

Microeconomic Theory II - UNI, Lima

Homework #5 - Answer key

1. **Cheap talk with two states.** State $\theta \in \{1, 2\}$ equally likely; receiver payoff $-(a - \theta)^2$; sender payoff $-(a - \theta - b)^2$, bias $b > 0$.

(a) *Receiver's action and babbling.*

- With quadratic loss, the receiver's optimal action equals the mean of the state under her posterior belief, $a = E[\theta]$. In a babbling equilibrium the sender's message is uninformative, so the posterior equals the prior and the receiver plays $a = E[\theta] = \frac{1}{2}(1) + \frac{1}{2}(2) = \frac{3}{2}$. This always exists: if the receiver ignores all messages, the sender is indifferent among messages, and sending uninformative messages is a best response, confirming the equilibrium.

(b) *Informative equilibrium.*

- In a separating equilibrium the receiver learns the state and plays $a = 1$ after the "low" message and $a = 2$ after the "high" message. Truth-telling requires each sender type to prefer the action its truthful message induces. The low type ($\theta = 1$), whose ideal action is $1 + b$, must prefer $a = 1$ to $a = 2$:

$$-(1 - 1 - b)^2 \geq -(2 - 1 - b)^2 \iff b^2 \leq (1 - b)^2 \iff b \leq \frac{1}{2}.$$

The high type ($\theta = 2$), whose ideal action is $2 + b$, must prefer $a = 2$ to $a = 1$: $-(2 - 2 - b)^2 \geq -(1 - 2 - b)^2$, i.e. $b^2 \leq (1 + b)^2$, which holds for all $b > 0$. Hence an informative equilibrium exists if and only if $b < \frac{1}{2}$ (the high type never wants to mislead downward; only the low type's upward temptation can bind).

(c) *Interpretation.*

- Informative communication survives only when the sender's bias is small relative to the distance between states. When $b < \frac{1}{2}$, as at $b = \frac{1}{4}$, the low type would rather have the receiver play 1 than 2, so honest messages are incentive compatible and the state can be communicated. When $b \geq \frac{1}{2}$, as at $b = 1$, the low type prefers to be mistaken for the high type, truthful messages unravel, and only babbling survives—the receiver acts on the prior and plays $\frac{3}{2}$. The larger the conflict of interest, the less information cheap talk can convey.

2. **Screening with two consumer types.** Net utility $U = \theta q - \frac{q^2}{2} - T$; $\theta_H = 4$ with probability $\mu = \frac{1}{3}$, $\theta_L = 2$ with probability $\frac{2}{3}$; zero cost.

(a) *Observable-types benchmark.*

- The efficient quantity solves $\theta - q = 0$, so $q_L = 2$ and $q_H = 4$. Extracting each type's whole gross surplus $\theta q - \frac{q^2}{2}$ gives $T_L = 2 \cdot 2 - \frac{4}{2} = 2$ and $T_H = 4 \cdot 4 - \frac{16}{2} = 8$. Expected profit is $\Pi^{FB} = \frac{1}{3}(8) + \frac{2}{3}(2) = 4$.

(b) *Optimal menu.*

- The binding constraints are the low type's participation constraint and the high type's incentive constraint. Setting IR_L to bind, $T_L = \theta_L q_L - \frac{q_L^2}{2}$; setting IC_H to bind leaves the high type her information rent $(\theta_H - \theta_L) q_L$, so $T_H = \theta_H q_H - \frac{q_H^2}{2} - (\theta_H - \theta_L) q_L$. Maximizing expected profit gives $q_H = \theta_H = 4$ and

$$q_L = \theta_L - \frac{\mu}{1 - \mu} (\theta_H - \theta_L) = 2 - \frac{1/3}{2/3} (2) = 1.$$

The payments are $T_L = 2 \cdot 1 - \frac{1}{2} = \frac{3}{2}$ and $T_H = 16 - 8 - (2)1 = 6$, and the high type's information rent is $(\theta_H - \theta_L) q_L = 2$.

(c) *Comparison and interpretation.*

- Expected profit is $\Pi = \frac{1}{3}(6) + \frac{2}{3}(\frac{3}{2}) = 2 + 1 = 3$, below the first-best 4: private information is costly to the monopolist. The high type's quantity is undistorted ($q_H = 4$), since there is no higher type whose rent a distortion could reduce. The low type's quantity is distorted downward ($q_L = 1 < 2$), because a smaller low-type bundle makes it less tempting for the high type to mimic, which lets the monopolist charge the high type more. The high type retains a positive information rent, 2, precisely the surplus she could obtain by taking the low type's bundle.

3. **Moral hazard with limited liability.** Risk-neutral principal and agent; high effort costs $c = 1$ and succeeds with $q_H = \frac{3}{4}$, low effort is free and succeeds with $q_L = \frac{1}{4}$; success yields $R = 8$; wages nonnegative.

(a) *Unobservable-effort contract.*

- Since the agent is risk-neutral and limited liability binds only from below, paying anything after failure merely relaxes incentives and wastes money, so $w_F = 0$ is optimal. To induce high effort the incentive-compatibility constraint is

$$q_H w_S - c \geq q_L w_S \iff w_S \geq \frac{c}{q_H - q_L} = \frac{1}{\frac{3}{4} - \frac{1}{4}} = 2.$$

The cheapest incentive-compatible wage is $w_S = 2$ (with $w_F = 0$). The agent's expected payoff is $q_H w_S - c = \frac{3}{4}(2) - 1 = \frac{1}{2} > 0$, the limited-liability rent. The principal's expected profit is $q_H(R - w_S) = \frac{3}{4}(8 - 2) = \frac{9}{2}$. (Inducing high effort is worthwhile, since paying nothing and accepting low effort would yield only $q_L R = \frac{1}{4}(8) = 2 < \frac{9}{2}$.)

(b) *The value of monitoring.*

- If the principal observes effort, she pays only to cover the cost of high effort, $w = c = 1$, conditional on high effort, leaving no rent. Profit becomes $q_H R - c = \frac{3}{4}(8) - 1 = 5$. The value of monitoring is $5 - \frac{9}{2} = \frac{1}{2}$, exactly the limited-liability rent conceded under unobservable effort.

(c) *Interpretation.*

- Under moral hazard the principal can reward only the observable outcome, not effort. Motivating the agent then requires a success wage high enough that the agent strictly gains from working hard, and limited liability blocks the principal from recovering that surplus after a failure, so the agent keeps a rent. Monitoring removes the informational friction and lets the principal pay for effort directly, eliminating the rent. Its value is precisely the rent saved, $\frac{1}{2}$. If monitoring cost m , the principal would monitor when $m < \frac{1}{2}$ and tolerate the moral-hazard rent otherwise.

4. **The market for lemons.** Quality $\theta \sim U[0, 1]$ known to the seller; seller values θ , buyer values $\lambda\theta$ with $\lambda > 1$.

(a) *Who sells, and average quality.*

- A seller offers her car only if the price covers her value, $p \geq \theta$, so the cars offered are those with $\theta \in [0, p]$ (for $p \leq 1$). Since quality is uniform, the average quality of the cars offered is the midpoint $E[\theta \mid \theta \leq p] = \frac{p}{2}$.

(b) *Unraveling.*

- The buyer pays at most her expected value of a car on offer, $\lambda \cdot \frac{p}{2}$. Trade requires $\lambda \frac{p}{2} \geq p$, i.e. $\lambda \geq 2$. If $\lambda < 2$ this fails for every $p > 0$, so the only price at which the market clears is $p = 0$: the presence of low-quality cars drags down the price buyers will pay, which drives the better cars out of the market, lowering average quality further—the classic unraveling.

(c) *Adverse selection and a remedy.*

- Trade is efficient for every car ($\lambda\theta > \theta$), yet asymmetric information destroys the market: any price attracts a selected, below-average sample of cars, and buyers rationally discount accordingly. A signaling or certification device—an inspection, a warranty, or a credible quality certificate—can let high-quality sellers separate themselves and restore the trades that adverse selection would otherwise eliminate. At $\lambda = \frac{3}{2}$, the buyer's willingness to pay is $\frac{3}{2} \cdot \frac{p}{2} = \frac{3}{4}p < p$, so no positive price sustains trade and the market collapses.