

Part III. Sources of market power

Chapter 6. Advertising and related marketing strategies



Slides

Industrial Organization: Markets and Strategies

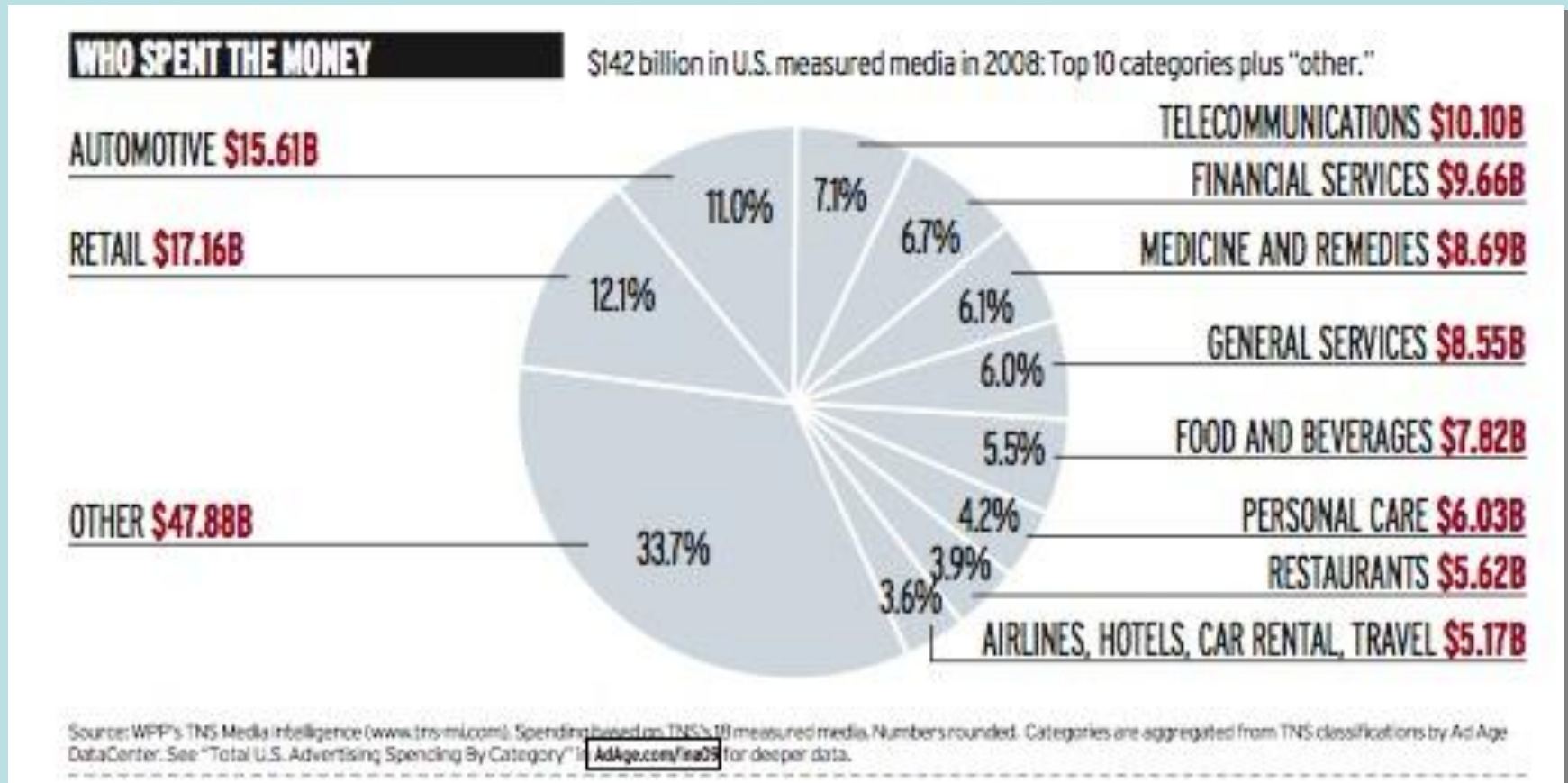
Paul Belleflamme and Martin Peitz, 2d Edition

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Chapter 6. Learning objectives

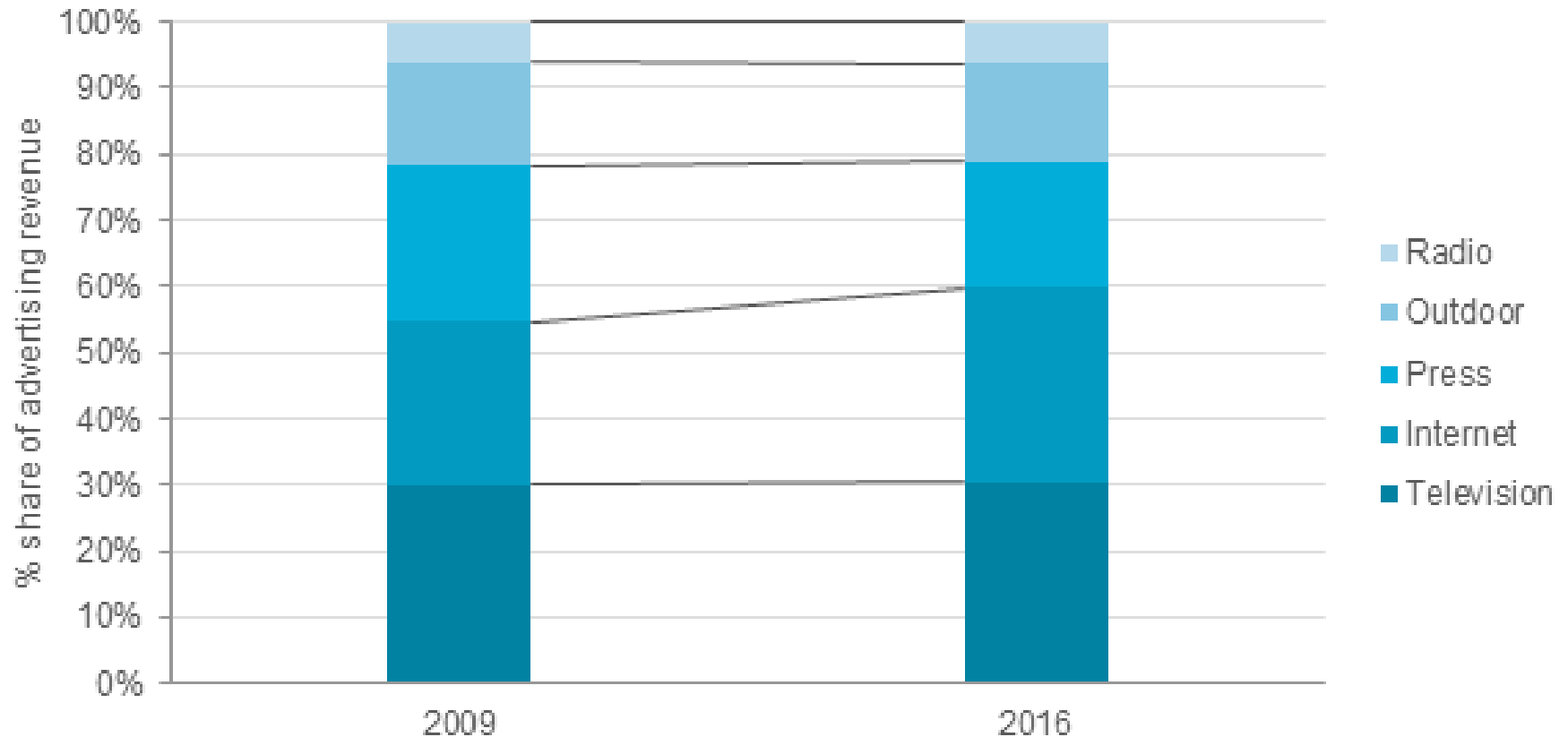
- Understand to what end and to what effect firms devote resources to advertising.
- Be able to distinguish between the different views on advertising: informative, persuasive and complementary advertising.
- Understand how a monopolist chooses advertising expenditures and how this choice affects welfare.
- Understand how advertising decisions are made in oligopoly settings and how they affect price competition.

Case. U.S. media spending on advertising



(Data from *AdvertisingAge*, July 2009)

Case. U.S. media spending on advertising



(From *AdvertisingAge*)

Why do firms advertise?

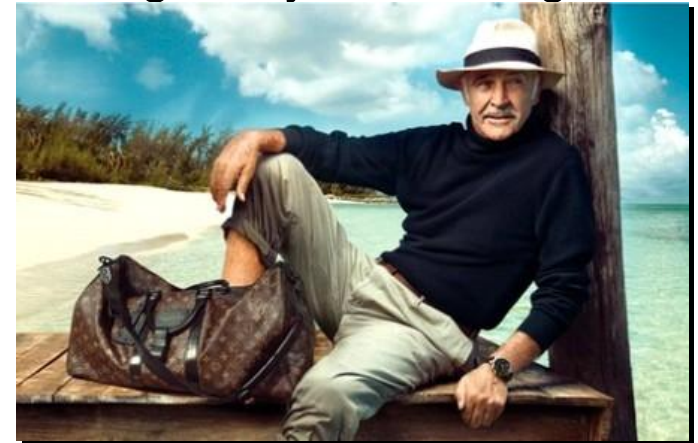
- Advertising raises the demand for the advertised product. It can do so in three ways:
 - **Inform** consumers about the product's existence, price, or characteristics.
 - **Persuade** consumers, changing their tastes for the product.
 - **Complement** the product, adding an image that consumers value.
- Why consumers respond to advertising determines its effects on prices, competition, and welfare (next slides).

Why do consumers respond to advertising?

- 1st view: advertising is persuasive
 - It alters consumers' tastes.
 - It $\uparrow$ product differentiation & consumers' loyalty.
 - Likely effects (to be confirmed)
 - Demand becomes less elastic; prices $\uparrow$; entry becomes more difficult; welfare $\downarrow$.
- 2nd view: advertising is informative
 - It conveys information about existence, prices and characteristics of products (directly or indirectly).
 - Likely effects (to be confirmed)
 - Demand becomes more elastic; prices $\downarrow$; welfare $\uparrow$.

Why do consumers respond to advertising? (cont'd)

- 3rd view: advertising is complementary
 - Complementary to the advertised product.
 - It enters into the consumer's utility function, in complement with the product itself.
 - Idea: consumers consume 'social images' by combining products and advertising.
- **Likely effects** (to be confirmed)
 - Similar to persuasive view.
 - Yet, it may be beneficial through its direct valuation by the consumers.



Having a Louis Vuitton bag may allow you to look as cool as Sean Connery if (1) you actually possess such a bag and (2) the ad links the bag to the cool attitude that you want to project in your relevant peer group.

How to distinguish the 3 views empirically?

- Classify advertising spots
 - Possible to identify directly informative advertising.
 - Much harder to separate indirectly informative and persuasive advertising.
- Find industries subject to a shock and analyze the effect of advertising on market outcomes
 - E.g., a certain type of advertising becomes (il)legal
- Look at implied purchasing behavior
 - Informative advertising is valuable for inexperienced consumers but not for experienced consumers.
 - Persuasive & complementary advertising affect both types in the same way.
 - **Testable hypothesis:** *informative advertising affects demand of inexperienced consumers more strongly.*

Case. Yoplait 150



- Data
 - Yoplait 150: 1st low calorie, low fat yogurt, introduced into the US market in 1987.
 - Scanner data collected at Sioux Falls, South Dakota and Springfield, Missouri (about 4,000 households)
 - Weekly prices at drugstores and supermarkets over three years (1986-1988).
 - A.C. Nielsen TV meters: household TV advertisement exposures.
- Main result
 - Advertisement affects initial purchases much more than repeat purchases.
 - Supports the view that *advertising was predominantly informative* in the Yoplait 150 case.

Price and non-price strategies in monopoly

- **Dorfman-Steiner model.** A monopolist chooses its price p and advertising expenditure A . Demand is $Q(p, A)$, with $Q_p < 0$ and $Q_A > 0$:

$$\max_{p,A} \Pi(p, A) = pQ(p, A) - C(Q(p, A)) - A$$

- **Step 1. Price.** $\frac{\partial \Pi}{\partial p} = (p - C')Q_p + Q = 0$. Dividing by pQ_p ,

$$\frac{p - C'}{p} = -\frac{Q}{pQ_p} = \frac{1}{\eta_{Q,p}}$$

- The usual Lerner rule, where $\eta_{Q,p} = -\frac{p}{Q}Q_p$ is the price elasticity of demand.

Price and non-price strategies in monopoly (cont'd)

- **Step 2. Advertising.** $\frac{\partial \Pi}{\partial A} = (p - C')Q_A - 1 = 0$. Dividing by p and multiplying and dividing by $\frac{A}{Q}$,

$$\frac{p - C'}{p} = \frac{1}{pQ_A} = \frac{1}{\eta_{Q,A}} \frac{A}{pQ}$$

- where $\eta_{Q,A} = \frac{A}{Q} Q_A$ is the advertising elasticity of demand.
- **Step 3. Combining Steps 1 and 2.** Both right-hand sides equal the Lerner index, so

$$\underbrace{\frac{A}{pQ}}_{\text{advertising intensity}} = \frac{\eta_{Q,A}}{\eta_{Q,p}}$$

Price and non-price strategies in monopoly (cont'd)

- **Example.** $Q(p, A) = A^\gamma p^{-\varepsilon}$, with constant elasticities $\eta_{Q,A} = \gamma$ and $\eta_{Q,p} = \varepsilon$.
 - With $\gamma = 0.1$ and $\varepsilon = 2$: the Lerner index is $\frac{1}{2}$, and advertising is $\frac{0.1}{2} = 0.05$, that is, 5% of revenue.
 - If demand becomes more responsive to advertising ($\gamma = 0.2$), advertising rises to 10% of revenue.
- **Lesson:** A monopoly sets its advertising intensity (advertising expenditure over revenue) equal to the ratio of the advertising elasticity of demand over the price elasticity of demand.

Closer look at how advertising affects demand

- **Persuasive advertising.** Consumers of mass 1 buy at most one unit; valuations θ are uniform on $[0,1]$.
- Advertising ‘inflates’ valuations: willingness to pay is $g(A)\theta$, with $g(0) = 1$ and $g'(A) > 0$.
- **Step 1. Demand.** A consumer buys if $g(A)\theta \geq p$, that is, if $\theta \geq \frac{p}{g(A)}$.

$$Q(p, A) = 1 - \frac{p}{g(A)}$$

- **Step 2. Price elasticity.**

$$\eta_{Q,p} = -\frac{p}{Q} \frac{\partial Q}{\partial p} = \frac{p}{g(A)} \times \frac{1}{1 - p/g(A)} = \frac{p}{g(A) - p}$$

Closer look at how advertising affects demand

(cont'd)

- **Step 3. Effect of advertising.** Since $g'(A) > 0$, $\eta_{Q,p} = \frac{p}{g(A)-p}$ decreases in A .
 - Example: $p = 1$. With $g(A) = 2$, $\eta_{Q,p} = 1$; with $g(A) = 3$, $\eta_{Q,p} = \frac{1}{2}$.
- More advertising makes demand **less elastic**, as predicted by the persuasive view.
- Intuitively, advertising raises all valuations proportionally, so the marginal consumer is further from the price, and a price increase loses fewer consumers.

Closer look at how advertising affects demand

(cont'd)

- **Informative advertising.** N consumers, each with demand $d(p)$, are initially unaware of the product; they learn about it only if they receive an ad.
- **Step 1. Reach.** The monopolist sends A ads, each reaching one consumer at random. A consumer receives none with probability

$$\left(1 - \frac{1}{N}\right)^A \approx e^{-A/N} \quad (\text{for large } N)$$

- **Step 2. Demand.**

$$Q(p, A) = \underbrace{N(1 - e^{-A/N})}_{G(A): \text{informed consumers}} d(p) = G(A)d(p)$$

- $G'(A) = e^{-A/N} > 0$ and $G''(A) = -\frac{1}{N}e^{-A/N} < 0$: diminishing returns to advertising.

Closer look at how advertising affects demand

(cont'd)

- **Step 3. Price elasticity.**

$$\eta_{Q,p} = -\frac{p}{Q} \frac{\partial Q}{\partial p} = -\frac{p G(A) d'(p)}{G(A) d(p)} = -\frac{p d'(p)}{d(p)}$$

- The elasticity does not depend on A : advertising enlarges the market without changing how each consumer responds to the price.
- More advertising **does not make demand less elastic**, as predicted by the informative view.

Some welfare effects of advertising

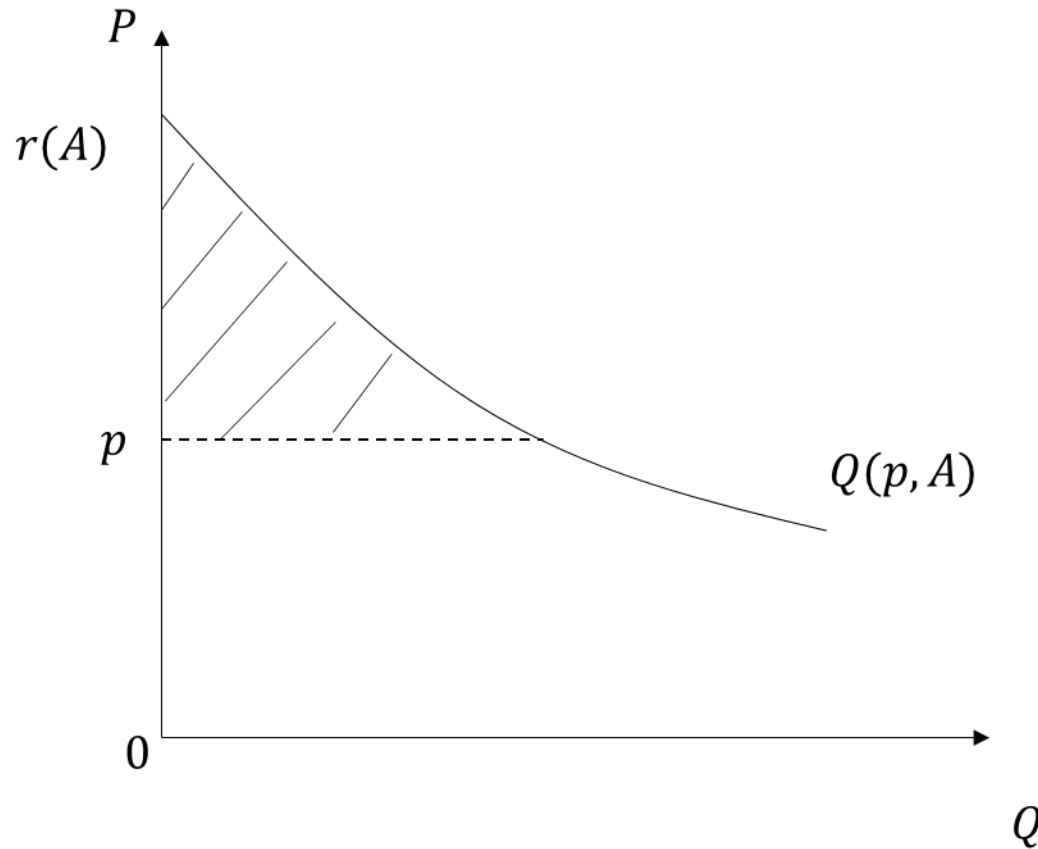
- Is advertising socially desirable?
 - We study the issue in the previous monopoly model
 - Starting point: monopoly solution, (p_m, A_m)
 - Change advertising to some nearby level A
 - Monopolist reacts with profit-maximizing price $p_m(A)$
 - Compute change in welfare, where welfare is defined by

$$W(p, A) = \Pi(p, A) + \int_p^{r(A)} Q(p, A) dp,$$

with $r(A)$ satisfying $Q(r(A), A) = 0$.

Maximum price consumers are willing to pay (may vary with A)

Some welfare effects of advertising (cont'd)



Some welfare effects of advertising (cont'd)

- We want to evaluate

$$\frac{dW(p_m(A), A)}{dA} \Big|_{A=A_m} = \left\{ \frac{d\Pi(p_m(A), A)}{dA} + Q(r(A), A) r'(A) - Q(p_m(A), A) p'_m(A) + \int_{p_m(A)}^{r(A)} Q_A(p, A) dp \right\} \Big|_{A=A_m} = 0$$

$$\Rightarrow \frac{dW(p_m(A), A)}{dA} \Big|_{A=A_m} = -Q(p_m, A_m) p'_m(A_m) + \int_{p_m}^{r(A_m)} Q_A(p, A_m) dp.$$

If advertising $\uparrow$ monopoly price, it $\downarrow$ consumer surplus and thus, welfare.

Additional consumer surplus that advertising generates on the infra-marginal units because of the demand shift.

Negative or positive effect

Positive effect

Net effect ?

Some welfare effects of advertising (cont'd)

- **Step 1. Sign of $p'_m(A)$.** The monopoly price $p_m(A)$ solves $\frac{\partial \Pi}{\partial p} = 0$.

Totally differentiating with respect to A ,

$$\frac{\partial^2 \Pi}{\partial p^2} dp + \frac{\partial^2 \Pi}{\partial p \partial A} dA = 0 \quad \Rightarrow \quad p'_m(A) = - \frac{\partial^2 \Pi / \partial p \partial A}{\partial^2 \Pi / \partial p^2}$$

- The denominator is negative (profit is concave in p), so $p'_m(A)$ has the sign of $\frac{\partial^2 \Pi}{\partial p \partial A}$.
- **Step 2. Cross derivative.** Differentiating $\frac{\partial \Pi}{\partial p} = (p - C')Q_p + Q$ with respect to A ,

$$\frac{\partial^2 \Pi}{\partial p \partial A} = Q_A + (p - C')Q_{pA} - C''Q_A Q_p$$

Some welfare effects of advertising (cont'd)

- **Informative advertising.** With $Q = G(A)d(p)$: $Q_p = Gd' < 0$, $Q_A = G'd > 0$, and $Q_{pA} = G'd'$. Substituting into Step 2,

$$\frac{\partial^2 \Pi}{\partial p \partial A} = G' \underbrace{[d + (p - C')d']}_{=0 \text{ by the price FOC}} - C'' \underbrace{Q_A Q_p}_{<0}$$

- Hence, $p'_m(A)$ has the same sign as C'' :
 - Constant or decreasing marginal cost ($C'' \leq 0$): more advertising does not raise the price, so the first welfare effect is nonnegative, and $\frac{dW}{dA} > 0$ at A_m . **Too little advertising.**
 - Increasing marginal cost ($C'' > 0$): more advertising raises the price, and the net welfare effect is ambiguous.

Some welfare effects of advertising (cont'd)

- **Persuasive advertising: example.** $Q(p, A) = 1 - \frac{p}{\alpha A}$ and $C(Q) = cQ$, with $\alpha > 4$.

- **Step 1. Price.** $\frac{\partial \Pi}{\partial p} = 1 - \frac{2p-c}{\alpha A} = 0$, so

$$p_m(A) = \frac{\alpha A + c}{2} \Rightarrow p'_m(A) = \frac{\alpha}{2} > 0$$

- More advertising raises the price: the first welfare effect is negative.

- **Step 2. Advertising.** $\frac{\partial \Pi}{\partial A} = \frac{p(p-c)}{\alpha A^2} - 1 = 0$. Substituting $p_m(A)$, $(\alpha A)^2 - c^2 = 4\alpha A^2$, so

$$A_m = \frac{c}{\sqrt{\alpha(\alpha - 4)}} \quad \text{and} \quad p_m = \frac{c}{2} \left(1 + \sqrt{\frac{\alpha}{\alpha - 4}} \right)$$

Some welfare effects of advertising (cont'd)

- **Numerical example.** $\alpha = 5$ and $c = 1$:

$$A_m = \frac{1}{\sqrt{5}} \approx 0.45 \quad \text{and} \quad p_m = \frac{1}{2}(1 + \sqrt{5}) \approx 1.62$$

- Check: $p_m(A_m) = \frac{5 \times 0.45 + 1}{2} \approx 1.62$.
- Advertising raises the price (first effect negative) but also raises consumers' valuations (second effect positive). Whether the monopolist advertises too much or too little depends on which effect dominates.

Some welfare effects of advertising (cont'd)

- **Lesson:** If additional advertising does not cause the monopolist to raise its price, then the monopolist will supply too little advertising. But if it does, then it induces 2 conflicting effects on welfare and the net effect is ambiguous.
- Effect of additional advertising on price?
 - Depends on the nature of advertising and on the monopolist's cost function.
 - Informative advertising: monopoly advertising is socially insufficient if marginal cost is constant or decreasing.
 - Persuasive advertising: even if advertising $\uparrow$ monopoly price, monopolist may provide too little advertising.

Does advertising toughen or soften competition?

- Advertising can play 2 roles
 - “Constructive” role
 - Informs consumers about existence, characteristics and price of products
 - ambivalent effect on price competition
 - Or: ↑ perceived differences between brands
 - likely to **soften** price competition
 - Combative role
 - Helps firms steal each other’s business
 - likely to **toughen** price competition
- This general intuition needs to be confirmed by looking at specific market settings.

Typologies of advertising

		Effect on rival firms?	
		Constructive role (Positive effect)	Combative role (Negative effect)
Effect on consumers?	Informative	<i>Promote a whole category of products</i>	<i>Inform about prices</i>
	Persuasive	<i>Increase perceived differentiation</i>	<i>Convince consumers to switch</i>

Informative advertising

- Intuition
 - In monopoly, more informative advertising → more informed consumers → more profits
 - In oligopoly, more informative advertising → more informed consumers **about several products** → **more intense competition** → more or less profits?
- Model: extension of Hotelling model
 - 2 firms located at extreme points of $[0,1]$
 - Mass 1 of consumers uniformly distributed on $[0,1]$
 - Utility of consumer x (assuming linear transport costs):

$$r - \tau x - p_1 \text{ if she buys 1 unit of good 1,}$$

$$r - \tau(1 - x) - p_2 \text{ if she buys 1 unit of good 2.}$$

Informative advertising (cont'd)

- Demands

- Only a share λ_i of consumers know about the existence of product i .
- Probability of being informed: independent of location
- 3 types of consumers

- Fully informed $\rightarrow$ share $\lambda_i \lambda_j \rightarrow$ indifferent consumer:

$$r - \tau \hat{x} - p_1 = r - \tau(1 - \hat{x}) - p_2 \Leftrightarrow \hat{x} = \frac{1}{2} - \frac{1}{2\tau}(p_1 - p_2)$$

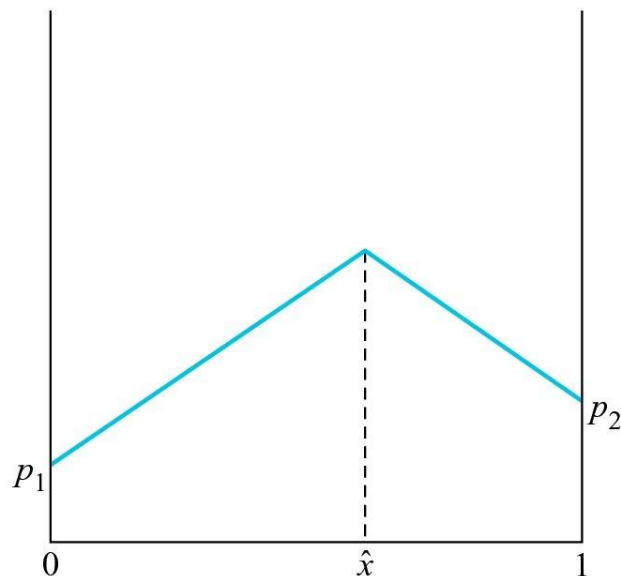
- Partially informed (know good i only) $\rightarrow$ share $\lambda_i(1 - \lambda_j) \rightarrow$ buy if

$$r - \tau - p_i \geq 0 \text{ (suppose } r \text{ large enough, so OK)}$$

- Uninformed $\rightarrow$ share $(1 - \lambda_i)(1 - \lambda_j) \rightarrow$ don't buy

Informative advertising (cont'd)

• Demands (cont'd)



$$\begin{aligned}
 Q_1(p_1, p_2, \lambda_1, \lambda_2) &= \lambda_1 \left[(1 - \lambda_2) + \lambda_2 \hat{x}(p_1, p_2) \right] \\
 &= \lambda_1 \left[(1 - \lambda_2) + \lambda_2 \frac{1}{2\tau} (\tau - p_1 + p_2) \right]
 \end{aligned}$$



More informative advertising
from both firms
→ larger share of fully
informed consumers
→ larger price elasticity of
demand

Informative advertising (cont'd)

- Price elasticity

$$\eta_{p_1, Q_1} = \frac{\partial Q_1}{\partial p_1} \frac{\partial p_1}{\partial Q_1} = -\frac{\lambda_1 \lambda_2}{2\tau} \frac{p_1}{Q_1}$$

- At symmetric prices $p_1 = p_2$

$$\eta_{p_1, Q_1} = \frac{1}{2\tau} \frac{\lambda_2 p}{(1 - \lambda_2) + \frac{\lambda_2}{p}} = -\frac{1}{2\tau} \frac{\lambda_2 p}{1 - \frac{\lambda_2}{p}} = -\frac{\lambda_2 p}{(2 - \lambda_2)\tau}$$

- Evaluated at symmetric adv $\lambda_1 = \lambda_2 = \lambda$

$$\eta_{p_1, Q_1} = -\frac{\lambda p}{(2 - \lambda)\tau}$$

$$\frac{\partial |\eta_{p_1, Q_1}|}{\partial \lambda} = \frac{2p}{(2 - \lambda)^2 \tau} > 0$$

$\Rightarrow \uparrow \lambda \Rightarrow$ demand becomes more elastic

Informative advertising (cont'd)

- **Equilibrium analysis.** Firms simultaneously set prices and the share λ_i of consumers they inform, at cost $A(\lambda_i) = \frac{a}{2}\lambda_i^2$:

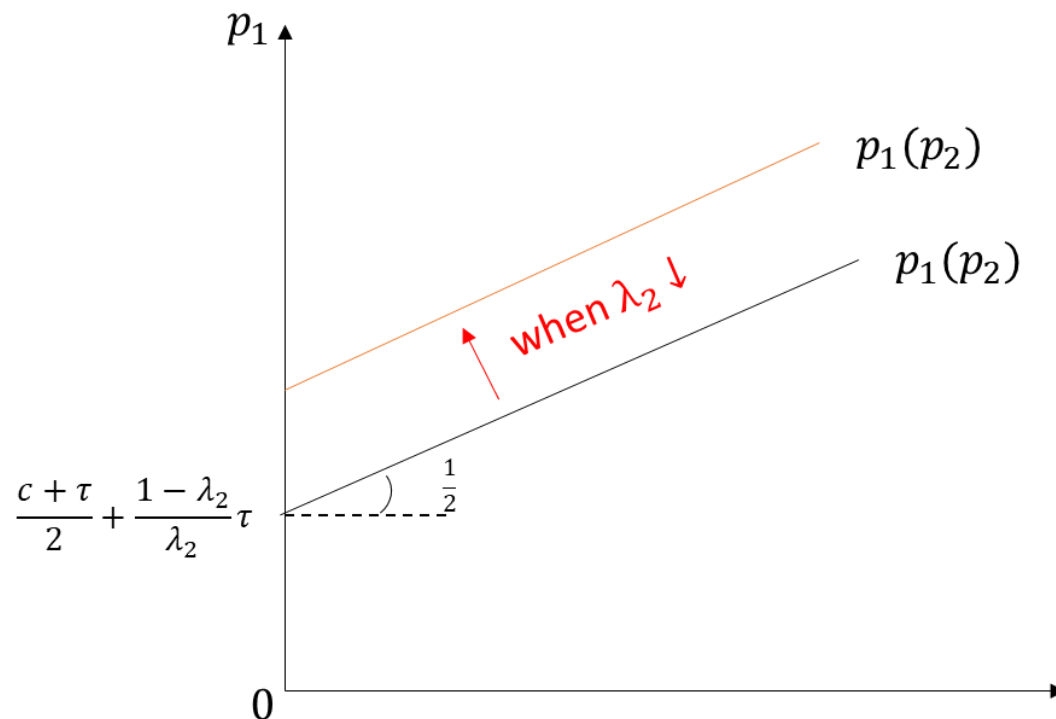
$$\max_{p_1, \lambda_1} (p_1 - c)Q_1(p_1, p_2, \lambda_1, \lambda_2) - \frac{a}{2}\lambda_1^2$$

- **Step 1. Price FOC.** $\lambda_1 \left[(1 - \lambda_2) + \lambda_2 \frac{\tau - 2p_1 + p_2 + c}{2\tau} \right] = 0$, which gives the best response

$$p_1(p_2) = \frac{p_2 + c + \tau}{2} + \underbrace{\frac{1 - \lambda_2}{\lambda_2} \tau}_{\geq 0}$$

- The extra term reflects captive consumers (informed only about firm 1), which push firm 1's price above the Hotelling best response.

Informative advertising (cont'd)



1) If $\lambda_2 = 1$, $\frac{1 - \lambda_2}{\lambda_2} \tau = 0$

2) If $\lambda_2 \in [0, 1)$, $\frac{1 - \lambda_2}{\lambda_2} \tau > 0$

Informative advertising (cont'd)

- **Step 2. Advertising FOC.** $(p_1 - c)[(1 - \lambda_2) + \lambda_2 \hat{x}] - a\lambda_1 = 0$.
- **Step 3. Symmetric equilibrium.** With $p_1 = p_2 = p^*$ and $\lambda_1 = \lambda_2 = \lambda^*$, $\hat{x} = \frac{1}{2}$, and the two FOCs become

$$p^* = c + \frac{2 - \lambda^*}{\lambda^*} \tau \quad \text{and} \quad a\lambda^* = (p^* - c) \frac{2 - \lambda^*}{2}$$

- Substituting the first into the second, $2a(\lambda^*)^2 = \tau(2 - \lambda^*)^2$, so

$$\lambda^* = \frac{2}{1 + \sqrt{2a/\tau}}, \quad p^* = c + \sqrt{2a\tau}, \quad \pi^* = \frac{2a}{(1 + \sqrt{2a/\tau})^2}$$

- Interior solution ($\lambda^* \leq 1$) requires $a \geq \frac{\tau}{2}$.

$$\lambda^* = \frac{2}{1 + \sqrt{2a/\tau}}, p^* = c + \sqrt{2a\tau}, \pi^* = \frac{2a}{(1 + \sqrt{2a/\tau})^2}$$

Informative advertising

- Observations
 - Higher price than under full information
 $(a > \tau/2 \Rightarrow p^* > c + \tau)$
 - Why? Lower elasticity of demand $\rightarrow$ higher markup
 - More product differentiation ($\tau \uparrow$) $\rightarrow$ higher prices
 - Stronger effect than under full information
 - Lower advertising cost ($a \downarrow$) $\rightarrow$ lower prices
 - Why? $a \downarrow \rightarrow$ advertising $\uparrow \rightarrow$ more informed consumers $\rightarrow$ more competition $\rightarrow$ prices $\downarrow$
 - Amount of advertising $\uparrow$ when $a \downarrow$ or $\tau \uparrow$
 - Profits increase as advertising becomes more costly
 - Why? Negative direct effect (higher costs) more than compensated by positive strategic effect (lower share of informed consumers $\lambda \rightarrow$ less intense competition)

Worked example: informative advertising

- Parameters.** $\tau = 1$; compare advertising costs $a = 2$ and $a = 8$. Under full information (Hotelling), $p = c + 1$ and $\pi = \frac{1}{2}$.

	λ^*	$p^* - c$	π^*
Hotelling (full information)	1	1	0.50
$a = 2$	0.67	2	0.44
$a = 8$	0.40	4	0.64

- Result:** Higher advertising costs mean fewer informed consumers and more captive demand, so prices rise. With $a = 8$, firms earn more than under full information, even after paying for ads.

Informative advertising (cont'd)

- **Lesson:** Due to strategic effects of informative advertising, higher advertising costs translate into more market power → Firms' profits can be higher in a market with higher advertising costs.
- Applications
 - **Testable hypothesis:** Internet search engines → lower advertising costs → lower profits?
 - Industry lobbying in favor of advertising restrictions?
 - High α seen as collusive device
 - Advertising restrictions self-imposed by certain professions (e.g., lawyers, accountants)

Persuasive advertising

- Intuition
 - In monopoly, more persuasive advertising → outward shift of demand → more profits
 - In oligopoly, does the increase in one firm's demand come at the expense of another firm's demand?
 - **Yes** → shift of demand between brands → business stealing → advertising may be excessive (prisoners' dilemma)
 - **No** → global demand expansion → advertising may be insufficient (public good nature)
- Modelling: 3 extensions of Hotelling model
 - Advertising increases willingness to pay
 - Advertising changes distribution of consumer tastes
 - Advertising increases perceived product differences

Persuasive advertising (sketch)

- Advertising $\uparrow$ willingness to pay
 - Utility of consumer x (with λ_i = advertising intensity)

$r + \beta\lambda_1 - \tau x - p_1$ if she buys 1 unit of good 1,
 $r + \beta\lambda_2 - \tau(1 - x) - p_2$ if she buys 1 unit of good 2.

- Advertising changes distribution of tastes
 - Symmetric distribution function

$$F(x; \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2)x - (\lambda_1 - \lambda_2)x^2$$

- **Lesson:** In both cases, advertising expenditures are simply a form of wasteful competition: if firms could cooperate, they would agree not to advertise.

Persuasive advertising (cont'd)

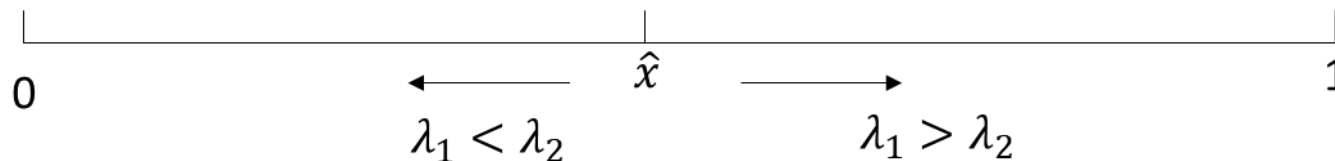
- Indifferent consumer

$$(r + \beta\lambda_1) - \tau\hat{x} - p_1 = (r + \beta\lambda_2) - \tau(1 - \hat{x}) - p_2$$

- Solve for $\hat{x}$

$$\hat{x} = \underbrace{\frac{1}{2} + \frac{p_1 - p_2}{2\tau}}_{\text{old}} + \underbrace{\beta \frac{\lambda_1 - \lambda_2}{2\tau}}_{\text{new}}$$

-



- If $\beta = 0$, the new term = 0;
- if $\lambda_1 = \lambda_2$, the new term = 0;
- if $\lambda_1 > \lambda_2$, the new term > 0 , $Q_1 \uparrow$;
- if $\lambda_1 < \lambda_2$, the new term < 0 , $Q_1 \downarrow$.

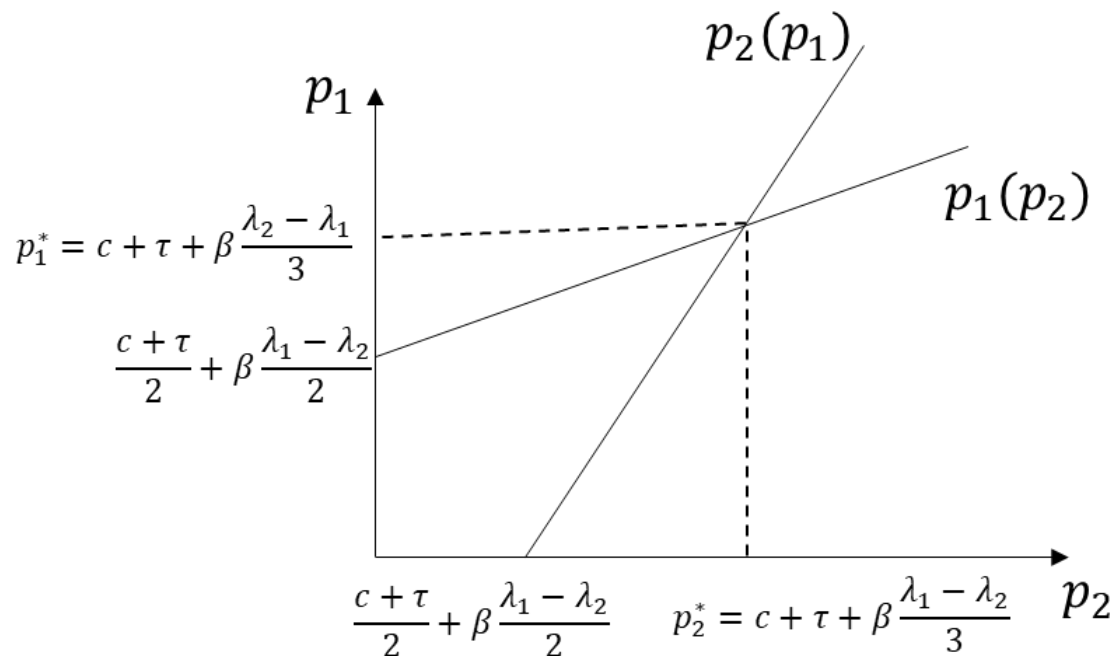
Persuasive advertising (cont'd)

- 2nd stage

$$\max_{p_1} \pi_1 = (p_1 - c) \hat{x}$$

- FOC = 0 $\Rightarrow p_1 = \frac{1}{2}(c + \tau + p_2 + \beta(\lambda_1 - \lambda_2))$

$$\pi_1^* = \frac{(3\tau + \beta(\lambda_1 - \lambda_2))^2}{18\tau}$$



Persuasive advertising (cont'd)

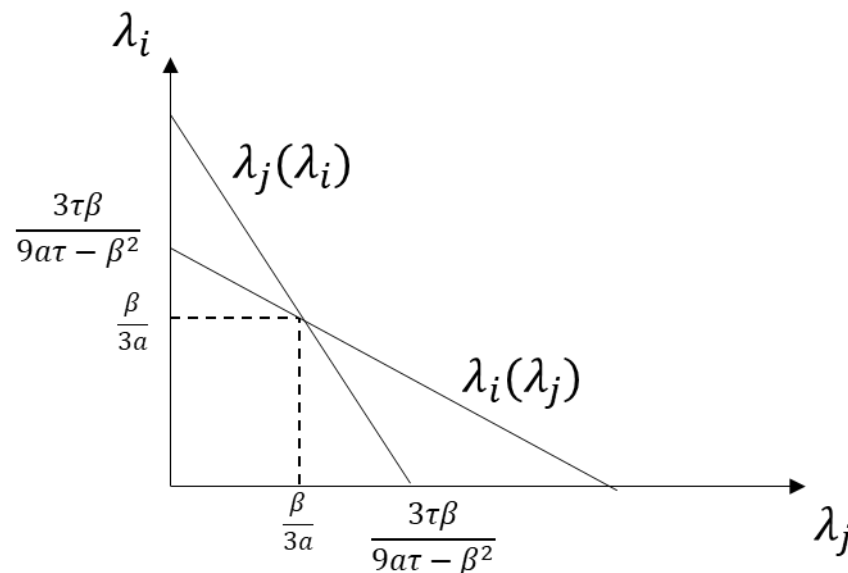
- 1st stage

$$\max_{\lambda_i} \pi_i^* - \frac{a}{2} \lambda_i^2$$

- FOC is

$$\frac{\beta}{9\tau} [3\tau + \beta(\lambda_i - \lambda_j)] - a\lambda_i = 0$$

- Solve for $\lambda_i(\lambda_j)$, and by symmetry, $\lambda_i^* = \lambda_j^* = \frac{\beta}{3a}$



Persuasive advertising (cont'd)

$$p_i^*(\lambda_i^*, \lambda_j^*) = c + \tau + \frac{\beta(\frac{\beta}{3a} - \frac{\beta}{3a})}{3} = c + \tau$$

same as Hotelling

$$\Rightarrow \pi^* = \underbrace{\frac{\tau}{2} - \frac{\beta^2}{18a}}_{\text{Profits with adv.}} < \frac{\tau}{2}$$

Profits with adv.

Hotelling profits

Persuasive advertising (cont'd)

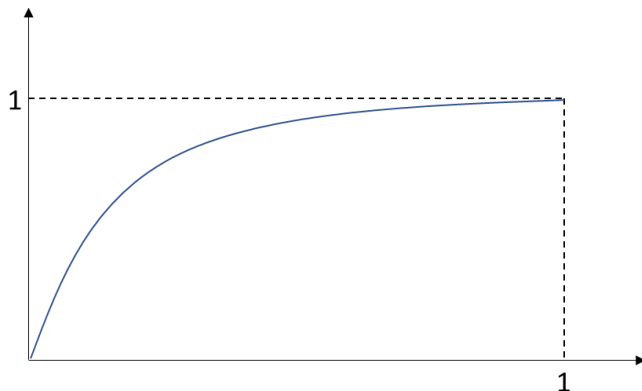
- **Example (1st extension).** $\tau = 1$, $\beta = 1$, and $a = 1$:

$$\lambda^* = \frac{\beta}{3a} = \frac{1}{3}, \quad p^* = c + 1, \quad \pi^* = \frac{1}{2} - \frac{1}{18} = \frac{4}{9} < \frac{1}{2}$$

- Each firm's advertising raises its own demand, but in equilibrium the effects cancel: prices are as in Hotelling, and firms only bear the advertising costs.
 - A prisoners' dilemma: both firms would be better off not advertising.

Persuasive advertising (cont'd)

- Advertising changes distribution of consumer tastes (2nd extension)
- $F(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2)x - (\lambda_1 - \lambda_2)x^2$
- $f(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2) - 2(\lambda_1 - \lambda_2)x$
- ① If $\lambda_1 = \lambda_2$, $\Rightarrow F(x) = x, f(x) = 1$, it is a uniform distribution.
- ② If $\lambda_1 > \lambda_2$, $F(x)$ is concave.



Firm 1

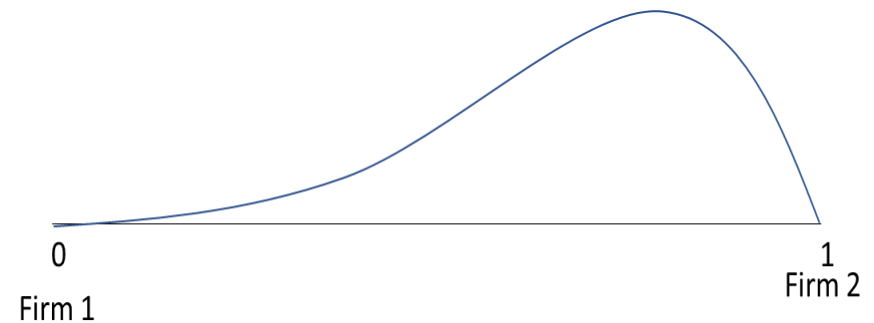
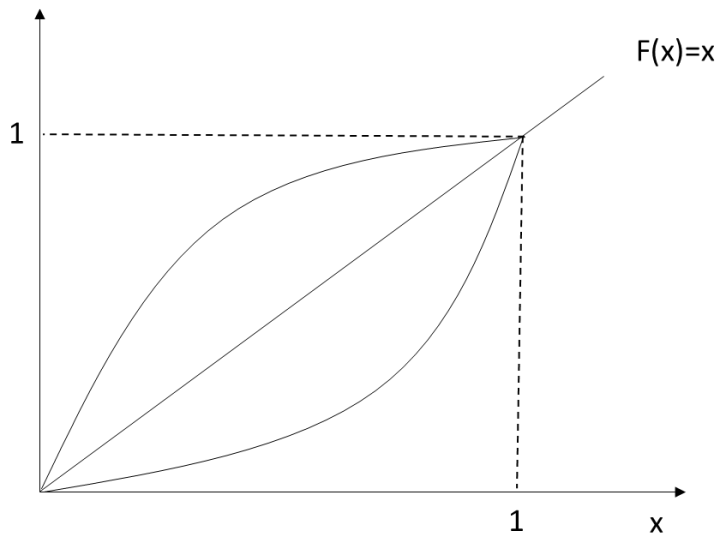
0

1

Firm 2

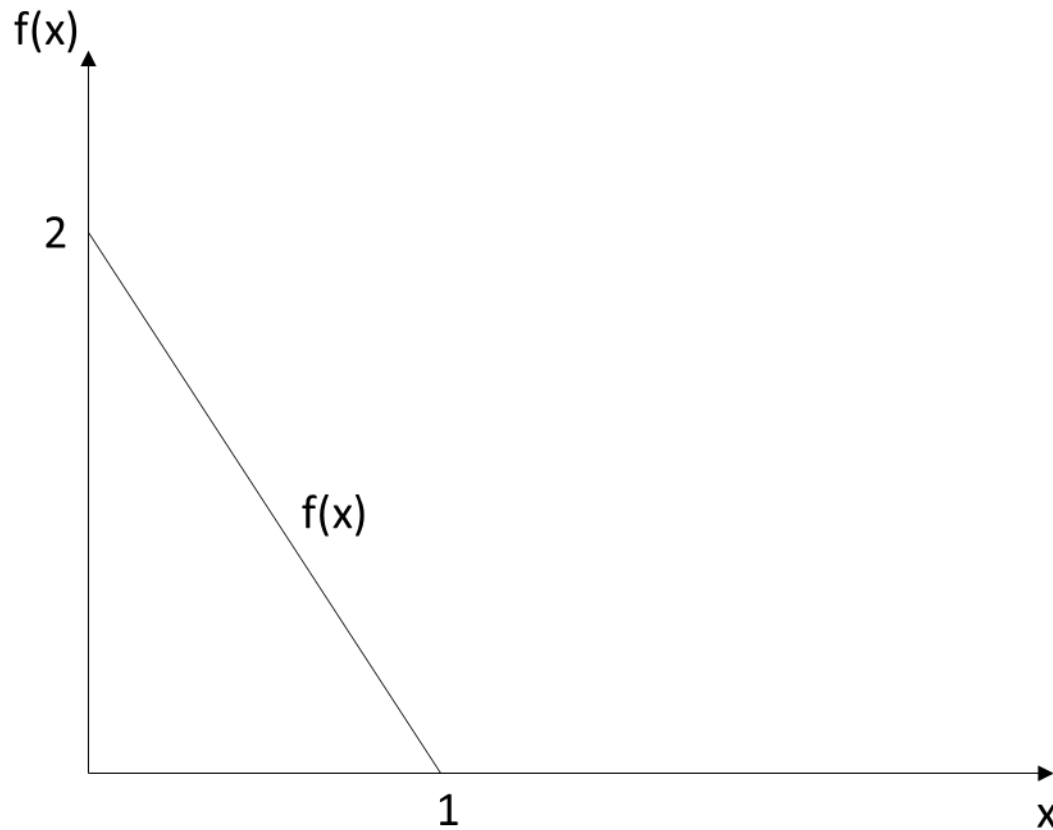
Persuasive advertising (cont'd)

① If $\lambda_1 < \lambda_2$, $F(x)$ is convex.



Persuasive advertising (cont'd)

- If $\lambda_1 = 1, \lambda_2 = 0$, then $f(x) = 2 - 2x$



Persuasive advertising (cont'd)

- **2nd extension: equilibrium.** Firm 1's demand is $Q_1 = F(\hat{x})$, with $\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$:

$$\max_{p_1, \lambda_1} (p_1 - c)F(\hat{x}; \lambda_1, \lambda_2) - \frac{a}{2}\lambda_1^2$$

- **Simultaneous choice** of prices and advertising. In a symmetric equilibrium, $F(x) = x$, so the price FOC gives $p^* = c + \tau$ (as in Hotelling), and the advertising FOC is

$$(p^* - c)(\hat{x} - \hat{x}^2) = a\lambda^* \quad \Rightarrow \quad \tau \times \frac{1}{4} = a\lambda^* \quad \Rightarrow \quad \lambda^* = \frac{\tau}{4a}$$

$$\pi^* = \frac{\tau}{2} - \frac{a}{2} \left(\frac{\tau}{4a} \right)^2 = \frac{\tau}{2} - \frac{\tau^2}{32a} < \frac{\tau}{2}$$

Persuasive advertising (cont'd)

- **Sequential choice:** advertising first, then prices. Now firm 1 anticipates that more advertising changes both prices: at the symmetric point, $\frac{\partial p_1}{\partial \lambda_1} = \frac{\tau}{6}$ and $\frac{\partial p_2}{\partial \lambda_1} = -\frac{\tau}{6}$.
- The rival's price cut makes advertising less attractive, so firms advertise less:

$$\lambda^* = \frac{\tau}{6a} \quad \text{and} \quad \pi^* = \frac{\tau}{2} - \frac{\tau^2}{72a}$$

- Prices are still $p^* = c + \tau$, as in Hotelling.
- In both timings, advertising is wasteful: firms would earn $\frac{\tau}{2}$ if neither advertised.

Persuasive advertising (cont'd)

- **3rd extension: advertising increases perceived differences.**
The transportation cost becomes $\tau + \beta\lambda_1 + \beta\lambda_2$.

- **Step 1. Indifferent consumer.**

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2(\tau + \beta\lambda_1 + \beta\lambda_2)}$$

- **Step 2. Price stage.** This is the Hotelling model with transportation cost $\tau + \beta(\lambda_1 + \lambda_2)$, so

$$p_1^* = p_2^* = c + \tau + \beta(\lambda_1 + \lambda_2) \quad \text{and} \quad \pi_i = \frac{\tau + \beta(\lambda_1 + \lambda_2)}{2} - \frac{a}{2}\lambda_i^2$$

Persuasive advertising (cont'd)

- **Step 3. Advertising stage.** Firm 1's FOC is $\frac{\beta}{2} - a\lambda_1 = 0$. By symmetry,

$$\lambda_1^* = \lambda_2^* = \frac{\beta}{2a}, \quad p^* = c + \tau + \frac{\beta^2}{a}, \quad \pi^* = \frac{\tau}{2} + \frac{3\beta^2}{8a}$$

- Prices and profits are higher than in Hotelling: advertising relaxes price competition.
- **Step 4. Cooperative advertising.** If firms maximize joint profits, $\tau + \beta(\lambda_1 + \lambda_2) - \frac{a}{2}(\lambda_1^2 + \lambda_2^2)$, then

$$\beta - a\lambda_i = 0 \quad \Rightarrow \quad \lambda_i = \frac{\beta}{a}$$

- Twice the equilibrium level: each firm's advertising also benefits its rival (a public good).

Persuasive advertising (cont'd)

- **Example (3rd extension).** $\tau = 1$, $\beta = 1$, and $a = 1$:

	λ each	$p - c$	Profit each
No advertising (Hotelling)	0	1	0.50
Equilibrium	0.5	2	0.875
Cooperative	1	3	1.00

- **Result:** Advertising that increases perceived differences raises prices and profits. Firms advertise too little from their own point of view, since each firm's ads also soften its rival's competition.

Persuasive advertising (cont'd)

- Advertising $\uparrow$ perceived product differences
 - Advertising intensities affect degree of product differentiation (i.e., transport cost):

$$\tau(\lambda_1, \lambda_2) = \tau + \beta\lambda_1 + \beta\lambda_2$$

- **Lesson:** Here, firms invest in advertising to relax price competition and, thereby, achieve higher profits. Because advertising is a public good, firms would even be better off by coordinating their advertising decisions.

Review questions

- Which industries advertise a lot? Give two examples and discuss the likely reasons for high advertising expenditures.
- Discuss the difference between persuasive advertising and advertising as a complement.
- Consider informative advertising about a product's existence. Does an increase in the advertising cost function necessarily lead to lower profits? Discuss.
- Discuss possible effects of persuasive advertising under imperfect competition.