communication, which generates hard evidence. The CA finds it with probability $\rho \in [0, 1]$ and imposes a fine $F > 0$.

1. Every firm chooses whether to communicate.
2. If both communicate, an infinitely repeated game of output competition follows; otherwise, firms compete à la Cournot.
3. The CA finds the evidence with probability ρ.

Payoffs with Competition Policy

- No communication: both firms earn $\pi^C(t)$. Collusion: $\pi^M(t) - \rho F$. Deviation: $\pi^D(t) - \rho F$.
- ρF is the **expected penalty** from collusion.
- We assume $\pi^M(t) - \rho F > \pi^C(t)$, that is,

$$\rho F < \pi^M(t) - \pi^C(t) = \frac{(1-c-t)^2}{72} \tag{7.79}$$

or, solving for t, $t < \bar{t} \equiv 1 - c - 6\sqrt{2\rho F}$.

Collusion with Competition Policy

- Collusion is sustained if (7.80)–(7.82) hold, which simplify to

$$\pi^D(t) - \pi^M(t) \leq \frac{\delta}{1-\delta}\left[\left(\pi^M(t) - \rho F\right) - \pi^C(t)\right] \quad (7.83)$$

- The gain from cheating (left side) is the same as in (7.74).
- The discounted value of continued collusion (right side) is **smaller**: collusion is less likely to arise with audits than without them.

The Cutoff $\overline{\delta}_{CA}$

- Solving (7.83) for δ:

$$\delta \geq \overline{\delta}_{CA} \equiv \frac{\pi^D(t) - \pi^M(t)}{(\pi^D(t) - \rho F) - \pi^C(t)} \tag{7.84}$$

- Inserting profits, (7.85), and simplifying:

$$\overline{\delta}_{CA} = \frac{9(1-c-t)^2}{17(1-c-t)^2 - 576\rho F} \tag{7.86}$$

- $\overline{\delta}_{CA} \in (0,1)$ if and only if (7.79) holds, that is, $t < \overline{t}$.

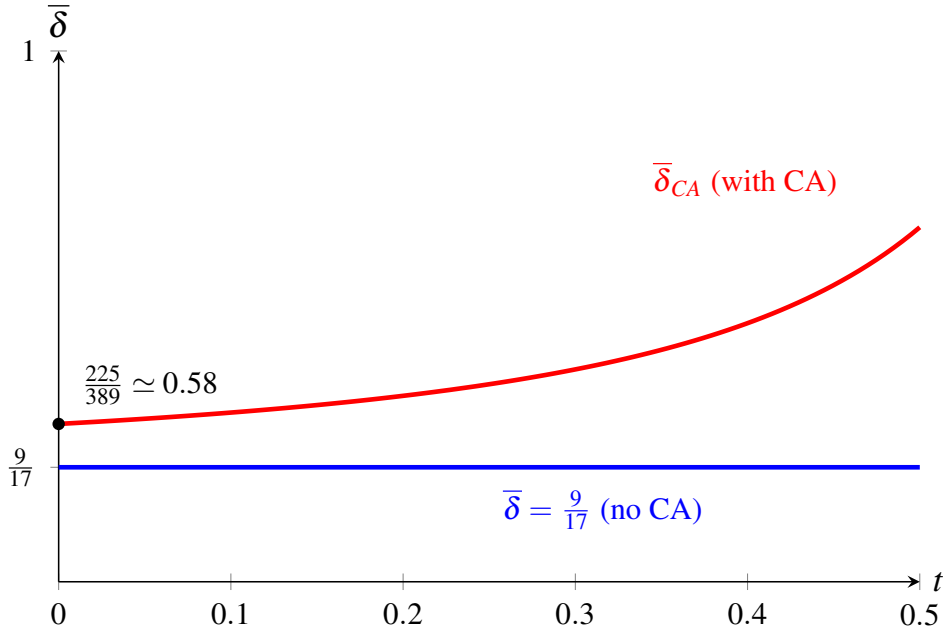
The CA Deters Collusion

- Comparing the cutoffs with and without the CA:

$$\overline{\delta}_{CA} - \overline{\delta} = \frac{5{,}184\rho F}{289(1-c-t)^2 - 9{,}792\rho F} \tag{7.87}$$

- Positive if and only if $\rho F < \frac{17(1-c-t)^2}{576}$, which holds since $\rho F < \frac{(1-c-t)^2}{72}$ by assumption.
- Collusion is harder to sustain when the CA is present: $\overline{\delta}_{CA} > \overline{\delta}$.

Figure 7.6: Cutoffs $\overline{\delta}$ and $\overline{\delta}_{CA}$



- At $c = 0$ and $\rho = F = \frac{1}{20}$: $\overline{\delta}_{CA} = \frac{225(1-t)^2}{425t^2-850t+389}$, interior for $t < 1 - \frac{3\sqrt{2}}{10} \simeq 0.58$.
- Without fees, $\overline{\delta}_{CA} = \frac{225}{389} \simeq 0.58 > \frac{9}{17}$; a more stringent fee widens the gap.

Emission Fees Hinder Collusion

- Cutoff $\overline{\delta}_{CA}$ increases in the fee:

$$\frac{\partial \overline{\delta}_{CA}}{\partial t} = \frac{10{,}368\rho F(1-c-t)}{\left[17(1-c-t)^2 - 576\rho F\right]^2} > 0 \qquad (7.88)$$

- Without the CA, the fee has no effect ($\overline{\delta} = \frac{9}{17}$).
- The expected fine creates a **cost asymmetry**: the fee reduces the collusive profit $\pi^M(t)$, the only scenario in (7.83) where the firm pays the expected fine, making collusion less attractive.

Fees, Monitoring, and Fines

- A more stringent fee works like more likely monitoring (higher ρ) or more severe fines (higher F):

$$\frac{\partial \overline{\delta}_{CA}}{\partial \rho} = \frac{5{,}184F(1-c-t)^2}{\left[17(1-c-t)^2 - 576\rho F\right]^2} > 0$$

  and similarly $\frac{\partial \overline{\delta}_{CA}}{\partial F} > 0$.
- Collusion is deterred when the expected penalty, ρF, increases.

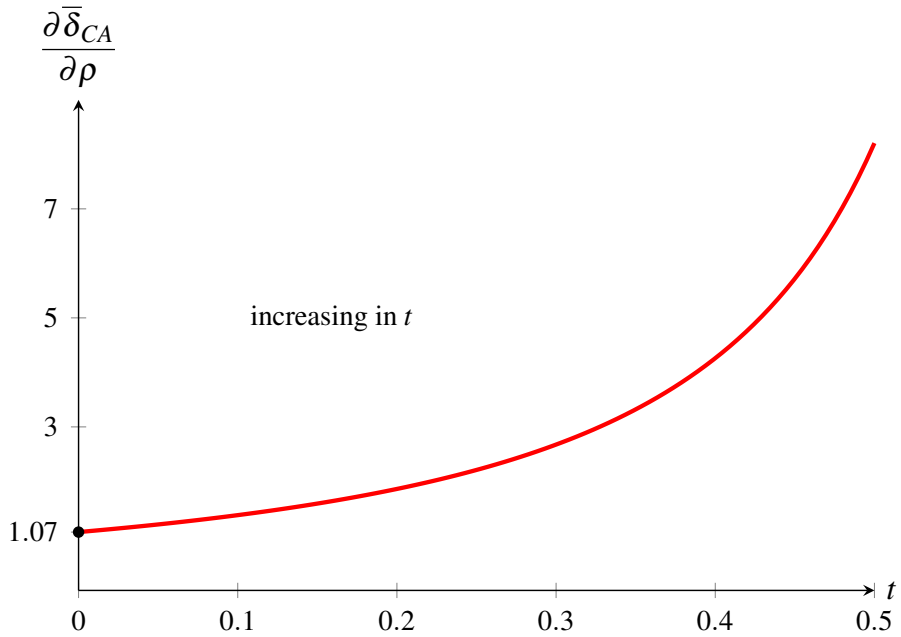
Fees Make Monitoring More Effective

- The cross-partial derivative, (7.89), is positive:

$$\frac{\partial^2 \overline{\delta}_{CA}}{\partial \rho \partial t} = \frac{10{,}368F(1-c-t)\left[17(1-c-t)^2+576\rho F\right]}{\left[17(1-c-t)^2-576\rho F\right]^3} > 0$$

- An increase in ρ raises $\overline{\delta}_{CA}$ by more when fees are more stringent.
- Similarly, $\frac{\partial^2 \overline{\delta}_{CA}}{\partial F \partial t} > 0$ (7.90): more severe fines are more effective when fees are more stringent.

Figure 7.7: Derivative $\frac{\partial \overline{\delta}_{CA}}{\partial \rho}$ as a Function of t



- Same parameters as Figure 7.6 ($c = 0$, $\rho = F = \frac{1}{20}$). Without fees, $\frac{\partial \overline{\delta}_{CA}}{\partial \rho} \simeq 1.07$.
- The derivative increases in t: fees make monitoring more effective at deterring collusion.

Section 7.11

Summary

Summary: Merger-to-Monopoly and Cost Reductions

- **Merger-to-monopoly:** output, consumer surplus, and damage decrease; producer surplus increases. Welfare enhancing if these gains outweigh the consumer surplus loss: $d > \frac{5}{7}$.
- **Cost-reducing mergers:** consumer surplus increases if and only if $x > x_{CS} = \frac{1-c}{3}$; producer surplus increases more; damage may increase.
- Substantial cost reductions make the merger more likely to be welfare enhancing, even if pollution is not extremely severe.

Summary: Non-Linear Costs and More Firms

- **Convex costs:** the merger reduces output, consumer surplus, and damage by less, and its profit gain shrinks. Welfare improving iff $d > \overline{d}(h) \in \left[\frac{5}{7}, \frac{3}{4}\right)$.
- **More firms:** larger mergers reduce output, consumer surplus, and damage more, and raise producer surplus more; they are less likely to be welfare enhancing.
- Profitable only if $\frac{k}{n} \geq \widehat{k}(n)$, never below 80%.

Summary: Regulation, Cartels, and Functional Forms

- **Regulation:** fees induce the same output with or without the merger, so the CA's decision is inconsequential; net profits double, and fee revenue falls by the same amount (without subsidies and $d < 1$, the CA blocks).
- **Cartels:** without the CA, fees do not affect collusion; with the CA, fees hinder collusion and facilitate the CA's task.
- **Functional forms:** cutoffs such as $\frac{5}{7}$ rely on linear demand and quadratic damage; for mergers that reduce aggregate output, the trade-off between lower CS and lower emissions holds more generally.

Summary: Merger Review in Practice

- **Market definition** (SSNIP test) determines market shares; the **HHI** rises by $2s_1s_2$ (U.S. thresholds: 1,800 and 100 points).
- **Unilateral** and **coordinated** effects are complementary theories of harm.
- Even modest **efficiencies** can offset a price increase, if merger-specific and verifiable.
- **Remedies**, mostly structural, resolve more cases than prohibitions.
- **Killer acquisitions** of clean technologies are often too small to notify; no emission fee restores the lost abatement.

Key Equations at a Glance (I)

Quantity	**Formula**
$W^{NM},\ W^{M}$	$\frac{2(1-c)^2(2-d)}{9},\ \frac{(1-c)^2(3-d)}{8}$
ΔW	$\frac{(1-c)^2(7d-5)}{72},\ \text{so } \overline{d} = \frac{5}{7}$
x_{CS}	$\frac{1-c}{3}$
$\overline{d}(h)$	$\frac{10+3h}{2(7+2h)} \in \left[\frac{5}{7}, \frac{3}{4}\right)$
$\overline{d}(k,n)$	$\frac{2n-k+3}{2n(n-k+2)-k+1}$
$\widehat{k}(n)$	$\frac{2n+3-\sqrt{4n+5}}{2n} \geq 0.8$

Key Equations at a Glance (II)

Quantity	**Formula**
Q^{SO}	$\frac{1-c}{1+d}$
$t^{NM}, \quad t^{M}$	$\frac{(1-c)(2d-1)}{2(1+d)}, \quad \frac{(1-c)(d-1)}{1+d}$
$t^{NM} - t^{M}$	$\frac{1-c}{2(1+d)}$
$\overline{\delta}$	$\frac{9}{17}$
$\overline{\delta}_{CA}$	$\frac{9(1-c-t)^2}{17(1-c-t)^2-576\rho F}$

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Questions? Comments?

Chapter 8

Managerial Incentives

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Introduction: Why Managerial Incentives Matter

- Chapters 2–7 assume that firms maximize **profits**.
- In practice, firms often pursue *other* objectives:
  - Public firms weigh social welfare (EDF, Baowu, Equinor).
  - Managers earn bonuses tied to *sales*.
  - Firms hold equity stakes in their rivals.
  - Firms commit to reduce their environmental damage (CSR).
- **Key question:** Does each incentive make the optimal emission fee *more* or *less* stringent?

Guiding Questions (Section 8.1.1)

1. How do managerial objectives beyond profit maximization affect regulatory outcomes?
2. What distinguishes mixed oligopolies from purely private markets?
3. How do sales-based incentives influence output and emissions?
4. Why might cross-ownership lead to softer competition and lower pollution?
5. Why does environmental CSR call for a less stringent emission fee, and is it profitable?
6. How should emission fees be adjusted when firms pursue non-standard objectives?
7. What real-world examples illustrate the need for tailored regulation under managerial incentives?

Chapter 8 at a Glance: Road Map

Section	Model	Main result
8.2	Mixed duopoly	Fee independent of α
8.4	Promoting sales	Fee increases in α
8.6	Cross-ownership	Fee decreases in α_i and α_j
8.8	Environmental CSR	Fee decreases in α
8.10	Summary	Comparison across incentives

- Case studies follow each model: Sections 8.3, 8.5, 8.7, and 8.9.

Profit Maximization: The Benchmark

Before studying managerial incentives, recall the Chapter 2 benchmark:

- Both firms maximize profits, $\pi_i = (1 - q_i - q_j)q_i - (c + t)q_i$.
- Cournot equilibrium output:

$$q^* = \frac{1 - c - t}{3}, \quad Q^* = \frac{2(1 - c - t)}{3}$$

- Emission fee inducing $Q^{SO} = \frac{1-c}{1+d}$, equation (2.10):

$$t = \frac{(1 - c)(2d - 1)}{2(1 + d)}$$

- Every model in Chapter 8 reduces to this benchmark when $\alpha = 0$.

Four Managerial Incentives

Incentive	**Who?**	**Objective**
Mixed duopoly	Public firm 2	$\alpha W + (1-\alpha)\pi_2$
Promoting sales	Manager of firm 2	$\alpha Rev_2 + (1-\alpha)\pi_2$
Cross-ownership	Every firm i	$(1-\alpha_j)\pi_i + \alpha_i\pi_j$
Environmental CSR	Firm 2	$\pi_2 - \alpha Env$

- Same demand, $p(Q) = 1 - Q$, and marginal cost c, where $1 > c \geq 0$.
- Parameter α measures how far the objective departs from profits; its meaning differs across sections.
- Timing: the EA sets fee t; firms then compete à la Cournot.

Section 8.2

Managerial Incentive I: Mixed Duopolies

Mixed Duopolies: Setup and Firm Objectives

- Firm 1 is *private*; firm 2 is *public*, with **nationalization level** $\alpha \in [0,1]$: weight on welfare, or the government's share ($\alpha = 0$: fully privatized; $\alpha = 1$: completely public).
- $p(Q) = 1 - Q$; cost c, where $1 > c \geq 0$; damage $Env = \frac{d}{2}Q^2$ with $d > \frac{1}{2}$. Timing: the EA sets t; firms observe t and choose output.

Private firm 1 maximizes its profit; public firm 2, a weighted objective:

$$\pi_1 = (1 - Q)q_1 - (c + t)q_1 \quad (8.1)$$

$$V = \alpha W + (1 - \alpha)\pi_2 \quad (8.2)$$

- $W = CS + PS + T - Env$, with $CS = \frac{Q^2}{2}$, $PS = \pi_1 + \pi_2$, and $T = tQ$. The fee burden tQ cancels out in W, so V only includes tq_2 through π_2.

Private Firm's Best Response (Section 8.2.1)

Private firm 1 differentiates (8.1):

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - c - t = 0$$

$$q_1(q_2) = \frac{1 - c - t}{2} - \frac{1}{2} q_2 \tag{8.3}$$

- Same as in standard Cournot models with private firms.
- Unaffected by pollution, d, or by nationalization, α.

Public Firm's Best Response Function

Differentiating $V = \alpha W + (1-\alpha)\pi_2$ with respect to q_2:

$$\alpha \underbrace{[1 - c - (1+d)(q_1+q_2)]}_{\partial W/\partial q_2} + (1-\alpha)\underbrace{(1 - 2q_2 - q_1 - c - t)}_{\partial \pi_2/\partial q_2} = 0$$

Solving for q_2:

$$q_2(q_1) = \frac{1 - c - t(1-\alpha)}{2 - \alpha(1-d)} - \frac{1 + \alpha d}{2 - \alpha(1-d)} q_1 \qquad (8.4)$$

- $\alpha = 0$: $q_2(q_1) = \frac{1-c-t}{2} - \frac{1}{2}q_1$, symmetric to firm 1's; $\alpha = 1$: $q_2(q_1) = \frac{1-c}{1+d} - q_1$.
- The fee shifts BRF_2 down, but this effect is attenuated as α rises, and nil at $\alpha = 1$ (fees are revenue neutral for welfare).

Slope of the Public Firm's Best Response

Define the slope $s \equiv -\frac{1+\alpha d}{2-\alpha(1-d)}$. Then

$$\begin{aligned} \frac{\partial s}{\partial \alpha} &= -\frac{1+d}{\left[2-\alpha(1-d)\right]^2} < 0 \quad \text{and} \\ \frac{\partial s}{\partial d} &= -\frac{\alpha(1-\alpha)}{\left[2-\alpha(1-d)\right]^2} \leq 0 \end{aligned} \tag{8.5}$$

- BRF_2 becomes steeper as α increases, and weakly steeper as d increases (strictly when $0 < \alpha < 1$): when firm 1 expands, the public firm cuts its output by more, to curb pollution.
- When firm 1 reduces q_1 by one unit, firm 2 increases q_2 by $\frac{1+\alpha d}{2-\alpha(1-d)}$, less than one unit if and only if $\alpha < 1$.
- A fully nationalized firm, $\alpha = 1$, offsets *every* unit, keeping aggregate output at $\frac{1-c}{1+d}$.

Special Cases: Two Market Failures

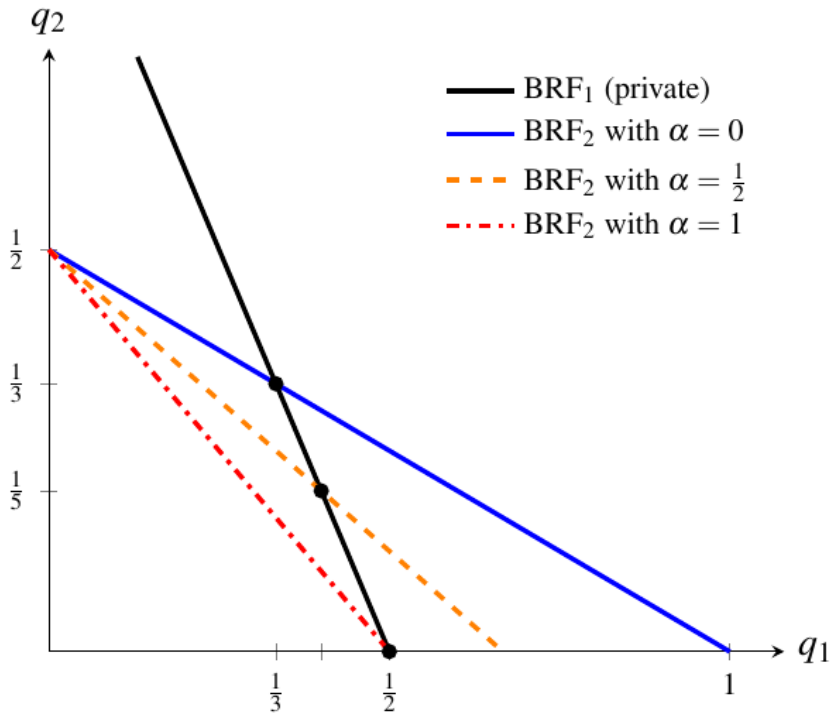
- **No pollution** ($d = 0$, $\alpha > 0$): $q_2(q_1) = \frac{1-c-t(1-\alpha)}{2-\alpha} - \frac{1}{2-\alpha} q_1$.
  - The public firm fills the output gap that the private firm leaves (oligopoly underproduction).
- **Pollution** ($d > 0$, $\alpha > 0$): BRF_2 shifts down and becomes steeper.
  - The public firm now fights two market failures: pollution and underproduction.
  - A lower q_1 mitigates pollution, so the public firm can expand more.

Table 8.1: Public Firm's Best Response Function

	$d = 0$	$d > 0$
$\alpha = 0$	$\frac{1-c-t}{2} - \frac{1}{2}q_1$	$\frac{1-c-t}{2} - \frac{1}{2}q_1$
$1 > \alpha > 0$	$\frac{1-c-t(1-\alpha)}{2-\alpha} - \frac{1}{2-\alpha}q_1$	$\frac{1-c-t(1-\alpha)}{2-\alpha(1-d)} - \frac{1+\alpha d}{2-\alpha(1-d)}q_1$
$\alpha = 1$	$(1-c) - q_1$	$\frac{1-c}{1+d} - q_1$

- Top row: whether $d = 0$ or $d > 0$ is inconsequential for a private firm.
- Bottom row: with pollution, the public firm seeks a lower aggregate output, $\frac{1-c}{1+d} < 1 - c$.

Best Response Functions: An Example



- Parameters $c = 0$, $d = 1$, $t = 0$: BRF_2 is $q_2 = \frac{1}{2} - \frac{1+\alpha}{2} q_1$, steeper as α rises.
- The equilibrium moves along BRF_1: q_2 falls and q_1 rises.

Finding Equilibrium Output (Section 8.2.2)

Firms are asymmetric, so we *cannot* invoke symmetry.

Step 1. Insert BRF_1 (8.3) into BRF_2 (8.4), as in (8.6).

Step 2. Collect the q_2 terms, (8.7)–(8.8):

$$\frac{3-\alpha(2-d)}{2[2-\alpha(1-d)]}q_2 = \frac{(1-c-t)(1-\alpha d)+2\alpha t}{2[2-\alpha(1-d)]}$$

Step 3. Cancel $2[2-\alpha(1-d)]$ and solve for q_2:

$$q_2(t) = \frac{(1-c-t)(1-\alpha d)+2\alpha t}{3-\alpha(2-d)} \tag{8.9}$$

Step 4. Insert (8.9) into BRF_1; (8.11) collects both outputs:

$$q_1(t) = \frac{(1-c-t)(1+\alpha d)-\alpha(1-c)}{3-\alpha(2-d)} \tag{8.10}$$

Equilibrium Output: Special Cases

- **Fully privatized**, $\alpha = 0$:

$$q_1(t) = q_2(t) = \frac{1 - c - t}{3} \quad \text{(standard Cournot)}$$

- **Fully nationalized**, $\alpha = 1$:

$$\begin{aligned} q_1(t) &= \frac{(1 - c - t)d - t}{1 + d} \\ q_2(t) &= \frac{(1 - c)(1 - d) + (1 + d)t}{1 + d} \end{aligned}$$

Effect of Nationalization on Each Firm

$$\frac{\partial q_1(t)}{\partial \alpha} = \frac{(1-c)(2d-1) - 2t(1+d)}{[3-\alpha(2-d)]^2} \tag{8.12}$$

$$\frac{\partial q_2(t)}{\partial \alpha} = -\frac{2\left[(1-c)(2d-1) - 2t(1+d)\right]}{[3-\alpha(2-d)]^2} \tag{8.13}$$

- Both derivatives have the same numerator, with opposite signs.
- An increase in α moves the two firms' outputs in *opposite* directions.

When Is Pollution Severe? Cutoff $\overline{d}$

- $q_1(t)$ increases in α if and only if $(1-c)(2d-1) > 2t(1+d)$, that is,

$$d > \overline{d} \equiv \frac{1-c+2t}{2(1-c-t)}$$

- $\overline{d} - \frac{1}{2} = \frac{3t}{2(1-c-t)} > 0$ for every positive fee.
- **Severe pollution**, $d > \overline{d}$: firm 2 cuts output to curb pollution; firm 1 expands.
- **Mild pollution**, $\overline{d} > d > \frac{1}{2}$: firm 2 expands to correct underproduction; firm 1 contracts.

Aggregate Output and Nationalization

$$Q(t) = q_1(t) + q_2(t) = \frac{2\left[1 - c - t(1-\alpha)\right] - \alpha(1-c)}{3 - \alpha(2-d)} \quad (8.14)$$

- $\alpha = 0$: $Q(t) = \frac{2(1-c-t)}{3}$, unaffected by d (firms ignore pollution); $\alpha = 1$: $Q(t) = \frac{1-c}{1+d}$, decreasing in d (firm 2 only considers welfare).

$$\frac{\partial Q(t)}{\partial \alpha} = -\frac{(1-c)(2d-1) - 2t(1+d)}{\left[3 - \alpha(2-d)\right]^2} \quad (8.15)$$

- Negative if and only if $d > \overline{d}$: from (8.12)–(8.13), the change in $q_2(t)$ is *twice* as large as the change in $q_1(t)$.
- Severe pollution: $Q(t)$ falls. Mild pollution: $Q(t)$ rises.

Table 8.2: Comparative Statics of Higher α

	$q_1(t)$	$q_2(t)$	$Q(t)$
Pollution is not severe, $d < \overline{d}$	$-$	$+$	$+$
Pollution is severe, $d > \overline{d}$	$+$	$-$	$-$

- When $d = \overline{d}$, an increase in α does not affect either firm's output.

Cutoff $\overline{d}$ Increases in t and c

$$\frac{\partial \overline{d}}{\partial t} = \frac{3(1-c)}{2(1-c-t)^2} > 0 \quad \text{and} \quad \frac{\partial \overline{d}}{\partial c} = \frac{3t}{2(1-c-t)^2} \tag{8.16}$$

- A more stringent fee or more costly production reduces aggregate output.
- Underproduction becomes a more pressing market failure for the public firm.
- The region $d < \overline{d}$ expands: a more nationalized firm increases its output, and $Q(t)$ increases.

When Are Both Firms Active?

- Firm 1: $q_1(t) > 0$ if and only if $t < \frac{(1-c)(1+\alpha d-\alpha)}{1+\alpha d}$.
- Firm 2, when $\alpha d < 1$: $q_2(t) > 0$ for every $t \in [0, 1-c)$.
- Firm 2, when $\alpha d \geq 1$: $q_2(t) > 0$ if and only if $t > \frac{(1-c)(\alpha d-1)}{\alpha d+2\alpha-1}$.
- Intuitively, at low fees a public firm that cares strongly about pollution would like a negative output, so it chooses $q_2 = 0$.

Social Welfare in the Mixed Duopoly

$$W = \underbrace{\frac{Q^2}{2}}_{CS} + \underbrace{\pi_1 + \pi_2}_{PS} + \underbrace{tQ}_{T} - \underbrace{\tfrac{d}{2}Q^2}_{Env}$$

- The fee burden in PS cancels with fee revenue T.
- Firms share the same marginal cost and sell a homogeneous good, so W depends only on *aggregate* output Q.
- The EA's goal: induce $Q^{SO} = \frac{1-c}{1+d}$, whatever the ownership structure.

Finding the Emission Fee (Section 8.2.3)

Setting $Q(t) = Q^{SO}$:

$$\frac{2\left[1 - c - t(1-\alpha)\right] - \alpha(1-c)}{3 - \alpha(2-d)} = \frac{1-c}{1+d} \tag{8.17}$$

Solving for t, which requires $\alpha < 1$:

$$t(\alpha) = \frac{(1-c)(2d-1)}{2(1+d)} \tag{8.18}$$

- *Independent* of α, and equal to the Chapter 2 fee (2.10): a **neutrality** result. Positive for all $d > \frac{1}{2}$.
- A higher α changes the *composition* of output, but the fee that steers aggregate output to Q^{SO} is the same for every α.
- Neutrality relies on cost-symmetric firms selling a homogeneous good; it breaks down under other managerial incentives (Section 8.4).

Output at the Fee

Inserting $t(\alpha)$ into (8.11):

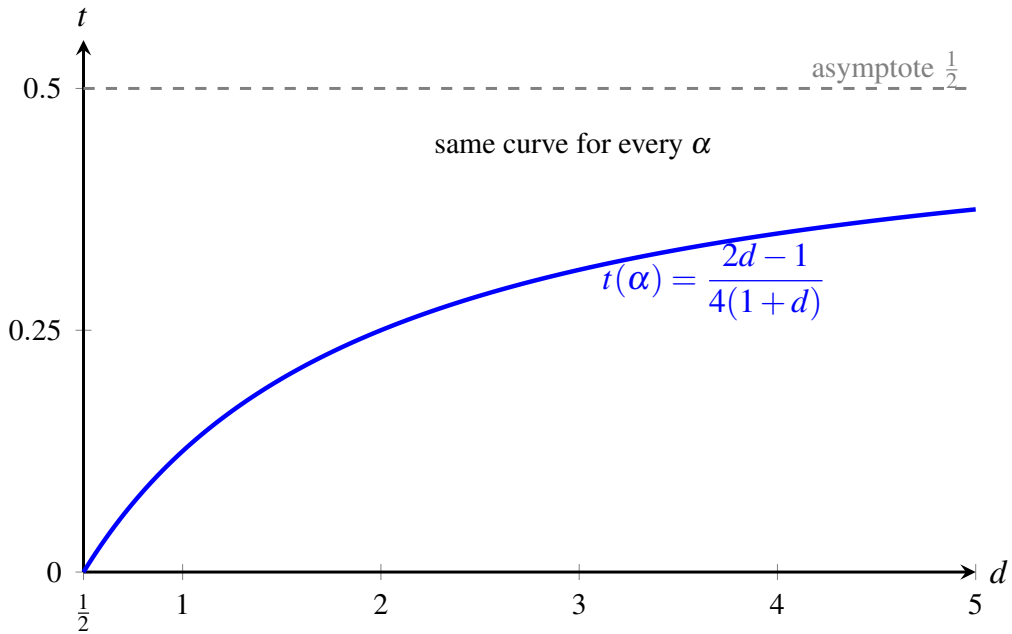
$$q_1(t(\alpha)) = q_2(t(\alpha)) = \frac{1-c}{2(1+d)} \tag{8.19}$$

- Each firm produces half of Q^{SO}, for every α.
- At Q^{SO}, $\frac{\partial W}{\partial q_2} = 1 - c - (1+d)Q^{SO} = 0$: the public firm's welfare motive plays no role at the margin.
- Cutoff $\overline{d}$ evaluated at fee (8.18) equals d, so the derivatives (8.12), (8.13), and (8.15) are zero at this fee.

A Fully Public Firm, $\alpha = 1$

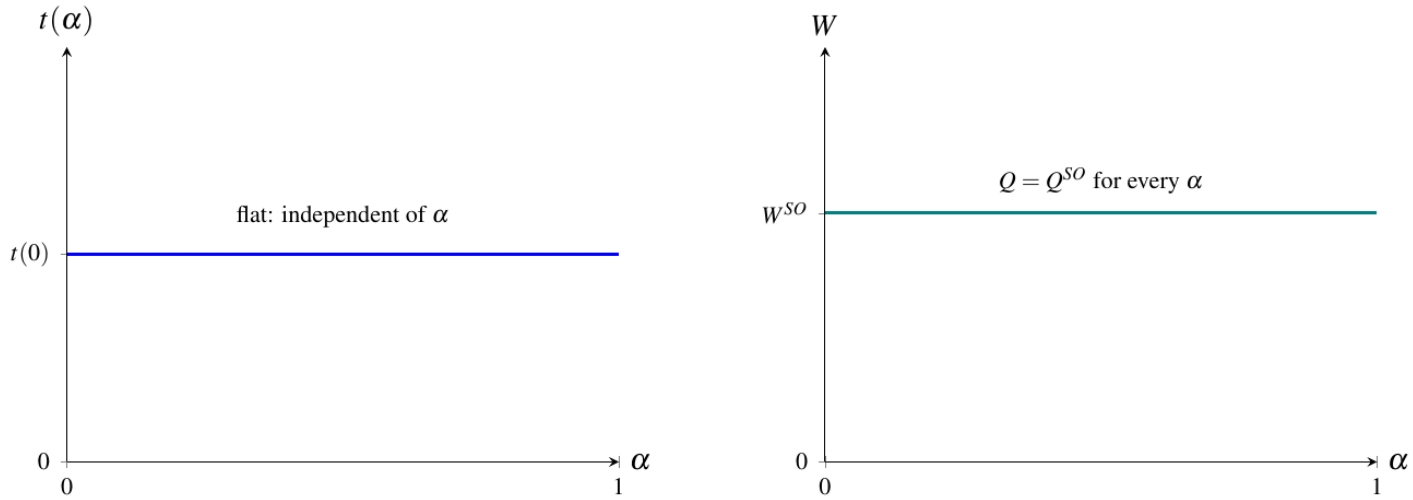
- Aggregate output (8.14) equals Q^{SO} for *every* fee: the public firm's best response, $q_2 = Q^{SO} - q_1$, offsets any change in q_1.
- Every fee at or above $\frac{(1-c)(d-1)}{1+d}$, where q_2 becomes positive, induces Q^{SO}.
- Both firms are active if the fee also lies below $\frac{d(1-c)}{1+d}$, where firm 1 exits.
- Fee (8.18) lies in this interval, and it is the only fee at which both firms produce the same output.

Figure 8.1: Emission Fee $t(\alpha)$, Independent of α



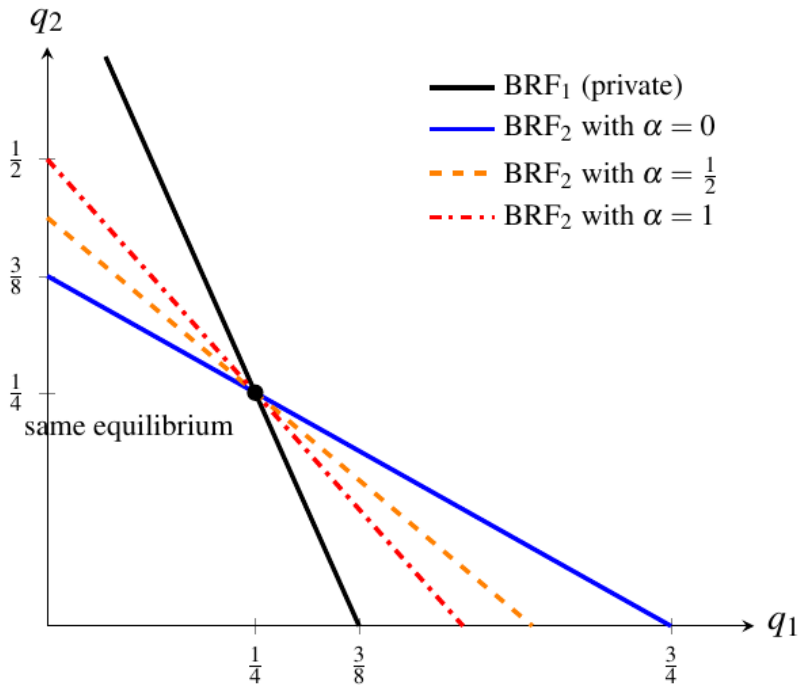
- Evaluated at $c = \frac{1}{2}$: $t(\alpha) = \frac{2d-1}{4(1+d)}$, the same curve for every α; d starts at $\frac{1}{2}$.

Fee and Welfare Under the Mixed Duopoly



- Left: the fee $t(\alpha)$ is **flat** in α, equal to the Chapter 2 fee.
- Right: at this fee $Q = Q^{SO}$ for every α, so welfare is the same for every α.
- Choosing α together with the fee would not raise welfare: at fee (8.18), nationalization changes neither output nor welfare.

Graphical Summary: The Equilibrium at the Fee



- With $c = 0$ and $d = 1$, the fee is $t = \frac{1}{4}$: every BRF$_2$ pivots around the same equilibrium, $q_1 = q_2 = \frac{1}{4}$.

8.3.1 EDF and Engie in France

- **EDF:** wholly owned by the French State since June 2023, after the buyout of minority shareholders.
- **Engie:** listed; the State holds about one quarter of the capital and one third of the voting rights.
- Carbon price from the EU ETS; the regulator (CRE) sets network and retail tariffs, not emission fees.
- **Connection to the model:** EDF is a fully nationalized firm, $\alpha = 1$; Engie a partially nationalized rival. With severe pollution, a more nationalized firm reduces its output while its rival expands, and the optimal fee does not depend on nationalization.
- Testing this neutrality result would require comparing fees across industries with different public shares, which this case does not do.

8.3.2 Baowu Steel in China

- China Baowu Steel Group (formed in 2016), the world's largest steelmaker, is a central state-owned enterprise; it competes with private steelmakers such as Jiangsu Shagang.
- Since 2018, the Environmental Protection Tax Law taxes air and water pollutants.
- Reduced rates (75% or 50%) for firms well below the discharge standard; since 2019, ultra-low-emission steel mills can qualify.
- **Connection to the model:** Baowu is the public firm, Shagang the private firm, and the tax an emission fee; the case does not measure how ownership affects output.

8.3.3 Equinor on the Norwegian Continental Shelf

- The Norwegian State owns 67% of Equinor, which operates on commercial terms.
- Equinor produces oil and gas alongside private firms such as Aker BP.
- Since 1991, Norway has taxed CO_2 from petroleum activities; producers also pay for EU ETS allowances.
- **Connection to the model:** a partially nationalized firm, $0 < \alpha < 1$; its objective may place little weight on welfare.

8.3.4 SAS and Norwegian Air in Scandinavia

- Until 2024, the Danish and Swedish states each held 21.8% of SAS.
- After its 2024 restructuring: Castlelake (about 32%), Danish State (about 26%), Air France-KLM (19.9%), Lind Invest (about 9%); Air France-KLM agreed in July 2025 to raise its stake to 60.5%.
- Norwegian is listed in Oslo; the Norwegian State sold its stake in June 2025.
- **Connection to the model:** SAS is partially nationalized, with a minority public owner; the case does not show how ownership changed output.

8.3.5 TEPCO and Regional Utilities in Japan

- Since 2012, TEPCO has been controlled by a state-backed corporation holding about half of its voting rights.
- Regional utilities such as Kansai Electric are listed private firms; since April 2016, they can compete across regions.
- Japan's Tax for Climate Change Mitigation (2012) was phased in until April 2016 to 289 yen per tonne of CO_2.
- Since April 2026: mandatory phase of the GX Emissions Trading System, for companies emitting at least 100,000 tonnes of CO_2 per year.

8.3.5 KEPCO in South Korea

- The Korean government holds about 51% of KEPCO, directly and through the Korea Development Bank.
- KEPCO's six generation subsidiaries sell power alongside independent producers.
- **Connection to the model:** TEPCO and KEPCO's subsidiaries play the role of the public firm; their rivals, the private firm.
- These cases illustrate mixed ownership in electricity, but they do not test the model's predictions.

Section 8.4

Managerial Incentive II: Promoting Sales

Promoting Sales: Motivation

- Managers often receive bonuses based on **sales** as well as profits (automobiles, fashion, consumer electronics).
- Builds on Fershtman and Judd (1987) and Sklivas (1987): owners reward managers for both profits and sales.
- Sales incentives induce more production and emissions.
- The EA responds with a more stringent emission fee.

Firm Objectives (Section 8.4.1)

Firm 1 maximizes its profit:

$$\pi_1 = (1 - Q)q_1 - (c + t)q_1 \tag{8.20}$$

Firm 2's manager maximizes a combination of sales revenue and profits:

$$V = \alpha Rev_2 + (1 - \alpha)\pi_2 \tag{8.21}$$

- $Rev_2 \equiv (1 - Q)q_2$ and $\pi_2 = (1 - Q)q_2 - (c + t)q_2$.
- $\alpha = 0$: only profits (standard Cournot); $\alpha = 1$: only sales.

Simplifying Firm 2's Objective

$$V = \alpha\left[(1-Q)q_2\right] + (1-\alpha)\left[(1-Q)q_2 - (c+t)q_2\right]$$

which simplifies to

$$V = (1-Q)q_2 - (1-\alpha)(c+t)q_2 \tag{8.22}$$

- The manager only internalizes a share $1-\alpha$ of the after-tax cost $c+t$, as in problem (8.23).
- A higher α gives the manager more incentives to boost sales.
- Timing: the EA sets t; firms observe t and choose output.

Best Response Functions (Section 8.4.2)

Firm 1, as in Section 8.2:

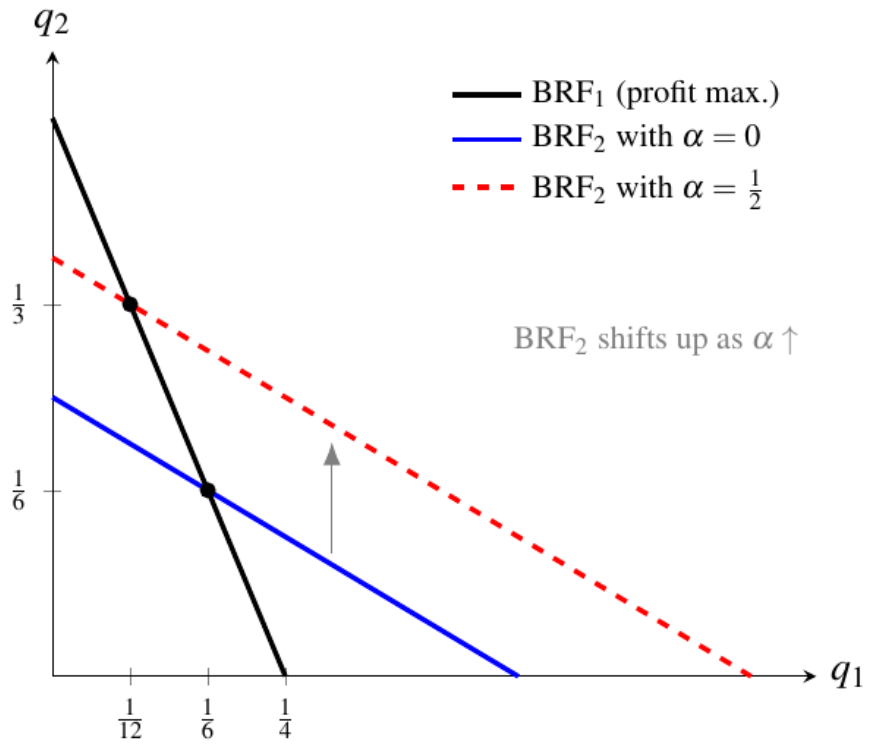
$$q_1(q_2) = \frac{1-c-t}{2} - \frac{1}{2}q_2 \tag{8.24}$$

Firm 2, from $1 - q_1 - 2q_2 - (1-\alpha)(c+t) = 0$:

$$q_2(q_1) = \frac{1-(1-\alpha)(c+t)}{2} - \frac{1}{2}q_1 \tag{8.25}$$

- Slope $-\frac{1}{2}$: unaffected by α, d, or t.
- Intercept shifts up as α increases, and down as $c+t$ increases.

Best Response Functions with Sales Incentives



- Example with $c + t = \frac{1}{2}$: only BRF_2 shifts up; firm 2 expands and firm 1 contracts.

Finding Equilibrium Output (Section 8.4.3)

Step 1. Insert (8.24) into (8.25):

$$q_2 = \frac{1-(1-\alpha)(c+t)}{2} - \frac{1}{2}\left(\frac{1-c-t}{2} - \frac{1}{2}q_2\right)$$

Step 2. Collect terms: $\frac{3}{4}q_2 = \frac{1-(1-2\alpha)(c+t)}{4}$.

Step 3. Solve for q_2, and insert it into (8.24) to find q_1:

$$q_1(\alpha) = \frac{1-(1+\alpha)(c+t)}{3} \text{ and} \tag{8.26}$$

$$q_2(\alpha) = \frac{1-(1-2\alpha)(c+t)}{3}$$

$$q_2(\alpha) - q_1(\alpha) = \alpha(c+t) > 0 \tag{8.27}$$

- Firm 2 produces more than firm 1 for all $\alpha > 0$; at $\alpha = 0$, $q_1(0) = q_2(0) = \frac{1-(c+t)}{3}$.

When Does Firm 1 Exit?

- $q_1(\alpha) > 0$ if and only if $(1+\alpha)(c+t) < 1$, that is,

$$t < \bar{t}(\alpha) \equiv \frac{1}{1+\alpha} - c \tag{8.28}$$

- Then $q_2(\alpha) > 0$ too, since $q_2(\alpha) > q_1(\alpha)$.
- Firm 2's manager only internalizes a share $1-\alpha$ of $c+t$, so a higher fee drives firm 1 out first.
- $\bar{t}(\alpha)$ decreases in α; Sections 8.4.3–8.4.4 study $t < \bar{t}(\alpha)$, Section 8.4.5 studies $t \geq \bar{t}(\alpha)$.

Effect of Sales Incentives, Costs, and the Fee

$$\frac{\partial q_1(\alpha)}{\partial \alpha} = -\frac{c+t}{3} < 0 \quad \text{and} \quad \frac{\partial q_2(\alpha)}{\partial \alpha} = \frac{2(c+t)}{3} > 0 \tag{8.29}$$

$$Q(\alpha) = q_1(\alpha) + q_2(\alpha) = \frac{2-(2-\alpha)(c+t)}{3} \tag{8.30}$$

$$\frac{\partial q_1(\alpha)}{\partial c} = -\frac{1+\alpha}{3} < 0 \quad \text{and} \quad \frac{\partial q_2(\alpha)}{\partial c} = \frac{2\alpha-1}{3} \tag{8.31}$$

- $\frac{\partial Q(\alpha)}{\partial \alpha} = \frac{c+t}{3} > 0$: the increase in q_2 dominates the decrease in q_1.
- $q_2(\alpha)$ increases in c if and only if $\alpha > \frac{1}{2}$; $\frac{\partial Q(\alpha)}{\partial c} = \frac{\alpha-2}{3} < 0$.
- The fee enters $c+t$ as c does: $\frac{\partial q_i}{\partial t} = \frac{\partial q_i}{\partial c}$.

Table 8.3: Equilibrium Output, Comparative Statics

	$q_1(\alpha)$	$q_2(\alpha)$	$Q(\alpha)$
Sales incentives, α	$-$	$+$	$+$
Marginal costs, c	$-$	$+$ if and only if $\alpha > \frac{1}{2}$	$-$
Emission fee, t	$-$	$+$ if and only if $\alpha > \frac{1}{2}$	$-$

Finding the Emission Fee (Section 8.4.4)

Setting $Q(\alpha) = Q^{SO}$:

$$\frac{2-(2-\alpha)(c+t)}{3} = \frac{1-c}{1+d} \tag{8.32}$$

Solving for t:

$$t(\alpha) = \frac{(2d-1)(1-c)+\alpha c(1+d)}{(1+d)(2-\alpha)} \tag{8.33}$$

- Positive for every $\alpha \in [0,1]$, since $d > \frac{1}{2}$.

The Fee Distortion

$$t(\alpha) = t(0) + \underbrace{\frac{\alpha(2d + 3c - 1)}{2(1 + d)(2 - \alpha)}}_{\text{Fee distortion, } +} \tag{8.34}$$

- $t(0) = \frac{(2d-1)(1-c)}{2(1+d)}$, the fee when all firms maximize profits.
- $2d + 3c - 1 > 0$ for all $d > \frac{1}{2}$, so the distortion is positive:

$$t(\alpha) > t(0) \tag{8.35}$$

- Anticipating more output and emissions, the EA sets a more stringent fee.

Comparative Statics of the Fee Distortion

With $FD(\alpha) \equiv \frac{\alpha(2d+3c-1)}{2(1+d)(2-\alpha)}$:

$$\frac{\partial FD(\alpha)}{\partial \alpha} = \frac{2d+3c-1}{(1+d)(2-\alpha)^2} > 0 \tag{8.36}$$

$$\frac{\partial FD(\alpha)}{\partial d} = \frac{3\alpha(1-c)}{2(1+d)^2(2-\alpha)} > 0$$

- Stronger sales incentives or more severe pollution increase the fee distortion.

Is Firm 1 Active at Fee $t(\alpha)$?

Inserting $t(\alpha)$ into $q_1(\alpha)$:

$$q_1(\alpha) = \frac{1 - c - \alpha(c + d)}{(2 - \alpha)(1 + d)} \tag{8.37}$$

which is positive if and only if

$$\alpha < \hat{\alpha} \equiv \frac{1 - c}{c + d} \tag{8.38}$$

- Equivalently, $t(\alpha) < \bar{t}(\alpha)$. When $\alpha \geq \hat{\alpha}$, $t(\alpha)$ would require a negative output from firm 1.
- $\hat{\alpha}$ decreases in d and in c: a more damaging pollutant calls for a higher fee, which drives firm 1 out at weaker sales incentives.
- $c = \frac{1}{2}$ gives $\hat{\alpha} = \frac{1}{1+2d} < \frac{1}{2}$. When $d \leq 1 - 2c$, $\hat{\alpha} \geq 1$: $t(\alpha)$ applies for all α.

What If Firm 1 Exits? (Section 8.4.5)

When $t \geq \bar{t}(\alpha)$, firm 1 produces zero and firm 2 becomes a monopolist, maximizing $V = (1 - q_2)q_2 - (1 - \alpha)(c + t)q_2$:

$$q_2^M(\alpha) = \frac{1 - (1 - \alpha)(c + t)}{2} \tag{8.39}$$

- Firm 1's best response to $q_2^M(\alpha)$ is weakly negative if and only if $t \geq \bar{t}(\alpha)$: it stays out.
- At $t = \bar{t}(\alpha)$, $q_2^M(\alpha) = Q(\alpha) = \frac{\alpha}{1+\alpha}$: aggregate output is continuous in the fee.

Firm 2 Produces Without Profit

- After firm 1 exits, output decreases in the fee at a slower rate, since $\frac{1-\alpha}{2} < \frac{2-\alpha}{3}$.
- Firm 2's profit margin: $p - c - t = \frac{1-(1+\alpha)(c+t)}{2} \leq 0$ for all $t \geq \bar{t}(\alpha)$.
- The manager only internalizes a share $1 - \alpha$ of the after-tax cost, so she keeps producing even when the firm earns no profit.
- We take α as given, but the owners would want to revise it in this case.

The Fee When Firm 1 Exits

Setting $q_2^M(\alpha) = Q^{SO}$ and solving for t:

$$t^M(\alpha) = \frac{(1-c)(d-1) + \alpha c(1+d)}{(1+d)(1-\alpha)} \tag{8.40}$$

The optimal emission fee is

$$t^*(\alpha) = \begin{cases} t(\alpha) & \text{if } \alpha < \hat{\alpha}, \\ t^M(\alpha) & \text{if } \hat{\alpha} \leq \alpha < 1, \end{cases} \tag{8.41}$$

- Continuous at $\hat{\alpha}$, where both fees equal the exit fee $\bar{t}(\hat{\alpha})$.

Fee $t^M(\alpha)$ Increases in Sales Incentives

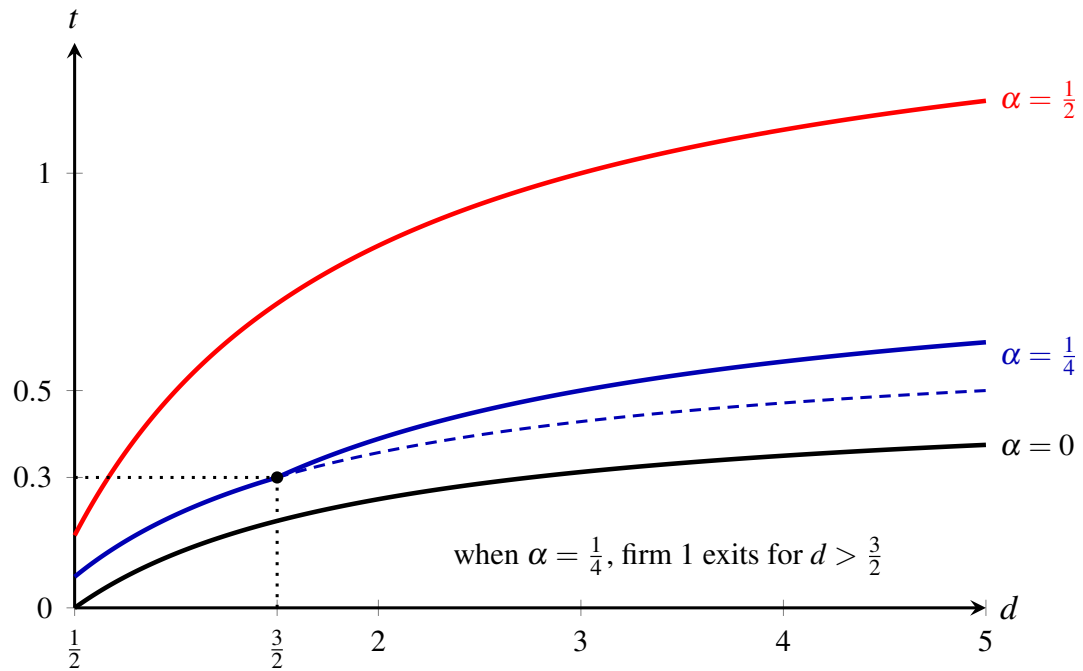
$$\frac{\partial t^M(\alpha)}{\partial \alpha} = \frac{2c + d - 1}{(1 + d)(1 - \alpha)^2} \tag{8.42}$$

- Positive: the monopoly case only arises when $\hat{\alpha} < 1$, that is, when $d > 1 - 2c$.
- Intuitively, a more sales-oriented manager responds less to the fee, so the EA needs a higher fee to reduce output to Q^{SO}.

Pure Sales Maximization, $\alpha = 1$

- The manager ignores the fee altogether.
- Firm 2 as a monopolist produces $q_2^M(1) = \frac{1}{2}$ for every fee; with both firms active, $Q = \frac{2-c-t}{3}$, which falls to $\frac{1}{2}$ at the exit fee $\bar{t}(1) = \frac{1}{2} - c$.
- Aggregate output is at least $\frac{1}{2}$ for every fee: the EA can induce Q^{SO} only if $d \leq 1 - 2c$, and then $t(1)$ applies.
- Otherwise, the EA needs a second instrument, such as an emission standard on firm 2.

Figure 8.2: Emission Fee $t^*(\alpha)$ Increases in α



- $c = \frac{1}{2}$, so $\hat{\alpha} = \frac{1}{1+2d}$; a higher α yields a more stringent fee. At $\alpha = \frac{1}{4}$, firm 1 exits for $d > \frac{3}{2}$, where the fee follows $t^M(\alpha)$.

What If Both Firms Promote Sales? (Section 8.4.6)

Firm i offers sales incentive α_i, possibly different from α_j:

$$\max_{q_i \geq 0} \; V = (1 - Q)q_i - (1 - \alpha_i)(c + t)q_i \tag{8.43}$$

$$q_i(q_j) = \frac{1 - (1 - \alpha_i)(c + t)}{2} - \frac{1}{2}q_j \tag{8.44}$$

- Analogous to (8.25), but now for both firms.

Finding Equilibrium Output (Section 8.4.7)

Inserting $q_j(q_i)$ into $q_i(q_j)$, as in (8.45), and solving:

$$q_i(\alpha_i, \alpha_j) = \frac{1 - (1 - 2\alpha_i + \alpha_j)(c + t)}{3} \tag{8.46}$$

$$Q(\alpha_i, \alpha_j) = \frac{2 - (2 - \alpha_i - \alpha_j)(c + t)}{3} \tag{8.47}$$

- q_i increases in its own α_i but decreases in its rival's α_j, as in (8.29); Q increases in both.
- Symmetric incentives, $\alpha_i = \alpha_j = \alpha$:

$$q_i(\alpha, \alpha) = \frac{1 - (1 - \alpha)(c + t)}{3} \tag{8.48}$$

increasing in α, with $Q(\alpha, \alpha) = \frac{2[1-(1-\alpha)(c+t)]}{3}$; at $\alpha = 0$, the Cournot output $\frac{1-c-t}{3}$.

Finding the Emission Fee (Section 8.4.8)

Setting $Q(\alpha_i, \alpha_j) = Q^{SO}$, as in (8.49), and solving:

$$t(\alpha_i, \alpha_j) = \frac{(2d-1)(1-c)}{2(1+d)} + \underbrace{\frac{(2d+3c-1)(\alpha_i+\alpha_j)}{2(1+d)(2-\alpha_i-\alpha_j)}}_{\text{Fee distortion, } +} \tag{8.50}$$

- The first term is $t(0,0)$, the fee when both firms maximize profits.
- The fee distortion is positive: $t(\alpha_i, \alpha_j) > t(0,0)$.

Fee Distortion with Two Sales-Oriented Firms

$$\frac{\partial FD}{\partial \alpha_i} = \frac{\partial FD}{\partial \alpha_j} = \frac{2d + 3c - 1}{(1+d)(2-\alpha_i-\alpha_j)^2} > 0 \tag{8.51}$$

$$\frac{\partial FD}{\partial d} = \frac{3(1-c)(\alpha_i+\alpha_j)}{2(1+d)^2(2-\alpha_i-\alpha_j)} > 0 \tag{8.52}$$

- Either firm's sales incentive has the same effect on the stringency of the fee.
- With $\alpha_i = \alpha_j = \alpha$, each manager perceives cost $(1-\alpha)(c+t)$; the fee sets it equal to $c + t(0,0)$, so $t = \frac{c+t(0,0)}{1-\alpha} - c$.

Are Both Firms Active at the Fee?

At fee (8.50), firm i produces

$$q_i(\alpha_i, \alpha_j) = \frac{1 - c + \alpha_i(2c + d - 1) - \alpha_j(c + d)}{(1 + d)(2 - \alpha_i - \alpha_j)} \tag{8.53}$$

- $\alpha_i = \alpha_j = \alpha < 1$: each firm produces $\frac{1-c}{2(1+d)}$, half of Q^{SO}.
- Otherwise, the firm with the weaker sales incentive may exit; then Section 8.4.5 applies.
- For $\alpha_i = 0$, firm i is active if and only if $\alpha_j < \hat{\alpha}$, as in (8.38).
- $\alpha_i = \alpha_j = 1$: $Q = \frac{2}{3}$ for every fee, so the fee cannot affect output.

8.5.1 Supermarkets

- **Walmart (US):** store managers' annual bonuses depended mainly on store sales until a January 2024 redesign gave store profit a larger role.
- **Mercadona (Spain):** 26.6% of Spanish grocery sales in 2024 (Kantar); pays all employees an annual bonus out of company results.
- EU Packaging and Packaging Waste Regulation (2025/40) applies from August 12, 2026.
- **Connection to the model:** a sales bonus is a positive weight α on revenue; the examples show such pay is used, not that it has increased emissions.

8.5.2 Fast Fashion Retailers

- **Inditex** (Zara's parent) has, since 2019, tied store teams' bonuses to each store's sales-growth targets.
- The revised Waste Framework Directive (2025/1892) requires extended producer responsibility schemes for textiles and footwear.
- Since July 19, 2026, large firms cannot destroy unsold apparel and footwear (Ecodesign for Sustainable Products Regulation).
- **Connection to the model:** a sales bonus induces managers to sell more than a profit-maximizing manager would; the case does not measure its effect on output or waste.

8.5.3 Automotive Sector: Fleet Standards

- **Japan:** fiscal-year 2030 standard of 25.4 km per liter, 32.4% above the fiscal-year 2016 average.
- **South Korea:** fleet-average standard of 70 g/km for 2030, plus annual sales targets for low-emission vehicles.
- **Connection to the model:** with strong sales incentives, a fee may be unable to reduce output enough (Section 8.4.5).
- Fleet standards cap the emissions that a sales-oriented manager can generate, whatever her incentives.

8.5.4 Consumer Electronics: Samsung and Xiaomi

- In 2024, Samsung held 18.0% and Xiaomi 13.6% of global smartphone shipments (IDC); Xiaomi delivered 136,854 electric vehicles in its first year.
- Samsung's main employee bonus is paid from the profit each division earns above its target.
- Since June 20, 2025, EU ecodesign rules require spare parts for seven years after a model leaves the market.
- **Connection to the model:** Samsung's profit-based bonus corresponds to $\alpha = 0$; the case does not show how pay affects output.

Section 8.6

Managerial Incentive III: Cross-Ownership

Cross-Ownership: Setup and Objectives (Section 8.6.1)

- Builds on Reynolds and Snapp (1986): partial equity interests soften Cournot competition. Kanjilal and Muñoz-Garcia (2019) let firms choose their equity shares.
- Firm i holds a share α_i of firm j's profits, and firm j a share α_j of firm i's, where $\frac{1}{2} \geq \alpha_i, \alpha_j \geq 0$. Every firm i chooses q_i to maximize

$$V_i = \underbrace{(1-\alpha_j)\pi_i}_{\text{Own profits}} + \underbrace{\alpha_i \pi_j}_{\text{Rival's profit}} \tag{8.54}$$

- Firm i keeps the share $1-\alpha_j$ of its own profits that firm j does not own; for example, if $\alpha_j = \frac{1}{4}$, firm i keeps $\frac{3}{4}$ of its profits.

Special Cases of the Objective Function

- $\alpha_i = \alpha_j = 0$: $V_i = \pi_i$, standard Cournot.
- $\alpha_i = \alpha_j = \frac{1}{2}$: $V_i = \frac{1}{2}(\pi_i + \pi_j)$, joint profit maximization, as in a merger between symmetric firms.
- $\alpha_i > 0 = \alpha_j$: $V_i = \pi_i + \alpha_i \pi_j$.
- $\alpha_j > 0 = \alpha_i$: $V_i = (1 - \alpha_j)\pi_i$.
- Symmetric shares $\alpha_i = \alpha_j = \alpha$: $V_i = \pi_i - \alpha(\pi_i - \pi_j)$.

Table 8.4: Objective Functions V_1 and V_2

	$\alpha_2 = 0$	$\frac{1}{2} > \alpha_2 > 0$	$\alpha_2 = \frac{1}{2}$
$\alpha_1 = 0$	π_1 π_2	$(1-\alpha_2)\pi_1$ $\pi_2 + \alpha_2\pi_1$	$\frac{1}{2}\pi_1$ $\pi_2 + \frac{1}{2}\pi_1$
$\frac{1}{2} > \alpha_1 > 0$	$\pi_1 + \alpha_1\pi_2$ $(1-\alpha_1)\pi_2$	$(1-\alpha_2)\pi_1 + \alpha_1\pi_2$ $(1-\alpha_1)\pi_2 + \alpha_2\pi_1$	$\frac{1}{2}\pi_1 + \alpha_1\pi_2$ $(1-\alpha_1)\pi_2 + \frac{1}{2}\pi_1$
$\alpha_1 = \frac{1}{2}$	$\pi_1 + \frac{1}{2}\pi_2$ $\frac{1}{2}\pi_2$	$(1-\alpha_2)\pi_1 + \frac{1}{2}\pi_2$ $\frac{1}{2}\pi_2 + \alpha_2\pi_1$	$\frac{1}{2}(\pi_1 + \pi_2)$ $\frac{1}{2}(\pi_1 + \pi_2)$

- Each cell: V_1 (top) and V_2 (bottom).

Best Response Function (Section 8.6.2)

Differentiating V_i with respect to q_i:

$$(1-\alpha_j)\underbrace{(1-c-t-2q_i-q_j)}_{\partial\pi_i/\partial q_i}+\alpha_i\underbrace{(-q_j)}_{\partial\pi_j/\partial q_i}=0 \qquad (8.55)$$

The second term is the negative externality that q_i imposes on firm j's profit, weighted by firm i's share α_i. Solving:

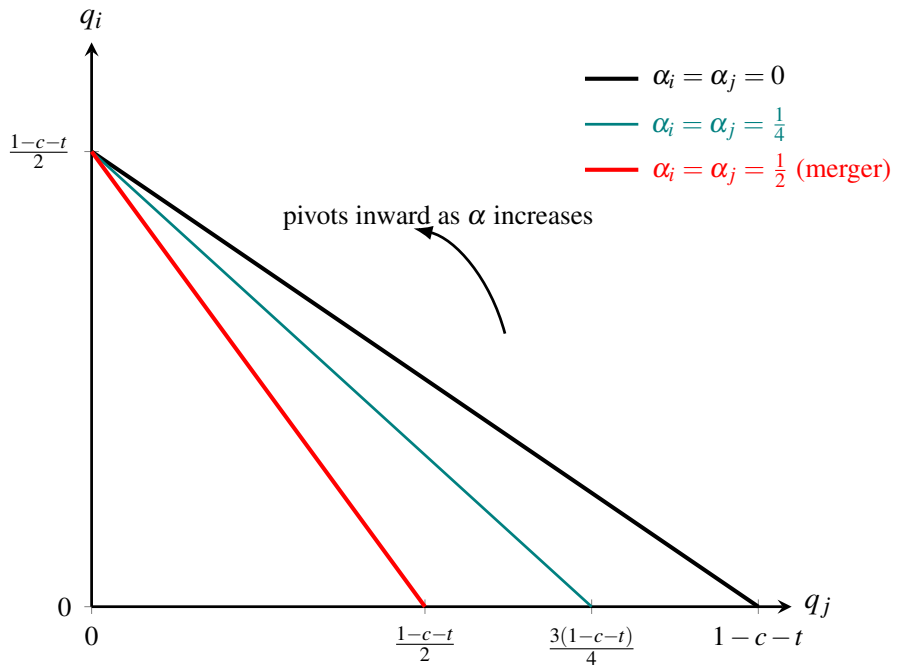
$$q_i(q_j)=\frac{1-c-t}{2}-\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}q_j \qquad (8.56)$$

- Vertical intercept $\frac{1-c-t}{2}$, as in standard Cournot. Slope increases in α_i, and in α_j when $\alpha_i > 0$ (derivative $\frac{\alpha_i}{2(1-\alpha_j)^2}$).
- Positive if and only if $q_j \leq \frac{(1-c-t)(1-\alpha_j)}{1+\alpha_i-\alpha_j}$.

Slopes in Special Cases

- Symmetric shares, $\alpha_i = \alpha_j = \alpha$: slope $\frac{1}{2(1-\alpha)}$.
  - $\alpha = 0$: $\frac{1}{2}$ (standard Cournot).
  - $\alpha = \frac{1}{4}$: $\frac{2}{3}$.
  - $\alpha = \frac{1}{2}$ (merger): 1.
- Only firm i holds equity, $\alpha_i > 0 = \alpha_j$: slope $\frac{1+\alpha_i}{2}$, also steeper.

Figure 8.3: Best Response with Cross-Ownership



- The best response function pivots inward as equity shares increase.

Why Does Cross-Ownership Soften Competition?

- Equity stakes help every firm internalize the negative externality its output imposes on its rival's profit.
- A higher α_i or α_j raises the weight that firm i places on its rival's profit relative to its own.
- Firm i responds to an increase in q_j by reducing q_i more than without equity shares.
- At $\alpha_i = \alpha_j = \frac{1}{2}$, firms fully internalize these effects, as in a symmetric merger.

Finding Equilibrium Output (Section 8.6.3)

Solving (8.56) and its counterpart for firm j simultaneously:

$$q_i(\alpha_i, \alpha_j) = \frac{(1-c-t)(1-\alpha_i)}{3-\alpha_i-\alpha_j} \tag{8.57}$$

strictly positive as long as $c + t < 1$. **Symmetric case**, $\alpha_i = \alpha_j = \alpha$:

Step 1. The slope in (8.56) is $\frac{1}{2(1-\alpha)}$. Invoke symmetry, $q_i = q_j = q$, so $q = \frac{1-c-t}{2} - \frac{1}{2(1-\alpha)}q$.

Step 2. Collect terms:

$$q\,\frac{3-2\alpha}{2(1-\alpha)} = \frac{1-c-t}{2}$$

Step 3. Solve: $q_i(\alpha, \alpha) = \frac{(1-c-t)(1-\alpha)}{3-2\alpha}$, which is (8.57) with $\alpha_i = \alpha_j = \alpha$.

Effect of Equity Shares on Individual Output

$$\frac{\partial q_i}{\partial \alpha_i} = -\frac{(1-c-t)(2-\alpha_j)}{(3-\alpha_i-\alpha_j)^2} < 0 \quad \text{and} \tag{8.58}$$

$$\frac{\partial q_i}{\partial \alpha_j} = \frac{(1-c-t)(1-\alpha_i)}{(3-\alpha_i-\alpha_j)^2} > 0 \tag{8.59}$$

- A larger stake in its rival leads firm i to internalize more of the price reduction its output imposes on firm j; a larger stake held by its rival raises firm i's output.

$$q_i(\alpha_i, \alpha_j) - q_j(\alpha_i, \alpha_j) = \frac{(1-c-t)(\alpha_j - \alpha_i)}{3-\alpha_i-\alpha_j} \tag{8.60}$$

- Positive if and only if $\alpha_i < \alpha_j$: the firm holding the smaller stake in its rival produces more.
- $\frac{\partial q_i}{\partial c} = \frac{\partial q_i}{\partial t} = -\frac{1-\alpha_i}{3-\alpha_i-\alpha_j} < 0.$

Equilibrium Output: Special Cases

- No cross-ownership, $\alpha_i = \alpha_j = 0$:

$$q_i(0,0) = \frac{1-c-t}{3}$$

- Symmetric shares: $q_i(\alpha,\alpha) = \frac{(1-c-t)(1-\alpha)}{3-2\alpha}$, decreasing in α.
- Maximum shares, $\alpha_i = \alpha_j = \frac{1}{2}$:

$$q_i\left(\tfrac{1}{2},\tfrac{1}{2}\right) = \frac{1-c-t}{4} \quad \text{(as in a merger)}$$

Aggregate Output

$$Q(\alpha_i, \alpha_j) = \frac{(1-c-t)(2-\alpha_i-\alpha_j)}{3-\alpha_i-\alpha_j} \tag{8.61}$$

$$\frac{\partial Q}{\partial \alpha_i} = \frac{\partial Q}{\partial \alpha_j} = -\frac{1-c-t}{(3-\alpha_i-\alpha_j)^2} < 0 \tag{8.62}$$

- A higher α_i decreases q_i and increases q_j; the first effect dominates.
- $\frac{\partial Q}{\partial c} = -\frac{2-\alpha_i-\alpha_j}{3-\alpha_i-\alpha_j} < 0.$

Table 8.5: Equilibrium Output, Comparative Statics

	q_i	q_j	Q
Firm i's equity share in firm j, α_i	$-$	$+$	$-$
Firm j's equity share in firm i, α_j	$+$	$-$	$-$
Marginal production cost, c	$-$	$-$	$-$
Emission fee, t	$-$	$-$	$-$

Finding the Emission Fee (Section 8.6.4)

Setting $Q(\alpha_i, \alpha_j) = Q^{SO}$:

$$\frac{(1-c-t)(2-\alpha_i-\alpha_j)}{3-\alpha_i-\alpha_j} = \frac{1-c}{1+d} \tag{8.63}$$

Solving for t:

$$t(\alpha_i, \alpha_j) = \frac{(1-c)[d(2-\alpha_i-\alpha_j)-1]}{(2-\alpha_i-\alpha_j)(1+d)} \tag{8.64}$$

The Fee Distortion

$$t(\alpha_i, \alpha_j) = \frac{(2d-1)(1-c)}{2(1+d)} - \underbrace{\frac{(1-c)(\alpha_i+\alpha_j)}{2(2-\alpha_i-\alpha_j)(1+d)}}_{\text{Fee distortion}} \tag{8.65}$$

- The distortion is positive whenever $\alpha_i + \alpha_j > 0$, and it enters with a negative sign:

$$t(\alpha_i, \alpha_j) < t(0,0) \tag{8.66}$$

- The EA anticipates lower output and emissions, and sets a less stringent fee.

Comparative Statics of the Fee Distortion

$$\frac{\partial FD}{\partial \alpha_i} = \frac{\partial FD}{\partial \alpha_j} = \frac{1-c}{(2-\alpha_i-\alpha_j)^2(1+d)} > 0 \qquad (8.67)$$

$$\frac{\partial FD}{\partial d} = -\frac{(1-c)(\alpha_i+\alpha_j)}{2(2-\alpha_i-\alpha_j)(1+d)^2} < 0$$

- The distortion increases in both equity shares, but decreases in pollution severity.

When Is the Fee Positive?

- The numerator of (8.64) is positive if and only if $d(2-\alpha_i-\alpha_j)-1>0$, that is,

$$d > \widehat{d}(\alpha_i,\alpha_j) \equiv \frac{1}{2-\alpha_i-\alpha_j} \tag{8.68}$$

- Otherwise, the fee is negative: the EA subsidizes production to offset the output reduction that cross-ownership induces.
- If subsidies are not available, the EA sets $t=0$ (Chapter 2).

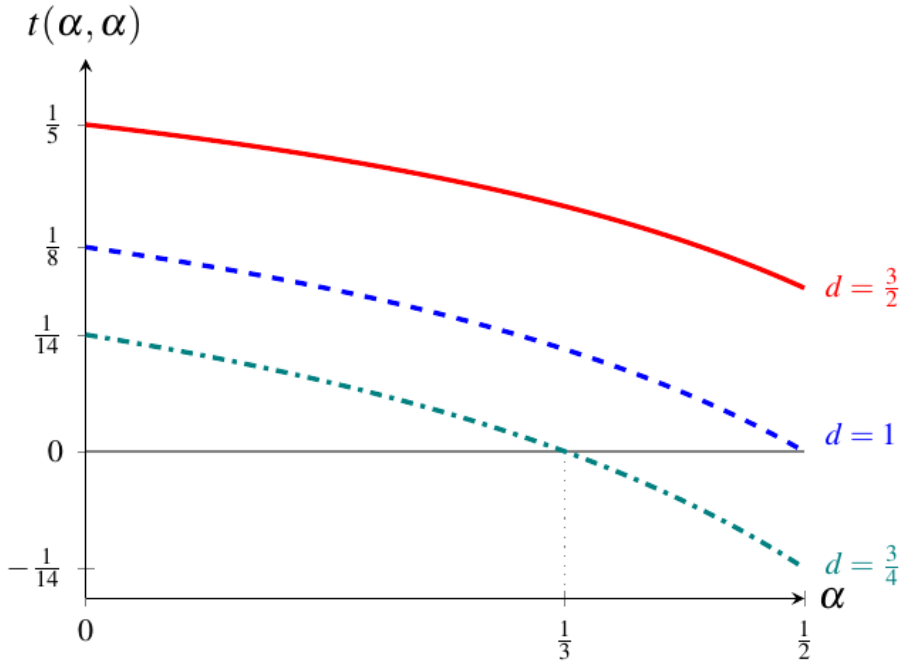
Fee and Cutoff: Special Cases

- No equity shares: $t(0,0) = \frac{(2d-1)(1-c)}{2(1+d)}$ and $\widehat{d}(0,0) = \frac{1}{2}$, as in Chapter 2.
- Symmetric shares:

$$t(\alpha,\alpha) = \frac{(1-c)[2d(1-\alpha)-1]}{2(1+d)(1-\alpha)} \tag{8.69}$$

  with $\widehat{d}(\alpha,\alpha) = \frac{1}{2(1-\alpha)}$, increasing in α.
- Maximum shares: $t(\frac{1}{2},\frac{1}{2}) = \frac{(1-c)(d-1)}{1+d}$ and $\widehat{d}(\frac{1}{2},\frac{1}{2}) = 1$, as in a merger.

Fee with Symmetric Equity Shares



- $c = \frac{1}{2}$: the fee (8.69) decreases in α; with $d = \frac{3}{4}$ it becomes a subsidy for $\alpha > \frac{1}{3}$, where $\widehat{d}(\alpha,\alpha) > \frac{3}{4}$.

More Equity, Less Stringent Fee

$$\frac{\partial t}{\partial \alpha_i} = \frac{\partial t}{\partial \alpha_j} = -\frac{1-c}{(2-\alpha_i-\alpha_j)^2(1+d)} < 0 \tag{8.70}$$

$$\frac{\partial \widehat{d}}{\partial \alpha_i} = \frac{\partial \widehat{d}}{\partial \alpha_j} = \frac{1}{(2-\alpha_i-\alpha_j)^2} > 0 \tag{8.71}$$

- Larger equity shares lower aggregate output, so the EA sets a less stringent fee.
- The fee is also less likely to be positive.

Both Firms Active and First Best

- At fee $t(\alpha_i, \alpha_j)$, firm i produces

$$\frac{(1-\alpha_i)(1-c)}{(2-\alpha_i-\alpha_j)(1+d)} > 0$$

  so both firms remain active for all admissible equity shares.
- The fee achieves the first best even when $\alpha_i \neq \alpha_j$.
- Firms produce different output levels, (8.60), but they share the same constant marginal cost, so the split of Q^{SO} does not affect total costs.
- Consumer surplus, $\frac{(Q^{SO})^2}{2}$, is unaffected by equity shares too: the lower fee raises profits and cuts tax revenue equally.

8.7.1 Automotive Sector

- **Mazda (2017):** Toyota acquired 5.05% of Mazda; Mazda acquired 0.25% of Toyota.
- **Suzuki (2019):** Toyota agreed to acquire 4.94% of Suzuki; Suzuki to buy about 0.2% of Toyota.
- **Subaru:** Toyota raised its stake to 20% in 2020 (21.46% in March 2026); Subaru bought about 0.3% of Toyota. The two firms jointly developed the GR86 and BRZ sports cars (2021).
- **Connection to the model:** each firm places weight α_i on its rivals' profits; effects on emissions have not been established.

8.7.2 Common Ownership in China

- Common ownership: the same investors hold shares in several firms of an industry.
- Liu et al. (2025): among Chinese listed firms, common ownership by state-owned investors is associated with better environmental performance.
- Channels: easier access to finance and green innovation that spreads across the industry; stronger in privately owned firms.
- **Connection to the model:** shared investors value several firms' profits, as in Section 8.6; the evidence is an association, not causation.

8.7.3 Energy Sector

- Benchekroun et al. (2022): fossil-fuel oligopoly with cross-ownership and a clean energy source of fixed capacity.
- Cross-ownership reduces fossil-fuel output.
- Large damages: more cross-ownership lowers the social value of the clean technology. Small damages: a large increase in cross-ownership can raise it.
- **Connection to the model:** complements Section 8.6, where the fee becomes a subsidy when pollution is mild; see also Bárcena-Ruiz and Sagasta (2021).

Section 8.8

Managerial Incentive IV: Corporate Social Responsibility

Environmental CSR: Motivation

- Many private firms commit to reduce their environmental impact beyond what regulation requires: **corporate social responsibility** (CSR).
- These commitments may respond to the preferences of owners, investors, employees, or customers.
- Unlike the public firm of Section 8.2, a CSR firm stays private, and ignores consumer surplus and its rival's profits.
- It only internalizes part of the industry's environmental damage.

Firm Objectives (Section 8.8.1)

Firm 1 maximizes its profit:

$$\pi_1 = (1 - Q)q_1 - (c + t)q_1 \tag{8.72}$$

Firm 2 maximizes its profit minus a share of the damage:

$$V = \pi_2 - \alpha Env \tag{8.73}$$

- $Env = \frac{d}{2}Q^2$, with $d > \frac{1}{2}$; $\alpha \in [0, 1]$ measures firm 2's environmental concern.
- $\alpha = 0$: profit maximization; $\alpha = 1$: firm 2 internalizes all the damage, but still ignores consumer surplus.
- We assume $\alpha d < 1$, so firm 2 is active.

Best Response Functions (Section 8.8.2)

Firm 1: as in (8.3), $q_1(q_2) = \frac{1-c-t}{2} - \frac{1}{2}q_2$.

Firm 2: first-order condition

$$\underbrace{(1 - 2q_2 - q_1 - c - t)}_{\partial \pi_2 / \partial q_2} - \alpha \underbrace{d(q_1 + q_2)}_{\partial Env / \partial q_2} = 0$$

$$q_2(q_1) = \frac{1 - c - t}{2 + \alpha d} - \frac{1 + \alpha d}{2 + \alpha d} q_1 \qquad (8.74)$$

- For $\alpha > 0$: lower intercept and steeper slope; firm 2 produces less for every q_1.

Equilibrium Output (Section 8.8.3)

$$q_1(t) = \frac{(1+\alpha d)(1-c-t)}{3+\alpha d} \quad \text{and} \tag{8.75}$$
$$q_2(t) = \frac{(1-\alpha d)(1-c-t)}{3+\alpha d}$$

$$Q(t) = \frac{2(1-c-t)}{3+\alpha d} \tag{8.76}$$

- $\alpha = 0$: the Chapter 2 output, $\frac{1-c-t}{3}$.
- $q_2(t) > 0$ for all $t < 1-c$ if and only if $\alpha d < 1$.

Effect of CSR on Output

$$\frac{\partial q_1(t)}{\partial \alpha} = \frac{2d(1-c-t)}{(3+\alpha d)^2} > 0 \quad \text{and} \tag{8.77}$$
$$\frac{\partial q_2(t)}{\partial \alpha} = -\frac{4d(1-c-t)}{(3+\alpha d)^2} < 0$$

- A more environmentally concerned firm 2 reduces its output.
- Output levels are strategic substitutes: firm 1 expands, offsetting half of firm 2's reduction.
- Aggregate output and emissions still decrease: $\frac{\partial Q(t)}{\partial \alpha} = -\frac{2d(1-c-t)}{(3+\alpha d)^2} < 0$.

Emission Fee (Section 8.8.4)

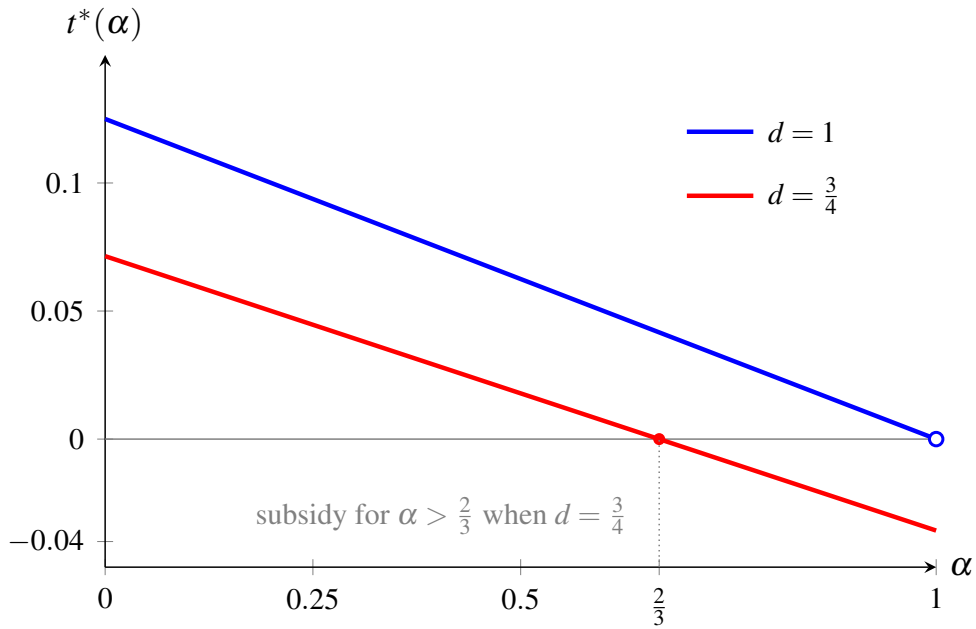
Welfare depends on aggregate output alone; setting $Q(t) = Q^{SO}$:

$$t^*(\alpha) = \frac{(1-c)(2d-1-\alpha d)}{2(1+d)} \tag{8.78}$$

$$\frac{\partial t^*(\alpha)}{\partial \alpha} = -\frac{(1-c)d}{2(1+d)} < 0 \tag{8.79}$$

- $\alpha = 0$: the Chapter 2 fee (2.10).
- Positive if and only if $d > \bar{d}(\alpha) \equiv \frac{1}{2-\alpha}$, which increases in α: a subsidy for a wider range of d.

Figure 8.4: Fee $t^*(\alpha)$ with Environmental CSR



- $c = \frac{1}{2}$, $d = \frac{3}{4}$ and $d = 1$: the fee decreases in α; with $d = \frac{3}{4}$, a subsidy for all $\alpha > \frac{2}{3}$.
- Open circle at $\alpha = 1$ for $d = 1$, since $\alpha d < 1$ is required.

Why a Less Stringent Fee?

- Firm 2's environmental concern acts as a *private fee* on its own output.
- Anticipating lower aggregate output, the EA sets a less stringent fee.
- The lower fee benefits both firms, including firm 1, which does not engage in CSR.
- Firm 2 also pays a lower fee, but this saving does not offset the cost of its own CSR (Section 8.8.5).

Output at the Fee: First Best

At fee $t^*(\alpha)$:

$$q_1^* = \frac{(1-c)(1+\alpha d)}{2(1+d)} \quad \text{and} \quad q_2^* = \frac{(1-c)(1-\alpha d)}{2(1+d)}$$

- Both positive since $\alpha d < 1$; firm 2 produces less than firm 1.
- Aggregate output equals Q^{SO}: the fee achieves the first best.
- The EA needs to know α: with the Chapter 2 fee, aggregate output would fall below Q^{SO}.

Is Environmental CSR Profitable? (Section 8.8.5)

The price-cost margin, $1 - Q(t) - c - t$, coincides with $q_1(t)$, so

$$\pi_2(t) = q_1(t)q_2(t) = \frac{(1+\alpha d)(1-\alpha d)(1-c-t)^2}{(3+\alpha d)^2} \qquad (8.80)$$

- Decreasing in α: $\frac{\partial \pi_2(t)}{\partial \alpha} = -\frac{2d(1+3\alpha d)(1-c-t)^2}{(3+\alpha d)^3} < 0$.
- At fee $t^*(\alpha)$: $\pi_2^* = \frac{(1-c)^2(1-\alpha^2 d^2)}{4(1+d)^2}$, decreasing in α; $\pi_1^* = (q_1^*)^2$, increasing in α.

What If Owners Choose the CSR Level? (Section 8.8.6)

Unlike sales incentives (Section 8.4), environmental CSR makes firm 2 *less* aggressive. Would its owners choose this commitment?
Timing: (1) owners choose $\alpha \geq 0$; (2) the EA sets $t^*(\alpha)$; (3) firms choose output.

$$\max_{\alpha \geq 0} \ \pi_2^*(\alpha) = \frac{(1-c)^2(1-\alpha^2 d^2)}{4(1+d)^2} \tag{8.81}$$

$$\frac{\partial \pi_2^*(\alpha)}{\partial \alpha} = -\frac{(1-c)^2 \alpha d^2}{2(1+d)^2} \tag{8.82}$$

- Zero at $\alpha = 0$ and negative for all $\alpha > 0$: owners choose $\alpha = 0$, no environmental CSR.

Why Do Owners Choose No CSR?

- Fee $t^*(\alpha)$ induces Q^{SO} for every α, so $\pi_2^* = q_1^* q_2^*$ is the product of two outputs adding up to Q^{SO}.
- This product is largest when both firms produce the same output, which only occurs at $\alpha = 0$.
- If the EA sets its fee first, (8.80) still decreases in α: owners choose $\alpha = 0$, a corner solution (Appendix A.4).
- Environmental CSR must stem from owners' or stakeholders' preferences, not strategic motives; in line with Hirose et al. (2020).

Related Literature on Environmental CSR

- **Liu et al. (2015):** CSR certified by a non-governmental organization; the competition mode affects firms' incentives to adopt it.
- **Kopel and Brand (2012):** with delegation and similar costs, a socially concerned firm obtains a higher market share and profit.
- **Lambertini et al. (2016):** dynamic duopoly; in a large enough market, the CSR firm sells more and earns a higher profit than its rival.

Further Reading (Beyond the Book)

- **Azar, Schmalz, and Tecu (2018),** "Anticompetitive Effects of Common Ownership," *Journal of Finance*: on US airline routes, higher common ownership is associated with higher ticket prices.
- **He and Huang (2017),** "Product Market Competition in a World of Cross-Ownership," *Review of Financial Studies*: firms sharing institutional blockholders gain market share faster and form more joint ventures and alliances.
- **Connection to Section 8.6:** shared owners value rivals' profits, as in cross-ownership.

8.9.1 Patagonia

- In September 2022, founder Yvon Chouinard and his family transferred the ownership of Patagonia.
- The Patagonia Purpose Trust received all voting shares (about 2%); the Holdfast Collective, a nonprofit, all nonvoting shares (about 98%).
- Every dollar the Holdfast Collective receives is used to fight the environmental crisis; expected dividends of about $100 million per year.
- **Connection to the model:** owners place a large weight α on environmental objectives; the model predicts lower own output and profit, and rivals that expand.

8.9.2 Danone

- In June 2020, Danone's shareholders approved, almost unanimously, the status of *entreprise à mission* (French law of 2019).
- Danone became the first listed company to adopt this status.
- The status writes social and environmental objectives into the articles of association, with a committee monitoring progress.
- **Connection to the model:** similar to a positive α in Section 8.8; approved by shareholders, so CSR reflects owners' preferences.

Section 8.10

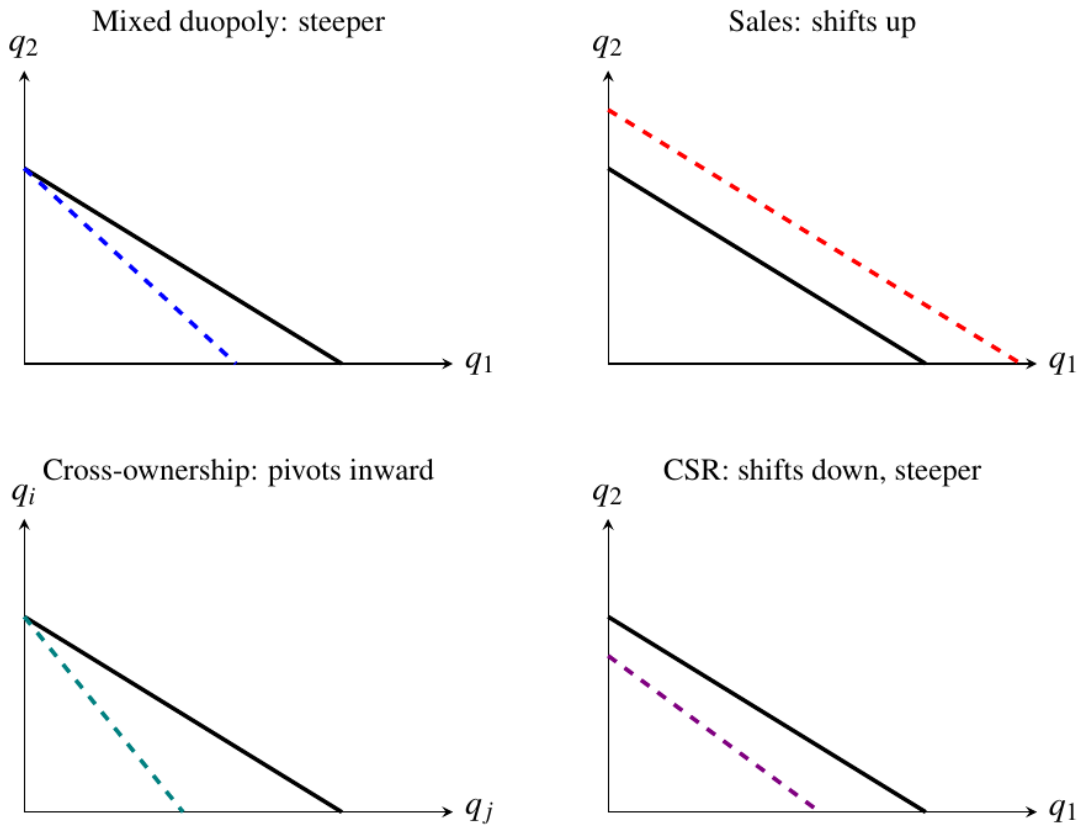
Summary

How Managerial Incentives Change the Fee

Incentive	**Best response**	Q	**Fee**
Benchmark	—	Cournot	(2.10)
Mixed duopoly	BRF_2 steeper	Mix changes	Unchanged
Sales	BRF_2 shifts up	$\uparrow$	$\uparrow$
Cross-ownership	Both pivot inward	$\downarrow$	$\downarrow$
CSR	BRF_2 down, steeper	$\downarrow$	$\downarrow$

- An incentive that raises output at fee (2.10) calls for a more stringent fee; one that lowers output, a less stringent fee.
- With fee (2.10), welfare falls except in the mixed duopoly.

Best Response Effects at a Glance



- Black: profit-maximizing benchmark. Dashed: the firm with the managerial incentive.

Aggregate Output Side by Side

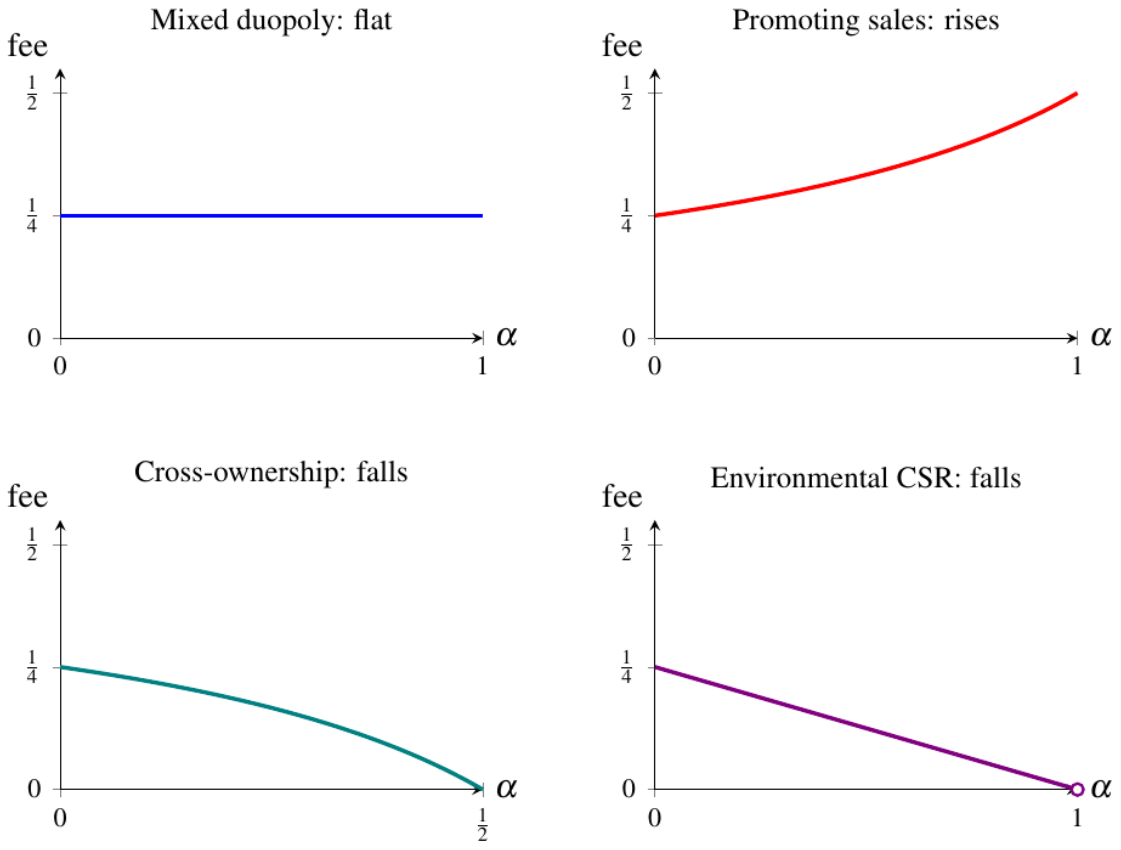
Setting	**Aggregate output**
Benchmark	$\frac{2(1-c-t)}{3}$
Mixed duopoly (8.14)	$\frac{2[1-c-t(1-\alpha)]-\alpha(1-c)}{3-\alpha(2-d)}$
Promoting sales (8.30)	$\frac{2-(2-\alpha)(c+t)}{3}$
Cross-ownership (8.61)	$\frac{(1-c-t)(2-\alpha_i-\alpha_j)}{3-\alpha_i-\alpha_j}$
Environmental CSR (8.76)	$\frac{2(1-c-t)}{3+\alpha d}$

Emission Fees Side by Side

Setting	**Fee inducing Q^{SO}**
Mixed duopoly (8.18)	$\frac{(1-c)(2d-1)}{2(1+d)}$
Promoting sales (8.33)	$\frac{(2d-1)(1-c)+\alpha c(1+d)}{(1+d)(2-\alpha)}$ if $\alpha < \hat{\alpha}$
Cross-ownership (8.64)	$\frac{(1-c)[d(2-\alpha_i-\alpha_j)-1]}{(2-\alpha_i-\alpha_j)(1+d)}$
Environmental CSR (8.78)	$\frac{(1-c)(2d-1-\alpha d)}{2(1+d)}$

- Sales incentives with $\alpha \geq \hat{\alpha}$: firm 1 exits, and the fee is $t^M(\alpha)$ in (8.40).

Optimal Fee as a Function of α



- $c = 0$ and $d = 1$: all four fees start at $\frac{1}{4}$ when $\alpha = 0$.

Summary: Mixed Oligopolies

- Public firms include social welfare in their objectives. When pollution is severe, a more nationalized firm reduces output and its rival expands; when it is not severe, the opposite occurs.
- Welfare depends only on aggregate output, so the regulator targets the same socially optimal output for every degree of public ownership.
- The optimal fee coincides with the purely private fee: a neutrality result. At this fee, both firms produce the same output.
- A fully nationalized firm alone steers aggregate output to the social optimum; the fee only divides output between the two firms.

Summary: Sales-Based Incentives

- Firms that maximize a weighted combination of profits and revenues produce more than profit-maximizing firms.
- This overproduction leads to higher emissions and damage, so the regulator increases the fee.
- With strong sales incentives, the optimal fee drives the profit-maximizing rival out of the market.
- If the manager only cares about sales, the fee may be unable to reduce output enough, and a second instrument is needed.

Summary: Cross-Ownership

- Firms that partially own competitors internalize rivals' profits, which softens competition.
- Softer competition leads to lower output and emissions than in standard oligopolies.
- The optimal emission fee is lower, as reduced output mitigates pollution; it becomes a subsidy when pollution is mild.

Summary: Corporate Social Responsibility

- A firm engaging in environmental CSR reduces its output, while its profit-maximizing rival expands, offsetting half of the reduction.
- The optimal fee decreases in the firm's environmental concern and achieves the first best, but the regulator needs to know the CSR level.
- Environmental CSR lowers the firm's profit under quantity competition; profit-maximizing owners would choose no CSR, so CSR reflects owners' or stakeholders' preferences.

Summary: Comparative Statics and Functional Forms

- Cross-ownership and CSR reduce output; sales incentives increase it; public ownership changes the composition of output.
- The optimal fee increases with sales incentives and decreases with cross-ownership and CSR, but is independent of public ownership.
- Welfare depends on the balance between market power and pollution; regulation must be tailored to firm objectives.
- Neutrality relies on symmetric, constant marginal costs and a homogeneous good; with asymmetric firms, the fee may depend on α.

Policy Implications: What Does the EA Need to Know?

1. **Ownership structure:** the fee is independent of the nationalization level α; state ownership only changes which firm produces.
2. **Executive pay:** the managers' sales weight α, and whether the profit-maximizing rival remains active; sales-weighted bonuses call for a more stringent fee, and pure sales maximization may require an emission standard.
3. **Equity cross-holdings:** both equity shares, α_i and α_j; a less stringent fee, possibly a subsidy when pollution is mild.
4. **CSR commitments:** firm 2's CSR level α; a less stringent fee, since with the Chapter 2 fee output would fall below Q^{SO}.

Connections to Other Chapters

- **Chapter 2:** every model reduces to the standard Cournot benchmark, with fee (2.10), when $\alpha = 0$ (or $\alpha_i = \alpha_j = 0$).
- **Chapter 7 (mergers):** cross-ownership with $\alpha_i = \alpha_j = \frac{1}{2}$ yields the output and the fee of a merger between both firms.
- **Within Chapter 8:** the public firm (Section 8.2) internalizes welfare, while the CSR firm (Section 8.8) only internalizes part of the damage.
- **Combining incentives:** if the private firm's manager in the mixed duopoly has sales weight s, the fee is (8.33) at s for every $\alpha < 1$, while both firms are active.

Review Questions

1. Why is BRF_1 unaffected by α in the mixed duopoly, while BRF_2 depends on both α and d?
2. Explain why fee (8.18) is independent of α. How much does each firm produce at this fee?
3. In Section 8.4, why does firm 1 exit first when the fee is high?
4. Show that $q_i(\frac{1}{2}, \frac{1}{2}) = \frac{1-c-t}{4}$. For which values of d is the fee a subsidy?
5. Why would firm 2's owners choose no environmental CSR?

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

End of Chapter 8 Slides