

Game Theory at the Movies 3:

Mixed Strategies and Strictly Competitive Games

Felix Munoz-Garcia

EconS 424, Strategy and Game Theory
Washington State University

Outline

- Mixed strategies and strictly competitive games.
- The penalty kick: Zidane against Buffon (2006).
- *Das Boot* (1981): a pursuit game.
- *Rounders* (1998): bluffing in poker.

Mixed-Strategy Equilibrium

- In some games no pure-strategy NE exists: every predictable player is exploited.
- Players then randomize, and each mixes so as to make the *opponent* indifferent.
- In a strictly competitive (zero-sum) game, the msNE coincides with each player's max-min strategy.

The Penalty Kick



Zinedine Zidane in the 2006 World Cup final, Berlin.

Image: David Ruddell, CC BY 2.0, via Wikimedia Commons.

The Penalty Kick as a Game

Keeper dives			
		Left	Right
Kicker (rows)	Left	0.5	0.9
	Right	0.9	0.5

- Entries: the kicker's probability of scoring; the keeper wants them low.
- No pure-strategy NE: the kicker wants to avoid the keeper, the keeper wants to match.

Finding the msNE

- Let the kicker shoot left with probability p .
- Keeper indifferent between diving left and right:
- $0.5p + 0.9(1 - p) = 0.9p + 0.5(1 - p)$, so $p = \frac{1}{2}$.
- By symmetry, the keeper dives left with probability $\frac{1}{2}$, and the kicker scores with probability 0.7.

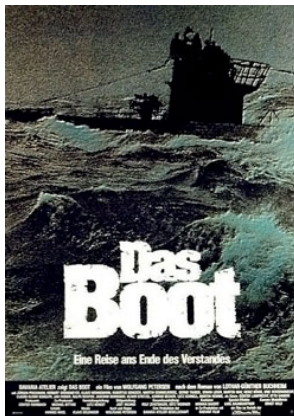
Zidane's Chip

- July 9, 2006, Berlin, seventh minute of the World Cup final.
- Zidane chips the ball straight down the middle.
- A bet on Buffon's own mixed strategy: keepers almost always dive to a side.
- Buffon dives; the ball comes down off the crossbar and in.

Do Professionals Mix?

- Chiappori, Levitt, and Groseclose (2002) and Palacios-Huerta (2003) studied professional penalties.
- Kickers and keepers mix in almost the proportions the theory predicts.
- They score at nearly equal rates whichever way they go.
- No exploitable pattern from one kick to the next.

Das Boot (1981)



Theatrical release poster.

Poster © the film's distributor; reproduced for teaching (fair use).

Hide and Seek Under Water

- A destroyer hunts the submarine from above, listening for it.
- The submarine runs silent below, trying to slip away.
- Neither can see the other: what one gains, the other loses.
- A strictly competitive game, like the penalty kick.

A Simple Pursuit Game

Destroyer searches			
		Shallow	Deep
Submarine hides (rows)	Shallow	0.2	1
	Deep	1	0.6

- Entries: the submarine's probability of surviving (illustrative numbers).
- Detection is easier in shallow water (0.8) than in deep water (0.4).

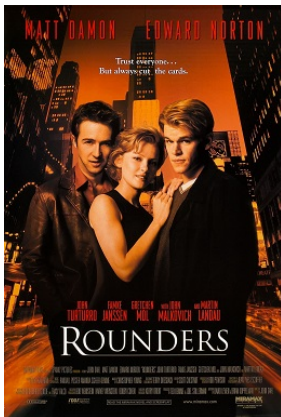
Solving the Pursuit Game

- Submarine hides in shallow water with probability q ; destroyer indifferent:
- $0.2q + (1 - q) = q + 0.6(1 - q)$, so $q = \frac{1}{3}$.
- The same equation for the destroyer gives: search shallow water with probability $\frac{1}{3}$.
- Survival probability: $0.2 \cdot \frac{1}{3} + \frac{2}{3} \approx 0.73$.

What the Pursuit Game Teaches

- The submarine hides more often where it is harder to find.
- The destroyer searches deep water more often, anticipating that.
- Each mixes to deny the other any information to exploit.
- Isaacs (1965) made pursuit and evasion a formal subject at RAND.

Rounders (1998)



Theatrical release poster.

Poster © Miramax Films; reproduced for teaching (fair use).

A Simple Bluffing Game

- Each player antes 1. Mike draws a strong or weak hand, each with probability $\frac{1}{2}$.
- Mike always bets 1 with a strong hand. With a weak hand he can *bluff* (bet) or check.
- His opponent sees a bet and chooses *Call* or *Fold*.
- Mike's expected payoffs (the opponent gets the negative):

Opponent, after a bet

	Call	Fold
Bluff	0	1
Check	0.5	0

Mike, with a weak hand (rows)

Bluffing in Equilibrium

- No pure-strategy NE: if Mike bluffs, his opponent calls; then Mike prefers to check; and so on.
- Mike bluffs with probability b ; opponent indifferent:
 $0.5(1 - b) = b$, so $b = \frac{1}{3}$.
- Opponent calls with probability c ; Mike indifferent:
 $1 - c = 0.5c$, so $c = \frac{2}{3}$.
- **Lesson:** bluffing is part of rational play (von Neumann and Morgenstern, 1944).

Teddy KGB's Tell

- Mike's opponent, Teddy KGB, eats Oreo cookies at the table.
- With a very strong hand, KGB breaks the cookie beside his ear.
- The tell reveals what his mixed strategy was meant to hide.
- Mike no longer has to guess: he best responds and wins the final hand.

Summary

- When predictable players are exploited, equilibrium requires mixing.
- Each player mixes to make the opponent indifferent.
- Penalty kicks and submarine hunts: strictly competitive games solved by randomizing.
- Poker: bluffing is an equilibrium strategy, and a tell destroys it.