

Microeconomic Theory II - UNI, Lima
Homework #5 - Due date: October 14th, in class.

1. **Cheap talk with two states.** A privately informed sender observes the state $\theta \in \{1, 2\}$, each equally likely, and sends a costless message to a receiver, who then chooses an action $a \in \mathbb{R}$. The receiver's payoff is $-(a - \theta)^2$, so she wants to match the state. The sender's payoff is $-(a - \theta - b)^2$, where $b > 0$ is a commonly known bias: the sender always prefers a higher action than the receiver.
 - (a) Given any posterior belief about θ , what action does the receiver choose? Describe the babbling (uninformative) equilibrium and the action it induces, and explain why it always exists.
 - (b) Consider an informative (separating) equilibrium in which the sender's message reveals the state, so the receiver plays $a = 1$ after one message and $a = 2$ after the other. Write the truth-telling conditions for both sender types, and show that such an equilibrium exists if and only if $b < \frac{1}{2}$.
 - (c) Interpret the cutoff. What happens to the possibility of informative communication as the sender's bias grows? Evaluate the cases $b = \frac{1}{4}$ and $b = 1$.

2. **Screening with two consumer types.** A monopolist with zero marginal cost sells a divisible good to two types of consumers. A type- θ consumer who buys quantity q and pays T obtains net utility $U = \theta q - \frac{q^2}{2} - T$. The high type has $\theta_H = 4$ and occurs with probability $\mu = \frac{1}{3}$; the low type has $\theta_L = 2$ and occurs with probability $\frac{2}{3}$. The monopolist offers a menu $\{(q_L, T_L), (q_H, T_H)\}$ and cannot observe a buyer's type.
 - (a) As a benchmark, find the quantities and the monopolist's profit if types were observable (the efficient quantity solves $\theta = q$, and the monopolist extracts each type's whole surplus).
 - (b) With hidden types, state which participation and incentive constraints bind, and derive the optimal quantities. Show that $q_H = 4$ (no distortion at the top) and $q_L = \theta_L - \frac{\mu}{1-\mu}(\theta_H - \theta_L) = 1$. Compute the payments T_L, T_H and the high type's information rent.
 - (c) Compare the monopolist's profit under asymmetric information with the first-best. Explain the two features of the optimal menu: the undistorted high-type quantity and the downward-distorted low-type quantity.

3. **Moral hazard with limited liability.** A risk-neutral principal hires a risk-neutral agent to run a project. The agent privately chooses effort $e \in \{H, L\}$. High effort costs $c = 1$ and yields success with probability $q_H = \frac{3}{4}$; low effort is costless and yields success with probability $q_L = \frac{1}{4}$. Success generates revenue $R = 8$ for the principal, failure generates 0. The agent is protected by limited liability, so wages must be nonnegative; the principal offers $w_S \geq 0$ after success and $w_F \geq 0$ after failure.

- (a) Suppose effort is unobservable and the principal wants to induce high effort. Argue that $w_F = 0$ is optimal, write the incentive-compatibility constraint, and find the smallest success wage w_S that induces high effort. Compute the agent's limited-liability rent and the principal's expected profit.
 - (b) Suppose instead the principal can monitor effort directly. Find the wage that now implements high effort, the principal's profit, and the value of monitoring.
 - (c) Explain why moral hazard forces the principal to leave the agent a rent, and relate the value of monitoring to that rent. If monitoring cost the principal an amount m , for which values of m would the principal choose to monitor?
4. **The market for lemons.** A used car's quality θ is uniformly distributed on $[0, 1]$ and is known only to the seller. A seller values a car of quality θ at θ ; a buyer values the same car at $\lambda\theta$, with $\lambda > 1$, so trade is efficient for every car. Cars trade at a single price p .
- (a) At price p , which sellers offer their cars, and what is the average quality of the cars offered?
 - (b) A buyer pays at most her expected value of a car on offer. Show that trade can occur at a positive price only if $\lambda \geq 2$, and that for $\lambda < 2$ the market unravels to $p = 0$.
 - (c) Even though trade is efficient for every car, little or no trade occurs when $\lambda < 2$. Explain the role of adverse selection, describe one mechanism that could restore trade, and evaluate the case $\lambda = \frac{3}{2}$.