EconS 501 - Microeconomic Theory I
Homework #6 - Due date: Thursday, November 12th, in class.

1. **Tax incidence and deadweight loss in partial equilibrium.** A competitive market has a differentiable demand function $x(p)$, with $x'(p) < 0$, and a differentiable supply function $q(p)$, with $q'(p) > 0$. The government imposes a specific tax of t dollars per unit, collected from sellers. Let p_c denote the price paid by consumers, so sellers receive $p_s = p_c - t$.

   (a) *Incidence, in general.* Write the equilibrium condition that determines $p_c(t)$. Using the implicit function theorem, show that

   $$\frac{dp_c}{dt} = \frac{q'(p_s)}{q'(p_s) - x'(p_c)} \in (0, 1),$$

   and that, evaluated at $t = 0$, this equals $\dfrac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}$, where $\varepsilon_D = -x'(p)\dfrac{p}{x}$ and $\varepsilon_S = q'(p)\dfrac{p}{q}$ are the price elasticities of demand and supply. Which side of the market bears more of the tax, and why? What happens when supply is perfectly inelastic?

   (b) *A linear market.* Let $x(p) = 100 - 2p$ and $q(p) = 3p - 20$. Find the equilibrium without the tax, and then show that, with a tax t, $p_c = 24 + \dfrac{3}{5}t$, $p_s = 24 - \dfrac{2}{5}t$, and the quantity traded is $52 - \dfrac{6}{5}t$. Verify that the share of the tax paid by consumers coincides with the elasticity formula in part (a).

   (c) *Deadweight loss.* Show that the deadweight loss of the tax in part (b) is $DWL(t) = \dfrac{3}{5}t^2$, and that it coincides with the Harberger formula $DWL = \dfrac{1}{2}t^2 \dfrac{x'(p)\,q'(p)}{x'(p) - q'(p)}$ (here exact, since both curves are linear). Compute DWL and tax revenue $R(t)$ at $t = 10$.

   (d) *The Laffer curve and the marginal excess burden.* Find the revenue-maximizing tax and the deadweight loss it generates. Then compute the marginal excess burden, $\dfrac{dDWL/dt}{dR/dt}$, as a function of t, evaluate it at $t = 10$, and explain why it grows without bound as t approaches the revenue-maximizing tax.

2. **Long-run competitive equilibrium with free entry.** Every active firm in a competitive industry has cost function

   $$c(q) = F + aq + \frac{h}{2}q^2 \quad \text{for } q > 0,$$

   with $c(0) = 0$ in the long run, where $F = 8$ is a fixed cost that the firm avoids by exiting, $a = 16$, and $h \geq 0$. Marginal cost is therefore $c'(q) = a + hq = 16 + hq$. Market demand is $x(p) = 400 - 10p$. Treat the number of firms J as a continuous variable.

(a) *Short-run supply.* With J fixed and the fixed cost sunk, show that, for $h > 0$, a firm's short-run supply function is $q(p) = \dfrac{p - 16}{h}$ for $p \geq 16$ (and zero otherwise), and find the industry short-run supply. Explain why, in the short run, a firm produces as long as the price covers its average *variable* cost. What does short-run supply look like when $h = 0$?

(b) *Long-run equilibrium.* For $h > 0$, find the efficient scale $\bar{q}$ (the output that minimizes average cost) and the long-run equilibrium price, aggregate output, and number of firms. Show that

$$\bar{q} = \frac{4}{\sqrt{h}}, \qquad p^* = 16 + 4\sqrt{h}, \qquad J^* = 60\sqrt{h} - 10h,$$

where we assume $h < 36$ so that the market is active. How do the efficient scale, the price, and the number of firms change with h? Explain why no long-run competitive equilibrium exists when $h = 0$.

(c) *A demand shock.* Demand permanently increases to $x(p) = 480 - 10p$. Holding $J = J^*$ fixed, show that the new short-run price is $p^{SR} = 16 + \dfrac{16}{3}\sqrt{h}$ and that each firm earns a profit of $\dfrac{56}{9}$, for every $h > 0$. Then find the new long-run equilibrium, and describe the long-run industry supply curve.

(d) *Efficiency of free entry.* A planner chooses the number of firms J and output per firm q to maximize Marshallian surplus, $W(J, q) = \int_0^{Jq} P(Q)\, dQ - J\, c(q)$, where $P(Q)$ is the inverse demand function from the original demand $x(p) = 400 - 10p$. Show that the planner's first-order conditions imply $P(Jq) = c'(q)$ and $P(Jq)\, q = c(q)$, so that each firm produces at the minimum of its average cost, and that the planner's solution coincides with the long-run competitive equilibrium of part (b). Contrast this result with the excess-entry result under Cournot competition.

3. **An exchange economy: equilibrium and the welfare theorems.** Two consumers trade two goods. Their utility functions are

$$u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha} \quad \text{and} \quad u_2(x_{12}, x_{22}) = x_{12}^{2/3} x_{22}^{1/3},$$

where $\alpha \in (0, 1)$ and x_{li} is consumer i's consumption of good l. Endowments are $\omega_1 = (4, 2)$ and $\omega_2 = (2, 4)$.

(a) *Contract curve.* Find the set of interior Pareto-efficient allocations and show that it can be written as

$$x_{21} = \frac{12(1 - \alpha)x_{11}}{6\alpha + (2 - 3\alpha)x_{11}}, \qquad x_{11} \in [0, 6].$$

Depict it in the Edgeworth box for $\alpha = \dfrac{1}{3}$, and show that it then becomes $x_{21} = \dfrac{8x_{11}}{2 + x_{11}}$.

(b) *Walrasian equilibrium.* Normalize $p_1 = 1$ and let $p_2 = p$. Find each consumer's Walrasian demand and the Walrasian equilibrium. Show that

$$p^* = \frac{7 - 6\alpha}{3\alpha + 4}, \qquad x_{11}^* = \frac{30\alpha}{3\alpha + 4}, \qquad x_{21}^* = \frac{30(1 - \alpha)}{7 - 6\alpha}.$$

How does the equilibrium price of good 2 change with α? Explain. Show that neither consumer is worse off than at her endowment. For which value of α is there no trade in equilibrium?

(c) *Walras' law and uniqueness.* Verify Walras' law, $z_1(p) + p\, z_2(p) = 0$, for every $p > 0$. Show that $z_1(p)$ is strictly increasing in p, so the equilibrium is unique, and relate this to the gross-substitutes property of Cobb-Douglas preferences.

(d) *Second Welfare Theorem.* Consider the Pareto-efficient allocation with $x_{11} = 3$. Show that the supporting price p and the lump-sum transfer T from consumer 2 to consumer 1 that decentralizes this allocation are

$$p = \frac{2 - \alpha}{4\alpha} \qquad \text{and} \qquad T = \frac{4 - 7\alpha}{2\alpha}.$$

For which values of α does the transfer go from consumer 1 to consumer 2 instead? Explain.

4. **A Robinson Crusoe economy.** Robinson has 24 hours a day, which he splits between leisure l and labor $L = 24 - l$. He consumes coconuts x, with utility $u(x, l) = \ln x + \ln l$. His firm turns labor into coconuts with technology $x = \beta\sqrt{L}$, where $\beta > 0$ measures productivity. Robinson is both the consumer and the owner of the firm. Take coconuts as numéraire ($p = 1$), and let w be the wage.

(a) *The firm.* Solve the firm's profit-maximization problem, finding its labor demand $L(w)$, coconut supply, and profit $\pi(w)$. Show that $L(w) = \dfrac{\beta^2}{4w^2}$ and $\pi(w) = \dfrac{\beta^2}{4w}$.

(b) *The consumer.* Robinson's budget constraint is $x + wl = 24w + \pi(w)$. Find his demands for coconuts and leisure, and his labor supply.

(c) *Equilibrium and the First Welfare Theorem.* Find the equilibrium wage and allocation, and show that $w^* = \dfrac{\beta}{4\sqrt{2}}$, $L^* = 8$, and $x^* = 2\sqrt{2}\,\beta$. Explain why the equilibrium amount of labor does not depend on β. Then solve the planner's problem, $\max_L \ln(\beta\sqrt{L}) + \ln(24 - L)$, and verify that the equilibrium is Pareto efficient, with $MRS_{l,x} = MRT$ at the optimum.

(d) *A distortionary labor tax.* The government taxes labor income at rate $\tau \in (0, 1)$ and returns the revenue to Robinson as a lump-sum transfer. Show that the equilibrium labor supply becomes $L^*(\tau) = \dfrac{24(1 - \tau)}{3 - \tau}$, which decreases in τ and does not depend on β. Compute Robinson's utility for $\tau = 0$ and $\tau = \dfrac{1}{2}$, show that the utility loss from the tax does not depend on β, and explain why the tax reduces welfare even though all of its revenue is returned to him.