Microeconomic Theory II - UNI, Lima
Homework #1 - Due date: August 6th, in class.

1. **Dominance, rationalizability and the order of elimination.** Consider the following simultaneous-move game between player 1, who chooses rows, and player 2, who chooses columns. In every cell, the first number denotes player 1's payoff and the second number denotes player 2's payoff.

		Player 2			
		L	C	R	X
Player 1	U	$8,3$	$4,3$	$0,1$	$0,0$
	M	$6,3$	$6,2$	$6,3$	$0,0$
	D	$8,3$	$0,1$	$4,3$	$0,0$
	B	$7,3$	$1,2$	$1,2$	$20,0$

Answer the following questions.

(a) Show that exactly one strategy is strictly dominated by a pure strategy in this game, and eliminate it. Show, in addition, that no strategy of player 1 is strictly dominated in the original game, either by a pure or by a mixed strategy.

(b) In the game that remains after the first round of elimination, show that one of player 1's strategies is strictly dominated by a mixed strategy even though no pure strategy of player 1 strictly dominates it. Find such a mixed strategy. Then show that no further strategy can be strictly eliminated. [*Hint*: search for a mixture of two of player 1's strategies.]

(c) Find the set of rationalizable strategies of every player.

(d) Find every pure-strategy Nash equilibrium (psNE) of the game. Compare the set of psNEs against the set of rationalizable strategy profiles you found in part (c), and identify a strategy that is rationalizable but that is not played in any psNE.

(e) Show that player 2's strategy L weakly dominates both C and R, but strictly dominates neither. Starting from the reduced game of part (c), apply iterated elimination of weakly dominated strategies twice, first deleting C before R and then deleting R before C. Do the two orders of elimination lead to the same prediction? Which psNEs survive each of them?

2. **Cournot competition with production spillovers among n firms.** Consider $n \geq 2$ firms competing à la Cournot, facing inverse demand function $p(Q) = a - Q$, where $Q = \sum_{j=1}^{n} q_j$ denotes aggregate output and $a > c > 0$. Every firm i's marginal cost is, instead of c, given by

$$c - \alpha \sum_{j \neq i} q_j,$$

so that the production of all its rivals shifts firm i's marginal cost. Parameter α measures the strength of these production spillovers. When $\alpha > 0$, rivals' output decreases firm i's marginal cost (as when a larger industry generates a common pool of trained workers or shared infrastructure); when $\alpha < 0$, rivals' output increases firm i's marginal cost (as when firms compete for a scarce input). Denote by $Q_{-i} = \sum_{j \neq i} q_j$ the aggregate output of firm i's rivals.

Answer the following questions.

(a) Find every firm i's best response function, $q_i(Q_{-i})$. How is its intercept and its slope affected by parameter α? For which values of α do firms regard their outputs as strategic substitutes, and for which values as strategic complements?

(b) Find every firm's equilibrium output, aggregate output, price, and profit in the symmetric equilibrium.

(c) Check the second-order condition of firm i's problem. Then find the restriction on α that guarantees that the symmetric equilibrium you found in part (b) is interior and positive, and the (tighter) restriction that guarantees that the best response dynamics converge to it. Is the binding restriction the second-order condition?

(d) *Comparative statics.* How are equilibrium output, price and profit affected by a marginal increase in α? Interpret.

(e) *Comparative statics.* How are equilibrium output per firm, aggregate output and profit affected by a marginal increase in the number of firms n? Can output per firm *increase* in the number of firms? If so, find the exact condition on α under which this occurs, and interpret it.

(f) *Numerical example.* Evaluate your results at $a = 10$ and $c = 4$ with $n = 3$ firms, for spillover levels $\alpha = 0$, $\alpha = \dfrac{1}{2}$ and $\alpha = 1$. Then set $a = 10$, $c = 9$ and $\alpha = \dfrac{3}{2}$, and evaluate equilibrium output, aggregate output, price and profit for $n = 2$, $n = 3$ and $n = 4$. What is the largest number of firms that this industry can sustain in an interior equilibrium?

3. **Bertrand competition with differentiated products and asymmetric costs.** Consider two firms competing in prices and selling differentiated products. Firm i's demand function is

$$q_i(p_i, p_j) = a - p_i + \gamma p_j,$$

where $a > 0$ and parameter $\gamma \in (0, 1]$ measures the degree of product substitutability. When γ approaches zero the goods are essentially unrelated, whereas when $\gamma = 1$ they are close substitutes. Firms face constant marginal costs c_1 and c_2, which may differ, so firm i's profit is $\pi_i = (p_i - c_i)q_i(p_i, p_j)$.

Answer the following questions.

(a) Find every firm i's best response function, $p_i(p_j)$, and check its second-order condition. Do firms regard prices as strategic substitutes or as strategic complements? How is the slope of the best response function affected by γ?

(b) Find equilibrium prices p_1^* and p_2^*, allowing for $c_1 \neq c_2$.

(c) Show that the low-cost firm charges a lower price than its rival. Then show that *both* equilibrium prices increase when *either* marginal cost increases, and compare the size of the own-cost effect against that of the cross-cost effect. Interpret.

(d) Find equilibrium output and profit for every firm. Then find the cutoff $\bar{c}_2$ such that firm 2 is priced out of the market, selling no units in equilibrium, if and only if $c_2 \geq \bar{c}_2$.

(e) Evaluate equilibrium prices in the two limiting cases $\gamma \to 0$ and $\gamma \to 1$. Interpret each of them.

(f) *Numerical example.* Evaluate your results at $a = 10$, $\gamma = \frac{1}{2}$, $c_1 = 1$ and $c_2 = 6$. Report equilibrium prices, quantities and profits, the two cost effects on p_1^*, and the cutoff $\bar{c}_2$.

4. **Tragedy of the commons with heterogeneous appropriators.** Consider $n \geq 2$ herders who share a common pasture. Herder i sends $g_i \geq 0$ goats to graze, and we let $G = \sum_{j=1}^{n} g_j$ denote the total number of goats in the pasture. The value of a goat decreases in congestion, and is given by $a - G$, where $a > 0$. Herders differ in how costly it is for them to raise goats: herder i's cost of sending g_i goats is $\frac{c_i}{2} g_i^2$, where $c_i > 0$. A herder with a low c_i is more efficient than a herder with a high c_i. Herder i's payoff is, therefore,

$$\pi_i(g_i, g_{-i}) = (a - G)g_i - \frac{c_i}{2} g_i^2.$$

Throughout, denote $K \equiv \sum_{j=1}^{n} \frac{1}{1 + c_j}$ and $S \equiv \sum_{j=1}^{n} \frac{1}{c_j}$.

Answer the following questions.

(a) Find herder i's best response function, $g_i(G_{-i})$, where $G_{-i} = \sum_{j \neq i} g_j$, and check its second-order condition. Do herders regard their grazing decisions as strategic substitutes or as strategic complements? How is the slope of the best response function affected by c_i?

(b) Find the Nash equilibrium of this game in closed form, g_i^* for every herder i, along with the equilibrium total number of goats, G^*. Confirm that your expression collapses to the familiar symmetric result when $c_i = c$ for every herder.

(c) Find the socially optimal grazing profile, g_i^{SO}, and the socially optimal total, G^{SO}.

(d) Show that $G^* > G^{SO}$ for every $n \geq 2$, so the pasture is unambiguously over-exploited, and that $G^* = G^{SO}$ when $n = 1$. [*Hint*: compare KS against $T \equiv \sum_{j=1}^{n} \frac{1}{c_j(1 + c_j)}$, and note that KS contains every term of T plus additional positive terms.]

(e) Which herder over-extracts by the most? Answer this question first in proportional terms, comparing the ratio $\frac{g_i^*}{g_i^{SO}}$ across herders, and then in absolute terms, comparing the difference $g_i^* - g_i^{SO}$. Do the two criteria give the same ranking?

(f) *Comparative statics.* Consider the symmetric case $c_i = c$ for every herder. How are g_i^* and G^* affected by a marginal increase in the number of herders n? What happens to the value of a goat, $a - G^*$, as n grows without bound?

(g) *Numerical example.* Evaluate your results with $n = 3$ herders, costs $c_1 = 1$, $c_2 = 2$ and $c_3 = 5$, and $a = 132$. Report K and S, the equilibrium and socially optimal grazing levels of every herder, the two totals, and the proportional and absolute over-extraction of each herder.

Additional practice (not graded). For further practice on this material, see the textbook, Chapters 2-4.