Two-Part Tariffs with Two Firms

Competition in Nonlinear Pricing on a Hotelling Line

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Roadmap

- **The model.** Two firms, a Hotelling line, and a two-part tariff.
- **Equilibrium.** Per-unit price and fixed fee, stage by stage.
- **A benchmark.** How competition looks under ordinary linear pricing.
- **Three comparisons.** Profits, welfare, and consumer surplus.
- **Intuition, then a worked numerical example.**

Primary source: Armstrong and Vickers (2001); textbook treatment in Belleflamme and Peitz (2015, §9.3.2). Full reference at the end.

Two-Part Tariffs Are Everywhere

- A *two-part tariff* charges a fixed fee plus a price per unit:

$$T(q) = m + p\,q.$$

- **Warehouse clubs.** Costco: a membership fee, then per-item prices.
- **Amusement parks, gyms, nightclubs.** An entry charge, then rides, classes, or drinks.
- **Razors and printers.** A fee-like durable, then blades or ink per unit.

The average price per unit, $\dfrac{T(q)}{q} = p + \dfrac{m}{q}$, falls the more you buy.

The Question, and What We Will Find

- **Question.** When two firms compete and can set *both* a per-unit price and a fixed fee, what tariff emerges, and who benefits relative to plain (linear) pricing?
- **We will show three comparisons** between two-part (NL) and linear (L) pricing:

$$W_{NL} > W_L, \qquad \pi_{NL} > \pi_L, \qquad \text{but} \qquad CS_{NL} < CS_L.$$

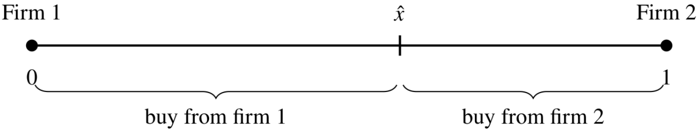
- Higher welfare, higher profits, yet lower consumer surplus.

Section 1

The Model

The Hotelling Market

- Two firms at the ends of a line of unit length: firm 1 at 0, firm 2 at 1.
- A unit mass of consumers is spread uniformly along the line.
- A consumer at x pays a mismatch cost τ per unit of distance; $\tau > 0$ measures *product differentiation*.
- Both firms produce at constant marginal cost c.



Timing

- **Stage 1.** Firms simultaneously choose their two-part tariffs (p_i, m_i), $i = 1, 2$.
- **Stage 2.** Each consumer observes both tariffs, picks a firm, and then chooses how many units to buy.

We solve by **backward induction**: the consumers' problem first, then the firms'.

Section 2

Stage 2: The Consumers

Indirect Utility and Demand

- Facing per-unit price p_i, a consumer chooses q to solve

$$v(p_i) \equiv \max_q \{u(q) - p_i\, q\}.$$

- First-order condition $u'(q) = p_i$ defines the *Walrasian demand* $q(p_i)$.
- **Key term.** $v(p_i)$ is the consumer's surplus from the good, before any fixed fee.

Roy's Identity (We Use It Twice)

- Differentiating $v(p_i) = \max_q\{u(q) - p_i q\}$ by the envelope theorem,
$$v'(p_i) = -q(p_i).$$
- **Intuitively,** a one-dollar rise in the price costs the consumer one dollar on each of the $q(p_i)$ units she already buys.

The Two "Deals" on Offer

- Net surplus from firm i's tariff is the good's surplus minus the fee:

$$w(p_i, m_i) = v(p_i) - m_i.$$

- Write $w_1 \equiv w(p_1, m_1)$ and $w_2 \equiv w(p_2, m_2)$.
- A consumer at x enjoys

$$r - \tau x + w_1 \;\; \text{(firm 1)}, \qquad r - \tau(1 - x) + w_2 \;\; \text{(firm 2)}.$$

- Market is *covered*: the constant r drops out of every comparison.

The Indifferent Consumer and Demands

- The consumer $\hat{x}$ indifferent between the firms solves

$$r - \tau\hat{x} + w_1 = r - \tau(1 - \hat{x}) + w_2.$$

- Solving,

$$\hat{x} = \frac{\tau + w_1 - w_2}{2\tau}.$$

- Demands: firm 1 serves $D_1 = \hat{x}$ and firm 2 serves $D_2 = 1 - \hat{x}$.
- A firm wins more consumers by offering a better deal w_i.

Section 3

Stage 1: The Firms

Firm 1's Problem

- Each of firm 1's $\hat{x}$ customers buys $q(p_1)$ units and pays the fee:
$$\max_{p_1, m_1} \ \pi_1 = \hat{x} \cdot \Big[\left(p_1 - c\right) q\left(p_1\right) + m_1 \Big].$$
- **Change of variables.** Let the firm choose the *deal* w_1 instead of the fee, using $m_1 = v(p_1) - w_1$:
$$\max_{p_1, w_1} \ \pi_1 = \frac{\tau + w_1 - w_2}{2\tau} \Big[\left(p_1 - c\right) q\left(p_1\right) + v\left(p_1\right) - w_1 \Big].$$
- Market share $\hat{x}$ depends on w_1, not on p_1.

First-Order Condition in p_1

- Since $\hat{x}$ is independent of p_1, maximize the bracket:

$$q(p_1) + q'(p_1)(p_1 - c) + v'(p_1) = 0.$$

- Use Roy's identity $v'(p_1) = -q(p_1)$; the outer terms cancel:

$$\underbrace{q'(p_1)}_{<0}(p_1 - c) = 0 \quad \Longrightarrow \quad p_1^* = c.$$

- Same for firm 2: **per-unit price equals marginal cost**.

Why Marginal-Cost Pricing?

- The per-unit price does *not* help win customers (that is the fee's job).
- It only sets how much surplus the firm creates with each customer it already has.
- That surplus is largest at $p = c$: **marginal-cost pricing makes the pie as large as possible.**
- The firm then takes its share of the pie through the fixed fee.

First-Order Condition in w_1

- Differentiating in w_1 (share rises at $\frac{1}{2\tau}$; fee falls one-for-one):

$$\Big[(p_1 - c)\, q(p_1) + v(p_1) - w_1\Big] - \big(\tau + w_1 - w_2\big) = 0.$$

- Insert $p_1 = c$ and symmetry $w_1 = w_2$, then use $v(c) - w_1 = m_1$:

$$v(c) - w_1 - \tau = 0 \quad \Longrightarrow \quad m_1^* = \tau.$$

- The fixed fee equals the differentiation parameter τ.

Equilibrium Tariff and Profit

- Each firm posts the two-part tariff

$$T(q) = \underbrace{\tau}_{\text{fee}} + \underbrace{c}_{\text{price per unit}} q.$$

- Each serves half the market, earns zero margin, collects the fee:

$$\pi_1 = \tfrac{1}{2} \cdot \tau = \frac{\tau}{2}, \qquad \pi_{NL} = \tau \quad \text{(industry)}.$$

- The fee is the reward for differentiation: larger τ, softer competition, higher fee.

Section 4

A Benchmark: Linear Pricing

A Useful Reparametrization

- Now firms may set only a price $(m_i = 0)$, so $w_i = v(p_i)$.
- With the rival at p, write the firm's price as $p_i = c + k(p - c)$, $k \in [0, 1]$, and let

$$s(k) \equiv v\big(c + k(p - c)\big), \quad s(0) = v(c), \quad s(1) = v(p).$$

- By Roy's identity, $s_k(k) = -(p - c)\, q\big(c + k(p - c)\big) < 0$.

Profit in Terms of s_k

- Per-customer profit at $p_i = c + k(p - c)$:

$$\pi_i = k(p - c)\, q\big(c + k(p - c)\big) = -\, k\, s_k(k).$$

- At $k = 1$ (price p), industry profit under linear pricing is

$$\pi_L \equiv \pi(p) = -\, s_k(1).$$

- Firm i chooses k to solve

$$\max_k \; \frac{\tau + s(k) - s(1)}{2\tau} \cdot \big(-\, k\, s_k(k)\big).$$

Equilibrium Condition

- The first-order condition, evaluated at the symmetric $k = 1$, gives

$$-s_k(1) = s_{kk}(1) + \frac{1}{\tau}\left[s_k(1)\right]^2.$$

- Writing $\pi_L = -s_k(1)$,

$$\pi_L = s_{kk}(1) + \frac{1}{\tau}\pi_L^2.$$

Result 1: Profits Are Higher Under NL

- The curvature term is positive:

$$s_{kk}(1) = -(p-c)^2\, q'(\cdot) > 0.$$

- Hence $\pi_L > \dfrac{1}{\tau}\pi_L^2$, i.e. $\tau > \pi_L$. Since $\pi_{NL} = \tau$,

$$\pi_{NL} = \tau > \pi_L.$$

- **Nonlinear pricing softens competition:** the fee recovers more than a single linear price can.

Section 5

Welfare

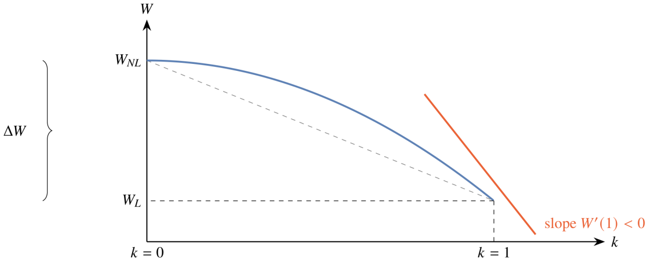
Total Welfare Along the Price Index

- Welfare is producer plus consumer surplus (the fee is a transfer):

$$W(k) = \underbrace{-\,k\,s_k(k)}_{\text{PS}} + \underbrace{s(k)}_{\text{CS}}.$$

- **Under NL** ($k = 0$): $W_{NL} = v(c)$ — the efficient surplus.
- **Under L** ($k = 1$): $W_L = s(1) - s_k(1)$.
- Since $W'(k) = -\,k\,s_{kk}(k) \le 0$, welfare peaks at $k = 0$.

Result 2: Welfare Is Higher Under NL



- W is concave and decreasing in k: $W_{NL} > W_L$.
- Two-part tariffs price the marginal unit at cost, avoiding the linear-price distortion.

Result 3: The Welfare Gain Is Smaller

- Concavity bounds the gap by the slope at $k = 1$:

$$\Delta W \leq -W'(1) = s_{kk}(1) = \frac{\pi_L}{\tau}(\tau - \pi_L).$$

- Since $\frac{\pi_L}{\tau} < 1$,

$$\Delta W \leq s_{kk}(1) < \tau - \pi_L = \pi_{NL} - \pi_L = \Delta\pi.$$

- $\Delta W < \Delta\pi$: firms gain more than society does.

Section 6

Consumer Surplus

Result 4: Consumers Are Worse Off Under NL

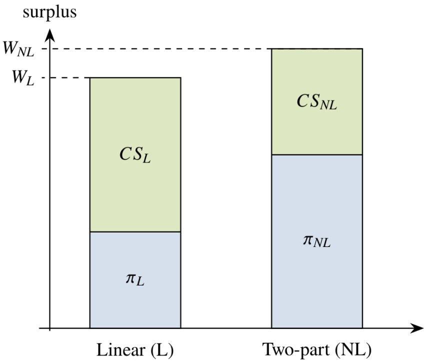
- Since $W = CS + \pi$, i.e. $CS = W - \pi$, rearrange $\Delta W < \Delta\pi$:

$$W_{NL} - \pi_{NL} < W_L - \pi_L \quad \Longrightarrow \quad CS_{NL} < CS_L.$$

- Collecting the three comparisons:

$$W_{NL} > W_L, \qquad \pi_{NL} > \pi_L, \qquad \text{but} \qquad CS_{NL} < CS_L.$$

Decomposition of Welfare



Two-part tariffs enlarge the pie and hand firms a larger slice, so much larger that consumers keep less than under linear pricing.

Section 7

Intuition: Why Consumers Prefer Linear Pricing

Two-Part Tariffs: Cutting the Fee

- The per-unit price is already at cost, so a firm competes only through the fee.
- A one-dollar cut in the fee has **two effects**:
  - (+) *Business stealing:* pulls consumers from the rival, raising $\hat{x}$.
  - (−) *Lost revenue:* one dollar less from every existing customer.

Linear Pricing: Cutting the Price

- Cutting the price has the same two effects **and a third:**
  (+) *Demand expansion:* each of the firm's own customers buys more units, and the firm earns its margin on the extra volume.
- This third effect gives firms a stronger reason to cut prices under L than to cut fees under NL.

Putting It Together

- Competition is **fiercer under linear pricing**: prices fall closer to cost, leaving more surplus with consumers.
- Under two-part tariffs the marginal price is pinned at cost and the fee moves no quantity, so this pro-competitive force is absent.
- The very efficiency of two-part tariffs removes the demand-expansion channel that would have disciplined the firms.

Section 8

A Numerical Example with Linear Demand

Setup: Linear Demand

- Quadratic utility $u(q) = a\,q - \frac{q^2}{2}$ gives

$$q(p) = a - p, \qquad v(p) = \frac{1}{2}(a-p)^2.$$

- Check: $v'(p) = -(a-p) = -q(p)$ (Roy's identity holds).
- **Parameters:** $a = 2$, $c = 1$, $\tau = \frac{9}{32}$.

Two-Part Tariff: The Numbers

- **Prices.** $p^* = c = 1, \;\; m^* = \tau = \dfrac{9}{32}$, so $T(q) = \dfrac{9}{32} + q$.
- Each consumer buys $q(c) = a - c = 1$ unit.
- **Outcomes.**

$$\pi_{NL} = \frac{9}{32}, \qquad W_{NL} = v(c) = \frac{1}{2}, \qquad CS_{NL} = v(c) - \tau = \frac{7}{32}.$$

Linear Pricing: The Numbers

- Symmetric FOC $\tau(a + c - 2p) = (a - p)^2(p - c)$; with $z \equiv p - c$,
$$(4z - 1)\left(8z^2 - 14z + 9\right) = 0.$$
- The quadratic has no real root, so $z = \frac{1}{4}$, i.e. $p_L = \frac{5}{4}$, and $q_L = \frac{3}{4}$.
- **Outcomes.**
$$\pi_L = \frac{6}{32}, \qquad CS_L = \frac{9}{32}, \qquad W_L = \frac{15}{32}.$$

Side by Side

	Two-part (NL)	Linear (L)
Per-unit price p	1	$\frac{5}{4}$
Units per consumer q	1	$\frac{3}{4}$
Industry profit π	$\frac{9}{32}$	$\frac{6}{32}$
Consumer surplus CS	$\frac{7}{32}$	$\frac{9}{32}$
Total welfare W	$\frac{16}{32}$	$\frac{15}{32}$

All figures over the common denominator 32: the numerators compare directly.

Reading the Numbers

- The per-unit price falls from $\frac{5}{4}$ to cost 1, so each purchase rises from $\frac{3}{4}$ to a full unit: total surplus grows by $\frac{1}{32}$.
- The fee lets firms collect $\frac{3}{32}$ more — that is $\frac{2}{32}$ beyond the whole welfare gain.
- That extra $\frac{2}{32}$ is exactly the fall in consumer surplus $\left(\frac{9}{32} \to \frac{7}{32}\right)$.

Key Takeaways

- Competition drives the **per-unit price to marginal cost**; firms earn through the **fixed fee** $m^* = \tau$, so $T(q) = \tau + cq$.
- Relative to linear pricing:

$$W_{NL} > W_L, \qquad \pi_{NL} > \pi_L, \qquad \text{but} \qquad CS_{NL} < CS_L.$$

- Two-part tariffs are **efficient** and **profitable**, yet leave consumers **worse off**.
- The reason: efficient marginal pricing removes the demand-expansion force that keeps linear competition sharp.

Reference

- **Primary source.** Armstrong, Mark, and John Vickers (2001). "Competitive Price Discrimination." *RAND Journal of Economics* 32(4): 579–605.
  - https://www.jstor.org/stable/2696383
- **Textbook treatment.** Belleflamme, Paul, and Martin Peitz (2015). *Industrial Organization: Markets and Strategies*, 2nd ed. Cambridge University Press (§9.3.2).