

Part VI. Theory of competition policy

Chapter 15. Horizontal mergers



Slides

Industrial Organization: Markets and Strategies

Paul Belleflamme and Martin Peitz, 2d Edition

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Chapter 15. Learning objectives

- Understand what influences the profitability of Cournot mergers.
- Assess how a merger affects welfare.
- Examine how modifications of the model change the results of our analyses.
 - Successive mergers
 - Entry-inducing mergers
 - Price competition
 - Impact on tacit collusion
- Understand how merger analyses are conducted empirically.

Horizontal mergers

- Merger control in the US
 - Long tradition: Clayton Act from 1914
 - Mergers are forbidden if substantial lessening of competition
 - Current approach: Horizontal Merger Guidelines (2010)
- Merger control in the European Union
 - Introduced in 1990
 - Before: individual member states were in charge
 - Revised in the 2004 Horizontal Merger Guidelines
- 2 main effects of mergers on competition
 - **Negative**: fewer decision makers
 - **Positive**: efficiency gains
- What if profitable but welfare-reducing merger?

Mergers between 2 firms

- Market for a homogeneous product
 - Monopoly profit $\pi^m = \pi(1)$ is greater than industry profits in duopoly $2\pi(2)$
 - Merger to monopoly is always profitable.
- For mergers which fall short of a merger to monopoly, the issue is less clear-cut.
- Compare profits at the post-merger and the pre-merger equilibria.
 - Suppose constant marginal cost and no synergy effects, so no cost advantage for the merged firm (merger equivalent to buying a firm and closing it down)

Mergers between 2 firms (cont'd)

- Consider a single merger between 2 firms
 - The other firms in the industry will always gain from the merger.
 - Why? The merging firms internalize their previous rivalry once they are placed under common control.
 - Benefits the firms outside the merger
 - Cournot competition, as a response to the merger,
 - the other firms increase output
 - detrimental to merging firm's profit.
- **Lesson:** Under Cournot competition, mergers of two firms are unlikely to be profitable if the market is fragmented but they are more likely to be profitable if the market is concentrated.

Mergers between 2 firms: the Cournot benchmark

- Each of the n symmetric firms chooses q_i to maximize

$$(a - q_i - Q_{-i} - c)q_i.$$

- The first-order condition $a - c - 2q_i - Q_{-i} = 0$ gives the best response

$$q_i(Q_{-i}) = \frac{a - c}{2} - \frac{1}{2}Q_{-i}.$$

- Imposing symmetry $Q_{-i} = (n - 1)q$ and solving for the common output,

$$q^*(n) = \frac{a - c}{n + 1},$$

$$\pi(n) = \left(\frac{a - c}{n + 1} \right)^2.$$

- Intuition.** More rivals (a larger n) shrink each firm's output and profit.

Mergers between 2 firms: is the merger profitable?

- **Two firms merge.** Without synergies the merged firm is an ordinary Cournot competitor, so only $n - 1$ firms remain.
- The merger pays only if the merged firm earns more than the two firms earned apart, $\pi(n - 1) \geq 2\pi(n)$:

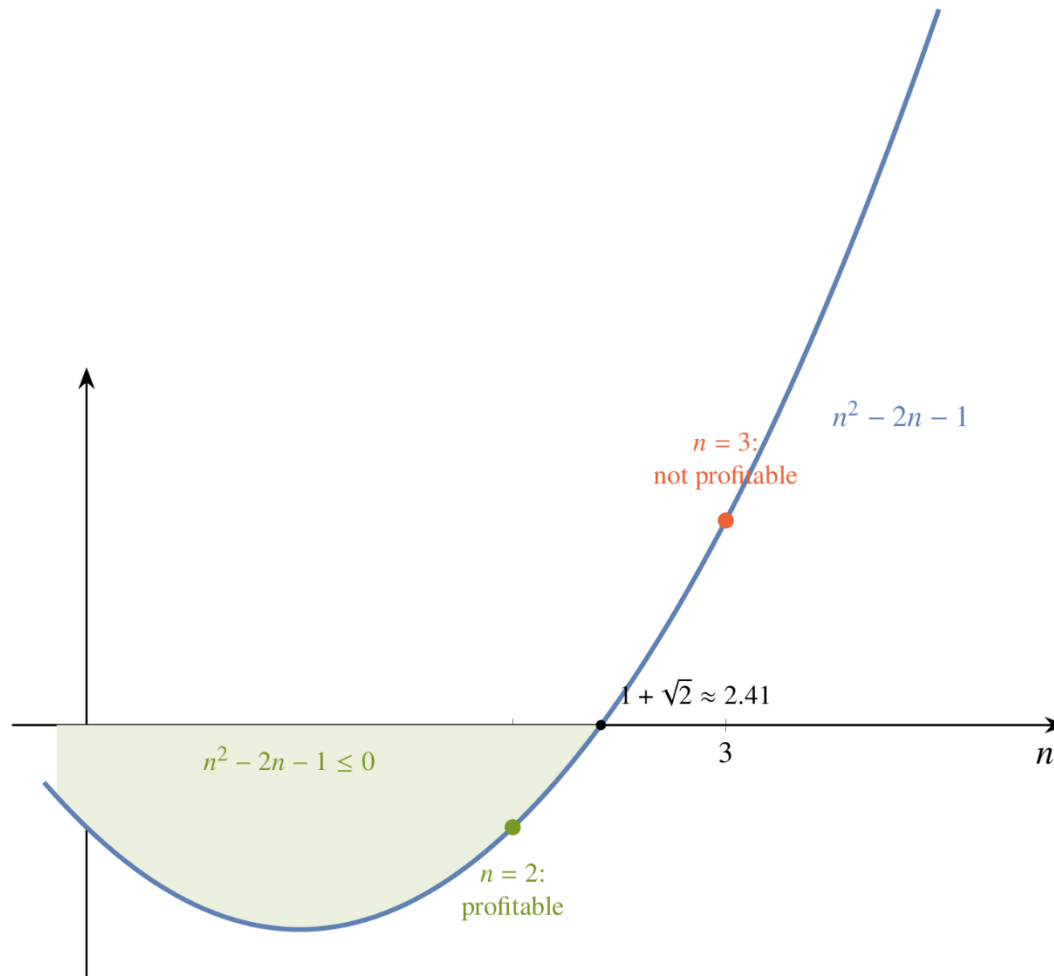
$$\left(\frac{a - c}{n}\right)^2 \geq 2\left(\frac{a - c}{n + 1}\right)^2,$$

$$(n + 1)^2 \geq 2n^2,$$

$$n^2 - 2n - 1 \leq 0.$$

- The positive root is $1 + \sqrt{2} \approx 2.41$, so a two-firm merger pays only when $n = 2$ (merger to monopoly) — the merger paradox.
- **Example.** With $a - c = 12$, a merger of two gives $\pi(2) = 16 < 2\pi(3) = 18$ at $n = 3$, and $\pi(9) = 1.44 < 2\pi(10) \approx 2.38$ at $n = 10$ — both unprofitable.

Mergers between 2 firms: the sign of $n^2 - 2n - 1$



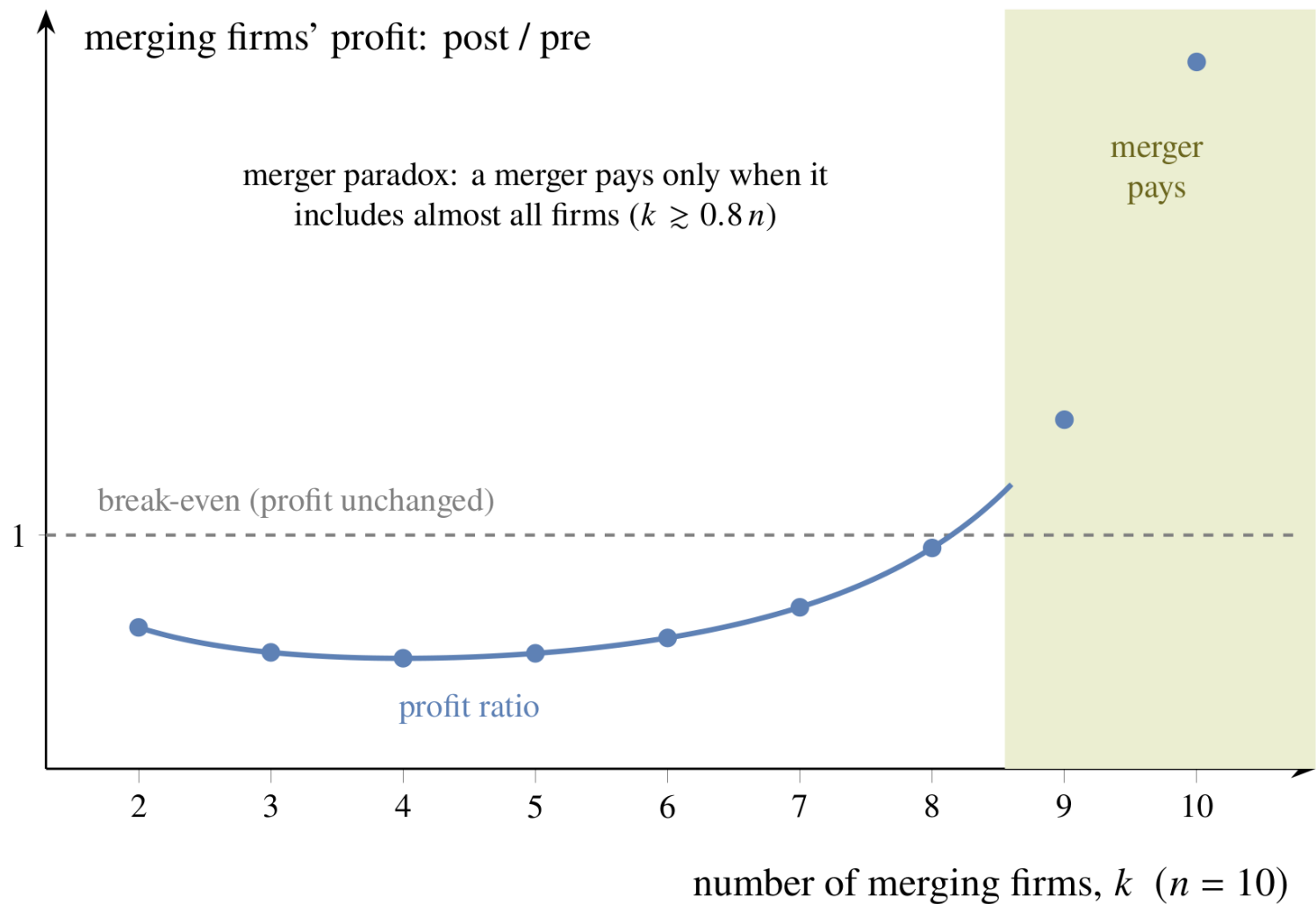
Mergers between 2 firms (cont'd)

- Previous result hinges on 6 assumptions
 - Only 2 firms merge.
 - No increasing marginal cost of production (no capacity constraints)
 - Merger does not affect efficiency of production.
 - Only a single merger is possible.
 - No additional firm can enter.
 - Firms are Cournot competitors.
- Relax these assumptions one by one.

Mergers between several firms

- Merger is profitable if it involves a sufficiently large number of firms.
- **Lesson:** Mergers between multiple firms are only profitable for Cournot competition if a highly concentrated market results.
- Profitability depends on 2 opposite forces
 - By internalizing previous rivalry, the merged entity reduces its quantity and thereby increases its profit.
 - But outside firms react by increasing their quantity, which reduces the profitability of the merger.
 - For the 1st effect to dominate, the number of outside firms must be small enough.

The merger paradox



Mergers between several firms: the profitability condition

- **Setting.** n firms, k of which merge, with no synergies. After the merger $n - k + 1$ firms remain.
- The merged firm beats the k pre-merger profits only if $\pi(n - k + 1) \geq k\pi(n)$:

$$\left(\frac{a - c}{n - k + 2}\right)^2 \geq k \left(\frac{a - c}{n + 1}\right)^2,$$

$$(n + 1)^2 \geq k(n - k + 2)^2.$$

- **Factoring.** Rearranged, this inequality becomes

$$(k - 1)[-k^2 + (2n + 3)k - (n + 1)^2] \geq 0,$$

$$k \geq \frac{1}{2}(2n + 3 - \sqrt{4n + 5}).$$

- The larger root exceeds n , so only the lower threshold binds.

Mergers between several firms: the 80% rule

- **The 80% rule.** The threshold is at least $0.8n$, with equality at $n = 5$:

$$\frac{1}{2}(2n + 3 - \sqrt{4n + 5}) \geq 0.8n,$$

$$0.16(n - 5)^2 \geq 0.$$

- A profitable merger must include roughly 80% of the firms.

n	Threshold	Minimum number of merging firms	Share of the industry
3	2.44	3	100%
5	4.00	4	80%
10	8.15	9	90%
20	16.9	17	85%

Efficiency-increasing mergers

- If merger confers an advantage to merged entity with respect to the outside firms:
 - Merged firm does not reduce its production so much so outside firms do not raise their production so much
- More mergers should become profitable in the presence of efficiency gains.
- 2 different types of efficiency gains
 - Scale economies
 - Synergies

Scale economies

- Merged entity has now access to the combined productive capacity of the merging partners.
- Assumptions
 - Each firm owns a certain amount of capital.
 - When some firms merge, the merged entity owns the combined capital stock.
 - Each firm's marginal cost increases linearly with that firm's output.
 - The larger the firm's capital stock, the lower the slope of the marginal cost curve.
- The merged entity has a lower marginal cost curve than either of the constituent firms.

Scale economies: the cost structure

- **Setup.** Inverse demand $P(Q) = a - Q$. Each firm owns capital K and produces at cost

$$C(q) = cq + \frac{h}{2K} q^2.$$

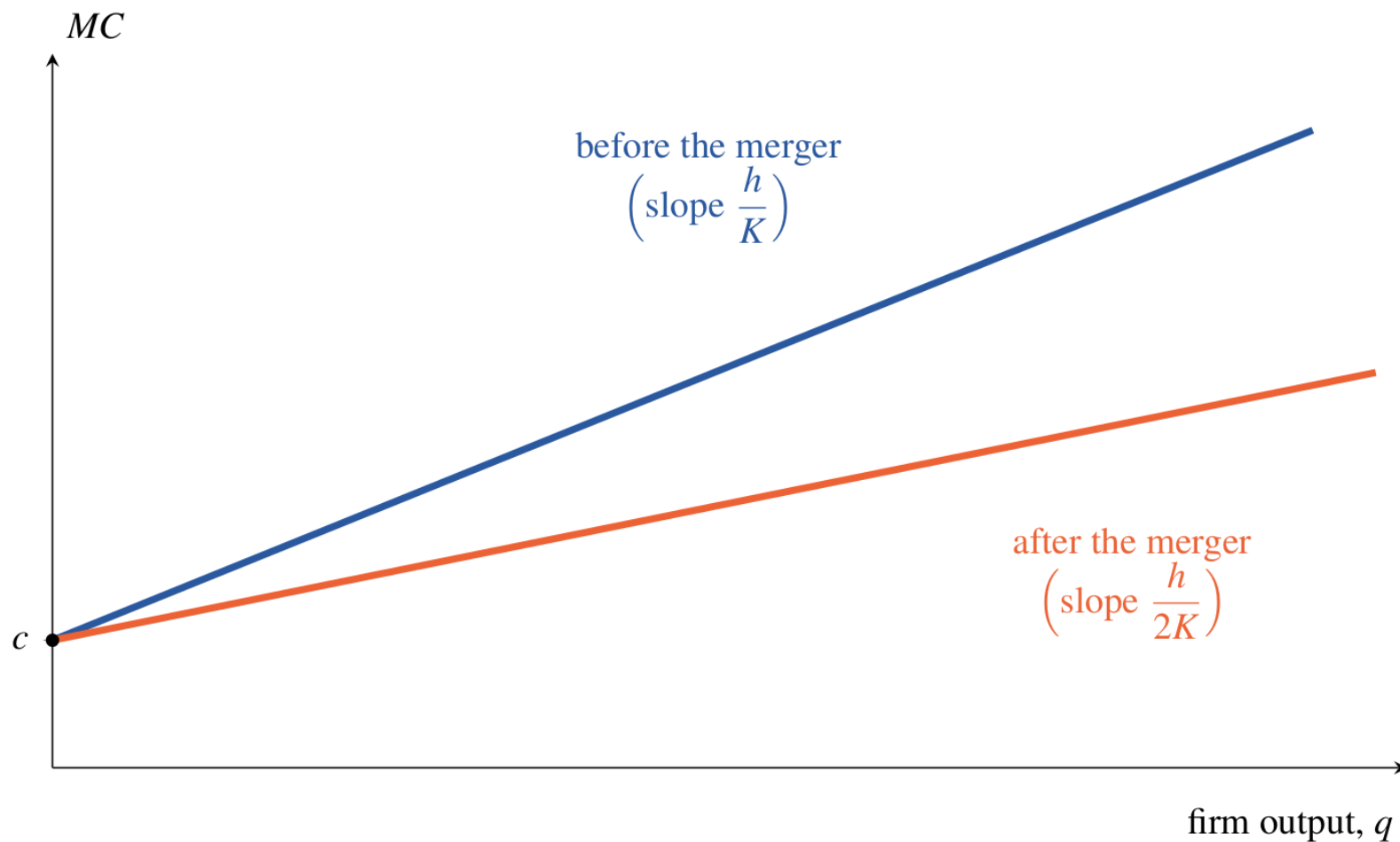
- Its marginal cost rises in output:

$$MC(q) = c + \frac{h}{K} q \equiv c + \kappa q,$$

$$\kappa \equiv \frac{h}{K}.$$

- **Intuition.** More capital K means a smaller slope κ : a larger firm has a flatter marginal-cost curve.

Scale economies: the marginal-cost curve



Scale economies: Cournot equilibrium without a merger

- Each of the n firms chooses q_i to maximize

$$(a - q_i - Q_{-i})q_i - cq_i - \frac{\kappa}{2}q_i^2.$$

- The first-order condition $a - c - (2 + \kappa)q_i - Q_{-i} = 0$ gives the best response

$$q_i(Q_{-i}) = \frac{a - c}{2 + \kappa} - \frac{1}{2 + \kappa}Q_{-i}.$$

- Imposing symmetry $Q_{-i} = (n - 1)q$:

$$q^{NM} = \frac{a - c}{n + 1 + \kappa},$$

$$\pi^{NM} = \frac{2 + \kappa}{2} \left(\frac{a - c}{n + 1 + \kappa} \right)^2.$$

Scale economies: the merged firm

- **Two firms merge and pool their capital**, so the merged entity has capital $2K$. Its marginal-cost slope halves:

$$MC_M(q_I) = c + \frac{h}{2K} q_I = c + \frac{\kappa}{2} q_I,$$

$$C_M(q_I) = cq_I + \frac{\kappa}{4} q_I^2.$$

- **Intuition.** The flatter marginal cost is the economy of scale: the merged firm contracts output less than two separate firms would.
- Post-merger, $n - 1$ firms compete: the merged firm (output q_I) and $n - 2$ outsiders (each q_O).

Scale economies: the merged firm's best response

- The merged firm chooses its output q_I to maximize

$$(a - q_I - (n - 2)q_O)q_I - cq_I - \frac{\kappa}{4}q_I^2.$$

- Its first-order condition is

$$a - c - \left(2 + \frac{\kappa}{2}\right)q_I - (n - 2)q_O = 0,$$

- so, solving for q_I , the merged firm's best response is

$$q_I = \frac{2(a - c)}{4 + \kappa} - \frac{2(n - 2)}{4 + \kappa}q_O.$$

Scale economies: an outsider's best response

- A representative outsider sets q_j ; its first-order condition is

$$a - q_I - 2q_j - (n - 3)q_O - c - \kappa q_j = 0.$$

- Imposing symmetry across outsiders, $q_j = q_O$, and rearranging,

$$a - c - q_I - (n - 1 + \kappa)q_O = 0,$$

- so each outsider's best response is

$$q_O = \frac{a - c}{n - 1 + \kappa} - \frac{1}{n - 1 + \kappa} q_I.$$

Scale economies: equilibrium outputs

- The two first-order conditions form a linear system in q_I and q_O :

$$\left(2 + \frac{\kappa}{2}\right) q_I + (n - 2)q_O = a - c,$$

$$q_I + (n - 1 + \kappa)q_O = a - c.$$

- Eliminating q_I (write $D \equiv \kappa^2 + (n + 3)\kappa + 2n$) leaves

$$\frac{D}{2} q_O = \left(1 + \frac{\kappa}{2}\right) (a - c),$$

$$q_O^* = \frac{(2 + \kappa)(a - c)}{D}.$$

- Back-substituting into the outsider's condition,

$$q_I^* = (a - c) - (n - 1 + \kappa)q_O^* = \frac{2(1 + \kappa)(a - c)}{D}.$$

Scale economies: the equilibrium price

- The price comes from inverse demand $P(Q) = a - Q$, with total output $Q = q_I + (n - 2)q_O$:

$$p^* = a - q_I^* - (n - 2)q_O^*.$$

- Substituting the equilibrium outputs and simplifying,

$$p^* = c + \frac{(1 + \kappa)(2 + \kappa)(a - c)}{D}.$$

- Intuition.** The equilibrium price stays above the cost intercept c .

Scale economies: the merged firm's profit

- Profit is revenue minus cost:

$$\pi_I = (p^* - c)q_I^* - \frac{\kappa}{4}(q_I^*)^2.$$

- Inserting the equilibrium values and factoring $(1 + \kappa)^2$,

$$\pi_I = (1 + \kappa)^2 [2(2 + \kappa) - \kappa] \frac{(a - c)^2}{D^2},$$

$$\pi_I = \frac{(1 + \kappa)^2 (4 + \kappa) (a - c)^2}{D^2}.$$

- Intuition.** A larger κ raises the merged firm's profit — the scale advantage.

Scale economies: when is the merger profitable?

- The merger is profitable only if the merged firm earns at least the two pre-merger profits, $\pi_I \geq 2\pi^{NM}$:

$$\frac{(1 + \kappa)^2(4 + \kappa)(a - c)^2}{D^2} \geq (2 + \kappa) \left(\frac{a - c}{n + 1 + \kappa} \right)^2.$$

- The factor $(a - c)^2$ cancels; clearing denominators leaves a single cubic in κ :

$$f(n, \kappa) \equiv \kappa^3 - 2(n - 5)\kappa^2 + (2n - 3n^2 + 17)\kappa - 4(n^2 - 2n - 1) \geq 0.$$

- Merger paradox** ($\kappa = 0$). Then $f(n, 0) = -4(n^2 - 2n - 1)$, nonnegative only for $n \leq 1 + \sqrt{2} \approx 2.41$: only a merger to monopoly is profitable.

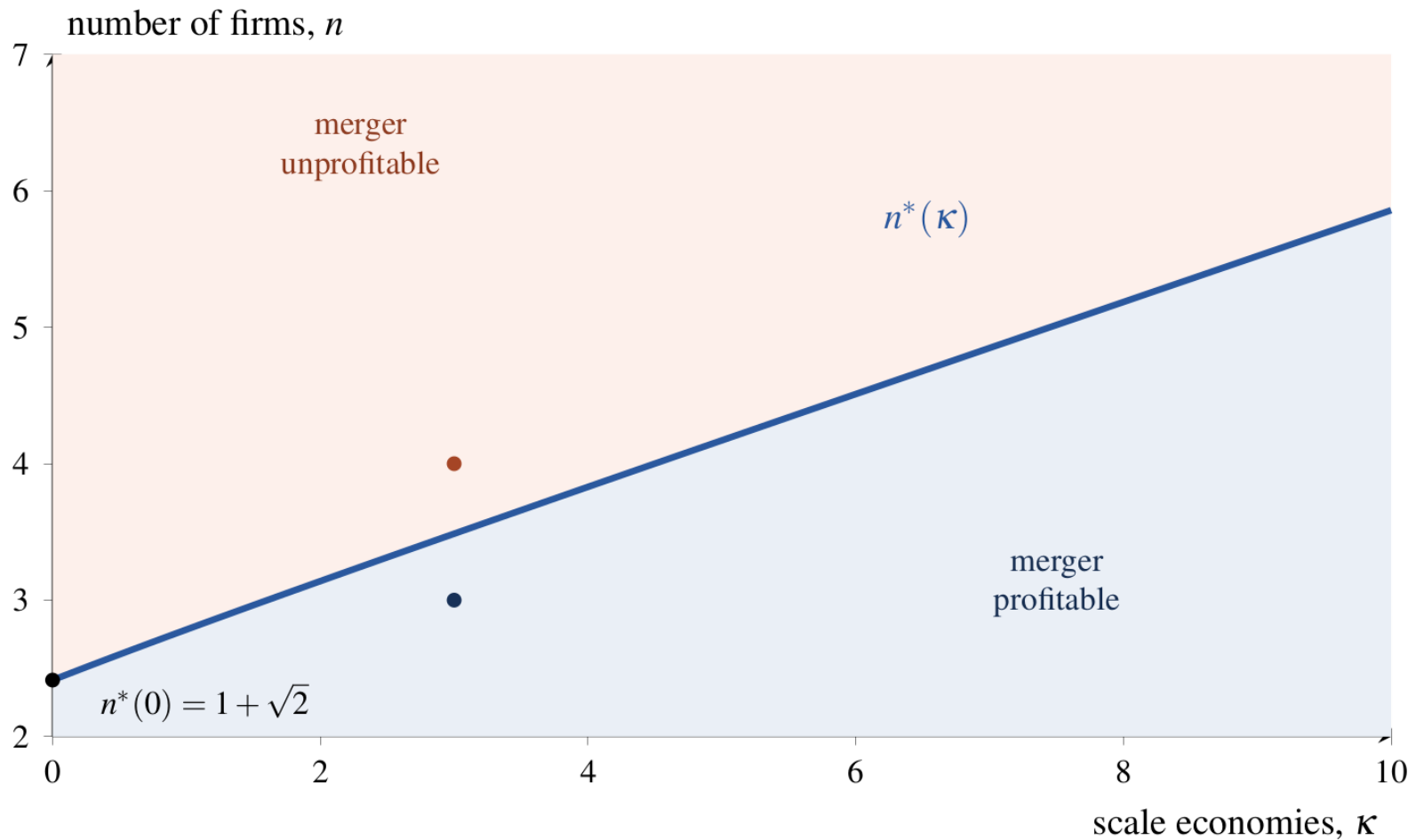
Scale economies: the profitability cutoff

- Solving $f(n, \kappa) = 0$ for n gives the cutoff

$$n^*(\kappa) = \frac{4 + \kappa - \kappa^2 + 2(1 + \kappa)\sqrt{(\kappa + 2)(\kappa + 4)}}{3\kappa + 4}.$$

- A two-firm merger is profitable if and only if $n \leq n^*(\kappa)$.
- Intuition.** $n^*(0) = 1 + \sqrt{2} \approx 2.41$ and $n^*(3) \approx 3.49$; the cutoff rises in κ . Stronger scale economies make mergers profitable in less concentrated markets.
- Lesson.** Combining fixed assets under increasing marginal cost enlarges the scope for profitable Cournot mergers.

Scale economies: profitable vs unprofitable mergers



Scale economies: a numerical example

- Take $a - c = 12$ and $\kappa = 3$, so the cutoff is $n^*(3) \approx 3.49$.
- $n = 3$: $f(3,3) = 43 > 0$, so $\pi_I = 14.8 > 2\pi^{NM} = 14.7$ — **profitable**.
- $n = 4$: $f(4,3) = -52 < 0$, so $\pi_I = 11.2 < 2\pi^{NM} = 11.3$ — **unprofitable**.
- So a two-firm merger is profitable up to $n = 3$ firms here, versus only $n = 2$ (merger to monopoly) when marginal costs are constant.

Synergies

- Assumptions
 - $P(q) = a - q$, n firms, k of which merge
 - Pre-merger constant marginal cost c
 - Merger reduces marginal cost of the merged firm from c to $c - x$ (x positive).
 - Redo the previous analysis under these assumptions.
- **Lesson:** Mergers between Cournot competitors that do not result in a highly concentrated market are only profitable if they entail sufficiently large synergies.

Synergies: the merged firm's best response

- **After the merger**, the merged entity has marginal cost $c - x$, while the $n - k$ outsiders keep c , so $n - k + 1$ firms compete.

- The merged firm chooses its output q_I to maximize

$$(a - q_I - Q_{-I} - (c - x))q_I,$$

- where $Q_{-I} = (n - k)q_O$ is total outsider output. Its first-order condition $a - (c - x) - 2q_I - Q_{-I} = 0$ gives the best response

$$q_I(Q_{-I}) = \frac{(a - c) + x}{2} - \frac{1}{2}Q_{-I}.$$

- **Intuition.** The synergy x lifts the merged firm's best-response intercept, so it supplies more for any outsider output.

Synergies: an outsider's best response

- A representative outsider sets q_o to maximize $(a - q_o - Q_{-o} - c)q_o$, with first-order condition

$$a - c - 2q_o - Q_{-o} = 0.$$

- Imposing symmetry across the $n - k$ outsiders, $Q_{-o} = q_I + (n - k - 1)q_o$, and rearranging,

$$a - c - q_I - (n - k + 1)q_o = 0,$$

- so each outsider's best response is

$$q_o(q_I) = \frac{a - c}{n - k + 1} - \frac{1}{n - k + 1} q_I.$$

Synergies: equilibrium outputs

- Insert the outsider's best response, with $Q_{-I} = (n - k)q_O$, into the merged firm's:

$$2q_I = (a - c) + x - (n - k) \frac{(a - c) - q_I}{n - k + 1}.$$

- Collecting the q_I terms,

$$(n - k + 2)q_I = (a - c) + (n - k + 1)x,$$

$$q_I = \frac{(a - c) + (n - k + 1)x}{n - k + 2}.$$

- Back-substituting gives each outsider's output,

$$q_O = \frac{(a - c) - x}{n - k + 2}.$$

- Intuition.** A larger synergy x raises q_I and lowers q_O : the merged firm expands at the outsiders' expense.

Synergies: profit and the profitability cutoff

- With linear demand the merged firm's margin equals its output, so its profit is $\pi_I = q_I^2$.

- It beats the k pre-merger profits $k\pi(n)$ when $\pi_I \geq k\pi(n)$, that is

$$\frac{(a - c) + (n - k + 1)x}{n - k + 2} \geq \sqrt{k} \frac{a - c}{n + 1}.$$

- In terms of the synergy intensity $\theta \equiv \frac{x}{a - c}$ (the cost saving as a fraction of the margin $a - c$), the merger is profitable when $\theta \geq \theta_p$, where

$$\theta_p = \frac{(n - k + 2)\sqrt{k} - (n + 1)}{(n - k + 1)(n + 1)}.$$

- **Intuition.** Larger or less concentrated markets need a bigger synergy to make the merger pay.

Synergies: a numerical example

- Take $n = 5$ firms, $k = 2$ of them merging, and $a - c = 12$. Each firm's pre-merger profit is $\pi(5) = 4$, so the two insiders together earn 8.
- The cutoff synergy is

$$\theta_p = \frac{5\sqrt{2} - 6}{24} \approx 0.045,$$

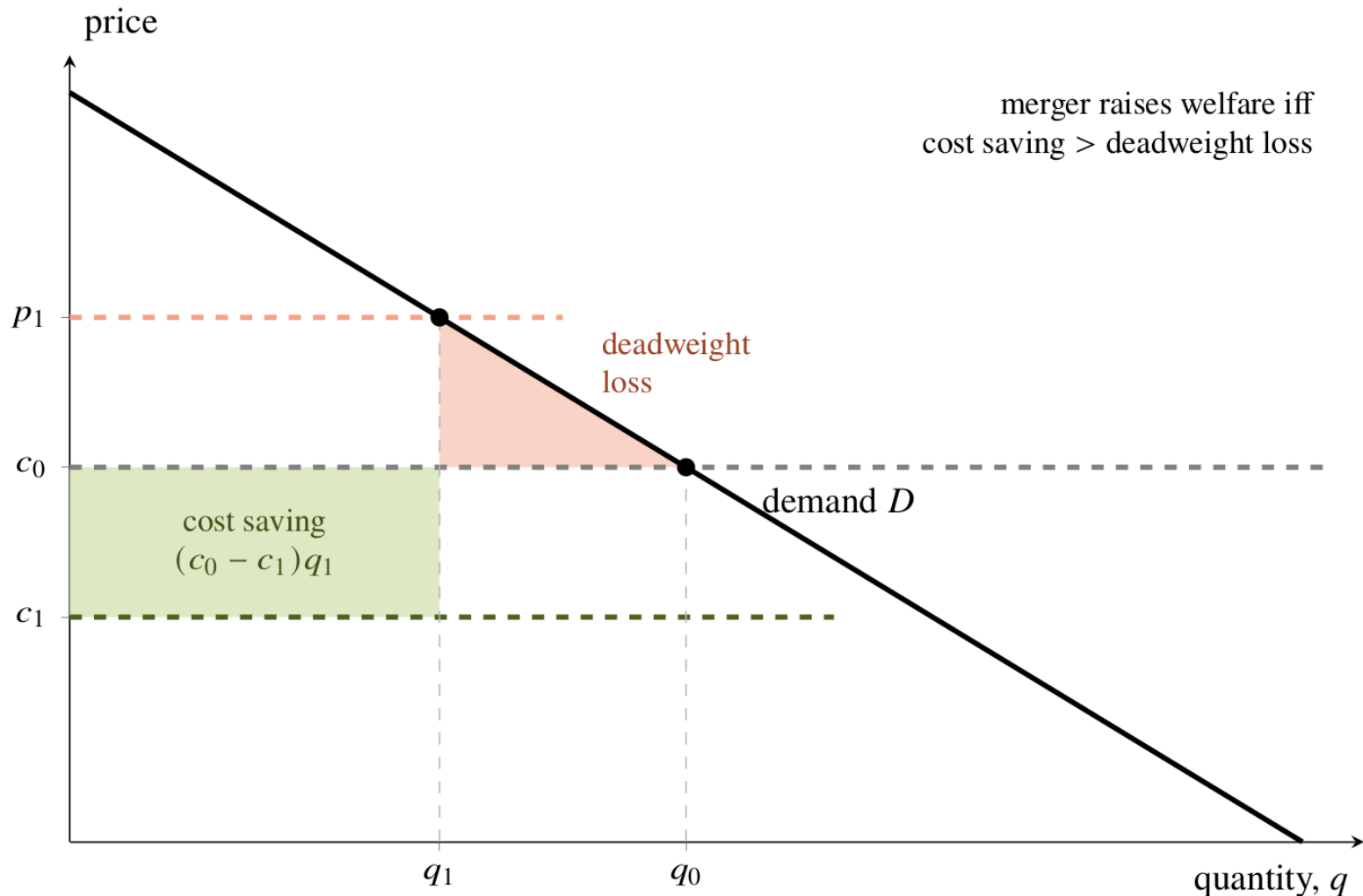
$$x_p = \theta_p(a - c) \approx 0.54.$$

- Strong synergy** ($x = 1$): $q_I = 3.2$, so $\pi_I = 10.24 > 8$ — **profitable**.
- Weak synergy** ($x = 0.3$): $q_I = 2.64$, so $\pi_I = 6.97 < 8$ — **unprofitable**.
- So a synergy above $x_p \approx 0.54$ makes this two-firm merger profitable, though without synergies it would not be.

Welfare analysis of Cournot mergers

- We use the **total surplus criterion**
 - Sum of consumer surplus, CS, and total profits (“insiders” and “outsiders” profits).
 - Allow merger if $\Delta W = \Delta CS + \Delta \Pi_I + \Delta \Pi_O > 0$
- Interplay between
 - Competition-reducing effect of mergers
 - Potential efficiency gains
- Merger can increase total surplus if efficiency gains are large enough.
 - ‘Efficiency defence’
- But, there also exist levels of synergies that make mergers profitable but, at the same time, detrimental to consumers (or to society as a whole).

Efficiency defense: Williamson trade-off





Case. The 'efficiency defence'

- 1998: *Superior Propane* and *ICG Propane* are the 2 largest propane distributors in Canada.
- Plan of merger
- 2 effects
 - Reduction of competition
 - Substantial in 66 of 74 local markets
 - Near monopoly in 16 local markets.
 - Estimated deadweight loss of C\$ 6 Mio. per year
 - Cost efficiencies
 - Estimated C\$ 29 Mio. per year
- Merger permitted using the total surplus criterion.

Welfare with synergies: the setup

- **We judge the merger by total surplus**, consumer surplus plus the merged firm's profit Π_I and the outsiders' profit Π_O :

$$W = CS + \Pi_I + \Pi_O.$$

- In the synergy model k of n firms merge and the merged firm's marginal cost falls from c to $c - x$. The synergy intensity

$$\theta \equiv \frac{x}{a - c} \in (0,1)$$

- is the cost saving as a fraction of the margin $a - c$; the bound $\theta < 1$ keeps every outsider producing ($q_O > 0$).
- We find three thresholds for θ : consumer surplus θ_c , profitability θ_p , and total welfare θ_w .
- **Intuition.** A larger θ means deeper cost savings, helping the merged firm, its rivals, and consumers to different degrees.

Welfare with synergies: post-merger equilibrium

- Recall the post-merger equilibrium of the merged firm and the $n - k$ outsiders:

$$q_I = \frac{(a - c) + (n - k + 1)x}{n - k + 2},$$

$$q_O = \frac{(a - c) - x}{n - k + 2}.$$

- Total output is

$$Q^{post} = q_I + (n - k)q_O = \frac{(a - c)(n - k + 1) + x}{n - k + 2}.$$

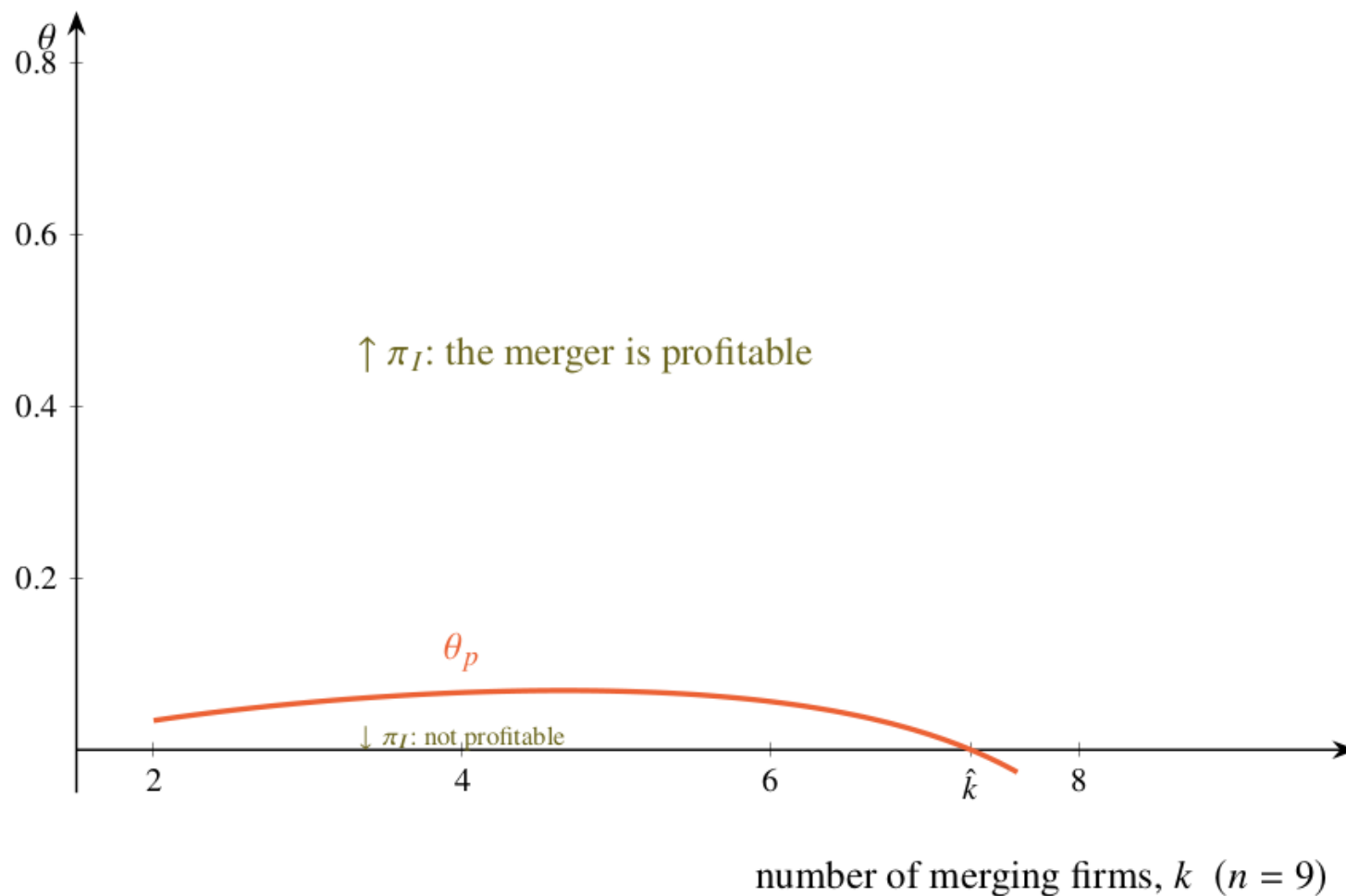
- Intuition.** The merged firm produces more and each outsider less; total output rises with the synergy x .

Welfare with synergies: finding the profitability cutoff

- Profitability.** With linear demand the merged firm's margin equals its output, so its profit is $\pi_I = q_I^2$. The k insiders earned $k\pi(n) = k \left(\frac{a-c}{n+1} \right)^2$ apart.
- The merger is profitable when $\pi_I \geq k\pi(n)$, that is $q_I \geq \sqrt{k} \frac{a-c}{n+1}$.
- Substituting $q_I = \frac{(a-c)+(n-k+1)x}{n-k+2}$ and solving for the synergy intensity $\theta = \frac{x}{a-c}$,

$$\theta \geq \frac{(n-k+2)\sqrt{k} - (n+1)}{(n-k+1)(n+1)} \equiv \theta_p.$$
- Intuition.** Larger or less concentrated markets need a bigger synergy to make the merger pay. The next figure plots θ_p alone.

Welfare with synergies: the profitability cutoff



Welfare with synergies: the consumer-surplus threshold

- **Consumers gain** only if total output does not fall:

$$Q^{post} \geq Q^{pre} = \frac{n(a - c)}{n + 1}.$$

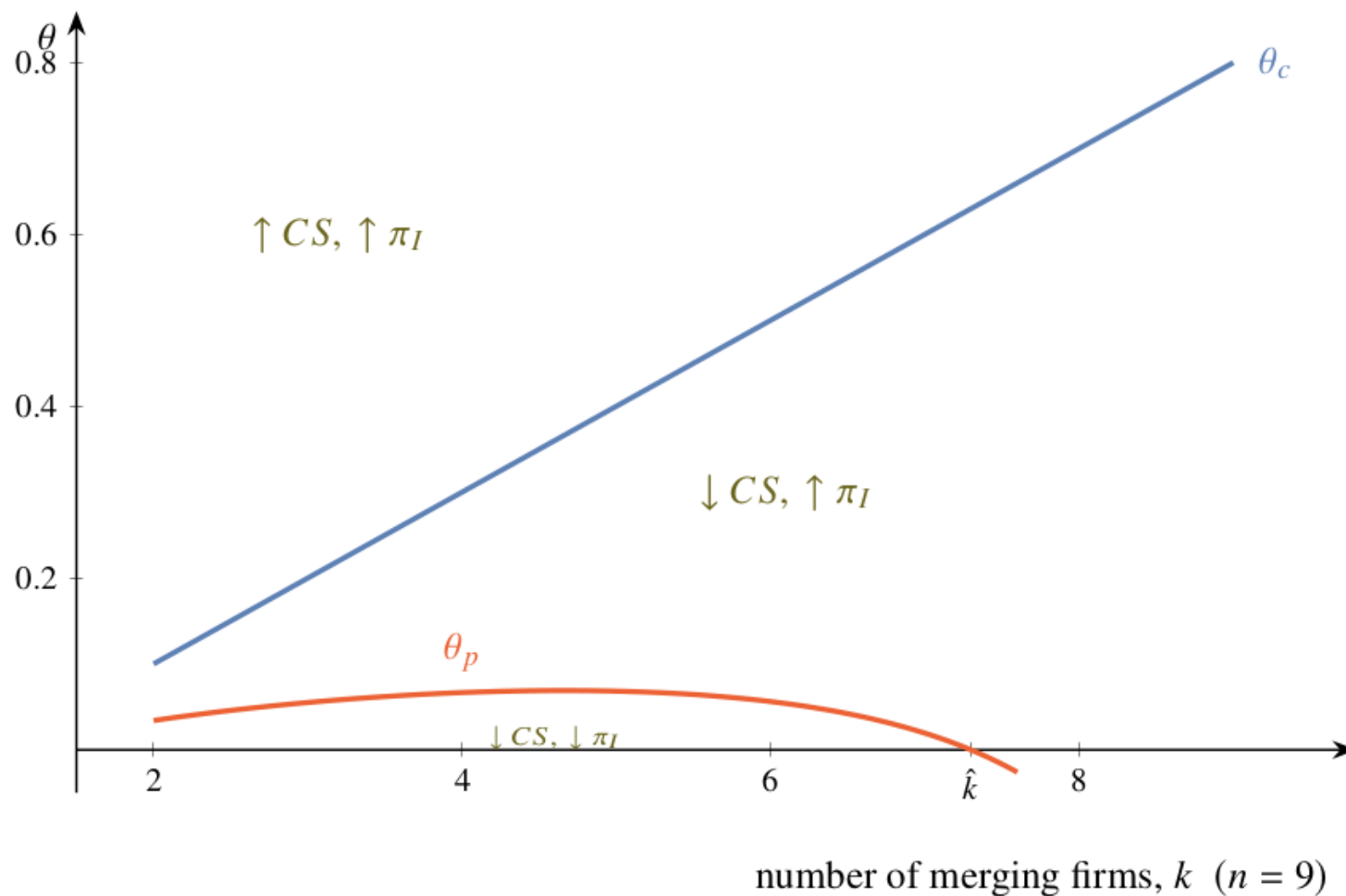
- Substituting Q^{post} and simplifying,

$$\frac{(a - c)(n - k + 1) + x}{n - k + 2} \geq \frac{n(a - c)}{n + 1},$$

$$\theta \geq \frac{k - 1}{n + 1} \equiv \theta_c.$$

- **Comparison.** Since $k \geq \sqrt{k}$, we have $\theta_p \leq \theta_c$: any merger that raises consumer surplus is already profitable, but not the reverse. A consumer-surplus authority approves only if $\theta \geq \theta_c$.

Welfare with synergies: adding consumer surplus



Welfare with synergies: the total-welfare threshold

- **Total welfare** is consumer surplus plus the merged firm's profit and the outsiders' profit:

$$W^{pre} = \frac{1}{2} \left(\frac{n(a-c)}{n+1} \right)^2 + n \left(\frac{a-c}{n+1} \right)^2,$$

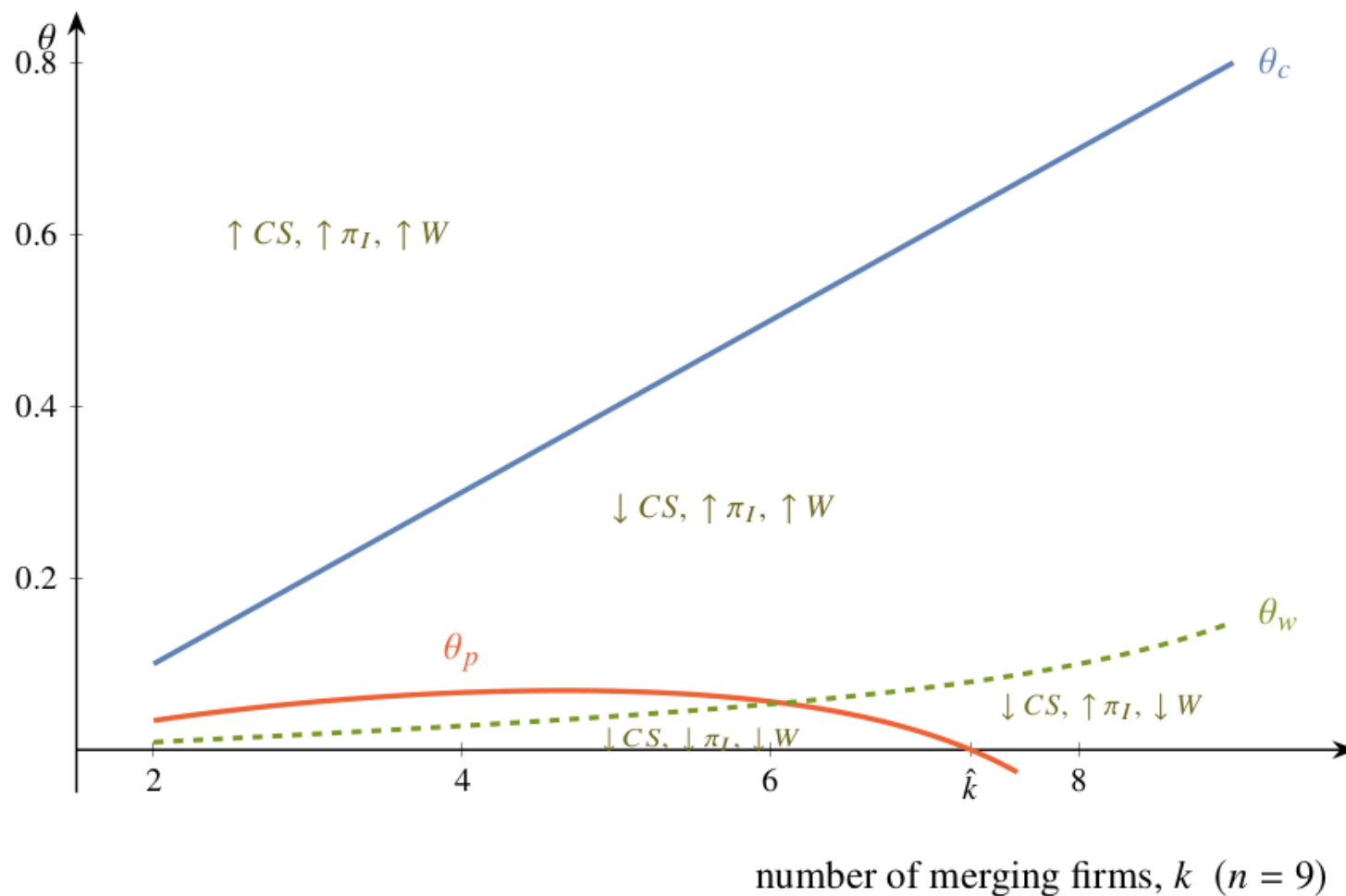
$$W^{post} = \frac{1}{2} (Q^{post})^2 + q_I^2 + (n-k)q_O^2.$$

- Setting $W^{post} \geq W^{pre}$ and solving for θ gives the welfare threshold

$$\theta_w = \frac{-(n+1)(n-k+3) + (n-k+2)\sqrt{n^2 - 2k^2 + 4kn + 6k}}{(n+1)(2n^2 + 2k^2 + 6n - 4kn - 6k + 3)}.$$

- **Ranking.** θ_c is always the highest bar. But θ_p and θ_w cross: $\theta_p > \theta_w$ for small k (a profitable merger raises welfare), and $\theta_p < \theta_w$ for large k (a profitable merger can lower welfare).

Welfare with synergies: the three thresholds



Worked example: merger with synergies

- Parameters.** $n = 5$ firms, $k = 2$ merging, with synergy intensity $\theta \equiv \frac{x}{a-c}$.

The three thresholds are:

Criterion	Threshold	Value
Total welfare	θ_w	0.024
Profitability	θ_p	0.045
Consumer surplus	$\theta_c = (k - 1)/(n + 1)$	0.167

- Result:** With $\theta = 0.03$, the merger raises total welfare but is not profitable, so it is not proposed. With $\theta = 0.10$, it is profitable and raises welfare, yet it harms consumers: an authority using the consumer surplus criterion would block it.

Welfare analysis of Cournot mergers (cont'd)

- welfare = total surplus
- **Lesson:** Suppose that mergers among Cournot competitors induce synergies. When a merger does not make the market too concentrated, profitability of the merger is a sufficient condition for welfare improvement. However, if the merger results in a highly concentrated market, it could be profitable and welfare-detrimental at the same time.

General welfare: why the external effect

- **The question.** Judge the merger by total surplus — consumer surplus plus the merged firm's profit Π_I and the outsiders' profit Π_O — and split it as

$$W = \Pi_I + (CS + \Pi_O).$$

- **A profitable merger** ($\Delta\Pi_I > 0$) raises welfare whenever its **external effect** on consumers and outsiders is positive,

$$\Delta(CS + \Pi_O) \geq 0.$$

- **Why split this way.** Estimating $\Delta\Pi_I$ needs the synergies, which are hard to observe; the external effect needs far less information.
- **What matters** for consumers and outsiders is only the change in the insiders' equilibrium output.

General welfare: the Cournot setup

- **A general model.** Drop the linear case: n Cournot firms, inverse demand $P(q)$ with q the industry output, and firm i has cost $C_i(q_i)$.
- **Assumptions.** Revenues satisfy $P'(q) + q_i P''(q) < 0$ (outputs are strategic substitutes) and costs are convex, $C_i'' \geq 0$.
- Profits are then concave, so each firm's first-order condition characterizes the equilibrium:

$$\frac{\partial \pi_i}{\partial q_i} = P(q) - C_i'(q_i) + P'(q)q_i = 0.$$

- **Intuition.** The markup equals $P - C_i' = -P'(q)q_i > 0$: each firm's price–cost margin is proportional to its own output.

General welfare: margins and market shares

- **Margins track shares.** Dividing the first-order conditions of any two firms i and j ,

$$\frac{P(q) - C'_i(q_i)}{P(q) - C'_j(q_j)} = \frac{q_i}{q_j} = \frac{s_i}{s_j}, s_i \equiv \frac{q_i}{q}.$$

- **The welfare object.** The merger's effect on everyone except the insiders is consumer surplus plus the outsiders' profit,

$$CS + \Pi_o = \int_0^q P(z) dz - P(q)q + \sum_{i \in O} [P(q)q_i - C_i(q_i)].$$

- The first two terms are consumer surplus; the sum is the outsiders' profit.
- **Intuition.** Shares summarize everything: a firm's margin, and so its weight in welfare, is proportional to its share.

General welfare: when is the external effect positive?

- **Outsiders' reaction.** Define $\lambda_i \equiv -\frac{dq_i}{dq} > 0$: when total output q falls, each outsider i expands.
- **The condition.** A merger among the insiders has a **positive external effect** on consumers and outsiders if

$$s_I < \sum_{i \in O} \lambda_i s_i,$$

- evaluated at the insiders' pre-merger output — an **upper bound** on the insiders' combined share s_I (Lesson 15.7).
- **Intuition.** If s_I is small the insiders barely cut output; the outsiders expand, shifting production to lower-cost firms — a welfare gain.

General welfare: proof step (i)

- **The proof has four steps.** Sign the change in $CS + \Pi_O$ as the insiders cut output, then integrate over the whole merger.
- **Step (i): the reaction term.** An outsider's best response $r_i = \frac{dq_i}{dq_{-i}}$ (with $q_{-i} = q - q_i$) satisfies $r_i \in (-1, 0)$. From $r_i dq_{-i} = dq_i$ and $dq = dq_i + dq_{-i}$:

$$r_i dq = r_i dq_i + r_i dq_{-i} = r_i dq_i + dq_i = (1 + r_i) dq_i.$$

$$\Rightarrow \frac{dq_i}{dq} = \frac{r_i}{1 + r_i} \equiv -\lambda_i < 0.$$

- Since $r_i < 0$ and $1 + r_i > 0$, we have $\lambda_i > 0$: outsiders expand when q falls (linear demand: $\lambda_i = 1$).

General welfare: proof (ii), differentiating consumer surplus

- **Step (ii): differentiate $CS + \Pi_o$.** Begin with consumer surplus,

$$CS = \int_0^q P(z) dz - P(q)q.$$

- The integral gives $P(q)dq$ (fundamental theorem of calculus), and the product $P(q)q$ gives $P'(q)q dq + P(q)dq$ (product rule). So

$$dCS = P(q)dq - [P'(q)q dq + P(q)dq].$$

- The two $P(q)dq$ terms cancel, leaving

$$dCS = -P'(q)q dq.$$

General welfare: proof (ii), differentiating the outsiders' profit

- **Now the outsiders' profit**, a sum over the $n - k$ outsiders:

$$\Pi_o = \sum_{i \in O} [P(q)q_i - C_i(q_i)].$$

- For each outsider, $d[P(q)q_i] = P'(q)q_i dq + P(q)dq_i$ (product rule) and $d[C_i(q_i)] = C'_i(q_i)dq_i$, so

$$d\Pi_o = \sum_{i \in O} [P'(q)q_i dq + (P(q) - C'_i(q_i))dq_i].$$

- Splitting the sum and writing $q_o \equiv \sum_{i \in O} q_i$,

$$d\Pi_o = P'(q)q_o dq + \sum_{i \in O} [P(q) - C'_i(q_i)]dq_i.$$

General welfare: proof (ii), combining and the first-order condition

- **Add the two pieces** ($dCS = -P'(q)q dq$ and the $d\Pi_O$ just found):

$$dCS + d\Pi_O = -P'(q)q dq + P'(q)q_O dq + \sum_{i \in O} [P(q) - C'_i(q_i)] dq_i.$$

- **Outsider's first-order condition.** Outsider i maximizes $\pi_i = P(q)q_i - C_i(q_i)$; $\partial \pi_i / \partial q_i = 0$ gives $P(q) + P'(q)q_i - C'_i(q_i) = 0$, so $P(q) - C'_i(q_i) = -P'(q)q_i$.
- Substituting into the last sum,

$$dCS + d\Pi_O = -P'(q)q dq + P'(q)q_O dq + \sum_{i \in O} [-P'(q)q_i] dq_i.$$

General welfare: proof (ii), substitute the reaction term

- **Regroup the first two terms.** Factor $P'(q)dq$: $-P'(q)q dq + P'(q)q_O dq = -P'(q)(q - q_O)dq$, and $q - q_O = q_I$, so they equal $-P'(q)q_I dq$.
- **The reaction term.** Write $dq_i = \frac{dq_i}{dq} dq = -\lambda_i dq$ from step (i); the sum becomes

$$\sum_{i \in O} [-P'(q)q_i] dq_i = \sum_{i \in O} [-P'(q)q_i](-\lambda_i) dq = P'(q) \sum_{i \in O} \lambda_i q_i dq.$$

- Putting the two pieces together,

$$dCS + d\Pi_O = -P'(q)q_I dq + P'(q) \sum_{i \in O} \lambda_i q_i dq = P'(q) \left(\sum_{i \in O} \lambda_i q_i - q_I \right) dq.$$

General welfare: proof (iii), integrating over the merger

- **Step (iii): integrate.** Divide the step-(ii) result by dq_I to get the rate of change along the merger:

$$\frac{d(CS + \Pi_O)}{dq_I} = P'(q) \left(\sum_{i \in O} \lambda_i q_i - q_I \right) \frac{dq}{dq_I}.$$

- The merger cuts the insiders' output from q_I^0 to q_I^f ($q_I^f < q_I^0$). Integrate:

$$\Delta(CS + \Pi_O) = \int_{q_I^0}^{q_I^f} \frac{d(CS + \Pi_O)}{dq_I} dq_I.$$

- Reverse the limits (flipping the sign) and insert the derivative:

$$\Delta(CS + \Pi_O) = \int_{q_I^f}^{q_I^0} [-P'(q)] \frac{dq}{dq_I} \left(\sum_{i \in O} \lambda_i q_i - q_I \right) dq_I.$$

General welfare: proof (iv), signing the effect

- **Step (iv): sign it.** From step (i) $\frac{dq}{dq_I} > 0$, and $-P'(q) > 0$, so the sign of $\Delta(CS + \Pi_O)$ equals the sign of the bracket:

$$\Delta(CS + \Pi_O) > 0 \Leftrightarrow \sum_{i \in O} \lambda_i q_i - q_I > 0.$$

- Dividing by total output q turns outputs into shares $s_i = q_i/q$:

$$\sum_{i \in O} \lambda_i \frac{q_i}{q} > \frac{q_I}{q} \Leftrightarrow s_I < \sum_{i \in O} \lambda_i s_i.$$

- **Conclusion.** A profitable merger raises welfare exactly when $s_I < \sum_{i \in O} \lambda_i s_i$. (Monotonicity needs $P'' \geq 0, P''' \geq 0, C_i''' \leq 0$.)

General welfare: linear Cournot and the fifty-percent rule

- **Linear Cournot.** With linear demand and constant marginal cost every $\lambda_i = 1$, so the condition becomes

$$s_I < \sum_{i \in O} s_i = 1 - s_I \Leftrightarrow s_I < \frac{1}{2}.$$

- **Farrell and Shapiro.** A positive external effect arises exactly when the insiders' combined pre-merger share is below 50%.
- **Tension.** Profitability (no synergies) needs a combined share above 80%, but a positive external effect needs it below 50%: a profitable no-synergy merger lowers welfare; only a highly unprofitable one raises it.
- **Lesson.** Only the insiders' share and the outsiders' reaction matter — not the size of the synergies.

Successive mergers

- If 2 mergers raise consumer surplus in isolation, they also raise consumer surplus when they take place together.
 - Conditional on one merger being proposed and approved, because (i) it is profitable, and (ii) it increases consumer surplus,
 - 2nd merger remains profitable and beneficial to consumers
- So the competition authority can use a myopic policy (i.e., look at one merger at a time).
- **Lesson:** In environments where multiple disjoint mergers can be proposed over time, an antitrust authority that is concerned with the maximization of (discounted) consumer surplus can use a myopic policy that approves mergers if they do not decrease consumer surplus at the time of approval.

Successive mergers: total output in linear Cournot

- **Inverse demand** $P(q) = a - q$. A Cournot competitor i with marginal cost c_i has first-order condition $a - c_i - 2q_i - q_{-i} = 0$.
- Summing over all n firms, $na - \sum_{i=1}^n c_i - 2q - (n-1)q = 0$, so total output is

$$q = \frac{na - C}{n+1}, C \equiv \sum_{i=1}^n c_i.$$

- Before any merger every firm has cost c , so $C = nc$ and

$$q_{ea} = \frac{n(a-c)}{n+1}.$$

- **Intuition.** A merger raises consumer surplus if and only if it raises total output q .

Successive mergers: when does one merger raise surplus?

- **One merger.** If the k_j firms in set I_j merge, $n - k_j + 1$ firms remain with cost sum $C_j = (n - k_j + 1)c - x_j$ (synergy x_j). Total output is

$$q_j = \frac{(n - k_j + 1)(a - c) + x_j}{n - k_j + 2}.$$

- It raises consumer surplus iff $q_j \geq q_{ea}$, that is

$$\theta_j \equiv \frac{x_j}{a - c} \geq \frac{k_j - 1}{n + 1} \equiv \theta_c.$$

- **Intuition.** This is exactly the consumer-surplus threshold θ_c from the welfare analysis: a merger helps consumers when its synergy clears that bar.

Successive mergers: two mergers together

- **Two mergers.** With both sets I_1 and I_2 merging, $n - k_1 - k_2 + 2$ firms remain and total output is

$$q_{12} = \frac{(n - k_1 - k_2 + 2)(a - c) + x_1 + x_2}{n - k_1 - k_2 + 3}.$$

- Together they raise consumer surplus iff $q_{12} \geq q_{ea}$:

$$\frac{x_1 + x_2}{a - c} \geq \frac{k_1 + k_2 - 2}{n + 1}.$$

- **Complementarity.** The bar is the **sum** of the two single-merger thresholds, since $(k_1 - 1) + (k_2 - 1) = k_1 + k_2 - 2$. So if each merger raises consumer surplus alone, the two do together.
- **Lesson.** A consumer-surplus-maximizing authority can approve mergers myopically, one at a time (Lesson 15.8).

Case. Consolidation in the US airline industry?

- February 2008:
 - Delta Airlines and Northwest Airlines announce their plan to merge.
 - Other rumours of mergers
 - United Airlines and Continental
 - Continental and American Airlines
- Airlines executives declared:
 - Mergers were conditional on one another.
 - 2 major airlines needed to merge for the other airlines to follow suit.
 - OK with status quo if 1st merger did not take place.



Mergers and entry

- The profitability of a merger should be questioned if entry can occur.
 - If the entry threat is immediate, there does not exist any rationale for a merger except for efficiency motives.
- Shows that entry barriers are at the heart of the competitive effects of mergers.
- **Lesson:** If a merger only temporarily leads to a smaller number of firms because there is subsequent entry, a merger in a Cournot model is not profitable when firms are sufficiently patient (unless the merged firm enjoys a much higher productive efficiency).

Mergers and entry: free entry and the merger incentive

- **Free entry.** With fixed cost f , a duopoly prevails before the merger when $\pi(2) > f > \pi(3)$, where $\pi(m)$ is per-firm profit (gross of f) with m active firms.
- **Without entry**, a merger to monopoly also saves one fixed cost, so it is profitable:

$$\pi(1) > 2\pi(2) \Rightarrow \pi(1) - f > 2\pi(2) - 2f.$$

- **With entry.** In an infinitely repeated Cournot game with a merger stage, entry becomes effective Δt periods after a successful takeover; firms discount at δ .
- **Intuition.** The merged firm is a monopolist only temporarily, until entry restores the duopoly.

Mergers and entry: value with delayed entry

- **No merger.** Each firm's discounted profit is

$$V(2) = \frac{1}{1 - \delta} (\pi(2) - f).$$

- **Two extremes.** If $\Delta t = \infty$ (entry impossible) the merger to monopoly is necessarily profitable. If $\Delta t = 0$ (immediate entry) the merged firm meets a new rival at once, nets zero, and there is no rationale to merge — entry barriers are at the heart of merger effects.
- **With delay Δt .** After a takeover bid of at least $V(2)$, firm 1 is a monopolist for Δt periods, then a duopolist:

$$V(1) = \sum_{t=0}^{\Delta t-1} \delta^t [\pi(1) - f] + \sum_{t=\Delta t}^{\infty} \delta^t [\pi(2) - f].$$

Mergers and entry: the profitability condition

- **Profitable iff** $V(1)$ minus the acquisition cost exceeds $V(2)$. Since the cost is $V(2)$, the condition is $V(1) > 2V(2)$.

- Take $\Delta t = 1$ (monopoly for one period). Then

$$\pi(1) - f > (2 - \delta) \frac{\pi(2) - f}{1 - \delta}.$$

- Rearranging for the discount factor,

$$\delta < 1 - \frac{\pi(2) - f}{\pi(1) - \pi(2)} \equiv \bar{\delta}.$$

- Since $\pi(2) > f$ gives $\bar{\delta} < 1$ and $\pi(1) + f > 2\pi(2)$ gives $\bar{\delta} > 0$, the merger pays only if firms are **impatient** ($\delta < \bar{\delta}$). Patient firms value the lasting duopoly, so a temporary monopoly is not worth it (Lesson 15.9).

Mergers under price competition

- Suppose that firms set prices in a differentiated product market.
which gives stronger incentives to merge
- **Why?** With price competition, firms compete less aggressively after a merger.
 - Outsiders set a higher price.
 - Merged firm then benefits from these higher prices.
- With price competition, there is a rationale for mergers even in a non-concentrated industry.

Price competition: demand and best responses

- **Price competition.** Products are differentiated and firms set prices. Demand for product i is

$$Q_i = a - p_i + d\bar{p}_{-i}, \bar{p}_{-i} = \frac{1}{n-1} \sum_{j \neq i} p_j.$$

- With zero marginal cost $\pi_i = p_i Q_i$; the first-order condition gives the best response

$$p_i = \frac{a}{2} + \frac{d}{2} \bar{p}_{-i}.$$

- Best responses slope up: prices are **strategic complements**. The symmetric pre-merger equilibrium ($p_i = \bar{p}_{-i}$) is

$$p_{ea}^* = \frac{a}{2-d}.$$

Price competition: the merged two-product firm

- **The merged firm.** Two firms merge into a two-product firm I charging one price p_I ; the $n - 2$ outsiders each set p_O . Its profit is

$$\pi_I = 2p_I \left(a - p_I + \frac{d}{n-1} (p_I + (n-2)p_O) \right),$$

- with first-order condition

$$a - 2p_I + \frac{2d}{n-1} p_I + \frac{d(n-2)}{n-1} p_O = 0.$$

- **Internalization.** Cutting one product's price steals demand from its **own** other product, so the merged firm prices less aggressively — its best response shifts **out**.
- With strategic complements the outsiders then raise prices too, so $p_I^* > p_O^* > p_{ea}^*$: a merger raises all prices.

Price competition: equilibrium prices

- **Solving** the two-equation system (the merged firm's condition and an outsider's best response) gives

$$p_I^* = \frac{a(n-1)(2n-2+d)}{2[(2-d)(n-1)^2 - d^2]},$$

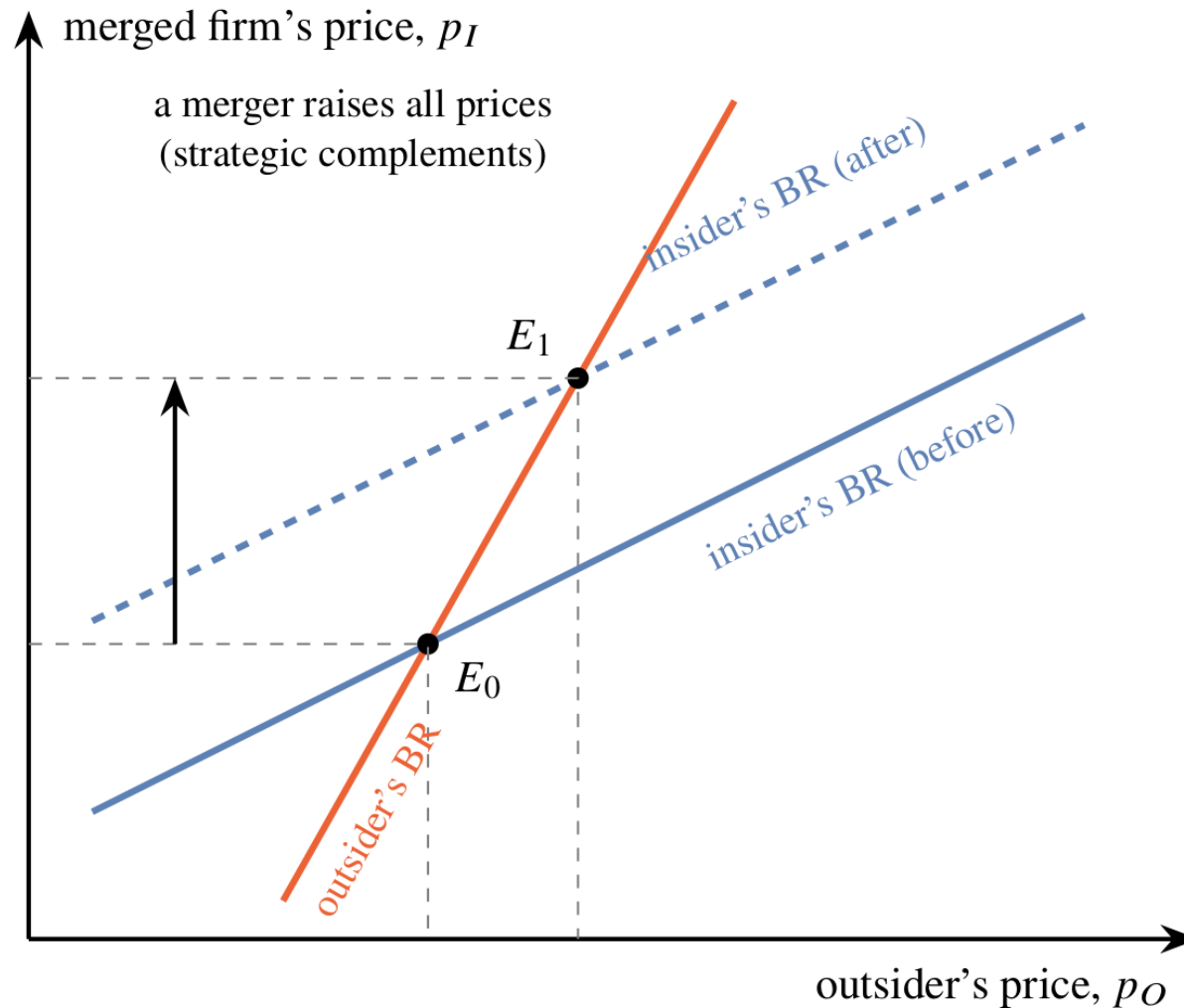
$$p_O^* = \frac{a(n-1)^2}{(2-d)(n-1)^2 - d^2}.$$

- One checks $p_I^* > p_O^* > p_{ea}^*$: a merger raises prices for **all** firms (Lesson 15.10).
- **Deneckere and Davidson.** The outsiders' higher prices raise the merged firm's profit even with **no** efficiency gains — strong private incentives to merge, unlike under quantity competition.

Mergers under price competition (cont'd)

- **Lesson:** If firms have differentiated products and compete in prices, a merger between two firms leads to higher prices for all firms.
- In contrast to quantity competition there are strong private incentives to merge absent any efficiency gains.
- Merger is profitable and necessarily welfare-reducing if there are no efficiency gains.

Price competition: a merger raises prices



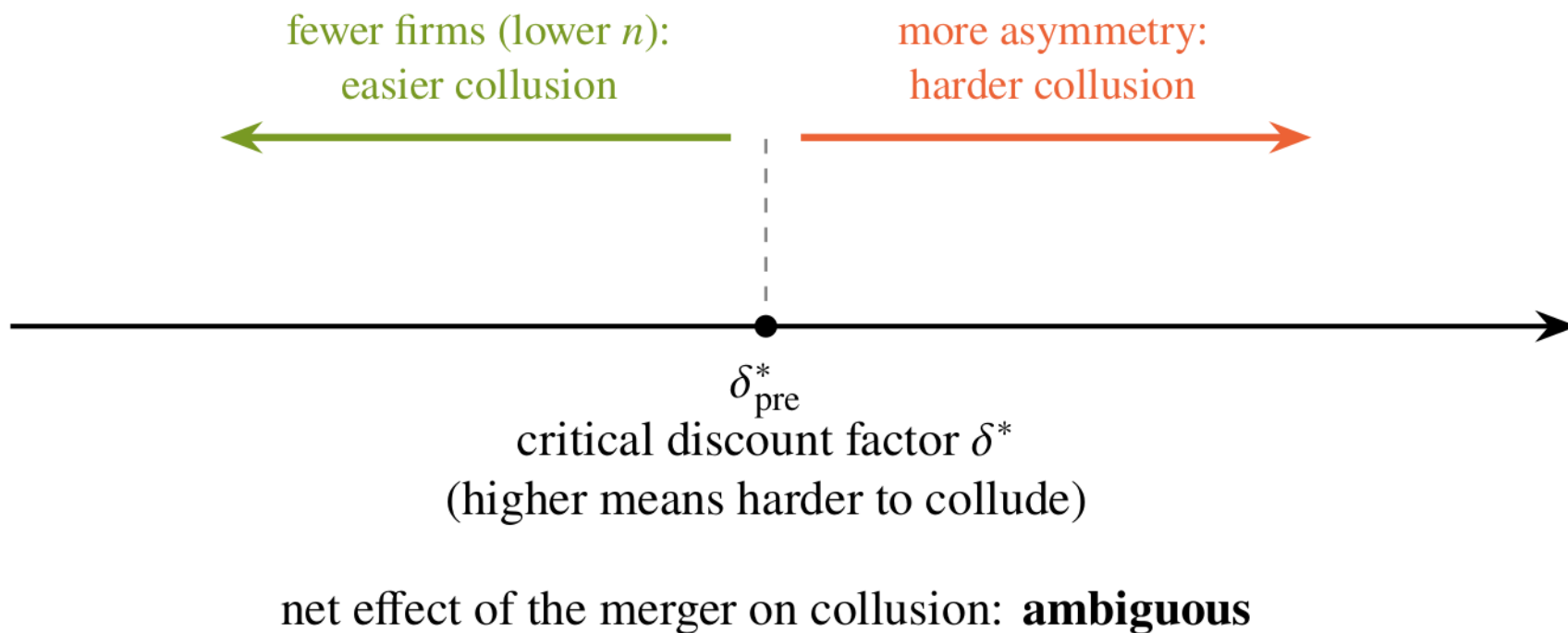
Coordinated effects

- Possibility that a merger could increase the incentives of the remaining firms to engage in tacit collusion (**coordinated effects**)
- Contrasting impacts on sustainability of collusion
 - Merger reduces the number of firms in the market, which makes tacit collusion easier to sustain.
 - Merger affects asymmetries among colluding firms.
 - In a market which is symmetric prior to the merger, a merger introduces asymmetries.
 - In the presence of synergies, merged entity may become a more efficient producer than outside firms.
 - In differentiated good industries, merged entity may own a larger set of brands.

Coordinated effects (cont'd)

- Contrasting impacts (cont'd)
 - Asymmetries among firms make tacit collusion harder to sustain.
 - More difficult to define the common collusive price (more efficient firms prefer lower prices than less efficient ones).
 - Allocation of production quotas is also harder to agree upon (equal sharing would typically be inefficient when some firms are more efficient than others).
- **Lesson:** A merger may exert 2 conflicting effects on the sustainability of collusion: a positive effect through the reduction in the number of firms and a negative effect if it increases the asymmetry among firms.

Coordinated effects: two conflicting forces





Case. Nestlé-Perrier merger

Pre-merger		Proposition 1		Proposition 2		Proposition 3	
Perrier	36%	Nestlé + Perrier	53%	Nestlé + Perrier – Volvic	38%	Nestlé + Perrier – Volvic – other brands	
Nestlé	17%						
BSN	23%	BSN	23%	BSN + Volvic	38%		
Others	24%	Others	24%	Others	24%		

Refused because
dominant position

Refused because
coordinated effects

OK

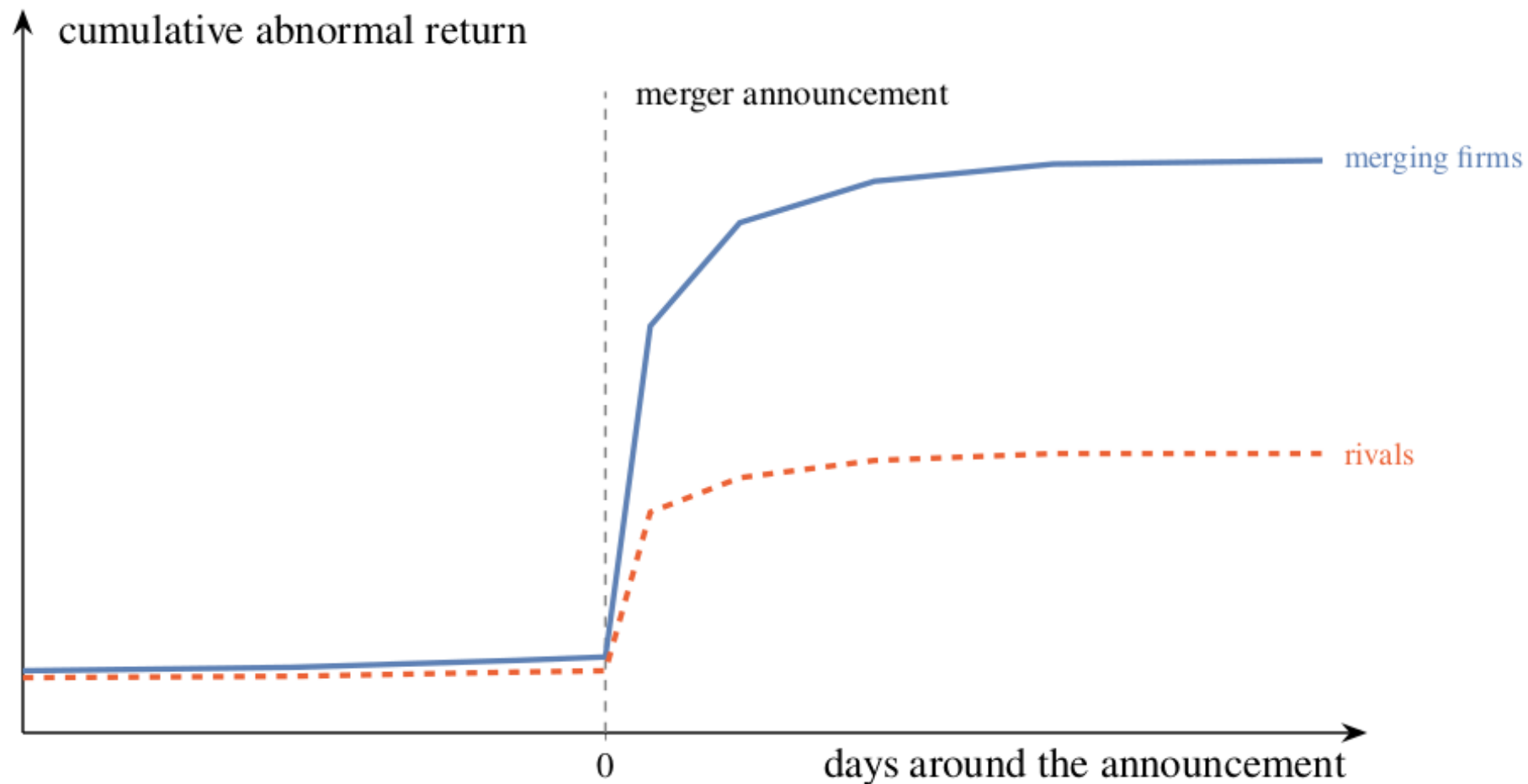
Event studies

- Analysis of stock market performance of merging firms and outsiders following
 - a merger announcement,
 - the eventual investigation and subsequent decision of the competition authority.
- Learn the potential effects of a merger from the reaction of the stock market.
 - Increase in the equity value of the merging firms and of the outsiders indicates that the merger is likely to result in
 - higher prices
 - thus higher profits,
 - at the expense of consumers.

Event studies (cont'd)

- Not clear
 - when stock market participants learn about a merger announcement
 - what they learn from it
 - the announcement may simply inform about the good health of the industry.
- **Intuition.** What matters is the rivals' reaction: if rivals gain, the market expects weaker competition and higher prices; if they lose, a tougher, more efficient merged firm.

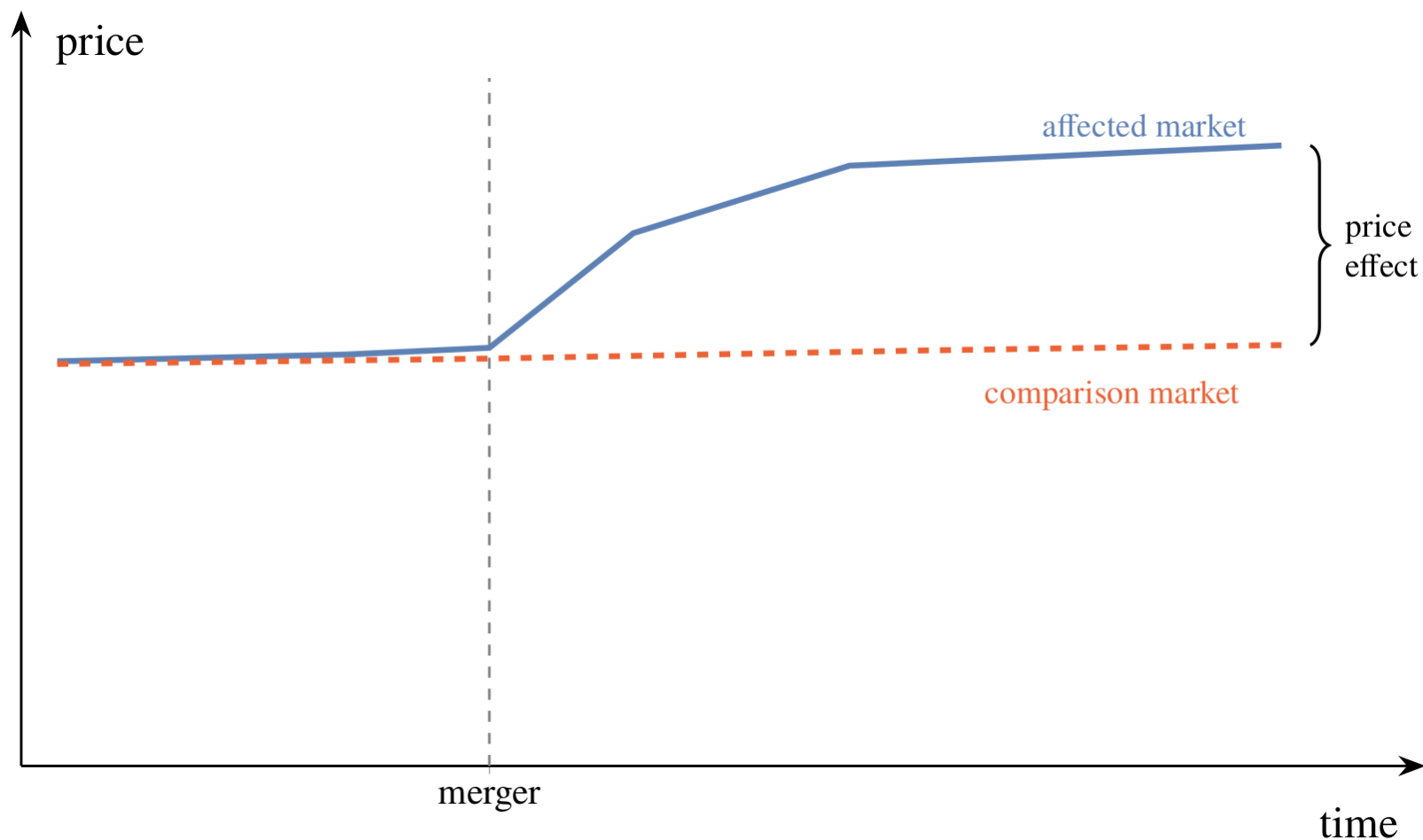
Event studies: abnormal returns around the announcement



Direct price comparisons

- Direct comparisons of prices before and after the merger
- Can help competition authorities to identify the characteristics that are likely to make a merger anticompetitive or efficiency-enhancing.
- Ex post analysis may be used to assess the performance of ex ante analyses.
- **Intuition.** The difficulty is finding a valid comparison, a market the merger did not affect, so the price change reflects the merger and not an industry-wide shock.

Direct price comparisons: the merger's price effect



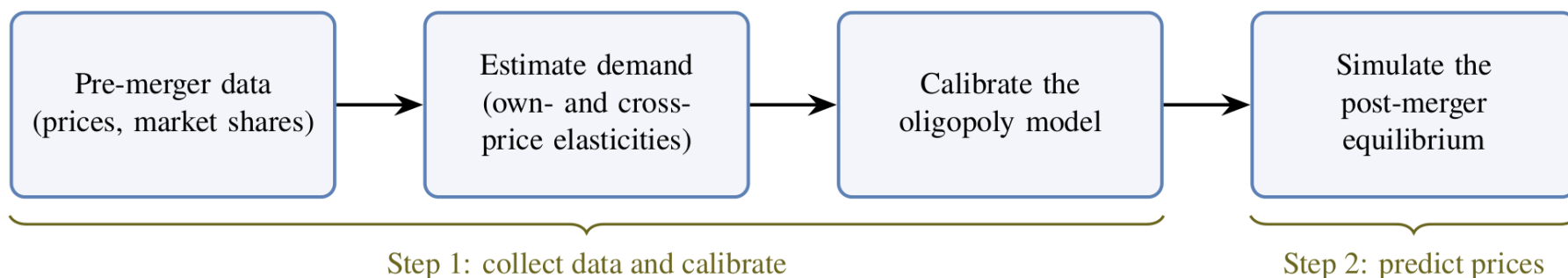
Merger simulations

- 2 steps
 - Collect pre-merger market information
 - To estimate direct and cross elasticities
 - To calibrate a one-shot-non-cooperative oligopoly model that matches the critical features of the industry under review.
 - Use calibrated model to calculate post-merger equilibrium
- *Almost Ideal Demand System (AIDS)* or nested / random coefficient logit models to analyse markets with differentiated consumer products
- Quantify and disentangle the impacts a merger is likely to have on concentration and efficiencies

Merger simulations (cont'd)

- Serious limitations
 - Price predictions subject to modelling error
 - In their current form, focus on the immediate price and output effects of mergers
 - Leave aside longer-term potential impacts of the merger
 - Usually assume that the firms' behaviour does not change as a result of the merger.
- **Intuition.** A simulation is only as good as its demand estimates and its assumption about how firms compete: wrong inputs give the wrong predicted price change.

Merger simulations: from data to predicted prices



Review questions

- Explain why mergers that do not involve efficiency gains are rarely profitable in Cournot industries.
- What is the main advantage of assessing the welfare impact of a merger by looking at its 'external effect'? Explain in words the condition under which this external effect is positive.
- Taking successive mergers and entry into account, discuss whether and, if yes, how merger analysis has to be reconsidered.
- Explain the fundamental differences between mergers in Cournot and in Bertrand industries.
- What is meant by 'coordinated effects' of a merger. Discuss.