

Part IV. Pricing strategies and market segmentation

Chapter 9. Menu pricing



Slides

Industrial Organization: Markets and Strategies

Paul Belleflamme and Martin Peitz, 2d Edition

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Chapter 9. Learning objectives

- Be able to make a clear difference between menu pricing and group pricing.
- Understand how a monopolist sets menu prices and under which conditions menu pricing leads to higher profits than uniform pricing.
- Assess the welfare effects of menu pricing.
- Analyze quality- and quantity-based menu pricing in oligopolistic settings.

Menu vs. group pricing

- Group (and personalized) pricing
 - Seller can infer consumers' willingness to pay from observable and verifiable characteristic (e.g., age)
- Menu pricing
 - Willingness to pay = private information
 - Seller must bring consumer to reveal this information.
 - How?
 - Identify product dimension valued differently by consumers
 - Design several versions of the product along that dimension
 - Price versions to induce consumers' self-selection
 - **Menu pricing** (a.k.a. versioning, 2nd-degree price discrimination, nonlinear pricing)
 - *Screening problem*: uninformed party brings informed parties to reveal their private information

Case. Menu pricing in the information economy

- Versioning based on quality
 - ‘Nagware’: software distributed freely but displaying ads or screen encouraging users to buy full version
→ annoyance = discriminating device
- Versioning based on time
 - Books: first in hardcover, later in paperback
 - Movies: first in theaters, next on DVD, finally on TV.
→ price decreases as delay increases
- Versioning based on quantity
 - Software site licenses
 - Newspaper subscription
→ quantity discounts





Case. Geographical pricing by LCCs

- Low Cost Carriers have abandoned many of the price discrimination tactics of the airline industry
 - 'Point-to-point' tickets, 'no-frills' flights
- But, geographical price discrimination on their website (Bachis and Piga, 2006)
 - Example: London-Madrid flight
 - 1st leg for British traveller, fare offered in £
 - Return leg for Spanish traveller, fare offered in €
 - If booking occurs at same time and no price discrimination, then ratio of prices = exchange rate
 - Yet, difference of at least 7£ for 450 000 observations
 - Despite possibility of arbitrage.

Monopoly menu pricing

- Quality-dependent prices
 - Consumer's indirect utility when buying one unit of quality s at price p : $U(\theta, s) - p$ (utility = 0 if not buying)
 - U increases in s and in θ (taste parameter)
 - Suppose 2 types of consumers
 - 'Low type', in proportion $1-\lambda$, with taste parameter θ_1
 - 'High type', in proportion λ , with taste parameter $\theta_2 > \theta_1$
 - High types care more about quality than low types:
 $U(\theta_2, s) > U(\theta_1, s)$
 - High types value more any *increase* in quality than low types:
 $U(\theta_2, s_2) - U(\theta_2, s_1) > U(\theta_1, s_2) - U(\theta_1, s_1)$ for $s_2 > s_1$
 $\rightarrow$ *Single-crossing property*
- Monopolist can produce s_1 and s_2 at constant marginal costs c_1 and c_2 .

Monopoly menu pricing (cont'd)

- **Assumption.** Each quality is worth producing for each type:
 $U(\theta_i, s_j) > c_j$.
- **Step 1. Uniform pricing (high quality only).** The monopolist sells s_2 at a single price. Two options:
 - Sell only to high types at $p = U(\theta_2, s_2)$: profit $[U(\theta_2, s_2) - c_2]\lambda$.
 - Sell to everyone at $p = U(\theta_1, s_2)$: profit $U(\theta_1, s_2) - c_2$.
- Selling only to high types is better if

$$\lambda > \frac{U(\theta_1, s_2) - c_2}{U(\theta_2, s_2) - c_2} \equiv \lambda_0$$

- Hence, uniform-pricing profit is

$$\Pi_s = [U(\theta_2, s_2) - c_2]\lambda \text{ if } \lambda \geq \lambda_0, \quad \Pi_s = U(\theta_1, s_2) - c_2 \text{ if } \lambda < \lambda_0$$

Monopoly menu pricing (cont'd)

- **Step 2. Menu pricing.** Sell s_1 at p_1 to low types and s_2 at p_2 to high types:

$$\max_{p_1, p_2} \Pi_m = \lambda(p_2 - c_2) + (1 - \lambda)(p_1 - c_1)$$

- subject to participation (PC) and incentive-compatibility (IC) constraints:

$$(PC_1) U(\theta_1, s_1) \geq p_1 \quad (PC_2) U(\theta_2, s_2) \geq p_2$$

$$(IC_1) p_1 \leq p_2 - [U(\theta_1, s_2) - U(\theta_1, s_1)]$$

$$(IC_2) p_2 \leq p_1 + [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

- (IC_i) : type i weakly prefers its own option to the other one.

Monopoly menu pricing (cont'd)

- **Step 3. Binding constraints.** The monopolist raises prices until PC_1 and IC_2 bind:

$$p_1^* = U(\theta_1, s_1)$$

$$p_2^* = U(\theta_1, s_1) + [U(\theta_2, s_2) - U(\theta_2, s_1)] = U(\theta_2, s_2) - \underbrace{[U(\theta_2, s_1) - U(\theta_1, s_1)]}_{\text{information rent}}$$

- By single crossing, the rent is positive, so $p_2^* < U(\theta_2, s_2)$: PC_2 holds, and IC_1 also holds.
- **Step 4. Menu profit.**

$$\Pi_m = \lambda[U(\theta_2, s_2) - (U(\theta_2, s_1) - U(\theta_1, s_1)) - c_2] + (1 - \lambda)[U(\theta_1, s_1) - c_1]$$

Monopoly menu pricing (cont'd)

- **Step 5. Menu vs. uniform pricing.** Define

$$a \equiv U(\theta_2, s_2) - c_2, \quad b \equiv U(\theta_2, s_1) - c_1, \quad x \equiv U(\theta_1, s_2) - c_2, \quad y \equiv U(\theta_1, s_1) - c_1$$

- Case $\lambda \geq \lambda_0$: $\Pi_m - \Pi_s = (1 - \lambda)y - \lambda(b - y) > 0$ if and only if $\lambda < \frac{y}{b} \equiv \bar{\lambda}$.
- Case $\lambda < \lambda_0$: $\Pi_m - \Pi_s > 0$ if and only if $\lambda > \frac{x-y}{a-b} \equiv \underline{\lambda}$.
- Both intervals are nonempty ($\underline{\lambda} < \lambda_0 < \bar{\lambda}$) if and only if

$$\frac{a}{b} > \frac{x}{y} \quad \Leftrightarrow \quad \frac{U(\theta_2, s_2) - c_2}{U(\theta_2, s_1) - c_1} > \frac{U(\theta_1, s_2) - c_2}{U(\theta_1, s_1) - c_1}$$
- Then menu pricing is optimal when $\underline{\lambda} < \lambda < \bar{\lambda}$.

Monopoly menu pricing (cont'd)

- **Distortion of quality.** Now the monopolist also chooses s_1 and s_2 , with cost $c(s)$ per unit. Using Step 4,

$$\max_{s_1, s_2} \lambda [U(\theta_2, s_2) - (U(\theta_2, s_1) - U(\theta_1, s_1)) - c(s_2)] + (1 - \lambda) [U(\theta_1, s_1) - c(s_1)]$$

- **FOC for s_2 .** Only the first bracket depends on s_2 :

$$c'(s_2) = \frac{\partial U(\theta_2, s_2)}{\partial s_2} \quad (\text{efficient: no distortion at the top})$$

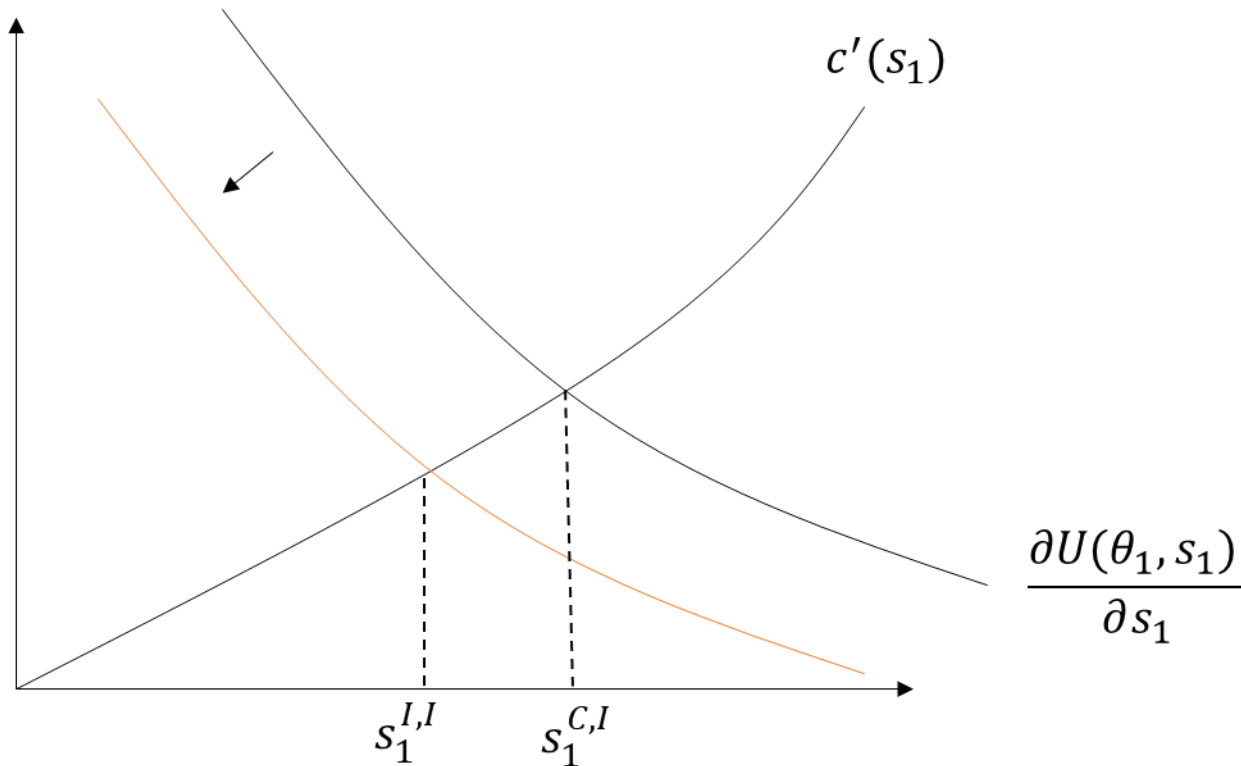
- **FOC for s_1 .** Raising s_1 also raises the high type's information rent:

$$c'(s_1) = \frac{\partial U(\theta_1, s_1)}{\partial s_1} - \frac{\lambda}{1 - \lambda} \underbrace{\left(\frac{\partial U(\theta_2, s_1)}{\partial s_1} - \frac{\partial U(\theta_1, s_1)}{\partial s_1} \right)}_{>0 \text{ by single crossing}}$$

- Marginal cost is below marginal utility: s_1 is distorted downward.

Monopoly menu pricing (cont'd)

- **Distortion of quality** (cont'd)



Monopoly menu pricing (cont'd)

- **Linear utility**, $U(\theta, s) = \theta s$. The condition from Step 5 becomes

$$\frac{\theta_2 s_2 - c_2}{\theta_2 s_1 - c_1} > \frac{\theta_1 s_2 - c_2}{\theta_1 s_1 - c_1} \Leftrightarrow (\theta_2 - \theta_1)(s_1 c_2 - s_2 c_1) > 0 \Leftrightarrow \frac{c_2}{s_2} > \frac{c_1}{s_1}$$
 - Menu pricing requires the cost per unit of quality to be higher for the high-quality version.
- **Information goods** ($c_1 = c_2 = 0$), or equal costs ($c_1 = c_2 = c$): the condition fails: $\frac{c}{s_2} < \frac{c}{s_1}$ if $c > 0$, and both sides are zero if $c = 0$.
- With linear utility, a monopolist selling information goods never uses menu pricing: selling a single version is optimal.

Monopoly menu pricing (cont'd)

- **Utility not linear in quality:** $U(\theta, s) = \theta s + k$, with $k \geq 0$ (a base utility from the good). The condition from Step 5 becomes

$$(\theta_2 - \theta_1)[k(s_2 - s_1) - (c_1 s_2 - c_2 s_1)] > 0 \quad \Leftrightarrow \quad k > \frac{c_1 s_2 - c_2 s_1}{s_2 - s_1}$$

- If $c_1 = c_2 = c$, this becomes $k > c$.
- **Information goods** ($c = 0$) with $k > 0$: the thresholds are

$$\underline{\lambda} = \frac{\theta_1(s_2 - s_1)}{\theta_2(s_2 - s_1)} = \frac{\theta_1}{\theta_2} \quad \text{and} \quad \bar{\lambda} = \frac{k + \theta_1 s_1}{k + \theta_2 s_1}$$

- So menu pricing is optimal when $\frac{\theta_1}{\theta_2} < \lambda < \frac{k + \theta_1 s_1}{k + \theta_2 s_1}$.

Monopoly menu pricing (cont'd)

- Quality-dependent prices: a numerical example
 - Monopolist produces software in 2 versions:
 - Basic version and Pro version (higher quality, with advanced computing functionalities); $c_{basic} = c_{pro} = 0$
 - 120 potential consumers
 - types: λ universities (high type) and $120 - \lambda$ businesses (low type)
 - Willingness to pay:

	Universities	Businesses
Pro	9	3
Basic	5	2

- Single-crossing: $U(\theta_2, s_2) - U(\theta_2, s_1) = 4 > U(\theta_1, s_2) - U(\theta_1, s_1) = 1$

Monopoly menu pricing (cont'd)

	Universities λ	Businesses $120 - \lambda$
Pro	9	3
Basic	5	2

- A numerical example (cont'd)
 - Optimal uniform pricing
 - Sell Pro version.
 - Either at $p_{pro} = 9 \rightarrow q_{pro} = \lambda$ & $\pi^{uni} = 9\lambda$
 - Or at $p_{pro} = 3 \rightarrow q_{pro} = 120$ & $\pi^{uni} = 360$
 - So, $\pi^{uni} = \max \{9\lambda, 360\}$
 - If seller can tell universities and businesses apart $\rightarrow$ personalized pricing
 - Sell Pro version at $p_{pro} = 9$ to universities and at $p_{pro} = 3$ to businesses $\rightarrow \pi^{pers} = 9\lambda + 3(120 - \lambda) = 360 + 6\lambda$
 - If seller **cannot** tell universities and businesses apart $\rightarrow$ menu pricing
 - Use the 2 versions to induce self-selection: sell Pro version to universities and Basic version to businesses
 - Problem: find incentive compatible prices

Monopoly menu pricing (cont'd)

	Universities λ	Businesses $120 - \lambda$
Pro	9	3
Basic	5	2

- A numerical example (cont'd)
 - Let's find menu prices by trial and error
 - 1st trial: charge each group its reservation price
 - $p_{pro} = 9$ and $p_{basic} = 2$
 - Problem: universities prefer Basic version as it yields larger surplus: $9 - 9 < 5 - 2 \rightarrow$ self-selection is not achieved
 - Self-selection (or incentive compatibility) constraint: price difference $\leq$ premium universities are willing to pay for upgrading to the Pro version: $p_{pro} - p_{basic} \leq 9 - 5 = 4$
 - 2nd trial: charge universities their reservation price and compute incentive compatible price of Basic version
 - $p_{pro} = 9$ and $p_{basic} = 9 - 4 = 5$
 - Problem: businesses don't buy!
 - Participation constraint: price of Basic version $\leq$ businesses' reservation price: $p_{basic} \leq 2$

Monopoly menu pricing (cont'd)

	Universities λ	Businesses $120 - \lambda$
Pro	9	3
Basic	5	2

- A numerical example (cont'd)
 - Optimum
 - Combining the 2 constraints: $p_{basic} = 2$ and $p_{pro} = 2 + 4 = 6$
 - Profits: $\pi^{menu} = 6\lambda + 2(120 - \lambda) = 240 + 4\lambda$
 - Menu vs. group pricing
 - Lower profits under menu pricing:
 $\pi^{menu} - \pi^{pers} = -(120 + 2\lambda) < 0$
 - Inducing self-selection induces two types of losses:
 - Businesses are offered a low-quality product instead of a high-quality one $\rightarrow$ loss: $(120 - \lambda)(2 - 3) = -(120 - \lambda)$
 - Universities are sold the high-quality product at a discount; they are left with an 'information rent' $\rightarrow$ loss: $\lambda(6 - 9) = -3\lambda$
 - Total loss: $-(120 - \lambda) - 3\lambda = -(120 + 2\lambda)$

Monopoly menu pricing: summary

- **Lesson:** Consider a monopolist who offers 2 pairs of price and quality to 2 types of consumers. Prices are chosen so as to fully appropriate low-type's consumer surplus. High-type consumers obtain a positive surplus ('information rent') as they can always choose the low-quality instead.

Monopoly menu pricing (cont'd)

- A numerical example (cont'd)
 - Menu vs. uniform pricing
 - Menu pricing *may* improve profits.
 - Scenario 1: $\lambda > 40 \rightarrow$ firm only sells to universities under uniform pricing $\rightarrow \pi^{uni} = 9\lambda$
 - **Cannibalization**: universities now pay less for Pro version $\rightarrow$ loss of $\lambda(6-9) = -3\lambda$
 - **Market expansion**: businesses now buy Basic version $\rightarrow$ gain of $(120 - \lambda)2$
 - Net gain if $-3\lambda + (120 - \lambda)2 > 0 \Leftrightarrow \lambda < 48$
 - *If so, menu pricing also increases welfare* (firm and universities strictly better off; businesses as well off)

	Universities λ	Businesses $120 - \lambda$
Pro	9	3
Basic	5	2

Monopoly menu pricing (cont'd)

- A numerical example (cont'd)
 - Menu vs. uniform pricing (cont'd)
 - Scenario 2: $\lambda < 40 \rightarrow$ firm sells to everyone under uniform pricing $\rightarrow \pi^{uni} = 360$
 - No market expansion in this case, but 2 opposite effects.
 - Businesses buy Basic instead of Pro version $\rightarrow$ loss of $(120 - \lambda)(2 - 3)$
 - Universities pay more for Pro version $\rightarrow$ gain of $\lambda(6 - 3)$
 - Net gain if $-(120 - \lambda) + 3\lambda > 0 \Leftrightarrow \lambda > 30$
 - If so, menu pricing reduces welfare (firm better off, but universities worse off; businesses as well off)

	Universities λ	Businesses $120 - \lambda$
Pro	9	3
Basic	5	2

Monopoly menu pricing (cont'd)

- **Summary of the numerical example** (120 consumers, λ universities).
- The thresholds match the general formulas: $\underline{\lambda} = \frac{3-2}{9-5} = \frac{1}{4}$ (30 of 120), $\lambda_0 = \frac{3}{9}$ (40 of 120), and $\bar{\lambda} = \frac{2}{5}$ (48 of 120).

Number of universities	Optimal pricing	Profit
$\lambda < 30$	Uniform: Pro at 3 to everyone	360
$30 < \lambda < 48$	Menu: Pro at 6, Basic at 2	$240 + 4\lambda$
$\lambda > 48$	Uniform: Pro at 9 to universities	9λ

Monopoly menu pricing: summary

- **Lesson:** Menu pricing is optimal (i) if proportion of high-type consumers is neither too small nor too large, and (ii) if going from low to high quality increases surplus proportionally more for high-type consumers than for low-type consumers.
- **Lesson:** Menu pricing improves welfare if selling the low quality leads to an expansion of the market; otherwise, menu pricing deteriorates welfare.

Monopoly menu pricing: further results

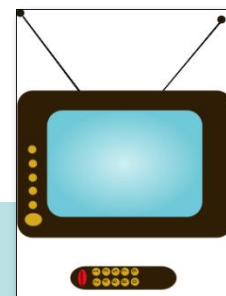
- If the monopolist also chooses the two qualities (see the distortion-of-quality slides):

$$c'(s_2) = \frac{\partial U(\theta_2, s_2)}{\partial s_2} \quad \text{and} \quad c'(s_1) < \frac{\partial U(\theta_1, s_1)}{\partial s_1}$$

- Lesson:** High-type consumers are offered the socially optimal quality, while low-type consumers are offered a quality that is distorted downward compared to the first best.

Monopoly menu pricing: further results

- Damaged good strategy may be profitable
 - Firm intentionally damages portion of the goods to price discriminate.



Case. Damaged goods

- IBM LaserPrinter E → identical to original printer, but software limited printing to 5 rather than 10 pages/minute
- Sony MiniDisc 60' → curbed 74' disc
- Sharp DVD players → DVE611 and DV740U are almost identical, but DV740U does not allow user to play output encoded in PAL format on NTSC televisions (a critical button is hidden on the remote)

Monopoly menu pricing: further results (cont'd)

- Previous quality model
 - Suppose linear utility: $U(\theta, s) = \theta s$
 - Cost of producing one unit of given quality: $c(s_i)$
- Transposition to time-dependent prices
 - Let $s = e^{-rt}$, where t = date when the good is produced and delivered, and r = interest rate

$$\max_{t_1, t_2} (1 - \lambda) [\theta_1 e^{-rt_1} - c(e^{-rt_1})] + \lambda [\theta_2 e^{-rt_2} - (\theta_2 - \theta_1) e^{-rt_1} - c(e^{-rt_2})]$$

Monopoly menu pricing: further results (cont'd)

- Transposition to quantity-dependent prices
 - Consumers can buy a certain quantity q_i at price p_i
 - Unit price may depend on quantity purchased (nonlinear pricing). Let $q_i = c(s_i)$
 $\rightarrow s_i = c^{-1}(q_i) = V(q_i)$

$$\max_{q_1, q_2} (1 - \lambda) [\theta_1 V(q_1) - q_1] + \lambda [\theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) - q_2]$$

Menu pricing under imperfect competition

- Monopoly setting gives useful insights.
- But, we want to know how menu pricing is affected by - and affects - competition.
 - E.g.: airline travel
 - Empirical studies suggest that competition tends to reinforce price discrimination
 - Borenstein (1991): number of stations offering leaded gas ↓
→ difference between margins on unleaded and leaded gas ↓
- 2 extensions of Hotelling model
 - Quality-based menu pricing
 - Two-part tariffs (quantity-based menu pricing)

Menu pricing under imperfect competition (cont'd)

- Competitive quality-based menu pricing
 - Sketch of the model
 - 2 firms located at the extremes of Hotelling line
 - Each firm can sell high-end & low-end versions of some good
 - Mass 1 of consumers uniformly distributed on the line
 - Heterogeneous in terms of transportation costs
 - Heterogeneous in terms of valuation of quality

Menu pricing under imperfect competition (cont'd)

- Competitive quality-based menu pricing (cont'd)**

- Utility of buying good k is $U = s_K - \alpha_k p_{iK}$ ← α_k is marginal utility of income, $\alpha_h < \alpha_l$

s_K is quality of good k , $s_H > s_L$

- If multiply by $\frac{1}{\alpha_k} = \theta_k$,

$$U = \frac{1}{\alpha_k} s_K - \frac{1}{\alpha_k} \alpha_k p_{iK} = \theta_k s_K - p_{iK}$$

- Utility of k -type consumer can be

$$\begin{aligned} & s_H - \alpha_k p_{1H} - x \\ & s_H - \alpha_k p_{2H} - (1 - x) \\ & s_L - \alpha_k p_{1L} - x \\ & s_L - \alpha_k p_{2L} - (1 - x) \end{aligned}$$

If buying H from firm 1

If buying H from firm 2

If buying L from firm 1

If buying L from firm 2

Menu pricing under imperfect competition (cont'd)

- **Competitive quality-based menu pricing** (cont'd)
- **Discriminatory equilibrium**
- Firm 1 sets p_{1H} and p_{1L} , firm 2 sets p_{2H} and p_{2L}
- h -type consumer buys H , he is indifferent between buying from 1 and 2 if

$$s_H - \alpha_h p_{1H} - x_h = s_H - \alpha_h p_{2H} - (1 - x_h)$$

$$\Rightarrow x_h = \frac{1 + \alpha_h(p_{2H} - p_{1H})}{2}$$

- l -type consumer buys L , he is indifferent between buying from 1 and 2 if

$$s_L - \alpha_l p_{1L} - x_l = s_L - \alpha_l p_{2L} - (1 - x_l)$$

$$\Rightarrow x_l = \frac{1 + \alpha_l(p_{2L} - p_{1L})}{2}$$

- We have 2 separate Hotelling models, one is selling H to h consumers only, the other is selling L to l consumers only.

Menu pricing under imperfect competition (cont'd)

- **Competitive quality-based menu pricing** (cont'd)
- Given demands, firm 1 solves

$$\max_{p_{1H}, p_{1L}} \pi_1 = p_{1H} x_h + p_{1L} x_l$$

- Firm 2 solves

$$\max_{p_{2H}, p_{2L}} \pi_2 = p_{2H} (1 - x_h) + p_{2L} (1 - x_l)$$

- The Nash equilibrium prices are

$$p_H^* = p_{1H}^* = p_{2H}^* = \frac{1}{\alpha_h}$$

$$p_L^* = p_{1L}^* = p_{2L}^* = \frac{1}{\alpha_l}$$

- If $\alpha_h = 1, \alpha_l = 5$

$$p_H^* = 1, p_L^* = \frac{1}{5}$$

$$x_h = \frac{1}{2}, x_l = \frac{1}{2}, \quad \pi^* = \pi_1^* = \pi_2^* = \frac{3}{5}$$

Menu pricing under imperfect competition (cont'd)

- **Competitive quality-based menu pricing** (cont'd)
- Are these prices incentive-compatible?
- Assume $s_H = 3$, $s_L = 2$
- h -type consumers

$$s_H - \alpha_h p_H^* \geq s_L - \alpha_h p_L^* \\ \Rightarrow 3 - 1 \cdot 1 \geq 2 - 1 \cdot \frac{1}{5} \Rightarrow 2 \geq \frac{9}{5} \quad \checkmark$$

- l -type consumers

$$s_L - \alpha_l p_L^* \geq s_H - \alpha_l p_H^* \\ \Rightarrow 2 - 5 \cdot \frac{1}{5} \geq 3 - 1 \cdot 5 \Rightarrow 1 \geq -2 \quad \checkmark$$

Menu pricing under imperfect competition (cont'd)

- **Competitive quality-based menu pricing** (cont'd)
- Other price deviation
 - i. Sell H to l -types, and L to h -types . $\rightarrow$ Not feasible, since h prefer H when l purchase H .
 - ii. Sell L to all consumers. $\rightarrow$ If the firm earns any profit from selling L at p_{1L} to some consumers, then it can further increase profit by selling H to the same consumers at $p_{1L} + \frac{s_H - s_L}{\alpha_l} - \epsilon$, where $\frac{s_H - s_L}{\alpha_l}$ is quality premium scaled by consumer's WTP.
 - iii. Sell H to everyone.

Menu pricing under imperfect competition (cont'd)

- **Deviation: sell H to everyone.** Firm 2 keeps $p_{2H}^* = 1$ and $p_{2L}^* = \frac{1}{5}$; firm 1 sets p_{1H} and sells only H .
 - h -types compare H from 1 and H from 2: $x_h = \frac{1 + \alpha_h(p_{2H}^* - p_{1H})}{2} = \frac{2 - p_{1H}}{2}$.
 - l -types compare H from 1 and L from 2: $\tilde{x}_l = \frac{1 + (s_H - s_L) + \alpha_l(p_{2L}^* - p_{1H})}{2} = \frac{3 - 5p_{1H}}{2}$.
- **Firm 1's problem.**

$$\max_{p_{1H}} p_{1H} \left[\frac{2 - p_{1H}}{2} + \frac{3 - 5p_{1H}}{2} \right] = \frac{p_{1H}(5 - 6p_{1H})}{2}$$

$$\Rightarrow p_{1H} = \frac{5}{12} \quad \Rightarrow \quad \tilde{\pi}_1 = \frac{5}{12} \times \frac{5}{4} \times \frac{1}{2} \approx 0.521 < 0.6 = \pi_1^*$$
 - The deviation is not profitable.

Menu pricing under imperfect competition (cont'd)

- **Competitive quality-based menu pricing** (cont'd)
- **Non-discriminatory equilibrium**

- Every firm i offers H only to all types of consumers

- h -type consumers comparing p_{1H} and p_{2H}

$$s_H - \alpha_h p_{1H} - x_h = s_H - \alpha_h p_{2H} - (1 - x_h)$$

$$\Rightarrow x_h = \frac{1 + \alpha_h(p_{2H} - p_{1H})}{2}$$

- l -type consumers comparing p_{1H} and p_{2H}

$$s_H - \alpha_l p_{1H} - \tilde{x}_l = s_H - \alpha_l p_{2H} - (1 - \tilde{x}_l)$$

$$\Rightarrow \tilde{x}_l = \frac{1 + \alpha_l(p_{2H} - p_{1H})}{2}$$

Menu pricing under imperfect competition (cont'd)

- **Non-discriminatory equilibrium.** Firm 1's profit from selling H to both types:

$$\max_{p_{1H}} p_{1H} \left[\frac{1 + \alpha_h(p_{2H} - p_{1H})}{2} + \frac{1 + \alpha_l(p_{2H} - p_{1H})}{2} \right] = p_{1H} \left[1 + \frac{\tilde{\alpha}}{2}(p_{2H} - p_{1H}) \right]$$

- where $\tilde{\alpha} \equiv \alpha_h + \alpha_l$.
- **Best response.** The FOC is $1 + \frac{\tilde{\alpha}}{2}p_{2H} - \tilde{\alpha}p_{1H} = 0$, so

$$p_{1H}(p_{2H}) = \frac{1}{\tilde{\alpha}} + \frac{1}{2}p_{2H}$$

- **Symmetric equilibrium** with $\alpha_h = 1$ and $\alpha_l = 5$ ($\tilde{\alpha} = 6$): each firm sells to half of each type, so

$$\hat{p}_H = \frac{2}{\tilde{\alpha}} = \frac{1}{3} \quad \text{and} \quad \hat{\pi} = \frac{2}{\tilde{\alpha}} = \frac{1}{3}$$

Menu pricing under imperfect competition (cont'd)

- **Deviation.** Firm 2 keeps $\hat{p}_H = \frac{1}{3}$; firm 1 raises p_{1H} to sell only to h -types.
- l -types do not buy from firm 1 (even at $x = 0$) if

$$s_H - \alpha_l \hat{p}_H - 1 > s_H - \alpha_l p_{1H} \quad \Leftrightarrow \quad p_{1H} > \hat{p}_H + \frac{1}{\alpha_l} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

- Firm 1 then solves

$$\max_{p_{1H}} p_{1H} \frac{1 + \alpha_h(\hat{p}_H - p_{1H})}{2} = \frac{p_{1H}(4 - 3p_{1H})}{6}$$

$$\Rightarrow p_{1H} = \frac{2}{3} > \frac{8}{15} \quad \Rightarrow \quad \pi_{1H} = \frac{2}{3} \times \frac{2}{6} = \frac{2}{9} < \frac{3}{9} = \hat{\pi}$$

- Firm 1 has no incentive to deviate.

Menu pricing under imperfect competition (cont'd)

- Competitive quality-based menu pricing (cont'd)
 - Main results
 - Multiple equilibria → Coexistence of:
 - ‘Discriminatory’ equilibrium: both firms offer 2 versions, consumers self-select (high types buy high-end version, low types buy low-end version)
 - ‘Non-discriminatory’ equilibrium: both firms produce only the high-end version

Menu pricing under imperfect competition (cont'd)

- Competitive quality-based menu pricing (cont'd)
 - Comparison with monopoly
 - Here, monopolist would optimally choose uniform pricing → introducing a competitor may lead to menu pricing by both firms.
 - In duopoly, firms may prefer to coordinate on the situation where they both price discriminate, while, as a monopolist, each firm would not price discriminate.
 - Incentive compatibility constraints may not be binding in duopoly.

Menu pricing under imperfect competition (cont'd)

- Competitive quantity-based menu pricing
 - Sketch of the model
 - 2 firms located at the extremes of Hotelling line
 - Each firm sets a two-part tariff: $T_i(q) = m_i + p_i q$
 - m_i : fixed fee; p_i : variable fee
 - E.g., telephony: subscription fee + price per minute
 - Mass 1 of consumers uniformly distributed on the line
 - One-stop shoppers, identical variable demand (consumers can consume any quantity from the firm they patronize)

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing (cont'd)**
- Utility is $u(q)$

$$\max_q u(q) - p_i q = v(p_i)$$

- Let $FOC_q = 0$

$$\Rightarrow u'(q) = p_i$$

Walrasian
demand

- Utility is then

$$w(p_i, m_i) = v(p_i) - m_i$$

Fixed fee

Utility from
consuming
 q_i unit

- If buys from firm 1, net utility is $r - \tau x + w(p_1, m_1)$.
- If buys from firm 2, net utility is $r - \tau(1 - x) + w(p_2, m_2)$.

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing** (cont'd)
- Indifferent consumer satisfies

$$r - \tau \hat{x} + w(p_1, m_1) = r - \tau(1 - \hat{x}) + w(p_2, m_2)$$

$$\Rightarrow \hat{x} = \frac{\tau + w(p_1, m_1) - w(p_2, m_2)}{2\tau} = \frac{\tau + w_1 - w_2}{2\tau}$$

- Demand for firm 1 is $\hat{x}$, demand for firm 2 is $1 - \hat{x}$.
- Firm 1 solves

$$\max_{p_1, m_1} \pi_1 = \hat{x}[(p_1 - c)q(p_1) + m_1]$$

- Since $w_1 = v(p_1) - m_1$, $m_1 = v(p_1) - w_1$. We can rewrite firm's PMP as

$$\max_{p_1, m_1} \pi_1 = \frac{\tau + w_1 - w_2}{2\tau} [(p_1 - c)q(p_1) + v(p_1) - w_1]$$

Menu pricing under imperfect competition (cont'd)

- Competitive quantity-based menu pricing (cont'd)**

- FOC_{p_1}

$$q(p_1) + q'(p_1)(p_1 - c) + v'(p_1) = 0$$

- Since $v'(p_1) = -q(p_1)$, we can rewrite FOC_{p_1} as

$$\begin{aligned} q(p_1) + q'(p_1)(p_1 - c) - q(p_1) &= 0 \\ \Rightarrow q'(p_1)(p_1 - c) &= 0 \\ \Rightarrow p_1^* &= c = p_2^* \end{aligned}$$

- FOC_{w_1}

$$[(p_1 - c)q(p_1) + v(p_1) - w_1] - (\tau + w_1 - w_2) = 0$$

- Plugging $p_1 = c$ and invoking symmetry, $w_1 = w_2$, yields

$$\boxed{m_1} \rightarrow \boxed{v(p_1) - w_1} - \tau = 0$$

$$\Rightarrow m_1^* = \tau = m_2^*$$

$$\Rightarrow \pi_1^* = \frac{1}{2}\tau = \pi_2^*$$

Menu pricing under imperfect competition (cont'd)

- **Linear pricing (no fixed fee).** Notation: $\pi(p) = (p - c)q(p)$, and index a price by k through $p(k) = c + k(p - c)$, so that $k = 0$ is marginal-cost pricing and $k = 1$ is price p .

$$s(k) \equiv v(c + k(p - c)) \Rightarrow s_k(k) = (p - c)v'(c + k(p - c))$$

- Since $v' = -q$, profit at price $p(k)$ is

$$\pi(c + k(p - c)) = k(p - c)q(c + k(p - c)) = -ks_k(k)$$

- In particular, $\pi(p) = -s_k(1)$.
- **Firm i 's problem.** With the rival at p , firm i chooses k :

$$\max_k \Pi_i = \frac{\tau + s(k) - s(1)}{2\tau} (-ks_k(k))$$

- A symmetric equilibrium requires $k = 1$ to be optimal.

Menu pricing under imperfect competition (cont'd)

- **Check that $k = 1$ is optimal.** Firm i maximizes

$\frac{\tau + s(k) - s(1)}{2\tau} (-ks_k(k))$. The FOC is

$$s_k(k)(-ks_k(k)) + (\tau + s(k) - s(1))(-s_k(k) - ks_{kk}(k)) = 0$$

- At $k = 1$, $s(k) - s(1) = 0$, so

$$-(s_k(1))^2 - \tau(s_k(1) + s_{kk}(1)) = 0 \quad \Rightarrow \quad -s_k(1) = s_{kk}(1) + \frac{(s_k(1))^2}{\tau}$$

- Since industry profit under linear pricing is $\pi(p) = -s_k(1)$,

$$\Pi_L = s_{kk}(1) + \frac{(s_k(1))^2}{\tau} = s_{kk}(1) + \frac{\Pi_L^2}{\tau}$$

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing** (cont'd)
- We know $s_k(k) = (p - c)v'(c + k(p - c))$ and $v'(p_i) = -q(p_i)$, then

$$s_k(k) = -(p - c)q(c + k(p - c))$$

- Differentiating $s_k(k)$ with respect to k , yields

$$\begin{aligned} s_{kk}(k) &= -(p - c)q'(c + k(p - c))(p - c) \\ &= -(p - c)^2 q'(c + k(p - c)) > 0 \end{aligned}$$

$$\Rightarrow \Pi_L = s_{kk}(1) + \frac{(s_k(1))^2}{\tau} > \frac{(s_k(1))^2}{\tau} = \frac{1}{\tau} \Pi_L^2$$

$$\Rightarrow \tau > \Pi_L$$

Industry profit
under non-
linear pricing

Industry profit
under linear
pricing

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing (cont'd)**

- At $\hat{p} = c + k(p - c)$, welfare is

$$\begin{aligned} W(k) &= PS(k) + CS(k) \\ &= \Pi(c + k(p - c)) + p(c + k(p - c)) \\ &= -ks_k(k) + s(k) \end{aligned}$$

- **Non-linear pricing** $\rightarrow k = 0, \hat{p} = c$.

$$W(0) = -0 \cdot s_k(0) + s(0) = s(0)$$

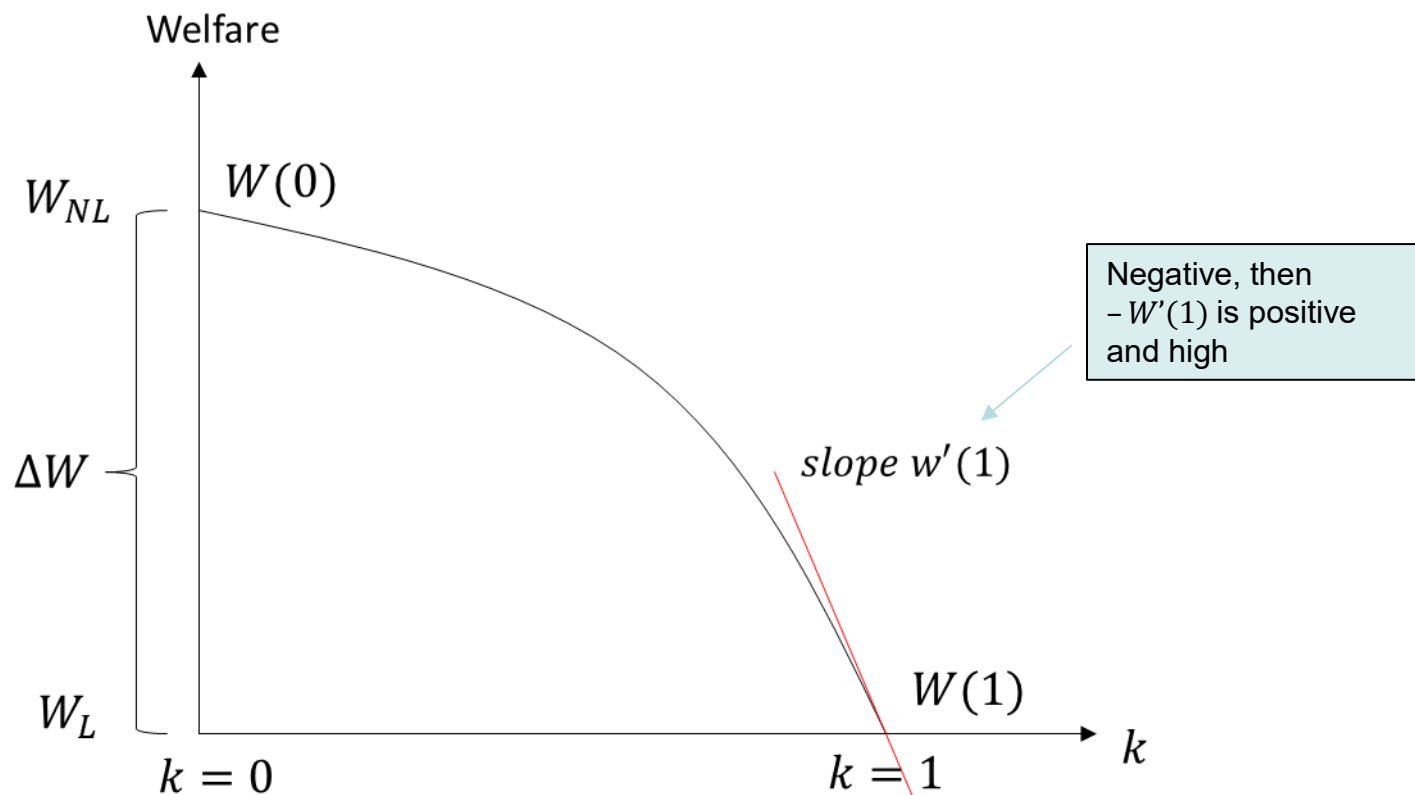
- **Linear pricing** $\rightarrow k = 1, \hat{p} = p$.

$$W(1) = -1 \cdot s_k(1) + s(1) = s(1) - s_k(1)$$

- Assume the welfare is concave in linear prices and thus concave in k .
- Let $\Delta W = W_{NL} - W_L = W(0) - W(1)$

Menu pricing under imperfect competition (cont'd)

- Competitive quantity-based menu pricing (cont'd)**



- Thus, $s_{kk}(1) = -W'(1) \geq \Delta W$

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing (cont'd)**
- Since $s_{kk}(1) = \Pi_L - \frac{1}{\tau} \Pi_L^2 = \frac{\Pi_L}{\tau} (\tau - \Pi_L)$, and $\Pi_L < \tau$, we have that $\frac{\Pi_L}{\tau} < 1$.

- Then $s_{kk}(1) = \frac{\Pi_L}{\tau} (\tau - \Pi_L) < \tau - \Pi_L$.
 $\Rightarrow \Delta W = W_{NL} - W_L < s_{kk}(1) < \tau - \Pi_L = \Pi_{NL} - \Pi_L$

- Finally, since $W = CS + \Pi$, we have $CS = W - \Pi$.

$$\Rightarrow CS_{NL} < CS_L$$

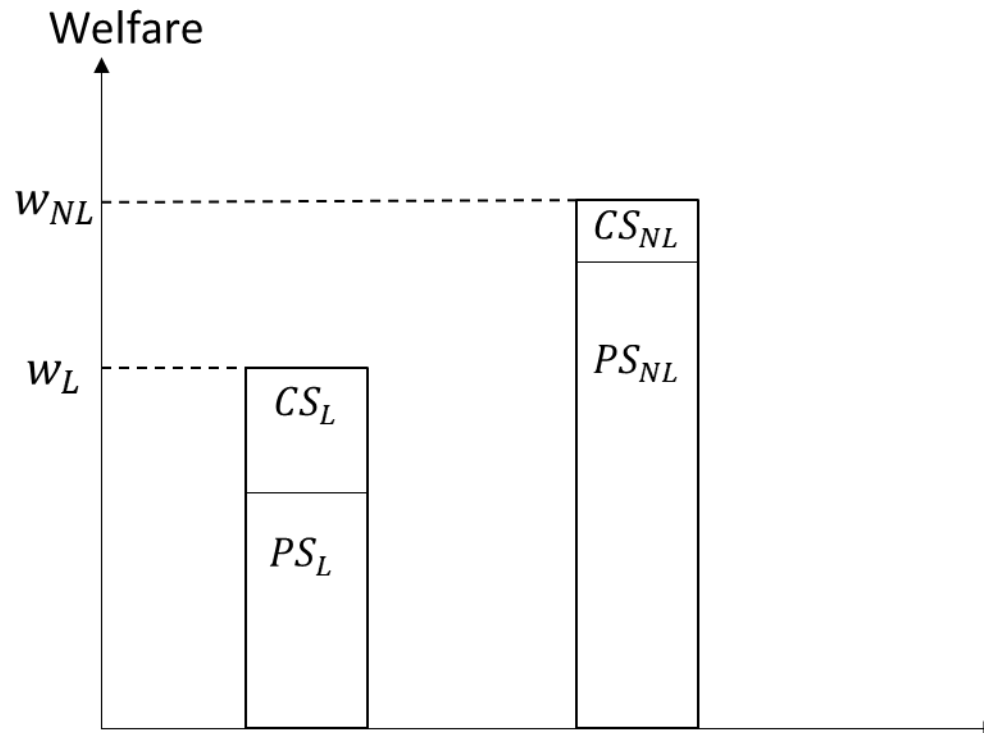
Consumers are worse off under NL than L

$$W_{NL} - \Pi_{NL}$$

$$W_L - \Pi_L$$

Menu pricing under imperfect competition (cont'd)

- Competitive quantity-based menu pricing (cont'd)



$$\begin{aligned} W_{NL} &> W_L \\ \Pi_{NL} &> \Pi_L \\ CS_{NL} &< CS_L \end{aligned}$$

Menu pricing under imperfect competition (cont'd)

- **Competitive quantity-based menu pricing (cont'd)**
- Under NL , lower fee has 2 effects
 - i. Positive effect → stealing consumers from rival.
 - ii. Negative effect → lower revenue.
- Under L , lower price has 3 effects
 - i. Positive effect → stealing consumers from rival.
 - ii. Negative effect → lower revenue.
 - iii. Positive effect → an increase in demand.

Menu pricing under imperfect competition (cont'd)

- Competitive quantity-based menu pricing (cont'd)
 - Main results
 - Unique symmetric equilibrium: firms offer tariffs
 $T(q) = \tau + cq$
 - τ : transport cost parameter
 - c : firms' marginal cost
 - Competition with two-part tariffs improves welfare compared to competition with linear tariffs

Review questions

- Suppose a firm can target two groups of consumers by a menu of prices with different qualities but that it can also offer different prices to different consumer groups. What should it do?
- When does menu pricing dominate uniform pricing in monopoly? Discuss the countervailing effects.
- How does competition affect the use of menu pricing? Discuss.
- What are the effects of competition on quantity-based menu pricing?