

Game Theory at the Movies 1:

Nash Equilibrium

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Outline

- Nash equilibrium: definition and how to find it.
- *A Beautiful Mind* (2001): the bar scene.
- *Rebel Without a Cause* (1955): the chickie run.
- *Waiting for Godot* (1953): coordination without a focal point.

Nash Equilibrium

- A strategy profile is a **Nash equilibrium** (NE) if no player can increase his payoff by changing his own strategy, while the others keep theirs fixed.
- **Test:** can anyone gain by deviating *alone*?
- If no one can, every player is playing a *best response* to the others.

Finding a Nash Equilibrium in a Matrix

- **Step 1.** For each column, underline the row player's highest payoff.
- **Step 2.** For each row, underline the column player's highest payoff.
- **Step 3.** Cells with both payoffs underlined are NEs.
- Intuitively, in those cells each player is already best responding.

A Beautiful Mind (2001)



Theatrical release poster.

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The Bar Scene

- A blonde woman and her friends walk into a bar where Nash sits with his classmates.
- Nash's proposal: nobody approaches the blonde, everyone approaches her friends.
- If all go for the blonde, they block each other, and her friends refuse to be a second choice.
- The film presents this proposal as the birth of Nash's equilibrium idea.

The Bar Scene as a Game

		Student 2	
		Blonde	Friend
Student 1 (rows)	Blonde	0, 0	3, 2
	Friend	2, 3	2, 2

- Approaching the blonde alone gives 3; approaching a friend gives 2.
- If both approach the blonde, both get 0.

Best Responses

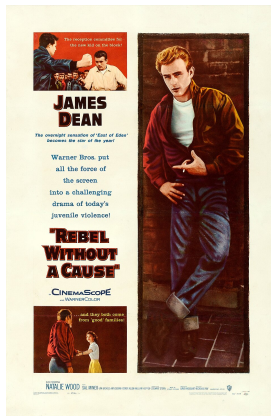
		Student 2	
		Blonde	Friend
Student 1 (rows)	Blonde	0, 0	<u>3</u> , <u>2</u>
	Friend	<u>2</u> , <u>3</u>	2, 2

- Two NEs: (Blonde, Friend) and (Friend, Blonde).
- In each, one student approaches the blonde and the other does not.

What the Film Got Wrong

- Nash's proposal is (Friend, Friend): a *cooperative* outcome.
- It is **not** a NE: if the other approaches a friend, approaching the blonde gives $3 > 2$.
- Every student faces the same temptation, so the pact unravels.
- **Lesson:** a NE is a profile no one wants to leave alone, whether or not it is good for the group.

Rebel Without a Cause (1955)



Theatrical release poster.

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The Chickie Run

- Jim Stark and Buzz Gunderson drive stolen cars at full speed toward a cliff edge.
- The first to jump out is the “chicken.”
- Jim jumps at the last moment.
- Buzz’s sleeve catches on the door handle, and he goes over the cliff.

The Game of Chicken

		Buzz	
		Jump	Stay
Jim (rows)	Jump	0, 0	<u>-1</u> , <u>1</u>
	Stay	<u>1</u> , <u>-1</u>	-10, -10

- Two NEs: (Jump, Stay) and (Stay, Jump).
- Each driver's best response is the opposite of the other's.

What the Scene Teaches

- The game has two NEs, one for each winner; the fight is over which one occurs.
- **Commitment:** a driver who convinces the rival that he will not jump wins.
- Buzz's commitment came by accident: a caught sleeve chose for him.
- Bertrand Russell (1959) used this contest to describe nuclear brinkmanship.

Waiting for Godot (1953)



Vladimir and Estragon in a production of Beckett's play, Avignon, 1978.

Image: Fernand Michaud, CC0, via Wikimedia Commons.

Waiting for Godot

- Two men wait by a bare tree for Godot, who has promised to come and never does.
- They do not know exactly where Godot said to wait, or when.
- A meeting needs both sides at the same place, at the same time.

A Meeting Without a Focal Point

		The other party	
		Tree	Crossroads
Tree		<u>1</u> , <u>1</u>	0, 0
Crossroads		0, 0	<u>1</u> , <u>1</u>
		You (rows)	

- Two NEs, (Tree, Tree) and (Crossroads, Crossroads), equally good.
- Nothing inside the game says which one to pick.

Focal Points

- Schelling (1960): players often coordinate on a *focal point*, an option that stands out.
- Examples: a natural meeting place, a customary time.
- Beckett's focal point (a tree, on an uncertain day) is too weak.
- **Lesson:** NE tells us which outcomes are stable, not which one players will reach.

Summary

- A NE passes one test: no player gains by deviating alone.
- *A Beautiful Mind*: a good outcome for the group need not be a NE.
- *Rebel Without a Cause*: in chicken, two NEs and a fight over which.
- *Waiting for Godot*: with several equally good NEs, coordination needs a focal point.