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Two-Part Tariffs with Two Firms

Competition in nonlinear pricing on a Hotelling line

Based on Armstrong and Vickers (2001); textbook treatment in Belleflamme and Peitz (2015, §9.3.2). Full references at the end.

1. The Model

We study two firms that compete in *two-part tariffs*. A two-part tariff asks a consumer who buys q units of the good to pay a fixed fee m_i regardless of the quantity, plus a constant per-unit price p_i for each unit consumed, so that the total payment to firm i is

$$T_i(q) = m_i + p_i\, q.$$

This is a simple form of nonlinear pricing: the average price paid per unit, $\dfrac{T_i(q)}{q} = p_i + \dfrac{m_i}{q}$, falls as the consumer buys more. Our question is what such a tariff looks like when firms compete, and how the outcome compares with ordinary (linear) pricing in which firms may set only the per-unit price.

The two firms sell horizontally differentiated products and sit at the two ends of a Hotelling line of unit length. Firm 1 is located at 0 and firm 2 at 1. A unit mass of consumers is spread uniformly along the line; a consumer at location $x \in [0, 1]$ incurs a "transportation" (mismatch) cost τ per unit of distance to the firm she buys from, where $\tau > 0$ measures the degree of product differentiation. Both firms produce at the same constant marginal cost c. Figure 1 depicts the market.

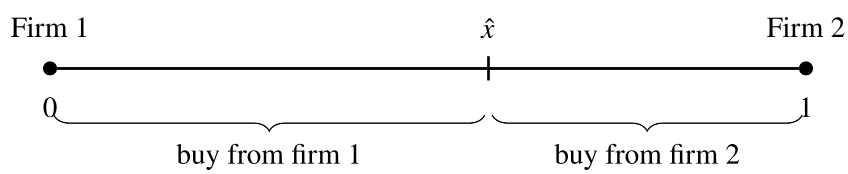

Figure 1: The Hotelling line. The consumer at $\hat{x}$ is indifferent between the two firms; consumers to her left buy from firm 1 and those to her right from firm 2.

The game has two stages, which we solve by backward induction:

Stage 1. Firms simultaneously choose their two-part tariffs (p_i, m_i), $i = 1, 2$.

Stage 2. Each consumer observes both tariffs, chooses the firm to patronize, and then decides how many units to buy.

We start with Stage 2 (the consumers' problem) and then move back to Stage 1 (the firms' problem).

2. Stage 2: The Consumers

Consider a consumer who has decided to buy from a firm charging per-unit price p_i. She chooses her quantity q to maximize the utility she gets from consuming the good, $u(q)$, net of the per-unit payment $p_i q$. Her *indirect utility* is the value of this problem,

$$v(p_i) \equiv \max_q \{\, u(q) - p_i\, q \,\}, \tag{1}$$

where $u(q)$ is increasing and concave. The first-order condition of (1) is

$$u'(q) = p_i,$$

which defines the consumer's *Walrasian demand* $q(p_i)$: she keeps buying units as long as the marginal utility of the good exceeds its per-unit price, and stops where the two are equal.

Roy's identity. A fact we use repeatedly is that the derivative of the indirect utility with respect to the per-unit price equals minus the quantity demanded,

$$v'(p_i) = -q(p_i). \tag{2}$$

This follows from the envelope theorem applied to (1): because q is chosen optimally, a marginal change in p_i affects v only through the direct term $-p_i q$, whose derivative is $-q(p_i)$. Intuitively, a one-dollar rise in the per-unit price costs the consumer exactly one dollar on each of the $q(p_i)$ units she was already buying.

Bringing back the fixed fee m_i, the net surplus a consumer obtains from firm i's tariff is her indirect utility from the variable good minus the fixed fee,

$$w(p_i, m_i) = \underbrace{v(p_i)}_{\text{surplus from } q(p_i) \text{ units}} - \underbrace{m_i}_{\text{fixed fee}}.$$

To keep the notation compact, write $w_1 \equiv w(p_1, m_1)$ and $w_2 \equiv w(p_2, m_2)$ for the two "deals" on offer.

A consumer located at x therefore enjoys total utility

$$\begin{aligned} &r - \tau x + w_1 && \text{if she buys from firm 1,} \\ &r - \tau(1-x) + w_2 && \text{if she buys from firm 2,} \end{aligned}$$

where r is the (large) gross value of consuming the product category at all. We assume the market is *covered*, so every consumer buys from one of the two firms; the constant r is then common to both options and drops out of every comparison below.

The indifferent consumer. The consumer $\hat{x}$ who is just indifferent between the two firms solves

$$r - \tau\hat{x} + w_1 = r - \tau(1-\hat{x}) + w_2.$$

Cancelling r and solving for $\hat{x}$,

$$\begin{aligned} w_1 - w_2 + \tau &= 2\tau\,\hat{x} \\ \hat{x} &= \frac{\tau + w_1 - w_2}{2\tau}. \end{aligned} \tag{3}$$

Everyone to the left of $\hat{x}$ prefers firm 1 and everyone to the right prefers firm 2. Because consumers are uniformly spread on a line of unit mass, firm 1's demand is exactly $\hat{x}$ and firm 2's demand is $1 - \hat{x}$:

$$D_1 = \hat{x} = \frac{\tau + w_1 - w_2}{2\tau}, \qquad D_2 = 1 - \hat{x} = \frac{\tau + w_2 - w_1}{2\tau}.$$

A firm attracts more consumers by offering a better deal (a larger w_i), whether it improves that deal through a lower per-unit price or a lower fixed fee.

3. Stage 1: The Firms

Take firm 1. Each of its $\hat{x}$ customers buys $q(p_1)$ units, on which the firm earns the per-unit margin $p_1 - c$, and each pays the fixed fee m_1. Firm 1 therefore solves

$$\max_{p_1, m_1} \quad \pi_1 \;=\; \underbrace{\hat{x}}_{\text{mass of buyers}} \cdot \Big[\underbrace{(p_1 - c)\, q(p_1)}_{\text{variable margin}} \;+\; \underbrace{m_1}_{\text{fixed fee}} \Big]. \tag{4}$$

It is cleaner to let the firm choose the *deal* w_1 it offers rather than the fee m_1 directly. Since $w_1 = v(p_1) - m_1$, we have $m_1 = v(p_1) - w_1$; substituting this and (3) into (4) turns the problem into a choice of the per-unit price p_1 and the deal w_1,

$$\max_{p_1, w_1} \quad \pi_1 \;=\; \underbrace{\frac{\tau + w_1 - w_2}{2\tau}}_{\hat{x}} \cdot \Big[(p_1 - c)\, q(p_1) + \underbrace{v(p_1) - w_1}_{m_1} \Big]. \tag{5}$$

This change of variables is what makes the model tractable, because the market share $\hat{x}$ now depends on w_1 but *not* on the per-unit price p_1. The two instruments split their jobs: p_1 shapes the size of the surplus firm 1 creates with each of its customers, while w_1 determines how many customers it wins.

First-order condition in p_1. Because $\hat{x}$ does not depend on p_1, differentiating (5) with respect to p_1 and dividing by $\hat{x} > 0$ leaves only the bracketed term,

$$q(p_1) + q'(p_1)\,(p_1 - c) + v'(p_1) = 0.$$

Now use Roy's identity (2), $v'(p_1) = -q(p_1)$. The first and last terms cancel,

$$\cancel{q(p_1)} + q'(p_1)\,(p_1 - c) \underbrace{-\cancel{q(p_1)}}_{v'(p_1)} = 0 \quad \Longrightarrow \quad \underbrace{q'(p_1)}_{<0 \text{ (law of demand)}} \quad (p_1 - c) = 0.$$

Since $q'(p_1) < 0$, the only way this can hold is $p_1 - c = 0$, that is,

$$p_1^* = c \qquad \text{(and, by the same argument, } p_2^* = c\text{)}. \tag{6}$$

Interpretation. Each firm sets its per-unit price equal to marginal cost, so consumption is efficient. The reason is that the per-unit price does not help firm 1 win customers (that is the job of w_1); it only sets how much surplus the firm generates with each customer it already has. That surplus, variable profit plus the consumer's own gross surplus, is maximized when the good is priced at marginal cost, and the firm then collects its share of it through the fixed fee. Marginal-cost pricing makes the pie as large as possible; the fee decides how it is split.

First-order condition in w_1. Differentiating (5) with respect to w_1 gives two terms, one from the market share $\hat{x}$ (which rises at rate $\frac{1}{2\tau}$) and one from the fee inside the bracket (which falls one-for-one with w_1):

$$\frac{1}{2\tau}\Big[(p_1 - c)\, q(p_1) + v(p_1) - w_1\Big] - \frac{\tau + w_1 - w_2}{2\tau} = 0.$$

Multiplying through by 2τ (and recognizing $2\tau\,\hat{x} = \tau + w_1 - w_2$),

$$\Big[(p_1 - c)\, q(p_1) + v(p_1) - w_1\Big] - \big(\tau + w_1 - w_2\big) = 0. \tag{7}$$

Impose the optimal per-unit price $p_1 = c$ from (6), so the variable margin $(c - c)\, q(p_1) = 0$ vanishes, and invoke symmetry, $w_1 = w_2$, so that $w_1 - w_2 = 0$:

$$\Big[\underbrace{(c - c)\, q(p_1)}_{0} + v(p_1) - w_1 \Big] - \big(\tau + \underbrace{w_1 - w_1}_{0}\big) = 0 \quad \Longrightarrow \quad v(p_1) - w_1 - \tau = 0.$$

Finally recall that $w_1 = v(p_1) - m_1$, i.e. $v(p_1) - w_1 = m_1$, so the last equality reads $m_1 - \tau = 0$, i.e.

$$m_1^* = \tau \qquad \text{(and, by the same argument, } m_2^* = \tau). \tag{8}$$

The equilibrium tariff. Putting (6) and (8) together, each firm charges the two-part tariff

$$T(q) = \underbrace{\tau}_{\text{fixed fee}} + \underbrace{c}_{\text{price per unit}}\, q.$$

Competition drives the per-unit price down to marginal cost, and the firms make their money entirely on the fixed fee, which they can raise up to the value of the differentiation parameter τ. The fee is the reward for being differentiated: the more differentiated the two products (the larger τ), the softer the competition and the higher the fee each firm can sustain.

Equilibrium profits. At the symmetric equilibrium each firm serves half the market, $\hat{x} = \frac{1}{2}$, earns a zero variable margin, and collects the fee $m^* = \tau$ from each customer:

$$\pi_1 = \underbrace{\frac{\tau + w_1 - w_2}{2\tau}}_{= \frac{1}{2}} \Big[\underbrace{(p_1 - c)\, q_1}_{0} + \underbrace{m_1}_{\tau} \Big] = \frac{\tau}{2}. \tag{9}$$

Adding the two firms, industry profit under the two-part (nonlinear) tariff is

$$\pi_{NL} = \tau.$$

We keep this number in mind: it is the benchmark against which linear pricing is measured.

4. A Benchmark: Linear Pricing

Consider instead a setting of ordinary *linear pricing*, in which firms may charge only a per-unit price, with no fixed fee ($m_i = 0$). The consumer's deal is simply $w_i = v(p_i)$, and firm i's profit per customer is the variable margin $\pi_i(p_i) \equiv (p_i - c)\, q(p_i)$. We want to compare the two regimes for a general demand curve $q(\cdot)$, so a small change of variables helps.

Let firm j charge the candidate symmetric price p, and write the rival's price p_i as a point between marginal cost and p,

$$p_i = c + k\,(p - c), \qquad k \in [0, 1].$$

The number k measures how far the price sits above marginal cost: $k = 0$ means $p_i = c$ and $k = 1$ means $p_i = p$. Define the consumer's surplus as a function of this index,

$$s(k) \equiv v\big(c + k(p - c)\big), \qquad \text{so that} \qquad \begin{cases} s(0) = v(c) & \text{“low price” (all surplus to the consumer),} \\ s(1) = v(p) & \text{“high price.”} \end{cases} \tag{10}$$

Differentiating s and using Roy's identity (2),

$$s_k(k) \equiv \frac{\partial s(k)}{\partial k} = (p-c)\, v'\big(c + k(p-c)\big) = -(p-c)\, q\big(c + k(p-c)\big) \;< 0. \tag{11}$$

The consumer is worse off as the price rises, so $s_k < 0$.

The point of this parametrization is that the firm's per-customer profit can be written in terms of s_k. At price $p_i = c + k(p-c)$,

$$\pi_i\big(c + k(p-c)\big) = \underbrace{\big[c + k(p-c) - c\big]}_{=k(p-c)}\, q\big(c + k(p-c)\big) = k(p-c)\, q\big(c + k(p-c)\big) = -\,k\, s_k(k), \tag{12}$$

where the last equality just reads off (11). In particular, at $k = 1$ the per-customer (equivalently, unit-mass industry) profit under price p is

$$\pi_L \equiv \pi(p) = -\, s_k(1). \tag{13}$$

The firm's problem. Facing the rival's price p, firm i chooses its own price, i.e. chooses k, to solve

$$\max_k \quad \pi_i = \frac{\tau + s(k) - s(1)}{2\tau} \cdot \big(-\; k\, s_k(k)\big), \tag{14}$$

where we used $v(p_i) = s(k)$ and $v(p) = s(1)$ in the market share (3), and the per-customer profit (12). At a symmetric equilibrium the rival's price is a best response to itself, so $p_i = p$, i.e. $k = 1$.

First-order condition in k. Differentiating the product in (14) and setting it to zero,

$$-\,k\,\big(s_k(k)\big)^2 - \big[\tau + s(k) - s(1)\big]\big[s_k(k) + k\, s_{kk}(k)\big] = 0. \tag{15}$$

Evaluate this at the symmetric equilibrium $k = 1$, where $s(1) - s(1) = 0$ and the first bracket collapses to τ,

$$\begin{aligned} &-\big[s_k(1)\big]^2 - \tau\big[s_k(1) + s_{kk}(1)\big] = 0 \\ &\qquad\qquad \Longrightarrow \quad -\, s_k(1) = s_{kk}(1) + \frac{1}{\tau}\big[s_k(1)\big]^2. \end{aligned} \tag{16}$$

Using $\pi_L = -s_k(1)$ from (13), and hence $[s_k(1)]^2 = \pi_L^2$, equation (16) says that industry profit under linear pricing satisfies

$$\pi_L = s_{kk}(1) + \frac{1}{\tau}\, \pi_L^2. \tag{17}$$

The sign of s_{kk}. Differentiate (11) once more,

$$s_{kk}(k) = -(p-c)\, q'\big(c + k(p-c)\big) \cdot (p-c) = -\,\underbrace{(p-c)^2}_{>0}\, \underbrace{q'\big(c + k(p-c)\big)}_{<0 \text{ (law of demand)}} \;> 0. \tag{18}$$

So $s_{kk}(1) > 0$: the consumer's surplus falls at an increasing rate as the price climbs. Feeding this into (17),

$$\pi_L = \underbrace{s_{kk}(1)}_{>0} + \frac{1}{\tau}\, \pi_L^2 \;>\; \frac{1}{\tau}\, \pi_L^2 \quad \Longrightarrow \quad \pi_L > \frac{1}{\tau}\, \pi_L^2 \quad \Longrightarrow \quad \tau > \pi_L,$$

where the last step divides by $\pi_L > 0$. Since industry profit under the two-part tariff is $\pi_{NL} = \tau$, we have established the first comparison,

$$\pi_{NL} = \tau \;>\; \pi_L. \tag{19}$$

Result 1. Industry profits are higher under two-part tariffs than under linear pricing. Even though competition forces the per-unit price down to marginal cost, the fixed fee lets firms recover more than they could through a single linear price. Nonlinear pricing softens competition.

5. Welfare

Turn to total welfare, the sum of producer surplus and consumer surplus. At price $p_i = c + k(p - c)$, welfare per consumer is the firm's variable profit plus the consumer's own surplus (the fixed fee is a pure transfer and cancels),

$$W(k) = \underbrace{\pi\big(c + k(p - c)\big)}_{= -k\, s_k(k) \quad \text{(PS)}} + \underbrace{v\big(c + k(p - c)\big)}_{= s(k) \quad \text{(CS)}} = -\,k\, s_k(k) + s(k). \tag{20}$$

Evaluate W in the two regimes.

- **Under the two-part tariff (NL):** the per-unit price is c, so $k = 0$, and

$$W(0) = \underbrace{-\,0 \cdot s_k(0)}_{0} + s(0) = s(0) = v(c) \equiv W_{NL}.$$

  Welfare is maximized here, since pricing at marginal cost yields the efficient quantity.

- **Under linear pricing (L):** the price is p, so $k = 1$, and

$$W(1) = -\,1 \cdot s_k(1) + s(1) = s(1) - s_k(1) \equiv W_L.$$

To rank the two, note from (20) that

$$W'(k) = -\,s_k(k) - k\, s_{kk}(k) + s_k(k) = -\,k\, s_{kk}(k),$$

so $W'(0) = 0$ and $W'(k) < 0$ for $k > 0$ (because $s_{kk} > 0$): welfare peaks at $k = 0$ and falls thereafter. Assuming, as is standard, that welfare is concave in the price (hence in k), the welfare curve looks like Figure 2. In particular $W_{NL} = W(0) > W(1) = W_L$.

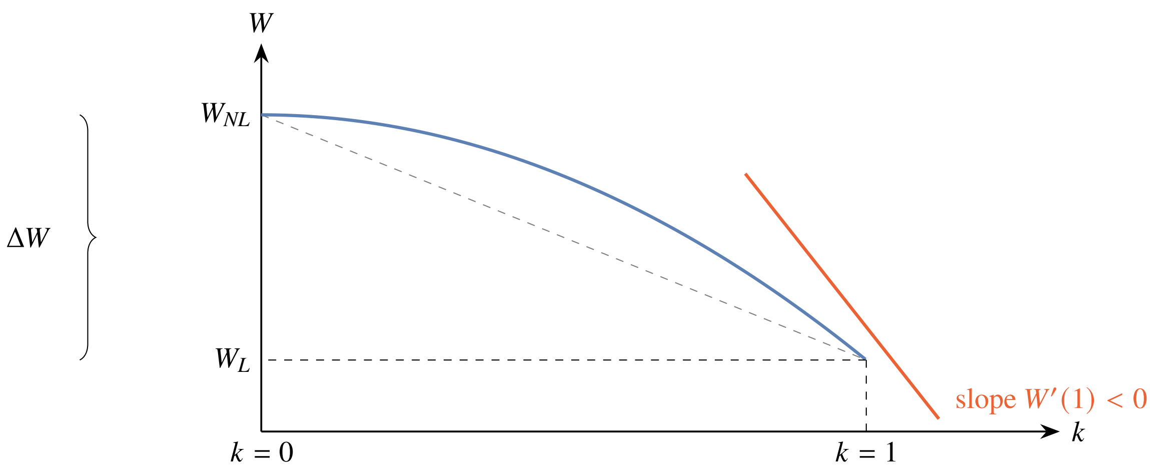

Figure 2: Welfare $W(k)$ is concave and decreasing in k. It is maximized at $k = 0$ (the two-part tariff, marginal-cost pricing) and falls to its linear-pricing value at $k = 1$. The gap is $\Delta W = W_{NL} - W_L$.

Result 2. Welfare is higher under two-part tariffs than under linear pricing, $W_{NL} > W_L$, because two-part tariffs price the marginal unit at cost and so avoid the quantity distortion that a linear price above cost creates.

How large is the welfare gap? Concavity lets us bound $\Delta W \equiv W_{NL} - W_L$. For a concave function the tangent at $k = 1$ lies above the curve, so evaluating that tangent back at $k = 0$ gives $W(0) \le W(1) - W'(1)$, i.e.

$$\Delta W = W(0) - W(1) \;\le\; -W'(1) = s_{kk}(1), \tag{21}$$

using $W'(1) = -s_{kk}(1)$. Now rewrite $s_{kk}(1)$ with the equilibrium condition (17), $s_{kk}(1) = \pi_L - \frac{1}{\tau}\pi_L^2$,

$$s_{kk}(1) = \pi_L - \frac{1}{\tau}\pi_L^2 = \frac{\pi_L}{\tau}\big(\tau - \pi_L\big). \tag{22}$$

Because $\pi_L < \tau$ from (19), the factor $\frac{\pi_L}{\tau} < 1$, while $\tau - \pi_L > 0$. Hence

$$s_{kk}(1) = \underbrace{\frac{\pi_L}{\tau}}_{<1}\underbrace{\big(\tau - \pi_L\big)}_{>0} < \tau - \pi_L.$$

Chaining this with the bound (21) and recalling $\tau - \pi_L = \pi_{NL} - \pi_L$,

$$\Delta W \;\le\; s_{kk}(1) \;<\; \tau - \pi_L \;=\; \pi_{NL} - \pi_L \;\equiv\; \Delta\pi. \tag{23}$$

Result 3. The welfare gain from two-part tariffs is smaller than the profit gain, $\Delta W < \Delta\pi$. Firms gain more, when moving from linear to two-part pricing, than society as a whole. That extra gain has to come from somewhere, and the next section shows it comes out of consumers' pockets.

6. Consumer Surplus and the Overall Comparison

Total welfare is consumer surplus plus industry profit, $W = CS + \pi$, so $CS = W - \pi$. Rearranging the welfare-versus-profit comparison (23),

$$\begin{aligned} & W_{NL} - W_L < \pi_{NL} - \pi_L \\ \Longleftrightarrow \quad & \underbrace{W_{NL} - \pi_{NL}}_{CS_{NL}} < \underbrace{W_L - \pi_L}_{CS_L} \\ \Longleftrightarrow \quad & CS_{NL} < CS_L. \end{aligned}$$

Result 4. Consumers are worse off under two-part tariffs than under linear pricing, $CS_{NL} < CS_L$, even though total welfare is higher. Collecting the three comparisons,

$$W_{NL} > W_L, \qquad \pi_{NL} > \pi_L, \qquad \text{but} \qquad CS_{NL} < CS_L.$$

Two-part tariffs enlarge the pie *and* hand firms a larger slice, so much larger that consumers are left with less than they had under linear pricing. Figure 3 shows the decomposition.

7. Intuition: Why Consumers Prefer Linear Pricing

The comparisons above are driven by how hard firms compete under each pricing regime, which in turn depends on what a firm gains by giving consumers a better deal.

Under **two-part tariffs**, the per-unit price is already at marginal cost, so the only way firm i improves its deal is by cutting the fixed fee. A one-dollar cut in the fee has two effects:

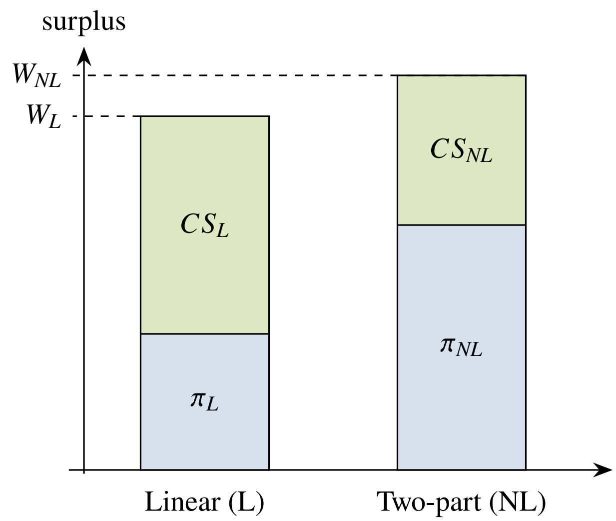

Figure 3: Decomposition of welfare into industry profit and consumer surplus. Moving from linear (L) to two-part (NL) pricing raises total welfare ($W_{NL} > W_L$) and profit ($\pi_{NL} > \pi_L$), but lowers consumer surplus ($CS_{NL} < CS_L$).

(+) *Business stealing.* A better deal pulls marginal consumers away from the rival, raising the firm's market share $\hat{x}$.

(−) *Lost revenue.* The firm collects one dollar less from every customer it already had.

Under **linear pricing**, the firm improves its deal by cutting the per-unit price, which has the same two effects *and* a third one:

(+) *Demand expansion.* A lower price makes each of the firm's own customers buy more units, and the firm earns its margin on the extra volume.

This third effect gives firms a stronger reason to cut prices under linear pricing than to cut fees under two-part tariffs. Competition is therefore fiercer under linear pricing: prices are driven closer to cost, and more surplus is left in consumers' hands. Under two-part tariffs the marginal price is pinned at cost and the fixed fee moves no quantity, so this extra pro-competitive force is absent, firms keep fees high, and consumers end up with less, exactly the ranking $CS_{NL} < CS_L$ found above. The very efficiency of two-part tariffs, pricing each unit at cost, is what removes the demand-expansion channel that would otherwise have disciplined the firms.

8. A Numerical Example with Linear Demand

To put numbers on the four results, take the quadratic gross utility $u(q) = a\,q - \frac{q^2}{2}$, whose marginal utility is $u'(q) = a - q$. Setting $u'(q) = p$ gives the *linear demand*

$$q(p) = a - p \qquad (\text{for } p \leq a),$$

and, evaluating $u(q) - p\,q$ at $q = a - p$, the indirect utility

$$v(p) = \max_q \left\{ a\,q - \frac{q^2}{2} - p\,q \right\} = \frac{1}{2}\,(a - p)^2.$$

As a check, $v'(p) = -(a-p) = -q(p)$, so Roy's identity (2) holds. Take $a = 2$, $c = 1$, and $\tau = \dfrac{9}{32}$.

Two-part tariff (NL). From Section 3 the per-unit price is $p^* = c = 1$ and the fee is $m^* = \tau = \dfrac{9}{32}$, so each firm posts $T(q) = \dfrac{9}{32} + q$. Every consumer buys $q(c) = a - c = 1$ unit. Industry profit is the fee collected from the unit mass of consumers, total welfare is the efficient surplus, and consumer surplus is that surplus net of the fee:

$$\pi_{NL} = \tau = \frac{9}{32}, \qquad W_{NL} = v(c) = \frac{1}{2}(a-c)^2 = \frac{1}{2}, \qquad CS_{NL} = v(c) - \tau = \frac{1}{2} - \frac{9}{32} = \frac{7}{32}.$$

Linear pricing (L). With no fee, the symmetric first-order condition from Section 4 is $\tau(a + c - 2p) = (a-p)^2(p-c)$. Writing the markup as $z \equiv p - c$ and inserting $a = 2$, $c = 1$, $\tau = \dfrac{9}{32}$,

$$\frac{9}{32}(1-2z) = (1-z)^2 z \quad \Longleftrightarrow \quad 32z^3 - 64z^2 + 50z - 9 = 0 \quad \Longleftrightarrow \quad (4z-1)\big(8z^2 - 14z + 9\big) = 0.$$

The quadratic factor has no real root (its discriminant is $14^2 - 4 \cdot 8 \cdot 9 = -92 < 0$), so the unique symmetric equilibrium markup is $z = \dfrac{1}{4}$, i.e. $p_L = c + \dfrac{1}{4} = \dfrac{5}{4}$. Each consumer then buys $q_L = a - p_L = \dfrac{3}{4}$, and

$$\pi_L = (p_L - c)\,q_L = \frac{1}{4} \cdot \frac{3}{4} = \frac{6}{32}, \qquad CS_L = v(p_L) = \frac{1}{2}\left(\frac{3}{4}\right)^2 = \frac{9}{32}, \qquad W_L = \pi_L + CS_L = \frac{15}{32}.$$

Comparison. Table 1 collects the two regimes.

	Two-part tariff (NL)	Linear pricing (L)
Per-unit price p	1	$\frac{5}{4}$
Units per consumer q	1	$\frac{3}{4}$
Industry profit π	$\frac{9}{32} \approx 0.281$	$\frac{6}{32} \approx 0.188$
Consumer surplus CS	$\frac{7}{32} \approx 0.219$	$\frac{9}{32} \approx 0.281$
Total welfare W	$\frac{16}{32} = 0.500$	$\frac{15}{32} \approx 0.469$

Table 1: Equilibrium outcomes under two-part and linear pricing, for $a = 2$, $c = 1$, and $\tau = \frac{9}{32}$. The profit, consumer-surplus, and welfare rows are written over the common denominator 32 so the numerators can be compared directly.

All four results hold:

$$\pi_{NL} = \frac{9}{32} > \frac{6}{32} = \pi_L, \qquad W_{NL} = \frac{16}{32} > \frac{15}{32} = W_L, \qquad CS_{NL} = \frac{7}{32} < \frac{9}{32} = CS_L,$$

and the welfare gain $\Delta W = \dfrac{1}{32}$ is smaller than the profit gain $\Delta\pi = \dfrac{3}{32}$.

Moving to the two-part tariff pushes the per-unit price down from $\dfrac{5}{4}$ to marginal cost 1, so each consumer's purchase rises from $\dfrac{3}{4}$ to a full unit: consumption becomes efficient and total surplus grows by $\dfrac{1}{32}$. The fixed fee, however, lets firms collect $\dfrac{3}{32}$ more than under linear pricing, which is $\dfrac{2}{32}$ more than the entire welfare gain. That extra $\dfrac{2}{32}$ is exactly the fall in consumer surplus $(\dfrac{9}{32} - \dfrac{7}{32})$. Consumption becomes efficient, industry profit rises, and consumers are left worse off.

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