

Signaling a Clean Product with Its Price

A step-by-step exercise in the spirit of Mahenc (2008)

Companion note to Mahenc, P. (2008), “Signaling the Environmental Performance of Polluting Products to Green Consumers,”
International Journal of Industrial Organization 26(1): 59–68. Full references at the end.

Overview. Many products claim to be “green,” but consumers cannot check how much pollution a product generates, and a firm’s claim is cheap to make. Mahenc (2008) asks whether the price can do the job that the claim cannot. His answer extends the classic result that a high price can signal high quality (Milgrom and Roberts, 1986) to a setting where consumers care about the *total* pollution associated with a product, so that demand and emissions are linked. When a cleaner product is more costly to produce, a clean firm signals with a price above its full-information level, and, because it sells less, it also pollutes less than under full information. When a cleaner product is cheaper to produce, as in the Porter hypothesis, the distortions go the other way. This note builds the mechanism with uniform tastes and linear emissions, so that every equilibrium has closed form. Step (a) solves the full-information benchmark; Step (b) states the firm’s incentives when consumers cannot observe cleanness; Step (c) finds the separating equilibrium with two types when cleaner products are costlier; Step (d) reverses the cost ranking; Step (e) solves the continuum-of-types model; Step (f) asks whether a label that reveals cleanness raises welfare; and Step (g) lists what the toy model keeps and drops.

1. The Toy Model

Emissions. A monopolist produces q units of a good. Production generates pollution

$$e = \frac{q}{\theta}, \quad (1)$$

where $\theta > 0$ is the environmental performance of the product: a higher θ means a cleaner product. This is the emission function $e = \frac{t(q)}{\theta}$ of the paper with $t(q) = q$.

Consumers. A unit mass of consumers have basic willingness to pay r , uniformly distributed on $[0, 1]$, and each buys at most one unit. A buyer bears a private damage $v(e) = e$ from the pollution of the product (a health cost, or the guilt from buying a dirty good), so a buyer of type r obtains $r - e - p$. The global damage of pollution falls on buyers and non-buyers alike and therefore does not affect who buys; it enters only the welfare analysis of Step (f).

Demand. The marginal buyer $\tilde{r}$ satisfies $\tilde{r} - e - p = 0$, and the buyers are those with $r \geq \tilde{r}$, so $q = 1 - \tilde{r}$. Substituting (1), $\tilde{r} - \frac{1-\tilde{r}}{\theta} - p = 0$, and solving,

$$D(p, \theta) = \alpha(\theta) (1 - p), \quad \alpha(\theta) \equiv \frac{\theta}{1 + \theta}. \quad (2)$$

Demand is the standard linear demand $1 - p$ scaled down by the factor $\alpha(\theta) < 1$: every extra buyer adds pollution that lowers the willingness to pay of all buyers, and this effect is weaker for a cleaner product. Pollution is then

$$e = \frac{D(p, \theta)}{\theta} = \frac{1 - p}{1 + \theta}, \quad (3)$$

which falls with the price and with cleanness. The inverse demand is $p = 1 - q - \frac{q}{\theta}$, and consumer surplus is $CS = \frac{q^2}{2}$.

Costs. The cost of production is $c(q, \theta) = m(\theta) q$, with a constant marginal cost $m(\theta)$. The paper's main case is that a cleaner product is more costly to produce, $m'(\theta) > 0$, so that $c_{q\theta} > 0$.

Numerical example. A dirty type $\theta_L = 1$ and a clean type $\theta_H = 2$, so $\alpha_L = \frac{1}{2}$ and $\alpha_H = \frac{2}{3}$. In Steps (a) to (c) the marginal costs are $m_L = 0$ and $m_H = \frac{1}{4}$.

Figure 1 depicts demand (2) and pollution (3) for the two types, with the price on the horizontal axis. Both demands vanish at $p = 1$, but the clean type's demand is higher at every lower price, because it scales the standard demand by $\alpha_H = \frac{2}{3}$ instead of $\alpha_L = \frac{1}{2}$. Pollution is lower for the clean type at every price, because each unit sold pollutes less, and it falls with the price for both types because fewer units are sold.

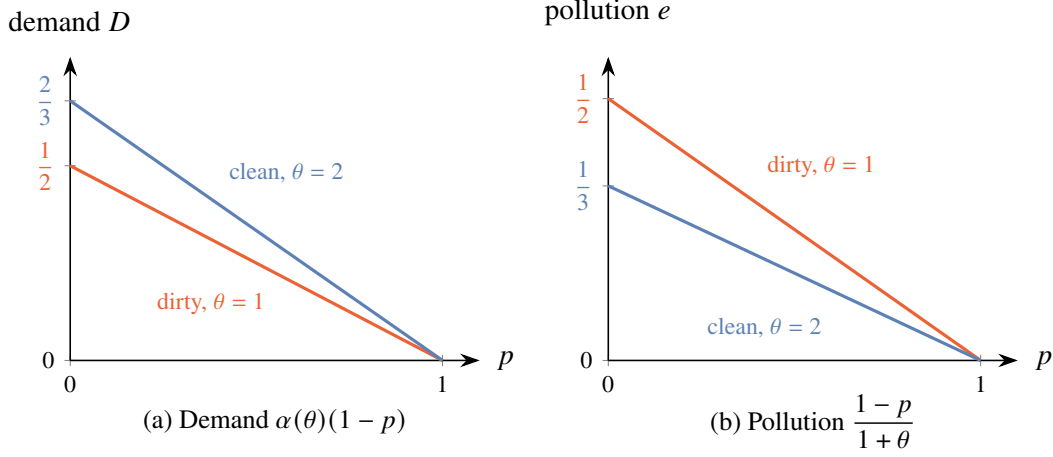


Figure 1: Demand and pollution against the price for the dirty type ($\theta_L = 1$) and the clean type ($\theta_H = 2$).

2. Step (a): Full Information

If consumers observe θ , the firm maximizes $(p - m)\alpha(\theta)(1 - p)$, so

$$p^*(\theta) = \frac{1 + m(\theta)}{2}, \quad q^*(\theta) = \frac{\alpha(\theta)(1 - m(\theta))}{2}, \quad \pi^*(\theta) = \frac{\alpha(\theta)(1 - m(\theta))^2}{4}. \quad (4)$$

The price elasticity of demand is $\eta = \frac{p}{1-p}$, and at p^* the Lerner index satisfies

$$\frac{p^* - m}{p^*} = \frac{1 - m}{1 + m} = \frac{1}{\eta(p^*)}, \quad (5)$$

the standard result that Lemma 1 of the paper records. Cleanness scales demand by the factor $\alpha(\theta)$, which rises with θ , without changing the elasticity at a given price, so it raises profit through the size of the market. In the example, the dirty type charges $p_L^* = \frac{1}{2}$, sells $\frac{1}{4}$, earns $\pi_L^* = \frac{1}{8}$, and generates $e_L^* = \frac{1}{4}$; the clean type charges $p_H^* = \frac{5}{8}$, sells $\frac{1}{4}$, earns $\pi_H^* = \frac{3}{32}$, and generates $e_H^* = \frac{1}{8}$.

Figure 2 depicts the profit of each type against its price. The dirty type's profit $\frac{1}{2} p (1 - p)$ is zero at $p = 0$ and at $p = 1$ and peaks at $p_L^* = \frac{1}{2}$ with height $\frac{1}{8}$. The clean type's profit $\frac{2}{3} (p - \frac{1}{4})(1 - p)$ is zero at its marginal cost $p = \frac{1}{4}$ and at $p = 1$, and peaks at $p_H^* = \frac{5}{8}$ with height $\frac{3}{32}$. The clean type charges more because its marginal cost is higher, and it earns less because its higher cost outweighs its larger market. These two curves, and their crossings with the profit levels in Step (c), are the ingredients of Figure 4.

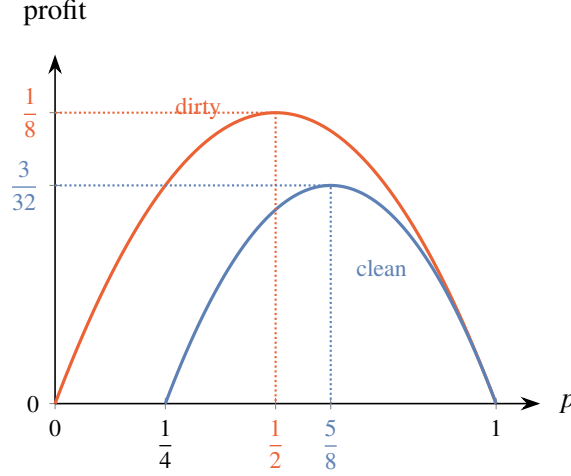


Figure 2: Profit of each type against its price under full information, with $(m_L, m_H) = (0, \frac{1}{4})$.

3. Step (b): Hidden Cleanness and the Firm's Incentives

Now consider that only the firm observes θ . Consumers see the price and form an inference $\hat{\theta}$, and they buy according to $D(p, \hat{\theta})$. A firm of type θ that is perceived as $\hat{\theta}$ earns

$$\pi(p, \theta, \hat{\theta}) = \alpha(\hat{\theta}) (p - m(\theta))(1 - p). \quad (6)$$

We look for separating perfect Bayesian equilibria, in which different types charge different prices and consumers infer the type from the price. Two properties of (6) drive the analysis.

Every type wants to be perceived as clean. Since α is increasing, $\pi_{\hat{\theta}} > 0$ whenever $p > m(\theta)$ (Condition 2 of the paper). The worst inference for any type is therefore the dirtiest one.

Single crossing. The perceived type multiplies profit, so a type's ranking of prices for a given perception depends only on $\ln[(p - m)(1 - p)]$, whose slope in p is

$$\frac{\partial}{\partial p} \ln[(p - m)(1 - p)] = \frac{1}{p - m} - \frac{1}{1 - p}, \quad \frac{\partial}{\partial m} \left[\frac{1}{p - m} - \frac{1}{1 - p} \right] = \frac{1}{(p - m)^2} > 0. \quad (7)$$

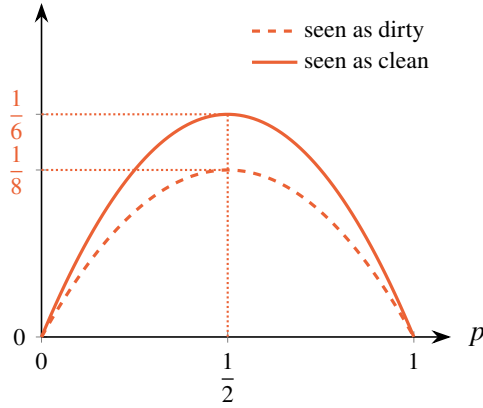
A firm with a higher marginal cost gains more, in proportional terms, from raising its price: a price increase raises its thinner margin by a larger proportion, while the proportional loss of sales, $\frac{1}{1-p}$ per unit of price, is the same for every type. When cleaner products are more costly ($m' > 0$), a clean firm is more willing than a dirty firm to trade a higher price for a better perception. In terms of the paper's Condition 4, the marginal rate of substitution between price and perceived type satisfies

$$\frac{\partial}{\partial \theta} \left[-\frac{\pi_{\hat{\theta}}}{\pi_p} \right] = \frac{c_{q\theta} D_{\hat{\theta}} D}{\pi_p^2}, \quad (8)$$

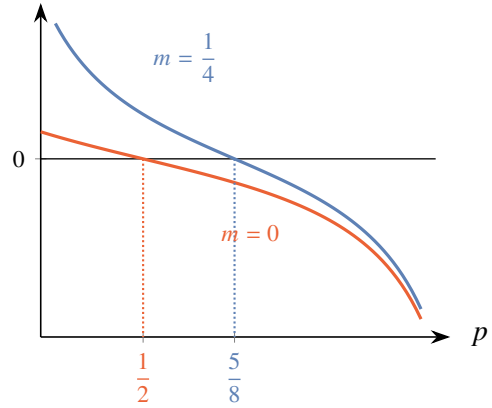
and since $D_{\hat{\theta}} = \frac{1-p}{(1+\theta)^2} > 0$, the sign of the single-crossing condition is the sign of $c_{q\theta}$.

Figure 3 depicts both properties. Panel (a) shows the dirty type's profit when it is perceived as dirty and when it is perceived as clean: the second curve is the first one multiplied by $\frac{\alpha_H}{\alpha_L} = \frac{4}{3}$, so it lies above it at every price and the two peak at the same price. Panel (b) shows the slope (7) of $\ln[(p - m)(1 - p)]$ for $m = 0$ and $m = \frac{1}{4}$. The slope is higher for the costlier type at every price, and it is zero at the price that maximizes $(p - m)(1 - p)$, which is $\frac{1}{2}$ for $m = 0$ and $\frac{5}{8}$ for $m = \frac{1}{4}$. At a price between $\frac{1}{2}$ and $\frac{5}{8}$, for example, the dirty type's profit falls in proportional terms while the clean type's profit is still rising.

profit of the dirty type



(a) Perception raises profit

slope of $\ln[(p-m)(1-p)]$ 

(b) Single crossing

Figure 3: Profit of the dirty type ($m = 0$) when perceived as dirty (dashed, α_L) and as clean (solid, α_H), and the slope of $\ln[(p-m)(1-p)]$ for $m = 0$ and $m = \frac{1}{4}$.

4. Step (c): Two Types when Cleaner Products Are Costlier

Let consumers believe that the firm is clean if it charges p_H and dirty otherwise. In a separating equilibrium the dirty type is recognized, so it charges its full-information price $p_L^* = \frac{1}{2}$ and earns $\frac{1}{8}$: there is no distortion at the bottom (Lemma 2 of the paper). The clean type's price is determined by two incentive constraints. Each one compares a profit that depends on the price, which is a curve in p_H , with a payoff that does not depend on it, which is a horizontal line, and Figure 4 is built from exactly these four objects.

The dirty type must not mimic. Charging p_H would make the dirty type look clean, and it would earn $\alpha_H(p_H - m_L)(1 - p_H) = \frac{2}{3}p_H(1 - p_H)$, a curve, instead of its equilibrium profit $\frac{1}{8}$, a line. Separation requires

$$\frac{2}{3}p_H(1 - p_H) \leq \frac{1}{8}. \quad (9)$$

The clean type must prefer to separate. If the clean type charges any other price, consumers believe it is dirty, and its best such deviation is its full-information price against the dirty type's demand, which earns $\alpha_L \frac{(1-m_H)^2}{4} = \frac{9}{128}$, a line. Charging p_H earns $\alpha_H(p_H - m_H)(1 - p_H) = \frac{2}{3}(p_H - \frac{1}{4})(1 - p_H)$, a curve. Separation requires

$$\frac{2}{3}\left(p_H - \frac{1}{4}\right)(1 - p_H) \geq \frac{9}{128}. \quad (10)$$

Rearranging the conditions. Each condition is a quadratic inequality in p_H . Moving everything to one side of (9), and multiplying (10) by $\frac{3}{2}$ to get $(p_H - \frac{1}{4})(1 - p_H) \geq \frac{27}{256}$ before doing the same,

$$\begin{aligned} (9) \quad &\iff p_H^2 - p_H + \frac{3}{16} \geq 0 \iff \left(p_H - \frac{1}{4}\right)\left(p_H - \frac{3}{4}\right) \geq 0 \\ &\iff p_H \leq \frac{1}{4} \text{ or } p_H \geq \frac{3}{4}, \end{aligned} \quad (11)$$

$$\begin{aligned} (10) \quad &\iff p_H^2 - \frac{5}{4}p_H + \frac{91}{256} \leq 0 \iff \left(p_H - \frac{7}{16}\right)\left(p_H - \frac{13}{16}\right) \leq 0 \\ &\iff \frac{7}{16} \leq p_H \leq \frac{13}{16}. \end{aligned} \quad (12)$$

The four roots are the prices at which a curve crosses its line, and they are the numbers marked on the horizontal axis of Figure 4(a).

Constructing Figure 4(a). Students can reproduce the panel in six steps, using the numbers of Table 1.

1. Draw the price p on the horizontal axis, from 0 to 1, and profit on the vertical axis, from 0 to about 0.2.
2. Plot the profit that the dirty type would earn if it were seen as clean, $\frac{2}{3}p(1-p)$. It is a downward parabola that is zero at $p = 0$ and $p = 1$ and peaks at $p = \frac{1}{2}$ with height $\frac{1}{6}$.
3. Draw the horizontal line at the dirty type's equilibrium profit, $\frac{1}{8}$. The parabola crosses it where $p(1-p) = \frac{3}{16}$, that is, at $p = \frac{1}{4}$ and $p = \frac{3}{4}$, and lies above it in between, where the dirty type would mimic.
4. Plot the profit of the clean type if it is seen as clean, $\frac{2}{3}(p - \frac{1}{4})(1-p)$, for $p \geq \frac{1}{4}$. It is zero at $p = \frac{1}{4}$ and $p = 1$ and peaks at $p = \frac{5}{8}$ with height $\frac{3}{32}$, its full-information profit.
5. Draw the horizontal line at the clean type's profit if it is seen as dirty, $\frac{9}{128}$. The curve crosses it where $(p - \frac{1}{4})(1-p) = \frac{27}{256}$, at $p = \frac{7}{16}$ and $p = \frac{13}{16}$, and lies above it in between, where the clean type prefers to separate.
6. Shade the prices at which the dirty type's curve is below its line and the clean type's curve is above its line, which gives $[\frac{3}{4}, \frac{13}{16}]$. The prices $p \leq \frac{1}{4}$ also satisfy (11), but there the clean type's curve is below its line.

Object in Figure 4(a)	Formula	Zeros or crossings	Peak
Dirty type, seen as clean (red curve)	$\frac{2}{3}p(1-p)$	0 and 1	$\frac{1}{6}$ at $p = \frac{1}{2}$
Dirty type's equilibrium profit (red line)	$\frac{1}{8}$	$\frac{1}{4}$ and $\frac{3}{4}$	—
Clean type, seen as clean (blue curve)	$\frac{2}{3}(p - \frac{1}{4})(1-p)$	$\frac{1}{4}$ and 1	$\frac{3}{32}$ at $p = \frac{5}{8}$
Clean type, seen as dirty (blue line)	$\frac{9}{128}$	$\frac{7}{16}$ and $\frac{13}{16}$	—

Table 1: Ingredients of Figure 4(a). The crossings are those of each curve with the line of the same color.

Combining (9) and (10), the separating prices for the clean type are $p_H \in [\frac{3}{4}, \frac{13}{16}]$. The low prices $p_H \leq \frac{1}{4}$ that would also deter the dirty type are useless to the clean type, since they leave it no margin over its marginal cost $\frac{1}{4}$. Among the separating equilibria, the Intuitive Criterion of Cho and Kreps (1987) selects the least costly one, $p_H = \frac{3}{4}$: if the clean type charged $p_H > \frac{3}{4}$, a deviation to a price in $(\frac{3}{4}, p_H)$ would not pay for the dirty type even if consumers believed it was clean, but it would pay for the clean type, so consumers should attribute the deviation to the clean type.

Result 1 (Proposition 1 of the paper, two types). When cleaner products are more costly to produce:

- the clean type signals with a price above its full-information level, $\frac{3}{4} > \frac{5}{8}$;
- the dirty type charges its full-information price, $\frac{1}{2}$;
- the clean type sells less, $\frac{1}{6}$ instead of $\frac{1}{4}$, and pollutes less, $\frac{1}{12}$ instead of $\frac{1}{8}$.

The clean type pays for its signal: its profit falls from $\frac{3}{32}$ to $\frac{1}{12}$, a loss of $\frac{1}{96}$. Pollution falls by one third, because the signal works by restricting output and output is what pollutes.

In Figure 4(a), the dirty type mimics at the prices where the red curve lies above the dotted red line, the clean type separates at the prices where the blue curve lies above the dotted blue line, and the separating prices, shaded, satisfy both conditions.

5. Step (d): When Cleaner Products Are Cheaper

Porter and van der Linde (1995) argue that cleaner products can be cheaper to produce. Swap the marginal costs: $m_L = \frac{1}{4}$ and $m_H = 0$, so that $c_{q\theta} < 0$. Under full information the dirty type charges $\frac{5}{8}$ and earns $\frac{9}{128}$, and the clean type charges $\frac{1}{2}$, sells $\frac{1}{3}$, earns $\frac{1}{6}$, and generates $e_H^* = \frac{1}{6}$. The constraint that the dirty type must not mimic compares its curve $\frac{2}{3}(p - \frac{1}{4})(1 - p)$ with its full-information profit $\frac{9}{128}$, and the constraint that the clean type must prefer to separate compares its curve $\frac{2}{3}p(1 - p)$ with the profit $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$ that it earns if it is seen as dirty. They become

$$\frac{2}{3}\left(p_H - \frac{1}{4}\right)(1 - p_H) \leq \frac{9}{128} \iff p_H \leq \frac{7}{16} \text{ or } p_H \geq \frac{13}{16}, \quad (13)$$

$$\frac{2}{3}p_H(1 - p_H) \geq \frac{1}{8} \iff \frac{1}{4} \leq p_H \leq \frac{3}{4}, \quad (14)$$

so the separating prices are $p_H \in [\frac{1}{4}, \frac{7}{16}]$. Panel (b) of Figure 4 is built with the same six steps, with the roles of the two types exchanged: the red curve is the same parabola as the blue curve of panel (a) and its line is $\frac{9}{128}$, and the blue curve is the same parabola as the red curve of panel (a), with the line $\frac{1}{8}$, so the crossings $\frac{1}{4}, \frac{3}{4}$ and $\frac{7}{16}, \frac{13}{16}$ appear again, now for the other type. The least costly separating price for the clean type is now the highest one in the interval, $p_H = \frac{7}{16}$, because its profit $\frac{2}{3}p(1 - p)$ rises up to $p = \frac{1}{2}$.

Result 2 (cleaner is cheaper). When cleaner products are cheaper to produce:

- the clean type signals with a price below its full-information level, $\frac{7}{16} < \frac{1}{2}$;
- the dirty type charges its full-information price, $\frac{5}{8}$;
- the clean type sells more, $\frac{3}{8}$ instead of $\frac{1}{3}$, and pollutes more, $\frac{3}{16}$ instead of $\frac{1}{6}$.

Intuitively, the clean firm now has the wider margin, so a price cut lowers its margin by a smaller proportion than it lowers the dirty firm's, while both gain the same proportion of sales. A price cut is therefore the signal that the dirty firm finds too costly to copy, and the direction of the signal follows the sign of $c_{q\theta}$, as (8) shows.

Table 2 reports the full-information and signaling outcomes in both cases.

Case	Type	p^*	p^s	q^*	q^s	e^*	e^s	π^*	π^s
Cleaner is costlier	Dirty	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
$(m_L, m_H) = (0, \frac{1}{4})$	Clean	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{3}{32}$	$\frac{1}{12}$
Cleaner is cheaper	Dirty	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{9}{128}$	$\frac{9}{128}$
$(m_L, m_H) = (\frac{1}{4}, 0)$	Clean	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{3}{16}$	$\frac{1}{6}$	$\frac{21}{128}$

Table 2: Price, output, pollution, and profit under full information (*) and in the least costly separating equilibrium (s), with $\theta_L = 1$ and $\theta_H = 2$.

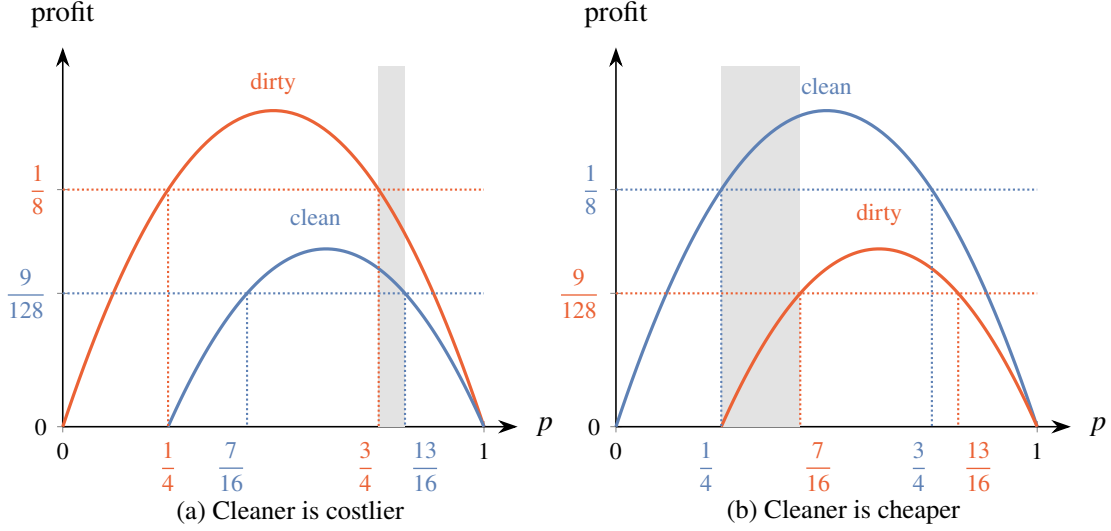


Figure 4: Profit from charging p and being perceived as clean, for each type, and each type's payoff outside the clean type's price (dotted). Shaded: separating prices for the clean type. Types $\theta_L = 1$ and $\theta_H = 2$; marginal costs $(m_L, m_H) = (0, \frac{1}{4})$ in panel (a) and $(\frac{1}{4}, 0)$ in panel (b).

6. Step (e): A Continuum of Types

The paper works with a continuum of types $\theta \in [\underline{\theta}, \bar{\theta}]$ and the method of Mailath (1987). Let $\theta \in [1, 2]$ with $m(\theta) = \frac{\theta-1}{4}$, which reproduces the two types of Step (c) at the end points. A separating equilibrium is a strictly monotone price function $\varphi(\theta)$, and a consumer who sees p infers $\hat{\theta} = \varphi^{-1}(p)$.

Local incentive compatibility. Type θ chooses which type $\hat{\theta}$ to imitate, earning $\Pi(\theta, \hat{\theta}) \equiv \pi(\varphi(\hat{\theta}), \theta, \hat{\theta})$. Truth-telling requires $\Pi_{\hat{\theta}}(\theta, \theta) = 0$, that is, $\pi_p \varphi'(\theta) + \pi_{\hat{\theta}} = 0$, which gives the differential equation

$$\varphi'(\theta) = -\frac{\pi_{\hat{\theta}}}{\pi_p} = \frac{(\varphi - m(\theta))(1 - \varphi)}{\theta(1 + \theta)(2\varphi - 1 - m(\theta))}, \quad (15)$$

where we used $\frac{\alpha'(\theta)}{\alpha(\theta)} = \frac{1}{\theta(1+\theta)}$ and $\pi_p = \alpha(\theta)(1 + m(\theta) - 2\varphi)$. The initial condition is $\varphi(1) = p^*(1) = \frac{1}{2}$: the dirtiest type is recognized as the dirtiest, so it has no reason to distort (Lemma 2 of the paper).

The price exceeds the full-information price. Along a separating equilibrium $\varphi' > 0$ and $\pi_{\hat{\theta}} > 0$, so (15) requires $\pi_p < 0$, that is, $\varphi(\theta) > \frac{1+m(\theta)}{2} = p^*(\theta)$ for every $\theta > 1$. Each type prices above the level at which its profit stops rising, which is what makes the price costly enough to deter the types just below it.

Result 3 (Proposition 1 of the paper). With a continuum of types and $c_{q\theta} > 0$:

- the separating price $\varphi(\theta)$ is strictly increasing;
- it exceeds $p^*(\theta)$ for every type except the dirtiest;
- every type except the dirtiest pollutes less than under full information.

Equation (15) has no closed-form solution, but it is easy to integrate numerically (as a function $\theta(\varphi)$, whose slope is zero at the initial point). Figure 5 depicts the solution. In panel (a), the signaling price rises from 0.500 at $\theta = 1$ to 0.714 at $\theta = 1.5$ and 0.765 at $\theta = 2$, while the full-information price $p^*(\theta) = \frac{\theta+3}{8}$ rises only from 0.500 to 0.625. In panel (b), pollution under signaling falls from 0.250 to 0.078, against 0.125

under full information for the cleanest type. The price function has an infinite slope at $\theta = 1$: the types just above the dirtiest one must move their prices away from $\frac{1}{2}$ quickly, because near p^* a small price change costs almost nothing.

The cleanest type charges 0.765 in the continuum, more than the $\frac{3}{4}$ it charges with two types. With two types, the clean firm only needs to deter the dirtiest firm, whose profit is undistorted. With a continuum, each type must deter the types just below it, which already charge distorted prices, and the distortions accumulate along the chain. A discrete chain of types confirms this: the top price is 0.750 with two types, 0.762 with 11 equally spaced types, and approaches 0.765 as the grid becomes finer.

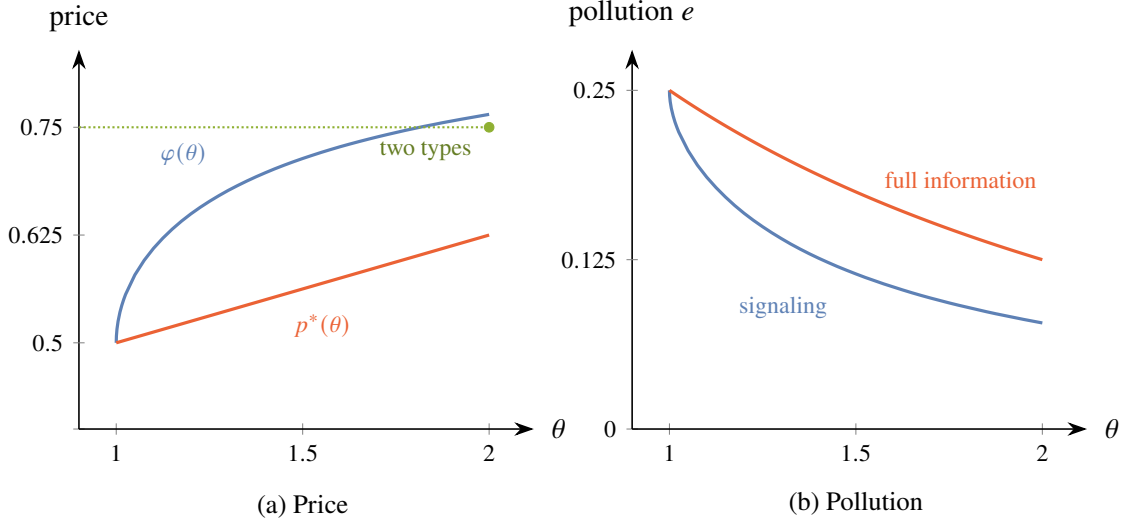


Figure 5: Separating price and pollution with a continuum of types $\theta \in [1, 2]$ and $m(\theta) = \frac{\theta-1}{4}$, against full information.

7. Step (f): Should a Label Reveal Cleanness?

The paper notes that a labeling program that resolves consumers' uncertainty removes one distortion but can worsen another: by ending the need to signal, it can raise pollution. The toy model makes the trade-off explicit. Let the global damage of pollution be δe , borne by everyone, and measure welfare as $CS + \pi - \delta e$, with $CS = \frac{q^2}{2}$. A label that reveals θ moves the clean type from its signaling outcome to its full-information outcome and leaves the dirty type unchanged.

Cleaner is costlier. With the label the clean type sells $\frac{1}{4}$ instead of $\frac{1}{6}$. Consumer surplus rises from $\frac{1}{72}$ to $\frac{1}{32}$ and profit from $\frac{1}{12}$ to $\frac{3}{32}$, a gain of $\frac{1}{36}$ in $CS + \pi$, while pollution rises by $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$. The label raises welfare if and only if

$$\frac{1}{36} > \frac{\delta}{24} \iff \delta < \frac{2}{3}. \quad (16)$$

Cleaner is cheaper. With the label the clean type raises its price from $\frac{7}{16}$ to $\frac{1}{2}$ and sells $\frac{1}{3}$ instead of $\frac{3}{8}$. $CS + \pi$ falls by $\frac{7}{576}$, because the signaling price cut had moved the monopoly output closer to the efficient level, while pollution falls by $\frac{3}{16} - \frac{1}{6} = \frac{1}{48}$. The label raises welfare if and only if

$$\frac{\delta}{48} > \frac{7}{576} \iff \delta > \frac{7}{12}. \quad (17)$$

Result 4 (labels). A label that reveals cleanness raises welfare:

- when cleaner products are costlier, if and only if the pollution damage is low, $\delta < \frac{2}{3}$;
- when cleaner products are cheaper, if and only if the pollution damage is high, $\delta > \frac{7}{12}$.

Signaling distorts the clean firm's output, and the direction of the distortion determines whether removing it helps the environment or the market. This illustrates the paper's point that a single instrument can mitigate one distortion and worsen another; with three distortions (the externality, market power, and asymmetric information), the paper calls for three instruments: a Pigouvian tax on emissions, a tax or subsidy that offsets the price distortion, and a lump-sum transfer.

8. Step (g): What the Toy Model Keeps and Drops

The toy model keeps the paper's main conclusions:

1. Every type wants to look clean (Condition 2), and the dirtiest type charges its full-information price (Lemma 2, Step c).
2. The direction of the signal follows the sign of $c_{q\theta}$: a price above the full-information level when cleaner products are costlier, below it when they are cheaper (Proposition 1 and its discussion, Steps c and d).
3. Signaling lowers pollution when cleaner products are costlier, from $\frac{1}{8}$ to $\frac{1}{12}$ for the clean type with two types, and for every type above the dirtiest with a continuum (Steps c and e).
4. Full disclosure can raise pollution, so a label alone need not raise welfare (Step f).

The linear-uniform specification adds two features to the general analysis. First, cleanness enters demand as the multiplicative factor $\alpha(\theta) = \frac{\theta}{1+\theta}$, which makes single crossing a one-line calculation (7). Second, the comparison between two types and a continuum shows that the distortion of the cleanest type grows with the number of types it must separate from, from 0.750 to 0.765.

Relative to Mahenc (2008), the toy model fixes $v(e) = e$, $t(q) = q$, a uniform distribution of basic willingness to pay on $[0, 1]$, and a constant marginal cost; the paper allows a convex private damage, a convex emission function, and a general cost $c(q, \theta)$. As in the paper, the firm is a monopolist, cleanness is exogenous, the price is the only signal, and the analysis focuses on separating equilibria. The paper's conclusion suggests advertising and green investment as additional signals; each of these features, and competition between a clean and a dirty firm, can be added one at a time.

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