

Part VI. Theory of competition policy

Chapter 14. Cartels and tacit collusion



Slides

Industrial Organization: Markets and Strategies

Paul Belleflamme and Martin Peitz, 2d Edition

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So far

- Followed mostly a *positive* approach
- Describe, explain workings of imperfectly competitive markets

In this part

- Follow mostly a *normative* approach
- Guidance for competition policy
 - **Basic postulate**: competition is desirable as it promotes economic efficiency
 - **Problem**: firms might be tempted to reduce competition
 - **Consequence**: set of rules aiming at maintaining competition → **competition (antitrust) policy**

Organization of Part VI

- Chapter 14. Collusive practices
 - Price-fixing, market-sharing → firms eliminate competition between them
 - How does collusion emerge? How is it sustained?
- Chapter 15. Horizontal mergers
 - Examine potential effects on welfare
 - Negative: fewer independent decision makers
 - Positive: increased efficiency (synergies)
- Chapter 16. Strategic incumbent firms
 - Incumbent may try to make entry more difficult on their market.
 - How? To what effect

Organization of Part VI (cont'd)

- Chapter 17. Vertically related markets
 - Market for intermediate products; successive steps in the value chain
 - Effects of vertical mergers?
 - Positive: elimination of successive margins
 - Negative: potential foreclosure
 - Effects of vertical restraints
 - Resale-price maintenance, exclusive dealing, ...

Chapter 14. Learning objectives

- Identify the incentives for a firm to collude.
 - Cartel formation
- Understand how cartels and other forms of collusion can be sustained.
 - Sustainability of tacit collusion
 - Repeated competition
- Understand how authorities can fight against collusion.
 - Detecting and fighting collusion

Case. The vitamin cartels



- Worldwide market for bulk vitamins
- In Europe, sales of bulk vitamins were 800m € in 1998
- Production of vitamins is highly concentrated
 - Largest firm is Hoffmann-La Roche: market share of 40-50%
 - BASF: 20-30%
 - Aventis: 5-15%
- Concentration on the production side
 - Slow and costly plant construction
 - Economies of scale in the production technology
- Buyer side is more fragmented.
- November 2001: European Commission imposes a fine of 855.22 m € to 8 companies for participating in secret market-sharing and price-fixing cartels.

Formation and stability of cartels

- Simple market structure
 - n symmetric firms produce a homogeneous good
 - Constant marginal cost c
 - Competition à la Cournot
 - Firms face an inverse demand given by $P(q) = a - q$, where q is the total quantity produced
- 3 alternative procedures
 - Firms decide **simultaneously** whether or not to participate in a single industry-wide cartel.
 - Endogenous formation of cartels in a **sequential** way
 - Bilateral **market-sharing agreements** (“I stay out of your market if you stay out of mine”)

Simultaneous cartel formation

- **Setting.** A cartel of k firms forms, with $1 < k \leq n$. The cartel acts as a single firm and splits its profit equally among its members.
- The Cournot game is then played among $n - k + 1$ symmetric players: the cartel and the $n - k$ independent firms (outsiders).
- **Question.** Is a cartel of size k stable, that is, does no member want to leave it?
 - If a member leaves, it becomes an outsider and the cartel shrinks to $k - 1$ members.

Simultaneous cartel formation (cont'd)

- **Step 1. Profits for a given cartel size.** With inverse demand $P(q) = a - q$ and N symmetric Cournot players, each player earns

$$\pi_i = \frac{(a - c)^2}{(N + 1)^2}$$

- Here $N = n - k + 1$, so $N + 1 = n - k + 2$. Since the cartel splits its profit among its k members,

$$\pi^{in}(k) = \frac{(a - c)^2}{k(n - k + 2)^2} \quad \text{and} \quad \pi^{out}(k) = \frac{(a - c)^2}{(n - k + 2)^2}$$

- Outsiders free-ride on the cartel: $\pi^{out}(k) > \pi^{in}(k)$ for all $k \geq 2$.

Simultaneous cartel formation (cont'd)

- **Step 2. Stability condition.** A member does not leave if its profit inside the cartel is at least the profit it earns as an outsider of a cartel with $k - 1$ members:

$$\pi^{in}(k) \geq \pi^{out}(k - 1) \quad \Leftrightarrow \quad \frac{(a - c)^2}{k(n - k + 2)^2} \geq \frac{(a - c)^2}{(n - k + 3)^2}$$

- With $k - 1$ members, there are $n - k + 2$ players, so the denominator of $\pi^{out}(k - 1)$ is $(n - k + 3)^2$.
- Cancelling $(a - c)^2$ and cross-multiplying,

$$(n - k + 3)^2 \geq k(n - k + 2)^2$$

Simultaneous cartel formation (cont'd)

- **Step 3. Rewriting the condition.** Let $m \equiv n - k \geq 0$ be the number of outsiders. The condition becomes

$$(m + 3)^2 \geq k(m + 2)^2 \quad \Leftrightarrow \quad (1 - k)m^2 + (6 - 4k)m + (9 - 4k) \geq 0$$

- If $k \geq 3$, the three coefficients, $1 - k$, $6 - 4k$, and $9 - 4k$, are negative, so the condition never holds.
- If $k = 2$, the condition becomes $-m^2 - 2m + 1 \geq 0$, which only holds at $m = 0$, that is, when $n = 2$.

Simultaneous cartel formation (cont'd)

- **Step 4. Conclusion.**

- With $n \geq 3$ firms, every member of a cartel prefers to leave it and free-ride as an outsider: no cartel is stable.
- With $n = 2$ firms, $\pi^{in}(2) = \frac{(a-c)^2}{8} > \pi^{out}(1) = \frac{(a-c)^2}{9}$, so both firms form a cartel.

- **Lesson:** Consider the formation of a single cartel on a Cournot market with homogeneous goods and constant marginal costs. If there are at least three firms in the industry, all firms remain independent. If there are just two firms in the industry, the two firms form a cartel.

- **Intuition** for this result:
 - Formation of the cartel induces positive externalities on the firms outside the cartel (higher market price).
 - All firms prefer to free-ride on the public good provided by cartel members.
- Result changes if firms produce **horizontally differentiated goods**
 - Competition and free-riding incentive are relaxed.
 - → It is possible to find stable cartels comprising not all firms but a strict subset of them (if goods are sufficiently differentiated).
 - **See next example**

Example: Stable partial cartels

- Recall:
 - Cournot market with homogeneous good, constant marginal cost
 - Simultaneous formation of a single cartel
 - All firms remain independent if at least 3 firms.
- However, if firms produce **horizontally differentiated goods** competition and free-riding incentive are relaxed.
- Consider the following inverse demand functions

$$p_i = a - q_i - \gamma \sum_{j \neq i} q_j, \text{ where } \gamma \in [0, 1]$$

measures the strength of product substitutability

Example: Stable partial cartels (cont'd)

- **Parameters.** $n = 3$ firms, $\gamma = \frac{1}{2}$, and $a - c = 1$. Is it stable for firms 1 and 2 to form a cartel while firm 3 remains independent?
- **Step 1. Cartel.** Firms 1 and 2 choose q_1 and q_2 to maximize their joint profits:

$$\Pi_{12} = \left(1 - q_1 - \frac{1}{2}(q_2 + q_3)\right)q_1 + \left(1 - q_2 - \frac{1}{2}(q_1 + q_3)\right)q_2$$

- Differentiating with respect to q_1 and imposing symmetry ($q_1 = q_2 = q_{12}$):

$$1 - 3q_{12} - \frac{1}{2}q_3 = 0 \quad \Leftrightarrow \quad q_{12}(q_3) = \frac{1}{6}(2 - q_3)$$

Example: Stable partial cartels (cont'd)

- **Step 2. Independent firm.** Firm 3 chooses q_3 to maximize

$$\pi_3 = \left(1 - q_3 - \frac{1}{2}(q_{12} + q_{12})\right) q_3 = (1 - q_3 - q_{12})q_3$$

- Its first-order condition, $1 - 2q_3 - q_{12} = 0$, yields the best response

$$q_3(q_{12}) = \frac{1}{2}(1 - q_{12})$$

- **Step 3. Equilibrium.** Solving the two best responses simultaneously,

$$q_{12} = \frac{3}{11} \quad \text{and} \quad q_3 = \frac{4}{11}$$

Example: Stable partial cartels (cont'd)

- **Step 4. Profits.** Price margins of a cartel member and of firm 3:

$$1 - \frac{3}{11} - \frac{1}{2} \left(\frac{3}{11} + \frac{4}{11} \right) = \frac{9}{22} \quad \text{and} \quad 1 - \frac{4}{11} - \frac{3}{11} = \frac{4}{11}$$

- Hence, profits are

$$\pi^{in}(2) = \frac{9}{22} \times \frac{3}{11} = \frac{27}{242} \quad \text{and} \quad \pi^{out}(2) = \frac{4}{11} \times \frac{4}{11} = \frac{32}{242}$$

- The free-riding effect is still present: the outsider earns more than each cartel member, $\pi^{out}(2) > \pi^{in}(2)$.

Example: Stable partial cartels (cont'd)

- Stability requires two conditions:
 - Internal stability:** no cartel member wants to leave.
 - External stability:** the outsider does not want to join.
- Step 5. Internal stability.** If firm 1 leaves, the three firms are independent. Each solves $\max_{q_i} \left(1 - q_i - \frac{1}{2}(q_j + q_k)\right) q_i$, which yields

$$q_i = \frac{1}{3} \quad \text{and} \quad \pi^{out}(1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
 - Staying in the cartel is better: $\pi^{in}(2) = \frac{27}{242} \approx 0.1116 > \frac{1}{9} \approx 0.1111$.

Example: Stable partial cartels (cont'd)

- **Step 6. External stability.** If firm 3 joins, the three firms maximize joint profits, so that $q_i = \frac{1}{4}$ and

$$\pi^{in}(3) = \left(1 - \frac{1}{4} - \frac{1}{2} \times \frac{2}{4}\right) \frac{1}{4} = \frac{1}{8} = 0.125$$

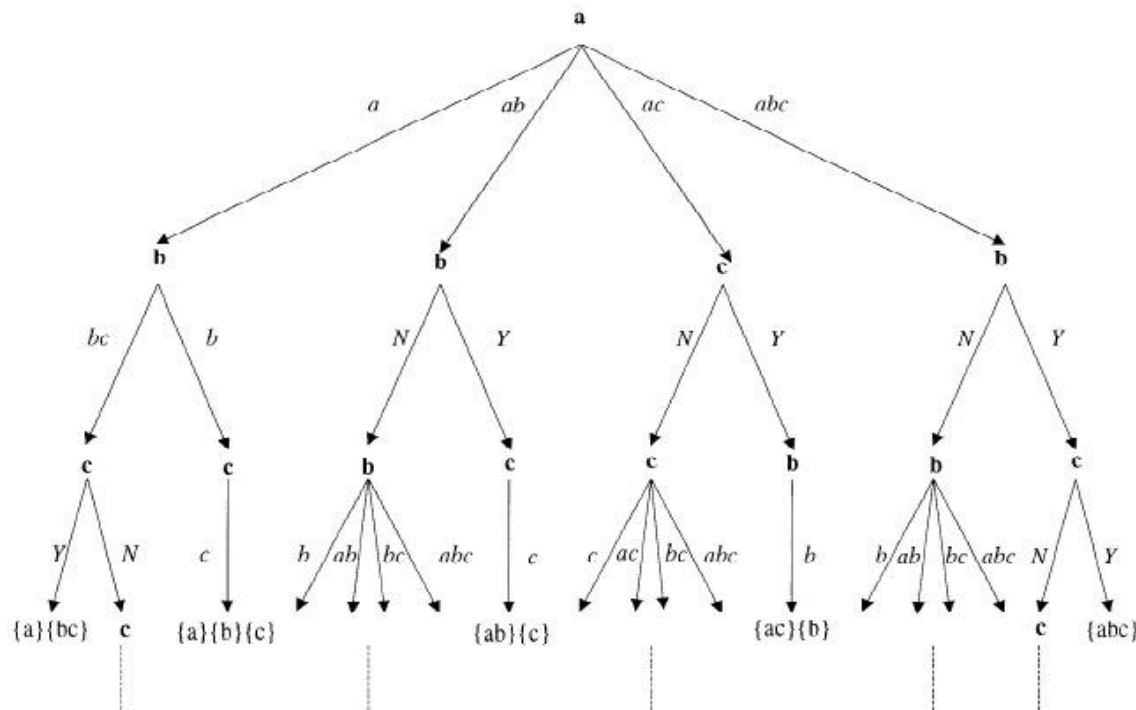
- Staying out is better: $\pi^{out}(2) = \frac{32}{242} \approx 0.132 > 0.125$.

Example: Stable partial cartels (cont'd)

- **Conclusion.** The cartel formed by firms 1 and 2 is stable, even though the outsider earns more than its members.
- With homogeneous goods ($\gamma = 1$) and $n = 3$, no cartel was stable. With differentiated goods ($\gamma = \frac{1}{2}$), a partial cartel is stable.
- Intuitively, product differentiation softens competition between the cartel and the outsider, so the gain from leaving the cartel and free-riding is smaller.

Sequential cartel formation

- Game where multiple cartels can be formed
 - Exogenous specification of the ordering of the firms; 1st firm proposes a cartel; if all prospective members accept, cartel is formed; otherwise, 1st firm in the ordering that refuses proposes another cartel; etc.



Source: Bloch, 1995

Sequential cartel formation (cont'd)

- Sequentiality entails important differences:
 - Firms can commit to stay out of the cartel.
 - In equilibrium, the first firms remain independent and free-ride on the cartel that the last firms form.
- **Step 1. Participation condition.** The last k firms prefer forming a cartel of size k to all firms remaining independent (Cournot with n firms) if

$$\pi^{in}(k) \geq \pi^{out}(1) \quad \Leftrightarrow \quad \frac{(a - c)^2}{k(n - k + 2)^2} \geq \frac{(a - c)^2}{(n + 1)^2}$$

Sequential cartel formation (cont'd)

- **Step 2. Solving for k .** Cross-multiplying and factoring,

$$(n + 1)^2 - k(n - k + 2)^2 \geq 0$$

$$\Leftrightarrow (k - 1)[-k^2 + (2n + 3)k - (n + 1)^2] \geq 0$$

- Since $k > 1$, the bracket must be nonnegative. Its roots are $\frac{1}{2}(2n + 3 \pm \sqrt{4n + 5})$, and the larger root exceeds n . Hence, the condition holds if and only if

$$k \geq k^* \equiv \frac{2n + 3 - \sqrt{4n + 5}}{2}$$

Sequential cartel formation (cont'd)

- **Step 3. Size of the cartel.** The cartel includes the last $\hat{k}$ firms, where $\hat{k}$ is the smallest integer satisfying $k \geq k^*$.
- The cartel always includes at least 80% of the firms: $\frac{k^*}{n} \geq 0.8$ is equivalent to $0.16(n - 5)^2 \geq 0$, which always holds.

n	k^*	k^*/n	Cartel size
2	1.70	0.85	2
3	2.44	0.81	3
4	3.21	0.80	4
5	4.00	0.80	4
6	4.81	0.80	5
10	8.15	0.81	9

Sequential cartel formation (cont'd)

- **Lesson:** Consider a Cournot market with homogeneous goods. The first $(n - k^*)$ firms remain independent while the last k^* firms form a cartel, with k^* being larger than 80% of the firms in the industry.
- **Intuition**
 - **Simultaneous** cartel formation: each firm has an incentive to leave the cartel.
 - **Sequential** cartel formation: first firms can commit to stay out and cartel can then form

Network of market-sharing agreements

- **Bilateral** collusive agreements
- Market-sharing agreements
 - 2 firms are active on different geographical markets or serve distinct consumer segments
 - Refraining from competing on the other firm's territory
 - Constitute a collusive structure, a **collusive network**
- Network stability if
 - No pair of firms has an incentive to form a new link.
 - No firm has an incentive to unilaterally destroy an existing link.
- **Lesson:** If collusive networks are negotiated bilaterally, they may lead to full collusion, with every firm a monopoly on its own market.

Network of market-sharing agreements (cont'd)

- **Setting.** n firms. Firm i is based in market i and can also sell in every other market. Every market features Cournot competition with inverse demand $P(q) = a - q$.
- A link ij is a market-sharing agreement: firm i stays out of market j , and firm j stays out of market i . The set of links forms the network g .
- Let $n_i(g)$ be the number of active firms in market i . Firm i 's profit is

$$\Pi_i(g) = \underbrace{\frac{(a - c)^2}{(n_i(g) + 1)^2}}_{\text{home market}} + \underbrace{\sum_{j: ij \notin g} \frac{(a - c)^2}{(n_j(g) + 1)^2}}_{\text{markets without agreement}}$$

Network of market-sharing agreements (cont'd)

- **Step 1. Gain from a new link.** If firms i and j sign an agreement, firm i faces one fewer rival at home, but it gives up its sales in market j :

$$\Pi_i(g + ij) - \Pi_i(g) = \underbrace{\frac{(a - c)^2}{n_i(g)^2} - \frac{(a - c)^2}{(n_i(g) + 1)^2}}_{\text{gain at home}} - \underbrace{\frac{(a - c)^2}{(n_j(g) + 1)^2}}_{\text{loss in market } j}$$

- Profits in all other markets are unaffected.
- **Step 2. Empty network.** Without agreements, $n_i = n_j = n$. The gain from the first link is

$$\frac{(a - c)^2}{n^2} - \frac{2(a - c)^2}{(n + 1)^2} = \frac{(a - c)^2(-n^2 + 2n + 1)}{n^2(n + 1)^2}$$

- This gain is positive only if $n < 1 + \sqrt{2} \approx 2.41$. With $n \geq 3$ firms, no pair signs the first agreement: the empty network is stable.

Network of market-sharing agreements (cont'd)

- **Step 3. Breaking a link.** Consider two linked firms, i and j , each facing x active firms at home. If firm i breaks the link, firm j enters market i and firm i enters market j . Firm i keeps the link if

$$\frac{(a-c)^2}{(x+1)^2} - \underbrace{\frac{(a-c)^2}{(x+2)^2}}_{\text{home, one more rival}} - \underbrace{\frac{(a-c)^2}{(x+2)^2}}_{\text{sales in market } j} \geq 0$$

- Rearranging, $(x+2)^2 \geq 2(x+1)^2$, that is, $x^2 \leq 2$, which holds at $x = 1$.
- **Step 4. Complete network.** Every pair has signed an agreement, so each firm is a monopolist at home ($x = 1$), and no firm wants to break a link.
 - For example, with $a - c = 1$: keeping the link yields $\frac{1}{4} = 0.25$, while breaking it yields $\frac{1}{9} + \frac{1}{9} \approx 0.22$.

Network of market-sharing agreements (cont'd)

- **Conclusion.** With $n \geq 3$ firms, two networks are stable:
 - the empty network, where firms compete in every market, and
 - the complete network, where every firm is a monopolist in its home market: **full collusion**.
- Bilateral market-sharing agreements can therefore sustain full collusion, although firms may also fail to sign any agreement.

Tacit collusion

- ‘Meeting of the minds’ between colluding firms
- Analysis of ‘tacit agreements’ is also highly relevant for explicit agreements
 - Sustainability necessary for cartels as long as punishments cannot be legally binding
- Considering 2 firms
 - Offer perfect substitutes at constant marginal costs c
 - Compete over time (each period $t = 1, 2, \dots, T$, firms repeat the ‘static’ game)
- **Lesson:** If competition is repeated over a **finite** number of periods, firms play according to the (unique) Nash equilibrium of the static game in each period. Tacit collusion cannot emerge.

Tacit collusion: Infinite horizon

- Infinite time horizon (no known end to the game)
- Tacit collusion may emerge.
- Consider the **grim trigger strategy**
 - Firm i starts by choosing the action that maximizes total profits.
 - Firm i keeps on choosing this action as long as both firms have done so in all previous periods.
→ *cooperation phase*
 - If one firm deviates, deviation ‘triggers’ the start of the *punishment phase*.
 - Firms choose the action that corresponds to the Nash equilibrium of the static game.

Grim trigger Strategy

- Cooperative action: both obtain $\pi^c = \pi^m/2$
- If one plays the cooperative action and the other optimally deviates, the deviating firm obtains π^d
- At the Nash equilibrium of the static game, both firms obtain π^n , with $\pi^d > \pi^c > \pi^n$.
- Trade-off between
 - immediate gain from deviation
 - future losses resulting from the other firm's punishment
- Trade-off depends on
 - magnitude of the deviation and the punishment profits with respect to the collusive profits
 - the firms' discount factor

Grim trigger strategy (cont'd)

- **Step 1. Value of colluding.** If both firms follow the grim trigger strategy, each earns π^c in every period:

$$V^C = \pi^c + \delta\pi^c + \delta^2\pi^c + \dots = \frac{1}{1-\delta}\pi^c$$

- **Step 2. Value of deviating.** If firm 1 deviates, it earns π^d today and π^n in every later period (punishment phase):

$$V^D = \pi^d + \delta\pi^n + \delta^2\pi^n + \dots = \pi^d + \frac{\delta}{1-\delta}\pi^n$$

Grim trigger strategy (cont'd)

- **Step 3. Incentive constraint.** Firms follow the grim trigger strategy if and only if $V^C \geq V^D$:

$$\frac{1}{1-\delta} \pi^c \geq \pi^d + \frac{\delta}{1-\delta} \pi^n \quad \Leftrightarrow \quad \underbrace{\frac{\delta}{1-\delta} (\pi^c - \pi^n)}_{\text{discounted future loss}} \geq \underbrace{\pi^d - \pi^c}_{\text{short-term gain}}$$

- **Step 4. Critical discount factor.** Multiplying both sides by $1 - \delta$ and solving for δ ,

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^n} \equiv \delta_{\min}$$

- $\delta_{\min}$ increases in the gain from deviating, $\pi^d - \pi^c$, and decreases in the loss from the punishment, $\pi^c - \pi^n$.

Application to price competition

- **Bertrand duopoly** with identical and constant marginal costs:
 - Collusion: both firms charge p^m and earn $\pi^c = \frac{\pi^m}{2}$.
 - Deviation: undercutting the rival captures the market, $\pi^d \approx \pi^m$.
 - Punishment: static Bertrand equilibrium, $\pi^n = 0$.
- **Minimum discount factor:**

$$\delta_{\min}^{Bert} = \frac{\pi^m - \frac{\pi^m}{2}}{\pi^m - 0} = \frac{1}{2}$$

- **Lesson:** In the infinitely repeated Bertrand duopoly game, any profit level between zero and the monopoly profit can be supported in a subgame perfect equilibrium if $\delta \geq 1/2$, that is, if firms are sufficiently patient.

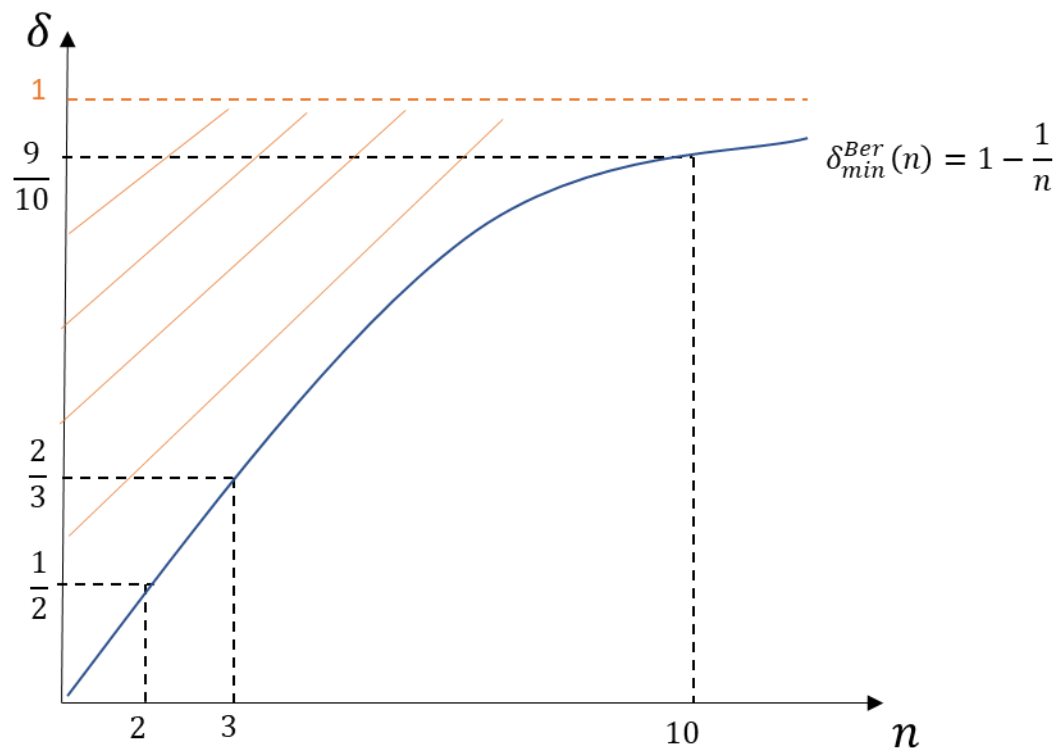
Application to price competition (cont'd)

- **n firms.** Collusive profits are shared among n firms, $\pi^c = \frac{\pi^m}{n}$, while $\pi^d = \pi^m$ and $\pi^n = 0$:

$$V^C \geq V^D \quad \Leftrightarrow \quad \frac{1}{1-\delta} \frac{\pi^m}{n} \geq \pi^m \quad \Leftrightarrow \quad \delta \geq 1 - \frac{1}{n} \equiv \delta_{\min}^{Bert}(n)$$
- For example, $\delta_{\min}^{Bert}(2) = \frac{1}{2}$, $\delta_{\min}^{Bert}(3) = \frac{2}{3}$, and $\delta_{\min}^{Bert}(10) = \frac{9}{10}$.
- The critical discount factor increases in n : each firm's share of the collusive profit falls, while the deviation profit, π^m , does not.

- **Lesson:** In the infinitely repeated Bertrand price-setting game, the set of discount factors that can support collusion is larger the smaller the number of firms in the market.

Application to price competition (cont'd)



Application to quantity competition

- **n -firm Cournot model** with inverse demand $P(q) = a - q$ and constant marginal cost c .
- **Step 1. Collusion.** Firms produce the monopoly output and share it equally:

$$q^m = \frac{a - c}{2} \quad \text{and} \quad \pi^c = \frac{\pi^m}{n} = \frac{(a - c)^2}{4n}$$

- **Step 2. Punishment.** Firms revert to the Cournot equilibrium:

$$\pi^n = \frac{(a - c)^2}{(n + 1)^2}$$

Application to quantity competition (cont'd)

- **Step 3. Deviation.** The other $n - 1$ firms produce $\frac{q^m}{n}$ each. The deviator's best response to their total output is

$$q^d = \frac{1}{2} \left(a - c - (n - 1) \frac{a - c}{2n} \right) = \frac{(n + 1)(a - c)}{4n}$$

- At the best response, the price margin equals q^d , so

$$\pi^d = (q^d)^2 = \frac{(n + 1)^2 (a - c)^2}{16n^2}$$

Application to quantity competition (cont'd)

- **Step 4. Critical discount factor.** Using the profits from Steps 1 to 3,

$$\pi^d - \pi^c = \frac{(a - c)^2(n - 1)^2}{16n^2} \quad \text{and} \quad \pi^d - \pi^n = \frac{(a - c)^2(n - 1)^2(n^2 + 6n + 1)}{16n^2(n + 1)^2}$$

- Dividing the first difference by the second,

$$\delta_{\min}^{Cour}(n) = \frac{\pi^d - \pi^c}{\pi^d - \pi^n} = \frac{(n + 1)^2}{n^2 + 6n + 1}$$

- $\delta_{\min}^{Cour}(n)$ increases in n and converges to 1 as $n \rightarrow \infty$.

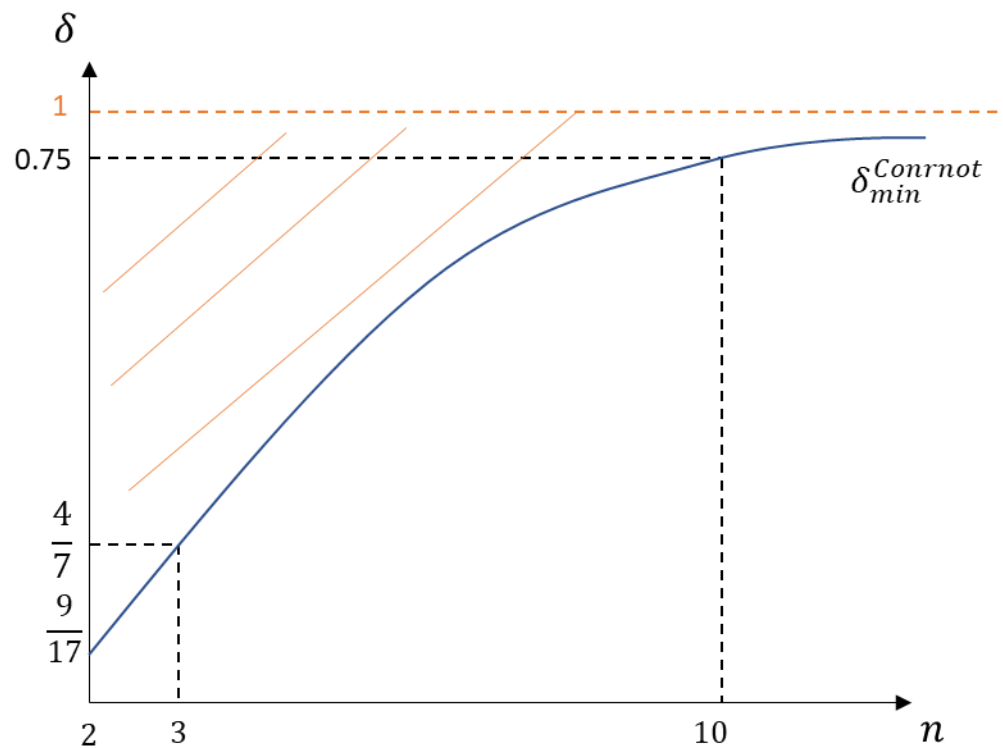
Application to quantity competition (cont'd)

- **Comparison with Bertrand competition.**

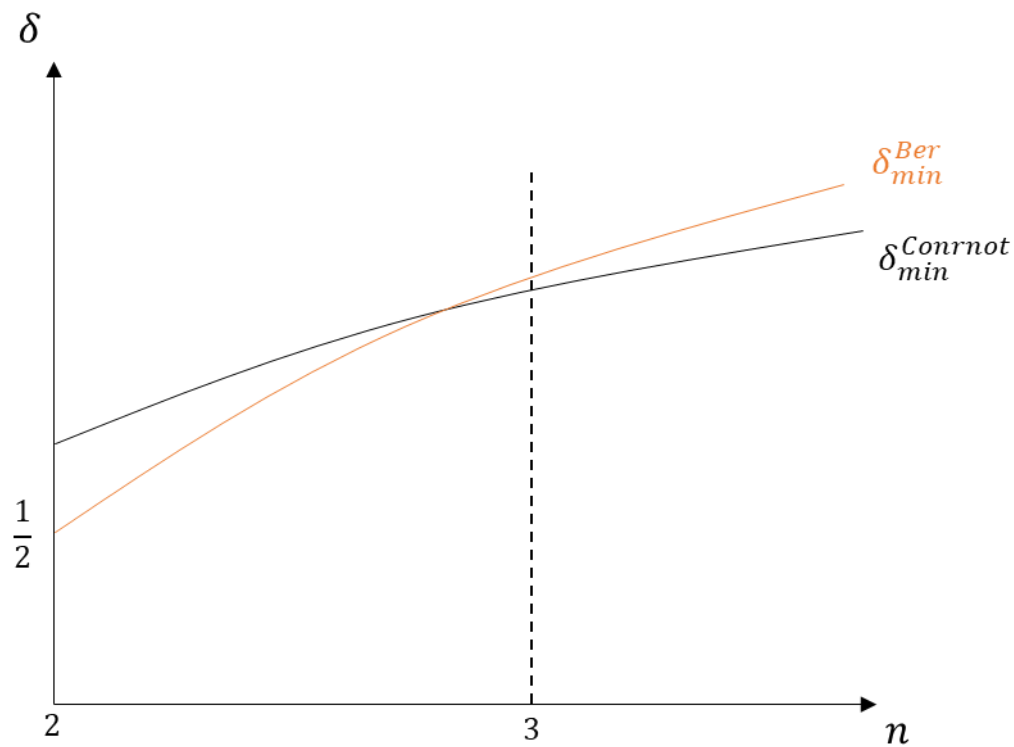
n	Bertrand: $1 - 1/n$	Cournot: $(n+1)^2/(n^2+6n+1)$
2	0.500	0.529
3	0.667	0.571
4	0.750	0.610
5	0.800	0.643
10	0.900	0.752

- **Result:** With two firms, collusion is easier to sustain under Bertrand competition. With three or more firms, it is easier under Cournot competition: Bertrand punishments are harsher (zero profits), but a Bertrand deviator captures the whole market, so the deviation gain grows quickly with n .

Application to quantity competition (cont'd)



Application to quantity competition (cont'd)



Optimal punishment of deviating firms

- Under **price competition**, reversion to the Nash equilibrium generates **zero** profits for all firms.
 - Most severe credible punishment
- Under **quantity competition**, reversion to the Nash equilibrium gives rise to **positive** profits.
 - Possible to design more severe punishment schemes
- **But, punishment must be credible**
 - Reversion to the Nash equilibrium forever ensures this credibility.
 - With more severe punishments, credibility can be restored by shortening the punishment phase, eventually coming back to the collusive outcome.

Optimal punishment of deviating firms (cont'd)

- Stick-and-carrot strategies
 1. Start the game by playing the collusive output q^* , as prescribed by the collusive agreement.
 2. Cooperate as long as the collusive output has been observed in all preceding periods.
 3. If one of the players deviates from the collusive agreement at period t , play $\hat{q}$ at period $t + 1$ (punishment phase) and return to the collusive agreement at period $t + 2$.
 4. If one of the players chooses a quantity $q \neq \hat{q}$ during the punishment phase, start the punishment phase again at the following period.

Optimal punishment of deviating firms (cont'd)

- **Notation.** q^* is the collusive output, $\hat{q}$ the punishment output, $\pi(q)$ each firm's profit when all firms produce q , and $\pi^d(q)$ the profit from a best response when all rivals produce q .
- **Step 1. Value of the punishment phase.** Firms produce $\hat{q}$ for one period and then return to collusion:

$$V^p = \underbrace{\pi(\hat{q})}_{\text{punishment}} + \underbrace{\delta\pi(q^*) + \delta^2\pi(q^*) + \cdots}_{\text{back to collusion}} = \pi(\hat{q}) + \frac{\delta}{1-\delta}\pi(q^*)$$

Optimal punishment of deviating firms (cont'd)

- **Step 2. Credible punishment.** A firm that deviates in the punishment phase earns $\pi^d(\hat{q})$ today, and the punishment restarts tomorrow. Firms carry out the punishment if

$$V^p \geq \pi^d(\hat{q}) + \delta V^p \quad \Leftrightarrow \quad (1 - \delta)V^p \geq \pi^d(\hat{q})$$

- Substituting V^p from Step 1,

$$\underbrace{\delta[\pi(q^*) - \pi(\hat{q})]}_{\text{profit loss}} \geq \underbrace{\pi^d(\hat{q}) - \pi(\hat{q})}_{\text{profit gain}} \quad (\text{C1})$$

- Intuitively, a harsh punishment is credible because it is short: firms accept a low profit today to return to collusion tomorrow.

Optimal punishment of deviating firms (cont'd)

- **Step 3. Sustainable collusion.** After a history of cooperation, a firm abides by the collusive agreement if

$$\pi(q^*) + \delta\pi(q^*) + \delta^2\pi(q^*) + \dots \geq \pi^d(q^*) + \delta\pi(\hat{q}) + \delta^2\pi(q^*) + \dots$$

- Both sides coincide from the third period on, since firms return to collusion after one period of punishment.
- Hence, $\pi(q^*) + \delta\pi(q^*) \geq \pi^d(q^*) + \delta\pi(\hat{q})$, or

$$\underbrace{\delta[\pi(q^*) - \pi(\hat{q})]}_{\text{profit loss}} \geq \underbrace{\pi^d(q^*) - \pi(q^*)}_{\text{profit gain}} \quad (\text{C2})$$

- A harsher punishment (lower $\pi(\hat{q})$) relaxes (C2), but it also raises the gain from deviating in the punishment phase in (C1).

Worked example: optimal punishment

- **Parameters.** Cournot duopoly with $P(q) = 1 - q$ and $c = 0$. Firms collude at the monopoly output, $q^* = \frac{1}{4}$ per firm.
- **Profits.** If both firms produce q , $\pi(q) = (1 - 2q)q$. The best response to q is $\frac{1-q}{2}$, so $\pi^d(q) = \frac{(1-q)^2}{4}$.
 - Collusion: $\pi(q^*) = \frac{1}{8}$. Deviation: $\pi^d(q^*) = \frac{9}{64}$, a gain of $\frac{9}{64} - \frac{8}{64} = \frac{1}{64}$.

- **Credible punishment (C1):**

$$\delta \left[\frac{1}{8} - (1 - 2\hat{q})\hat{q} \right] \geq \frac{(1 - \hat{q})^2}{4} - (1 - 2\hat{q})\hat{q}$$

- **Sustainable collusion (C2):**

$$\delta \left[\frac{1}{8} - (1 - 2\hat{q})\hat{q} \right] \geq \frac{1}{64}$$

Worked example: optimal punishment (cont'd)

- **Most severe credible punishment.** Firms choose the largest $\hat{q}$ satisfying (C1), so (C1) binds. Solving this quadratic equation in $\hat{q}$,

$$\hat{q}(\delta) = \frac{6 - 4\delta + \sqrt{2\delta}}{2(9 - 8\delta)}$$

- A larger $\hat{q}$ means a harsher punishment, since $\pi(\hat{q}) = (1 - 2\hat{q})\hat{q}$ decreases in $\hat{q}$ for $\hat{q} > \frac{1}{4}$.
- For $\delta \geq \frac{1}{2}$, $\hat{q} = \frac{1}{2}$: the price drops to zero, and so do punishment profits.

Worked example: optimal punishment (cont'd)

- **Minimum discount factor.** Substituting $\hat{q}(\delta)$ into (C2), collusion at the monopoly output is sustainable if and only if

$$\delta \geq \frac{9}{32} \approx 0.28$$

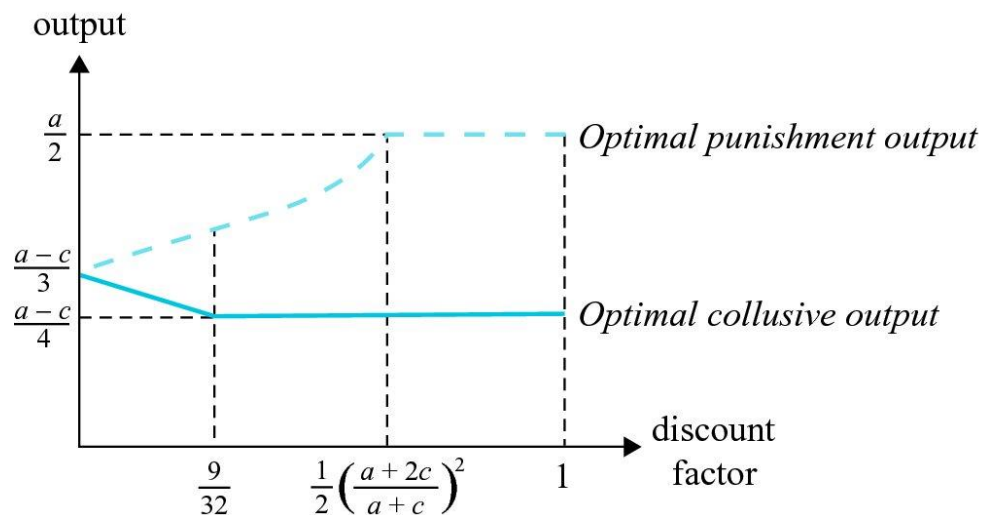
- At this threshold, the punishment is harsher than reverting to the Cournot equilibrium:

$$\hat{q} = \frac{5}{12} \quad \text{and} \quad \pi(\hat{q}) = \frac{5}{72} < \pi^n = \frac{1}{9} = \frac{8}{72}$$

- With grim trigger strategies, full collusion requires $\delta \geq \frac{9}{17} \approx 0.53$. Stick-and-carrot strategies sustain full collusion for a much larger set of discount factors.

Optimal punishment of deviating firms (cont'd)

- Lesson:** In the infinitely repeated Cournot quantity setting game, the set of discount factors that can support full collusion is larger when firms use the optimal punishment of the stick-and-carrot strategy rather than the reversion to the Nash equilibrium of the grim trigger strategy.



Collusion and frequency of interaction

- **Setting.** n firms compete in prices and follow grim trigger strategies. We compare two scenarios:
 - **Scenario 1:** firms interact only every k periods (in periods $1, k + 1, 2k + 1, \dots$).
 - **Scenario 2:** firms interact every period, but prices are set for k periods.
- With $k = 1$, both scenarios coincide with the standard model, where collusion requires $\delta \geq 1 - \frac{1}{n}$.

Collusion and frequency of interaction (cont'd)

- **Scenario 1: interaction every k periods.**
- **Step 1. Value of colluding.** Firms earn $\frac{\pi^m}{n}$ in periods 1, $k + 1$, $2k + 1$, ..., and nothing in between:

$$V_1^c = \frac{\pi^m}{n} (1 + \delta^k + \delta^{2k} + \dots) = \frac{1}{1 - \delta^k} \frac{\pi^m}{n}$$

- **Step 2. Value of deviating.** A deviator earns π^m once and zero afterward (static Bertrand punishment): $V_1^d = \pi^m$.
- **Step 3. Incentive constraint.**

$$V_1^c \geq V_1^d \quad \Leftrightarrow \quad \delta^k \geq 1 - \frac{1}{n} \quad \Leftrightarrow \quad \delta \geq \left(1 - \frac{1}{n}\right)^{1/k}$$

Collusion and frequency of interaction (cont'd)

- **Scenario 2: prices set for k periods.**
- **Step 1. Value of colluding.** Firms interact every period:

$$V_2^c = \frac{1}{1 - \delta} \frac{\pi^m}{n}$$

- **Step 2. Value of deviating.** The rivals cannot react for k periods, so the deviator earns π^m during k periods and zero afterward:

$$V_2^d = \pi^m (1 + \delta + \dots + \delta^{k-1}) = \frac{1 - \delta^k}{1 - \delta} \pi^m$$

- **Step 3. Incentive constraint.**

$$V_2^c \geq V_2^d \quad \Leftrightarrow \quad \frac{1}{n} \geq 1 - \delta^k \quad \Leftrightarrow \quad \delta \geq \left(1 - \frac{1}{n}\right)^{1/k}$$

Collusion and frequency of interaction (cont'd)

- Both scenarios yield the same critical discount factor, which increases in k :

$$\delta \geq \left(1 - \frac{1}{n}\right)^{1/k}$$

- For example, with $n = 2$: $\delta \geq 0.5$ if $k = 1$, $\delta \geq 0.71$ if $k = 2$, and $\delta \geq 0.79$ if $k = 3$.

- Lesson:** Firms find it harder to sustain collusion when they interact less frequently or when price adjustments are less frequent.

Collusion and multimarket contact

- What if firms face same competitors on several markets? Does this facilitate tacit collusion?
- 2 effects
 - Deviation more profitable → it can take place on all markets at the same time
 - Deviation more costly → deviators can be punished on all markets
- The 2 effects cancel out if identical markets, identical firms and constant returns to scale.
- Otherwise, 2nd effect dominates
 - Collusion easier to sustain with multimarket contacts
 - Applications
 - Differing markets
 - Differing firms

Application: differing markets

- Different frequency of interaction
 - Suppose that firms can change prices more frequently in market 1 than in market 2
 - Multimarket contact opens up the possibility that a deviation in market 2 can be (immediately) punished in market 1
 - Pooling of incentive constraints across the two markets facilitates collusion
- Different number of firms
 - Suppose that firms *A* and *B* are both present in markets 1 and 2, firm *C* is only present on market 2
 - Idea: to induce firm *C* to collude by
 - leaving it a larger market share on market 2
 - Using the interaction on market 1 as a disciplining device

Application: differing markets (cont'd)

Different frequency of interaction

- 2 firms A and B interacting in both markets 1 and 2.

- Market 1: $k = 1$

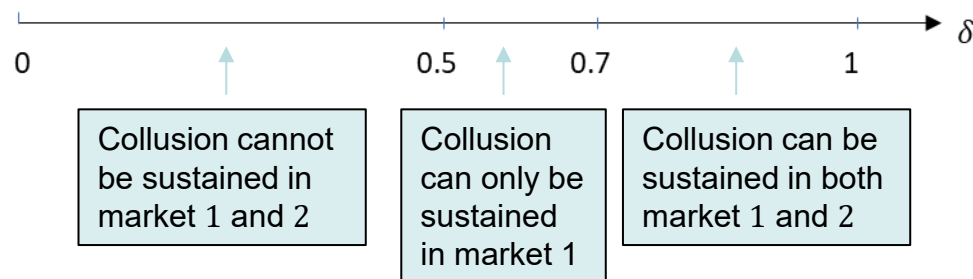
- Market 2: $k = 2$

- In market 1, collusion can be sustained if

$$\delta \geq 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

- In market 2, collusion can be sustained if

$$\delta \geq \left(1 - \frac{1}{2}\right)^{\frac{1}{2}} \approx 0.7$$



Application: differing markets (cont'd)

- **Pooling the incentive constraints.** Firms A and B collude in both markets, and a deviation in either market triggers the punishment in both markets.
- **Step 1. Value of colluding.** Each firm earns $\frac{\pi^m}{2}$ per period in each market:

$$V^c = \frac{1}{1-\delta} \frac{\pi^m}{2} + \frac{1}{1-\delta} \frac{\pi^m}{2} = \frac{\pi^m}{1-\delta}$$

- **Step 2. Value of deviating in both markets.** In market 1, the deviator earns π^m once. In market 2, prices are set for two periods, so it earns π^m twice:

$$V^d = \underbrace{\pi^m}_{\text{market 1}} + \underbrace{\pi^m + \delta\pi^m}_{\text{market 2}} = (2 + \delta)\pi^m$$

Application: differing markets (cont'd)

- Step 3. Incentive constraint.**

$$\frac{\pi^m}{1-\delta} \geq (2+\delta)\pi^m \quad \Leftrightarrow \quad \delta^2 + \delta - 1 \geq 0 \quad \Leftrightarrow \quad \delta \geq \frac{\sqrt{5}-1}{2} \approx 0.62$$

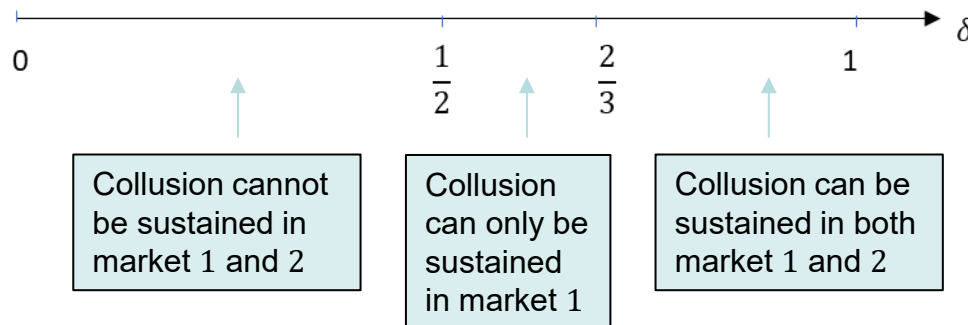
- For δ between 0.62 and 0.71, collusion in market 2 is sustainable only because of multimarket contact: the slack in the incentive constraint of market 1 disciplines behavior in market 2.

Collusion in	Requires
Market 1 alone	$\delta \geq 0.50$
Market 2 alone	$\delta \geq 0.71$
Both markets (pooled constraints)	$\delta \geq 0.62$

Application: differing markets (cont'd)

Different number of firms

- 3 firms A, B and C .
- Market 1: $A, B \rightarrow \delta \geq 1 - \frac{1}{2} = \frac{1}{2}$
- Market 2: $A, B, C \rightarrow \delta \geq 1 - \frac{1}{3} = \frac{2}{3}$



Application: differing markets (cont'd)

- **Idea.** Firms A and B give firm C a share s of the monopoly profit in market 2, and split the rest, $1 - s$, equally.
- **Step 1. Firm C .** It is only active in market 2, and it does not deviate if

$$\frac{s\pi^m}{1 - \delta} \geq \pi^m \quad \Leftrightarrow \quad s \geq 1 - \delta$$

- Firms A and B give C the smallest share that keeps it in the agreement, $s = 1 - \delta$. Each of them then receives a share $\frac{\delta}{2}$ of the monopoly profit in market 2.

Application: differing markets (cont'd)

- **Step 2. Firms A and B.** A deviation in both markets yields π^m in each market once, and zero afterward. Firm A does not deviate if

$$\underbrace{\frac{1}{1-\delta} \frac{\pi^m}{2}}_{\text{profit from } M1} + \underbrace{\frac{1}{1-\delta} \frac{\delta \pi^m}{2}}_{\text{profit from } M2} \geq \underbrace{2\pi^m}_{\text{deviation in } M1 \text{ and } M2}$$

- Simplifying, $1 + \delta \geq 4(1 - \delta)$, that is, $\delta \geq \frac{3}{5}$.

Collusion in	Requires
Market 1 alone (A, B)	$\delta \geq 1/2$
Market 2 alone (A, B, C)	$\delta \geq 2/3$
Both markets, with $s = 1 - \delta$ for C	$\delta \geq 3/5$

Application: differing markets (cont'd)

- Firms *A* and *B* are able to sustain collusion in both markets although they would not be able to collude on market 2 were they only active on that market

- **Lesson:** With multimarket contact on different markets, collusion may become sustainable in several markets, even though deviations would be profitable if firms were active only in one of the markets.

Application: Differing firms

- 2 markets (1 and 2)
- 2 firms (A and B)
 - firm A is installed in market 1
 - firm B is installed in market 2
- Each firm
 - produces a homogeneous good at constant marginal cost c
 - faces a transportation cost of τ to move one unit of output from its 'home' to the 'foreign' market
- Demand on both markets is given by $Q(p) = a - p$
- Monopoly price for home firm is $p^m(c) = (a - c) / 2$
- To ensure competition, we assume that the marginal cost of the foreign firm is smaller than the monopoly price

$$c + \tau < (a + c) / 2 \text{ or } 2\tau < a - c$$

Benchmark: Market-sharing agreements in one market

- As a benchmark, we first examine the optimal collusive outcome that would prevail if firms were only competing in one market.
- Assuming Bertrand competition, the optimal punishment consists for both firms in setting their price equal to c in every future period following a deviation
- Let $s_h + s_f$ denote the respective market shares of the home and the foreign firm
(by definition, $s_h + s_f = 1$)

Benchmark: sharing one market (cont'd)

- **Step 1. Deviation.** At a collusive price $p \geq c + \tau$, a deviating firm with cost c_i slightly undercuts its rival and serves the whole market:

$$\pi^d(p, c_i) = (p - c_i)(a - p)$$

- with $c_i = c$ for the home firm and $c_i = c + \tau$ for the foreign firm.
- **Step 2. Incentive constraints.** After a deviation, both firms price at c forever and earn zero. Each firm abides by the agreement if

$$\text{Home: } \frac{1}{1 - \delta} s_h (p - c)(a - p) \geq (p - c)(a - p) \quad \Leftrightarrow \quad s_h \geq 1 - \delta$$

$$\text{Foreign: } \frac{1}{1 - \delta} s_f (p - c - \tau)(a - p) \geq (p - c - \tau)(a - p) \quad \Leftrightarrow \quad s_f \geq 1 - \delta$$

Benchmark: sharing one market (cont'd)

- **Step 3. Conclusion.** Adding the two conditions, since market shares sum to one,

$$1 = s_h + s_f \geq 2(1 - \delta) \quad \Leftrightarrow \quad \delta \geq \frac{1}{2}$$

- Firms can sustain collusive prices above $c + \tau$ only if $\delta \geq \frac{1}{2}$.
- In addition, the inefficient foreign firm must receive a large enough market share to keep it from deviating, $s_f \geq 1 - \delta$.

Market-sharing agreements in two markets

- **Setting.** Symmetric collusive outcomes: both firms set the same price p in both markets, and the home firm receives a share s_h in its home market.
- For a given $p \geq c + \tau$, the most profitable agreement sets $s_h = 1$: each firm withdraws from its foreign market, where its cost is higher.
- The best collusive price is then the home monopoly price, and each firm earns the home monopoly profit in every period:

$$p^m(c) = \frac{a + c}{2} \quad \text{and} \quad V^c = \frac{1}{1 - \delta} \frac{(a - c)^2}{4}$$

Market-sharing agreements in two markets (cont'd)

- **Step 1. Best deviation.** A firm enters its foreign market and slightly undercuts the home firm, while it keeps the monopoly profit at home this period. In the foreign market, it earns

$$(p^m(c) - c - \tau)(a - p^m(c)) = \frac{(a - c)(a - c - 2\tau)}{4}$$

- After the deviation, both firms earn zero forever, so

$$V^d = \frac{(a - c)^2}{4} + \frac{(a - c)(a - c - 2\tau)}{4}$$

Market-sharing agreements in two markets (cont'd)

- **Step 2. Incentive constraint.** $V^c \geq V^d$ if and only if

$$\frac{\delta}{1-\delta} \frac{(a-c)^2}{4} \geq \frac{(a-c)(a-c-2\tau)}{4} \quad \Leftrightarrow \quad \delta \geq \frac{1}{2} \frac{a-c-2\tau}{a-c-\tau}$$

- The threshold is below $\frac{1}{2}$, and it decreases in τ : the higher the cost of serving the foreign market, the less profitable it is to deviate into it.
- For example, with $a - c = 1$ and $\tau = \frac{1}{4}$, collusion requires

$$\delta \geq \frac{1}{2} \times \frac{1/2}{3/4} = \frac{1}{3} \quad \text{compared with} \quad \delta \geq \frac{1}{2} \quad \text{in one market}$$

Market-sharing agreements with differing firms

- **Lesson:** The optimal market-sharing agreement can be sustained over a larger set of discount factors than the most profitable collusive outcome that firms can achieve when they are present on one market only.



Case. Multimarket contact in the U.S. airline industry

- Markets: different city-pair routes
- Multimarket contact may facilitate collusion when
 - Firms differ in their production costs across markets
 - Markets themselves differ
- Both conditions are relevant in this industry.
- Hub and spoke model
 - Significant cost advantage to the carrier operating the hub
- Significant cross-route differences
- Experts claimed: airlines refrain from pricing aggressively in a given route for fear of retaliation in another jointly contested route.
- Fares are (on average) higher on routes where the competing carriers have extensive interroute contacts.



Tacit collusion and cyclical demand

- Many markets are characterized by demand fluctuations
- Extension of the single-market analysis
 - 2 demand states: demand is either good $Q_G(p)$ or bad $Q_B(p)$ with $Q_G(p) > Q_B(p)$, for all p
 - Firms observe state of demand before setting their price.
- **Lesson:** Under demand uncertainty, the critical discount factor above which the fully collusive outcome can be sustained is larger than under demand certainty.
- Intuition
 - Punishment entails loss of an average of high and low profits
→ it is less severe than if the good demand state persisted
 - Hence, fully collusive outcome is more difficult to sustain than under demand certainty.

Tacit collusion and cyclical demand (cont'd)

- **Setting.** Bertrand duopoly. In each period, demand is bad (B) or good (G) with probability $\frac{1}{2}$ each, independently across periods. Firms observe the state before setting prices.
- Firms look for a pair of collusive prices (p_B, p_G) that makes deviations unprofitable in both states and, among all such pairs, maximizes their joint profits.
- Let $\pi_s(p) \equiv (p - c)Q_s(p)$ be the industry profit in state s . The expected value of colluding, before the state is known, is

$$V^c = \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{2} \frac{\pi_B(p_B)}{2} + \frac{1}{2} \frac{\pi_G(p_G)}{2} \right] = \frac{1}{1-\delta} \frac{\pi_B(p_B) + \pi_G(p_G)}{4}$$

Tacit collusion and cyclical demand (cont'd)

- **Step 1. Full collusion.** Firms charge the monopoly price of each state, p_B^m and p_G^m , with monopoly profits $\pi_B^m < \pi_G^m$:

$$V^c = \frac{1}{1 - \delta} \frac{\pi_B^m + \pi_G^m}{4}$$

- **Step 2. Incentive constraint in state s .** A deviator earns π_s^m today and zero afterward. Colluding yields half of π_s^m today, plus δV^c :

$$\frac{\pi_s^m}{2} + \delta V^c \geq \pi_s^m \quad \Leftrightarrow \quad \delta(\pi_B^m + \pi_G^m) \geq 2(1 - \delta)\pi_s^m$$

- The constraint is tighter in the good state, $s = G$, where the gain from deviating is larger.

Tacit collusion and cyclical demand (cont'd)

- **Step 3. Critical discount factor.** Solving the good-state constraint for δ ,

$$\delta \geq \frac{2\pi_G^m}{3\pi_G^m + \pi_B^m} \equiv \delta^0$$

- Since $0 < \pi_B^m < \pi_G^m$, we obtain $\frac{1}{2} < \delta^0 < \frac{2}{3}$. With demand certainty ($\pi_B^m = \pi_G^m$), the threshold would be $\frac{1}{2}$.

Discount factor	Outcome
$\delta < 1/2$	No collusion
$1/2 \leq \delta < \delta^0$	Partial collusion (next slides)
$\delta \geq \delta^0$	Full collusion

Tacit collusion and cyclical demand (cont'd)

- Partial collusion for lower discount factors
 - Full collusion sustained in the bad demand state but not in the good demand state
 - Bad demand: firms charge the monopoly price.
 - Good demand: firms set prices more aggressively than under monopoly.
 - Possible result: firms price lower in the good than in the bad demand state
 - Model can generate lower prices after a positive demand shock and countercyclical prices and markups.

Tacit collusion and cyclical demand (cont'd)

- **Step 4. Partial collusion for $\frac{1}{2} \leq \delta < \delta^0$.** Firms choose the industry profits π_B and π_G to solve

$$\max_{\pi_B, \pi_G} \frac{\pi_B + \pi_G}{4(1 - \delta)} \quad \text{s. t.} \quad \frac{\pi_s}{2} \leq \frac{\delta}{1 - \delta} \frac{\pi_B + \pi_G}{4} \quad \text{for } s = B, G$$

- The bad-state constraint holds whenever $\pi_G \geq \pi_B$ and $\delta \geq \frac{1}{2}$, so we can ignore it.
- Hence, firms set $p_B = p_B^m$ and choose the highest π_G that satisfies the good-state constraint.

Tacit collusion and cyclical demand (cont'd)

- **Step 5. Good-state profit.** Multiplying the good-state constraint by $4(1 - \delta)$,

$$2(1 - \delta)\pi_G \leq \delta(\pi_B^m + \pi_G) \quad \Leftrightarrow \quad \pi_G \leq \frac{\delta}{2 - 3\delta} \pi_B^m$$

- Firms set the price $\tilde{p}_G < p_G^m$ at which this constraint binds:

$$\pi_G(\tilde{p}_G) = \frac{\delta}{2 - 3\delta} \pi_B^m$$

- Since $\frac{\delta}{2 - 3\delta} \geq 1$ when $\delta \geq \frac{1}{2}$, industry profit is still higher in the good state.
- As δ approaches δ^0 , $\tilde{p}_G$ approaches p_G^m .

Tacit collusion and cyclical demand (cont'd)

- **Step 6. Comparing prices across states.** Profits are higher in the good state, but the price may be lower:
 - If $\tilde{p}_G < p_B^m$: **countercyclical** prices and price margins.
 - If $\tilde{p}_G > p_B^m$: **procyclical** prices and price margins.
- Intuitively, a larger demand makes a deviation more attractive, so firms lower the price in the good state to keep collusion sustainable.

Worked example: cyclical demand

- **Parameters.** $Q_B(p) = 1 - p$, $Q_G(p) = 2 - p$, and $c = 0$. Monopoly prices and profits: $p_B^m = \frac{1}{2}$, $\pi_B^m = \frac{1}{4}$, $p_G^m = 1$, and $\pi_G^m = 1$.
- **Full collusion** requires

$$\delta \geq \delta^0 = \frac{2 \times 1}{3 \times 1 + \frac{1}{4}} = \frac{8}{13} \approx 0.615$$

- **Partial collusion** with $\delta = 0.55$. The good-state profit is

$$\pi_G(\tilde{p}_G) = \frac{0.55}{2 - 1.65} \times \frac{1}{4} \approx 0.393$$

- Solving for the price below $p_G^m = 1$ that yields this profit,

$$\tilde{p}_G(2 - \tilde{p}_G) = 0.393 \quad \Rightarrow \quad \tilde{p}_G \approx 0.22 < p_B^m = 0.5$$

- Prices are countercyclical: firms charge less when demand is good.

Tacit collusion and cyclical demand (cont'd)

- **Lesson:** When full collusion cannot be sustained, firms may partially collude by setting the respective monopoly price in the bad demand state and setting a price lower than the respective monopoly price in the good demand state. This result is compatible with counter-cyclical prices and markups.
- **Illustration:** market for antibiotic tetracycline
 - Firms were confronted with a positive demand shock when the Armed Service Medical Procurement Agency placed a large order in October 1956.
 - Thereafter, price discipline among firms broke down.

Tacit collusion with unobservable actions

- Here, firms do not observe deviations
 - As before, demand uncertainty. **But**, no observation of state of demand, nor of rivals' prices → no way to infer deviations from market outcomes.
- Model
 - 2 firms, homogeneous product, constant marginal cost c , Bertrand competition
 - Bad demand state (probability α): no demand
 - Good demand state (probability $1-\alpha$): $Q(p) > 0$ for all $p \geq c$.
 - If firm faces zero demand, signal extraction problem:
 - Is it because rival deviated or because of bad demand?

Tacit collusion with unobservable actions (cont'd)

- Structure of punishment?
 - Punishment forever after zero demand? Too harsh as zero demand may result from bad state.
 - But milder punishment → lower discipline...
- Proposed strategies for both firms:
 1. Start with the collusive phase and charge price p^m until one firm makes zero profit (which along the equilibrium path, occurs in the low demand state)
 2. If one firm makes zero profits, this triggers a punishment phase for T periods in which each firm charges c . After T periods firms return to step 1.
 Grim trigger is the special case $T = \infty$. A finite T is a milder punishment.

Tacit collusion with unobservable actions (cont'd)

- **Step 1. Value of colluding.** Let V^c be a firm's expected discounted profit at the start of a collusive period, and V^p its value at the start of a punishment phase.

$$V^c = \underbrace{(1 - \alpha) \left[\frac{1}{2} \pi^m + \delta V^c \right]}_{\text{good demand}} + \underbrace{\alpha \left[0 + \delta V^p \right]}_{\text{bad demand}}$$

- Good demand: both firms charge p^m , share the monopoly profit, and stay in the collusive phase tomorrow.
- Bad demand: both firms sell nothing, so the punishment phase starts tomorrow.
- **Punishment occurs along the equilibrium path:** it is triggered by bad demand, not by cheating.

Tacit collusion with unobservable actions (cont'd)

- **Step 2. Value of the punishment phase.** Firms price at c (zero profit) for T periods and then return to the collusive phase:

$$V^p = 0 + \delta \cdot 0 + \dots + \delta^{T-1} \cdot 0 + \delta^T V^c = \delta^T V^c$$

- Substituting $V^p = \delta^T V^c$ into Step 1,

$$V^c = (1 - \alpha) \frac{1}{2} \pi^m + [(1 - \alpha)\delta + \alpha\delta^{T+1}]V^c$$

- Solving for V^c yields

$$V^c = \frac{(1 - \alpha)\pi^m}{2[1 - (1 - \alpha)\delta - \alpha\delta^{T+1}]}$$

- V^c decreases in T : since punishments happen after every bad-demand period, longer punishments lower collusive profits.

Tacit collusion with unobservable actions (cont'd)

- **Step 3. Value of deviating.** In a collusive period, a firm undercuts p^m slightly, without knowing the state of demand:

$$V^d = \underbrace{(1 - \alpha)[\pi^m + \delta V^p]}_{\text{good demand}} + \underbrace{\alpha[0 + \delta V^p]}_{\text{bad demand}}$$

- Good demand: the deviator serves the whole market; its rival sells nothing, which starts the punishment phase tomorrow.
 - Bad demand: no firm sells, so the punishment phase starts tomorrow as well.
- In the punishment phase, $p = c$ is the static Bertrand equilibrium, so no firm gains by deviating there. We only need to check the collusive phase.

Tacit collusion with unobservable actions (cont'd)

- **Step 4. Incentive constraint.** Collusion is sustainable if $V^c \geq V^d$. Using the expression of V^c from Step 1,

$$(1 - \alpha) \left[\frac{1}{2} \pi^m + \delta V^c \right] + \alpha \delta V^p \geq (1 - \alpha) [\pi^m + \delta V^p] + \alpha \delta V^p$$

- Cancelling $\alpha \delta V^p$ on both sides and dividing by $1 - \alpha$,

$$\underbrace{\delta(V^c - V^p)}_{\text{loss from punishment}} \geq \underbrace{\frac{1}{2} \pi^m}_{\text{gain from deviating}}$$

- Intuitively, a deviation increases profits by $\frac{1}{2} \pi^m$ today (in the good state), but it moves the firm to the punishment phase tomorrow for sure.

Tacit collusion with unobservable actions (cont'd)

- **Step 5. Solving the incentive constraint.** Since $V^p = \delta^T V^c$, then $V^c - V^p = (1 - \delta^T)V^c$. Using V^c from Step 2:

$$\delta(1 - \delta^T) \frac{(1 - \alpha)\pi^m}{2[1 - (1 - \alpha)\delta - \alpha\delta^{T+1}]} \geq \frac{1}{2}\pi^m$$

- Cancelling $\frac{1}{2}\pi^m$ and multiplying both sides by the denominator,

$$(1 - \alpha)\delta(1 - \delta^T) \geq 1 - (1 - \alpha)\delta - \alpha\delta^{T+1}$$

- Rearranging terms,

$$2(1 - \alpha)\delta - (1 - 2\alpha)\delta^{T+1} \geq 1 \quad (\text{IC})$$

Tacit collusion with unobservable actions (cont'd)

- **Step 6. When can collusion be sustained?**
 - If $\alpha \geq \frac{1}{2}$: since $\delta^{T+1} \leq \delta$, the left side of (IC) is at most $\delta < 1$, so (IC) never holds.
 - If $\alpha < \frac{1}{2}$: the left side of (IC) increases in T , and it tends to $2(1 - \alpha)\delta$ as $T \rightarrow \infty$.
- Hence, a punishment length T (possibly $T = \infty$) satisfying (IC) exists if and only if

$$(1 - \alpha)\delta \geq \frac{1}{2}$$

- With observable deviations (Bertrand duopoly), collusion requires $\delta \geq \frac{1}{2}$. Now it requires $\delta \geq \frac{1}{2(1-\alpha)}$, which is **more demanding**, and more so when bad demand is more likely.

Tacit collusion with unobservable actions (cont'd)

- **Step 7. Optimal length of the punishment phase.** Firms choose T to maximize V^c subject to (IC).
 - V^c decreases in T (Step 2), so firms prefer short punishments.
 - (IC) requires the punishment to be long enough. For $\alpha < \frac{1}{2}$, (IC) is equivalent to

$$\delta^{T+1} \leq \frac{2(1 - \alpha)\delta - 1}{1 - 2\alpha}$$

- The optimal T^* is the **smallest** integer satisfying this inequality: the punishment is just long enough to deter deviations.
 - A grim-trigger strategy ($T = \infty$) also deters deviations, but it yields lower profits than T^* .

Worked example

- **Parameters.** $\alpha = 0.2$, $\delta = 0.9$, and $\pi^m = 100$.
- **Feasibility.** $(1 - \alpha)\delta = 0.8 \times 0.9 = 0.72 \geq 0.5$, so collusion can be sustained for some T .
- **Optimal punishment.** (IC) becomes

$$2(0.8)(0.9) - (0.6)(0.9)^{T+1} \geq 1 \quad \Leftrightarrow \quad 0.9^{T+1} \leq \frac{0.44}{0.6} \approx 0.733$$
 - $T = 1$: $0.9^2 = 0.81 > 0.733$, so (IC) fails.
 - $T = 2$: $0.9^3 = 0.729 \leq 0.733$, so (IC) holds.
- Therefore, $T^* = 2$: after a period with zero sales, firms price at cost for two periods.

Worked example (cont'd)

- **Value of colluding** (Step 2, with $T^* = 2$):

$$V^c = \frac{0.8 \times 100}{2[1 - 0.72 - 0.2(0.9)^3]} = \frac{80}{2 \times 0.1342} \approx 298.1$$

- **Value of the punishment phase:** $V^p = 0.9^2 \times 298.1 \approx 241.4$.

- **Check (IC):** deviating yields

$$V^d = 0.8(100 + 0.9 \times 241.4) + 0.2(0.9 \times 241.4) \approx 297.3 < 298.1$$

- With $T = 1$ instead, $V^c \approx 339.0$ but $V^d \approx 354.6$: the punishment is too short and firms deviate.

Worked example (cont'd)

- **Comparison.** Expected discounted profit of each firm:
 - Observable deviations (no price wars): $\frac{0.8 \times 50}{1 - 0.9} = 400$.
 - Unobservable deviations, $T^* = 2$: $V^c \approx 298.1$.
 - Unobservable deviations, $T = \infty$ (grim trigger): $\frac{0.8 \times 50}{1 - 0.72} \approx 142.9$.
- Firms lose about one-fourth of their collusive profits because price wars occur after bad demand, even though no firm ever cheats.
- Choosing the shortest punishment that deters deviations more than doubles profits relative to grim trigger.

Tacit collusion with unobservable actions (cont'd)

- Price wars are part of the equilibrium: they follow bad demand, not cheating (Green and Porter, 1984).
- Profits exceed those in the repeated static Bertrand equilibrium (zero), but fall short of those with observable deviations.

- **Lesson:** Even if firms cannot observe deviations of other firms from equilibrium play, collusion can still be supported to some extent. However, the conditions for collusion to be sustainable are stricter than in a world in which deviations can be immediately observed and punished. In addition, profits are lower.

Detecting and fighting collusion

- Why do competition authorities mainly try to uncover explicit agreements such as price-fixing cartels if firms have other means to sustain collusion?
- **Answer (Lesson):** Without the information sharing within a cartel, collusion is more likely to be infeasible. Even if it is feasible, collusion may not be supported over the whole time-horizon but firms may alternate between collusive and punishment phases (over varying length) in which they switch between a high and a low price.

Detecting and fighting collusion

- Communication is central to collusion; thus it can be used to detect collusion.
 - Collusion might leave significant pieces of evidence.
 - Permanent records of meetings or agreements
 - Telephone conversations may have been tapped.
- Competition authorities have designed policies to encourage cartel members to bring evidence to the authorities by themselves.
- Simply looking at possible high price-cost margins is not sensible.
 - as we have seen in *Chapter 3* about market power

The difficulty in detecting collusion

- Four methods to detect collusion

SCREENING

1. Is the firm's behaviour consistent or not with properties or behaviour that are supposed to hold under a wide class of competitive models?
2. Are there structural breaks in the firm's behaviour?

VERIFICATION

3. Does the behaviour of suspected colluding firms differ from competitive firm's behaviour?
 - If only a subset of firms in the industry colludes and they can be identified a comparison can be directly carried out.
 - Otherwise one can resort to comparison across markets and across periods.
4. Which model (competitive or collusive) better fits the data?

The difficulty in detecting collusion (cont'd)

- All 4 methods suffer from 2 general problems:
 1. Necessary data to identify firm's behaviour often not available (cost is often unobservable).
 2. Firms have incentive to misreport private information.
 - Authorities are likely to suffer from the so-called **indistinguishability theorem**
 - → firms misreport cost information to make prices appear as resulting from competitive – and not collusive – behaviour
- Even if needed data are available, estimation of firm's behaviour may be extremely sensitive to model specifications.

Leniency and whistleblowing programmes

- To obtain evidence for the existence of cartels and collusion, competition authorities introduced
 - Corporate leniency programmes
 - Reduced sentences for firms that cooperate with the authorities and provide evidence for the existence of a cartel
 - Whistleblowing programmes
 - Shielding individual informants from criminal sanctions
- Results of law enforcement
 - Make cartels less stable
 - Break-up existing cartels
 - Prevent the formation of cartels



Case. The vitamin cartels (cont'd)

- For violation of article 81 of the Community Treaty and article 53 of the European Economic Area Agreement, 8 companies were fined for forming a cartel.
- Hoffmann-La Roche (462m €) and BASF (296.16m €) were considered the joint leaders and instigators of the cartels.
- Aventis received significantly lower fines as the first company to cooperate both with the US Department of Justice and the European Commission.



Case. The beer cartel in the Netherlands

- 4 Dutch beer brewers formed a cartel on the beer market (a branded consumer good market) in the Netherlands.
 - Coordinated prices and price changes (at the wholesale level)
 - Operated in 2 market segments
 - Consumption on the premises (bars, restaurants, hotels) → Coordinated rebate policies, market-sharing agreements
 - Retail (mostly supermarkets) → Market-sharing agreements
- In 2007 the European Commission fined the 3 leading Dutch brewers Heineken, Grolsch and Bavaria a total amount of 274m EUR for operating this cartel between at least 1996 and 1999.
- InBev (also part of the cartel) provided essential information that led to surprise inspections and hard evidence (handwritten notes, proof of secret meeting dates)
- Under the Commissions leniency programme, InBev did not have to pay fines.

Leniency programme

- Considering a corporate leniency programme whereby a cooperating firm is granted a reward (or, at least, a reduced fine).
 - The condition for the sustainability of collusion becomes more stringent.
- **Lesson:** Reducing fines for firms which report incriminating evidence may deter collusion. However, for some cartels, competition authorities have to grant sufficiently large positive rewards to deter collusion.

Whistleblowing programme

- Complements the corporate leniency programme by granting a positive reward to employees reporting incriminating evidence.
 - Firms will have to bribe informed employees to prevent them from disclosing information.
 - Collusion becomes less profitable and thus harder to sustain.
- **Lesson:** Corporate leniency and individual whistleblowing programmes are complementary in the fight against collusion.

Review questions

- Contrast the conditions for cartels to be stable when they form in a simultaneous versus sequential way.
- Explain how a collusive outcome may emerge noncooperatively as the equilibrium of a repeated game. Discuss how the horizon of the game, the number of firms, the frequency of interaction and multimarket contact affect the sustainability of collusion.
- Explain why tacit collusion is harder to sustain when demand fluctuates and the rival's actions cannot be observed.
- Explain why competition authorities encourage colluding firms and their employees to report incriminating evidence through leniency and whistleblowing programmes.