Advanced Microeconomic Theory

Chapter 9: Externalities and Public Goods

Outline

- Externalities
- The Coase Theorem
- Quotas
- Pigouvian Taxes and Subsidies
- Tradable Permits
- Regulating a Polluting Monopolist or Oligopoly

Outline (continued)

- Asymmetric Information
- Tragedy of the Commons
- Pollution Abatement
- Public Goods
- Lindahl Equilibria

Externalities

Externalities

- ***Externality*** emerges when the well-being of a consumer or the production possibilities of a firm is directly affected by the actions of another agent in the economy.
  - Example: the production possibilities of a fishery are affected by the pollutants that a refinery dumps into a lake.
  - The effects from one agent to another are not captured by the price system.
- The effects transmitted through the price system are referred to as "***pecuniary externalities***."

Externalities

- Consider a polluting firm (agent 1) and an individual affected by such pollution (agent 2).
- The firm's profit function is

$$\pi(p, x)$$

where p is the price vector and x is the amount of externality generated.

Externalities

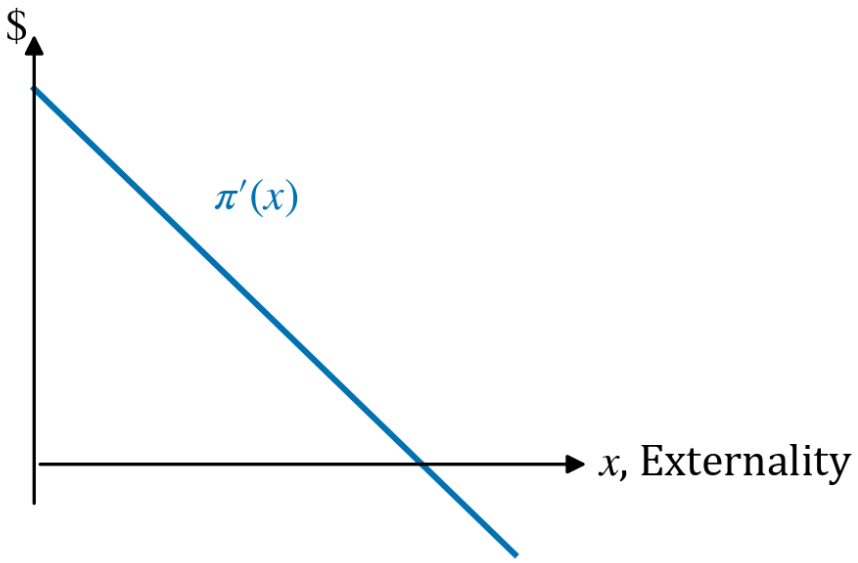
- Assume that p is given (i.e., p is a parameter). Then, the profit function becomes

$$\pi(x)$$

where $\pi'(0) > 0$ and $\pi''(x) < 0$, so marginal profit decreases in x and eventually becomes negative.

Externalities

- The firm obtains a positive and significant benefit from the first unit of the externality-generating activity.
- But the additional benefit from further units is decreasing.



Externalities

- The individual's (i.e., agent 2's) utility is given by

$$u(q, x)$$

where $q \in \mathbb{R}_+^N$ is a vector of N tradable goods and $x \in \mathbb{R}_+$ is the negative externality, with

$$\frac{\partial u}{\partial x} < 0 \quad \text{and} \quad \frac{\partial u}{\partial q_k} \geq 0 \quad \text{in every good } k$$

Externalities

- Let $q^*(p, w, x)$ denote the individual's Walrasian demand. Then,

$$v(x) = u(q^*(p, w, x), x)$$

  is the indirect utility function with $v'(x) < 0$ for all $x > 0$.
- We assume utility is quasilinear in wealth (no wealth effects), so the individual's utility is $v(x) + w$.

Externalities

- ***Example***:
  - Consider a firm whose profit function is $\pi = py - cy^2$, where $y \in \mathbb{R}_+$ is output and $p, c > 0$.
  - If every unit of output generates a unit of pollution, i.e., $x = y$, the profit function becomes $\pi(x) = px - cx^2$.
  - FOC wrt x yields $\pi'(x^*) = p - 2cx^* = 0$, producing $p = 2cx^*$ or $x^* = \frac{p}{2c}$.

Externalities

- ***Example*** (continued):
  - If every unit of output y generates α units of pollution, i.e., $x = \alpha y$ (so $y = \frac{x}{\alpha}$), where $\alpha > 0$, the profit function becomes

$$\pi(x) = p\frac{x}{\alpha} - c\left(\frac{x}{\alpha}\right)^2.$$

  - Taking FOC with respect to x yields

$$\pi'(x^*) = \frac{p}{\alpha} - 2c\frac{x^*}{\alpha}\frac{1}{\alpha} = 0,$$

with a competitive equilibrium level of pollution of

$$x^* = \alpha\frac{p}{2c}.$$

Externalities

- ***Competitive equilibrium***: All agents independently and simultaneously solve their PMP (for firms) or UMP (for consumers).
  - The firm chooses the level of the externality-generating activity, x, that solves its PMP

$$\max_{x \geq 0} \ \pi(x)$$

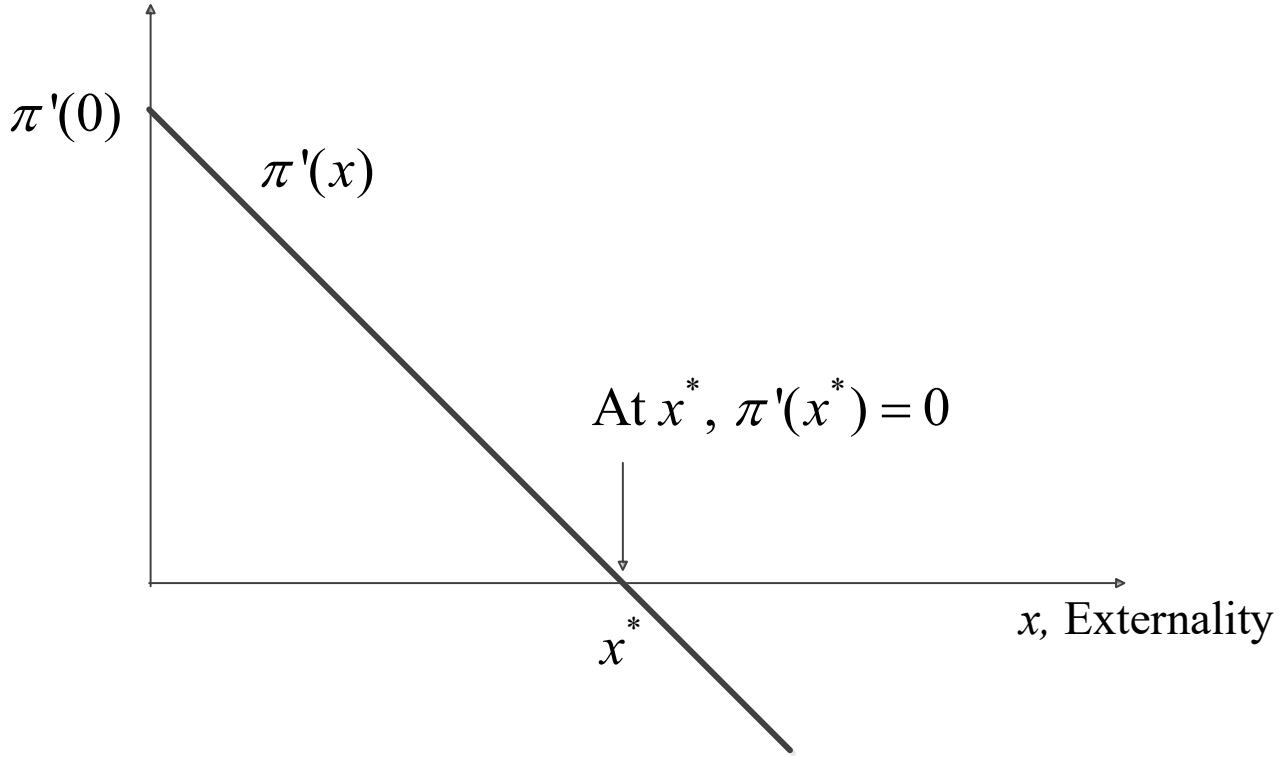
  - Taking FOC with respect to x yields

$$\pi'(x^*) \leq 0$$

with equality if $x^* > 0$ (interior solution).

Externalities

– The firm increases the externality-generating activity until the marginal benefit from an additional unit is zero, i.e., $\pi'(x^*) = 0$.



Externalities

– The UMP of the individual affected by pollution is

$$\max_{q \geq 0} \; u(q, x) \quad \text{s.t.} \quad pq \leq w$$

where $p \in \mathbb{R}_+^N$ is the given price vector.

– Notice that $q \in \mathbb{R}_+^N$ does not include pollution as one of the N tradable goods.

– Hence the individual cannot affect the level of the externality-generating activity x.

- Uninteresting case
- We relax this assumption when we assign property rights over x (Coase theorem section).

Externalities

- ***Pareto optimum***:
  - The social planner selects the level of x that maximizes social welfare (by quasilinearity, Pareto optimality is equivalent to maximizing aggregate surplus)

$$\max_{x \geq 0} \ \pi(x) + v(x)$$

  - Taking FOC with respect to x yields

$$\pi'(x^0) \leq -v'(x^0) \text{ with equality if } x^0 > 0$$

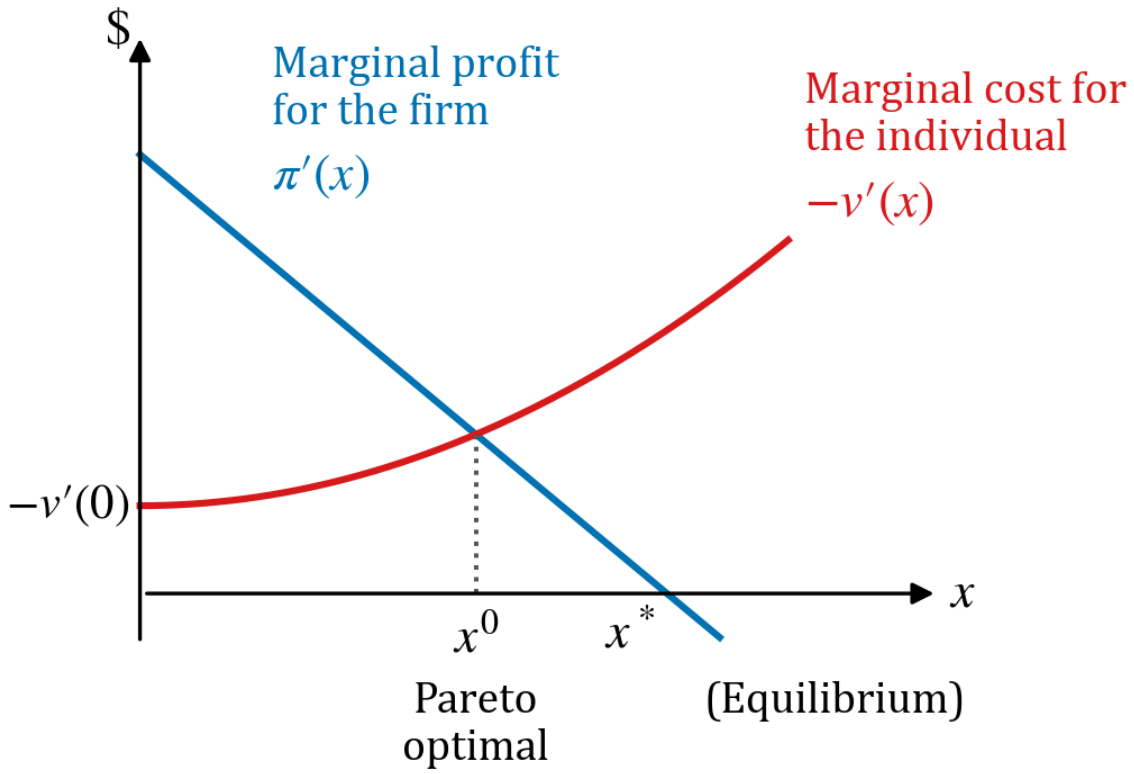
where x^0 is the Pareto optimal amount of the externality.

Externalities

- ***Pareto optimum*** (continued):
  - Intuitively, at a Pareto optimal (and interior) solution, the marginal benefit of the externality-generating activity, $\pi'(x^0)$, is equal to its marginal cost, $-v'(x^0)$.

Externalities

- Figure: Pareto optimal and equilibrium externality levels (negative externality)
- Too much externality x is produced in the competitive equilibrium relative to the Pareto optimum, i.e., $x^* > x^0$.



Externalities

- ***Example***:
  - Consider a firm with marginal profits of

$$\pi'(x) = a - bx, \text{ where } a, b > 0$$

    which is decreasing in x.

  - Assume a consumer whose marginal damage is

$$-v'(x) = c + dx, \qquad \text{where} \quad c, d > 0$$

    which is increasing in x.

Externalities

- ***Example*** (continued):
  - The competitive equilibrium amount of externality x^* solves $\pi'(x^*) = 0$, i.e., $a - bx^* = 0$. Hence,

$$x^* = \frac{a}{b}$$

  - The socially optimal level of the externality x^0 solves $\pi'(x^0) = -v'(x^0)$, i.e., $a - bx^0 = c + dx^0$. Thus,

$$x^0 = \frac{a - c}{b + d}$$

Externalities

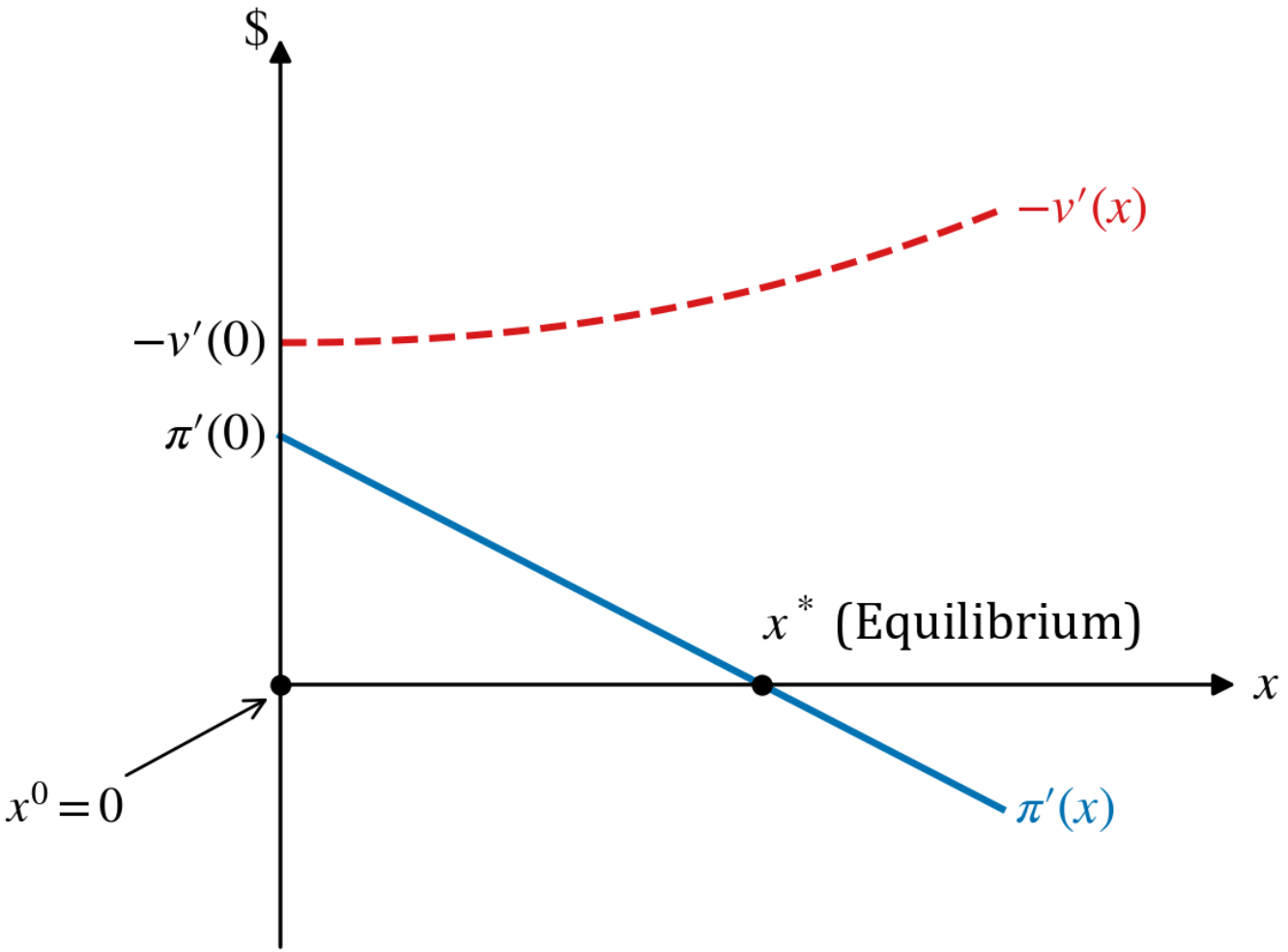
- ***Example*** (continued):
  - The level x^0 is positive if and only if $\pi'(0) > -v'(0)$, i.e., $a > c$; otherwise $x^0 = 0$.
  - Hence, for an interior optimum ($a > c$),

$$x^* - x^0 = \frac{a}{b} - \frac{a-c}{b+d} = \frac{ad+bc}{b(b+d)} > 0$$

Externalities

- Negative externalities are not necessarily eliminated at the Pareto optimal solution.
- This occurs if and only if $-v'(0) \geq \pi'(0)$.
- In this setting, the curves $\pi'(x)$ and $-v'(x)$ do not cross, and the Pareto optimal solution only occurs at the corner where $x^0 = 0$.

Externalities



Externalities

- If the firm's production activities produce a ***positive externality*** in the individual's well-being, then

$$v'(x) > 0 \text{ and } -v'(x) < 0$$

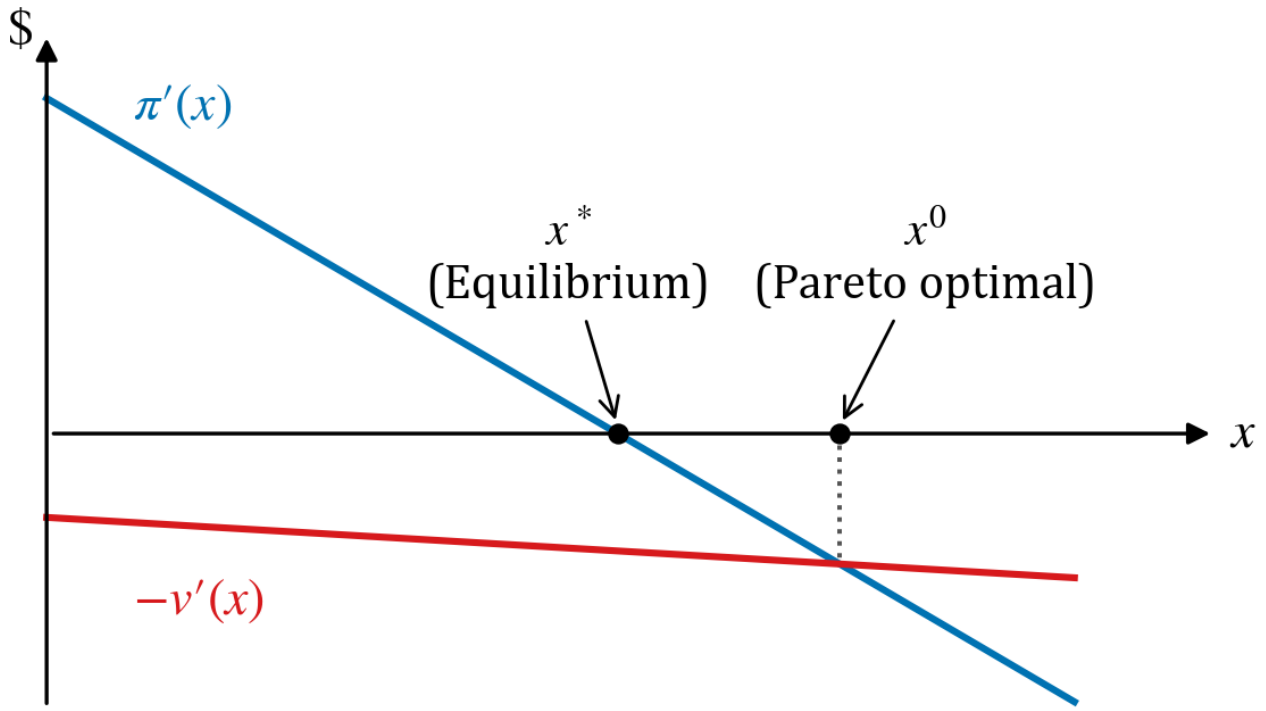
  - That is, the curve $-v'(x)$ lies below the horizontal axis.
  - $\pi'(x)$ remains unaffected.

Externalities

- In this setting, there is an underproduction of the externality-generating activity relative to the Pareto optimum, i.e., $x^* < x^0$.
- Since $\pi'(x^0) = -v'(x^0) < 0 = \pi'(x^*)$ and π' is decreasing, $x^0 > x^*$.

Externalities

- Pareto optimal and equilibrium externality level (positive externality).



Externalities

- ***Example*** (Positive externalities):
  - Consider two neighboring countries, $i \in \{1,2\}$, simultaneously choosing how many hours to spend on recycling activities, r_i.
  - The net benefit from recycling is:

$$\pi_i(r_i, r_j) = \left(a - r_i + \frac{r_j}{2}\right) r_i - b r_i$$

where $a > b > 0$, and b denotes the marginal cost of recycling.

Externalities

- ***Example*** (continued):
  - Country i's average benefit, $\left(a - r_i + \frac{r_j}{2}\right)$, is increasing in r_j because a clean environment produces positive external effects on the other country.
  - Let us first find the competitive equilibrium allocation.

Externalities

- ***Example*** (continued):
  - Taking FOC with respect to r_i yields $a - 2r_i + \frac{r_j}{2} - b = 0$, so country i's BRF is

$$r_i(r_j) = \frac{a-b}{2} + \frac{1}{4}r_j$$

  - Symmetrically, country j's BRF is

$$r_j(r_i) = \frac{a-b}{2} + \frac{1}{4}r_i$$

    - The positive slope of the BRFs indicates that countries' recycling activities are strategic complements.

Externalities

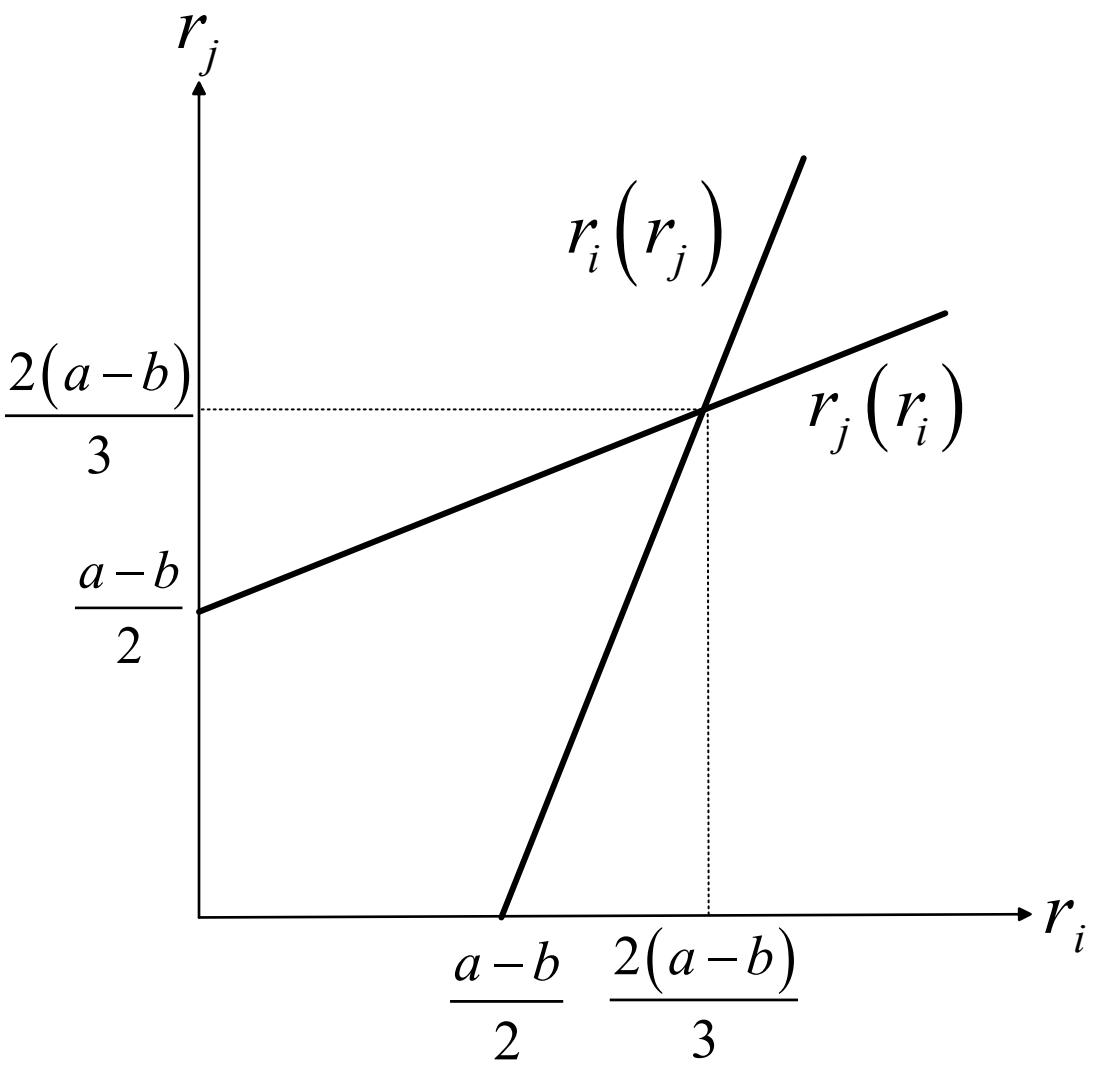
- ***Example*** (continued):
  - Simultaneously solving the two BRFs yields

$$r_i = \frac{1}{4}\left[\frac{a-b}{2} + \frac{r_i}{4}\right] + \frac{a-b}{2}$$

  - That is, $\frac{15}{16} r_i = \frac{5}{8}(a-b)$. Rearranging, we obtain an equilibrium level of recycling

$$r_i^* = \frac{2}{3}(a-b) \quad \text{for} \quad i \in \{1,2\}$$

Externalities



Externalities

- ***Example*** (continued):
  - A social planner simultaneously selects r_i and r_j in order to maximize social welfare

$$\max_{r_i, r_j}(a - r_i + \frac{r_j}{2})r_i - br_i + (a - r_j + \frac{r_i}{2})r_j - br_j$$

  - FOC with respect to r_i:

$$a - 2r_i + \frac{r_j}{2} - b + \frac{r_j}{2} = 0, \qquad \text{or} \quad a - b - 2r_i + r_j = 0$$

  and symmetrically for country j.

Externalities

- ***Example*** (continued):
  - The second $\frac{r_j}{2}$ term is the external benefit of r_i on country j, which the planner internalizes.

Externalities

- ***Example*** (continued):
  - Simultaneously solving the two FOCs yields the socially optimal levels of recycling

$$r_i^0 = a - b \quad \text{for} \quad i \in \{1,2\}$$

  - Note that, since $a > b$,

$$r_i^0 = a - b > \frac{2}{3}(a - b) = r_i^*$$

Solutions to the Externality Problem:

Property Rights

Property Rights

- This is a less intrusive approach:
  - let the parties bargain over the externality
  - no government intervention
- Key assumptions:
  - The property rights over the externality-generating activity must be:
    - Easy to identify and enforceable.
  - No bargaining costs.
  - Complete information about π and v.

Property Rights

- ***Property rights assigned to the affected individual (agent 2)***:
  - Let us assign property rights to the individual suffering the negative externality
    - "externality-free" environment: at the initial state no externality is generated, i.e., $x = 0$
  - The firm must then pay the affected individual if it wants to increase the externality above zero.

Property Rights

- ***Property rights assigned to the affected individual (agent 2)***:
  - In particular, let us assume that the affected individual makes a take-it-or-leave-it offer where the firm must pay $\$T$ in exchange for x units of pollution.

Property Rights

– The firm agrees to pay \$$T$ to the affected individual iff

$$\pi(x) + w_1 - T \geq \underbrace{\pi(0)}_{\text{status quo}} + w_1$$

where w_h is agent h's wealth, or

$$\pi(x) - T \geq \pi(0)$$

– Hence, the affected individual's UMP becomes that of choosing (x, T) that solves

$$\max_{x \geq 0, T} v(x) + w_2 + T$$
$$\text{s.t. } \pi(x) - T \geq \pi(0)$$

Property Rights

- The constraint is binding, since the affected individual will raise $\$T$ until the point where the firm is indifferent between accepting and rejecting the offer.
- Hence, $\pi(x) - T = \pi(0) \quad \text{or} \quad \pi(x) - \pi(0) = T.$
- Plugging this result into the affected individual's UMP, we obtain

$$\max_{x \geq 0} v(x) + w_2 + \underbrace{\pi(x) - \pi(0)}_{T}$$

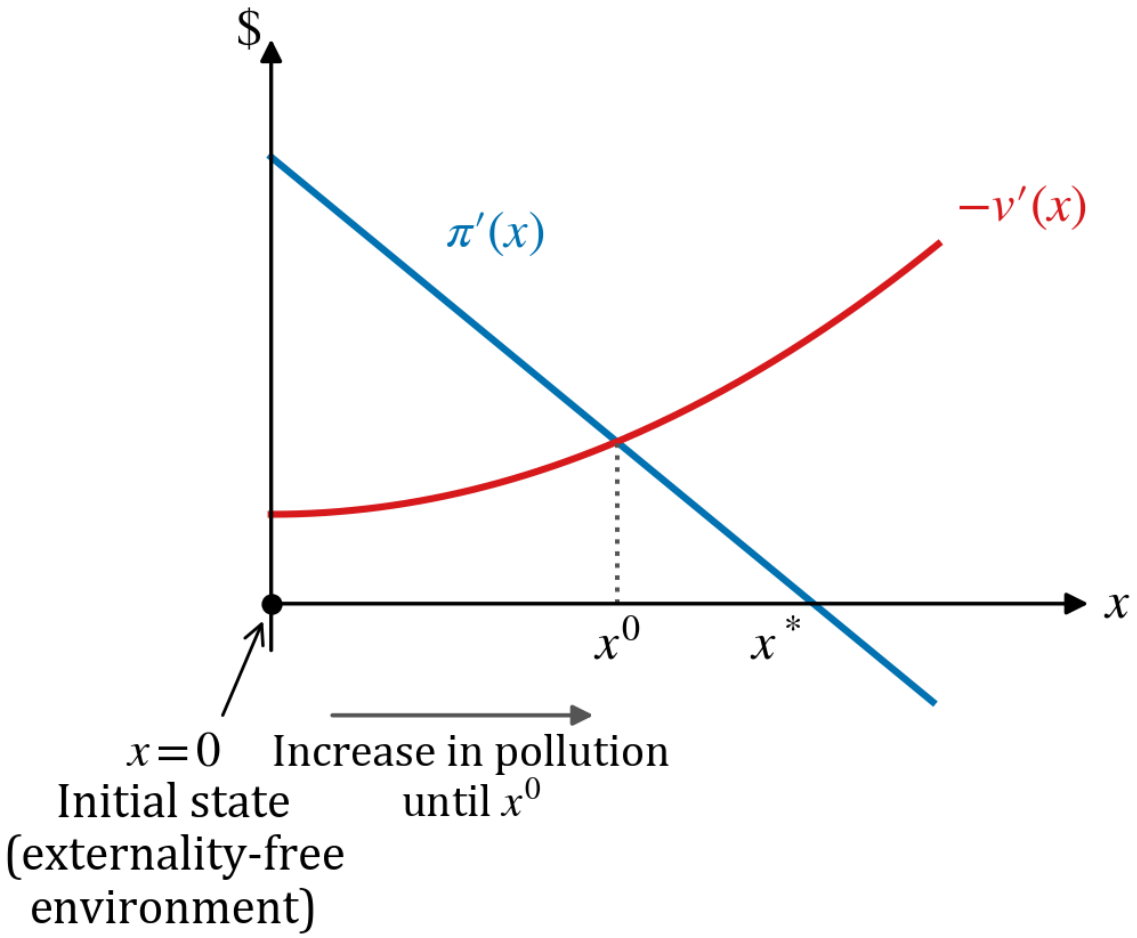
Property Rights

- The FOC with respect to x yields

$$v'(x) + \pi'(x) \leq 0 \quad \Leftrightarrow \quad \pi'(x) \leq -v'(x)$$

  with equality if $x > 0$.
- This coincides with the FOC of the social planner's problem.
- Therefore, bargaining allows for the level of the externality x to reach the optimal level, i.e., $x = x^0$.

Property Rights



Property Rights

- ***Property rights assigned to the firm***:
  - What if the property rights were assigned to the firm (i.e., the polluter)?
  - If there is no bargaining between the firm and the affected individual, the firm would pollute until $x = x^*$, where $\pi'(x^*) = 0$.
  - However, the affected individual can pay $\$T$ to the firm in exchange for a lower level of pollution, x, where $x < x^*$.

Property Rights

– The polluter is willing to accept this offer iff

$$\pi(x) + w_1 + T \geq \underbrace{\pi(x^*)}_{\text{status quo}} + w_1$$

$$\text{or} \quad \pi(x) + T \geq \pi(x^*)$$

– Thus, the affected individual's UMP becomes that of choosing (x, T) that solves

$$\max_{x \geq 0, T} v(x) + w_2 - T$$

$$\text{s.t.} \quad \pi(x) + T \geq \pi(x^*)$$

– Note that \$$T$ now enters negatively into the affected individual's utility, but positively into the firm's.

Property Rights

- The constraint is binding, since the affected individual reduces the $\$T$ until the point where the firm is indifferent between accepting and rejecting the offer T.
- Hence, $\pi(x) + T = \pi(x^*)$ or $T = \pi(x^*) - \pi(x)$.
- Inserting this result into the affected individual's UMP, we obtain

$$\max_{x \geq 0} v(x) + w_2 - \underbrace{[\pi(x^*) - \pi(x)]}_{T}$$

Property Rights

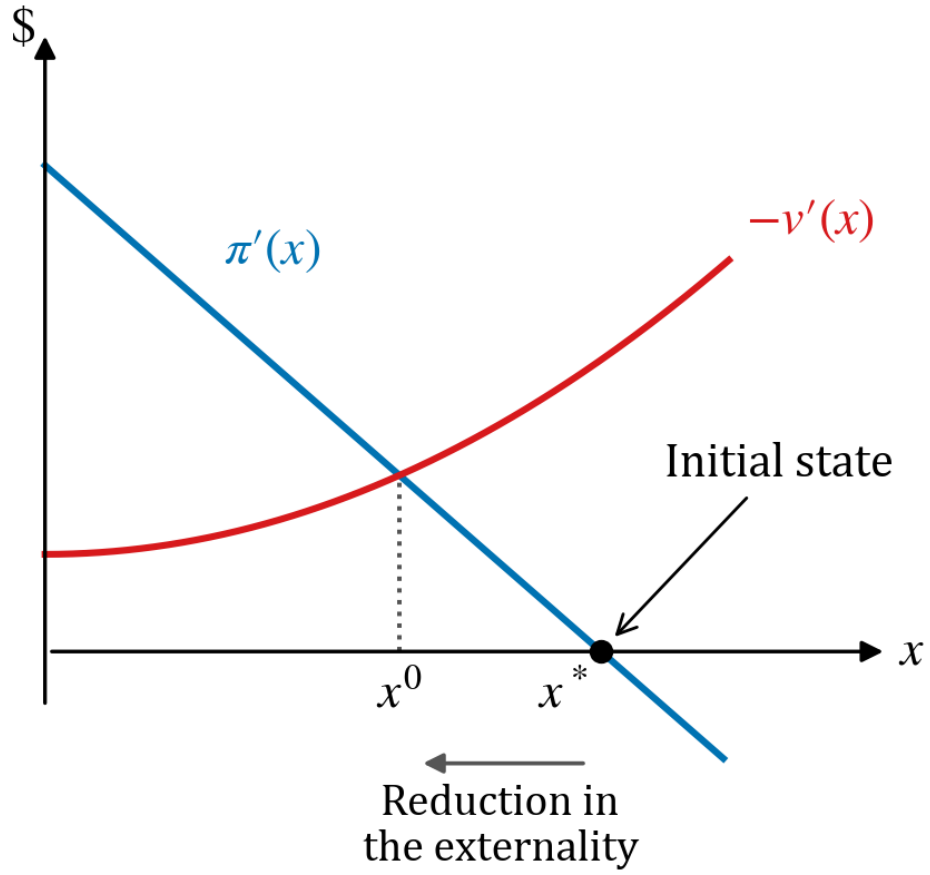
- The FOC with respect to x yields

$$v'(x) + \pi'(x) \leq 0 \quad \Leftrightarrow \quad \pi'(x) \leq -v'(x)$$

  with equality if $x > 0$.

- Again, the above coincides with the FOC at the optimal level of the externality (i.e., social planner's problem), where $x = x^0$.

Property Rights



Property Rights

- In summary, regardless of the initial assignment of property rights over the externality-generating activity…
  - agents can negotiate to increase or decrease the externality, x, until reaching the Pareto optimal level x^0.

Property Rights

- ***Coase Theorem***: If property rights are well defined and enforceable, bargaining is costless, information is complete, and preferences are quasilinear (no wealth effects), then the equilibrium level of the externality is x^0, regardless of the initial allocation of property rights.
  - That is, as long as property rights are clearly assigned, bargaining sets the level of the externality at the Pareto optimal level $x = x^0$.

Property Rights

- The allocation of property rights, however, affects the final wealth of the two agents!

Property Rights

- If property rights are assigned to the individual affected by the externality:
  - the firm must pay $T = \pi(x^0) - \pi(0)$ to the affected individual in order to increase the externality from $x = 0$ to $x = x^0$
  - the affected individual's utility is then

    $$v(x^0) + T \text{ or } v(x^0) + \pi(x^0) - \pi(0)$$

    while that of the firm is

    $$\pi(x^0) - T \text{ or } \pi(x^0) - \pi(x^0) + \pi(0) = \pi(0)$$

  - with $w_1 = w_2$, the affected individual's utility is higher than the firm's iff

    $$v(x^0) + \pi(x^0) - \pi(0) > \pi(0) \Leftrightarrow \pi(x^0) + v(x^0) > 2\pi(0)$$

Property Rights

- If property rights are assigned to the firm generating the externality:
  - the affected individual must pay $T = \pi(x^*) - \pi(x^0)$ to the firm in order to reduce the externality from $x = x^*$ to $x = x^0$
  - the firm's utility is then

    $$\pi(x^0) + T \text{ or } \pi(x^0) + \pi(x^*) - \pi(x^0) = \pi(x^*)$$

    while that of the affected individual is

    $$v(x^0) - T \text{ or } v(x^0) - \pi(x^*) + \pi(x^0)$$

  - with $w_1 = w_2$, the firm's utility is higher than the affected individual's iff

    $$\pi(x^*) > v(x^0) - \pi(x^*) + \pi(x^0) \Leftrightarrow 2\pi(x^*) > \pi(x^0) + v(x^0)$$

Property Rights

- Combining the two inequalities, we can conclude that the agent with the bargaining power (i.e., the property right over x) has a total utility higher than the agent without the bargaining power iff

$$2\pi(x^*) > \pi(x^0) + v(x^0) > 2\pi(0)$$

where $\pi(x^0) + v(x^0)$ measures the aggregate welfare at the social optimum, i.e., x^0.

Property Rights

- Each agent is better off holding the property right:
  - When the right moves from the individual to the firm, the individual's payoff falls by $\pi(x^*) - \pi(0) > 0$, and the firm's payoff rises by the same amount.

Property Rights

- ***Disadvantages of the Coase Theorem***:
  - Property rights must be perfectly defined
    - With whom should the firm bargain?
  - Bargaining must be costless.
    - This fails when many agents are involved.

Property Rights

- ***Disadvantages of the Coase Theorem*** (continued):
  - Property rights must be perfectly enforced:
    - The level of x must be perfectly observable and measurable by both parties
    - If a party does not comply with the agreement, it can be brought to court at no cost.
  - These assumptions are not satisfied in many cases, which limits the possibility of using negotiations.

Property Rights

- ***Advantages of the Coase Theorem***:
  - Only the parties involved need to know the marginal benefits and costs associated with the externality
  - The regulator only needs to assign and enforce property rights; it does not need to know $\pi(\cdot)$ or $v(\cdot)$.

Property Rights

- ***Remark***:
  - If the two parties are firms (e.g., a fishery and a refinery), a form of bargaining could be the sale of one firm to the other, i.e., a merger.
  - This is efficient as now the merged firm would internalize the effects that pollution imposes on the production process of the fishery.

Solutions to the Externality Problem:

More Intrusive Approaches

Quota

- Setting a quota (emission standard) that bans externality (emission) levels above x^0.
- The regulator must know the benefits and damages of the externality for all firms and consumers.

Pigouvian Taxation

- This policy sets a tax t_x per unit of the externality-generating activity x.
- What is the level of tax t_x that restores efficiency?
- Let us start by rewriting the firm's PMP

$$\max_{x \geq 0} \ \pi(x) - t_x \cdot x$$

Pigouvian Taxation

- FOC with respect to x:

$$\pi'(x) - t_x \leq 0, \qquad \text{or} \quad \pi'(x) \leq t_x$$

  with equality if $x > 0$.

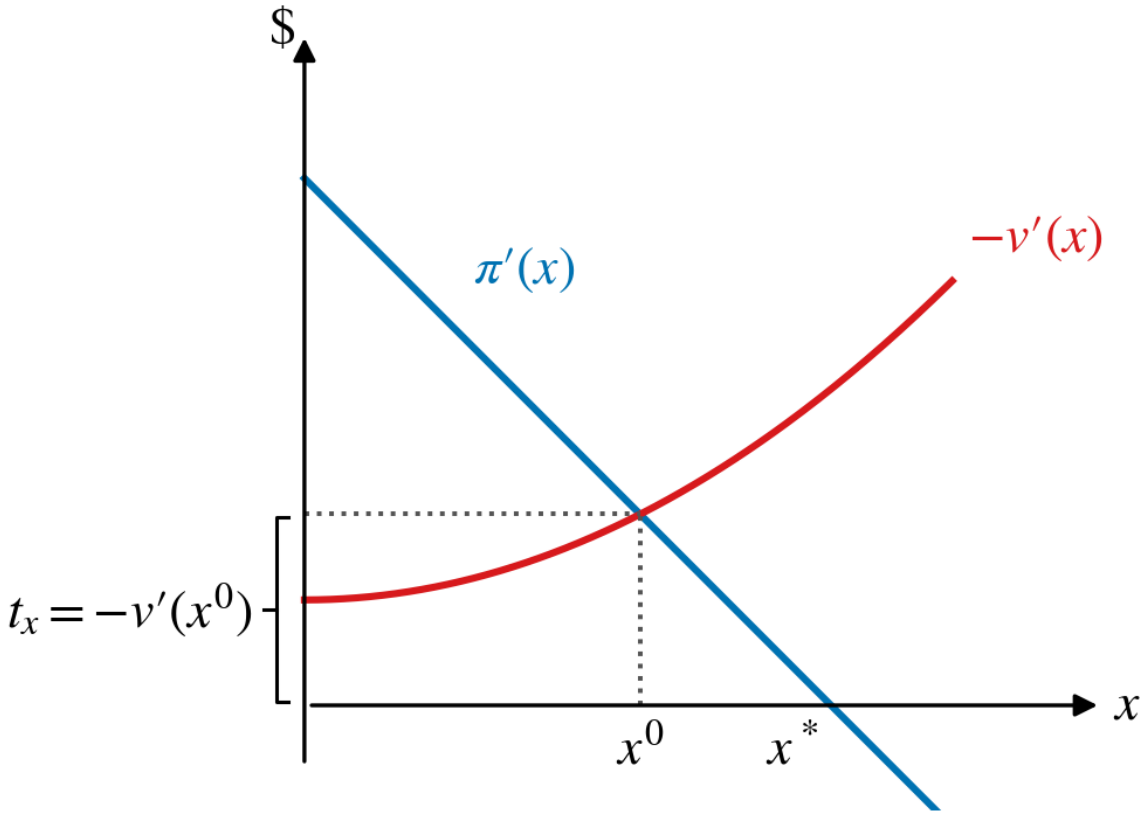
- *Intuition*: the firm increases x until the point where the marginal benefit from an additional unit of x coincides with the per-unit tax t_x.

Pigouvian Taxation

- We know that at the social optimum (i.e., x^0)
$$\pi'(x^0) = -v'(x^0)$$
- Hence, the tax t_x needs to be set at
$$t_x = -v'(x^0)$$
- This forces the firm to internalize the damage that the externality generates on the consumer's well-being at x^0.
- Since $\pi'' < 0$, $\pi'(x) = t_x$ has a unique solution, so $t_x = -v'(x^0)$ induces $x = x^0$.

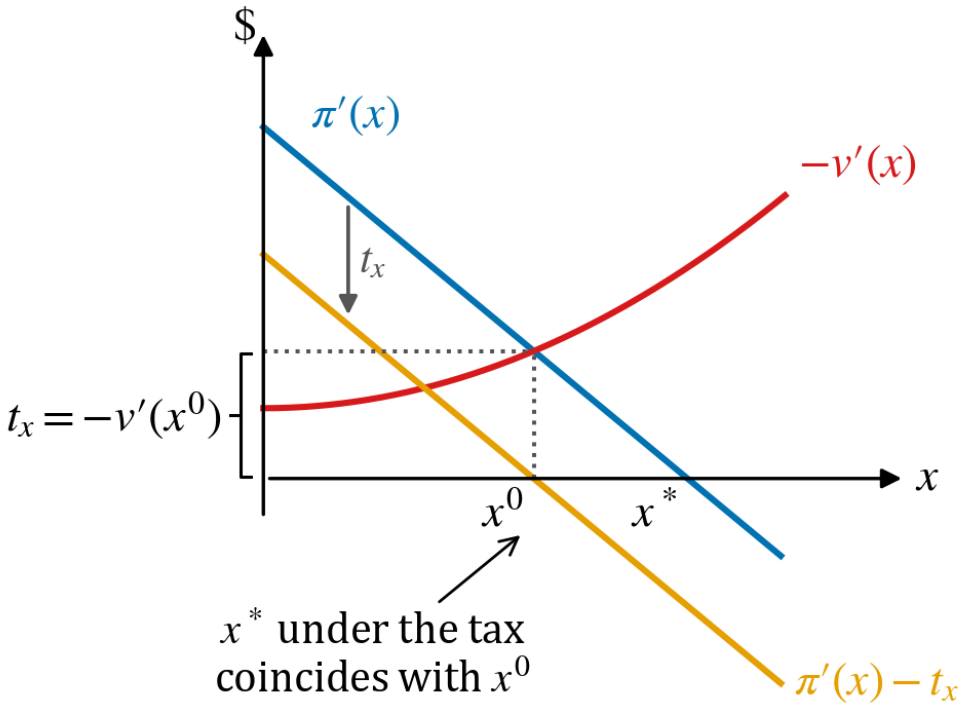
Pigouvian Taxation

- The tax t_x leads the firm to choose a level of x equal to x^0



Pigouvian Taxation

- The tax produces a downward shift in $\pi'(x)$.
- The new marginal benefit curve $\pi'(x) - t_x$ crosses the horizontal axis at x^0.

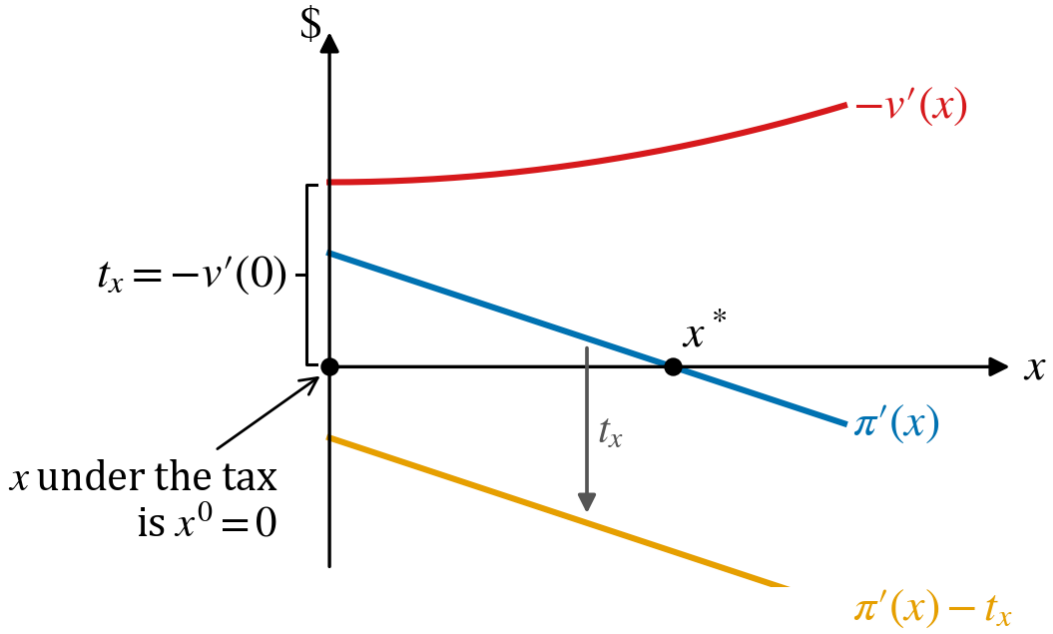


Pigouvian Taxation

- The optimality-restoring tax t_x is equal to the marginal damage at the optimal level x^0.
  - That is, it is equal to the amount of money that the affected individual would be willing to pay in order to reduce x slightly from its optimal level x^0.
- The tax t_x induces the firm to internalize the externality that it causes on the individual.

Pigouvian Taxation

- If the negative externality is large enough that $-v'(0) \geq \pi'(0)$ (see the corner case above), the social optimum is at $x^0 = 0$, and the tax $t_x = -v'(0)$ induces $x = 0$ (any $t_x \geq \pi'(0)$ does).



Pigouvian Subsidy

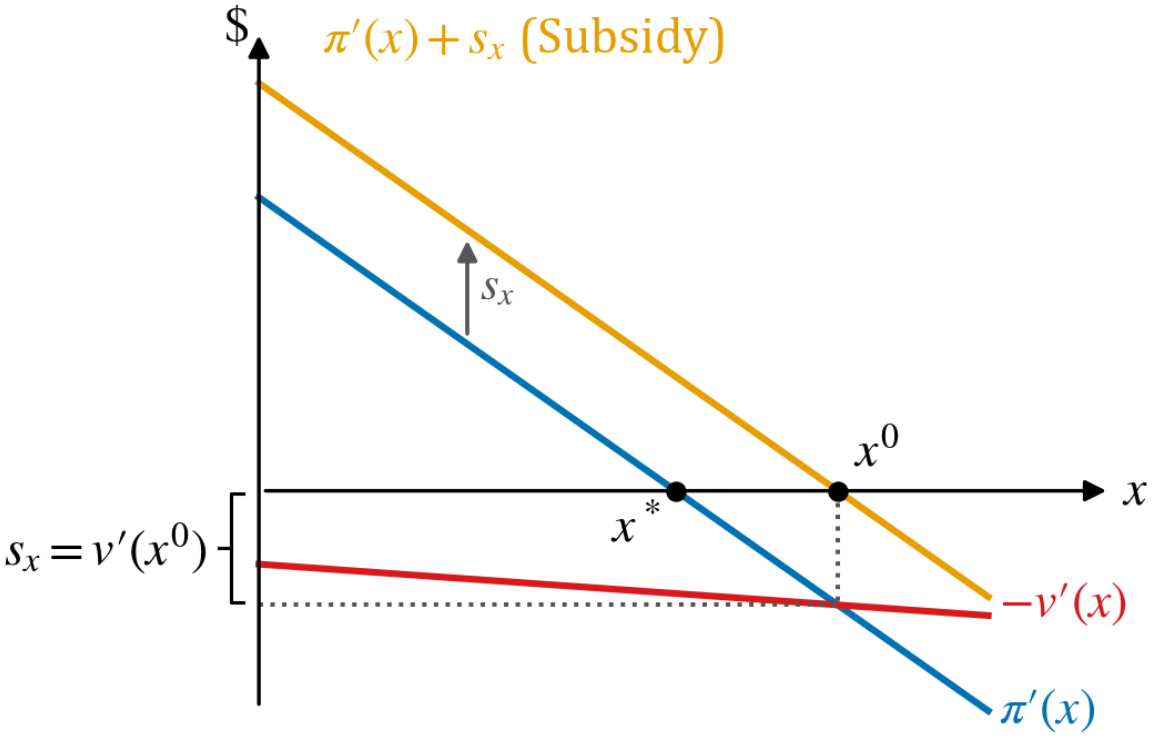
- This analysis extends to *positive externalities*.
- Since now $v'(x^0) > 0$ (i.e., x increases the individual's welfare), the optimality-restoring tax is

$$t_x = -v'(x^0) < 0$$

- We thus set "negative taxes" on the externality: a per-unit subsidy (s_x).
- The firm receives a payment of $s_x = -t_x = v'(x^0) > 0$ for each unit of the positive externality it generates.

Pigouvian Subsidy

- The per-unit subsidy produces an upward shift in the marginal benefits of the firm.
- The firm has incentives to increase x beyond the competitive equilibrium level x^* until reaching the Pareto optimal level x^0.



Pigouvian Policy: Important Points

a) A tax on the negative externality is equivalent to a subsidy inducing agents to reduce the externality.

- Consider a subsidy $s_x = -v'(x^0) > 0$ for every unit that the firm's choice of x is below x^*.
- The firm's PMP becomes:

$$\max_{x \geq 0} \pi(x) + s_x(x^* - x) = \pi(x) + \underbrace{s_x x^*}_{\text{lump-sum transfer}} - \underbrace{s_x x}_{\text{per-unit tax}}$$

Pigouvian Policy: Important Points

a) A tax on the negative externality is equivalent to a subsidy inducing agents to reduce the externality.

– FOC with respect to x yields

$$\pi'(x) - s_x \leq 0$$

with equality if $x > 0$, which holds at $x = x^0$ since $s_x = -v'(x^0)$.

Pigouvian Policy: Important Points

– This objective equals the Pigouvian-tax objective plus a lump-sum transfer $t_x x^*$ to the firm (with $s_x = t_x$), so the FOC is the same:

$$\pi(x) + s_x(x^* - x) = [\pi(x) - t_x x] + t_x x^*$$

– Hence, a subsidy for the reduction of the externality (combined with a lump-sum tax $t_x x^*$) can replicate the outcome of the Pigouvian tax.

Pigouvian Policy: Important Points

b) The Pigouvian tax levies a tax on the externality-generating activity (e.g., pollution) but not on the output that generated such pollution.

- Taxing output might lead the firm to reduce output, but it does not guarantee a reduction in pollutant emissions.
- A tax on output is equivalent to a tax on emissions only if emissions are proportional to output (as in the example with $x = \alpha y$).

Pigouvian Policy: Important Points

c) *The quota and the Pigouvian tax are equally effective under complete information.*

- They might not be equivalent when regulators face incomplete information about the benefits and costs of the externality for consumers and firms.

Solutions to the Externality Problem:

Tradable Externality Permits

Tradable Externality Permits

- Consider J firms and I consumers, where consumer i's utility is $v_i(x)$ with $x = \sum_j x_j$.
- Every permit grants the right to generate one unit of the externality.
- Suppose that $x^0 = \sum_j x_j^0$ total permits are given to the firms, with every firm receiving $\overline{x}_j$ of them.
- Let p_x^* denote the equilibrium price of these permits.

Tradable Externality Permits

- Firm j's PMP is

$$\max_{x_j \geq 0} \pi_j(x_j) + p_x^*(\overline{x}_j - x_j)$$

where firm j must pay a price p_x^* for every permit it needs to buy in excess of its initial endowment $\overline{x}_j$, and receives p_x^* for every permit it sells.

Tradable Externality Permits

- FOC wrt x_j yields

$$\pi_j'(x_j) - p_x^* \leq 0$$

with equality if $x_j > 0$ (interior solution).

- If all J firms carry out this PMP, we need the market clearing condition

$$\sum_j x_j = \sum_j \overline{x}_j = x^0$$

Tradable Externality Permits

- With the supply of permits fixed at x^0, the market-clearing price satisfies

$$p_x^* = -\sum_{i=1}^{I} v_i'(x^0)$$

Tradable Externality Permits

- Thus, firm j's FOCs become

$$\pi_j'(x_j^0) + \sum_{i=1}^{I} v_i'(x^0) \leq 0$$

  with equality if $x_j^0 > 0$ (interior solution).
- This condition coincides with the FOC that solves the social planner's problem.

Tradable Externality Permits

- The social planner solves

$$\max_{(x_1,\ldots,x_J)} \sum_j \pi_j(x_j) + \sum_i v_i\left(\sum_j x_j\right)$$

with FOC

$$\pi_j'(x_j) + \sum_i v_i'(x) \leq 0$$

with equality if $x_j > 0$.

- Therefore, every firm j is induced to voluntarily choose $x_j = x_j^0$.

Tradable Externality Permits

- The advantage of tradable externality permits:
  - Government officials need less information.
  - Relative to firm-specific quotas, the regulator only needs x^0, not each firm's π_j.
    - Specifically, data on aggregate industry profits and damages from the externality
    - But not on individual firms and consumers

Regulating a Polluting Monopolist

Regulating a Polluting Monopolist

- Consider a monopolist facing a linear inverse demand curve $p(x) = 1 - x$ and constant marginal production costs $0 \leq c < 1$.
- Each unit of output generates one unit of emissions, which cause an environmental damage $ED(x) = dx^2$, where $d > 0$.
  - Damage is convex in output.

Regulating a Polluting Monopolist

- The regulator charges an emission fee t per unit of emissions.
- The social planner seeks x_{SO} that maximizes

$$W(x) = CS(x) + PS(x) + T - ED(x)$$

where $CS(x)$ and $PS(x)$ denote consumer and producer surplus, respectively, and T represents tax revenue from the emission fee.

Regulating a Polluting Monopolist

- Let us first find the monopolist's profit-maximizing level of x for a given emission fee t.
- The PMP of the monopolist is

$$\max_{x \geq 0} (1 - x)x - (c + t)x$$

- FOC: $1 - 2x - (c + t) = 0$
- Solving for x yields an output function

$$x(t) = \frac{1 - (c + t)}{2}, \qquad x'(t) = -\frac{1}{2} < 0$$

where $x(t) > 0$ requires $t < 1 - c$.

Regulating a Polluting Monopolist

- The social planner solves

$$\max_{x \geq 0} W(x) = CS(x) + PS(x) + T - ED(x)$$

where $CS(x) \equiv \frac{1}{2}x^2$, $PS(x) \equiv (1-x)x - (c+t)x$, $T \equiv tx$, and $ED(x) \equiv dx^2$.

- T and the $-tx$ in $PS(x)$ cancel, so $W(x)$ does not depend on t directly.

Regulating a Polluting Monopolist

- FOC:

$$2\frac{1}{2}x + [1 - 2x - (c + t)] + t - 2dx = 0$$

or, rearranging, $1 - c = x(1 + 2d)$.

- The SOC holds, since $-(1 + 2d) < 0$.
- Solving for x yields

$$x_{\text{SO}} = \frac{1 - c}{1 + 2d}$$

Regulating a Polluting Monopolist

- The emission fee t that induces the monopolist to produce x_{SO} is found by solving

$$\frac{1-(c+t)}{2} = \frac{1-c}{1+2d}$$

which yields

$$t = (2d-1)\frac{1-c}{1+2d} \quad \text{or} \quad t = (2d-1)x_{SO}$$

Regulating a Polluting Monopolist

- Notice that
  - $t > 0$ (a tax) iff $d > 1/2$
  - $t < 0$ (a subsidy) iff $d < 1/2$
  - $t = 0$ iff $d = 1/2$
- The fee can be decomposed as

$$t = ED'(x_{\mathrm{SO}}) - x_{\mathrm{SO}} = 2dx_{\mathrm{SO}} - x_{\mathrm{SO}}$$

that is, the Pigouvian term minus a correction for the monopolist's market power.

Regulating a Polluting Monopolist

- *Intuition*:
  - If the market failure arising from the environmental externality is sufficiently large, i.e., $d > 1/2$, the regulator imposes a tax policy in order to reduce the production from the polluting monopolist.
  - If, in contrast, the market failure from the externality is smaller, i.e., $d < 1/2$, the regulator subsidizes the monopolist's production.

Regulating a Polluting Monopolist

- *Intuition* (continued):
  - The monopolist underproduces, $x_m = \frac{1-c}{2}$, while pollution calls for lower output.
  - Since

$$x_m - x_{\mathrm{SO}} = \frac{(2d-1)(1-c)}{2(1+2d)}$$

the monopolist overproduces relative to x_{SO}, and is taxed, if and only if $d > 1/2$.

Regulating a Polluting Oligopoly

Regulating a Polluting Oligopoly

- Consider a Cournot oligopoly. Same demand, damage $ED(X) = dX^2$ and welfare as in the monopoly case, now with an incumbent and an entrant, where $X \equiv x_{inc} + x_{ent}$ is aggregate output.
- Assume that the incumbent can
  - enjoy a cost advantage relative to the entrant, i.e., $c_{inc} < c_{ent} < 1$, or
  - face the same production costs, i.e., $c_{inc} = c_{ent} < 1$.
- We assume $d > \frac{1}{2}$ when comparing with the monopoly fee, so that both fees are positive.

Regulating a Polluting Oligopoly

- Let us first find the oligopolists' profit-maximizing output levels for a given emission fee t.
- The PMP of the incumbent is

$$\max_{x_{inc}} \ (1 - x_{inc} - x_{ent})x_{inc} - (c_{inc} + t)x_{inc}$$

  while that of the entrant's

$$\max_{x_{ent}} \ (1 - x_{ent} - x_{inc})x_{ent} - (c_{ent} + t)x_{ent}$$

- FOCs wrt x_i, where $i \in \{inc, \ ent\}$, yields

$$1 - 2x_i - x_j - (c_i + t) = 0 \ \text{ for } \ j \neq i$$

Regulating a Polluting Oligopoly

- Solving for x_i yields firm i's BRF

$$x_i(x_j) = \frac{1-(c_i+t)}{2} - \frac{1}{2}x_j$$

- Plugging firm j's BRF into firm i's, we obtain

$$x_i = \frac{1-(c_i+t)}{2} - \frac{1}{2} \cdot \left(\frac{1-(c_j+t)}{2} - \frac{1}{2}x_i \right)$$

- Solving for x_i yields an equilibrium output function

$$x_i(t) = \frac{1-2c_i+c_j-t}{3}$$

which is decreasing in c_i and t, but increasing in c_j.

Regulating a Polluting Oligopoly

- The social planner solves

$$\max_{X \geq 0} \; W(X) = CS(X) + PS(X) + T - ED(X)$$

where $X \equiv x_{inc} + x_{ent}$, $CS(X) \equiv \frac{1}{2}(X)^2$, $PS(X) \equiv (1 - X)X - (c_{inc} + t)X$, $T \equiv tX$, and $ED(X) \equiv d(X)^2$.

Regulating a Polluting Oligopoly

- FOC:

$$\frac{\partial W}{\partial X} = X + (1 - 2X) - c_{inc} - 2dX = 0$$

(the fee cancels, since $T = tX$ is a transfer), so

$$X_{SO} = \frac{1 - c_{inc}}{1 + 2d}$$

  - X_{SO} is the first-best output, produced at the lowest marginal cost c_{inc}.

Regulating a Polluting Oligopoly

- The emission fee t that induces the incumbent and entrant to produce $x_{inc,SO}$ and $x_{ent,SO}$, respectively, is found by simultaneously solving

$$x_{inc,SO} + x_{ent,SO} = \frac{1-c_{inc}}{1+2d}$$

$$x_{inc}(t) = \frac{1-2c_{inc}+c_{ent}-t}{3}$$

$$x_{ent}(t) = \frac{1-2c_{ent}+c_{inc}-t}{3}$$

Regulating a Polluting Oligopoly

- Two cases:
  - Case 1, cost symmetry ($c_{inc} = c_{ent}$): this system pins down t.
  - Case 2, cost asymmetry ($c_{inc} < c_{ent}$): a single fee cannot induce first-best production, so the regulator chooses t to maximize welfare evaluated at the equilibrium outputs $x_i(t)$, with each firm producing at its own cost (second-best).

Regulating a Polluting Oligopoly

- ***Case 1: Cost symmetry,*** $c_{inc} = c_{ent}$
  - Simultaneously solving the three equations yields

$$t = \frac{4d-1}{2}\frac{1-c_{inc}}{1+2d} \quad \text{or} \quad t = (4d-1)\frac{X_{SO}}{2}$$

where $t > 0$ iff $d > 1/4$.

- The regulator imposes emission fees on the oligopoly even in settings where a monopolist would receive a subsidy, i.e., $d \in \left(\frac{1}{4}, \frac{1}{2}\right)$.

  - Substituting t into the output function $x_i(t)$ yields

$$x_{inc,SO} = x_{ent,SO} = \frac{1}{2} \cdot \frac{1-c_{inc}}{1+2d} = \frac{X_{SO}}{2}$$

Regulating a Polluting Oligopoly

- ***Case 2: Cost asymmetry,*** $c_{inc} < c_{ent}$
  - Plugging $X(t) = \frac{2(1-t)-c_{inc}-c_{ent}}{3}$ into W (with each firm's own cost) and differentiating with respect to t yields

$$t = \frac{(2-c_{inc}-c_{ent})(4d-1)}{4A}$$

where $A \equiv 1 + 2d$ and $t > 0$ iff $d > \frac{1}{4}$.

Regulating a Polluting Oligopoly

- ***Case 2: Cost asymmetry (continued)***
  - Substituting this fee, t^*, into the output functions $x_i(t)$ yields the second-best outputs

$$x_{inc}(t^*) = \frac{2 - (3 + 4d)c_{inc} + (1 + 4d)c_{ent}}{4A}$$

$$x_{ent}(t^*) = \frac{2 - (3 + 4d)c_{ent} + (1 + 4d)c_{inc}}{4A}$$

  - The first is positive if and only if $c_{ent} > \frac{(3+4d)c_{inc}-2}{1+4d}$, and the second if and only if $c_{ent} < \frac{2+(1+4d)c_{inc}}{3+4d}$.

Regulating a Polluting Oligopoly

- Let us check whether the two conditions on c_{ent} bind.
- (1) The condition that guarantees $x_{inc}(t^*) > 0$, i.e., $c_{ent} > \frac{(3+4d)c_{inc}-2}{1+4d}$, holds for all $c_{inc} < c_{ent}$.
  - The cutoff has a negative vertical intercept, $-\frac{2}{1+4d}$, and reaches 1 at $c_{inc} = 1$, so it lies below the 45°-line for all $c_{inc} < 1$.
  - Hence, the cutoff is not binding.

Regulating a Polluting Oligopoly

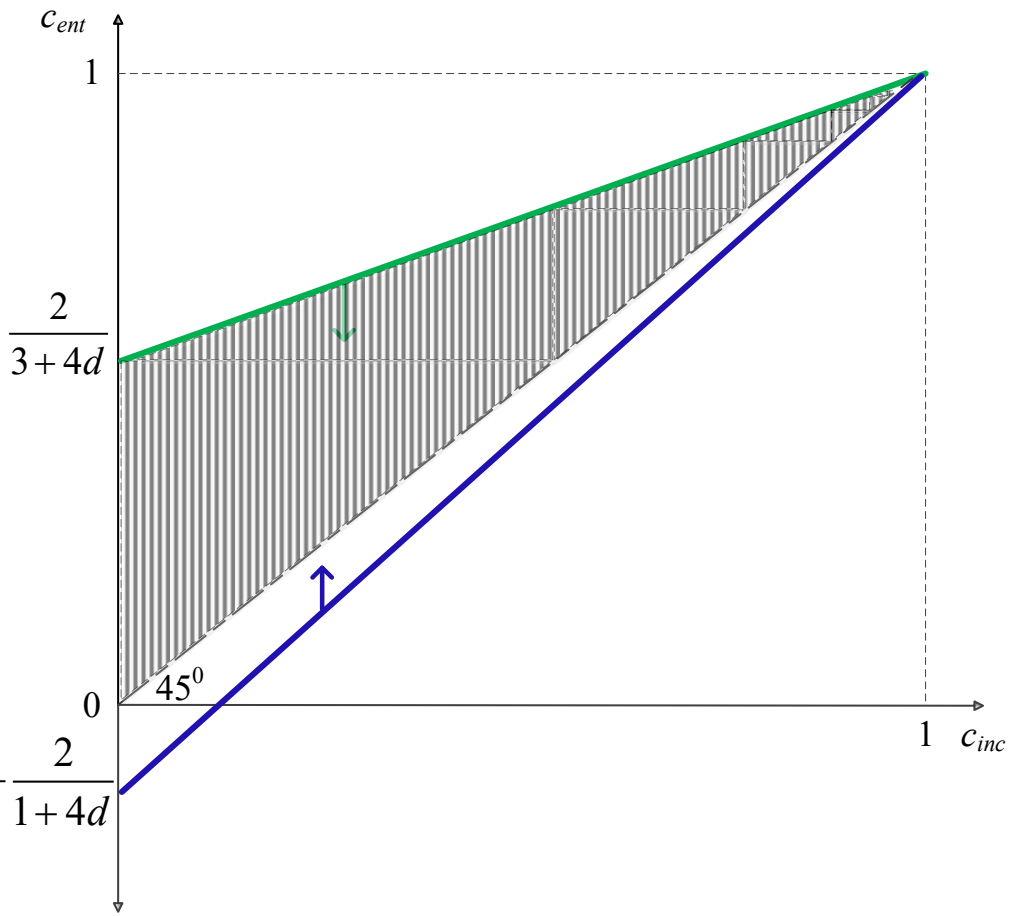
- (2) The condition that guarantees $x_{ent}(t^*) > 0$, i.e., $c_{ent} < \frac{2+(1+4d)c_{inc}}{3+4d}$, originates at $\frac{2}{3+4d}$, and reaches $c_{ent} = 1$ when $c_{inc} = 1$, thus lying above the 45°-line for all $c_{inc} < 1$.
  - Hence, the cutoff $c_{ent} < \frac{2+(1+4d)c_{inc}}{3+4d}$ is binding.

Regulating a Polluting Oligopoly

- For a positive emission fee that induces positive output from both firms, we need

$$c_{inc} < c_{ent} < \frac{2 + (1 + 4d)c_{inc}}{3 + 4d}$$

and $d > \frac{1}{4}$, where the entrant faces a higher, but not too much higher, cost than the incumbent.



Regulating a Polluting Oligopoly

- Fee comparison:
  - The regulator sets more stringent fees on the duopolists than to the monopolist
  - For the case of cost symmetry $c_{inc} = c_{ent}$,

$$(4d-1)\frac{X_{SO}}{2} > (2d-1)X_{SO}$$

$$\Leftrightarrow \; 4d-1 > 4d-2$$

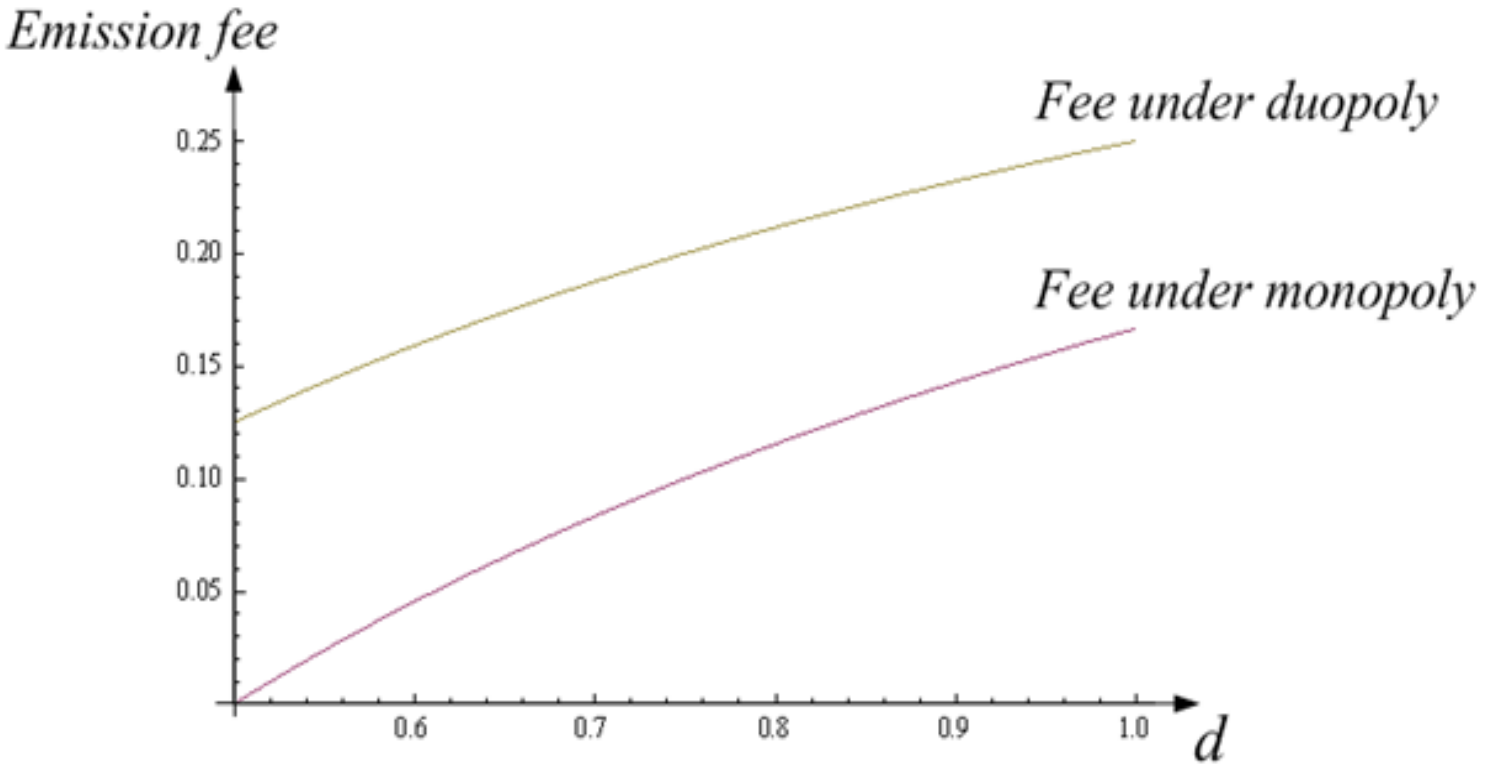
  - The same ranking holds when $c_{inc} < c_{ent}$ (compared with the monopoly fee at $c = c_{inc}$).

Regulating a Polluting Oligopoly

- Fee comparison:
  - *Intuition*:
    - The unregulated duopoly generates a larger amount of pollution than unregulated monopoly.
    - Hence, the regulator sets a more stringent fee on the former.

Regulating a Polluting Oligopoly

- Fee comparison ($c_{inc} = c_{ent} = \frac{1}{2}$):



Presence of Asymmetric Information in Externality Problems

Presence of Asymmetric Information in Externality Problems

- Consider a setting in which firms generate the externality whereas consumers are affected by that externality.
- Let $v(x, \eta)$ be the derived utility of a consumer of type $\eta \in \mathbb{R}$ from x amount of externality.
- Let $\pi(x, \theta)$ be the derived profit function of a firm of type $\theta \in \mathbb{R}$ which generates x amount of externality.
- Consider that parameters η and θ are privately observed by the consumer and firm, respectively.
- Functions $v(x, \eta)$ and $\pi(x, \theta)$ are strictly concave in the externality x for any value of η and θ.

Presence of Asymmetric Information in Externality Problems

- Agents do not observe each other's types, but know the ex-ante likelihoods of η and θ.
- For simplicity, we consider that parameters η and θ are independently distributed.

Presence of Asymmetric Information in Externality Problems

- Let us first consider the decentralized bargaining procedure.
- Bargaining in the presence of asymmetric information does not necessarily lead to an efficient level of the externality x^0.
- Suppose that the consumer has the right to an externality-free environment, and he makes a take-it-or-leave-it offer to the firm.
- Assume that there are two levels of the negative externality: $x = 0$ and $x = \bar{x}$
  - the consumer prefers $x = 0$ to $x = \bar{x}$, whereas the firm prefers $x = \bar{x}$ to $x = 0$.

Presence of Asymmetric Information in Externality Problems

- The benefits that a firm of type θ obtains from having an externality level $x = \bar{x}$ is

$$b(\theta) = \pi(\bar{x}, \theta) - \pi(0, \theta) > 0$$

- The costs that a consumer of type η bears from having an externality level $x = \bar{x}$ is

$$c(\eta) = v(0, \eta) - v(\bar{x}, \eta) > 0$$

Presence of Asymmetric Information in Externality Problems

- What matters in the negotiation between the consumer and the firm are the precise values of $b(\theta)$ and $c(\eta)$.
  - The CDFs of $b(\theta)$ and $c(\eta)$ are $G(b)$ and $F(c)$, respectively.
  - The PDFs of $b(\theta)$ and $c(\eta)$ are $g(b) > 0$ for all $b > 0$ and $f(c) > 0$ for all $c > 0$, respectively.

Presence of Asymmetric Information in Externality Problems

- In the absence of an agreement, the level of the externality remains at $x = 0$.
  - Consumer has the right to the resource

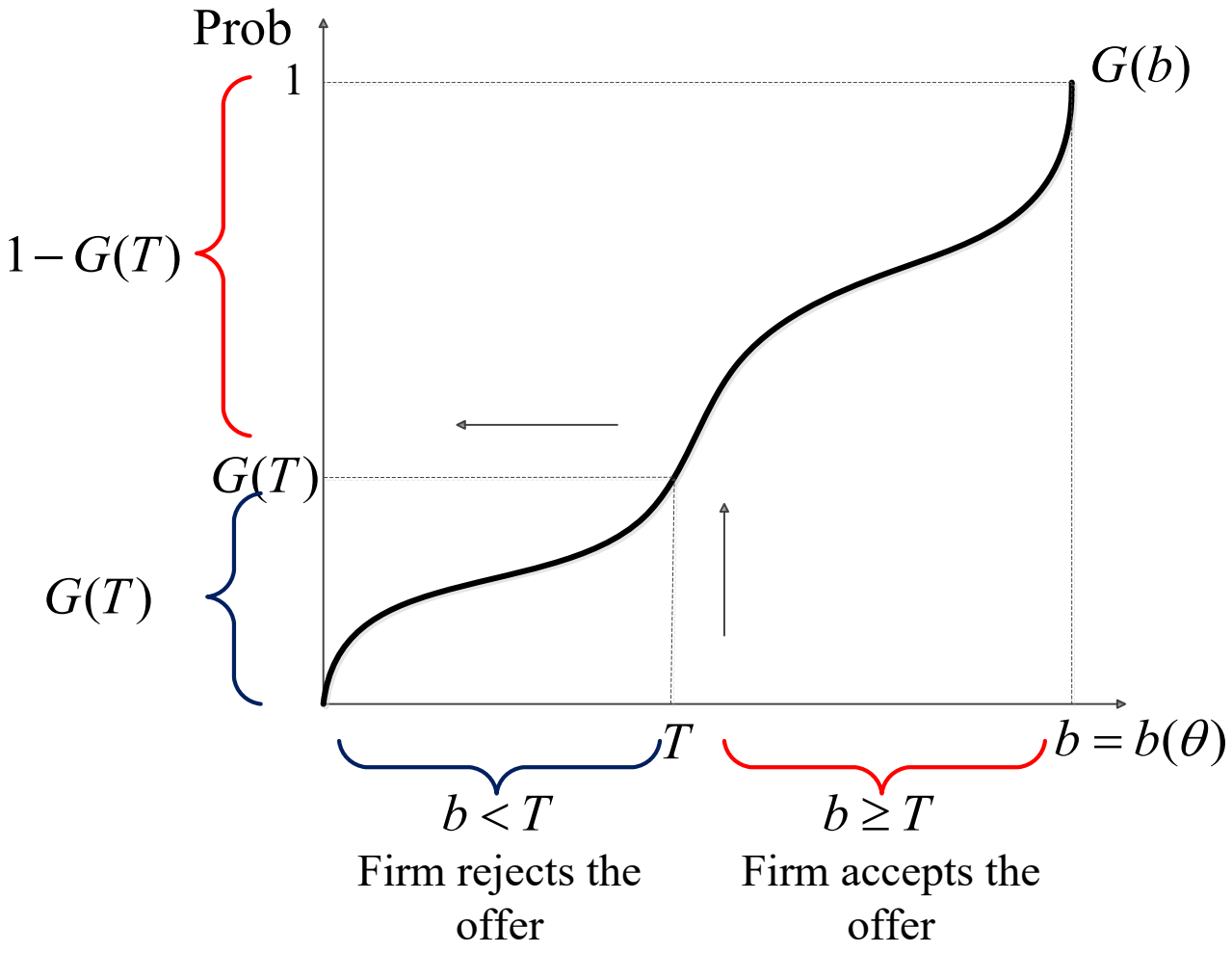
Presence of Asymmetric Information in Externality Problems

- *Pareto optimal outcome*: the firm should be allowed to set a level of the externality $x = \bar{x}$ whenever $b > c$.
  - Intuitively, the firm is willing to pay the consumer more than the damage that the consumer suffers from the externality.
  - Hence, $x = \bar{x}$ would be agreed upon by a firm and consumer if they were perfectly informed about each other's benefit b and cost c.

Presence of Asymmetric Information in Externality Problems

- Let us now start analyzing equilibrium strategies in this context.
- What amount should the consumer demand from the firm (a take-it-or-leave-it-offer) when his cost of the externality is $c(\eta) = c$?
- A θ-type firm will agree to pay T iff its benefits, $b(\theta) = b$, satisfy $b \geq T$.
- Hence, the consumer knows the probability of the firm accepting the payment of T is equal to the probability that $b \geq T$, i.e., $1 - G(T)$.

Presence of Asymmetric Information in Externality Problems



Presence of Asymmetric Information in Externality Problems

- Hence, the consumer chooses the value of the offer T that maximizes his expected net gain

$$\max_{T \geq 0} \; [1 - G(T)](T - c)$$

where

  - $1 - G(T)$ is the probability that an offer of T is accepted by the firm
  - $T - c$ is the net gain that a consumer (with cost c) obtains if the offer is accepted

Presence of Asymmetric Information in Externality Problems

- FOC wrt T yields

$$[1 - G(T_c^*)] - g(T_c^*)(T_c^* - c) \leq 0$$

and at an interior solution, $[1 - G(T_c^*)] = g(T_c^*)(T_c^* - c)$.

Presence of Asymmetric Information in Externality Problems

- Re-arranging,

$$\frac{1-G(T_c^*)}{g(T_c^*)} + c = T_c^*$$

- Since $1 - G(T_c^*) > 0$ and $g(T_c^*) > 0$, the ratio is positive, so $T_c^* > c$.

Presence of Asymmetric Information in Externality Problems

- This implies the firm rejects the consumer's offer when b satisfies $T_c^* > b > c$.
  - However, since $b > c$, Pareto optimality requires that the externality is increased until $x = \bar{x}$.
  - Hence, with probability $G(T_c^*) - G(c) > 0$, b lies in (c, T_c^*): the firm rejects the offer although agreement is efficient.

Presence of Asymmetric Information in Externality Problems

- Complete information:
  - The firm and the consumer are willing to bargain and have the externality produced when they are perfectly informed about their benefits and costs.
  - A welfare improvement for both parties.
- Asymmetric information:
  - The lack of information hinders the success of the mutually beneficial agreements.
  - Decentralized bargaining does not necessarily yield efficient outcomes.

Quotas under Incomplete Information

Quotas under Incomplete Information

- Unlike complete information settings:
  - Government intervention (quotas or taxes) does not necessarily achieve efficient outcomes when the agents are asymmetrically informed.
  - In addition, quotas or taxes are not perfect substitutes for one another.
- For given η and θ, the aggregate surplus resulting from externality level x is

$$v(x, \eta) + \pi(x, \theta)$$

Quotas under Incomplete Information

- The Pareto optimal level of the externality $x^0(\theta, \eta)$ solves

$$\max_{x \geq 0} \; v(x, \eta) + \pi(x, \theta)$$

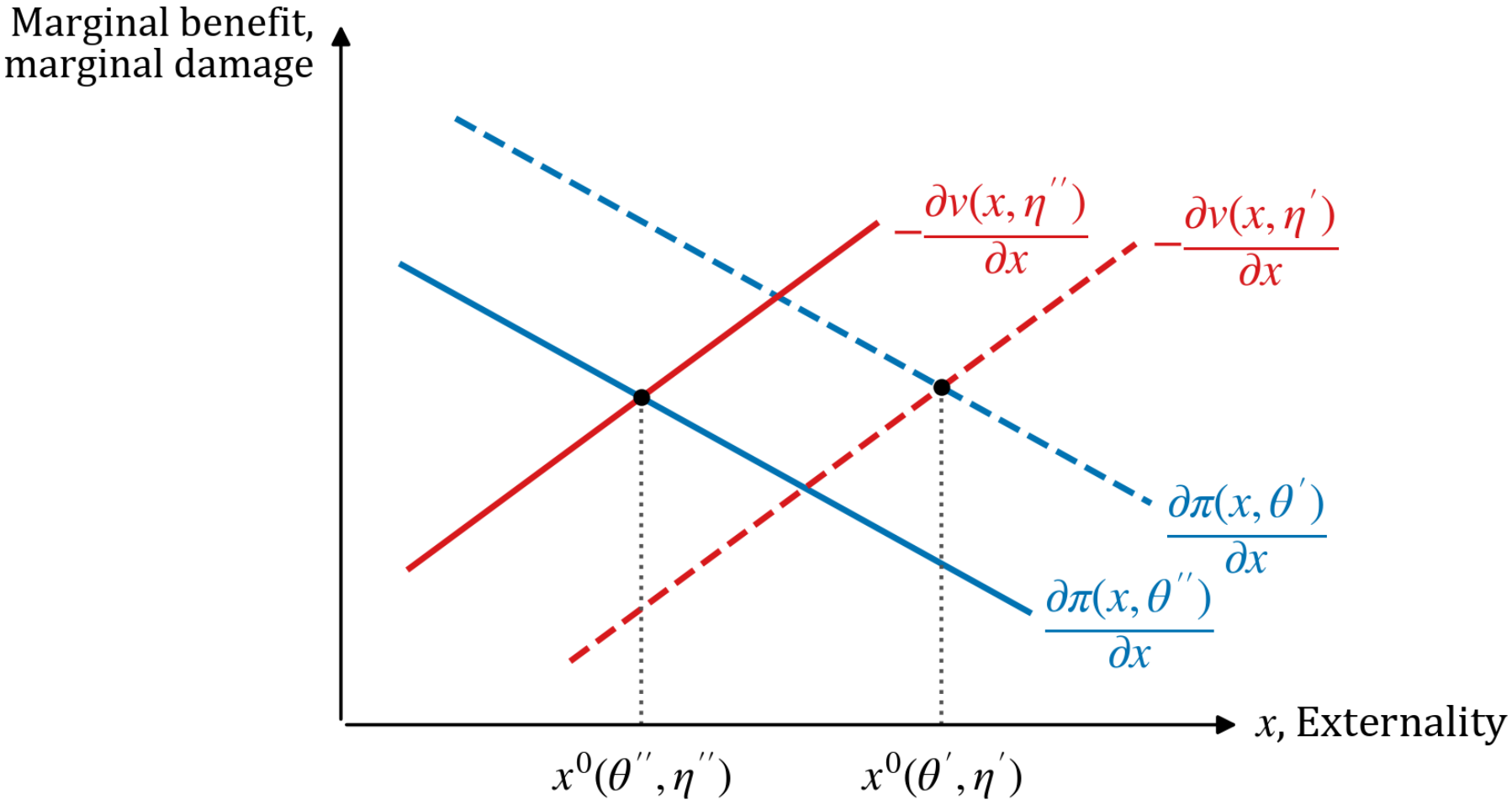
- FOC wrt x yields

$$\frac{\partial v(x,\eta)}{\partial x} + \frac{\partial \pi(x,\theta)}{\partial x} \leq 0$$

or, at an interior optimum,

$$\frac{\partial v(x,\eta)}{\partial x} + \frac{\partial \pi(x,\theta)}{\partial x} = 0$$

Quotas under Incomplete Information



Quotas under Incomplete Information

- Suppose that the regulator sets a quota $\hat{x}$ on the externality.
- The firm's PMP becomes

$$\max_{x \geq 0} \ \pi(x, \theta) \quad \text{s.t. } x \leq \hat{x}$$

- Let $x^q(\hat{x}, \theta)$ be the externality level that solves this PMP.
  - Since the PMP does not depend on η, $x^q(\hat{x}, \theta)$ is completely insensitive to η.
  - Thus, $x^q(\hat{x}, \theta)$ cannot coincide with $x^0(\theta, \eta)$ for every η.
  - The efficient level of externality is $x^0(\theta, \eta)$.

Quotas under Incomplete Information

- The regulator sets $\hat{x} = x^0(\overline{\theta}, \overline{\eta})$, where $\overline{\theta}$ and $\overline{\eta}$ are the expected values of θ and η.
- Assume the quota binds for every firm type, i.e., $\frac{\partial \pi(\hat{x},\theta)}{\partial x} > 0$ for all θ.
- Thus, the profit-maximizing level of the externality is $x^q(\hat{x}, \theta) = \hat{x}$.
- That is, the firm would like to increase the externality x beyond $\hat{x}$, but it cannot since it has already reached the quota.

Quotas under Incomplete Information

- The welfare change from quota $\hat{x}$ relative to the socially optimal level of externality $x^0(\theta,\eta)$ is

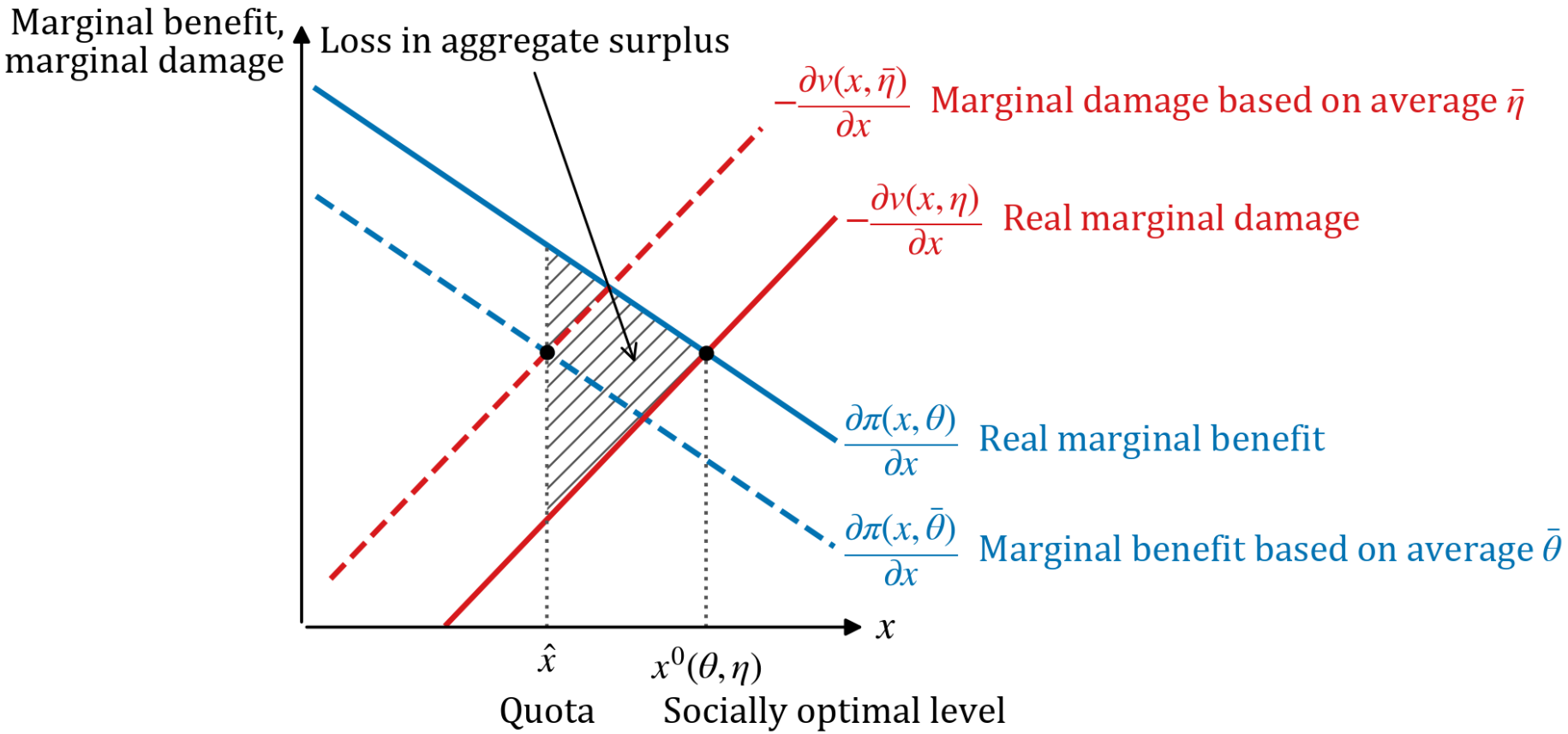
$$\overbrace{\left[v(x^q(\hat{x},\theta),\eta)+\pi(x^q(\hat{x},\theta),\theta)\right]}^{\text{Aggregate surplus with the quota } \hat{x}} - \underbrace{\left[v(x^0(\theta,\eta),\eta)+\pi(x^0(\theta,\eta),\theta)\right]}_{\text{Aggregate surplus at the PO}}$$

or, more compactly,

$$\int_{x^0(\theta,\eta)}^{x^q(\hat{x},\theta)} \left(\frac{\partial \pi(x,\theta)}{\partial x}+\frac{\partial v(x,\eta)}{\partial x}\right) dx$$

- This change is negative or zero, since $x^0(\theta,\eta)$ maximizes aggregate surplus.

Quotas under Incomplete Information



Taxes under Incomplete Information

Taxes under Incomplete Information

- Suppose that the regulator imposes a tax t per unit of the externality.
- The firm's PMP becomes

$$\max_{x \geq 0} \ \pi(x, \theta) - tx$$

- FOC yields $\frac{\partial \pi(x,\theta)}{\partial x} \leq t$ or, in an interior solution, $\frac{\partial \pi(x,\theta)}{\partial x} = t$.
- Let $x^t(t, \theta)$ denote the amount of the externality that solves the FOC (interior solution).
  - $x^t(t, \theta)$ is completely insensitive to η.
  - Thus, $x^t(t, \theta)$ cannot coincide with $x^0(\theta, \eta)$ for every η.

Taxes under Incomplete Information

- The welfare change caused by a tax t relative to the socially optimal level of externality $x^0(\theta, \eta)$ is

$$\overbrace{[v(x^t(t,\theta),\eta) + \pi(x^t(t,\theta),\theta)]}^{\text{Aggregate surplus with tax } t} - \underbrace{[v(x^0(\theta,\eta),\eta) + \pi(x^0(\theta,\eta),\theta)]}_{\text{Aggregate surplus at the PO}}$$

or, more compactly,

$$\int_{x^0(\theta,\eta)}^{x^t(t,\theta)} \left(\frac{\partial \pi(x,\theta)}{\partial x} + \frac{\partial v(x,\eta)}{\partial x} \right) dx$$

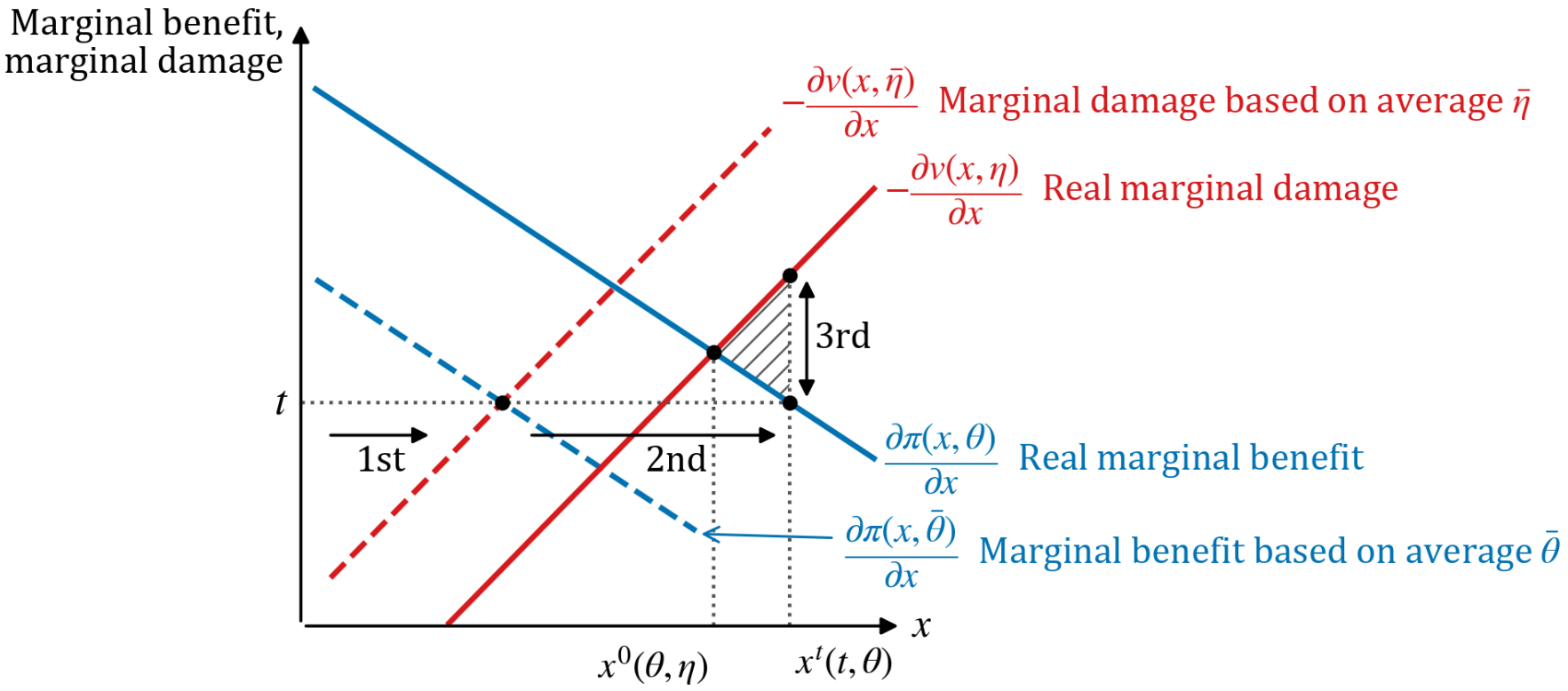
- This change is negative or zero, since $x^0(\theta, \eta)$ maximizes aggregate surplus.

Taxes under Incomplete Information

- The regulator sets the tax at the point that maximizes aggregate surplus, evaluated at the average values of θ and η, $(\overline{\theta}, \overline{\eta})$, that is

$$t = -\frac{\partial v(x^0(\overline{\theta}, \overline{\eta}), \overline{\eta})}{\partial x}$$

Taxes under Incomplete Information



1st: regulator sets t; 2nd: the firm responds along its true marginal benefit; 3rd: welfare loss

Comparing Policy Instruments under Incomplete Information

Policy Comparison

- Both quotas and emission fees create inefficiencies under incomplete information.
- Which instrument, despite being imperfect, performs better?
  - "second-best" policy
- It depends on the relative slopes of the marginal damage and marginal benefit functions.

Policy Comparison

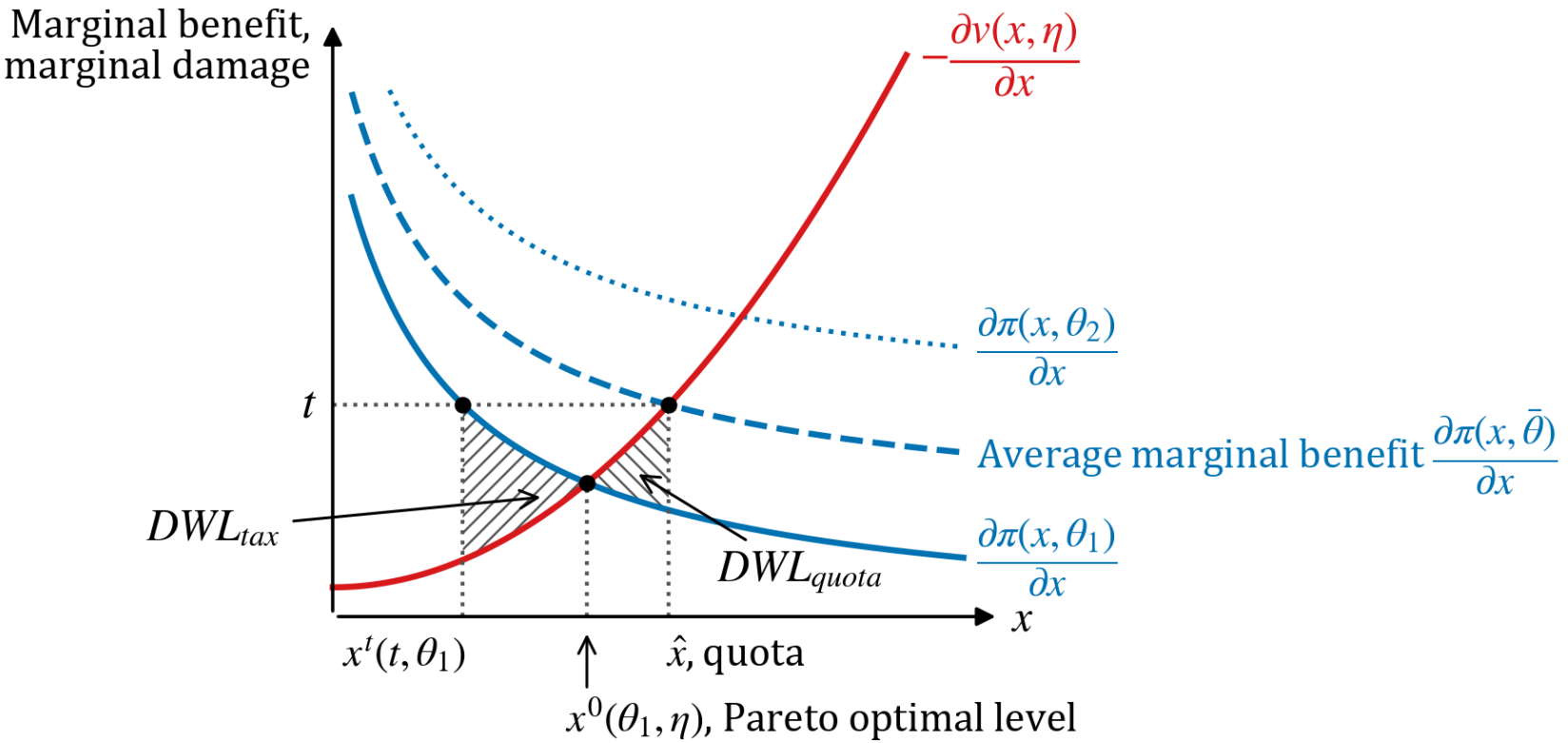
- Consider a setting where:
  - the realization of parameter θ is $\theta = \theta_1 < \overline{\theta}$ (true marginal benefit below its average)
  - the regulator has relatively precise information about the marginal damage function, but he is uncertain about the firm's marginal benefit function.

Policy Comparison

- The regulator sets:
  - a quota $\hat{x}$ at the point where the observed marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, crosses the average marginal benefit function, i.e., $\frac{\partial \pi(x,\overline{\theta})}{\partial x}$.
  - an emission fee t at the height at which the observed marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, crosses the average marginal benefit function, i.e., $\frac{\partial \pi(x,\overline{\theta})}{\partial x}$.

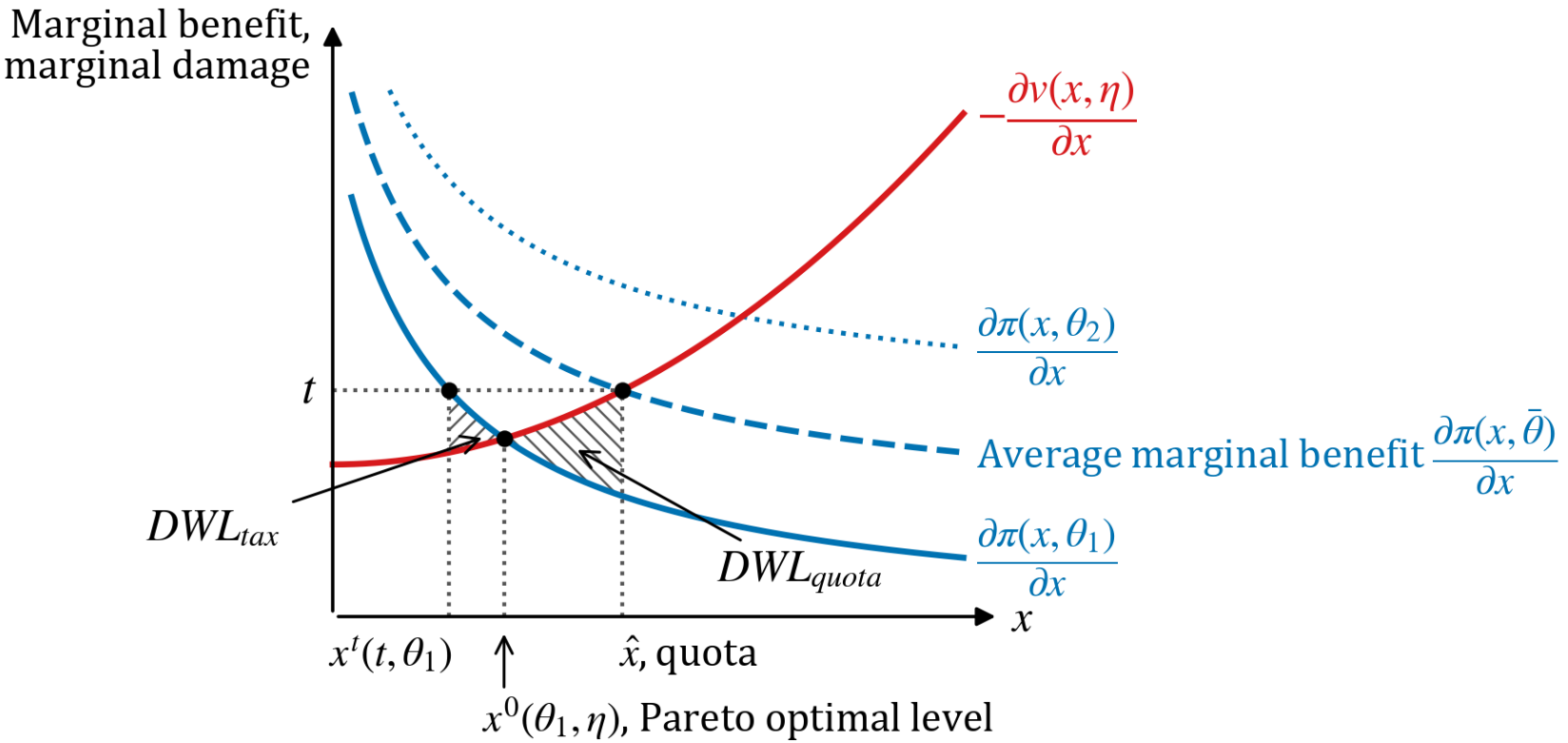
Policy Comparison

- The marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, is relatively sensitive to x.



Policy Comparison

- The marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, is not very sensitive to x.



Policy Comparison

- A quota performs better than an emission fee when the marginal damage curve is steeper (in absolute value) than the marginal benefit curve.
- The emission fee performs better when the marginal damage curve is flatter.

Tragedy of the Commons

Tragedy of the Commons

- "Tragedy of the Commons" considers a common pool resource (CPR, e.g., a fishing ground or an aquifer) exploited by $N \geq 2$ firms, and shows that firms' exploitation is socially excessive.
  - That is, the sum of the individual exploitation levels that each firm independently and simultaneously chooses exceeds the aggregate exploitation level that a social planner would select.

Tragedy of the Commons

- Consider a setting in which each firm exploiting the CPR chooses an individual appropriation level $x_i \geq 0$.
- In particular, every firm i's profit function is

$$\pi_i(x_i, X_{-i}) = p(X)x_i - c(x_i, X_{-i})$$

where $p(X)$ denotes the inverse demand function, with $p'(X) < 0$ and $p''(X) \leq 0$, and $X \equiv \sum_{i=1}^{N} x_i$ represents aggregate appropriation.

Tragedy of the Commons

- $c(x_i, X_{-i})$ represents firm i's cost of appropriating x_i units of the resource when its rivals extract $X_{-i} \equiv \sum_{j \neq i} x_j$, with

$$\frac{\partial c}{\partial x_i} > 0, \qquad \frac{\partial^2 c}{\partial {x_i}^2} \geq 0$$

- $\frac{\partial c}{\partial X_{-i}} > 0$ (scarcity: more appropriation by others raises i's cost), and
- $\frac{\partial^2 c}{\partial x_i \partial X_{-i}} > 0$, implying that the CPR becomes more scarce and more difficult to exploit by firm i.

Tragedy of the Commons

- In equilibrium, every firm i simultaneously and independently chooses its appropriation level x_i to maximize its profit $\pi_i(x_i, X_{-i})$.
- Taking FOC wrt x_i, and assuming an interior solution, we obtain

$$p(X^*) + p'(X^*)x_i^* = \frac{\partial c(x_i^*, X_{-i}^*)}{\partial x_i}$$

where $X^* = x_i^* + X_{-i}^*$.

Tragedy of the Commons

- Intuitively, the marginal revenue and marginal cost from individual appropriation x_i must coincide in equilibrium.
- Solving for x_i in the FOC above, we find firm i's BRF, $x_i^*(X_{-i}^*)$, describing firm i's appropriation as a function of its rivals' aggregate appropriation X_{-i}.

Tragedy of the Commons

- Consider a commons with:
  - linear inverse demand $p(X) = a - bX$, where $a, b > 0$; and
  - cost function $c(x_i, X_{-i}) = cx_i(1 + \alpha X_{-i})$, where $c > 0$, $a > c$, and $\alpha \geq 0$ represents how other firms' appropriation, X_{-i}, affects firm i's costs.
- The FOC for equilibrium appropriation becomes

$$a - b(x_i + X_{-i}) - bx_i = c(1 + \alpha X_{-i})$$

Tragedy of the Commons

- Solving for firm i's appropriation x_i yields a BRF of

$$x_i^*(X_{-i}) = \frac{a-c}{2b} - \frac{b+c\alpha}{2b} X_{-i}$$

which is decreasing in other firms' appropriation X_{-i}, thus reflecting that the appropriation levels of firms i and j are strategic substitutes; that is, as firm j increases its appropriation, firm i responds by decreasing its own.

Tragedy of the Commons

- In a symmetric equilibrium where $x_i^* = x_j^*$ for every pair of firms $i \neq j$, the preceding BRF becomes

$$x_i^* = \frac{a-c}{2b} - \frac{b+c\alpha}{2b}(N-1)x_i^*$$

since $X_{-i} = (N-1)x_i^*$. Solving for x_i^*,

$$x_i^* = \frac{a-c}{(N+1)b + c\alpha(N-1)}$$

which is decreasing in the number of firms N and in the external effect α.

Tragedy of the Commons

- For instance, when $N = 2$, this equilibrium appropriation becomes $x_i^* = \frac{a-c}{3b+c\alpha}$.
- When demand is perfectly elastic ($b = 0$), so firms are price takers with $p(X) = a$, and $\alpha > 0$, the FOC becomes $a = c(1 + \alpha X_{-i})$, so $X_{-i} = \frac{a-c}{c\alpha}$, and symmetry gives

$$x_i^* = \frac{a-c}{c\alpha(N-1)}$$

Tragedy of the Commons

- To more generally identify the conditions where the BRF of firm i, $x_i^*(X_{-i})$, decreases in X_{-i}, we differentiate its FOC wrt x_j,

$$\underbrace{\frac{\partial p(X^*)}{\partial x_j}}_{-} + \underbrace{\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j}}_{+} \cdot x_i - \underbrace{\frac{\partial^2 c(x_i^*, X_{-i}^*)}{\partial x_i \partial x_j}}_{+}$$

  which is negative if

$$\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j} \cdot x_i < -\frac{\partial p(X^*)}{\partial x_j} + \frac{\partial^2 c(x_i^*, X_{-i}^*)}{\partial x_i \partial x_j}$$

- In the special case of linear demand, $p(X) = a - bX$, $\frac{\partial p(X^*)}{\partial x_i} = -b$, implying that the cross-partial derivative is zero, $\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j} = 0$, such that the inequality above unambiguously holds.

Tragedy of the Commons

- The expression above is the cross-partial $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j}$.

  By the implicit function theorem,

$$\frac{dx_i}{dx_j} = -\frac{\frac{\partial^2 \pi_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \pi_i}{\partial {x_i}^2}}$$

- Since the SOC holds, $\frac{\partial^2 \pi_i}{\partial {x_i}^2} < 0$, the best response decreases if and only if the cross-partial is negative.

Tragedy of the Commons

- We next show that equilibrium appropriation is excessive relative to the joint-profit benchmark.
  - This benchmark ignores consumer surplus; it captures the firms' own loss from overusing the resource.
- If all firms maximize their joint profits, their maximization problem becomes

$$\max_{x_1,\ldots,x_N} p(X)X - \sum_{i=1}^{N} c(x_i, X_{-i})$$

since $\sum_{i=1}^{N} p(X)x_i = p(X)X$.

Tragedy of the Commons

- Taking FOC wrt x_i,

$$p(X^{JP}) + p'(X^{JP})x_i^{JP} + p'(X^{JP})\sum_{j\neq i} x_j^{JP}$$

$$= \frac{\partial c(x_i^{JP}, X_{-i}^{JP})}{\partial x_i} + \sum_{j\neq i} \frac{\partial c(x_j^{JP}, X_{-j}^{JP})}{\partial x_i}$$

for every firm i and $j \neq i$.

- This FOC differs from the equilibrium condition in two ways:

Tragedy of the Commons

- It considers aggregate (rather than individual) marginal revenue on the left-hand side because firms now internalize the effect that selling more units has on the revenues of all other firms (see the third term on the left-hand side) rather than on their own revenues alone.
- It includes the increase in marginal costs that other firms experience as a result of a larger appropriation by firm i (i.e., a negative externality in costs).

Tragedy of the Commons

- Writing down the difference between firm i's FOC and the FOC from the joint maximization problem of all firms,

$$\left[p(X) + p'(X)x_i - \frac{\partial c(x_i, X_{-i})}{\partial x_i}\right]$$
$$-[p(X) + p'(X)x_i + p'(X)\sum_{j \neq i} x_j$$
$$-\frac{\partial c(x_i, X_{-i})}{\partial x_i} - \sum_{j \neq i} \frac{\partial c(x_j, X_{-j})}{\partial x_i}]$$

Tragedy of the Commons

- This difference simplifies to

$$-p'(X)\sum_{j\neq i} x_j + \sum_{j\neq i}\frac{\partial c(x_j, X_{-j})}{\partial x_i}$$

- Since $p'(X) < 0$ (by the law of demand) and $\frac{\partial c(x_j, X_{-j})}{\partial x_i} > 0$ (appropriation by firm i raises its rivals' costs), both terms in this expression are positive.

Tragedy of the Commons

- At any x, firm i's marginal profit then exceeds the joint marginal profit; with concave objectives, the Nash level exceeds the joint-profit level, $x_i^* > x_i^{JP}$ for every firm i.
- Such a result is often referred to as the "tragedy of the commons."

Tragedy of the Commons

- Consider the following two extreme cases:

1. When $\frac{\partial c(x_j, X_{-j})}{\partial x_i} = 0$, firm *i*'s appropriation does not increase its rivals' costs, but demand is negatively sloped, $p'(X) < 0$.
   - Then, the above problem coincides with that for the standard cartel, where firms collude to reduce their production and increase profits.

Tragedy of the Commons

- Consider the following two extreme cases:

2. When $p'(X) = 0$, firms take prices as given, but every firm *i*'s appropriation increases its rivals' costs, that is, $\frac{\partial c(x_j, X_{-j})}{\partial x_i} > 0$.
   - This may occur when many firms are selling the same product in the international market. In this case, the inverse demand function collapses to an exogenous price *p*, and the result $x_i^* \geq x_i^{JP}$ now indicates that the joint-profit solution internalizes the external effect that each firm's appropriation generates on its rivals' costs.

Tragedy of the Commons

- Return to the example above with $N = 2$.
- In this context, the joint profit maximization problem becomes

$$\max_{x_i, x_j} \left[a - b\left(x_i + x_j\right)\right] \cdot \left(x_i + x_j\right) - cx_i\left(1 + \alpha x_j\right) - cx_j(1 + \alpha x_i)$$

Tragedy of the Commons

- Taking FOC with respect to every x_i yields

$$a - 2b(x_i + x_j) - c(1 + 2\alpha x_j) \leq 0 \text{ for all } i \in \{1,2\} \text{ and } j \neq i$$

- Assuming an interior solution and solving for x_i, we obtain

$$x_i = \frac{a - c}{2b} - \frac{2(b + c\alpha)}{2b} x_j$$

Tragedy of the Commons

- By symmetry, we can simultaneously solve for x_i and x_j to obtain the joint-profit appropriation levels

$$x_i^{JP} = x_j^{JP} = \frac{a-c}{2(2b+c\alpha)}$$

which are lower than the equilibrium appropriation x_i^*, since

$$\frac{a-c}{2(2b+c\alpha)} < \frac{a-c}{3b+c\alpha}$$

simplifies to $-c\alpha < b$, which holds for $c, b > 0$ and $\alpha \geq 0$ by assumption.

Tragedy of the Commons

- For example, consider $a = b = 1$, $c = 1/2$, and $\alpha = 1/4$.
  - In this setting, $x_i^* = 0.16 > 0.1176 \approx x_i^{JP}$.

Tragedy of the Commons

- In both extreme cases described above, $x_i^{JP} < x_i^*$ holds:
  1. When $\alpha = 0$ but $b > 0$, since
  $$x_i^{JP} = \frac{a-c}{4b} < \frac{a-c}{3b} = x_i^*$$
  2. When $\alpha > 0$ but $b = 0$, which yields
     - $p = a$, and
     - $x_i^{JP} = \frac{a-c}{2c\alpha} < \frac{a-c}{c\alpha} = x_i^*$

Tragedy of the Commons

- Consider a setting with $n \geq 2$ symmetric firms, facing a market demand function $p(Q) = 1 - Q$, where Q is aggregate output.
- Each firm j has a convex cost function, $c(q_j) = \theta q_j^2$, where $\theta \geq 1$ is a cost parameter, unrelated to the firm type above.
- Here the externality is environmental damage (below), not a cost externality across firms.

Tragedy of the Commons

- The PMP of every firm j is

$$\max_{q_j \geq 0} (1 - q_j - q_{-j}) q_j - \theta q_j^2$$

where $q_{-j} = \sum_{k \neq j} q_k$ represents the aggregate output of all firms but j.

Tragedy of the Commons

- Taking FOC with respect to q_j :

$$1 - 2(1 + \theta)q_j - q_{-j} = 0$$

- Solving for q_j yields firm j's BRF

$$q_j(q_{-j}) = \frac{1}{2(1+\theta)} - \frac{1}{2(1+\theta)} q_{-j}$$

- Note:
  - If firm j were alone in the commons, $q_{-j} = 0$, it would produce

$$q_j(0) = \frac{1}{2(1+\theta)}$$

  - Firm j's output decreases as the aggregate output of other firms, q_{-j}, increases.

Tragedy of the Commons

- Since all firms are symmetric, in equilibrium

$$q_j = q_i = q^*$$

- Hence, the BRF of firm j can be written as

$$q^* = \frac{1}{2(1+\theta)} - \frac{n-1}{2(1+\theta)} q^*$$

which, solving for q^*, entails an equilibrium output of

$$q^* = \frac{1}{n+1+2\theta}$$

with equilibrium profits of

$$\pi^* = [1 - q^* - (n-1)q^*]q^* - \theta(q^*)^2 = \frac{1+\theta}{(n+1+2\theta)^2}$$

Tragedy of the Commons

- Aggregate output is

$$Q^* = nq^* = \frac{n}{n + 1 + 2\theta}$$

- Aggregate profits are

$$\Pi^* = n\pi^* = \frac{n(1 + \theta)}{(n + 1 + 2\theta)^2}$$

Tragedy of the Commons

- These profits reach their maximum when

$$\frac{\partial \Pi^*}{\partial n} = \frac{(1+\theta)(1+2\theta-n)}{(n+1+2\theta)^3} = 0$$

where solving for n yields $n^* = 1 + 2\theta$.

- Π^* increases in n for $n < 1 + 2\theta$ and decreases for $n > 1 + 2\theta$, so n^* maximizes industry profits.

Tragedy of the Commons

- Since the inverse demand function is linear, consumer surplus is

$$CS^* = \frac{(Q^*)^2}{2} = \frac{n^2}{2(n+1+2\theta)^2}$$

Tragedy of the Commons

- The exploitation of the commons entails a negative environmental externality (e.g., reduced biodiversity).
  - We consider a convex environmental damage function

$$ED = dQ^2,\ d > 0$$

  - Thus, the equilibrium aggregate environmental damage is

$$ED^* = d(Q^*)^2 = \frac{dn^2}{(n+1+2\theta)^2}$$

Tragedy of the Commons

- The resulting social welfare is

$$SW^* = CS^* + \Pi^* - ED^*$$

$$= \frac{n^2}{2(n+1+2\theta)^2} + \frac{n(1+\theta)}{(n+1+2\theta)^2} - \frac{dn^2}{(n+1+2\theta)^2}$$

- Consumer surplus increases in the number of firms n, i.e.,

$$\frac{\partial CS^*}{\partial n} = \frac{n(1+2\theta)}{(n+1+2\theta)^3} > 0$$

Tragedy of the Commons

- The derivative of $\Pi^* - ED^*$ with respect to n is

$$\frac{\partial[\Pi^* - ED^*]}{\partial n} = -\frac{(1+\theta)(n-1-2\theta)+2dn(1+2\theta)}{(n+1+2\theta)^3}$$

which is negative if $(1+\theta)(n-1-2\theta)+2dn(1+2\theta) > 0$, for example for all $n \geq 1+2\theta$.

Tragedy of the Commons

- *Intuition*: increasing the number of firms has two opposing effects on welfare:
  1) *Positive effect*: it increases consumer surplus (as a larger output entails lower prices);
  2) *Negative effects*: it reduces each firm's profit and, for $n > 1 + 2\theta$, industry profits AND generates more environmental damage.

Tragedy of the Commons

- The positive effect offsets the negative effects when $\frac{\partial SW}{\partial n} = 0$, which occurs when

$$\frac{n(1+2\theta)}{(n+1+2\theta)^3} = \frac{(1+\theta)(n-1-2\theta)+2dn(1+2\theta)}{(n+1+2\theta)^3}$$

Tragedy of the Commons

- Solving for n,

$$\overline{n} = \frac{(1+\theta)(1+2\theta)}{2d(1+2\theta)-\theta}$$

where $\overline{n} > 0$ requires $2d(1+2\theta) > \theta$.

- Comparing, $\overline{n} < n^*$, and solving for d, yields $d > \frac{1}{2}$.

Tragedy of the Commons

- *Intuition:*
  - If the environmental damage from exploitation is sufficiently large, i.e., $d > \frac{1}{2}$, the welfare-maximizing number of firms $\overline{n}$ is below the number that maximizes industry profits, n^*, and firm concentration would be welfare-improving.
  - The opposite applies when $d < \frac{1}{2}$.

Pollution Abatement

Pollution Abatement

- Let us analyze emission fees that induce firms to reduce their emissions in the least costly way.
  - *Examples*: using end-of-pipe technologies, redesigning their production process, or just reducing their output.
- We examine settings where environmental damages are:
  a) uniformly distributed
  b) non-uniformly distributed

Pollution Abatement: Uniform Pollutants

- Consider a regulator seeking to limit total pollution to a maximum level $\overline{x}$, so

$$\overline{x} \geq \sum_j x_j$$

- Firm j's production function is $y_j = f_j(z_j)$, where z_j is the vector of inputs, with prices w.

Pollution Abatement: Uniform Pollutants

- Firm j invests in abatement technology to limit its emissions to a given level x_j. The abatement function is

$$a_j(y_j, V_j) = x_j$$

where V_j denotes the use of abatement inputs, each with a price p_V.

Pollution Abatement: Uniform Pollutants

- Firm j can reach a target emission level x_j by
  - producing few units of output and using few abatement inputs (i.e., low y_j and V_j), or
  - producing a large amount of output but using large amounts of abatement inputs (i.e., high y_j and V_j).

- The planner below fixes each firm's output at $\overline{y}_j$.

  Fixing output rules out the output-reduction channel; we focus on abatement.

Pollution Abatement: Uniform Pollutants

- If a social planner had the ability to choose the use of production and abatement inputs across firms, i.e., $(z_1, \ldots, z_N)$ and $(V_1, \ldots, V_N)$, he would solve

$$\min_{z_j, V_j} \sum_j (w z_j + p_V V_j)$$

$$\text{s.t. } f_j(z_j) = \overline{y}_j,$$

$$\sum_j a_j(\overline{y}_j, V_j) \leq \overline{x}, \text{and}$$

$$V_j \geq 0 \text{ for every firm } j.$$

Pollution Abatement: Uniform Pollutants

- The Lagrangian of this constrained minimization problem is

$$\mathcal{L} = \sum_j \left(wz_j + p_V V_j\right) + \sum_j \lambda_j \left[\overline{y}_j - f_j(z_j)\right]$$
$$+\mu\left[\sum_j a_j(\overline{y}_j, V_j) - \overline{x}\right]$$

- FOC with respect to z_j yields

$$w = \lambda_j \frac{\partial f_j(z_j)}{\partial z_j} \text{ for every firm } j$$

Pollution Abatement: Uniform Pollutants

- FOC with respect to V_j yields

$$p_V = -\mu \frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j} \text{ for every firm } j$$

Pollution Abatement: Uniform Pollutants

- That is,
  - production input z_j should be increased until the cost of an additional unit, w, coincides with its marginal benefit in terms of additional production.
  - abatement input V_j should be increased until its cost, p_V, coincides with its marginal benefit, $-\mu \frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}$.

Pollution Abatement: Uniform Pollutants

- For a cost-minimizing firm to voluntarily select V_j at the socially optimal level, we need to set a uniform emission fee on pollution, $t^* = \mu$, for every firm j.

Pollution Abatement: Uniform Pollutants

- Consider the cost minimization problem of firm j

$$\min_{z_j, V_j} wz_j + p_V V_j + t_j x_j$$

$$\text{s.t. } f_j(z_j) = \overline{y}_j \text{ and } a_j(\overline{y}_j, V_j) = x_j$$

which can be rewritten as

$$\min_{z_j, V_j} wz_j + p_V V_j + t_j a_j(\overline{y}_j, V_j)$$

$$\text{s.t. } f_j(z_j) = \overline{y}_j$$

Pollution Abatement: Uniform Pollutants

- Hence, the Lagrangian for firm j is

$$\mathcal{L} = wz_j + p_V V_j + t_j a_j(\overline{y}_j, V_j) + \lambda_j \left[\overline{y}_j - f_j(z_j)\right]$$

- FOC with respect to z_j yields

$$w = \lambda_j \frac{\partial f_j(z_j)}{\partial z_j}$$

which coincides with the planner's FOC when the fee equals the multiplier, $t_j = \mu$.

Pollution Abatement: Uniform Pollutants

- FOC with respect to V_j yields

$$p_V = -t_j \frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}$$

which coincides with the FOC we found for the social planner if $t_j = \mu$.

Pollution Abatement: Uniform Pollutants

- Solving for t_j in the latter equation yields

$$t_j = -\frac{p_V}{\frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}}$$

- For optimality, we need $t_j = \mu$. Thus, for every pair of firms j and k, $k \neq j$, we must have

$$-\frac{p_V}{\frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}} = -\frac{p_V}{\frac{\partial a_k(\overline{y}_k, V_k)}{\partial V_k}}$$

Pollution Abatement: Uniform Pollutants

- Rearranging,

$$\frac{\partial a_k(\overline{y}_k, V_k)}{\partial V_k} = \frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}$$

- That is, the marginal emission reduction from one more unit of abatement input is equal across firms at the social optimum.

Pollution Abatement: Non-Uniform Pollutants

- Consider now non-uniform pollution sources
  - *Example*: rivers.
- The amount of pollution measured at a particular measuring station, m_k, depends on:
  a) each firm's emissions x_j, and
  b) how pollution from a firm j transfers to the monitoring station located near firm k, as captured by d_{kj}.
- The measurement at station k, m_k, is given by

$$m_k = \sum_j d_{kj} x_j$$

Pollution Abatement: Non-Uniform Pollutants

- The regulator seeks to limit the measurement in each station k so it does not exceed a cutoff $\overline{m}_k$:

$$m_k = \sum_j d_{kj} x_j \leq \overline{m}_k$$

- The social planner problem is

$$\min_{z_j, V_j} \sum_j (w z_j + p_V V_j)$$

$$\text{s.t. } f_j(z_j) = \overline{y}_j \text{ for every firm } j,$$

$$\sum_j d_{kj} a_j(\overline{y}_j, V_j) \leq \overline{m}_k \text{ for every station } k$$

where $a_j(\overline{y}_j, V_j) = x_j$.

Pollution Abatement: Non-Uniform Pollutants

- The Lagrangian to this program is

$$\mathcal{L} = \sum_j \left(wz_j + p_V V_j\right) + \sum_j \lambda_j \left[\overline{y}_j - f_j(z_j)\right]$$
$$+ \sum_k \mu_k \left[\sum_j d_{kj} a_j(\overline{y}_j, V_j) - \overline{m}_k\right]$$

- FOC with respect to z_j yields

$$w = \lambda_j \frac{\partial f_j(z_j)}{\partial z_j} \text{ for every firm } j$$

Pollution Abatement: Non-Uniform Pollutants

- FOC with respect to V_j yields

$$p_V = -\sum_k \mu_k d_{kj} \frac{\partial a_j(\overline{y}_j, V_j)}{\partial V_j}$$

Pollution Abatement: Non-Uniform Pollutants

- In the case of two firms, the planner's FOC for firm 2 is

$$p_V = -\mu_1 d_{12} \frac{\partial a_2(\overline{y}_2, V_2)}{\partial V_2} - \mu_2 d_{22} \frac{\partial a_2(\overline{y}_2, V_2)}{\partial V_2}$$

- For firm 2's FOC to replicate the planner's, its fee must satisfy

$$-t_2 \frac{\partial a_2(\overline{y}_2, V_2)}{\partial V_2}$$
$$= -\mu_1 d_{12} \frac{\partial a_2(\overline{y}_2, V_2)}{\partial V_2} - \mu_2 d_{22} \frac{\partial a_2(\overline{y}_2, V_2)}{\partial V_2}$$

Pollution Abatement: Non-Uniform Pollutants

- Solving for t_2 yields

$$t_2 = \mu_1 d_{12} + \mu_2 d_{22}$$

- Hence, each firm faces its own fee $t_j = \sum_k \mu_k d_{kj}$, unlike the uniform fee $t^* = \mu$ under uniform pollution.

Pollution Abatement: Non-Uniform Pollutants

- The regulator can set taxes based on the pollution recorded at each monitoring point, m_k, rather than a single uniform emission fee.
- That is, firm j pays $\mu_k d_{kj}$ per unit of emissions for each station k, or $t_j = \sum_k \mu_k d_{kj}$ in total.

Public Goods

Public Goods

- Before defining public goods, let us define two properties:
  - ***Non-excludability***: If the good is provided, no consumer can be excluded from consuming it.
  - ***Non-rivalry***: Consumption of the good by one consumer does not reduce the quantity available to other consumers.

	Rivalrous	Non-rivalrous
Excludable	Private good	Club good
Non-excludable	Common property resource	Public good

Public Goods

- ***Private goods***, e.g., an apple. These goods are rival and excludable in consumption.
- ***Club goods***, e.g., a golf course. These goods are non-rival but excludable in consumption.
- ***Common property resources***, e.g., fishing grounds. These goods are rival but non-excludable in consumption.
- ***Public goods***, e.g., national defense. These goods are non-rival and non-excludable in consumption.

Public Goods

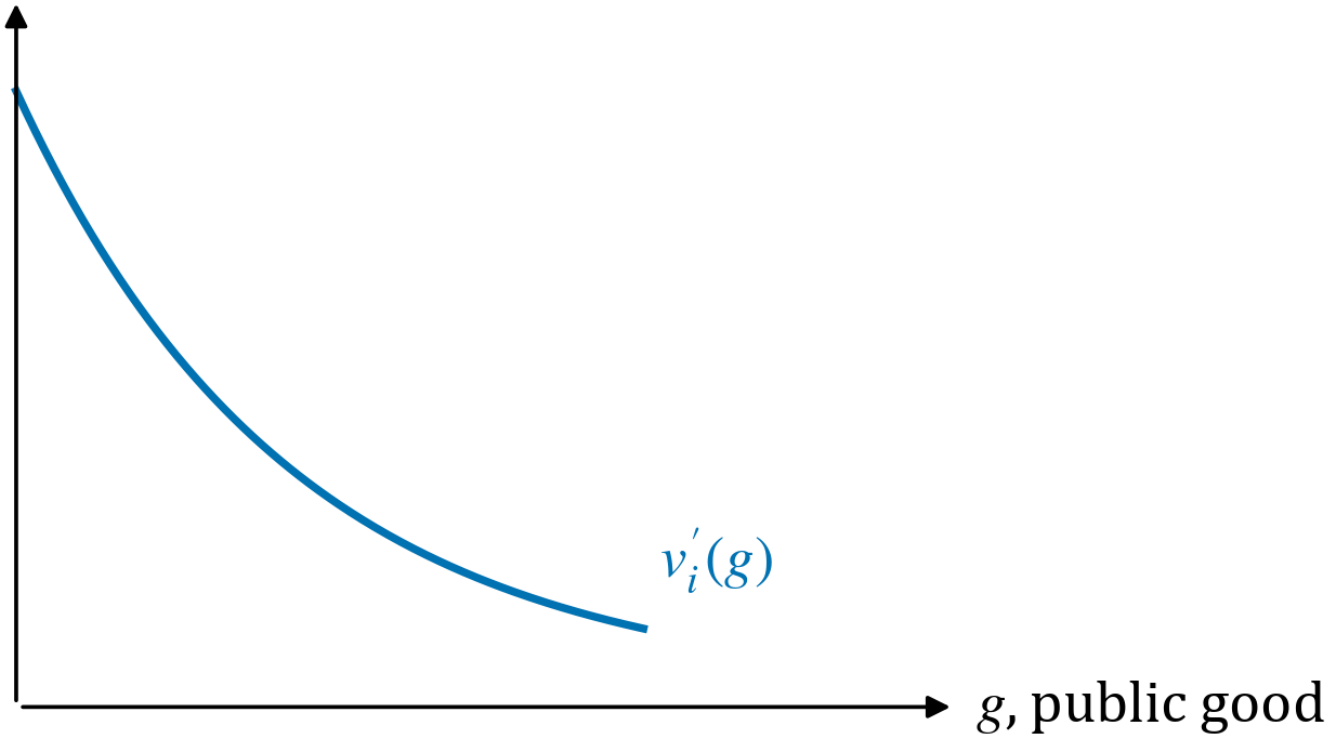
- Consider I consumers, one public good g and L traded private goods.
- Consumer i's utility from g units of the public good is $v_i(g)$, with marginal utility $v_i'(g)$.
  - Note that g does not have a subscript because of non-rivalry (every individual can enjoy g units of the public good).

Public Goods

- We consider the case of a public good, where $v_i'(g) > 0$ for every individual i.
  - A "public bad" would instead have $v_i'(g) < 0$.
- We assume that $v_i''(g) < 0$, which represents a positive but decreasing marginal utility from additional units of the public good.

Public Goods

- Marginal benefit from the public good

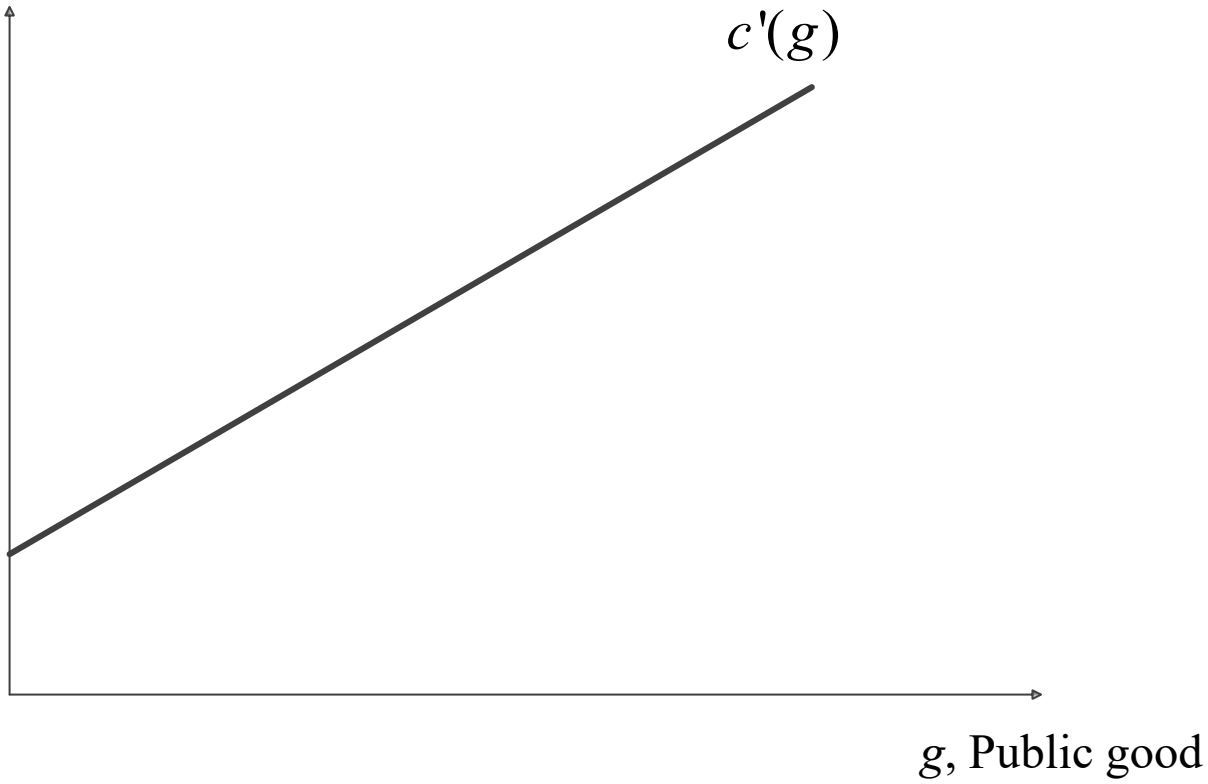


Public Goods

- We assume that the marginal utility from the public good, $v_i'(g)$, is independent of the private good (utility is quasilinear: $v_i(g) + x_i$).
- The cost of supplying g units of the public good is $c(g)$, where $c'(g) > 0$ and $c''(g) > 0$ for all g.
  - That is, the costs of providing the public good are increasing and convex in g.

Public Goods

- Marginal costs from providing the public good



Public Goods

- Let us first find the Pareto optimal allocation

$$\max_{g \geq 0} \sum_{i=1}^{I} v_i(g) - c(g)$$

- FOC with respect to g yields

$$\sum_{i=1}^{I} v_i'(g^0) - c'(g^0) \leq 0$$

with equality if $g^0 > 0$.

Public Goods

- SOCs hold since $v_i''(g) < 0$ for every i and $c''(g) > 0$, so

$$\sum_{i=1}^{I} v_i''(g) - c''(g) < 0 \quad \text{for all} \quad g$$

Public Goods

- In case of an interior solution, the optimal level of public good is achieved for the level of g^0 that solves

$$\sum_{i=1}^{I} v_i'(g^0) = c'(g^0)$$

- That is, the sum of the consumers' marginal benefits from an additional unit of the public good is equal to its marginal cost (***Samuelson rule***).

Public Goods

- The Pareto optimal level of public goods does not coincide with that of private goods.
- If g were a private good, each consumer would buy until

$$v_i'(g_i^*) = c'(g_i^*)$$

so only his own marginal benefit would count.

Public Goods

- ***Example*** (Discrete public good):
  - Consider a public good with $g \in \{0,1\}$, i.e., it is either produced or not.
  - Individual i's valuation of the good is $v_i(g) = \alpha_i g$, where $\alpha_i \geq 0$.
  - Total cost of producing the public good is cg, where $c > 0$.
  - Producing the good is Pareto optimal if and only if

$$\sum_{i=1}^{I} \alpha_i \geq c$$

Public Goods

- ***Example*** (continued):
  - In the discrete setting, the public good is produced if

$$\sum_{i=1}^{I} \alpha_i \geq c$$

  - That is, if the sum of individual valuations weakly exceeds the cost of the good.

Inefficiency of the Private Provision of Public Goods

Inefficiency of the Private Provision of Public Goods

- Let us consider the case in which a market exists for the public good and that each consumer i chooses how much of the public good to buy, denoted as $g_i \geq 0$, taking as given a market price of p.
- The total amount of the public good purchased by all I individuals is $g = \sum_{i=1}^{I} g_i$.
- Consider a single producer of the public good with a cost function $c(g)$.

Inefficiency of the Private Provision of Public Goods

- Formally, at a competitive equilibrium price p^*, each consumer i's purchase of the public good, g_i^*, must solve

$$\max_{g_i \geq 0} \; v_i\left(g_i + \sum\nolimits_{k \neq i} g_k^*\right) + w_i - p^* g_i$$

where w_i denotes i's wealth.

Inefficiency of the Private Provision of Public Goods

- The first term reflects that individual i benefits from both the g_i units of the public good he purchases and the $\sum_{k \neq i} g_k^*$ units that all other individuals acquire.
- Individual i takes others' purchases $\sum_{k \neq i} g_k^*$ as given and pays $p^* g_i$.

Inefficiency of the Private Provision of Public Goods

- FOC with respect to g_i yields
$$v_i'(g_i^* + \sum_{k \neq i} g_k^*) - p^* \leq 0$$
with equality if $g_i^* > 0$ (interior solution).
- For compactness, let g^* denote the total purchases of the public good, that is,
$$g^* = g_i^* + \sum_{k \neq i} g_k^*.$$
- Hence, the above FOC can be expressed as
$$v_i'(g^*) - p^* \leq 0$$
with equality if $g_i^* > 0$ (interior solution)

Inefficiency of the Private Provision of Public Goods

- On the other hand, the firm's PMP is

$$\max_{g \geq 0} \; p^* g - c(g)$$

- FOC with respect to g yields

$$p^* - c'(g^*) \leq 0$$

with equality if $g^* > 0$ (interior solution).

Inefficiency of the Private Provision of Public Goods

- Finally, the market clearing condition implies that the total amount of the public good produced coincides with the amount consumed by all individuals,

$$g^* = \sum_{i=1}^{I} g_i^*$$

Inefficiency of the Private Provision of Public Goods

- Combining the FOCs for consumers and the firm, we obtain

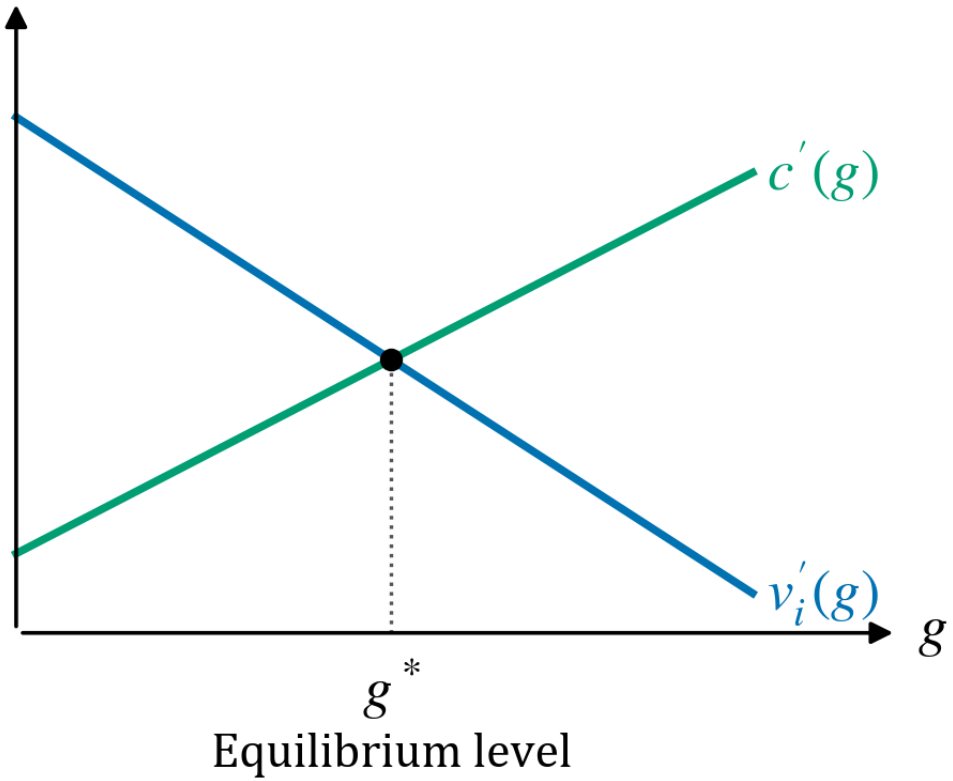
$$v_i'(g^*) \leq c'(g^*)$$

for every i, with equality if $g_i^* > 0$.

- Intuitively, individual i increases his consumption of the public good until his marginal benefit from the public good equals its marginal cost.

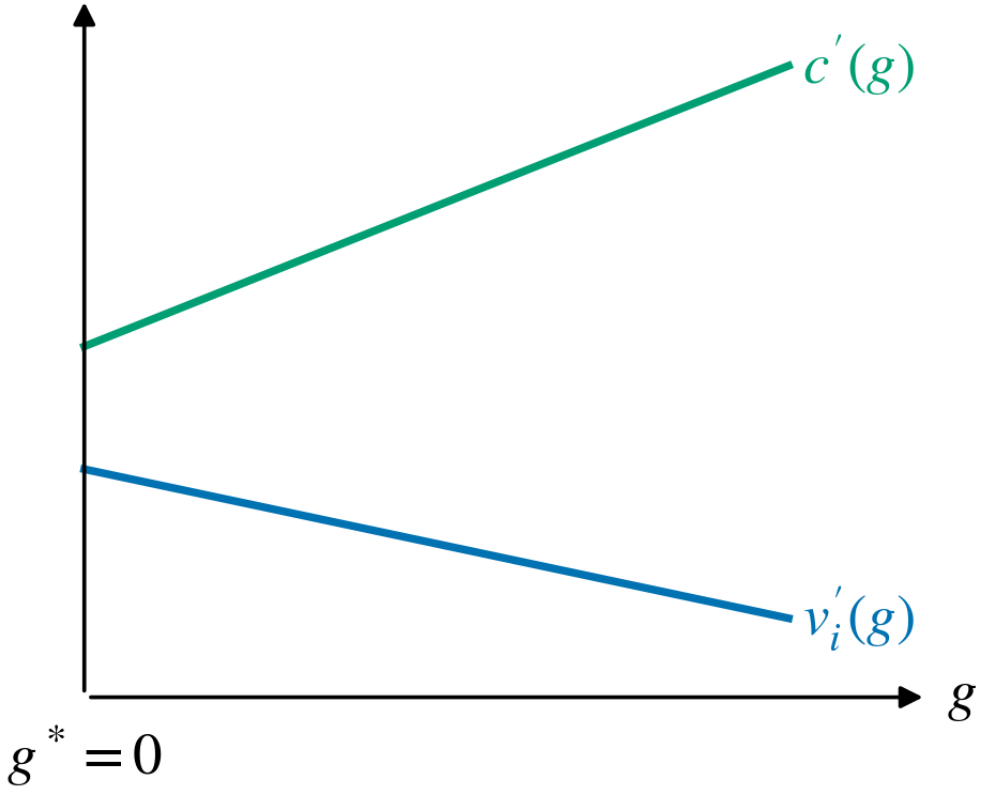
Inefficiency of the Private Provision of Public Goods

- Equilibrium level of public good (interior solution).



Inefficiency of the Private Provision of Public Goods

- Equilibrium level of public good (corner solution).



Inefficiency of the Private Provision of Public Goods

- However, at the Pareto optimum, we must have

$$\sum_{i=1}^{I} v_i'(g^0) = c'(g^0)$$

- That is, the sum of the marginal benefits that all individuals obtain from the public good must equal the marginal cost.

Inefficiency of the Private Provision of Public Goods

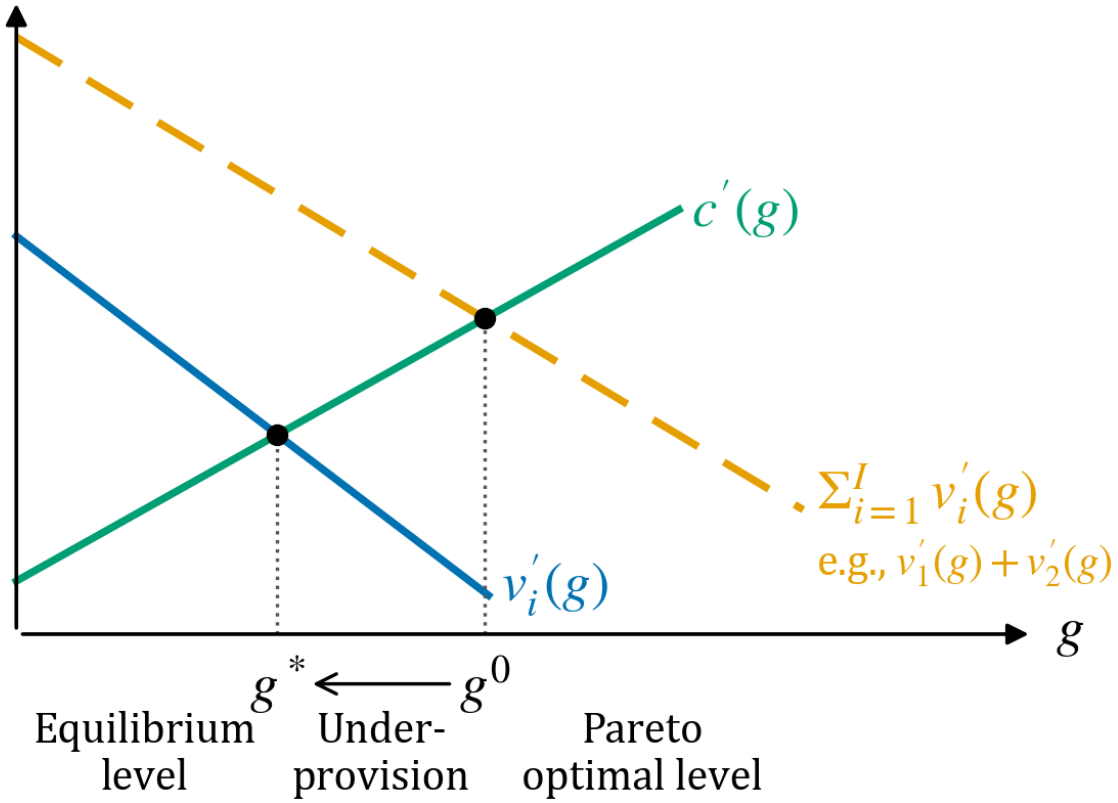
- At g^*, a buyer i has $v_i'(g^*) = c'(g^*)$, and $v_j'(g^*) > 0$ for all $j \neq i$, so

$$\sum_{j=1}^{I} v_j'(g^*) > c'(g^*)$$

- Since $\sum_j v_j' - c'$ is decreasing, $g^0 > g^*$: there is an *underprovision* of the public good.
  - *Exception*: when the marginal cost becomes vertical at a capacity $\overline{g}$ ($c'(g) = +\infty$ for $g > \overline{g}$). In this case, $g^* = g^0$.

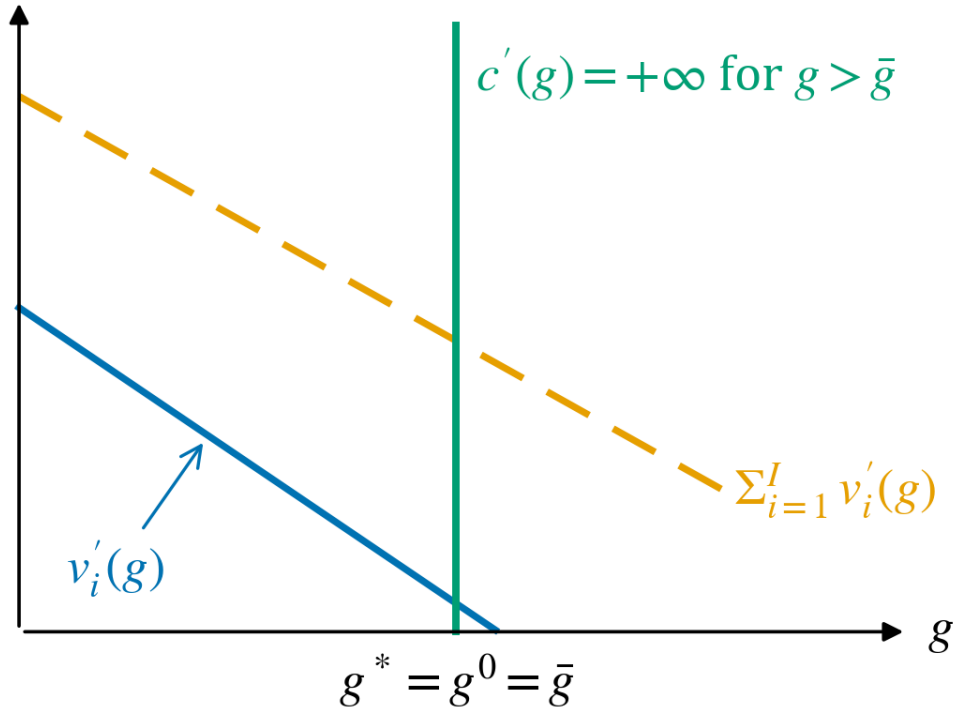
Inefficiency of the Private Provision of Public Goods

- Pareto optimal and equilibrium level of public good



Inefficiency of the Private Provision of Public Goods

- Pareto optimal level coincides with the equilibrium level of public good



Inefficiency of the Private Provision of Public Goods

- *Intuition*:
  - Each individual's purchase of the public good benefits not only him, but also all other individuals in the economy.
  - No individual internalizes the positive externality that his purchase generates on others.
  - Hence, each individual does not have enough incentives to purchase sufficient amounts of the public good.
  - This leads to the *free-rider problem*, whereby the public good is underprovided.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (Private contributions to a public good):
  - Consider an economy with two individuals $i \in \{1,2\}$, with quasilinear utility function
$$u_i(G, y_i) = y_i + m_i \log(G)$$
  - where $m_i > 0$ denotes the value that individual i assigns to total contributions to the public good, $G = g_i + g_j$, and y_i is a composite private good.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Assume that $m_1 > m_2$.
  - For simplicity, the price of both private and public good is 1, so each individual i faces the budget constraint $g_i + y_i = w$, where $w > m_1$.
  - Individual i's UMP is then

$$\max_{g_i \geq 0} \; y_i + m_i \log\left(g_i + g_j\right)$$
$$\text{s.t.} \;\; g_i + y_i = w$$

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Using the budget constraint, $y_i = w - g_i$, and the fact that $G = g_i + g_j$, we can rewrite this UMP as the unconstrained program

$$\max_{g_i \geq 0} \; w - g_i + m_i \log\left(g_i + g_j\right)$$

  - Taking FOC with respect to g_i yields

$$-1 + \frac{m_i}{g_i + g_j} \leq 0$$

with equality if $g_i > 0$.

Inefficiency of the Private Provision of Public Goods

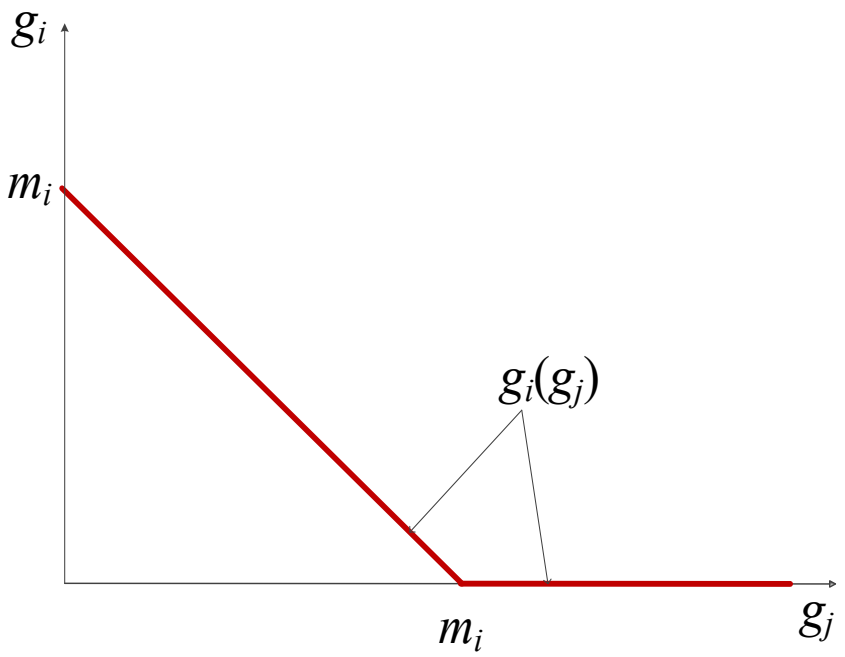
- ***Example*** (continued):
  - Solving for g_i produces BRF $g_i(g_j)$

$$g_i(g_j) = \begin{cases} m_i - g_j & \text{if } g_j < m_i \\ 0 & \text{otherwise} \end{cases}$$

  - If $g_j \geq m_i$, the left-hand side of the FOC is negative at every $g_i > 0$, so $g_i = 0$.

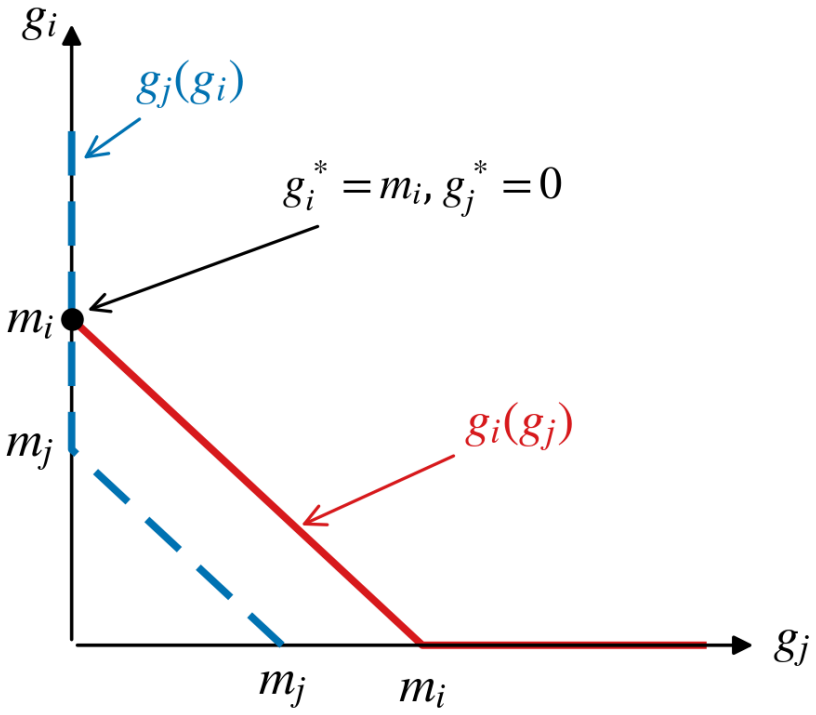
Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Individual i's BRF $g_i(g_j)$.
  - Individual j's BRF $g_j(g_i)$ is analogous.



Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - The equilibrium level of (g_i^*, g_j^*) is obtained by simultaneously solving the two BRFs, $g_i(g_j)$ and $g_j(g_i)$.
  - Hence, $g_1^* = m_1 > 0$ and $g_2^* = 0$, since $m_1 > m_2$.



Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Both individuals cannot contribute: that would require $g_1 + g_2 = m_1$ and $g_1 + g_2 = m_2$, impossible since $m_1 > m_2$.
  - With $g_2^* = 0$, individual 1 contributes $g_1^* = g_1(0) = m_1$; and individual 2's best response is $g_2(m_1) = \max\{m_2 - m_1, 0\} = 0$.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - In contrast, a social planner would maximize total welfare by solving

$$\max_{g_i, g_j} \; w - g_i + m_i \log(g_i + g_j) + w - g_j + m_j \log(g_j + g_i)$$

  - FOC:

$$-1 + \frac{m_i + m_j}{g_i + g_j} = 0$$

Inefficiency of the Private Provision of Public Goods

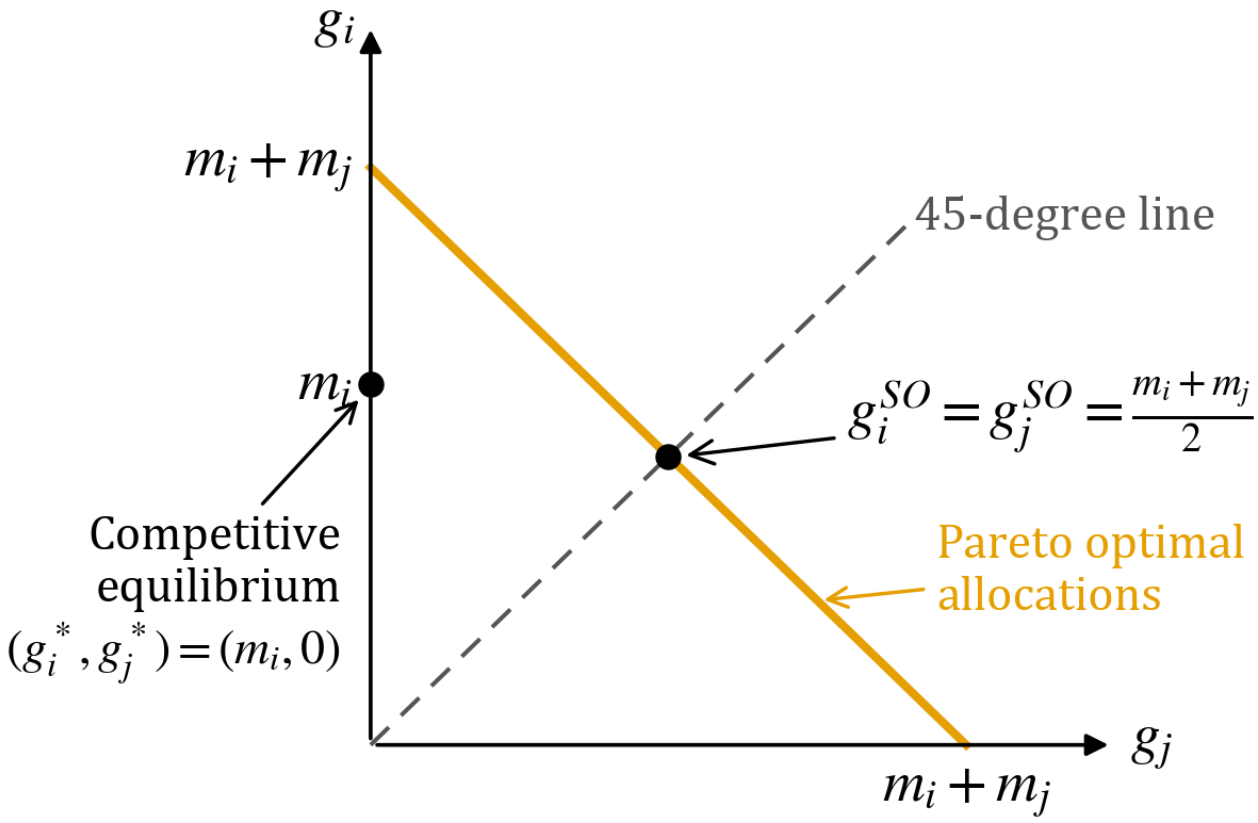
- ***Example*** (continued):
  - Solving for g_i, we obtain a continuum of Pareto optimal allocations

$$g_i^{SO} = m_i + m_j - g_j^{SO}$$

  - Hence $G^{SO} = m_1 + m_2 > m_1 = G^*$: the public good is underprovided.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):



Neutrality and Crowding out

Neutrality and Crowding out

- ***Revenue neutral policies***: an income tax on one individual whose revenue is transferred in full to another individual.
- Consider a Cobb-Douglas utility function

$$u_i(x_i, G) = x_i^{\alpha} G^{1-\alpha}$$

  where
  - x_i denotes the private good
  - G represents total contributions to the public good.
- Assume for simplicity that the price of the private and public good is 1, and that individual i's income is w_i.

Neutrality and Crowding out

- Every individual $i \in \{1,2\}$ solves the utility maximization problem

$$\max_{x_i, g_i} \; x_i^{\alpha}\left(g_i + g_j\right)^{1-\alpha}$$
$$\text{s.t.} \;\; x_i + g_i = w_i$$

- Since $x_i = w_i - g_i$, the above maximization problem can be reduced to an unconstrained program

$$\max_{g_i \geq 0} \; (w_i - g_i)^{\alpha}\left(g_i + g_j\right)^{1-\alpha}$$

Neutrality and Crowding out

- FOC wrt g_i yields

$$-\alpha(w_i - g_i)^{\alpha-1}\left(g_i + g_j\right)^{1-\alpha}$$
$$+(1-\alpha)(w_i - g_i)^{\alpha}\left(g_i + g_j\right)^{-\alpha} \leq 0$$

  with equality if $g_i > 0$.

- In the case of interior solutions, solving for g_i produces individual i's BRF

$$g_i\left(g_j\right) = (1-\alpha)w_i - \alpha g_j$$

Neutrality and Crowding out

- Inserting j's BRF into i's BRF yields

$$g_i = (1-\alpha)w_i - \alpha[(1-\alpha)w_j - \alpha g_i]$$

so that

$$g_i(1-\alpha^2) = (1-\alpha)(w_i - \alpha w_j)$$

- Hence, the equilibrium contribution is

$$g_i^* = \frac{w_i - \alpha w_j}{1+\alpha}$$

where $g_i^* > 0$ if and only if $w_i > \alpha w_j$.

Neutrality and Crowding out

- Equilibrium aggregate donation is

$$G^* = g_i^* + g_j^* = \frac{(1-\alpha)(w_1 + w_2)}{1+\alpha}$$

which increases in w_1 and w_2 but decreases in α because

$$\frac{\partial G^*}{\partial \alpha} = -\frac{2(w_1 + w_2)}{(1+\alpha)^2} < 0$$

Neutrality and Crowding out

- A benevolent planner maximizes $\sum_{i=1}^{2} {x_i}^{\alpha} G^{1-\alpha}$ subject to $x_1 + x_2 + G = w_1 + w_2$.
- For a given G, strict concavity of ${x_i}^{\alpha}$ gives $x_1 = x_2 = \frac{w_1+w_2-G}{2}$, and the FOC with respect to G becomes

$$\frac{\alpha}{w_1 + w_2 - G} = \frac{1-\alpha}{G}$$

so the socially optimal donation is

$$G^{SO} = (1-\alpha)(w_1 + w_2) > G^*$$

Neutrality and Crowding out

- Now, consider transfer $dw_1 > 0$ to individual 1 and a tax $dw_2 < 0$ so that

$$dw_1 + dw_2 = 0 \text{ or } dw_1 = -dw_2$$

- Individual i's equilibrium contribution g_i^* is affected as follows:

$$dg_i^* = \frac{dw_i - \alpha dw_j}{1+\alpha}$$

- Since $dw_i = -dw_j$, the above expression can be re-written as

$$dg_i^* = \frac{dw_i + \alpha dw_i}{1+\alpha} = \frac{(1+\alpha)dw_i}{1+\alpha} = dw_i$$

Neutrality and Crowding out

- Hence, individual i's contribution increases by the transfer he receives (if $dw_i > 0$) or falls by the tax he pays (if $dw_i < 0$).
- In addition, his contribution change dg_i^* is unaffected by the initial income distribution (i.e., w_i and w_j).

Neutrality and Crowding out

- As a consequence, aggregate donations are unaffected by income redistributions, since
$$dG^* = dg_1^* + dg_2^* = dw_1 + dw_2$$
which is zero by definition (i.e., $dw_1 = -dw_2$).
- This holds for redistributions that keep both individuals contributing, i.e., $w_i + dw_i > \alpha(w_j + dw_j)$ for $i, j \in \{1,2\}, i \neq j$ (Warr, 1983; Bergstrom, Blume and Varian, 1986).

Neutrality and Crowding out

- This condition shows the crowding out effect of levying taxes to fund the production of the public good.
- That is, a \$1 tax, i.e., $dw_i = -1 < 0$, reduces individual i's private contribution to the public good by \$1 (as long as i keeps contributing), since $dg_i^* = dw_i = -1$.

Neutrality and Crowding out

- ***Example*** (Increasing the number of contributors):
  - Let us extend the previous setting of two individuals with Cobb-Douglas preferences to a context with N individuals.
  - Assume all donors have the same income

$$w_1 = w_2 = \cdots = w_N = w$$

  - Every individual i chooses g_i to solve

$$\max_{g_i \geq 0} \alpha \log(w - g_i) + (1 - \alpha) \log\left(g_i + \sum_{j \neq i} g_j\right)$$

Neutrality and Crowding out

- ***Example*** (continued):
  - Differentiating with respect to g_i, we obtain

$$-\frac{\alpha}{w-g_i}+\frac{1-\alpha}{g_i+\sum_{j\neq i}g_j}\leq 0$$

with equality if $g_i > 0$. Rearranging the equality yields

$$\alpha g_i + \alpha G_{-i} = (1-\alpha)w - (1-\alpha)g_i$$

where $G_{-i} \equiv \sum_{j\neq i} g_j$ stands for the aggregate donations of all individuals other than i.

Neutrality and Crowding out

- ***Example*** (continued):
  - Solving for g_i, every individual i's best response function becomes

$$g_i(G_{-i}) = \begin{cases} (1-\alpha)w - \alpha G_{-i} & \text{if } G_{-i} < \dfrac{1-\alpha}{\alpha} w \\ 0 & \text{otherwise} \end{cases}$$

Neutrality and Crowding out

- ***Example*** (continued):
  - Invoking symmetry in equilibrium, i.e., $g_1 = g_2 = \cdots = g_N = g$, we have $G_{-i} = (N-1)g$, yielding
$$g = (1-\alpha)w - \alpha(N-1)g$$
  - Solving for g, we obtain an equilibrium donation
$$g^* = \frac{(1-\alpha)w}{1+\alpha(N-1)}$$
  - The aggregate contribution is
$$G^* = Ng^* = \frac{N(1-\alpha)w}{1+\alpha(N-1)}$$

Neutrality and Crowding out

- ***Example*** (continued):
  - The individual contribution is decreasing in α because

$$\frac{\partial g^*}{\partial \alpha} = -\frac{Nw}{[1+\alpha(N-1)]^2} < 0$$

so as the individual's utility is more sensitive to the private good, his contribution to the public good falls.

  - The individual contribution is increasing in w because

$$\frac{\partial g^*}{\partial w} = \frac{1-\alpha}{1+\alpha(N-1)} > 0$$

so as the individual becomes more wealthy, he can afford to make more contributions to the public good.

Neutrality and Crowding out

- ***Example*** (continued):
  - The effect of increasing the number of contributors N on the equilibrium results:

$$\frac{\partial g^*}{\partial N} = -\frac{\alpha(1-\alpha)w}{[1+\alpha(N-1)]^2} < 0$$

$$\frac{\partial G^*}{\partial N} = \frac{(1-\alpha)^2 w}{[1+\alpha(N-1)]^2} > 0$$

  - That is, while individual contributions decrease as a result of more donors potentially contributing to the public good, the overall effect of adding more donors is still positive.

Neutrality and Crowding out

- ***Example*** (continued):
  - In the limit of infinitely many contributors,

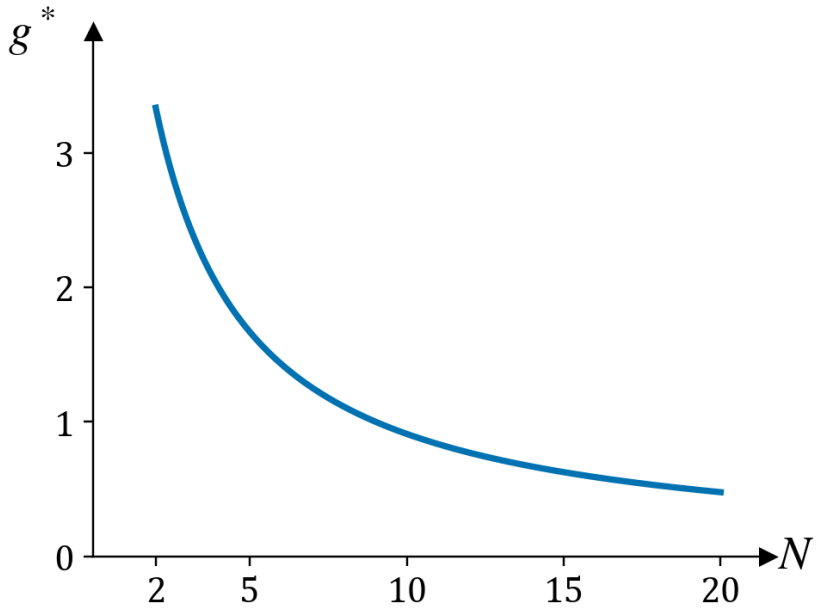
$$\lim_{N\to\infty} g^* = 0$$

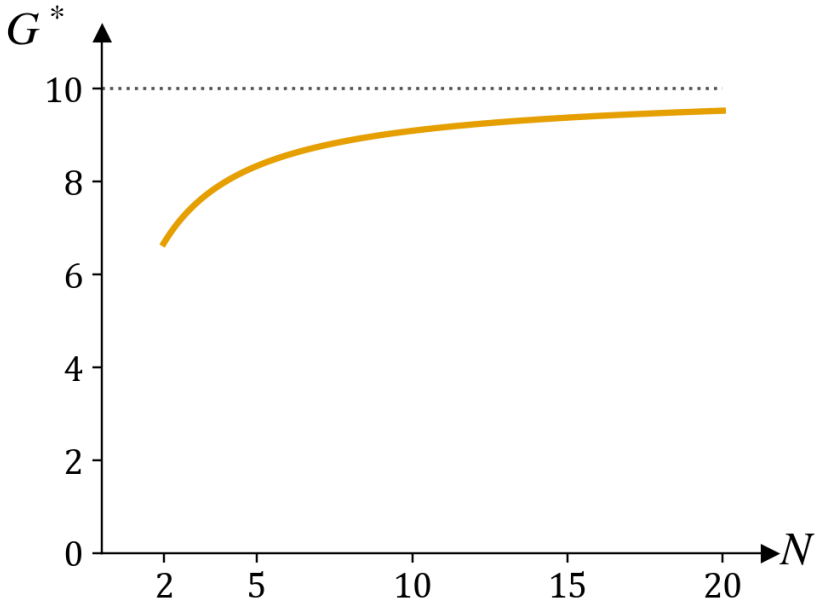
so each individual's contribution converges to zero, while

$$\lim_{N\to\infty} G^* = \frac{(1-\alpha)w}{\alpha}$$

Neutrality and Crowding out

- ***Example*** (continued):
  - We assume $\alpha = \frac{1}{2}$ and $w = 10$. Hence, the above expressions become $g^* = \frac{10}{1+N}$ and $G^* = N\frac{10}{1+N}$.





Neutrality and Crowding out

- In contrast, the social planner chooses $(g_1, \dots, g_N)$ to solve

$$\max_{g_1,\dots,g_N \geq 0} \alpha \log(w - g_i) + \alpha \sum_{j \neq i} \log(w - g_j)$$
$$+ N(1-\alpha) \log\left(g_i + \sum_{j \neq i} g_j\right)$$

- Differentiating with respect to g_i, an interior solution yields

$$-\frac{\alpha}{w - g_i} + \frac{N(1-\alpha)}{g_i + \sum_{j \neq i} g_j} = 0$$

Neutrality and Crowding out

- Rearranging and solving for g_i, we obtain

$$[\alpha + N(1-\alpha)]g_i = N(1-\alpha)w - \alpha \sum_{j \neq i} g_j$$

- Invoking symmetry, $g_i = g_j$ for all $j \neq i$, the socially optimal contribution is

$$g^{so} = \frac{N(1-\alpha)w}{\alpha + N(1-\alpha) + \alpha(N-1)}$$

Since $\alpha + N(1-\alpha) + \alpha(N-1) = N$, we obtain

$$g^{so} = (1-\alpha)w$$

Neutrality and Crowding out

- Every individual's contribution falls short of the socially optimal one,

$$g^{SO} = (1-\alpha)w > \frac{(1-\alpha)w}{1+\alpha(N-1)} = g^{*}$$

since this inequality reduces to $1+\alpha(N-1) > 1$, which holds because $N \geq 2$ and $0 < \alpha < 1$.

Neutrality and Crowding out

- The difference in contribution becomes

$$g^{SO} - g^* = (1-\alpha)w - \frac{(1-\alpha)w}{1+\alpha(N-1)}$$
$$= \frac{\alpha(1-\alpha)(N-1)w}{1+\alpha(N-1)}$$

which is increasing in w and N since

$$\frac{\partial(g^{SO} - g^*)}{\partial w} = \frac{\alpha(1-\alpha)(N-1)}{1+\alpha(N-1)} > 0$$
$$\frac{\partial(g^{SO} - g^*)}{\partial N} = \frac{\alpha(1-\alpha)w}{[1+\alpha(N-1)]^2} > 0$$

Neutrality and Crowding out

- Furthermore, this difference in contribution varies in α as

$$\frac{\partial(g^{SO} - g^{*})}{\partial\alpha} = \frac{(N-1)[1 - 2\alpha - \alpha^2(N-1)]w}{[1 + \alpha(N-1)]^2}$$

which is negative if

$$\alpha > \hat{\alpha} \equiv \frac{\sqrt{N} - 1}{N - 1} = \frac{1}{1 + \sqrt{N}}$$

Neutrality and Crowding out

- This is a condition that becomes less restrictive as N increases, since $\hat{\alpha}$ decreases in N:

$$\frac{\partial \hat{\alpha}}{\partial N} = -\frac{1}{2\sqrt{N}(1+\sqrt{N})^2} < 0$$

Neutrality and Crowding out

- When utility is more sensitive to the consumption of the private good (α increases), the difference between the equilibrium and socially optimal contributions narrows when $\alpha > \hat{\alpha}$.
- In addition,

$$\lim_{N\to\infty} \hat{\alpha} = \lim_{N\to\infty} \frac{1}{1+\sqrt{N}} = 0$$

so, in the limit of infinitely many contributors, $\frac{\partial(g^{SO}-g^{*})}{\partial\alpha} < 0$ holds for all values of α.

Neutrality and Crowding out

- Both the socially optimal contribution g^{SO} and the equilibrium contribution g^* decrease in α, but the former decreases faster when $\alpha > \hat{\alpha}$:

$$\frac{\partial g^{SO}}{\partial \alpha} = -w < \frac{\partial g^*}{\partial \alpha} = -\frac{Nw}{[1+\alpha(N-1)]^2}$$

so their difference shrinks.

Remedies to the Under-Provision of Public Goods

Remedies to the Under-Provision

- ***Quantity-based intervention***: a direct governmental provision of the public good
- ***Price-based intervention***: taxes or subsidies
  - We now denote the public good by x.
  - Assume two consumers with benefit functions $v_1(x_1 + x_2)$ and $v_2(x_1 + x_2)$, respectively, where x_i denotes the amount of the public good purchased by consumer i.

Remedies to the Under-Provision

- As in our analysis of externalities, we can design a subsidy s_i per unit of the public good purchased by every consumer i that induces him to take into account the positive external effect of his purchases of the public good on the other individual's welfare.
- We next show that the subsidy must be $s_i = v'_{-i}(x^0)$.

Remedies to the Under-Provision

- Here, $v'_{-i}(x) \equiv \sum_{j \neq i} v'_j(x)$ is the marginal benefit that all other individuals obtain from x units of the public good.
- Equivalently, the regulator can impose a per-unit tax $t_i = -v'_{-i}(x^0) < 0$ (a negative tax).

Remedies to the Under-Provision

- Every consumer i's UMP becomes that of selecting $\tilde{x}_i$ for a given level of $\tilde{x}_j$

$$\max_{x_i \geq 0} \underbrace{v_i\left(x_i + \tilde{x}_j\right) + \underbrace{s_i x_i}_{\text{subsidy}}}_{\text{benefit from } x_i} - \underbrace{\tilde{p} x_i}_{\text{cost}}$$

- Taking FOC with respect to x_i yields

$$v_i'\left(\tilde{x}_i + \tilde{x}_j\right) + s_i - \tilde{p} \leq 0$$

with equality if $\tilde{x}_i > 0$ (interior solution).

Remedies to the Under-Provision

- Using the market clearing condition $\tilde{x} = \tilde{x}_i + \tilde{x}_j$, and the fact that in a competitive equilibrium the firm's PMP implies $\tilde{p} = c'(\tilde{x})$ (if $\tilde{x} > 0$), the above FOC becomes

$$v_i'(\tilde{x}) + s_i \leq c'(\tilde{x})$$

Remedies to the Under-Provision

- For the equilibrium to reach $\tilde{x} = x^0$, the FOC must coincide with the Samuelson condition

$$v_i'(x^0) + v_j'(x^0) = c'(x^0)$$

which requires

$$s_i = v_j'(x^0)$$

Remedies to the Under-Provision

- Hence, we need a subsidy $s_i = v'_{-i}(x^0)$ which, for the case of only two consumers i and j, implies $s_i = v'_j(x^0)$.
- In the case of N individuals, the subsidy to consumer i would be

$$s_i = v'_j(x^0) + v'_k(x^0) + \cdots = \sum_{j \neq i} v'_j(x^0)$$

- The subsidies are financed with lump-sum taxes to avoid new distortions.

Remedies to the Under-Provision

- The introduction of a subsidy might seem an effective and easy solution to the underprovision problem in public goods.
- However, the regulator might not have access to information about the marginal benefits of the public good for every consumer.

Lindahl Equilibria

Lindahl Equilibria

- Private provision of a public good results in inefficiencies, i.e., $x^* < x^0$.
  - This inefficiency can be corrected by the use of quantity-based or price-based regulation.
- There is, however, a market solution that *in principle* can achieve optimality.
- Consider a market where every individual's consumption of the public good is a different commodity with its own market.
- Denote the price of this personalized good by p_i, which can differ across consumers.

Lindahl Equilibria

- If consumer i faces a price p_i^{**}, his UMP is

$$\max_{x_i \geq 0} \; v_i(x_i) + w_i - p_i^{**} x_i$$

where x_i denotes the amount of the public good consumer i demands at his personalized price p_i^{**}.

Lindahl Equilibria

- FOC wrt x_i yields

$$v_i'(x_i^{**}) - p_i^{**} \leq 0$$

with equality if $x_i^{**} > 0$.

- Hence, at the aggregate level,

$$\sum_{i=1}^{I} v_i'(x_i^{**}) \leq \sum_{i=1}^{I} p_i^{**}$$

Lindahl Equilibria

- On the other hand, the firm produces a bundle of I goods (one for each consumer), with PMP

$$\max_{x \geq 0} \underbrace{\sum_{i=1}^{I} p_i^{**} x}_{\text{Total revenue}} - c(x)$$

- Each unit of x jointly supplies one unit of every personalized good, so revenue is $(\sum_i p_i^{**})x$.

Lindahl Equilibria

- FOC wrt x yields

$$\sum_{i=1}^{I} p_i^{**} - c'(x^{**}) \leq 0, \qquad \text{or}$$

$$\sum_{i=1}^{I} p_i^{**} \leq c'(x^{**})$$

with equality if $x^{**} > 0$ (interior solution).

Lindahl Equilibria

- Market clearing in every personalized market gives $x_i^{**} = x^{**}$ for all i.
- For $x^{**} > 0$, the firm's FOC and each consumer's FOC hold with equality, so

$$\sum_{i=1}^{I} v_i'(x^{**}) = \sum_{i=1}^{I} p_i^{**} = c'(x^{**})$$

  which is the Samuelson condition; hence, $x^{**} = x^0$.
- This equilibrium is known as the ***Lindahl equilibrium***.

Lindahl Equilibria

- *Why do we obtain efficiency?*
  - First, we define personalized markets for the public good.
  - Second, each consumer, taking the price of his personalized good as given, fully determines his own level of consumption of the public good.
  - Each consumer pays for every unit he enjoys, so the externality is internalized and no one free-rides.

Lindahl Equilibria

- *Are these personalized markets for the public good realistic?*
  - We need excludability between the different personalized public goods, which might only be applicable to very specific public goods
    - e.g., some forms of health care, college education, etc.
  - Even if excludability were possible, personalized markets would be monopsonistic (there is only one buyer on the demand side)
    - Thus, the price-taking assumption is difficult to support.

Lindahl Equilibria

- ***Example*** (Calculating a Lindahl equilibrium):
  - Consider three first-year economics graduate students, Eric (E), Chris (C), and Matt (M) deciding to purchase a microwave (a public good) for their office.
  - The utility function of each student is

$$u_i(x_i, y) = \ln x_i + \alpha_i \ln y$$

Lindahl Equilibria

- ***Example*** (continued):
  - x_i denotes student i's consumption of the private good, $i \in \{E, C, M\}$;
  - y denotes the amount of the public good (spending on the microwave);
  - α_i measures how much student i values the microwave.

Lindahl Equilibria

- ***Example*** (continued):
  - For simplicity, assume that the price of the private good, x_i, and the wealth of each student are 1.
  - The microwave's unit cost is 1, so Lindahl prices must sum to marginal cost: $p_E + p_C + p_M = 1$.

Lindahl Equilibria

- ***Example*** (continued):
  - The UMP for student i is

$$\max_{x_i, y} \ln x_i + \alpha_i \ln y$$
$$\text{s.t.} \quad x_i + p_i y \leq 1$$

where p_i represents the Lindahl price each student pays.

Lindahl Equilibria

- ***Example*** (continued):
  - Since the budget constraint holds with equality, the UMP becomes

$$\max_{y} \ \ln(1 - p_i y) + \alpha_i \ln y$$

  - FOC wrt y yields

$$\frac{p_i}{1 - p_i y} = \frac{\alpha_i}{y}$$

  - Rearranging, we obtain

$$p_i y = \frac{\alpha_i}{1 + \alpha_i}$$

Lindahl Equilibria

- ***Example*** (continued):
  - Summing across all three students yields

$$(p_E + p_C + p_M)y = y = \frac{\alpha_E}{1+\alpha_E} + \frac{\alpha_C}{1+\alpha_C} + \frac{\alpha_M}{1+\alpha_M}$$

  - Substituting this value for y in equation $p_i y = \frac{\alpha_i}{1+\alpha_i}$ and solving for the Lindahl prices produces

$$p_i^{**} = \frac{\frac{\alpha_i}{1+\alpha_i}}{\frac{\alpha_E}{1+\alpha_E} + \frac{\alpha_C}{1+\alpha_C} + \frac{\alpha_M}{1+\alpha_M}}$$

Lindahl Equilibria

- ***Example*** (continued):
  - For instance, if $\alpha_E = 1$ and $\alpha_C = \alpha_M = 0.6$, then $y = 1.25$, with Lindahl prices $p_E^{**} = 0.4$ and $p_C^{**} = p_M^{**} = 0.3$.

Public Goods that Experience Congestion

Public Goods that Experience Congestion

- Consider that consumption by other individuals *reduces* the benefit each user i enjoys from the good.
- Hence, the utility function becomes

$$v_i(x_i, x_{-i}) + w_i$$

where x_i is individual i's use of the good, and $x_{-i} = (x_j)_{j \neq i}$ is the use by all others.

Public Goods that Experience Congestion

- The use by others, x_{-i}, enters $v_i(\cdot)$ negatively, thus capturing the ***congestion effect*** (as in CPR goods, e.g., the tragedy of the commons).
- Utility increases in x_i, but decreases in the use of the good by other individuals, x_{-i}.

Public Goods that Experience Congestion

- Let $x \equiv \sum_{i=1}^{N} x_i$ denote total use. The social planner's maximization problem is

$$\max_{x_1,\ldots,x_N} \sum_{i=1}^{N} [v_i(x_i, x_{-i}) + w_i] - c(x)$$

- Since $\frac{\partial c(x)}{\partial x_i} = c'(x)$, the FOC wrt x_i yields

$$\frac{\partial v_i(x_i, x_{-i})}{\partial x_i} + \sum_{j \neq i} \frac{\partial v_j(x_j, x_{-j})}{\partial x_i} - c'(x) \leq 0$$

for all i, with equality in the case of interior solutions.

Public Goods that Experience Congestion

- Each FOC equates individual i's marginal benefit, minus the marginal congestion cost that his use imposes on all others, to the marginal cost, where this congestion term is

$$\sum_{j \neq i} \frac{\partial v_j(x_j, x_{-j})}{\partial x_i} < 0$$

Public Goods that Experience Congestion

- In equilibrium, each individual ignores this term,

$$\frac{\partial v_i(x_i, x_{-i})}{\partial x_i} = c'(x)$$

so the socially optimal use is lower than the equilibrium use.

Behavioral Motives in Public Good Games

Warm-Glow Benefits

- Individuals obtain a "warm glow" benefit from their donations to the public good (see Andreoni, 1990)
- Individual i's utility function $u_i(x_i, G, g_i)$ is increasing in:
  - his private consumption, x_i;
  - total contributions to the public good, G; and
  - his own donation, g_i (the source of warm glow).

Warm-Glow Benefits

- Warm glow makes crowding-out incomplete: a $1 tax-financed transfer no longer reduces private donations by $1 (compare with the neutrality result above).

Warm-Glow Benefits

- Donor i's UMP is

$$\max_{x_i, g_i, G} \; u_i(x_i, G, g_i)$$
$$\text{s.t.} \quad x_i + g_i = w_i$$
$$g_i + G_{-i} = G$$

  where $G_{-i} = \sum_{j \neq i} g_j$ is total donations from all other individuals.
- Since $g_i = G - G_{-i}$ and $x_i = w_i - g_i = w_i - G + G_{-i}$, the UMP becomes

$$\max_{G_{-i} \leq G \leq w_i + G_{-i}} \; u_i(w_i - G + G_{-i}, G, G - G_{-i})$$

Warm-Glow Benefits

- FOC wrt G yields

$$-\frac{\partial u_i}{\partial x_i}+\frac{\partial u_i}{\partial G}+\frac{\partial u_i}{\partial g_i}=0$$

evaluated at $(w_i - G + G_{-i}, G, G - G_{-i})$, which defines a total donations function

$$G = f_i(w_i + G_{-i}, G_{-i})$$

where $w_i + G_{-i}$ is i's social income, and G_{-i} enters separately only because of warm glow ($g_i = G - G_{-i}$).

Warm-Glow Benefits

- The individual donation function of player i, $g_i = G - G_{-i}$, is

$$g_i = f_i(w_i + G_{-i}, G_{-i}) - G_{-i}$$

  - The first argument of $f_i(\cdot)$ is common to public good games in which donors do not benefit from warm glow.
  - In contrast, the second argument of $f_i(\cdot)$ arises because of the warm glow benefits that donors obtain.

Warm-Glow Benefits

- *Intuition*:
  - the first argument captures *altruistic* motivations in public goods; whereas
  - the second argument measures *egoistic* motivations (because the warm glow benefit is private).

Warm-Glow Benefits

- Let f_{ia} (f_{ie}) denote the partial derivative of $f_i(\cdot)$ wrt the first argument (altruism) and the second argument (egoism), where

$$f_{ia} \in (0,1),\ f_{ie} \geq 0 \text{ and } 0 < f_{ia} + f_{ie} \leq 1$$

Warm-Glow Benefits

- This notation helps us obtain the following "altruism coefficient"

$$\alpha_i = \frac{\frac{\partial f_i}{\partial w_i}}{\frac{\partial f_i}{\partial G_{-i}}} = \frac{f_{ia}}{f_{ia}+f_{ie}}$$

  - $\alpha_i = 1$ for pure altruists, $u_i(x_i, G)$, where $f_{ie} = 0$
  - $\alpha_i = f_{ia}$ for pure egoists, $u_i(x_i, g_i)$, since $f_{ia} + f_{ie} = 1$

Warm-Glow Benefits

- Let us now consider a transfer from individual 2 to individual 1, i.e., $dw_1 > 0$ and $dw_2 < 0$ where $dw_1 = -dw_2$.
- Let us examine how total donations G and individual contributions are affected by the transfer.
- Totally differentiating g_i yields

$$dg_i = f_{ia}(dw_i + dG_{-i}) + f_{ie}dG_{-i} - dG_{-i}$$

- Factoring out dG_{-i} yields

$$dg_i = f_{ia}\, dw_i + (f_{ia} + f_{ie} - 1)dG_{-i}$$

Warm-Glow Benefits

- Since $G_{-i} = G - g_i$ (by definition), substituting $dG_{-i} = dG - dg_i$ in the above expression yields

$$dg_i = f_{ia}dw_i + (f_{ia} + f_{ie} - 1)dG - (f_{ia} + f_{ie} - 1)dg_i$$

- Rearranging,

$$(f_{ia} + f_{ie})dg_i = (f_{ia} + f_{ie} - 1)dG + f_{ia}dw_i$$

- Dividing both sides by $(f_{ia} + f_{ie})$ and using the definition of α_i yields

$$dg_i = \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}dG + \alpha_i dw_i$$

Warm-Glow Benefits

- This captures how the donation of individual i is affected by a change in his wealth dw_i.
- Aggregating across donors

$$\sum_{i=1}^{I} dg_i = \sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}} dG + \sum_{i=1}^{I} \alpha_i \, dw_i$$

- Since $\sum_{i=1}^{I} dg_i = dG$, then the change in aggregate contributions, dG, is

$$dG = \sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}} dG + \sum_{i=1}^{I} \alpha_i \, dw_i$$

Warm-Glow Benefits

- We can now rearrange and solve for dG to obtain

$$dG = \frac{1}{1-\sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}} \sum_{i=1}^{I} \alpha_i \, dw_i$$

- For compactness, let

$$\kappa \equiv \frac{1}{1-\sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}}$$

Warm-Glow Benefits

- Since $f_{ia} + f_{ie} \leq 1$, every term $\frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}} \leq 0$, so $\kappa \in (0, 1]$.
- Thus, the expression for dG can be expressed as

$$dG = \kappa \sum_{i=1}^{I} \alpha_i \, dw_i$$

Warm-Glow Benefits

- Finally, since $dw_1 = -dw_2$ and $dw_j = 0$ for all other individuals $j \notin \{1, 2\}$, the above expression becomes

$$dG = \kappa\left[\alpha_1 dw_1 + \alpha_2(-dw_1)\right] = \kappa(\alpha_1 - \alpha_2)dw_1$$

- Since $\kappa > 0$, the transfer from individual 2 to individual 1 increases total donations if and only if individual 1 is more altruistic ($\alpha_1 > \alpha_2$), leaves them unchanged if $\alpha_1 = \alpha_2$, and reduces them otherwise.

Warm-Glow Benefits

- ***Example*** (Warm glow in public goods):
  - Consider a public good game with two individuals $i \in \{1, 2\}$, each with Cobb-Douglas utility function

$$u_i(x_i, G, g_i) = a \log x_i + b \log G + c_i \log g_i$$

where $c_i > 0$ represents the warm-glow benefit that individual i obtains from his contribution to the public good, with $c_1 > c_2$.

Warm-Glow Benefits

- ***Example*** (continued):
  - Since $x_i = w_i - g_i$ and $G = g_i + g_j$, we can rewrite this utility function as

$$u_i(w_i - g_i, g_i + g_j, g_i)$$
$$= a\log(w_i - g_i) + b\log(g_i + g_j) + c_i \log g_i$$

  - FOC wrt g_i yields

$$-\frac{a}{w_i - g_i} + \frac{b}{g_i + g_j} + \frac{c_i}{g_i} = 0$$

Warm-Glow Benefits

- ***Example*** (continued):
  - For simplicity, consider $w_i = 10$ for both individuals, $a = 1, b = \frac{1}{3}$, and warm-glow parameters $c_1 = \frac{1}{4}$ and $c_2 = \frac{1}{5}$, so player 1 obtains a larger warm-glow benefit than player 2.
  - Solving for g_1 (the positive root of a quadratic equation in g_1) yields player 1's best response function, $g_1(g_2)$. Operating similarly for player 2, we find his best response function $g_2(g_1)$.

Warm-Glow Benefits

- ***Example*** (continued):
  - Simultaneously solving for g_1 and g_2 yields equilibrium contribution levels of

$$g_1^* = 2.996 \quad \text{and} \quad g_2^* = 2.623$$

with aggregate donations $G^* = 5.619$.

Warm-Glow Benefits

- ***Example*** (continued):
  - At the equilibrium above, player 1 is the more altruistic donor in the sense of the previous result ($\alpha_1 \approx 0.42 > \alpha_2 \approx 0.39$): a larger warm-glow parameter makes his donation respond more to his own wealth.
  - Hence, a transfer from player 2 to player 1 should increase total donations.

Warm-Glow Benefits

- ***Example*** (continued):
  - A \$2 transfer from player 2 to player 1 gives

$$w_1 = \$12 \quad \text{and} \quad w_2 = \$8$$

which changes individual donations to

$$g_1^* = 3.857 \quad \text{and} \quad g_2^* = 1.892$$

with aggregate donations $G^* = 5.749 > 5.619$.

Social Preferences

- Consider the public good game where two players simultaneously and independently choose between contributing (C) and not contributing (NC) to the public good.

		Player 2	
		C	NC
Player 1	C	a, a	c, b
	NC	b, c	d, d

- Both players' payoffs satisfy $b > a > d > c$, thus indicating that both players have incentives to free ride.

Social Preferences

- Every player's best response is NC, both when his opponent contributes (since $b > a$), and when he does not (since $d > c$).
- In fact, both players find C to be strictly dominated by NC, ultimately implying that the strategy profile (NC,NC) is the unique Nash equilibrium of the game.

Social Preferences

- Consider that players exhibit Fehr and Schmidt (1999)-type social preferences:

$$U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$$

where

  - x_i and x_j are player i's and j's monetary payoffs, respectively
  - α_i is player i's disutility from *envy*, which occurs when $x_i < x_j$ implying $U_i(x_i, x_j) = x_i - \alpha_i(x_j - x_i)$
  - β_i is player i's disutility from *guilt* which occurs when $x_i > x_j$ implying $U_i(x_i, x_j) = x_i - \beta_i(x_i - x_j)$
- Fehr and Schmidt (1999) assume that players' envy is at least as strong as their guilt, i.e., $\alpha_i \geq \beta_i$ and $0 \leq \beta_i < 1$.

Social Preferences

- Hence, the above payoff matrix can be reformulated as follows

		Player 2: C	Player 2: NC
Player 1	C	a, a	$c - \alpha_1(b-c), b - \beta_2(b-c)$
	NC	$b - \beta_1(b-c), c - \alpha_2(b-c)$	d, d

- Every player's utility level decreases when he is either:
  - the player with the higher payoff (due to guilt), e.g., player 1 under outcome (NC,C), or
  - the player with the lower payoff (due to envy), e.g., player 1 under outcome (C,NC).

Social Preferences

- Let us first analyze player i's BRF:
  - When player j chooses C, player i prefers NC if $a \leq b - \beta_i(b-c)$, or, more compactly, $\beta_i \leq \frac{b-a}{b-c}$, where $\frac{b-a}{b-c} \in (0,1)$ since $a > c$.
  - When player j chooses NC, player i prefers NC if $c - \alpha_i(b-c) \leq d$, equivalently, $\alpha_i \geq \frac{c-d}{b-c}$, which holds since $\frac{c-d}{b-c} < 0 \leq \alpha_i$.

Social Preferences

- Therefore, if $\beta_i < \frac{b-a}{b-c}$, NC is a strictly dominant strategy for player i.
- If, instead, $\beta_i > \frac{b-a}{b-c}$, then player i's best response to C is C, but his best response to NC is still NC.

Social Preferences

- Hence, Nash equilibria of the game in pure strategies are:
  - (NC,NC) is the unique equilibrium if

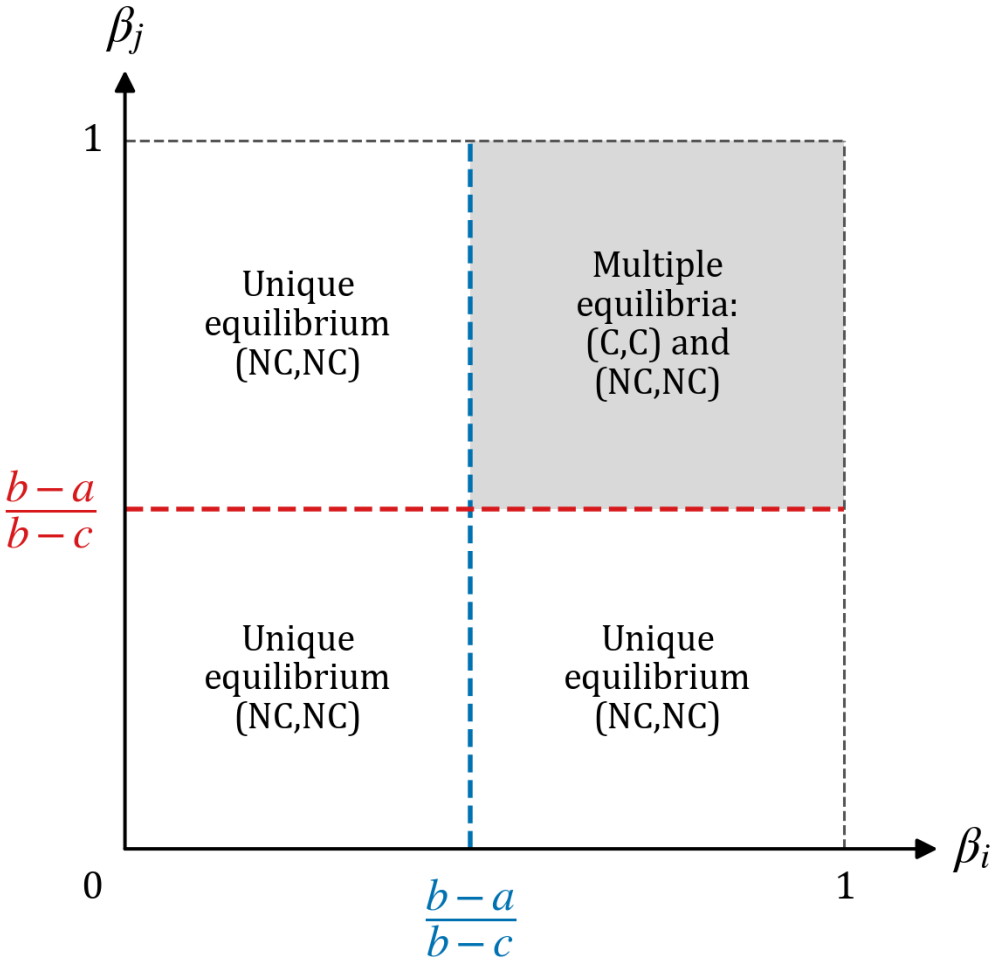
$$\beta_i < \frac{b-a}{b-c} \quad \text{or} \quad \beta_j < \frac{b-a}{b-c}$$

  - (C,C) and (NC,NC) if $\beta_i, \beta_j \geq \frac{b-a}{b-c}$

Social Preferences

- Intuitively,
  - if at least one player has a guilt parameter below $\frac{b-a}{b-c}$, the unique Nash equilibrium of the game coincides with that where players have no concerns about the fairness of the payoff distribution (standard preferences).
  - if both individuals are sufficiently concerned about fairness, we can identify two different Nash equilibria. In this region, each player's best response is to choose the same action as his opponent.

Social Preferences



Competition for Status Acquisition

Competition for Status Acquisition

- Consider a public good game with two agents privately contributing to its provision.
- Let g_i denote individual i's voluntary contributions to the public good, and $x_i \geq 0$ represent his consumption of private goods.
- Every player obtains the return $m > 0$ from total contributions to the public good, i.e., $G = g_i + g_j$.

Competition for Status Acquisition

- In addition, each player benefits from *status acquisition* if his donation is larger than his rival's.
- Assume that every player is endowed with w monetary units that can be distributed between private and public good consumption, with w large enough that $g_i \leq w$ never binds.
- The marginal utility individual i derives from his consumption of the private good is one.

Competition for Status Acquisition

- Contributor i's utility function is

$$u_i(x_i, g_i, g_j) = x_i + \ln[mG + \alpha_i(g_i - g_j)]$$

- In this setting, the *status* subject i acquires by contributing g_i is given by $g_i - g_j$.
  - That is, subject i enhances his relative status if his contribution is greater than individual j's; otherwise, subject i perceives himself as an individual with lower status than subject j.

Competition for Status Acquisition

- The difference is scaled by $\alpha_i \in [0, +\infty)$, indicating the importance of relative status for subject i.
- Players' parameters (m, α_i, α_j) are common knowledge.

Competition for Status Acquisition

- The UMP of every player i is

$$\max_{x_i, g_i} \; x_i + \ln\left[mG + \alpha_i\left(g_i - g_j\right)\right]$$

$$\text{s. t.} \;\; x_i + g_i = w,$$

$$g_i + g_j = G, \text{ and}$$

$$g_i, g_j \geq 0$$

- Using $x_i = w - g_i \geq 0$ and $g_i + g_j = G$, we obtain the following problem in g_i alone

$$\max_{g_i \geq 0} \; w - g_i + \ln\left[m\left(g_i + g_j\right) + \alpha_i\left(g_i - g_j\right)\right]$$

Competition for Status Acquisition

- Note that an increase in player j's contribution, g_j, imposes two types of externalities on player i's utility level:
  - ***Positive externality***: arising from the public good nature of player j's contributions.
  - ***Negative externality***: player j's donations reduce the status perception of player i.

Competition for Status Acquisition

- FOC wrt g_i yields player i's BRF

$$g_i(g_j) = \begin{cases} 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left[0, \frac{m+\alpha_i}{m-\alpha_i}\right] \\ 0 & \text{if } g_j > \frac{m+\alpha_i}{m-\alpha_i} \end{cases}$$

- This expression applies when $\alpha_i < m$.

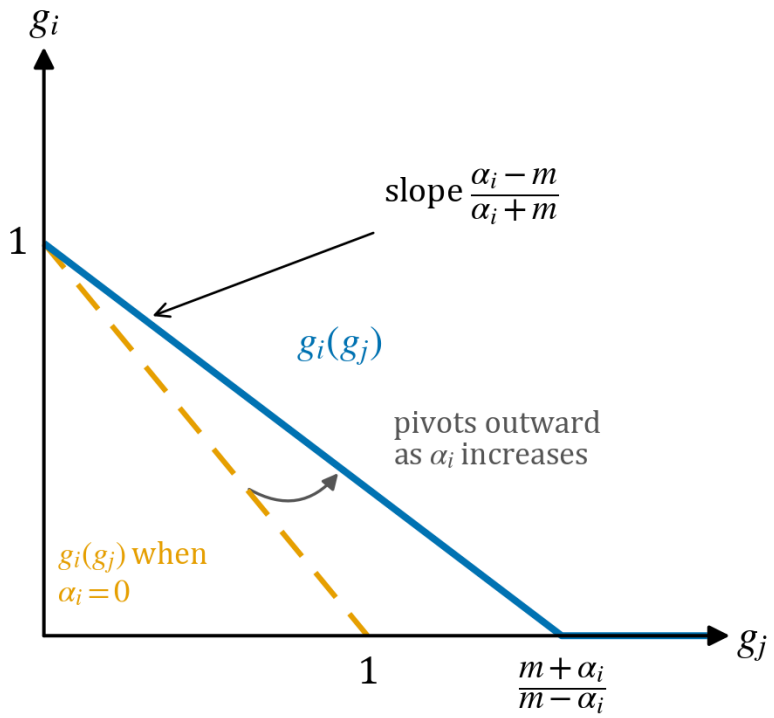
Competition for Status Acquisition

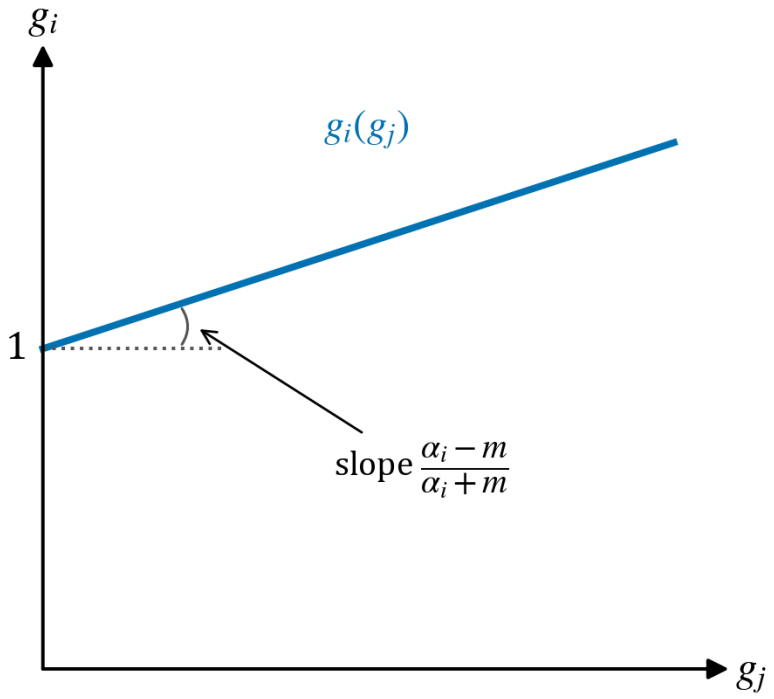
- When $\alpha_i \geq m$, the BRF is

$$g_i(g_j) = 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j \quad \text{for all} \quad g_j \geq 0$$

- When $\alpha_i = m$, $g_i = 1$ regardless of g_j.
- Note that player i's BRF is
  - weakly decreasing in g_j if $\alpha_i < m$
  - increasing in g_j if $\alpha_i > m$
  - weakly increasing in α_i (strictly if $g_j > 0$)

Competition for Status Acquisition





$g_i(g_j)$ when $\alpha_i < m$

$g_i(g_j)$ when $\alpha_i > m$

Competition for Status Acquisition

- *Remarks*:
  - When $\alpha_i < m$, the positive externality that player j's contributions impose on player i's utility dominates the negative externality.
    - Player i views player j's contributions as strategic *substitutes* for his own.
  - When $\alpha_i > m$, the negative externality that player j's contributions impose on player i's utility dominates the positive externality.
    - Player i views player j's donations as strategic *complements* for his own.

Competition for Status Acquisition

- Solving the two linear BRFs $g_i = 1 + s_i g_j$, where $s_i = \frac{\alpha_i - m}{\alpha_i + m}$, gives, whenever $\alpha_i + \alpha_j > 0$,

$$g_i = \frac{1 + s_i}{1 - s_i s_j} = \frac{\alpha_i(\alpha_j + m)}{m(\alpha_i + \alpha_j)}$$

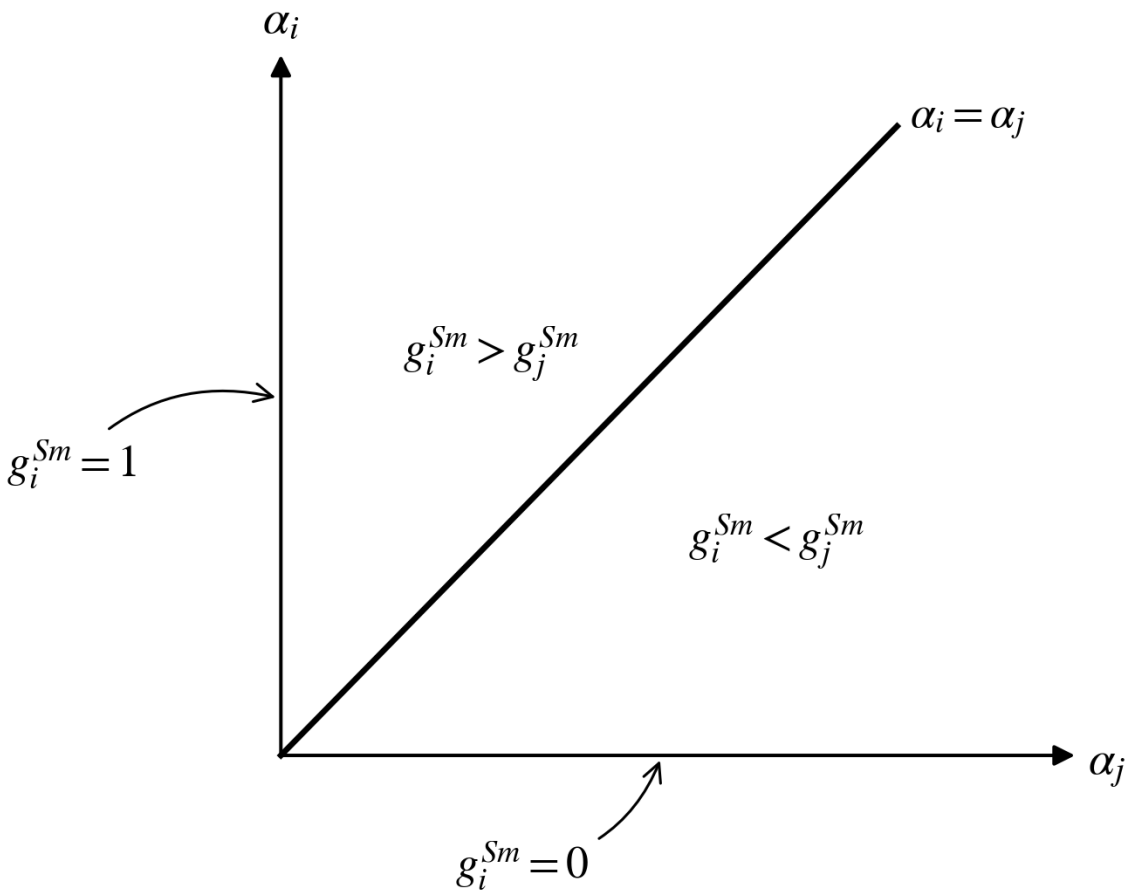
Competition for Status Acquisition

- Nash equilibrium contribution of player i (superscript Sm: status motive):

$$g_i^{Sm} = \begin{cases} 1 \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j = 0 \\ \dfrac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m} \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 0 \quad \text{if} \quad \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

where $g_i^{Sm} + g_j^{Sm} = 1$ if $\alpha_i = \alpha_j = 0$.

Competition for Status Acquisition



Competition for Status Acquisition

- Total contributions to the public good are

$$G^{Sm} = \begin{cases} 1 \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j = 0 \\ 1 + \dfrac{2\alpha_i\alpha_j}{(\alpha_i+\alpha_j)m} \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 1 \quad \text{if} \quad \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

where G^{Sm} is weakly increasing in α_i and α_j and, for a given total status concern $\alpha_i + \alpha_j$, is highest when $\alpha_i = \alpha_j$.

Competition for Status Acquisition

- *Intuition*:
  - Total contributions when *only one* player values status coincide with total contributions when *neither* player does, $G^{Sm} = 1$.
  - An increase in the status concerns of only one individual (α_i or α_j) raises total contributions, provided the other individual also values status.
  - Finally, for a given total status concern, G^{Sm} is highest when $\alpha_i = \alpha_j$.

Appendix 1:
More General Policy Mechanisms

More General Policy Mechanisms

- In the presence of incomplete information, standard policy tools generally entail welfare losses (see quotas and taxes under incomplete information above).
- Let us examine more general policy mechanisms that try to maximize social surplus in the context of incomplete information.
- We consider mechanisms in which we ask agents to self-report their types.

More General Policy Mechanisms

- Recall the setting with incomplete information above: benefit $b(\theta)$, cost $c(\eta)$, and externality level $\overline{x}$.
- We ask the firm:
  - What is your benefit from increasing the externality level from $x = 0$ to $x = \bar{x}$, i.e., $b = b(\theta)$, given your private observation of θ?
- We ask the consumer:
  - What is your damage from the externality, i.e., $c = c(\eta)$, given your private observation of η?

More General Policy Mechanisms

- The mechanism we are interested in focuses on providing incentives to all parties to guarantee that a truth-telling equilibrium emerges.
- ***Groves-Clarke-Vickrey (GCV) mechanism***:
  - The regulator declares that it will set the level of the externality at $x = \bar{x}$ if $\hat{b} > \hat{c}$.
    - If this is the case, the regulator pays $\hat{b}$ to the consumer and charges $\hat{c}$ to the firm.
    - Not a typo!
  - Otherwise, the regulator keeps the level of the externality at $x = 0$.

More General Policy Mechanisms

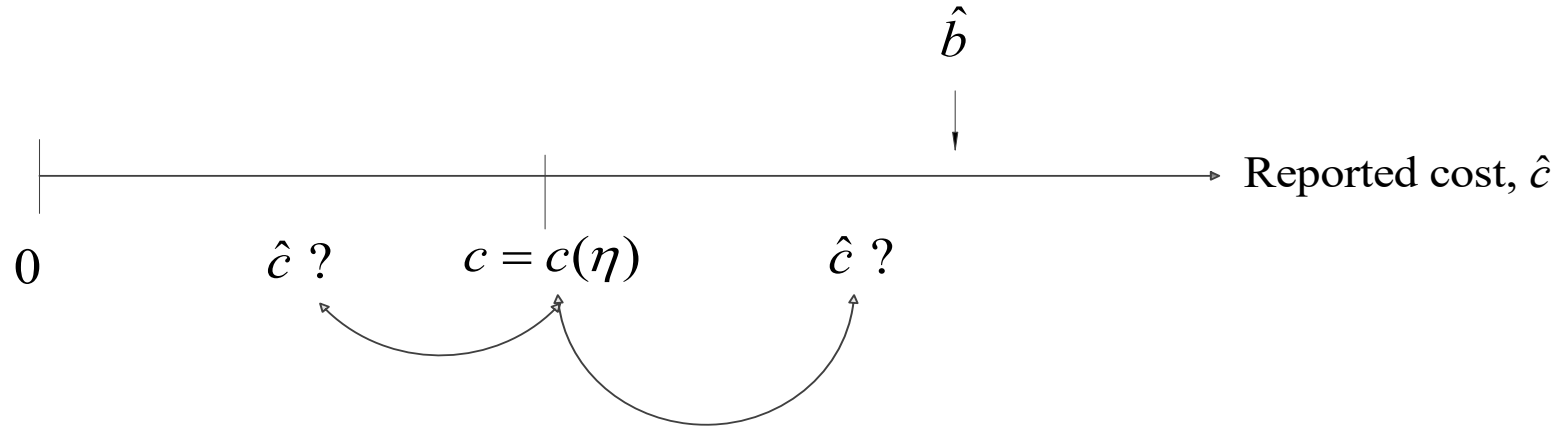
- ***Payoffs*** under the GCV mechanism:
  - Consumer: $\hat{b} - c$ if $\hat{c} < \hat{b}$, and 0 otherwise.
  - Firm: $b - \hat{c}$ if $\hat{c} < \hat{b}$, and 0 otherwise.

More General Policy Mechanisms

- Wouldn't this type of mechanism induce firms to *underreport* their benefits?
  - That is, stating a benefit $\hat{b} < b$, in order to reduce the compensation that they have to provide to those consumers affected by the externality.
- Also, wouldn't this type of mechanism induce consumers to *overreport* their damages?
  - That is, stating a cost $\hat{c} > c$, in order to guarantee that the externality is not allowed or, if allowed, they are substantially compensated for the cost they suffer.

More General Policy Mechanisms

- ***Consumer***:
  - Consider a consumer with a real cost $c = c(\eta)$.
  - Let us examine the consumer's optimal announcement, $\hat{c}$, given a firm's report of a benefit $\hat{b} > c$.



More General Policy Mechanisms

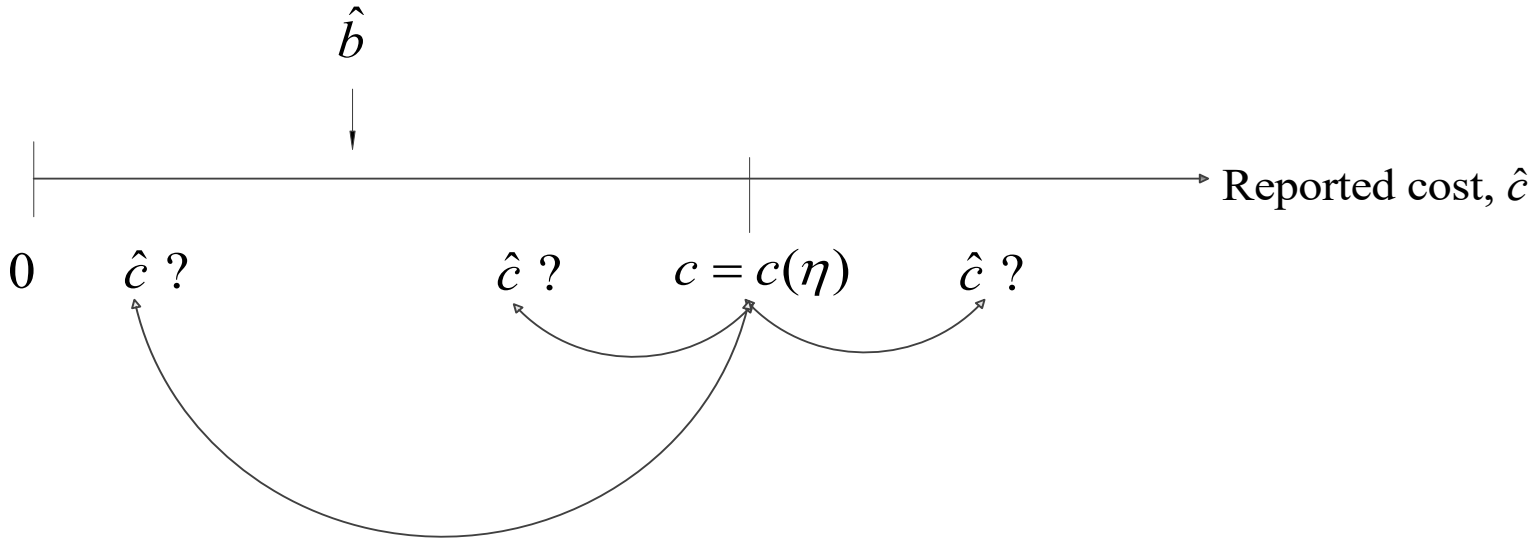
- The consumer does not have incentives to overreport her cost while keeping $\hat{c} < \hat{b}$, i.e., $c < \hat{c} < \hat{b}$, or underreport it, i.e., $\hat{c} < c$, since in both cases the payment she receives is $\hat{b}$.
  - The payment that the consumer receives is unaffected by her report, so truthful reporting is optimal; any other report $\hat{c} < \hat{b}$ does equally well.

More General Policy Mechanisms

- If the consumer overreports her costs, i.e., $\hat{c} > \hat{b}$, the regulator would decide to not allow the externality, i.e., $x = 0$.
  - Such an outcome yields a lower payoff for the consumer than the above outcomes, whereby a report $\hat{c} = c$ yields a payment of $\hat{b}$ from the regulator.

More General Policy Mechanisms

– Let us now examine the consumer's optimal announcement, $\hat{c}$, given a firm's announcement of a benefit $\hat{b} < c$.



More General Policy Mechanisms

- If the consumer overreports her costs, i.e., $\hat{c} > \hat{b}$, the regulator would decide to not allow the externality, i.e., $x = 0$.
- If the consumer slightly underreports her cost $\hat{c}$, i.e., $\hat{b} < \hat{c} < c$, the externality is still not allowed by the regulator, given that reports satisfy $\hat{c} > \hat{b}$.

More General Policy Mechanisms

– Finally, an extreme underreport of her costs, i.e., $\hat{c} < \hat{b}$, is not sensible either:

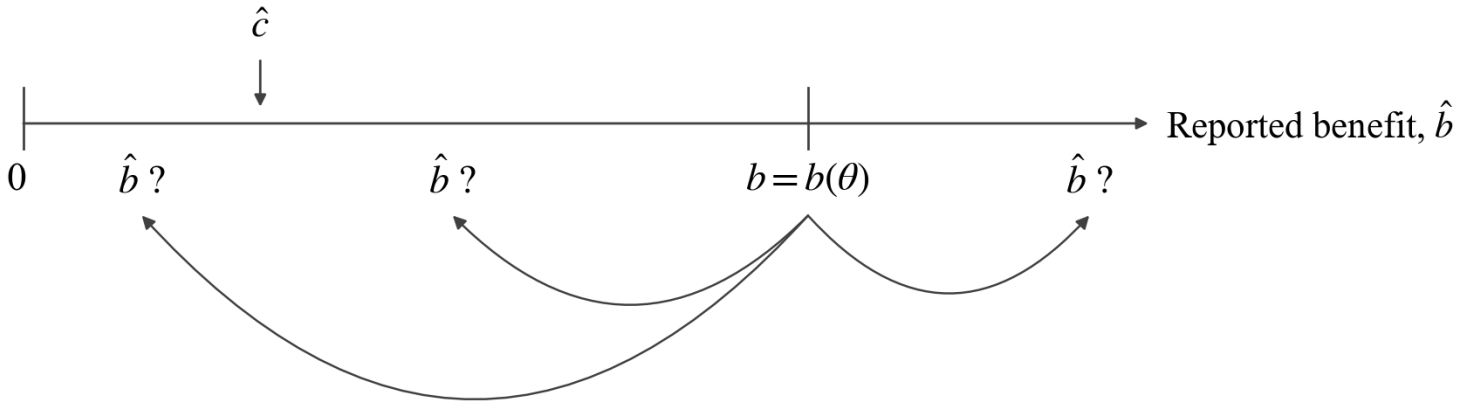
  - While the externality is now allowed (since $\hat{b} > \hat{c}$), the consumer receives a payment $\hat{b}$ below her true cost c, i.e., $c > \hat{b}$.

More General Policy Mechanisms

- Hence, the consumer has incentives to truthfully reveal the damage she suffers from the externality, $\hat{c} = c(\eta)$, regardless of the precise report $\hat{b}$ that the firm makes.
  - That is, truthfully reporting her cost is a weakly dominant strategy for the consumer.

More General Policy Mechanisms

- ***Firm***:
  - Consider a firm with a real benefit $b = b(\theta)$.
  - Let us first examine the firm's optimal announcement, $\hat{b}$, given a consumer's report of a cost $\hat{c} < b$.



More General Policy Mechanisms

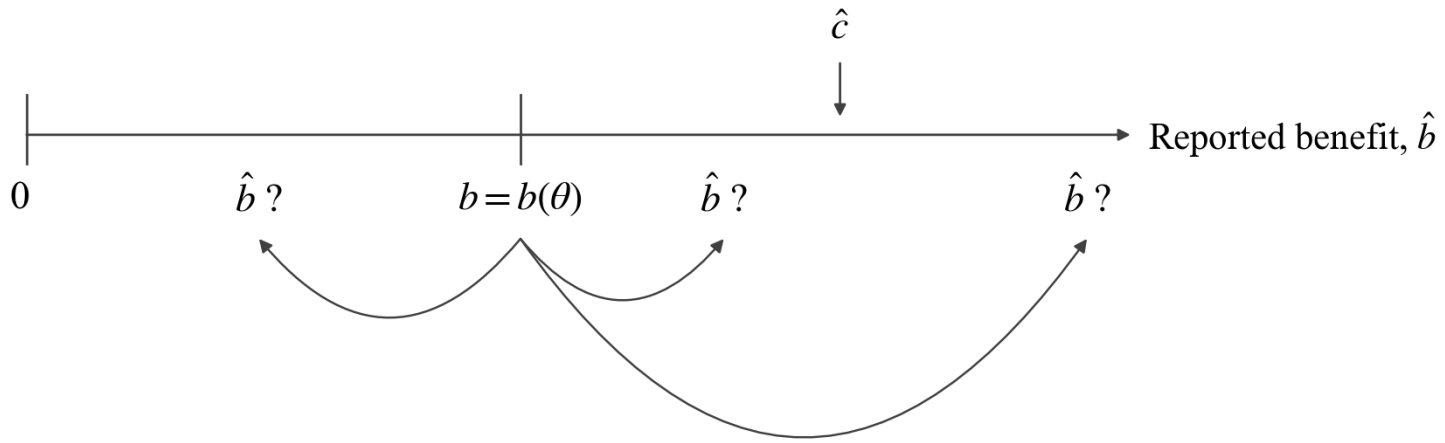
- The firm has no incentives to overreport its true benefit b, i.e., $\hat{b} > b$.
  - The firm would have to pay the same charge, $\hat{c}$, to the regulator, and the externality would still be allowed since reports satisfy $\hat{b} > \hat{c}$.
- The firm has no incentives to slightly underreport its true benefit, i.e., $\hat{c} < \hat{b} < b$.
  - The charge that the firm has to pay is still $\hat{c}$ and the externality is allowed, since reports still satisfy $\hat{b} > \hat{c}$.

More General Policy Mechanisms

- Finally, the firm has no incentives to extremely underreport its true benefit, i.e., $\hat{b} < \hat{c}$.
  - In this case, the externality would not be allowed by the regulator given that reports satisfy $\hat{b} < \hat{c}$, so the firm forgoes $b - \hat{c} > 0$.

More General Policy Mechanisms

– Let us now consider the case where consumer's report $\hat{c}$ lies above the firm's true benefit b, i.e., $\hat{c} > b$.



More General Policy Mechanisms

- If the firm overreports its benefit, i.e., $b < \hat{c} < \hat{b}$, the externality would be allowed (since $\hat{b} > \hat{c}$).
  - However, the firm has to pay a charge $\hat{c}$ to the regulator, which is higher than the real benefit that the firm obtains from the externality, i.e., $b < \hat{c}$.
- If the firm slightly overreports its benefits, i.e., $b < \hat{b} < \hat{c}$, or underreports it, i.e., $\hat{b} < b < \hat{c}$, the externality will not be allowed given that reports would now satisfy $\hat{b} < \hat{c}$.

More General Policy Mechanisms

- Hence, the firm prefers no externality whatsoever, i.e., $x = 0$.
  - The true benefit that the firm obtains from the externality b lies below the cost $\hat{c}$ that the consumer reported.
- Therefore, truthfully reporting its benefit from the externality is a weakly dominant strategy for the firm.

More General Policy Mechanisms

- Truth-telling by both parties gives $x = \overline{x}$ if and only if $b > c$, the efficient outcome.
- When $x = \overline{x}$, the regulator pays $\hat{b}$ to the consumer and collects $\hat{c}$ from the firm, a net outlay of $b - c > 0$: the mechanism is not budget balanced.