

Chapter 1

Mixed Oligopoly and Environmental Regulation

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-García

Outline of Chapter 1

- A non-technical introduction to mixed oligopoly and why environmental regulation works differently when a public firm is present.
- Sections:
 - 1.1 What is a mixed oligopoly?
 - 1.2 Public firms in polluting industries.
 - 1.3 Two market failures.
 - 1.4 The public firm's objective.
 - 1.5 Instruments of environmental policy.
 - 1.6 Related literature.
 - 1.7 Organization of the book.

Section 1.1

What Is a Mixed Oligopoly?

What Is a Mixed Oligopoly?

- An *oligopoly*: a few firms whose decisions affect one another.
- A *mixed* oligopoly: at least one firm is **publicly owned**.
- The defining feature is the **objective**, not ownership as such:
 - Private firm $\Rightarrow$ maximizes its own *profit*.
 - Public firm $\Rightarrow$ places weight on *social welfare*, not on profit alone.
- Key insight: even with the same demand and costs, the two firms behave differently because they pursue different goals.

Section 1.2

Public Firms in Polluting Industries

Public Firms in Polluting Industries

- Public firms are common exactly where pollution matters most.
- **Electricity:** Électricité de France (fully state-owned since 2023) competes with private generators across Europe.
- **Oil and gas:** Equinor (about two-thirds Norwegian state), Petrobras, Saudi Aramco: majority state-owned, competing with private majors.
- **Transport:** national rail and postal operators, alongside private carriers.
- Lesson: a regulator who treats such an industry as purely private will misjudge how much regulation it needs.

Two Kinds of State Involvement

- Tables 1.1 and 1.2 place these firms alongside their peers.
- **Majority stake (Table 1.1):** the state holds at least half of the shares and directs the firm's decisions. A fully public firm competes with private rivals.
- **Minority stake (Table 1.2):** the state shares control with private investors. The firm's own objective already blends public and private aims.
- Chapter 9 returns to the minority-stake case and asks what degree of nationalization the state should choose.

Majority State-Owned Firms (Table 1.1)

Firm	Country	Industry	State stake
Saudi Aramco	Saudi Arabia	Oil and gas	$\approx 98\%$
Equinor	Norway	Oil and gas	67.0%
Petrobras	Brazil	Oil and gas	50.3% voting
Pemex	Mexico	Oil and gas	100%
EDF	France	Electricity	100%
Eskom	South Africa	Electricity	100%

- All in oil, gas, or electricity: sectors with the largest emissions.
- Saudi Aramco: 82% held directly by the state, a further 16% through the Public Investment Fund.
- Petrobras: 50.3% of voting shares but only $\approx 36.6\%$ of capital (Brazilian law reserves the majority of voting stock for the state).

Minority State Stakes (Table 1.2)

Firm	Country	Industry	State stake
Eni	Italy	Oil and gas	$\approx 33\%$
Deutsche Telekom	Germany	Telecommunications	$\approx 28\%$
Airbus	FR/DE/ES	Aerospace	25.7%
Enel	Italy	Electricity	23.6%
Renault	France	Automobiles	15.0% cap, 29% vote
Volkswagen	Germany	Automobiles	11.8% cap, 20% vote

- Capital-vs-voting wedge (Orange, Engie, Renault, Volkswagen): double-voting or special-share rules give the state more say than its capital share suggests.
- Fits the partial nationalization case with $\alpha < 1$: the firm's own objective already blends profit and welfare (Chapter 9).

Section 1.3

Two Market Failures

First Failure: Market Power

- With only a few firms, output is held **below** the competitive level and price exceeds marginal cost.
- Consumer surplus is lower than it would be under competition.
- Taken on its own, this failure calls for policies that *raise* output, such as a production subsidy.

Second Failure: The Pollution Externality

- Each unit of output causes environmental harm the firm does not bear.
- Output and emissions are **too high** from society's point of view.
- Taken on its own, this failure calls for policies that *reduce* output, such as an emission fee.

The Two Failures Pull in Opposite Directions

- A regulator must weigh both together:
 - If pollution is mild $\Rightarrow$ market power dominates $\Rightarrow$ a subsidy may be preferred.
 - If pollution is severe $\Rightarrow$ the externality dominates $\Rightarrow$ an emission fee is called for.
- A mixed oligopoly adds a third element: the public firm's objective already contains **both** concerns.
- Key insight: the public firm carries part of the correction inside its own objective.

Section 1.4

The Public Firm's Objective

The Public Firm's Objective

- Social welfare = consumer surplus + producer surplus – environmental damage:

$$W = CS + PS - Env.$$

- The public firm need not be fully nationalized. It maximizes a weighted average of its own profit and welfare:

$$V = (1 - \alpha) \pi_0 + \alpha W, \quad \alpha \in [0, 1].$$

- The weight α is the degree of nationalization: $\alpha = 1$ fully public (maximizes W), $\alpha = 0$ fully private (maximizes profit).

What the Weight α Does

- For $\alpha > 0$ the firm **internalizes** the externality (damage enters with a minus sign) and **values rivals' profit** (part of producer surplus).
- The larger α , the more the firm restrains output and corrects the externality on its own.
- This book carries a **general** α throughout; Chapter 9 makes α the object of choice (*partial privatization*).

Section 1.5

Instruments of Environmental Policy

The Emission Fee

- A per-unit charge on emissions. The instrument we use most often in the book.
- Gives every firm a continuous incentive to reduce output and emissions.
- A well-chosen fee can align private incentives with social welfare.
- Chapter 2 derives the fee that a regulator would set in the mixed-duopoly benchmark.

Quotas and Tradable Permits

- **Emission quotas:** a direct cap on each firm's emissions.
- **Tradable permits:** a cap on total emissions plus the right to trade allowances among firms.
- Under full information the three instruments can reach the same outcome (compared at the end of Chapter 2).
- What is new: they now operate in a market that already contains a **partial corrector** of the externality, the public firm.

Related Literature

- **Foundations:** De Fraja and Delbono (1989, 1990) on the public firm's strategy; Matsumura (1998) on partial privatization.
- **Ownership and subsidies:** White (1996); **timing:** Pal (1998).
- **Environmental branch:** Bárcena-Ruiz and Garzón (2006): the optimal tax is *lower* in a mixed than in a private oligopoly.
- **Recent synthesis:** Poyago-Theotoky (2024), with an emphasis on privatization.
- This book follows one thread: the **substitution** between public ownership and environmental regulation.

Organization of the Book

- Ch. 2: benchmark mixed duopoly (the result that organizes the book).
- Ch. 3–5: cost asymmetry; more private firms; several public firms.
- Ch. 6–8: private mergers; cost-reducing mergers; public-private mergers.
- Ch. 9–11: partial privatization; further extensions; policy lessons.
- As the market grows richer, the public firm's reach shrinks and the case for regulation returns.

Key Takeaways

- A mixed oligopoly pairs a public firm that places weight α on welfare with profit-maximizing rivals.
- A polluting industry suffers two market failures at once: market power and the pollution externality.
- The public firm's objective already weighs both, so public ownership acts as a partial substitute for environmental regulation.
- The book asks when that substitution suffices, and when a fee is still needed.

Chapter 2

The Mixed Duopoly Benchmark

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 2

- 2.2 Benchmark: unregulated mixed duopoly.
- 2.3 Regulated mixed duopoly (emission fee).
- 2.4 Public ownership as implicit regulation.
- 2.5 Other regulations.
- 2.6 Case study.
- 2.7 The public firm's weight on emissions.
- 2.8 Convex costs.
- 2.9 Numerical illustration.
- 2.10 Summary.

The Central Problem

- One **public** firm and one **private** firm, same demand and cost. The public firm places weight α on welfare, $1 - \alpha$ on its own profit (Matsumura, 1998).
- $\alpha = 1$: fully public. $\alpha = 0$: fully private. Interior α : partially privatized (e.g., Airbus).
- Key insight: the degree of nationalization α sets how much of the regulator's job the firm does on its own.

Guiding Questions

- ① How does α change the outcome, from a private duopoly to a fully public firm?
- ② Can a fully public firm reach the socially optimal output on its own?
- ③ When public ownership corrects the externality, is a fee still useful, and how does its role change with privatization?
- ④ How severe must pollution be before the public firm exits?

Real-World Context

- Large commercial aircraft: **Boeing** (fully private) vs **Airbus** (about a quarter held by the French, German, and Spanish states).
- Airbus is a firm with an intermediate weight α , neither fully public nor fully private.
- The market's output carries a clear environmental cost (aviation emissions).

Section 2.2

Benchmark: Unregulated Mixed Duopoly

Setup (Section 2.2)

- Inverse demand $p(Q) = 1 - Q$, $Q = q_0 + q_1$; common marginal cost c .
- Emissions = output; damage $Env = \frac{d}{2}Q^2$.
- Welfare $W = CS + PS - Env$, with $CS = \frac{Q^2}{2}$.
- Public firm maximizes $V = (1 - \alpha)\pi_0 + \alpha W$; private firm maximizes profit.

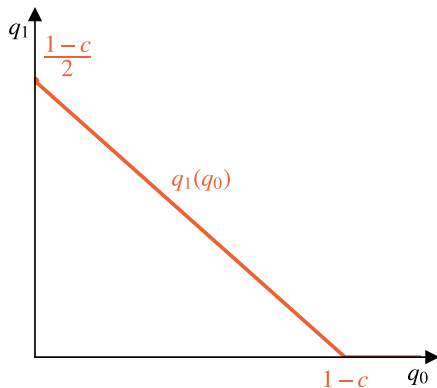
The Private Firm's Best Response (Section 2.2.1)

- Firm 1 maximizes $\pi_1 = (1 - q_0 - q_1)q_1 - cq_1$.
- Best response (independent of α):

$$q_1(q_0) = \frac{1 - c}{2} - \frac{1}{2}q_0.$$

- Slope $-\frac{1}{2}$: the private firm cuts one unit for every two the public firm adds.

Figure 2.1: The Private Firm's Best Response



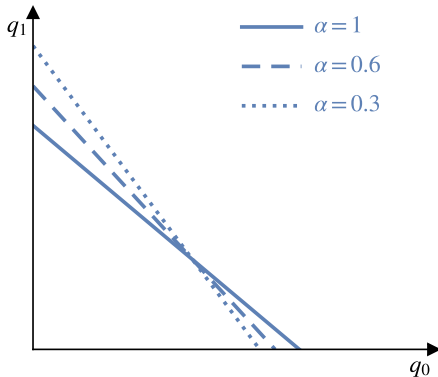
The Public Firm's Best Response (Section 2.2.2)

- FOC: $(1 - \alpha)(1 - Q - q_0 - c) + \alpha(1 - Q - c - dQ) = 0$,
giving

$$q_0(q_1) = \frac{(1 - c) - (1 + \alpha d)q_1}{2 - \alpha + \alpha d}.$$

- At $\alpha = 0$ it is the private reaction; at $\alpha = 1$ it becomes $\frac{1 - c}{1 + d} - q_1$.
- Higher α steepens the reaction (from $-\frac{1}{2}$ toward -1): a more public firm offsets the private firm more fully.

Figure 2.2: The Public Firm's Best Response over α



The Fully Public Case: $SMB = SMC$

- At $\alpha = 1$ the FOC reduces to

$$\underbrace{1 - Q}_{SMB} = \underbrace{c + dQ}_{SMC},$$

so aggregate output is $Q^{SO} = \frac{1 - c}{1 + d}$: the fully public firm reaches the social optimum.

- For $\alpha < 1$ the firm keeps some weight on profit and no longer targets Q^{SO} exactly.

Finding the Equilibrium (Section 2.2.3)

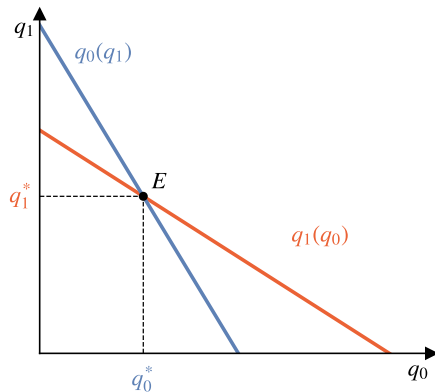
- With $D \equiv 3 - 2\alpha + \alpha d$,

$$q_1^{NR} = \frac{(1-c)(1-\alpha+\alpha d)}{D}, \quad q_0^{NR} = \frac{(1-c)(1-\alpha d)}{D},$$

$$\text{and } Q^{NR} = \frac{(1-c)(2-\alpha)}{D}.$$

- $\alpha = 0$: private duopoly $\frac{2(1-c)}{3}$. $\alpha = 1$: Q^{SO} .
- A more public firm moves the total toward the planner's level.

Figure 2.3: Equilibrium (Interior α)



Outputs as α Rises

- As the firm is nationalized, its output rises, the private firm's falls, and the total approaches Q^{SO} .
- The public firm exits only when $\alpha d \geq 1$: severe pollution and a green-enough firm.

Figure 2.4: Unregulated Outputs as α Rises

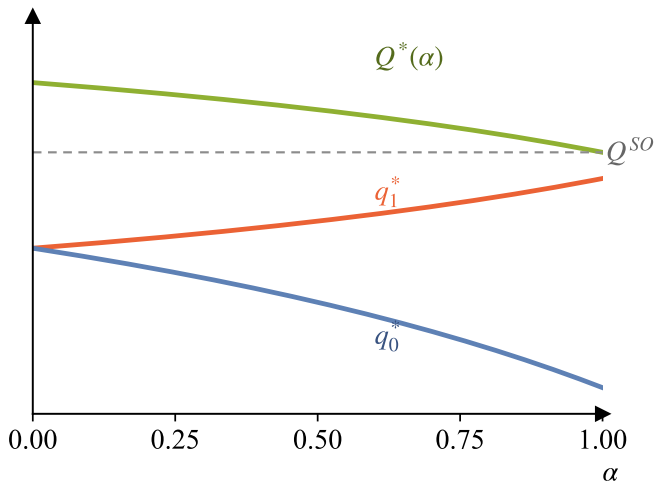
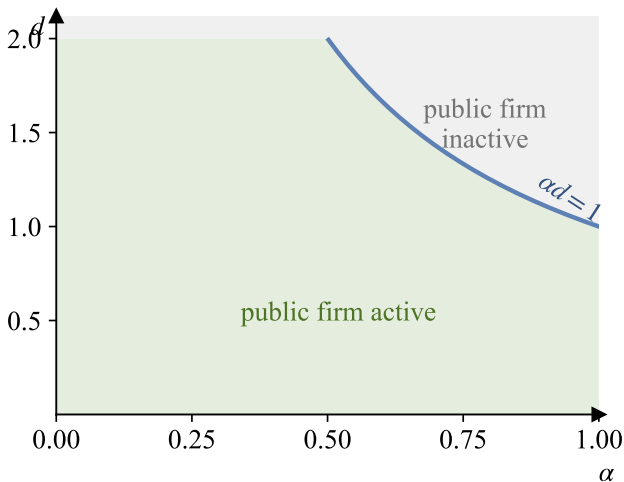


Figure 2.5: Active and Inactive Regimes, (α, d)



Welfare without Regulation (Section 2.2.4)

- The welfare loss

$$W^{SO} - W^{NR} = \frac{(1 - \alpha)^2(1 - c)^2(2d - 1)^2}{2(1 + d)D^2}$$

is zero only at $\alpha = 1$ and grows as the firm is privatized.

- Full public ownership reaches the first best; any privatization falls short, the more so the further d lies from $\frac{1}{2}$.

Section 2.3

Regulated Mixed Duopoly

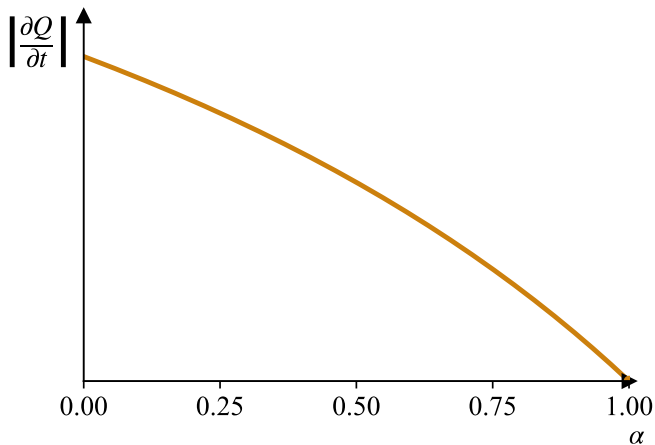
Stage 2: The Fee and the Public Firm (Section 2.3.1)

- Stage 1: the Environmental Agency sets t . Stage 2: firms choose output.
- Aggregate output under the fee:

$$Q(t) = Q^{NR} - \frac{2(1-\alpha)}{D} t.$$

- The fee bites more the more private the firm: traction rises from 0 at $\alpha = 1$ to $\frac{2}{3}$ at $\alpha = 0$. A fully public firm offsets it entirely.

Figure 2.6: The Fee Loses Traction as $\alpha \rightarrow 1$



Stage 1: The Optimal Fee (Section 2.3.2)

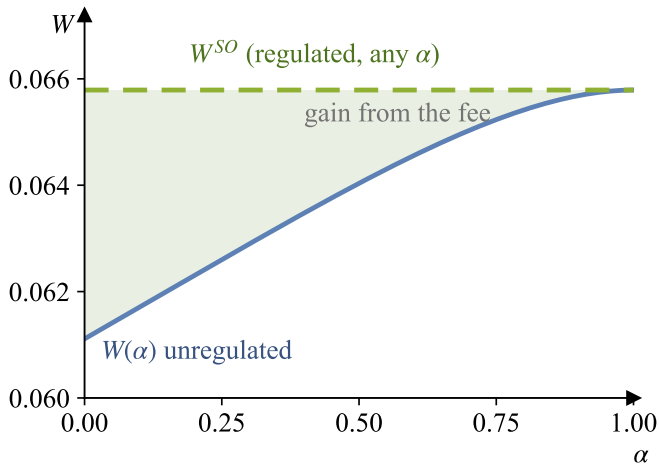
- The fee that restores the first best,

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)},$$

does **not** depend on α , and works for any $\alpha < 1$.

- It is a subsidy for mild pollution ($d < \frac{1}{2}$) and a charge for severe pollution.
- At $\alpha = 1$ the fee has no traction, but none is needed: the fully public firm already reaches Q^{SO} , so the fee is redundant.

Figure 2.7: Welfare against α



Section 2.4

Public Ownership as Implicit Regulation

Public Ownership as Implicit Regulation

- Both instruments enforce $SMB = SMC$: a fee raises private cost; public ownership imposes the correction from inside the objective.
- They are **substitutes**, and α sets how much each supplies: at $\alpha = 1$ the firm does it all; as α falls the fee does more, though the corrective fee itself is unchanged.
- The substitution breaks at the corner, where severe pollution drives the firm out and the fee takes over.

Other Regulations (Section 2.5)

- **Quotas:** redundant while the public firm is active; in the corner an aggregate cap $= Q^{SO}$ replicates the fee.
- **Tradable permits:** with symmetric costs there are no gains from trade; trade matters only under cost asymmetry (Chapter 3).

2.6 Case Study - Électricité de France

- EDF, France's dominant producer, was returned to full state ownership in 2023; it competes with private and foreign suppliers.
- Its low-emission (nuclear, hydro) fleet and public aims let it anchor output near the planner's level, so an added carbon charge does little.
- When its nuclear availability fell in 2022, it could no longer steer the market: the corner regime, where regulation must return.

Section 2.7

The Public Firm's Weight on Emissions

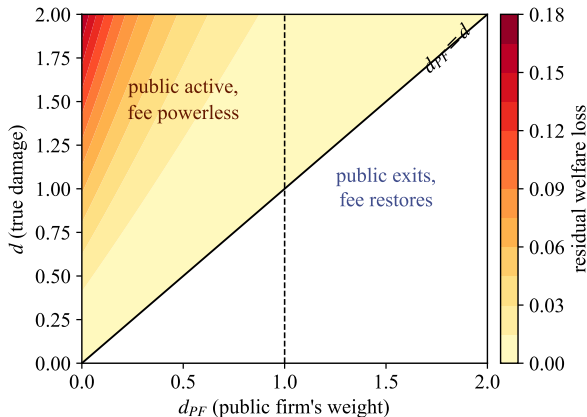
A Public Firm That Weighs Emissions Less

- Fix $\alpha = 1$ to isolate a second dial: let the firm maximize its own welfare $W_{PF} = CS + PS - \frac{d_{PF}}{2} Q^2$, with $d_{PF} \leq d$.
- It targets $Q = \frac{1-c}{1+d_{PF}}$: for $d_{PF} < d$ it over-produces.
- The fee is **powerless** (the firm offsets it); the loss

$$W^{SO} - W^{NR} = \frac{(1-c)^2(d-d_{PF})^2}{2(1+d)(1+d_{PF})^2}$$

persists. The corrective lever is the firm's **mandate**, not the fee.

Figure 2.8: Regimes in the Space (d_{PF} , d)



Section 2.8

Convex Costs

Convex Costs: Setup and Best Responses (Section 2.8)

- Let marginal cost rise with output: $C(q_i) = cq_i + \frac{\beta}{2}q_i^2$, so $MC = c + \beta q_i$, with $\beta \geq 0$; $\beta = 0$ is the baseline.
- The private reaction flattens: intercept $\frac{1-c}{2+\beta}$, slope $-\frac{1}{2+\beta}$.
- The public reaction adds β to its denominator, $2+\beta-\alpha(1-d)$.

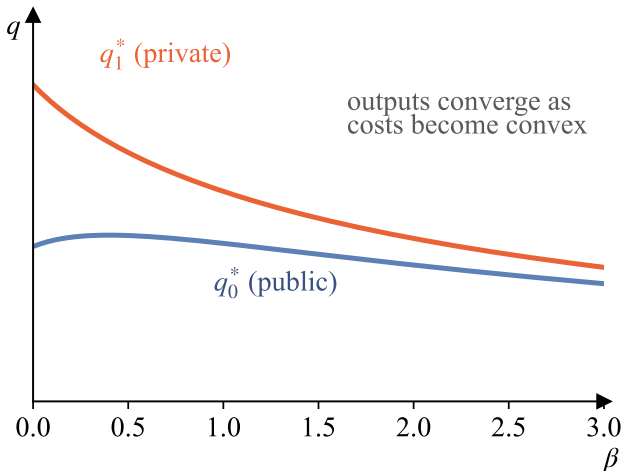
Equilibrium: Outputs Converge (Section 2.8)

- With $D_\beta = (1 + \beta)(3 + \beta) - \alpha(2 + \beta) + \alpha d(1 + \beta)$,

$$q_0^* = \frac{(1 - c)(1 + \beta - \alpha d)}{D_\beta}, \quad q_1^* = \frac{(1 - c)(1 + \beta - \alpha + \alpha d)}{D_\beta}.$$

- The gap $q_1^* - q_0^* = \frac{(1 - c)\alpha(2d - 1)}{D_\beta}$ shrinks as β grows:
convex costs pull the two outputs together.

Figure 2.9: Convex Costs Draw the Outputs Together



Social Optimum and a Limit of Public Ownership (Section 2.8)

- The planner now splits output equally:

$$q_0^{SO} = q_1^{SO} = \frac{1 - c}{2(1 + d) + \beta}.$$

- With $\beta > 0$ a fully public firm ($\alpha = 1$) no longer reaches the first best on its own: the loss is zero at $\beta = 0$ but positive for $\beta > 0$.
- Public ownership sets the level but cannot enforce the efficient split.

The Fee Regains a Role (Section 2.8)

- The fee bites even on a fully public firm: traction $\frac{\beta}{D_\beta} > 0$ at $\alpha = 1$, since it raises the firm's own marginal cost.
- The corrective fee $t^* = \frac{(1-c)(2d-1)}{2(1+d)+\beta}$ is independent of α and **falls** as β grows.
- Setting $t = t^*$ restores the first best, the level and the equal split, for every α and β .

Numerical Illustration (Section 2.9)

$c = \frac{1}{2}$, $d = 0.9$, **varying** α :

α	q_0	q_1	p	CS	Env	W
0.00	0.167	0.167	0.667	0.056	0.050	0.061
0.50	0.112	0.194	0.694	0.047	0.042	0.064
1.00	0.026	0.237	0.737	0.035	0.031	0.066

Welfare rises toward the first best $W^{SO} = 0.066$ as $\alpha \rightarrow 1$; the optimal fee $t^* = 0.105$ restores it for any $\alpha < 1$.

Summary / Key Takeaways

- The public firm maximizes $V = (1 - \alpha)\pi_0 + \alpha W$; its reaction steepens toward -1 as $\alpha \rightarrow 1$.
- Aggregate output $Q^{NR} = \frac{(1 - c)(2 - \alpha)}{D}$ runs from the private duopoly to Q^{SO} ; full public ownership is first best.
- The fee $t^* = \frac{(1 - c)(2d - 1)}{2(1 + d)}$ restores the first best for any $\alpha < 1$ and is redundant at $\alpha = 1$.
- Public ownership and the fee are substitutes; α sets how much each does.

Chapter 3

Cost Asymmetry

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 3

- 3.2 Unregulated mixed duopoly with asymmetric costs.
- 3.3 Cost advantage and cost disadvantage.
- 3.4 Regulation with asymmetric costs (incl. privatization and the cost gap).
- 3.5 Case study.
- 3.6 The public firm's weight on emissions.
- 3.7 Numerical illustration.
- 3.8 Summary.

The Central Problem

- Chapter 2 gave both firms the same cost, so only *total* output mattered.
- Now the public firm produces at c_0 , the private firm at c_1 , and the public firm keeps the general objective $V = (1 - \alpha)\pi_0 + \alpha W$.
- With a cost gap the **split** of output matters: a unit made by the high-cost firm wastes resources.
- We give the equilibrium for general α , then read the cost gap at the fully public case $\alpha = 1$.

Real-World Context

- Italian high-speed rail: **Trenitalia** (fully state-owned, via Ferrovie dello Stato) vs **Italo** (private), about a third of the market.
- Same lines, but the state incumbent and the private entrant carry different cost bases (legacy assets and labor vs newer stock).
- This cost gap between a public and a private firm is the subject of the chapter.

Section 3.2

Unregulated Mixed Duopoly with Asymmetric Costs

Private Firm's Best Response (Section 3.2.1)

- Private firm takes q_0 as given and maximizes $\pi_1 = (1 - q_0 - q_1)q_1 - c_1q_1$.
- FOC: $1 - q_0 - 2q_1 - c_1 = 0$, giving

$$q_1(q_0) = \frac{1 - c_1}{2} - \frac{1}{2}q_0.$$

- Same slope as Chapter 2, now with the private firm's own cost c_1 setting the intercept.

Public Firm's Best Response (Section 3.2.1)

- Public firm maximizes $V = (1 - \alpha)\pi_0 + \alpha W$; FOC combines the profit condition and the welfare condition,

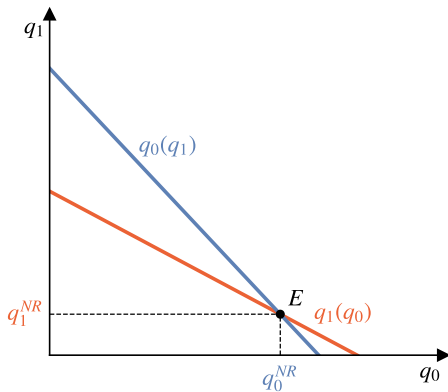
$$(1 - \alpha)(1 - Q - q_0 - c_0) + \alpha(1 - Q - c_0 - dQ) = 0.$$

- Solving,

$$q_0(q_1) = \frac{1 - c_0}{2 - \alpha + \alpha d} - \frac{1 + \alpha d}{2 - \alpha + \alpha d} q_1.$$

- At $\alpha = 1$ this simplifies to $\frac{1 - c_0}{1 + d} - q_1$: a fully public firm targets $Q = \frac{1 - c_0}{1 + d}$, using its **own** cost c_0 .

Figure 3.1: Equilibrium under a Public Cost Advantage



Equilibrium: System of Best Responses (Section 3.2.2)

- Substitute the private BR into the public BR and let $D \equiv 3 - 2\alpha + \alpha d$.
- Individual equilibrium outputs:

$$q_0^{NR} = \frac{1 + c_1 - 2c_0 - \alpha d(1 - c_1)}{D}, \quad q_1^{NR} = \frac{1 + c_0 - 2c_1 - \alpha(1 - c_1)}{D}$$

- Both are linear in the cost gap and in the weight α ; the denominator D is common.

Equilibrium: Aggregate Output

- Adding the two individual outputs,

$$Q^{NR} = \frac{(2 - c_0 - c_1) - \alpha(1 - c_1)}{D}.$$

- Symmetric costs, $c_0 = c_1 = c$: reduces to $\frac{(1 - c)(2 - \alpha)}{D}$, the Chapter 2 benchmark.
- Aggregate output falls with α when the public firm is efficient because it also shifts *who* produces.

Equilibrium at Full Nationalization ($\alpha = 1$)

- Aggregate output collapses to the planner's target:

$$Q^{NR} = \frac{1 - c_0}{1 + d}.$$

- Individual outputs use only c_0 , c_1 and d :

$$q_1^{NR} = \frac{d(1 - c_1) - (c_1 - c_0)}{1 + d}, \quad q_0^{NR} = \frac{1 - 2c_0 + c_1 - d(1 - c_1)}{1 + d}.$$

- The private firm remains active when $d(1 - c_1) > c_1 - c_0$, i.e., its cost handicap is not too large.

Comparative Statics: Efficient Public Firm (Section 3.2.3)

- Case $c_0 < c_1$: nationalizing (higher α) raises q_0 and lowers q_1 .
- Two effects work together:
 - *Level*: aggregate output moves toward the planner's target.
 - *Split*: production shifts from the high-cost private firm to the low-cost public firm.
- Welfare rises on both margins; the two effects reinforce one another.

Comparative Statics: Inefficient Public Firm (Section 3.2.3)

- Case $c_0 > c_1$: nationalizing again raises q_0 and lowers q_1 .
- Now the two effects pull against each other:
 - Level moves toward a target that is itself too low.
 - Split moves output to the *higher*-cost firm.
- Tension motivates the rest of the chapter; to read the cost gap cleanly we set $\alpha = 1$.

Advantage versus Disadvantage (Section 3.3)

- **Public cost advantage** ($c_0 < c_1$): total output is right, but part comes from the high-cost private firm $\Rightarrow$ productive inefficiency, welfare below first best.
- **Public cost disadvantage** ($c_0 > c_1$): the target is too low and the public firm produces high-cost output; if the gap is large it exits.
- Intuitively: an inefficient public firm may do better by withdrawing than by producing.

Section 3.4

Regulation with Asymmetric Costs

The Fee Reshuffles Output (Section 3.4.1, $\alpha = 1$)

- Fee t raises the private firm's marginal cost to $c_1 + t$ and leaves the public firm's target $Q = \frac{1 - c_0}{1 + d}$ unchanged.
- Equilibrium outputs under the fee,

$$q_1(t) = q_1^{NR} - t, \quad q_0(t) = q_0^{NR} + t.$$

- Aggregate output stays at Q^{NR} : the fee only moves production between firms.

Welfare Derivative: How the Fee Moves Welfare

- Substitute the outputs above into $W = CS + PS - Env$; since Q is fixed only production costs change:

$$\frac{\partial W}{\partial t} = c_1 - c_0.$$

- Each unit shifted from the private to the public firm saves the cost difference $c_1 - c_0$.
- The sign of the cost gap decides everything: fee helps if the public firm is efficient, hurts if it is not.

Optimal Fee under Cost Advantage: Corner Solution (Section 3.4.2)

- With $c_0 < c_1$, $\partial W / \partial t > 0$: welfare rises with t until the private firm exits.
- Set $q_1(t) = q_1^{NR} - t = 0$:

$$t^* = q_1^{NR} = \frac{d(1 - c_1) - (c_1 - c_0)}{1 + d}.$$

- Above t^* the private firm is out and further increases in t are inactive.

Optimal Fee under Cost Advantage: First Best

- At t^* the public firm produces the whole industry output at cost c_0 ; welfare reaches

$$W^{SO} = \frac{(1 - c_0)^2}{2(1 + d)}.$$

- Key contrast with Chapter 2: the fee is **not** redundant in the interior; it moves production to the efficient firm.
- Regulation strictly raises welfare even where public ownership already delivers the right total output.

Figure 3.2: Outputs as the Private Cost Rises

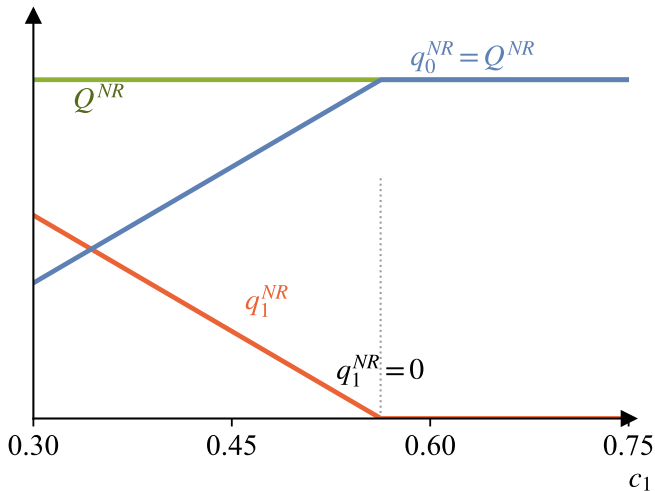
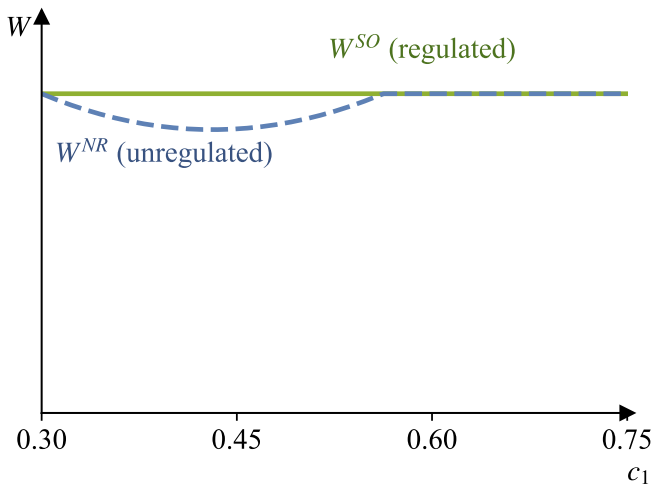


Figure 3.3: Welfare Gap Closed by Regulation



When the Fee Cannot Help (Section 3.4.3)

- Under a disadvantage ($c_0 > c_1$), $\frac{\partial W}{\partial t} < 0$: a fee shifts output to the inefficient public firm and lowers welfare.
- The optimal fee is $t^* = 0$.
- The remedy is the opposite of a fee: a subsidy to the efficient private firm, or **partial privatization** (Chapter 9; De Fraja and Delbono, 1989).

Privatization and the Cost Gap: Under Advantage (Section 3.4.4)

- For $\alpha < 1$ the public firm no longer sets the total by itself, so the fee regains traction on the **level** as well as the split (Chapter 2).
- Under a cost advantage the fee then works on both margins at once:
 - corrects the level distortion inherited from privatization;
 - continues to move output to the efficient public firm.
- A partially privatized efficient firm gains from the fee even more than a fully public one.

Privatization and the Cost Gap: Under Disadvantage

- Under a disadvantage the fee remains counterproductive, but lowering α is itself corrective.
- A smaller weight on the inefficient public firm's production shifts output to the efficient private firm.
- Chapter 9 develops partial privatization as the instrument of choice when the public firm is inefficient.

3.5 Case Study - Petróleos Mexicanos

- Pemex, wholly state-owned, competes with private and foreign operators since the 2013 reform and is widely seen as a high-cost producer.
- An inefficient public firm pulls output to its own expensive plants; a fee would make that worse, shifting production toward it.
- What raises welfare is improving Pemex's efficiency or letting the efficient private firms carry more of the output.

Public Firm's Weight on Emissions: Setup (Section 3.6)

- Fix $\alpha = 1$; the public firm weighs emissions at $d_{PF} \leq d$ rather than at the true damage.
- It maximizes its own welfare $W_{PF} = CS + PS - \frac{d_{PF}}{2} Q^2$ and targets

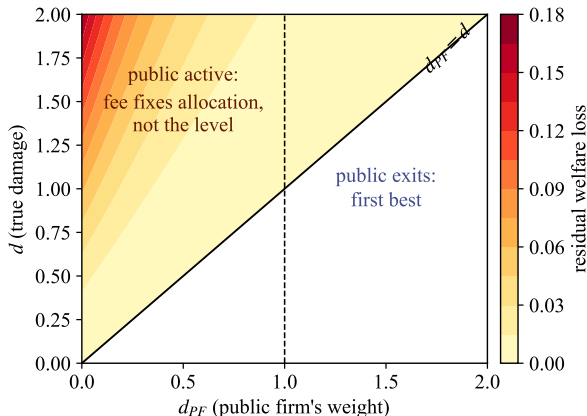
$$Q = \frac{1 - c_0}{1 + d_{PF}}.$$

- Both its *own* cost and its *own* weight enter the target.

Public Firm's Weight on Emissions: Two Distortions

- **Level distortion** (from $d_{PF} < d$): target output is too high; the fee cannot correct it while the public firm is active.
- **Allocation distortion** (cost gap): still corrected by the fee at rate $\frac{\partial W}{\partial t} = c_1 - c_0$, for any d_{PF} .
- A cost-advantaged public firm leaves the regulator able to fix the allocation but not the level.

Figure 3.4: Residual Welfare Loss in (d_{PF}, d)



Numerical Illustration: Parameters (Section 3.7)

- Costs: $c_0 = \frac{3}{10}$ (public), $c_1 = \frac{1}{2}$ (private) $\Rightarrow$ cost advantage, gap $c_1 - c_0 = 0.2$.
- Emission damage: $d = \frac{6}{10}$.
- Vary the degree of nationalization $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$; no fee.
- Report equilibrium outputs, price, consumer and producer surplus, damage, and welfare.

Numerical Illustration: Table 3.1

α	q_0	q_1	p	CS	PS	Env	W
0.00	0.300	0.100	0.600	0.080	0.100	0.048	0.132
0.25	0.311	0.094	0.594	0.082	0.101	0.049	0.133
0.50	0.326	0.087	0.587	0.085	0.101	0.051	0.135
0.75	0.346	0.077	0.577	0.089	0.102	0.054	0.138
1.00	0.375	0.063	0.562	0.096	0.102	0.057	0.141

Numerical Illustration: Reading the Table

- As α rises: q_0 rises, q_1 falls, aggregate output rises, price falls.
- Both margins push welfare up:
 - Level: aggregate output approaches the planner's target.
 - Split: production shifts from the high-cost private firm to the low-cost public firm.
- W rises from 0.132 at $\alpha = 0$ to 0.141 at $\alpha = 1$; nationalizing an efficient public firm raises welfare on both margins.

Summary / Key Takeaways

- The public firm (weight α , cost c_0) competes with a private firm at c_1 ; at $\alpha = 1$ it targets $\frac{1 - c_0}{1 + d}$.
- Welfare moves with the fee at rate $c_1 - c_0$: the fee reaches the first best under a cost advantage and is counterproductive under a disadvantage.
- Nationalizing an efficient public firm raises welfare; privatizing an inefficient one does.

Chapter 4

More Private Firms

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 4

- 4.2 Unregulated mixed oligopoly with n private firms.
- 4.3 Regulation with n private firms.
- 4.4 Case study.
- 4.5 The public firm's weight on emissions.
- 4.6 Numerical illustration.
- 4.7 Summary.

The Central Problem

- Most markets with a public firm contain several private competitors.
- One public firm (weight α on welfare) now faces $n \geq 1$ private firms at cost c ; $n = 1$ recovers Chapter 2.
- Key insight: there is a threshold, in n and α , beyond which the public firm can no longer hold the market to the social optimum.

Real-World Context

- Power generation in India: the state-owned **NTPC** (majority government-owned) is the largest generator.
- It competes with private generators: **Adani Power, Tata Power, JSW Energy**.
- One public firm against a growing number of private rivals, in a coal-heavy, high-emission industry.

Section 4.2

Unregulated Mixed Oligopoly

Setup (Section 4.2)

- One public firm (index 0) and n symmetric private firms at cost c .
- Inverse demand $p = 1 - Q$, with $Q = q_0 + \sum_{i=1}^n q_i$; environmental damage $\frac{d}{2}Q^2$.
- Private firm i maximizes $\pi_i = (1 - Q)q_i - cq_i$; the public firm maximizes $V = (1 - \alpha)\pi_0 + \alpha W$.
- Focal parameters: the number of private firms n and the degree of nationalization α .

Private Firm's Best Response (Section 4.2.1)

- FOC: $1 - c - q_0 - \sum_j q_j - q_i = 0$; with symmetric private output q_p this becomes $1 - c - q_0 - nq_p - q_p = 0$.
- Solving for q_p gives the two-term best response

$$q_p = \frac{1 - c}{n + 1} - \frac{1}{n + 1} q_0.$$

- Vertical intercept $\frac{1 - c}{n + 1}$ (private output when the public firm is absent); slope $\frac{1}{n + 1}$ measures the crowding by the public firm's output.

Public Firm's FOC (Section 4.2.1)

- Welfare component pulls toward marginal social cost; profit component pulls toward marginal revenue.
- Differentiating $V = (1 - \alpha)\pi_0 + \alpha W$ with respect to q_0 ,

$$(1 - \alpha)(1 - Q - q_0 - c) + \alpha(1 - Q - c - dQ) = 0.$$

- At $\alpha = 1$ the profit term drops out and the condition reduces to $1 - Q - c - dQ = 0$, i.e. $Q = \frac{1 - c}{1 + d} = Q^{SO}$, the first best.

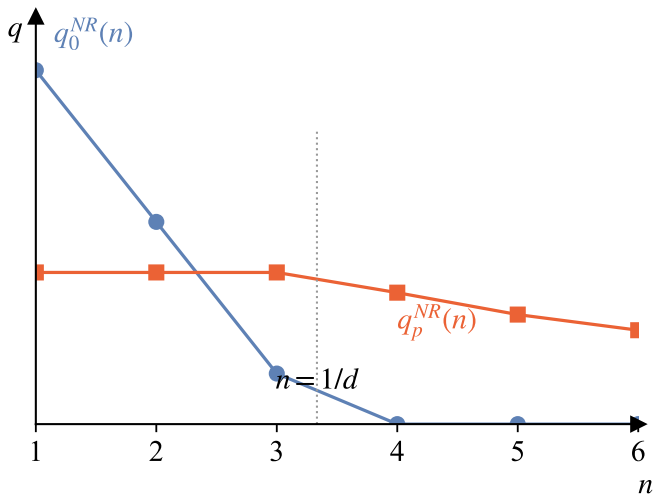
Public Firm's Best Response (Two-Term Form)

- Grouping the FOC in q_0 and the aggregate private output $Q_p = nq_p$,

$$q_0(Q_p) = \frac{1 - c}{2 - \alpha + \alpha d} - \frac{1 + \alpha d}{2 - \alpha + \alpha d} Q_p.$$

- Vertical intercept falls with α (more weight on welfare $\Rightarrow$ smaller intercept only when d is not extreme); slope rises with αd .
- At $\alpha = 1$: $q_0(Q_p) = \frac{1 - c}{1 + d} - Q_p$, so the fully public firm holds total output at Q^{SO} whatever the number of private firms.

Figure 4.1: Outputs as n Grows (fully public firm)



Solving the System of Best Responses

- Two equations in q_p and q_0 :

$$q_p = \frac{1-c}{n+1} - \frac{1}{n+1} q_0, \quad q_0 = \frac{1-c}{2-\alpha+\alpha d} - \frac{1+\alpha d}{2-\alpha+\alpha d} n q_p.$$

- Substitute the private best response into the public best response and collect terms in q_0 ; the denominator that appears is

$$D \equiv 1 + (n+1)(1-\alpha) + \alpha d.$$

- Then solve back for q_p and the aggregate $Q = q_0 + n q_p$.

Equilibrium Outputs (Section 4.2.2)

- Each private firm produces

$$q_p^{NR} = \frac{(1-c)(1-\alpha+\alpha d)}{D}.$$

- The public firm produces the residual

$$q_0^{NR} = \frac{(1-c)(1-\alpha dn)}{D}.$$

- Aggregate output $Q^{NR} = \frac{(1-c)[1+n(1-\alpha)]}{D}$; setting $n = 1$ returns the Chapter 2 duopoly.

The Fully Public Benchmark ($\alpha = 1$)

- At $\alpha = 1$, the denominator collapses to $D = 1 + d$ and

$$q_p^{NR} = \frac{(1-c)d}{1+d}, \quad q_0^{NR} = \frac{(1-c)(1-nd)}{1+d},$$

so aggregate output equals $Q^{NR} = \frac{1-c}{1+d} = Q^{SO}$.

- Each private firm always produces the positive quantity $\frac{(1-c)d}{1+d}$; the fully public firm produces the residual and can turn negative once $nd \geq 1$.

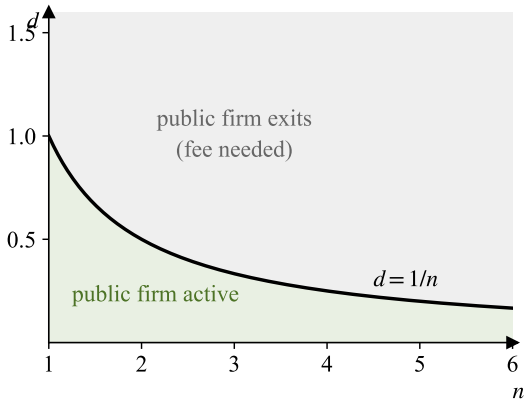
The Exit Threshold

- From the residual expression,

$$q_0^{NR} > 0 \iff \alpha dn < 1.$$

- Beyond $\alpha dn \geq 1$ the public firm shuts down: more private firms (higher n), or a greener firm under more severe pollution (higher αd), push it out.
- At full nationalization the boundary is $d = \frac{1}{n}$; a partially privatized firm cares less about damage and stays in the market longer.

Figure 4.2: Where a Fully Public Firm Suffices



Comparative Statics I: Effect of n

- While the public firm is active, each private firm's output $q_p^{NR} = \frac{(1-c)(1-\alpha+\alpha d)}{D}$ is independent of n (the n -terms sit only in D through the public firm's response).
- Adding a private firm therefore raises aggregate private output by q_p^{NR} and lowers q_0^{NR} by the same amount.
- The public firm absorbs the entire crowding: total output stays at Q^{SO} (under $\alpha = 1$) until the residual is exhausted.

Comparative Statics II: Effect of α

- A higher α shifts the public firm's intercept down and its slope up, so it produces less at each level of Q_p except when welfare so demands.
- Nationalizing the firm moves aggregate output toward the social optimum, as in Chapter 2, until the exit threshold binds.
- Past the threshold, further nationalization has no effect: the firm is already inactive and the market is a pure private oligopoly.

Section 4.3

Regulation with n Private Firms

Interior Regime: The Optimal Fee (Section 4.3.1)

- While the public firm is active ($\alpha dn < 1$), the emission fee restores the first best for any $\alpha < 1$ at

$$t^* = \frac{(1 - c)[d(n + 1) - 1]}{(1 + d)(n + 1)}.$$

- Setting $n = 1$ returns the duopoly fee of Chapter 2; t^* does not depend on α : the same charge corrects the market at every degree of privatization.
- Redundant at $\alpha = 1$: the fully public firm already reaches Q^{SO} without a fee.

Interior Regime: Reading the Fee

- More private firms, whose output the fee must curb, call for a larger charge: $\frac{\partial t^*}{\partial n} = \frac{(1-c)(1+d)}{(1+d)(n+1)^2} > 0$.
- Public ownership and the fee remain substitutes: the fee does more of the work the more privatized the firm.
- The number of private firms changes the size of the fee but not the principle behind it.

Corner Regime: Setup (Section 4.3.2)

- Once $\alpha dn \geq 1$ (at $\alpha = 1$: $d \geq \frac{1}{n}$), the public firm has exited and the market is a pure private oligopoly of n firms.
- Under a fee t , each private firm's FOC yields

$$q = \frac{1 - c - t}{n + 1},$$

so aggregate output is $\frac{n(1 - c - t)}{n + 1}$.

- The Environmental Agency chooses t to align aggregate output with the social optimum $\frac{1 - c}{1 + d}$.

Corner Regime: Deriving the Fee

- Set $\frac{n(1-c-t)}{n+1} = \frac{1-c}{1+d}$ and solve for t :

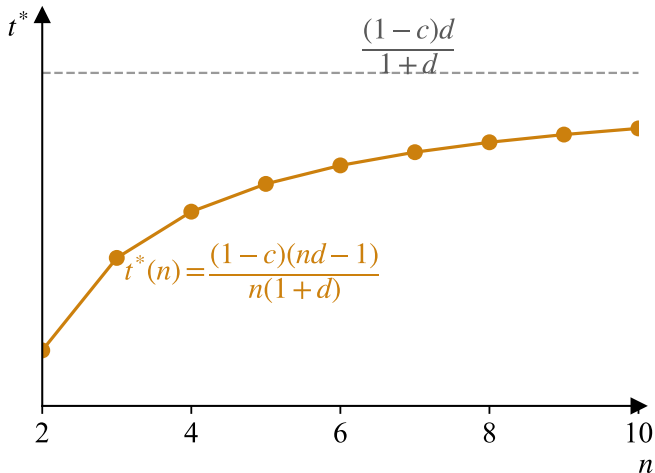
$$t^*(n) = \frac{(1-c)(nd-1)}{n(1+d)} = \frac{1-c}{1+d} \left(d - \frac{1}{n} \right).$$

- Zero at the boundary $d = \frac{1}{n}$ and positive beyond it, joining the interior fee continuously.
- Setting $n = 1$ returns the Chapter 2 corner fee.

Corner Regime: Comparative Statics

- Rising in the number of private firms: $\frac{\partial t^*}{\partial n} = \frac{1 - c}{n^2(1 + d)} > 0$.
- Rising in the pollution weight d : $\frac{\partial t^*}{\partial d} = \frac{(1 - c)(n + 1)}{n(1 + d)^2} > 0$.
- As $n \rightarrow \infty$, $t^* \rightarrow \frac{(1 - c)d}{1 + d}$: the fee needed to correct a fully competitive polluting industry.

Figure 4.3: The Optimal Fee in the Corner Regime



Case Study - India's Power Generation and NTPC (Section 4.4)

- **Market structure.** NTPC (majority state-owned) is India's largest generator; it competes with Adani Power, Tata Power, and JSW Energy.
- As private capacity has grown, NTPC's share of new additions has fallen, though it remains the single largest producer.
- Much of the country's thermal capacity, public and private, is coal-fired, so the environmental stakes are large.

Case Study - Reading through the Model

- A public firm holds output near the social optimum only while its rivals are few; as n grows, it is crowded to the margin.
- Once the exit threshold binds, the fee regains its role: relying on the public firm's mandate to discipline emissions becomes less effective.
- India's trajectory, with private and renewable capacity growing faster than NTPC's, follows the comparative statics of this chapter.

Section 4.5

The Public Firm's Weight on Emissions

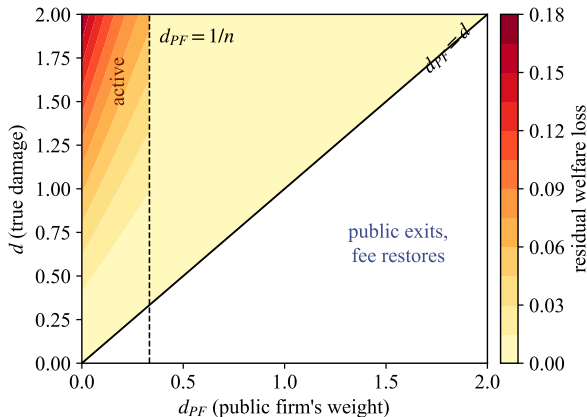
Setup: Own Emissions Weight d_{PF}

- Fix $\alpha = 1$ to isolate the emissions-weight dial; let the public firm maximize W_{PF} with weight $d_{PF} \leq d$.
- With n private firms it targets $Q = \frac{1-c}{1+d_{PF}}$; each private firm produces $\frac{(1-c)d_{PF}}{1+d_{PF}}$, and the public firm the residual $\frac{(1-c)(1-nd_{PF})}{1+d_{PF}}$.
- The public firm stays active while $d_{PF} < \frac{1}{n}$; a firm that undervalues emissions is pushed out at a smaller n .

Regulation and the Regime Boundary

- While the public firm is active the emission fee is powerless (Section 2.7), and the loss from $d_{PF} < d$ persists.
- Once $d_{PF} \geq \frac{1}{n}$ the public firm exits and the fee restores the first best.
- The regime boundary sits at $d_{PF} = \frac{1}{n}$ rather than at $d_{PF} = 1$ as in Chapter 2: more private firms shift the boundary inward.

Figure 4.4: Regimes in (d_{PF}, d) with n Private Firms



Section 4.6

Numerical Illustration

Parameters

- Marginal cost $c = \frac{1}{2}$; two private firms $n = 2$; severe pollution $d = \frac{9}{10}$.
- Vary the degree of nationalization $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$.
- Exit threshold at these parameters:
$$\alpha dn \geq 1 \iff \alpha \geq \frac{1}{1.8} \approx 0.556.$$

Table 4.1: Unregulated Mixed Oligopoly

α	q_0	q_p	p	CS	PS	Env	W
0.00	0.125	0.125	0.625	0.047	0.063	0.054	0.054
0.25	0.079	0.140	0.640	0.050	0.058	0.052	0.057
0.50	0.017	0.161	0.661	0.055	0.052	0.048	0.060
0.75	0.000	0.167	0.667	0.056	0.050	0.050	0.061
1.00	0.000	0.167	0.667	0.056	0.050	0.050	0.061

Reading the Table

- As α rises the public firm restrains its output and welfare increases.
- Under this severe pollution, a firm public enough ($\alpha > 0.556$) withdraws entirely: two private firms serve the market.
- Welfare then plateaus below the first best 0.066: past the exit threshold, nationalization can do no more and a fee is needed.

Summary / Key Takeaways

- Each private firm produces $\frac{(1-c)(1-\alpha+\alpha d)}{D}$; the public firm the residual; at $\alpha = 1$ these are the fully public outputs.
- The public firm stays active only while $\alpha dn < 1$: more private firms, or a greener firm under severe pollution, push it out.
- The interior fee $\frac{(1-c)[d(n+1)-1]}{(1+d)(n+1)}$ restores the first best for any $\alpha < 1$; the corner fee $\frac{1-c}{1+d}(d - \frac{1}{n})$ rises with n and d .
- Public ownership substitutes well for regulation in concentrated markets, poorly in crowded ones.

Chapter 5

Several Public Firms

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 5

- 5.2 Benchmark: several public firms.
- 5.3 Regulation with several public firms.
- 5.4 Cost-asymmetric public firms.
- 5.5 Case studies.
- 5.6 The public firms' weights on emissions.
- 5.7 Numerical illustration and summary.

The Central Problem

- Many polluting industries hold **several** public firms, each answering to a different government.
- Each public firm i maximizes $V_i = (1 - \alpha_i)\pi_i + \alpha_i[CS + PS - \frac{d_i}{2}Q^2]$; we fix $\alpha_i = 1$ to isolate the differing emissions weights $d_1, \dots, d_k$. k public firms plus $n - k$ private; $k = 1$ recovers Chapter 4.
- Key question: when the public firms differ in how much they value the environment, which one determines the market outcome?

Real-World Context

- **Oil and gas:** Aramco, Equinor ($\approx$ two-thirds Norwegian state), Petrobras, Pemex, ADNOC, the Chinese majors, all alongside ExxonMobil, Shell, Chevron.
- **Copper mining:** Chile's fully state-owned **Codelco**, the world's largest producer, with private majors (Freeport-McMoRan, BHP).
- These public producers carry different climate commitments.

Section 5.2

Benchmark: Several Public Firms

Model Setup (Section 5.2)

- k public firms and $n - k$ private firms, all with marginal cost c ; inverse demand $p = 1 - Q$ and damage $\frac{d}{2}Q^2$.
- Public firm i : $V_i = (1 - \alpha_i)\pi_i + \alpha_i[CS + PS - \frac{d_i}{2}Q^2]$, with $0 \leq d_i \leq d$.
- Baseline: $\alpha_i = 1$ for every public firm, so each maximizes

$$CS + PS - \frac{d_i}{2}Q^2,$$

while every private firm maximizes profit.

Private Firm's Best Response (Section 5.2.1)

- Private firm j maximizes $\pi_j = (1 - Q - c)q_j$, with $Q = q_j + Q_{-j}$.
- Its first-order condition, $1 - 2q_j - Q_{-j} - c = 0$, gives

$$q_j = \frac{1 - c}{2} - \frac{1}{2} Q_{-j}.$$

- Intercept $\frac{1 - c}{2}$; slope $-\frac{1}{2}$ against rival output, as in Cournot.

Public Firm's Target for Aggregate Output (Section 5.2.1)

- Public firm i 's objective is
 $CS + PS - \frac{d_i}{2} Q^2 = (1 - c)Q - \frac{1+d_i}{2} Q^2$.
- First-order condition: $1 - Q - c - d_i Q = 0$, giving the target

$$Q = \frac{1 - c}{1 + d_i}. \quad (5.1)$$

- The public firm aims at a **total**, not its own output. Greener firms (higher d_i) seek a lower total.

Equal Weights: Coordination (Section 5.2.2)

- If every public firm shares the same weight d_{PF} , all target the same total $\frac{1 - c}{1 + d_{PF}}$.
- Any split of production among them that reaches this total is an equilibrium.
- They act as a single welfare-minded producer split into k parts: the market matches the one-public-firm case of Chapter 4 with weight d_{PF} .

Different Weights: The Laggard Sets the Total (Section 5.2.3)

- Order $d_1 \leq d_2 \leq \dots \leq d_k$; the targets cannot all be met.
- Firm 1 seeks the highest total, $\frac{1-c}{1+d_1}$; a greener firm can push Q down only by cutting its own output, and cannot go below zero.
- Greener firms are **crowded out** to zero; firm 1 stays active and in equilibrium

$$Q = \frac{1-c}{1+d_{\min}}, \quad d_{\min} = \min_i d_i. \quad (5.2)$$

Individual Outputs at Equilibrium

- Each private firm produces

$$q_p = \frac{(1 - c) d_{\min}}{1 + d_{\min}},$$

as in the Chapter 4 single-public-firm market.

- The least green public firm supplies the residual $Q - (n - k)q_p$; the greener public firms produce zero.
- The industry's output and emissions are set by the *least* committed public firm.

Figure 5.1: Greener Public Firms Are Crowded Out

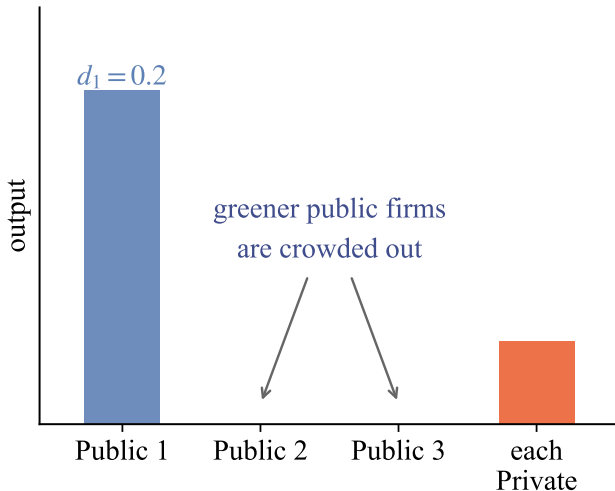
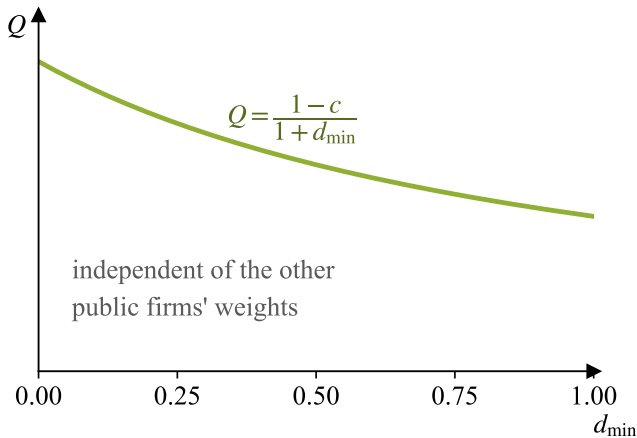


Figure 5.2: Only the Laggard's Weight Matters



Section 5.3

Regulation with Several Public Firms

Why the Emission Fee Is Powerless (Section 5.3)

- While the least green public firm is active, its target $\frac{1 - c}{1 + d_{\min}}$ depends on $d_{\min}$ and c , not on the fee.
- Whatever t the regulator sets, the laggard adjusts its own output to hold Q at its target. The fee cannot move aggregate output.
- If $d_{\min} = d$, the market is already at the first best and no fee is needed. If $d_{\min} < d$, the market overproduces.

Residual Welfare Loss (Section 5.3)

- When $d_{\min} < d$, welfare falls short of the first best by

$$\Delta W = \frac{(1 - c)^2 (d - d_{\min})^2}{2(1 + d)(1 + d_{\min})^2}.$$

- The gap grows as the laggard's weight drifts further from the true damage d .
- The fee cannot close ΔW so long as a public firm sets Q ; only once the laggard exits does the fee reach the remaining private firms.

The Effective Lever: The Laggard's Mandate

- Raising $d_{\min}$ (a greener mandate for the least committed firm) lowers Q and emissions.
- Making an already greener public firm greener does nothing: it is already sidelined at zero output.
- Environmental policy should aim at the *least* committed public producer, not at the greenest, and not, while a public firm is active, at the emission fee.

Section 5.4

Cost-Asymmetric Public Firms

Cost-Asymmetric Public Firms: Combined Target

- With heterogeneous costs c_i , public firm i 's first-order condition gives the target

$$Q_i^* = \frac{1 - c_i}{1 + d_i},$$

which combines its cost and its environmental weight.

- Both a low c_i and a low d_i push the target up.

Cost-Asymmetric Public Firms: Outcome (Section 5.4)

- The public firm with the highest $\frac{1 - c_i}{1 + d_i}$ stays active and sets aggregate output; the others are crowded out to zero.
- A low-cost, environmentally indifferent public firm is the most consequential: it produces the most, and it pulls emissions up.
- Same lesson as the symmetric-cost case, now read in terms of the combined target $\frac{1 - c_i}{1 + d_i}$ rather than d_i alone.

5.5 Case Study - China's Oil and Refining Majors

- Sinopec, PetroChina (CNPC), and CNOOC are three state-controlled majors that differ in focus, plus a fringe of independent refiners.
- The least environmentally committed active major sets the industry's output; greening one major does little while a less green one remains active.
- An economy-wide instrument, such as China's national ETS (2021), reaches the externality more reliably than any single producer's mandate.

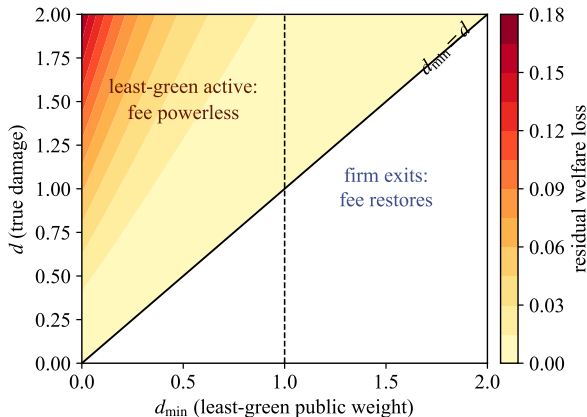
Section 5.6

The Public Firms' Weights on Emissions

Reading the Weights (Section 5.6)

- Only the least green weight $d_{\min}$ enters the outcome.
- The residual loss $\frac{(1-c)^2(d-d_{\min})^2}{2(1+d)(1+d_{\min})^2}$ is set by the gap between $d_{\min}$ and the true damage d .
- The fee restores the first best only once enough private firms crowd the laggard out.

Figure 5.3: Residual Welfare Loss in $(d_{\min}, d)$



Compounding α_i and d_i

- A lower degree of nationalization α_i works in the same direction as a lower d_i : both raise the total that firm i seeks.
- The firm that sets the market is the least committed on **both** counts, that is, the one with the highest target given its pair (α_i, d_i) .
- Nationalizing or greening any other public firm leaves the outcome unchanged.

Numerical Illustration: Parameters (Section 5.7)

- $c = \frac{1}{2}$, one private firm, true damage $d = \frac{6}{10}$.
- Two public firms with weights $d_1 = \frac{3}{10}$ (least green) and d_2 varying from $\frac{3}{10}$ to $\frac{9}{10}$.
- Aggregate target: $Q = \frac{1 - c}{1 + d_{\min}} = \frac{0.5}{1.3} \approx 0.385$; price $p = 1 - Q \approx 0.615$; private output $q_p = \frac{(1 - c)d_{\min}}{1 + d_{\min}} \approx 0.115$.

Numerical Illustration: Table 5.1 and Invariance

d_2	q_0^1	q_0^2	q_p	p	t^*	CS	PS	Env	W
0.3	0.135	0.135	0.115	0.615	0	0.074	0.044	0.044	0.074
0.5	0.269	0.000	0.115	0.615	0	0.074	0.044	0.044	0.074
0.7	0.269	0.000	0.115	0.615	0	0.074	0.044	0.044	0.074
0.9	0.269	0.000	0.115	0.615	0	0.074	0.044	0.044	0.074

Only the split between the two public firms changes with d_2 ; aggregate output, price, and every welfare component are pinned by d_1 .

Summary / Key Takeaways

- Equal weights: the public firms act as one; the total is pinned at $\frac{1 - c}{1 + d_{PF}}$.
- Different weights: the least green firm sets $Q = \frac{1 - c}{1 + d_{\min}}$; greener firms are crowded out.
- Only $d_{\min}$ matters; the fee is powerless while the laggard is active, and the effective lever is the laggard's mandate.

Chapter 6

Private Mergers

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 6

- 6.2 A private merger with no synergies.
- 6.3 When a merger brings the public firm back.
- 6.4 Regulation and the merger.
- 6.5 Mergers of different size.
- 6.6 Case study - India's power sector.
- 6.7 The public firm's weight on emissions.
- 6.8 Numerical illustration.
- 6.9 Summary.

The Central Problem

- What happens when some private firms merge, with a public firm present?
- Focal parameter: the number k of firms merging; $k = 1$ is no merger.
- The public firm keeps the general objective $V = (1 - \alpha)\pi_0 + \alpha W$; we develop the fully public case ($\alpha = 1$), where neutrality is cleanest, and Chapter 10 studies how privatization changes which mergers are approved.
- Key insight: a public firm **cushions** private mergers, so concentration can rise without any loss of welfare.

Guiding Questions (Section 6.1.1)

- How does a merger among private firms change total output and welfare when a public firm is present?
- Can market concentration rise from a merger without any loss of welfare?
- Might a merger return a public firm to a market it had left?
- How does the public firm's presence change the case for blocking a merger on concentration grounds?

Real-World Context

- India's power sector: private generators such as **Adani Power** have absorbed rival private plants through acquisition.
- The state-owned **NTPC** has stayed the largest producer.
- A private merger with a public firm still present: how does its response shape the effect on price, emissions, and welfare?

Section 6.2

A Private Merger with No Synergies

The Merger as a Fall in the Number of Firms (Section 6.2.1)

- With no cost savings, k merging firms become one firm at cost c .
- The market is Chapter 4 with $m \equiv n - k + 1$ private firms.
- Every equilibrium object carries over with this substitution.

Private Firm's Best Response (Chapter 4 Recap)

- Each of the m private firms maximizes $(1 - Q - c) q_i$, with $Q = q_0 + Q_p$.
- The FOC $1 - c - q_0 - 2q_i - Q_{-i} = 0$ yields the two-term best response

$$q_i(q_0, Q_{-i}) = \frac{1 - c}{2} - \frac{1}{2} (q_0 + Q_{-i}).$$

- Intercept $(1 - c)/2$; slope $1/2$ on rival output. Both are independent of m , so the merger acts through the number of firms alone.

Public Firm's Best Response (Chapter 4 Recap)

- With $\alpha = 1$ the public firm maximizes welfare
$$W = (1 - c)Q - \frac{1}{2}(1 + d)Q^2.$$
- Its FOC $(1 - c) - (1 + d)Q = 0$ pins total output at the social optimum $Q^{SO} = \frac{1 - c}{1 + d}.$
- Solving for q_0 gives the two-term best response

$$q_0(Q_p) = \frac{1 - c}{1 + d} - Q_p,$$

which the public firm uses to fill whatever aggregate private output leaves open.

Solving the Post-Merger Equilibrium

- By symmetry each private firm produces q_p , so $Q_p = m q_p$.
- Substituting $q_0 = \frac{1-c}{1+d} - m q_p$ into a private firm's FOC and collecting terms,

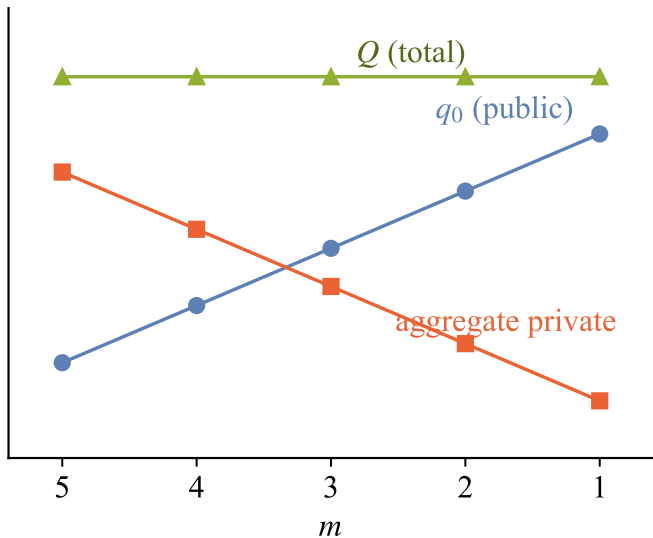
$$q_p = \frac{(1-c)d}{1+d}.$$

- Back into $q_0 = Q^{SO} - m q_p$ gives
 $q_0^{NR}(m) = \frac{(1-c)(1-md)}{1+d}$: private output is independent of m , while the public firm absorbs the change in m .

Equilibrium and Merger Neutrality (Section 6.2.2)

- Each private firm still makes $\frac{(1-c)d}{1+d}$; the public firm produces the residual $q_0^{NR}(m) = \frac{(1-c)(1-md)}{1+d}$.
- Total output stays at $Q^{SO} = \frac{1-c}{1+d}$.
- **Merger neutrality:** while the public firm is active, output, price, emissions, and welfare are all unchanged.

Figure 6.1: Outputs as the Merger Lowers m



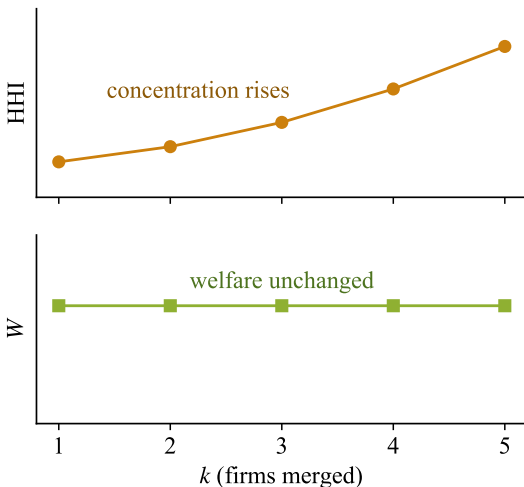
Concentration versus Welfare (Section 6.2.3)

- Each private firm's share is d ; the public firm's is $1 - md$.
- Herfindahl index $H(m) = (1 - md)^2 + md^2$ rises as the merger lowers m .
- Key insight: concentration climbs while welfare stays flat. A screen that reads a rising index as harm would misjudge a mixed market.

Herfindahl at the Merger Extremes

- Each private firm's share equals d ; the public firm's share equals $1 - md$.
- **No merger** ($m = n$): $H = (1 - nd)^2 + nd^2$. A small d leaves the public firm large and H close to $(1 - nd)^2$.
- **All firms merge** ($m = 1$): $H = (1 - d)^2 + d^2$. One large public firm faces one small private firm and H is close to $(1 - d)^2$.
- Concentration climbs monotonically as m falls, yet welfare stays at the first best throughout.

Figure 6.2: Concentration Rises, Welfare Unchanged



Section 6.3

When a Merger Brings the Public Firm Back

When a Merger Brings the Public Firm Back (Section 6.3)

- Neutrality holds while $d < \frac{1}{m}$; the merger raises $\frac{1}{m}$.
- A merger can move the market from the corner into the interior, so the public firm **re-enters** and resumes steering output to Q^{SO} .
- Then the merger is not merely neutral but welfare improving.

The Interior/Corner Boundary

- The public firm's residual output $q_0^{NR}(m) = \frac{(1-c)(1-md)}{1+d}$ is positive if and only if $d < \frac{1}{m}$.
- **Interior** ($d < 1/m$): public firm active; aggregate output Q^{SO} ; welfare at the first best.
- **Corner** ($d \geq 1/m$): $q_0 = 0$; the market behaves as a private Cournot oligopoly of m firms.
- A merger lowers m , so it raises $\frac{1}{m}$: an exited public firm can re-enter after the merger.

Sections 6.4-6.5

Regulation and Merger Size

Regulation and Merger Size (Sections 6.4–6.5)

- Interior: the fee is redundant before and after. Corner: the optimal fee $t^*(m) = \frac{(1-c)(md-1)}{m(1+d)}$ **falls** with the merger.
- $k = 2$: $m = n - 1$; generic k : $m = n - k + 1$; all merge ($k = n$): $m = 1$ (the mixed duopoly of Chapter 2).
- A private merger weakens, not strengthens, the case for a fee.

Optimal Fee in the Corner Regime

- With the public firm inactive, the m remaining private firms play Cournot at marginal cost $c + t$. Each produces $q = \frac{1 - c - t}{m + 1}$, so $Q(t) = \frac{m(1 - c - t)}{m + 1}$.
- The regulator sets t so that $Q(t) = Q^{SO} = \frac{1 - c}{1 + d}$. Solving,

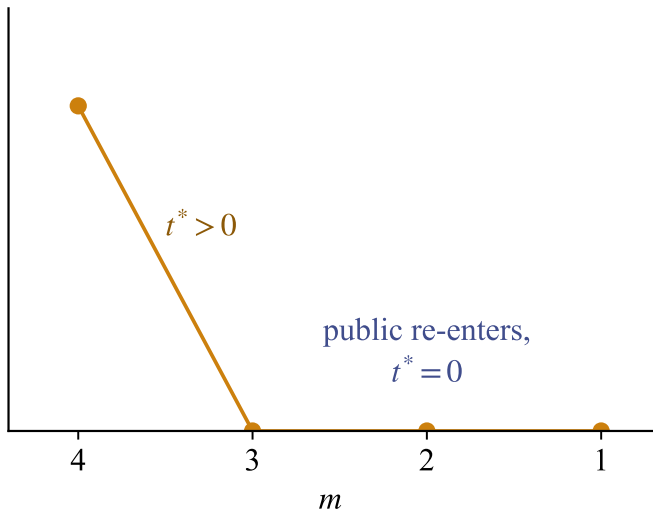
$$t^*(m) = \frac{(1 - c)(md - 1)}{m(1 + d)}.$$

- As the merger lowers m , t^* falls: the merger already cuts private output toward Q^{SO} , so a smaller fee is enough.

Mergers of Different Size

- **Two-firm merger** ($k = 2$): $m = n - 1$; one step along the Chapter 4 comparative statics.
- **Generic merger** ($k \leq n$): $m = n - k + 1$; the merger widens the interior region and lowers t^* .
- **Merger of all private firms** ($k = n$): $m = 1$; the market returns to the mixed duopoly of Chapter 2, one public firm against one private firm.
- Every equilibrium object of Chapter 4 carries over with the substitution $n \rightarrow m = n - k + 1$.

Figure 6.3: A Merger Can Remove the Need for a Fee



6.6 Case Study - Consolidation in India's Power Sector

- Adani Power has grown by acquiring rival private plants, while the state-owned NTPC has stayed the largest producer.
- Concentration screens can mislead where NTPC absorbs the merger and holds price and welfare fixed (Kanjilal et al., 2026).
- The comfort is conditional: NTPC offsets a merger only while it is an active, low-cost producer with room to expand.

Case Study - Reading the Limits

- Where the public firm is capacity-constrained or high-cost, a private merger raises price as it would in a purely private market.
- Merger review then cannot rely on NTPC alone; the fee returns as the main instrument, and $t^*(m)$ falls with the size of the merger.
- The chapter's caution against concentration screens is conditional on the public firm's continued ability to expand.

Section 6.7

The Public Firm's Weight on Emissions

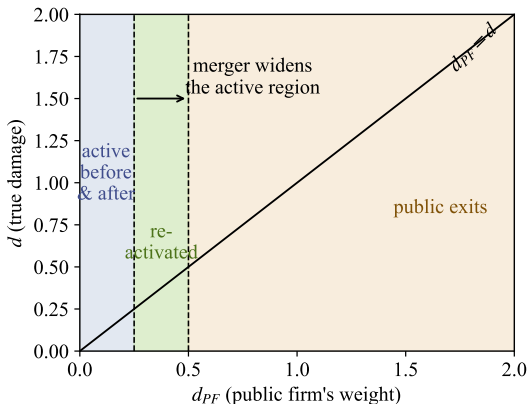
The Public Firm's Weight on Emissions (Section 6.7)

- With weight $d_{PF} \leq d$, the public firm stays active while $d_{PF} < \frac{1}{m}$.
- A merger raises $\frac{1}{m}$, widening the range of weights over which the public firm remains and the merger stays neutral.
- A merger can return even a *partly* green public firm to the market.

Active-Region Boundary $d_{PF} = 1/m$

- Repeating the derivation with weight d_{PF} on emissions gives $q_0 = \frac{(1-c)(1-md_{PF})}{1+d_{PF}}$, positive iff $d_{PF} < \frac{1}{m}$.
- Before the merger the boundary sits at $\frac{1}{n}$; after a merger of k firms it sits at $\frac{1}{n-k+1} > \frac{1}{n}$.
- Reading in the (d_{PF}, d) plane: the active region is the rectangle $d_{PF} < \frac{1}{m}$, and it widens rightward as m falls.

Figure 6.4: A Merger Widens the Active Region



Section 6.8

Numerical Illustration

Table 6.1: A Merger of k Firms
 ($c = \frac{1}{2}$, $d = 0.15$, $n = 5$)

k	q_0	q_p	p	HHI	CS	PS	Env	W
1	0.109	0.065	0.565	0.175	0.095	0.028	0.014	0.109
2	0.174	0.065	0.565	0.250	0.095	0.028	0.014	0.109
3	0.239	0.065	0.565	0.370	0.095	0.028	0.014	0.109
4	0.304	0.065	0.565	0.535	0.095	0.028	0.014	0.109
5	0.370	0.065	0.565	0.745	0.095	0.028	0.014	0.109

- Public output rises to fill the gap; each private firm holds at q_p .
- Price, CS , PS , emissions, and W do not move; the Herfindahl index climbs from 0.175 to 0.745.

Summary / Key Takeaways

- A no-synergy private merger lowers the number of private firms to $m = n - k + 1$.
- While the public firm is active the merger is welfare-neutral, though the Herfindahl index rises.
- A large enough merger can return the public firm to a market it had left, raising welfare back to the first best.
- The optimal fee falls with the merger, or drops to zero when the public firm re-enters; concentration-based merger screens can mislead in a mixed market.

Chapter 7

Cost-Reducing Mergers

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 7

- 7.2 A cost-reducing merger.
- 7.3 The welfare effect of the merger.
- 7.4 Regulation and the cost-reducing merger.
- 7.5 The Williamson trade-off in a mixed market.
- 7.6 Case studies.
- 7.7 The public firm's weight on emissions.
- 7.8 Numerical illustration.
- 7.9 Summary.

The Central Problem

- Real mergers often lower costs. The **Williamson (1968) trade-off**: less competition (harms consumers) versus lower cost (helps them).
- Two private firms ($n = 2$) merge into one at cost $c - e$; e is the synergy; $e = 0$ recovers Chapter 6.
- The public firm keeps the general objective $V = (1 - \alpha)\pi_0 + \alpha W$; we develop the fully public case ($\alpha = 1$); a lower α loosens its hold, so part of the trade-off reappears (Chapter 10).
- Key insight: with an active fully public firm the trade-off **collapses**, and the merger is welfare improving for any synergy.

Real-World Context

- Cement: in 2015 **Lafarge** and **Holcim** merged, citing about €1.4 billion in annual cost savings, forming the largest cement maker.
- In several North African and Middle Eastern markets, state-owned cement producers operate alongside such private firms.
- A cost-reducing private merger in a polluting industry with public producers.

Section 7.2

A Cost-Reducing Merger

The Pre-Merger Benchmark (Section 7.2)

- Before the merger: one public firm and two private firms, all at cost c ; pollution mild, $d < \frac{1}{2}$, so the public firm is active.
- The public firm holds aggregate output at the first best,

$$Q^{pre} = \frac{1 - c}{1 + d}, \quad W^{pre} = \frac{(1 - c)^2}{2(1 + d)}.$$

- The merger question then reduces to how these two quantities move once the two private firms combine into one firm at cost $c - e$.

The Merged Firm as an Asymmetric Duopoly

- After the merger: one public firm at cost $c_0 = c$ and one merged private firm at cost $c_1 = c - e$.
- Because $c_1 < c_0$, the merged firm is now the *efficient* producer, the mirror image of Section 3.4.
- So the post-merger market is the asymmetric mixed duopoly of Chapter 3, and we can read the equilibrium off those results.

The Merged Firm's Best Response

- The merged private firm solves $\max_{q_1} (1 - q_0 - q_1 - c_1) q_1$, with $c_1 = c - e$.
- Its first-order condition, $1 - q_0 - 2q_1 - (c - e) = 0$, yields the best response

$$q_1(q_0) = \frac{1 - c + e}{2} - \frac{1}{2} q_0.$$

- Intercept and slope: the intercept grows one for one with the synergy; the slope is the standard Cournot $\frac{1}{2}$.

The Public Firm's Best Response ($\alpha = 1$)

- At $\alpha = 1$ the public firm maximizes $W = Q - \frac{1}{2}Q^2 - \sum c_i q_i - \frac{d}{2}Q^2$.
- Its FOC, $1 - Q - c - dQ = 0$, targets $Q = \frac{1 - c}{1 + d}$, so its best response is

$$q_0(q_1) = \frac{1 - c}{1 + d} - 1 \cdot q_1.$$

- Intercept and slope: the public firm anchors the socially optimal total; the slope is 1, so it absorbs any change in q_1 unit for unit.

Equilibrium Outputs (Equation 7.1)

- Substituting the public firm's rule into the private FOC, and simplifying,

$$q_1(e) = \frac{(1-c)d}{1+d} + e, \quad q_0(e) = \frac{(1-c)(1-d)}{1+d} - e. \quad (7.1)$$

- The merged firm produces *more* than a symmetric private firm would, by exactly the synergy; the public firm produces correspondingly less.
- Aggregate output stays at $Q = \frac{1-c}{1+d}$, so price does not rise.

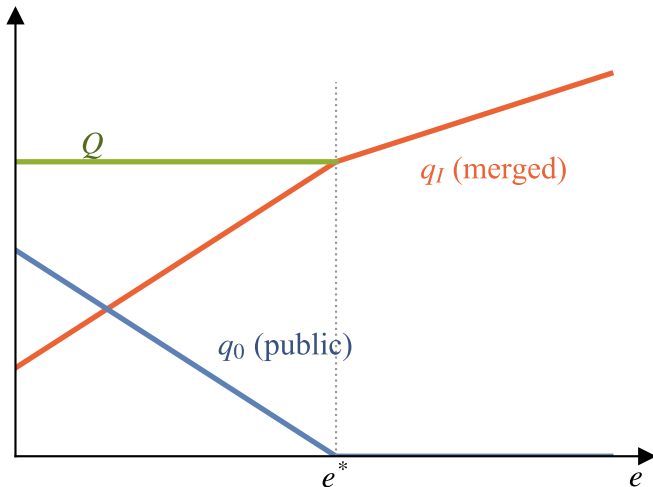
When Does the Public Firm Exit?

- From (7.1), $q_0(e) > 0$ requires

$$e < e^* \equiv \frac{(1-c)(1-d)}{1+d}. \quad (7.2)$$

- Beyond e^* the public firm withdraws and the efficient merged firm becomes a monopolist at cost $c - e$.
- The exit synergy e^* shrinks as d rises: a market with heavier damage assigns the public firm a smaller share, so a smaller cost saving already pushes it out.

Figure 7.1: Outputs as the Synergy e Grows



Section 7.3

The Welfare Effect of the Merger

Welfare While the Public Firm Is Active

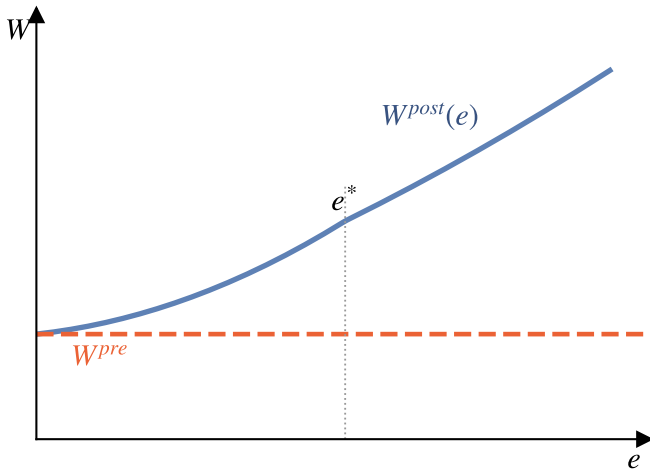
- Aggregate output is unchanged, so consumer surplus and damage are unchanged.
- Only the cost side moves: the merged firm now produces $q_1(e)$ at cost $c - e$ rather than at c , saving e per unit.
- Substituting into W , the pre- and post-merger levels compare as

$$W^{post} = W^{pre} + e q_1(e) > W^{pre} \quad \text{for any } e > 0. \quad (7.3)$$

Beyond the Exit Point e^*

- For $e \geq e^*$ the public firm withdraws and an efficient monopolist remains at cost $c - e$, producing
$$Q^{mon} = \frac{1 - (c - e)}{2}.$$
- Total output rises with e : welfare keeps increasing past e^* , now because a very efficient monopolist supplies a lot at low cost.
- The transition at e^* is smooth: the merger is welfare improving on both sides of it.

Figure 7.2: Welfare Rises in the Synergy



Regulation While the Public Firm Is Active (7.4)

- By (3.5), the fee changes welfare at the rate

$$\frac{\partial W}{\partial t} = c_1 - c_0 = -e < 0.$$

- A positive fee would shift output from the efficient merged firm to the less efficient public firm, so the optimal fee is $t^* = 0$.
- While the public firm anchors total output, regulation is unnecessary.

Regulation Once the Public Firm Exits

- For $e \geq e^*$ the merged firm is an unregulated monopolist producing $Q^{mon} = \frac{1 - (c - e)}{2}$.
- A fee then regains its usual role: it curbs the monopolist's emissions, trading off the externality against an already restrained output.
- So regulation of the cost-reducing merger becomes useful only once the merger has displaced the public firm.

The Williamson Trade-off Collapses (7.5)

- Without a public firm, two private firms merging into an efficient monopolist face the Williamson trade-off in full: welfare rises only if the synergy clears a threshold.
- With an active public firm, that threshold disappears, since the public firm prevents the price from rising in the first place.
- In a mixed market welfare rises from the first unit of cost saving.

Section 7.6

Case Study: The Lafarge-Holcim Merger in Cement

Case Study - Context

- In 2015 France's Lafarge and Switzerland's Holcim merged to form LafargeHolcim, citing about €1.4 billion in expected annual cost savings.
- Cement accounts for a large share of industrial carbon emissions; in North Africa, the Middle East, and parts of Asia state-owned cement producers operate alongside private ones.
- A textbook fit for the model: a cost-reducing merger between private firms in a polluting industry with public producers.

Case Study - Model Reading

- With an active public producer holding aggregate output fixed, the Williamson trade-off collapses and the merger is welfare improving for any $e > 0$.
- A cost-reducing merger of this kind is easier to justify in markets anchored by a state cement producer than in purely private markets.
- Environmental caveat: once the efficient merged firm is large enough to displace the public firm, an emission charge regains its role.

The Public Firm's Weight on Emissions (7.7)

- Let the public firm weigh emissions at its own $d_{PF} \leq d$; it still holds aggregate output at $Q = \frac{1 - c}{1 + d_{PF}}$.
- While it is active the merger still raises welfare by $e q_1$ for any $e > 0$, exactly as in Section 7.3.
- What changes is the exit synergy,

$$e^* = \frac{(1 - c)(1 - d_{PF})}{1 + d_{PF}}. \quad (7.4)$$

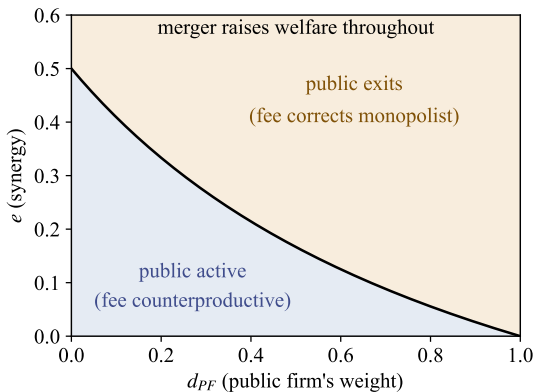
Comparative Statics of the Exit Synergy

- Differentiating (7.4),

$$\frac{\partial e^*}{\partial d_{PF}} = -\frac{2(1-c)}{(1+d_{PF})^2} < 0.$$

- A greener public firm, already targeting a lower output, is displaced by a smaller cost saving.
- A laxer public firm, at a lower d_{PF} , holds more space and is displaced only by a larger synergy.

Figure 7.3: The Exit Synergy $e^*(d_{PF})$



Section 7.8

Numerical Illustration

Parameters and Exit Point

- Take $c = \frac{1}{2}$ and $d = \frac{3}{10}$, and vary the synergy e ; the merged private firm produces at cost $c - e$.
- Aggregate output while the public firm is active:
$$Q = \frac{1 - c}{1 + d} = \frac{0.5}{1.3} \approx 0.385.$$
- Exit synergy: $e^* = \frac{(1 - c)(1 - d)}{1 + d} = \frac{0.35}{1.3} \approx 0.269.$

Table 7.1: Outputs, Price, and Welfare in e

e	q_0	q_1	p	t^*	CS	PS	Env	W
0.000	0.269	0.115	0.615	0	0.074	0.044	0.022	0.096
0.100	0.169	0.215	0.615	0	0.074	0.066	0.022	0.118
0.200	0.069	0.315	0.615	0	0.074	0.107	0.022	0.159
0.269	0.000	0.384	0.615	0	0.074	0.148	0.022	0.200
0.350	0.000	0.425	0.575	0	0.090	0.181	0.027	0.244

Table 7.1. A cost-reducing merger as the synergy e rises ($c = \frac{1}{2}$, $d = 0.3$).

Reading the Table

- While $e < e^* \approx 0.269$: price and CS are flat, damage is flat, and welfare rises by exactly $e q_1$ (e.g. $0.1 \times 0.215 \approx 0.022$ matches $0.118 - 0.096$).
- At $e = e^*$ the public firm exits, having handed all output to the efficient merged firm.
- Past e^* price falls, CS rises, and welfare keeps rising, now driven by the efficient monopolist. The fee stays at zero throughout the table.

Summary / Key Takeaways

- While the public firm is active, a cost-reducing merger raises welfare by $e q_1(e)$ for any synergy.
- Beyond e^* the public firm exits; welfare still rises in e .
- The Williamson trade-off, present in a private market, disappears in a mixed one; a fee is counterproductive while the public firm remains.

Chapter 8

Public-Private Mergers

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 8

- 8.2 The merged entity's objective.
- 8.3 A merger that keeps the public objective.
- 8.4 A merger that removes the public objective.
- 8.5 Cost-asymmetric public-private mergers.
- 8.6 Case studies.
- 8.7 The public firm's weight on emissions.
- 8.8 Summary.

The Central Problem

- Now the public firm itself merges with one private rival.
- Key question: what does the merged firm maximize, welfare or profit?
- That choice, not the change in the number of firms, decides the merger's effect on output, emissions, and welfare.

Real-World Context

- In January 2025 the private **Lufthansa Group** acquired a 41% stake in **ITA Airways**, Italy's flag carrier (the state kept the majority), and has since moved to a majority stake.
- A private firm and a state-owned one combine; the merged carrier's character depends on who holds control.
- Air travel is a growing source of emissions.

The Merged Entity's Objective (Section 8.2)

- If the state keeps control $\Rightarrow$ the merged firm maximizes welfare (**nationalization** of a private rival).
- If private owners take control $\Rightarrow$ it maximizes profit (**privatization** of the public firm).
- In the book's notation, state control is $\alpha = 1$ and private control $\alpha = 0$; the realistic case is an intermediate α (partial privatization, Chapter 9). The two poles bracket every outcome.

Sections 8.3–8.4

The Two Polar Cases

A Merger That Keeps the Public Objective (Section 8.3)

- The merged public firm runs both plants; the market is Chapter 4 with $n - 1$ private firms.
- It still targets $Q = \frac{1 - c}{1 + d}$: output, emissions, and welfare are **unchanged**. The merger is neutral.
- It only widens the range over which the public firm is active, from $d < \frac{1}{n}$ to $d < \frac{1}{n - 1}$. The fee stays redundant.

Privatizing Merger: The Merged Firm's Problem

- After privatization, the market is a pure private oligopoly of n firms.
- The merged firm M picks q_M to maximize

$$\pi_M = (1 - q_M - \sum_{j \neq M} q_j - c) q_M.$$

- FOC:

$$1 - c - 2q_M - \sum_{j \neq M} q_j = 0.$$

Best Response and the Symmetric Solution

- Each firm's best response takes the two-term form

$$q_i = \frac{1-c}{2} - \frac{\sum_{j \neq i} q_j}{2}.$$

- Imposing symmetry across the n private firms,

$$q^* = \frac{1-c}{n+1}, \quad Q^{priv} = \frac{n(1-c)}{n+1}.$$

- Aggregate output exceeds the social optimum $Q^{SO} = \frac{1-c}{1+d}$ whenever $nd > 1$.

A Merger That Removes the Public Objective (Section 8.4)

- The market becomes a pure private oligopoly of n firms, producing $\frac{n(1-c)}{n+1} > Q^{SO}$.

- Welfare falls by

$$W^{SO} - W^{priv} = \frac{(1-c)^2(nd-1)^2}{2(1+d)(n+1)^2}.$$

- But the fee now **works**: $t^*(n)$ restores the first best. A privatizing merger trades the public firm's discipline for the regulator's fee.

Welfare Before the Privatizing Merger

- With the public firm present, aggregate output is $Q^{SO} = \frac{1-c}{1+d}$ and welfare reaches the first best

$$W^{SO} = \frac{(1-c)^2}{2(1+d)}.$$

- The public firm's discipline internalizes the externality, so an emission fee is redundant and $t^* = 0$.

Welfare After the Privatizing Merger, With the Fee

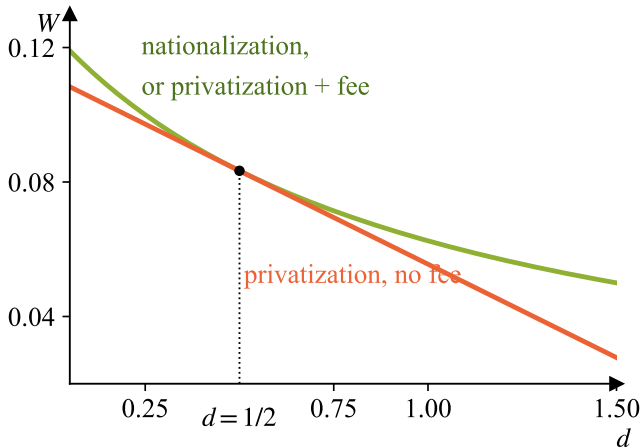
- Without a fee, welfare falls by $\frac{(1-c)^2(nd-1)^2}{2(1+d)(n+1)^2}$.
- Applied to a pure private oligopoly, the fee

$$t^*(n) = \frac{(1-c)(nd-1)}{n(1+d)}$$

holds aggregate output at Q^{SO} and restores W^{SO} .

- The regulator's instrument replaces the public firm's direct discipline.

Figure 8.1: Welfare after a Public-Private Merger



Section 8.5

Cost-Asymmetric Public-Private Mergers

Cost-Asymmetric Mergers (Section 8.5)

- If the firms differ in cost, the merged firm produces at the more efficient cost.
- Nationalization + efficiency: a pure gain (discipline kept, cost lowered).
- Privatization + efficiency: the gain offsets the loss of discipline; a large enough synergy makes even a privatizing merger welfare improving.

Section 8.6

Case Studies

8.6 Case Study - The Statoil-Hydro Merger in Norway

- In 2007 Norway's state-controlled Statoil merged with Norsk Hydro's oil and gas division (later Equinor); the state kept about two-thirds.
- Because control stayed public, the merged firm kept a welfare orientation and preserved the discipline a public firm brings.
- Had control passed to private owners, the market would have lost its corrector and a fee would have become necessary: the ownership share, not the combination, decides.

Case Study - Lufthansa and ITA Airways: Context

- January 2025: Lufthansa acquired a 41% stake in ITA Airways; the Italian state kept the majority.
- Lufthansa has since moved to majority control, leaving the state a minority holder.
- European aviation is a rising source of CO_2 emissions, priced through the EU ETS.

Case Study - Lufthansa and ITA Airways: Model Reading

- As long as the state kept majority control, ITA sat close to the nationalization pole and the merger was welfare neutral.
- Once Lufthansa took majority control, the outcome moved toward the privatization pole: the ETS price on aviation now carries the discipline the state previously provided.
- The lesson mirrors Statoil-Hydro: the ownership share, not the combination, decides the merger's welfare effect.

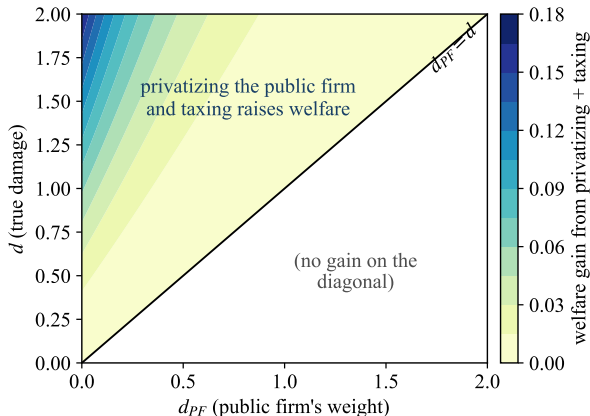
Section 8.7

The Public Firm's Weight on Emissions

The Public Firm's Weight on Emissions (Section 8.7)

- If the public firm undervalues emissions ($d_{PF} < d$), keeping it blocks the fee, and welfare stays below the first best.
- Privatizing it removes the obstruction, and the fee then restores the first best.
- Key insight: privatizing + taxing beats keeping the firm whenever $d_{PF} < d$; a public firm earns its place only if $d_{PF} \approx d$.

Figure 8.2: Gain from Privatizing and Taxing, in (d_{PF}, d)



8.8 Numerical Illustration: Setup

- Mixed duopoly with $c = \frac{1}{2}$ and $d = \frac{3}{2}$; the public firm merges with the private firm.
- Socially optimal aggregate output

$$Q^{SO} = \frac{1 - c}{1 + d} = \frac{1/2}{5/2} = 0.20, \quad W^{SO} = 0.050.$$

- Three regimes to compare: nationalization, privatization, and privatization with the emission fee.

Numerical Illustration: Outputs and the Fee

- Nationalization: the merged public firm produces $q_0 = 0.200$, private $q_1 = 0.000$, price $p = 0.800$.
- Privatization: the merged private firm produces $q_1 = 0.250$, price $p = 0.750$, no fee.
- Fee that restores the first best after privatization:

$$t^* = \frac{(1-c)(nd-1)}{n(1+d)} = \frac{(1/2)(1/2)}{1 \cdot (5/2)} = 0.100.$$

Numerical Illustration: Welfare Comparison (Table 8.1)

Regime	q_0	q_1	p	t^*	W
Nationalization	0.200	0.000	0.800	0.000	0.050
Privatization	0.000	0.250	0.750	0.000	0.047
Privatization + fee	0.000	0.200	0.800	0.100	0.050

- Privatization alone loses $W^{SO} - W^{priv} = 0.003$; the fee closes the gap.
- A privatizing merger trades the public firm's discipline for the regulator's fee.

Summary / Key Takeaways

- A public-private merger's effect turns on the merged firm's objective.
- Nationalization is neutral; privatization lowers welfare by $\frac{(1 - c)^2(nd - 1)^2}{2(1 + d)(n + 1)^2}$, which the fee then restores.
- Privatizing an inefficient or environmentally lax public firm, paired with a fee, can raise welfare.

Chapter 9

Partial Privatization

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-García

Outline of Chapter 9

- 9.2 The government's problem and the symmetric benchmark.
- 9.3 When privatization raises welfare.
- 9.4 Nationalization and the emission fee.
- 9.5 Case study.
- 9.6 The firm's weights: welfare and emissions.
- 9.7 Numerical illustration.
- 9.8 Summary.

The Central Problem

- The degree of nationalization α has run through the book as a parameter; here it is the **object of choice**.
- The government picks α : the weight the firm places on welfare ($1 - \alpha$ on its own profit). $\alpha = 1$ fully public, $\alpha = 0$ fully private.
- Key question: what α should a government choose, and how does the answer depend on efficiency, pollution, and the availability of a fee?

Real-World Context

- Partial state ownership is the rule among large firms:
 - Deutsche Telekom ($\approx 30\%$, incl. KfW), Enel ($\approx 24\%$), Engie ($\approx 24\%$), Airbus ($\approx$ a quarter).
- Each sits at an interior α , answering partly to shareholders, partly to a government.

Section 9.2

The Government's Problem and the Symmetric Benchmark

The Government's Problem (Section 9.2)

- Firm 0 maximizes $V = (1 - \alpha)\pi_0 + \alpha[CS + PS - \frac{d}{2}Q^2]$,
with (Chapter 2)

$$Q(\alpha) = \frac{(1 - c)(2 - \alpha)}{3 - 2\alpha + \alpha d},$$

from the private duopoly at $\alpha = 0$ to Q^{SO} at $\alpha = 1$.

- The welfare loss $\frac{(1 - \alpha)^2(1 - c)^2(2d - 1)^2}{2(1 + d)D^2}$ is zero only at $\alpha = 1$: with symmetric efficient firms, $\alpha^* = 1$.

The Private Firm's Best Response (Recall)

- The private firm ignores welfare and maximizes profit, giving the two-term best response

$$q_1(q_0) = \frac{1-c}{2} - \frac{1}{2}q_0.$$

- Intercept $\frac{1-c}{2}$ (Cournot monopoly on the residual margin),
slope $-\frac{1}{2}$; independent of α .

The Public Firm's Best Response as a Function of α

- The public firm's FOC weighs its own margin and the social margin $c + dQ$:

$$q_0(q_1) = \frac{1 - c}{2 - \alpha + \alpha d} - \frac{1 + \alpha d}{2 - \alpha + \alpha d} q_1.$$

- At $\alpha = 0$ the reaction matches the private firm's; at $\alpha = 1$ its slope steepens to -1 , so a fully public firm offsets the private firm one for one.

Solving the System: Aggregate Output $Q(\alpha)$

- Solving the two best responses at each α (with $D \equiv 3 - 2\alpha + \alpha d$),

$$q_0^{NR} = \frac{(1-c)(1-\alpha d)}{D}, \quad q_1^{NR} = \frac{(1-c)(1-\alpha + \alpha d)}{D},$$

so aggregate output $Q(\alpha) = \frac{(1-c)(2-\alpha)}{D}$.

- A more public firm raises q_1 , cuts q_0 , and moves total output toward the social optimum.

Two Anchors: $\alpha = 0$ and $\alpha = 1$

- At $\alpha = 0$: private duopoly, $Q^{NR} = \frac{2(1-c)}{3}$, above the social optimum whenever $d > \frac{1}{2}$.
- At $\alpha = 1$: $Q^{NR} = \frac{1-c}{1+d} = Q^{SO}$, so the fully public firm reaches the planner's aggregate output on its own.
- Between them, $Q(\alpha)$ moves monotonically toward Q^{SO} as α rises.

Welfare Loss as a Function of α

- Comparing $W(\alpha)$ to $W^{SO} = \frac{(1-c)^2}{2(1+d)}$,

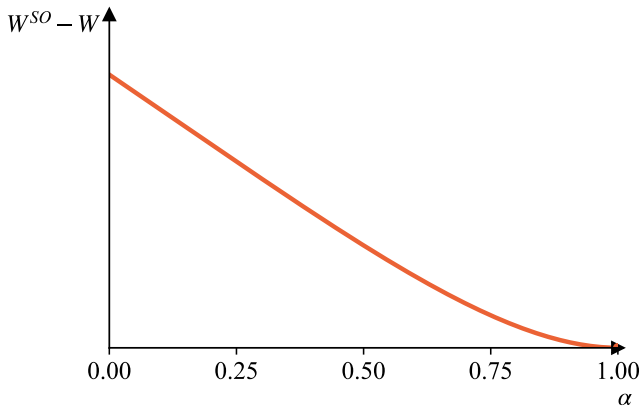
$$W^{SO} - W(\alpha) = \frac{(1-\alpha)^2(1-c)^2(2d-1)^2}{2(1+d)D^2}.$$

- The loss is zero only at $\alpha = 1$, and grows with $(1-\alpha)^2$ and with $(2d-1)^2$: the further pollution severity lies from $d = \frac{1}{2}$, the more privatization costs.

The Optimum in the Symmetric Benchmark: $\alpha^* = 1$

- Maximize $W(\alpha)$ in $\alpha \in [0, 1]$: since $W^{SO} - W(\alpha) \geq 0$ and vanishes only at $\alpha = 1$, the FOC $\partial W / \partial \alpha = 0$ selects the corner $\alpha^* = 1$.
- With symmetric efficient firms, full public ownership dominates. Any $\alpha < 1$ lowers welfare by an amount rising with $(1 - \alpha)^2$.
- An emission fee is not needed here: the fully public firm delivers the first best on its own.

Figure 9.1: Welfare Loss as the Firm Is Privatized



Section 9.3

When Privatization Raises Welfare

When Privatization Raises Welfare (Section 9.3)

- Full public ownership is best only if the public firm is efficient.
- With an **inefficient** public firm, or **convex costs**, privatizing in part moves output to the efficient private firm, so the optimal α is **interior**.
- This goes back to De Fraja and Delbono (1989) and Matsumura (1998); the fee still addresses the externality.

De Fraja–Delbono and Matsumura: What Changes

- Inefficient public firm ($c_0 > c$): a public firm that produces more draws output onto a high-cost plant; a lower α moves output onto the efficient private firm.
- Convex costs: raising q_0 becomes more expensive at the margin, again pushing toward an interior α^* .
- In both cases privatization earns its place through productive efficiency, and the emission fee is left to address the environmental externality.

Section 9.4

Nationalization and the Emission Fee

The Fee Gains Traction as the Firm Is Privatized

- Aggregate output under the fee falls by

$$\left| \frac{\partial Q}{\partial t} \right| = \frac{2(1 - \alpha)}{3 - 2\alpha + \alpha d},$$

zero at $\alpha = 1$ and rising to $\frac{2}{3}$ at $\alpha = 0$.

- A more public firm offsets the fee within its own output, so the fee is powerless at $\alpha = 1$ and works fully at $\alpha = 0$.

Nationalization and the Fee Are Substitutes

- As α falls, output and emissions rise *and* the fee's traction rises: the more the firm is privatized, the more a fee is needed and the more it delivers.
- Public ownership and regulation therefore substitute for one another: the government can rely on either channel to correct the externality.

The Corrective Fee Is Independent of α

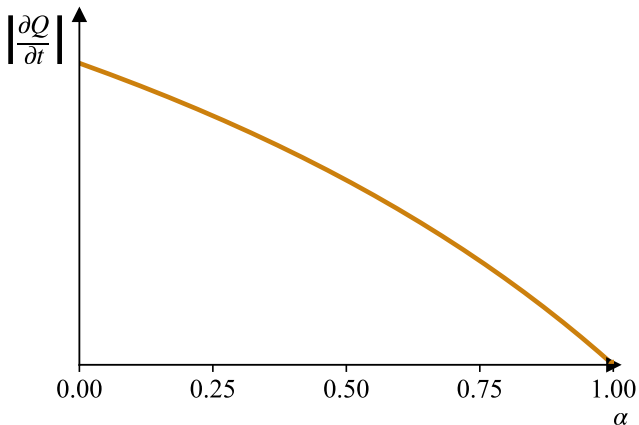
- The fee that restores the first best is

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)},$$

the same formula as for a private duopoly.

- A more privatized firm both over-produces more and responds to the fee more; the two effects cancel, so the same fee returns output to Q^{SO} at every $\alpha < 1$.

Figure 9.2: The Fee Gains Traction as α Falls



Section 9.5

Case Study — Enel and Partial State Ownership

9.5 Case Study — Enel: The Setting

- Enel is one of Europe's largest utilities; the Italian Treasury holds about a quarter of the shares, enough to be the largest shareholder.
- Enel is therefore a partially nationalized firm with an interior α , answering partly to the state and partly to private shareholders.
- It competes with fully private energy firms across Europe and Latin America.

9.5 Case Study — Enel: Reading the Model

- As Enel is privatized (lower α), its output and emissions rise and the case for a carbon charge on it strengthens in step.
- Enel's large investment in renewables points to the other dial, the firm's weight on emissions: both dials set how much correction it supplies.
- A partially nationalized firm that also weighs the environment corrects more than its ownership share alone would suggest.

Section 9.6

The Firm's Weights: Welfare and Emissions

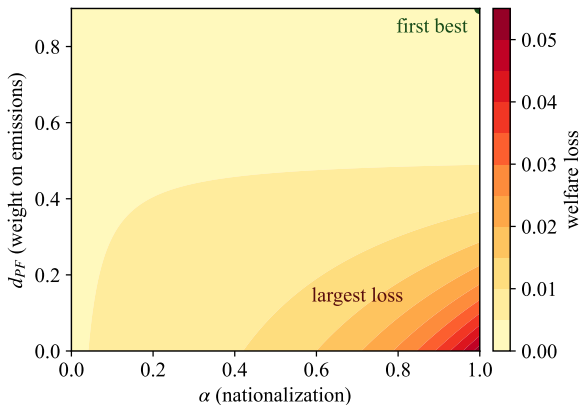
Two Dials of “Non-Greenness”

- Weight on welfare α : how much of society's objective the firm carries.
- Weight on emissions $d_{PF} \leq d$: how much of the true damage d the firm recognizes within the welfare it does weigh.
- Objective: $V = (1 - \alpha)\pi_0 + \alpha W_{PF}$, with
$$W_{PF} = CS + PS - \frac{d_{PF}}{2} Q^2.$$

The First Best on the Firm's Own

- Both dials push in the same direction: a lower α or a lower d_{PF} raises output and emissions, lowers welfare, and makes the fee more necessary.
- The first best on the firm's own requires *both* corners: $\alpha = 1$ **and** $d_{PF} = d$: a fully public firm as green as the regulator.
- A firm can be a poor environmental steward either by caring about profit or by undervaluing emissions, and the two failings compound.

Figure 9.3: Welfare Loss in (α, d_{PF})



Section 9.7

Numerical Illustration

Numerical Illustration: Setup

- Parameters: $c = \frac{1}{2}$, true damage $d = \frac{9}{10}$, firm undervalues emissions at $d_{PF} = \frac{7}{10}$.
- Vary the degree of nationalization $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$ and track outputs, price, surpluses, damage, and welfare.
- Reference: the first best is $W^{SO} = 0.066$ at the true damage d .

Numerical Illustration: Table

α	q_0	q_1	p	CS	PS	Env	W
0.00	0.167	0.167	0.667	0.056	0.056	0.050	0.061
0.25	0.154	0.173	0.673	0.053	0.057	0.048	0.062
0.50	0.138	0.181	0.681	0.051	0.058	0.046	0.063
0.75	0.117	0.191	0.691	0.048	0.059	0.043	0.064
1.00	0.088	0.206	0.706	0.043	0.061	0.039	0.065

Nationalizing raises welfare from 0.061 at $\alpha = 0$ toward 0.065 at $\alpha = 1$.

Numerical Illustration: Reading the Table

- As α rises, q_0 falls and q_1 rises, the price rises, and emissions fall.
- Welfare rises, but the firm's undervaluation of emissions caps it at 0.065, below the first best 0.066.
- Even a fully public firm falls short: it still weighs damage at $d_{PF} < d$. Full nationalization is no substitute for a green mandate.

Summary — The Government's Choice of α

- The degree of nationalization α is a policy instrument; with symmetric efficient firms $\alpha^* = 1$, and the welfare loss from privatization grows with $(1 - \alpha)^2$ and with $(2d - 1)^2$.
- With an inefficient public firm or convex costs, the optimal α is interior (De Fraja–Delbono; Matsumura): privatization then improves productive efficiency.

Summary — Fee and the Two Weights

- Nationalization and the fee are substitutes: the fee is powerless at $\alpha = 1$ and gains traction as the firm is privatized.
- The corrective fee $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ is independent of α : the same fee as for a private market.
- A lower α and a lower d_{PF} compound; the first best on the firm's own requires $\alpha = 1$ and $d_{PF} = d$.

Chapter 10

Merger Approval and the Competition Authority

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-García

Outline of Chapter 10

- 10.2 The approval game.
- 10.3 The authority with an active public firm.
- 10.4 The firms' decision to merge.
- 10.5 Agreement and disagreement.
- 10.6 Privatization and blocked mergers.
- 10.7 Case study.
- 10.8 The environmental standard.
- 10.9 Numerical illustration.
- 10.10 Summary.

The Central Problem

- A merger needs two yeses: the firms must **propose** it, and a competition authority must **approve** it.
- The authority approves if consumer surplus does not fall; firms propose if combined profit rises. The two tests need not agree.
- Question: does the presence of a public firm make approval easier, and how does privatization change that?

Guiding Questions

- ① Does a public firm make a merger easier or harder to approve?
- ② Can a merger be profitable yet blocked, and when?
- ③ How does privatizing the public firm change which mergers occur?
- ④ Should the authority weigh emissions alongside consumer surplus?

Real-World Context

- **FedEx**–Genco (2015), FedEx–TNT (2016), with the public **USPS** in the market.
- **Sunrise**–UPC (2020), behind majority-state-owned **Swisscom**.
- **Lafarge**–Holcim cement (2015), alongside state-owned producers, justified by about 1.4 billion euros in yearly savings.

Section 10.2

The Approval Game

The Approval Game – Setup (Section 10.2)

- One public firm, n private firms; inverse demand $p = 1 - Q$; damage $\frac{d}{2}Q^2$; every firm produces at marginal cost c .
- A group of k private firms considers merging into a single entity that produces at $c - e$, with synergy $e \geq 0$; the remaining $m = n - k + 1$ private firms plus the public firm keep cost c .
- $e = 0$ recovers the no-synergy merger of Chapter 6; $e > 0$ the cost-reducing merger of Chapter 7.

Timing and Solution Method

- Three-stage game:
 - ① the k firms decide whether to submit a merger request;
 - ② the competition authority approves or blocks;
 - ③ firms compete in quantities.
- The authority approves when the merger does not reduce consumer surplus, equivalently when aggregate output does not fall.
- We solve by backward induction; the public firm is fully welfare-maximizing ($\alpha = 1$) until Section 10.6.

Section 10.3

The Authority with an Active Public Firm

The Public Firm Holds Aggregate Output (Section 10.3)

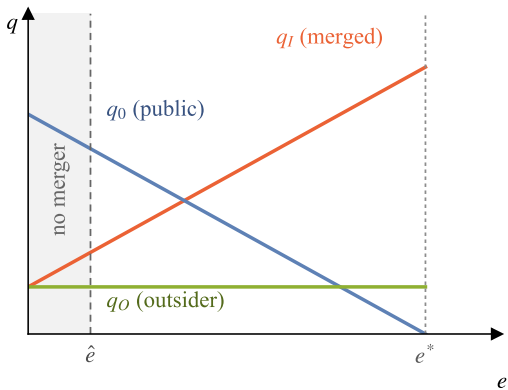
- While active, the public firm targets the social-optimum aggregate $Q = \frac{1-c}{1+d}$, so the merger leaves total output, price, and emissions untouched.
- The merged firm, efficient at $c - e$, produces more; the public firm makes room by producing less:

$$q_I(e) = \frac{(1-c)d}{1+d} + e, \quad q_O = \frac{(1-c)d}{1+d}, \quad q_0(e) = \frac{(1-c)(1-m)}{1+d}$$

with $m = n - k + 1$.

- Consumer surplus is unchanged; only the identity of the producer shifts.

Figure 10.1: Outputs as the Synergy e Grows



Why the Authority Approves Every Merger

- Consumer surplus is unchanged; welfare rises by the cost saving on the merged firm's output:

$$W^{post} = W^{pre} + e q_I \geq W^{pre}.$$

- Under the consumer-surplus standard the authority passes the merger; under the welfare standard it prefers it.
- The public firm's habit of steering total output removes the anticompetitive effect the authority exists to guard against.

The Active-Public-Firm Bound e^*

- The public firm stays active only while it produces a positive amount, that is while

$$e < e^* \equiv \frac{(1-c)(1-md)}{1+d}.$$

- Beyond e^* the efficient merged firm displaces the public firm and becomes a monopolist; welfare keeps rising in e , as in Section 7.3.
- The results in this chapter apply on the active-firm side of that bound.

Section 10.4

The Firms' Decision to Merge

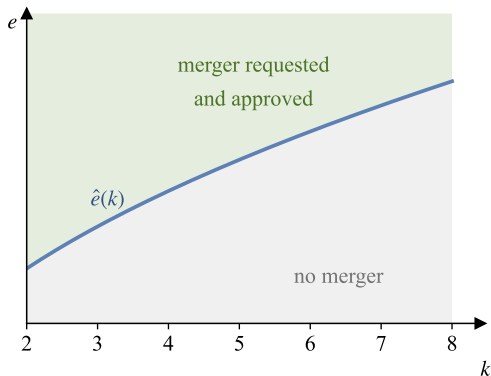
The Firms' Decision to Merge (Section 10.4)

- The k firms trade k separate outputs for one; each earned $\left[\frac{(1-c)d}{1+d} \right]^2$ before, the merged firm earns q_I^2 .
- The merger raises combined profit when q_I reaches $\sqrt{k}$ times the pre-merger output, that is when

$$e \geq \hat{e} \equiv \left(\sqrt{k} - 1 \right) \frac{(1-c)d}{1+d}.$$

- Below $\hat{e}$ the merger is not proposed; at or above it, the firms propose and the authority approves.

Figure 10.2: The Synergy Needed, $\hat{e}(k)$



Comparative Statics of $\hat{e}$

- $\hat{e}$ rises with k : the more firms combine, the more independent output they surrender, so the larger the saving they need (the merger paradox of Salant et al., 1983, in a mixed market).
- $\hat{e}$ rises with d : a greener target output leaves each private firm producing more, so each merging firm gives up more.
- $\hat{e}$ is **independent of** n : while the public firm pins the price, a private firm's profit is the same however many rivals surround it.

Section 10.5

Agreement and Disagreement

The Mixed-Market Alignment (Section 10.5)

- Placing the two decisions together: for $e < \hat{e}$ the firms do not propose; for $e \geq \hat{e}$ they propose and the authority approves.
- No synergy makes a merger *profitable yet blocked*: the region of profitable-but-blocked mergers is empty while the public firm is active.
- A welfare-minded public firm aligns the firms' incentive to merge with the authority's mandate to protect consumers.

The Private-Market Benchmark

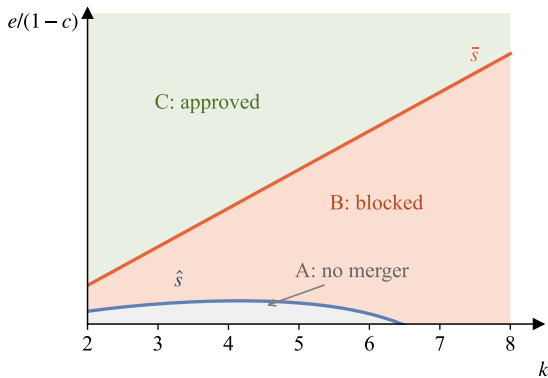
- Without a public firm, a merger lowers output and raises price; the authority approves only when the relative synergy $\frac{\bar{e}}{1-c}$ reaches

$$\bar{e} = \frac{(1-c)(k-1)}{n+1},$$

the cutoff of Perry and Porter (1985).

- The firms' profitability cutoff is smaller than $\bar{e}$ over a wide range of merger sizes, so a set of mergers is profitable yet blocked.

Figure 10.3: Three Regions in a Private Market



Section 10.6

Privatization and Blocked Mergers

Privatization Weakens the Offset (Section 10.6)

- The alignment of Section 10.5 rests on the public firm holding output fixed, which it does only while fully public.
- As α falls below one, the firm places less weight on welfare and offsets the merger less; aggregate output now falls when the private firms combine.
- Consumer surplus drops, and the authority's test starts to matter.

The Blocked-Merger Region Reopens

- The empty region of profitable-but-blocked mergers reopens and widens as α falls; at full privatization it reaches the private-market region of Figure 10.3.
- A cost of privatization the welfare comparison of Chapter 9 does not capture: profitable mergers get blocked.
- Counterpart, for merger review, of Kanjilal et al. (2022): a public firm weighing welfare more heavily makes welfare-improving mergers easier to approve.

Section 10.7

Case Study – Sunrise-UPC and Swisscom

Sunrise-UPC and Swisscom (Section 10.7)

- In 2020 the Swiss operators Sunrise and UPC Switzerland merged, forming the country's second-largest telecommunications firm behind **Swisscom**, about 51 percent state-owned.
- Two private firms combined in a market anchored by a large public incumbent – the setting of the chapter.

Reading the Merger Through the Model

- With Swisscom majority-state-owned, its output stabilizes total supply, so a welfare-improving private merger clears review even at a modest cost saving.
- As states sell down telecom stakes, the discipline that eased approvals fades; mergers among the remaining private operators warrant closer scrutiny.
- Switzerland's retention of a majority in Swisscom – against the wider European trend of privatizing incumbents – is the condition under which the chapter's permissive reading of private mergers holds.

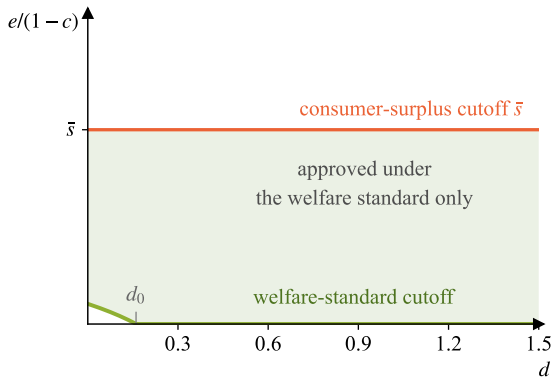
Section 10.8

The Environmental Standard

Consumer Surplus vs. Welfare Standard (Section 10.8)

- A consumer-surplus authority ignores the externality; a welfare standard credits a merger that lowers output with the emissions it avoids.
- While the public firm is active the two standards agree, since output does not move; in a private market they part.
- The welfare-standard cutoff sits below $\bar{e}$, and beyond a threshold d_0 every merger clears the welfare standard, since the avoided emissions outweigh the consumer loss.
- An authority weighing emissions, as the green merger guidelines of Kanjilal et al. (2026) propose, would allow mergers a consumer-surplus authority blocks.

Figure 10.4: Consumer-Surplus vs. Welfare Standard



A Public Firm That Undervalues Emissions

- A public firm that weighs emissions at $d_{PF} < d$ (Section 2.7) behaves, for merger purposes, like a partly privatized one: it holds output less firmly, and the region of blocked mergers opens as in Section 10.6.
- A greener public firm, by contrast, targets a lower output and exits at a smaller merger; the market is then private and region B returns.
- The public firm eases merger approval only while it is both active and committed to welfare.

Section 10.9

Numerical Illustration

Numerical Illustration (Section 10.9)

$c = \frac{1}{2}$, $d = 0.15$, $n = 4$, $k = 3$: each outsider produces 0.065, the fee is zero, $\hat{e} = 0.048$.

e	q_0	q_I	p	CS	PS	W	Decision
0.000	0.304	0.065	0.565	0.095	0.028	0.109	None
0.048	0.256	0.113	0.565	0.095	0.034	0.114	Approved
0.100	0.204	0.165	0.565	0.095	0.045	0.125	Approved
0.200	0.104	0.265	0.565	0.095	0.081	0.162	Approved

Below $\hat{e}$ no merger is proposed; above it, every merger is approved.

Reading the Table

- Price and consumer surplus are constant across rows: the public firm absorbs the merger one-for-one, so consumers see no change.
- The merged firm's output rises with e ; the public firm's falls by the same amount, keeping aggregate output at $\frac{1-c}{1+d}$.
- Welfare rises entirely through the cost saving $e q_I$; no row shows a profitable merger blocked.

Summary / Key Takeaways

- While the public firm is active it aligns firms and the authority: no profitable merger is blocked, and the authority approves welfare-improving mergers.
- The request cutoff $\hat{e} = (\sqrt{k} - 1) \frac{(1 - c)d}{1 + d}$ rises in k and d , independent of n .
- Privatization reopens the blocked-merger region; a welfare (environmental) standard approves more mergers than a consumer-surplus one.

Chapter 11

Further Extensions

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-García

Outline of Chapter 11

- 11.2 Price competition.
- 11.3 Public leaders and followers.
- 11.4 Free entry.
- 11.5 Foreign ownership.
- 11.6 Abatement investment.
- 11.7 Case study.
- 11.8 Numerical illustration.
- 11.9 Summary.
- Each extension takes the firm fully public ($\alpha = 1$) to isolate the new feature, except Section 11.3, which carries α throughout.

Introduction (Section 11.1)

- So far: firms compete in quantities, move at the same time, are given in number, are domestic, and have fixed emissions per unit.
- This chapter relaxes each feature one at a time; each new parameter returns the earlier model at a special value.
- The reader sees exactly what the new feature adds; the central mechanism, the public firm using its objective to steer the market, is the same throughout.

Section 11.2

Allowing for Price Competition

Price Competition: Setup

- Differentiated products; each firm posts a price and serves the demand that follows.
- The private firm prices to maximize profit; the public firm prices to reflect the full social marginal cost (production cost plus marginal damage).
- The public firm therefore prices above its own marginal cost when pollution is severe, and below the profit-maximizing level when market power is the larger concern.

Bertrand versus Cournot in a Mixed Duopoly

- **Ghosh and Mitra (2010):** the private-market ranking (Bertrand yields lower prices and higher welfare than Cournot) can *reverse* once one firm is public.
- Reason: the public firm already restrains output through its objective, so sharper Bertrand rivalry can push the market past the social target.
- Intuitions carried from private markets, including the presumption that Bertrand serves consumers better, need not hold once a public firm competes.

Endogenous Choice of Strategic Variable

- **Matsumura and Ogawa (2012):** let each firm choose price or quantity; setting a price is dominant for both the public and the private firm.
- Holds whether the goods are substitutes or complements, so price competition arises endogenously.
- The intensity of rivalry is sharper under prices, so the private firm's choices respond more strongly to the public firm than under quantity competition.

Section 11.3

Public Leaders and Followers

Three Timings (Section 11.3)

- Market of Chapter 4: one public firm and n private firms at marginal cost c ; public weight α on welfare.
- **SIM**: public and private firms choose at the same time (Cournot benchmark).
- **PubL**: public firm commits first, private firms respond (Enel, National Grid).
- **PubF**: private firms commit first, public firm adjusts (La Poste, a legacy telecom).

Output Under the Three Timings

- Solving by backward induction with damage set aside,

$$Q^{SIM} = \frac{(1-c)[(n+1) - n\alpha]}{(n+2) - \alpha(n+1)}, \quad Q^{PubF} = \frac{(1-c)[n(2-\alpha) + 1]}{(n+1)(2-\alpha)}$$

- All three rise with α and reach $1 - c$ at $\alpha = 1$; the general ranking is

$$Q^{SO} \geq Q^{PubF} \geq Q^{PubL} \geq Q^{SIM}.$$

- A public follower, observing private output, sets its own to fill the gap to the optimum: the second-mover advantage best corrects underproduction.

Welfare and the Best Timing (Clean Market)

- Welfare is highest when output is closest to $1 - c$, and each regime reaches $1 - c$ only at $\alpha = 1$: full public ownership is optimal under every timing.
- For any interior α ,

$$W^{PubF} > W^{PubL} > W^{SIM}.$$

- A regulator that can arrange the sequence, through licensing or the scheduling of capacity, would place the public firm last.

Timing and the Environment (Section 11.3.3)

- With damage $\frac{d}{2}Q^2$, the social optimum is the lower output $\frac{1-c}{1+d}$, so producing more is no longer unambiguously better.
- A public follower accommodates, so private leaders expand: aggregate output is pushed above the damage-adjusted optimum.
- The second-mover advantage that helped in a clean market is now a liability; full nationalization is no longer optimal under a public follower.
- Welfare ranking can invert as pollution grows.

Privatization as a Second-Best Instrument

- When a fee is available it corrects the externality directly, and the order of moves matters little.
- When a fee is politically out of reach, the regulator can still restrain a polluting industry by privatizing the public firm in part.
- Timing and α , chosen together, then stand in for the fee (Choi et al., 2026): they trade some output correction for lower emissions and improve on an unregulated industry while pollution is moderate.

Section 11.4

Allowing for Free Entry

Free Entry: Setup and Entry Condition

- The number of private firms was fixed; suppose instead private firms enter until profits are exhausted, $\pi_i(n) = F$ at the free-entry n^* .
- A larger public output leaves less room for private entrants, so the public firm shapes the market structure and not only the split within it.
- Two forces balance: the public firm's output shifts private profitability, and free entry adjusts n to restore the zero-profit condition.

Matsumura and Kanda (2005): The Full-Public Case Returns

- De Fraja and Delbono (1989) warn: at fixed n , full welfare-maximization by the public firm can lower welfare, since its large output erodes private profits.
- Matsumura and Kanda (2005): once entry is free, that concern lapses; the public firm's output now disciplines entry and offsets the excess entry that free markets otherwise bring.
- **Result:** a fully welfare-maximizing public firm is always optimal under free entry; the case for a fully public, fully green firm returns.

Section 11.5

Allowing for Foreign Ownership

Foreign Ownership: Setup

- Let a share φ of the private firm be foreign-owned; the public firm counts only the domestic part of the private firm's profit.
- $\varphi = 0$ returns the benchmark; as φ rises, the public firm cares less about the private firm's profit and more about domestic consumer surplus.
- Aggregate output becomes

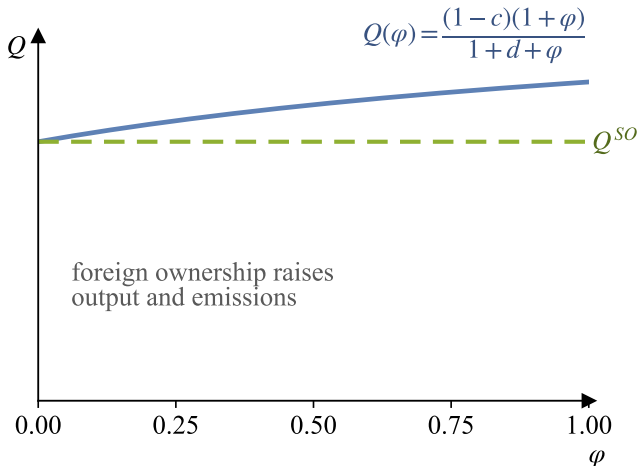
$$Q(\varphi) = \frac{(1 - c)(1 + \varphi)}{1 + d + \varphi},$$

rising in φ from the socially optimal level at $\varphi = 0$.

Fjell and Pal (1996): Welfare Reading

- A domestic welfare-maximizing public firm, discounting foreign profit, weights consumer surplus more heavily and expands output.
- Foreign ownership leads the public firm to over-produce and raises emissions; the environmental discipline of public ownership is eroded, and the case for a fee revives.
- In effect, foreign ownership acts like a lower environmental commitment: the public firm behaves less greenly because the surplus it now chases sits abroad.

Figure 11.1: Foreign Ownership Raises Output and Emissions



Section 11.6

Allowing for Abatement Investment

Two Margins: Output and Abatement

- Two-stage game: firms first invest in abatement a_i , then choose output q_i ; emissions per unit fall in a_i .
- The public firm invests to the level that maximizes welfare, weighing the cost of abatement against the environmental harm it avoids.
- The private firm invests less: it pays the cost of abatement but not the external benefit; a fee raises its incentive and aligns it with the public firm's.

Interaction of the Two Margins

- Restraining output already lowers emissions, which lowers the marginal value of abatement: output discipline and abatement partly substitute.
- **Bárcena-Ruiz and Garzón (2006)**: the optimal fee is lower in a mixed market than in a private one (Chapter 2 result).
- **Kato (2006)**: tradable versus non-tradable permits can raise or lower welfare depending on the cost gap between firms; the permit-market advantage of private settings is not automatic.
- **Poyago-Theotoky (2007, 2024)**: environmental R&D, cooperative or not, interacts with the fee; the recent synthesis surveys the field.

Section 11.7

Case Study: Timing in Liberalized Electricity Markets

Case Study: Context

- In several liberalized systems the legacy public generator set output or capacity before private entrants, acting as an industry leader.
- In others the public firm adjusted after private producers had committed, acting as a residual balancer, and so as a follower.
- The order was often a byproduct of how liberalization was sequenced: incumbents scheduling capacity ahead of entrants, or filling in the residual.

Case Study: Reading the Model

- A public follower best corrects ordinary underproduction through its second-mover advantage, but overshoots once emissions matter.
- Anticipating the accommodating follower, private leaders expand, pushing output above the damage-adjusted optimum.
- Where carbon pricing is politically constrained, adjusting the sequence of moves or the degree of public ownership substitutes in part for the missing fee.
- Timing and ownership are environmental instruments in their own right.

Numerical Illustration (Section 11.8)

- Foreign-ownership extension, with $c = \frac{1}{2}$ and $d = \frac{6}{10}$.

φ	q_0	q_1	p	CS	Env	W
0.00	0.125	0.187	0.688	0.049	0.029	0.078
0.25	0.176	0.162	0.662	0.057	0.034	0.071
0.50	0.214	0.143	0.643	0.064	0.038	0.066
0.75	0.245	0.128	0.628	0.069	0.042	0.063
1.00	0.269	0.115	0.615	0.074	0.044	0.061

- The public firm expands as φ rises: discounting the foreign-owned rival's profit, it produces more, so emissions rise and domestic welfare falls.

Summary / Key Takeaways

- Each extension adds one parameter and returns the earlier model at a special value of it.
- Price competition intensifies rivalry but leaves the mechanism intact; the Bertrand-Cournot ranking can reverse under a public firm.
- A public follower best corrects market power in a clean market, but is a liability under severe pollution; timing and α can substitute for a missing fee.
- Free entry returns the case for a fully welfare-maximizing public firm.
- Foreign ownership makes the public firm over-produce, reviving the fee.
- Abatement adds a second margin the public firm internalizes.

Chapter 12

Collusion in Mixed Oligopoly

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 12

- 12.1 Introduction.
- 12.2 Collusion between two private firms.
- 12.3 Collusion among n private firms.
- 12.4 Collusion and the emission fee.
- 12.5 Collusion under convex costs.
- 12.6 An efficient or lax public firm.
- 12.7 Case study.
- 12.8 Summary.

The Central Problem

- Private firms may collude; does a public competitor make it easier or harder?
- The public firm does **not** join the cartel: it best-responds each period, expanding when the cartel restricts.
- Key question: how does the degree of nationalization α change the minimal discount factor $\bar{\delta}$ that sustains collusion?

Real-World Context

- Cartels are often found where a state-owned firm competes with private ones: fuel distribution, cement, electricity.
- In Brazil, the state producer **Petrobras** sits alongside private distributors; the authority CADE has fined fuel-reseller cartels in many cities.
- Framework: infinitely repeated game, Cournot-Nash reversion after a defection (Friedman, 1971).

Section 12.2

Collusion Between Two Private Firms

Setup and the Residual Margin (Section 12.2)

- One public firm (weight α), two private firms, constant cost c , demand $p = 1 - Q$, damage $\frac{d}{2}Q^2$.
- The public firm best-responds to private output Q_p , so each private firm faces the residual margin

$$p - c = A - BQ_p, \quad B = \frac{1 - \alpha}{2 - \alpha + \alpha d}.$$

- B measures how far the public firm offsets the cartel: $B \rightarrow 0$ as $\alpha \rightarrow 1$, and B falls as d rises.

Collusion and Punishment Profits

- **Collusion.** The two private firms share the monopoly output on the residual margin: each produces $q^C = \frac{A}{4B}$ and earns

$$\pi^C = \frac{A^2}{8B}.$$

- **Punishment.** Static Cournot on the residual margin: each produces $q^N = \frac{A}{3B}$ and earns

$$\pi^N = \frac{A^2}{9B}.$$

- Both are proportional to A^2/B : comparisons among them do not depend on the intercept.

The Deviation Profit

- Moves are simultaneous, so during a one-shot defection the public firm's output is **fixed** at its collusive-phase level q_0^C .
- The defector best-responds against q_0^C and the other firm's collusive output q^C ; solving the FOC,

$$q^D = \frac{A(2B+1)}{8B}, \quad \pi^D = \frac{A^2(2B+1)^2}{64B^2}.$$

- The defector escapes the public firm's offsetting for one period: the greater the offsetting (smaller B), the larger this windfall.

Colluding versus Defecting

- Colluding forever: $V^C = \frac{\pi^C}{1-\delta}$.
- Defecting now, then punished forever: $V^D = \pi^D + \frac{\delta \pi^N}{1-\delta}$.
- Collusion holds when $V^C \geq V^D$, i.e.

$$\bar{\delta} = \frac{\pi^D - \pi^C}{\pi^D - \pi^N}.$$

The Collusion Threshold (Section 12.2)

- Substituting the three profits, the factor A^2/B cancels and

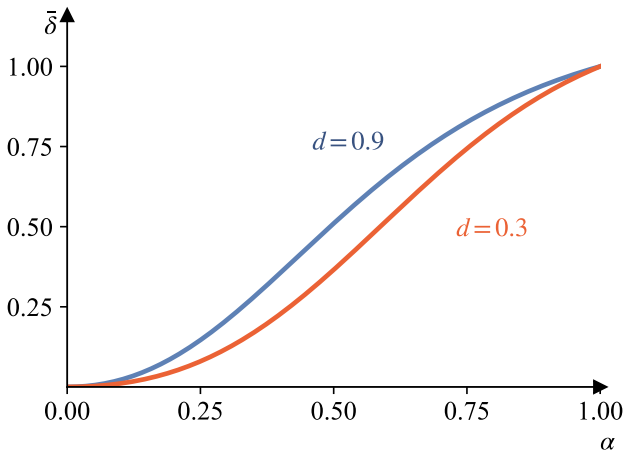
$$\bar{\delta} = \frac{9\alpha^2(1+d)^2}{9\alpha^2d^2 + 10\alpha^2d + 17\alpha^2 + 8\alpha d - 24\alpha + 16}.$$

- It rises monotonically in α and in d : a more welfare-oriented public firm makes collusion harder.

Three Reference Points

- **No public firm** ($B = 1$): $\bar{\delta} = \frac{9}{17}$, the textbook Cournot-duopoly threshold (Friedman, 1971).
- **Fully private public firm** ($\alpha = 0$): $\bar{\delta} = 0$; a two-firm cartel with a symmetric outsider is unprofitable (Salant, Switzer, Reynolds, 1983).
- **Fully nationalized** ($\alpha = 1$): $\bar{\delta} = 1$; collusion cannot be sustained at any discount factor.

Figure 12.1: The Threshold Rises with Nationalization



Intuition

- A more nationalized firm offsets the cartel more in normal times, holding the price down.
- But during the single period of a defection it cannot react, so the defector captures a market it would otherwise have contested.
- The larger the usual offsetting, the larger this windfall, so the temptation to defect, and the patience collusion needs, grow with α .
- Colombo (2015) finds the opposite sign in a related model; the direction turns on how the public firm responds to a defection.

Section 12.3

Collusion Among n Private Firms

Extending to n Colluders (Section 12.3)

- All n private firms collude (no partial cartels). The threshold becomes

$$\bar{\delta} = \frac{(Bn - 1)^2(n + 1)^2}{(Bn + 1)^2(n + 1)^2 - 16Bn^2},$$

with $B = \frac{1 - \alpha}{2 - \alpha + \alpha d}$.

- No public firm ($B = 1$): $\bar{\delta} = \frac{(n + 1)^2}{n^2 + 6n + 1}$, the textbook n -firm cartel threshold.

The Locus $Bn = 1$

- Setting the numerator to zero yields $Bn = 1$, i.e.

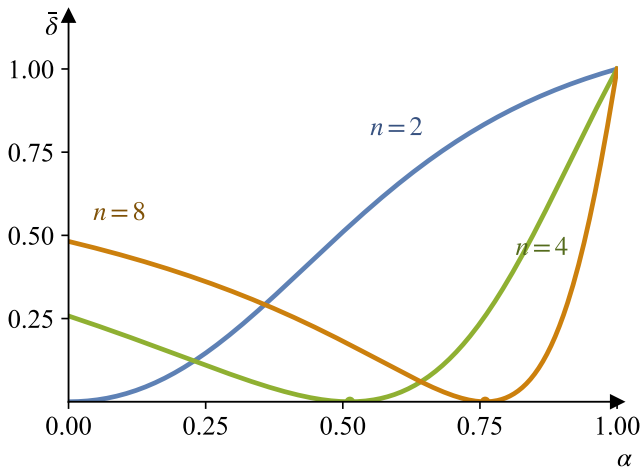
$$\alpha^* = \frac{n-2}{n-1+d} \in (0,1) \text{ for } n \geq 3.$$

- There the public firm's offsetting exactly cancels the cartel's restraint: the collusive output coincides with the static best response.
- A defection then yields no gain, so $\bar{\delta} = 0$: the cartel is trivially sustainable but has nothing to sustain.

A U-Shape in α

- Away from α^* , $\bar{\delta}$ rises: too little offsetting when α is small, too much when α is large.
- For $n \geq 3$, $\bar{\delta}$ is therefore **U-shaped** in α : it peaks near the extremes and vanishes at α^* .
- Cartel size and nationalization interact non-monotonically: adding firms shifts α^* to the right, so the interior of the U moves with n .

Figure 12.2: The Threshold for $n = 2, 4, 8$



The Comparative Statics

- Write $\bar{\delta}$ through B , with $\frac{\partial B}{\partial \alpha} = -\frac{1+d}{(2-\alpha+\alpha d)^2} < 0$ and $\frac{d\bar{\delta}}{dB} \propto (Bn-1)$.
- Then $\frac{\partial \bar{\delta}}{\partial \alpha} = \frac{d\bar{\delta}}{dB} \frac{\partial B}{\partial \alpha}$, and $\frac{\partial^2 \bar{\delta}}{\partial \alpha \partial n}$ shares the factor $Bn-1$.
- All switch sign around α^* : cartel size and nationalization are substitutes in one region, complements in another.

Section 12.4

Collusion and the Emission Fee

The Fee Shifts the Public Firm's Best Response

- Add an emission fee t . Each firm's private cost becomes $c + t$; the public firm still weighs the true damage d but now pays t on its own output.
- Its best response becomes, in two-term form,

$$q_0(Q_p) = \frac{(1 - c) - (1 - \alpha)t}{2 - \alpha + \alpha d} - \frac{1 + \alpha d}{2 - \alpha + \alpha d} Q_p,$$

where Q_p is the aggregate private output.

- Substituting into the residual margin, $p - c - t = A(t) - BQ_p$ with $A(t) = A - \frac{(1 + \alpha d)t}{2 - \alpha + \alpha d}$; slope B is unchanged.

Why the Fee Does Not Move $\bar{\delta}$

- All three profits scale with the common factor $A(t)^2/B$:

$$\pi^C = \frac{A(t)^2}{8B}, \quad \pi^N = \frac{A(t)^2}{9B}, \quad \pi^D = \frac{A(t)^2(2B+1)^2}{64B^2}.$$

- The factor $A(t)^2$ cancels in the ratio, leaving

$$\bar{\delta} = \frac{9(2B-1)^2}{36B^2 - 28B + 9} \text{ : the same threshold as in Section 12.2, independent of } t.$$

- **Result:** $\frac{\partial \bar{\delta}}{\partial t} = 0$. The fee cuts output and emissions but does not, on its own, make collusion harder to sustain.

Endogenous Fee: A Booster of Competition Policy

- If the environmental agency anticipates cheating, it sets a lower fee to protect the profit stream (Espinola-Arredondo and Munoz-Garcia, 2026).
- Fees then act as a **booster of competition policy**: audits, leniency programs, and whistleblower rewards deter collusion more strongly when a fee is in place, because the fee thins the profit from defection.
- Practical reading: in polluting industries, competition tools can be set less aggressively and still deter cartels as effectively as in an unregulated industry.

Section 12.5

Collusion under Convex Costs

Private Firms with Convex Costs

- Let $C(q_i) = cq_i + \frac{\beta}{2}q_i^2$, so $MC = c + \beta q_i$.
- Solving each private firm's FOC gives the two-term best response

$$q_i(Q_{-i}) = \frac{1-c}{2+\beta} - \frac{1}{2+\beta} Q_{-i}.$$

- Both the intercept and the slope shrink: each firm starts lower and expands less for any rival output. The best response is flatter.

The Public Firm's Best Response with β

- The public firm's problem gains the same term, and its best response becomes

$$q_0(Q_p) = \frac{1 - c}{2 + \beta - \alpha + \alpha d} - \frac{1 + \alpha d}{2 + \beta - \alpha + \alpha d} Q_p.$$

- The residual-margin shortcut of Section 12.2 no longer applies: each firm's profit carries its own $-\frac{\beta}{2}q_i^2$ term, so profit is no longer a common margin times output.
- The three profits must be computed **directly**.

Assembling the Three Profits

- Collect the factor common to all three into

$$K = \frac{(1-c)^2(1-\alpha+\beta+\alpha d)^2}{2G}, \quad G = 2 - \alpha + \beta + \alpha d.$$

- Introduce four polynomials in β :

$$\Delta_N = \beta^2 + (5 - \alpha + \alpha d)\beta + 3(1 - \alpha), \quad \Delta_C = \beta^2 + (6 - \alpha + \alpha d)\beta -$$

$$\Delta_0 = \beta^2 + (4 - \alpha + \alpha d)\beta + 2(1 - \alpha), \quad \Delta_D = \beta^2 + (5 - \alpha + \alpha d)\beta -$$

- Then $\pi^C = \frac{K}{\Delta_C}$, $\pi^N = K \frac{\Delta_0}{\Delta_N^2}$, $\pi^D = K \frac{\Delta_D^2}{(2 + \beta) G \Delta_C^2}$.

The Explicit $\bar{\delta}(\beta)$ and its Benchmark

- K enters every profit, so it cancels in the ratio:

$$\bar{\delta}(\beta) = \frac{\frac{\Delta_D^2}{(2+\beta) G \Delta_C^2} - \frac{1}{\Delta_C}}{\frac{\Delta_D^2}{(2+\beta) G \Delta_C^2} - \frac{\Delta_0}{\Delta_N^2}}.$$

- At $\beta = 0$ the four polynomials collapse to constants and $\bar{\delta}(0)$ reduces to the Section-12.2 threshold: setting $\beta = 0$ recovers the earlier benchmark, as it must.
- Signing $\frac{\partial \bar{\delta}}{\partial \beta}$ analytically is not straightforward; we rely on numerics and Figure 12.3.

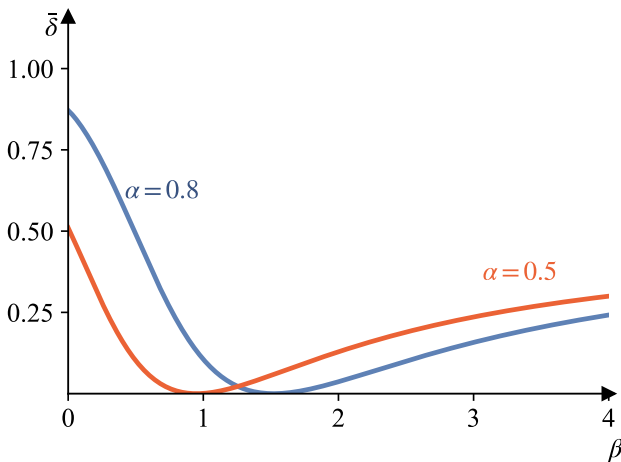
Numerical Evaluation

- At $c = \frac{1}{2}$, $d = \frac{9}{10}$, $\alpha = \frac{7}{10}$:

β	0	$\frac{1}{2}$	1	2	4
$\bar{\delta}(\beta)$	0.77	0.34	0.04	0.06	0.26

- Modest convexity roughly halves the patience collusion requires; near $\beta = 1$ sustaining the cartel is almost costless.
- A rising marginal cost reduces the defection gain: expanding beyond q^C is more expensive, so π^D falls toward π^C .

Figure 12.3: $\bar{\delta}$ against β



The effect is **non-monotone**: convex costs first ease collusion, then, past a minimum, make it harder again (Correia-da-Silva and Pinho, 2018).

Section 12.6

An Efficient or Lax Public Firm

A More or Less Efficient Public Firm

- Let the public firm have its own cost c_0 , possibly $c_0 \neq c$. Its best response gains c_0 in the intercept only:

$$q_0(Q_p) = \frac{1 - c_0}{2 - \alpha + \alpha d} - \frac{1 + \alpha d}{2 - \alpha + \alpha d} Q_p.$$

- Repeating Section 12.4's substitution, $p - c = A(c_0) - BQ_p$ with the slope B unchanged. All three profits scale with $A(c_0)^2/B$.
- In the ratio $A(c_0)^2$ cancels, leaving

$$\bar{\delta} = \frac{9(2B - 1)^2}{36B^2 - 28B + 9} = \frac{9\alpha^2(1 + d)^2}{9\alpha^2d^2 + 10\alpha^2d + 17\alpha^2 + 8\alpha d - 24\alpha + 16}.$$

A Greener or Laxer Public Firm

- Let the public firm weigh emissions at its own d_{PF} rather than the true d . Its best response becomes

$$q_0(Q_p) = \frac{1 - c}{2 - \alpha + \alpha d_{PF}} - \frac{1 + \alpha d_{PF}}{2 - \alpha + \alpha d_{PF}} Q_p,$$

and the residual slope $B = \frac{1 - \alpha}{2 - \alpha + \alpha d_{PF}}$ now moves with the mandate.

- Substituting into (12.16),

$$\bar{\delta} = \frac{9\alpha^2(1 + d_{PF})^2}{9\alpha^2 d_{PF}^2 + 10\alpha^2 d_{PF} + 17\alpha^2 + 8\alpha d_{PF} - 24\alpha + 16}.$$

The Sign of $\partial \bar{\delta} / \partial d_{PF}$

- Differentiating,

$$\frac{\partial \bar{\delta}}{\partial d_{PF}} = \frac{72\alpha^2(1-\alpha)(1+d_{PF})(4-3\alpha+\alpha d_{PF})}{(9\alpha^2 d_{PF}^2 + 10\alpha^2 d_{PF} + 17\alpha^2 + 8\alpha d_{PF} - 24\alpha + 16)^2} > 0.$$

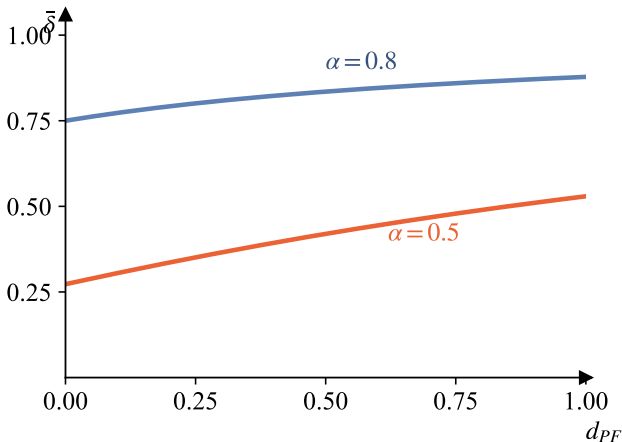
- Every factor in the numerator is positive for $\alpha \in (0, 1)$ and $d_{PF} \in [0, 1]$: a greener public firm raises the threshold; a laxer one lowers it.
- The lever that disciplines a private cartel is the public firm's **environmental mandate**, not its efficiency.

Table 12.1: $\bar{\delta}$ across c_0 and d_{PF}

c_0	d_{PF}				
	0.1	0.3	0.5	0.7	0.9
0.3	0.456	0.520	0.573	0.617	0.653
0.5	0.456	0.520	0.573	0.617	0.653
0.7	0.456	0.520	0.573	0.617	0.653

- Reading *across*: $\bar{\delta}$ rises with d_{PF} .
- Reading *down*: $\bar{\delta}$ is flat in c_0 (values coincide).
- $\alpha = 0.6$ throughout.

Figure 12.4: $\bar{\delta}$ against d_{PF}



Both curves slope upward: for a mixed oligopoly, private cartels are harder to sustain the greener the public firm's mandate.

Case Study (Section 12.7): Fuel Distribution in Brazil

- Petrobras, majority state-owned, is the large upstream producer; private distributors and resellers compete downstream.
- CADE has repeatedly prosecuted service-station cartels; a disciplining state producer raises the temptation to defect and makes such cartels harder to sustain.
- Petrobras's mandate has swung between commercial and political aims, so its effective α is intermediate, and the U-shape decides which way a change tips.

Summary / Key Takeaways

- A public firm outside the cartel still shapes it, by offsetting its restraint; absent the public firm the model returns $\frac{9}{17}$ and $\frac{(n+1)^2}{n^2+6n+1}$.
- With two colluders $\bar{\delta}$ rises with α and d ; with n it is U-shaped in α , vanishing at $\alpha^* = \frac{n-2}{n-1+d}$.
- An exogenous fee leaves $\bar{\delta}$ unchanged; endogenous fees act as a booster of competition policy.
- Convex costs make the effect non-monotone; the disciplining lever is the public firm's environmental mandate, not its efficiency.

Chapter 13

Policy Lessons

Instructor Slides

Regulation in Mixed Oligopoly: A Game-Theoretic Approach

Felix Muñoz-Garcia

Outline of Chapter 13

- 13.1 The question, restated.
- 13.2 When the fee is redundant, powerless, or necessary.
- 13.3 Lessons for merger review.
- 13.4 Lessons for privatization and public ownership.
- 13.5 The public firm's environmental mandate.
- 13.6 Limitations and directions.
- 13.7 Closing.

The Question, Restated (Section 13.1)

- How much of the regulator's job does a welfare-minded public firm do on its own, and when is a fee still needed?
- A graded answer: in the simplest market the public firm does the whole job; each complication erodes its reach.
- **Public ownership is a partial substitute for environmental regulation**, strong where the market is simple and weak where it is not.

Three States of the Emission Fee (Section 13.2)

- **Redundant:** an active, green, fully public firm ($\alpha = 1$) holds output at Q^{SO} (symmetric benchmark; more private firms; a private merger).
- **Powerless:** an active public firm that undervalues emissions offsets any fee. The lever is its mandate, not a fee.
- **Necessary:** the public firm is pushed out, or privatized ($\alpha < 1$), and the fee restores the first best.

Merger Review: Concentration and Efficiency (Section 13.3)

- Concentration screens can mislead: a private merger can raise the Herfindahl index while leaving price, output, and welfare unchanged, since the public firm absorbs it.
- Cost-reducing mergers: the Williamson (1968) trade-off collapses while an active public firm holds output fixed; the efficiency gain passes through.
- A review should ask whether the merger changes the public firm's capacity to discipline the industry, not read a rising index as harm (Kanjilal et al., 2026).

Merger Review: Public–Private Mergers and the Authority (Section 13.3)

- A public–private merger: welfare turns on whether the merged firm keeps a public objective or takes on a private one.
- A merger that privatizes the public firm removes the market's built-in corrector and makes an emission fee necessary in its place.
- A competition authority clears welfare-improving mergers at a modest cost saving while a public firm is active; privatizing that firm can leave the same merger blocked (Chapter 10).

Privatization: When to Keep Public, When to Privatize (Section 13.4)

- Keep public a firm that is efficient and as green as the regulator; privatizing it (a lower α) raises output and emissions and lowers welfare.
- Privatizing an inefficient or lax public firm can raise welfare, the more so when paired with a fee that corrects the externality the privatized firm no longer restrains (De Fraja and Delbono, 1989; Matsumura, 1998).
- A public firm earns its place only when it is both efficient and committed to the environment.

Privatization: Two Routes to the First Best (Section 13.4)

- Route 1: keep the firm public and green.
- Route 2: privatize the firm and levy an emission fee.
- The choice rests on the firm's efficiency and mandate, not on ownership for its own sake.
- With free entry the balance tips back to public ownership, since free entry restores the optimality of a fully welfare-maximizing public firm (Matsumura and Kanda, 2005).

The Public Firm's Environmental Mandate (Section 13.5)

- The lever that stands out is the public firm's own weight on emissions.
- With several public firms it narrows to the **least green active one**, which sets the industry's output.
- Greening an already-careful firm, or counting more public firms, does nothing; align the mandate of the laggard, a lever internal to the firm, not a tax.

Limitations and Directions (Section 13.6)

- Simple models: linear demand, constant marginal costs, and one externality, so that each mechanism stands out.
- Real markets add product differentiation, dynamics, uncertainty about costs and damages, investment in cleaner technology, and international competition.
- The wider mixed-oligopoly literature is surveyed by Poyago-Theotoky (2024), which points to research fronts we take one at a time in the frames ahead.

Direction 1: Market Structure and Conduct

- Product differentiation and price competition are mapped only in part: Ghosh and Mitra (2010) and Fujiwara (2007) study Bertrand and differentiated-Cournot mixed duopoly, without the emission fee.
- Timing structures beyond simultaneous moves: public leaders and followers and endogenous timing (Pal, 1998), studied for output but not for regulation.
- Vertical and multi-market structures in energy and transport invite transfer prices, self-preferencing, and access regulation alongside the fee.

Direction 2: Dynamics and Investment

- Free entry with a public firm: Matsumura and Kanda (2005); entry when the public firm can commit to output or fee levels remains open.
- Exit dynamics around a private cartel that faces an intermittent public presence are open.
- Investment in cleaner technology: whether a greener public mandate speeds up or crowds out private abatement R&D depends on complementarities the static models here cannot capture.
- Dynamic games with learning-by-doing look most useful for the joint choice of output and abatement.

Direction 3: Uncertainty and Information

- Regulation under damage uncertainty: extend the framework to a regulator that learns the true externality over time.
- Regulation with private information about the public firm's own weight on emissions.
- Greenwashing, whistleblower programs, and audits interact with the mixed structure in ways Espinola-Arredondo and Muñoz-Garcia (2026) begin to develop.

Direction 4: Collusion and Merger Review

- Chapter 12 opened a line following Colombo (2015), Correia-da-Silva and Pinho (2018), and Escrihuela-Villar and Guillén (2024).
- Whether the fee can be redesigned to deter collusion actively, rather than leave it neutral as in Chapter 12, is open.
- Merger review under a mixed structure is developed for horizontal and cost-reducing mergers (Kanjilal, Espinola-Arredondo, and Muñoz-Garcia, 2022, 2026), and open for vertical and portfolio mergers.

Direction 5: Political Economy of the Mandate

- Every result in the book conditions on a given pair (α, d_{PF}) : the chosen degree of nationalization and the public firm's weight on emissions.
- How are these two set? By elected officials facing lobbies, by corporate boards under fiduciary rules, by shifting political majorities.
- Bringing that endogeneity inside the model would connect the results here to the wider political-economy-of-regulation literature.

Direction 6: International and Open-Economy

- Fjell and Pal (1996) study a public firm competing with foreign private rivals.
- Natural next steps: an emission fee, cross-border pollution, and public firms in several countries that set fees non-cooperatively.
- Carbon leakage, border adjustments, and the role of state-owned exporters extend the substitution question from a single country to the international stage.

Direction 7: Instrument Design

- The book takes the emission fee as the leading instrument.
- Emissions permits in a mixed oligopoly (Kato, 2006): open questions when the public firm can bid strategically or when permit allocations are political.
- Standards, feed-in tariffs, and clean-technology subsidies once a public firm sits in the market are open on the same terms.

Closing (Section 13.7)

- A public firm is neither a substitute for environmental policy nor an obstacle to it in general.
- It is one or the other depending on how much it values the environment, how efficient it is, and the market around it.
- The book's contribution: to say **which, and when**.