

# Part II. Market power

## Chapter 3. Static imperfect competition



Slides

*Industrial Organization: Markets and Strategies*

Paul Belleflamme and Martin Peitz, 2d Edition

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## Oligopolies

- Industries in which a few firms compete
- Market power is collectively shared.
- Firms can't ignore their competitors' behaviour.
- **Strategic interaction** → Game theory

## Oligopoly *theories*

- *Cournot* (1838) → quantity competition
- *Bertrand* (1883) → price competition
- Not competing but **complementary theories**
  - Relevant for different industries or circumstances

## Organization of Part II

- Chapter 3
  - Simple settings: unique decision at single point in time
  - How does the nature of strategic variable (price or quantity) affect
    - strategic interaction?
    - extent of market power?
- Chapter 4
  - Incorporates time dimension: sequential decisions
  - Effects on strategic interaction?
  - What happens before and after strategic interaction takes place?



## Case. DVD-by-mail industry

- Facts
  - < 2004: *Netflix* almost only active firm
  - 2004: entry by *Wal-Mart* and *Blockbuster* (and later *Amazon*), not correctly foreseen by *Netflix*
- Sequential decisions
  - Leader: *Netflix*
  - Followers: *Wal-Mart*, *Blockbuster*, *Amazon*
- Price competition
  - *Wal-Mart* and *Blockbuster* undercut *Netflix*
  - *Netflix* reacts by reducing its prices too.
- Quantity competition?
  - Need to store more copies of latest movies

## Chapter 3. Learning objectives

- Get (re)acquainted with basic models of oligopoly theory
  - Price competition: Bertrand model
  - Quantity competition: Cournot model
- Be able to compare the two models
  - Quantity competition may be mimicked by a two-stage model (capacity-then-price competition)
  - Unified model to analyze price & quantity competition
- Understand the notions of strategic complements and strategic substitutes
- See how to measure market power empirically

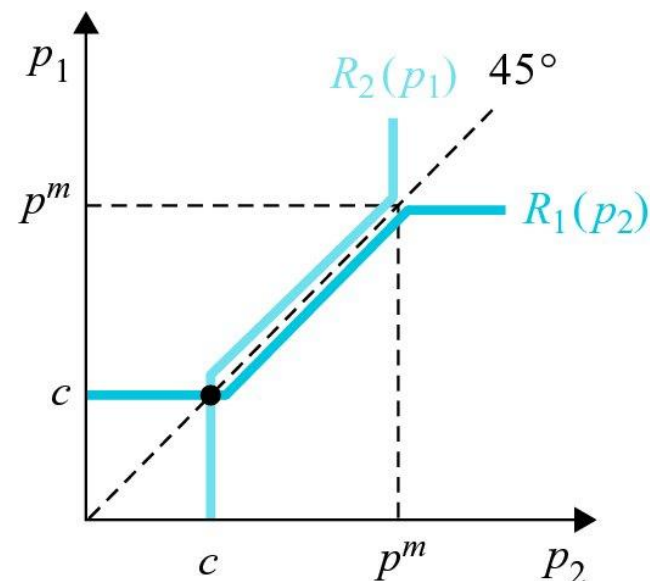
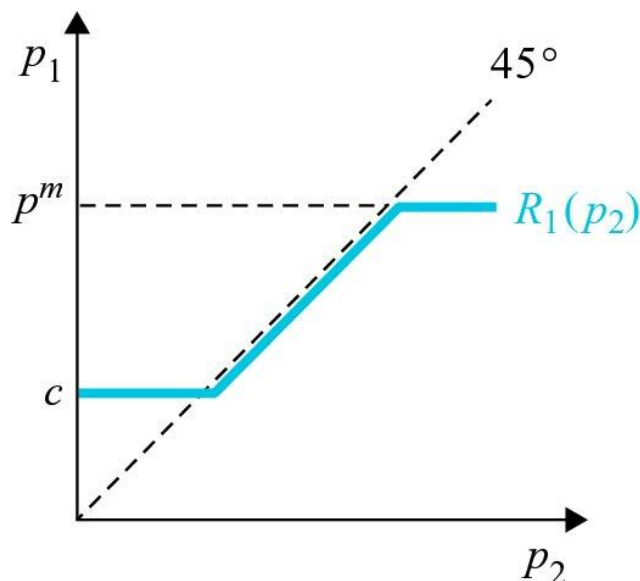
## The standard Bertrand model

- 2 firms
  - Homogeneous products
  - Identical constant marginal cost:  $c$
  - Set price simultaneously to maximize profits
- Consumers
  - Firm with lower price attracts all demand,  $Q(p)$
  - At equal prices, market splits at  $\alpha_1$  and  $\alpha_2=1-\alpha_1$
- → Firm  $i$  faces demand

$$Q_i(p_i) = \begin{cases} Q(p_i) & \text{if } p_i < p_j \\ \alpha_i Q(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

## The standard Bertrand model (cont'd)

- Unique Nash equilibrium
  - Both firms set price = marginal cost:  $p_1 = p_2 = c$
  - *Proof*
    - For any other  $(p_1, p_2)$ , a profitable deviation exists.
    - Or: unique intersection of firms' *best-response functions*



## The standard Bertrand model (cont'd)

- 'Bertrand Paradox'
  - Only 2 firms **but** perfectly competitive outcome
  - Message: there exist circumstances under which duopoly competitive pressure can be very strong
- **Lesson:** In a homogeneous product Bertrand duopoly with identical and constant marginal costs, the equilibrium is such that
  - firms set price equal to marginal costs;
  - firms do not enjoy any market power.

## Bertrand competition with asymmetric costs

- **Setting.** Two firms with constant marginal costs  $c_1 < c_2$ , homogeneous products, and demand  $Q(p)$ .
- **Step 1. Which prices can be an equilibrium?** Firm 2 never sells at a price below  $c_2$ , and firm 1 never prices below  $c_1$ . Any common price  $p \in [c_1, c_2]$ , with firm 1 serving the whole market, is an equilibrium.
- **Step 2. Selection.** Ruling out prices below cost for firm 2 (a weakly dominated strategy), the equilibrium is

$$p^* = c_2 \quad \text{and} \quad \pi_1^* = (c_2 - c_1)Q(c_2) > 0$$

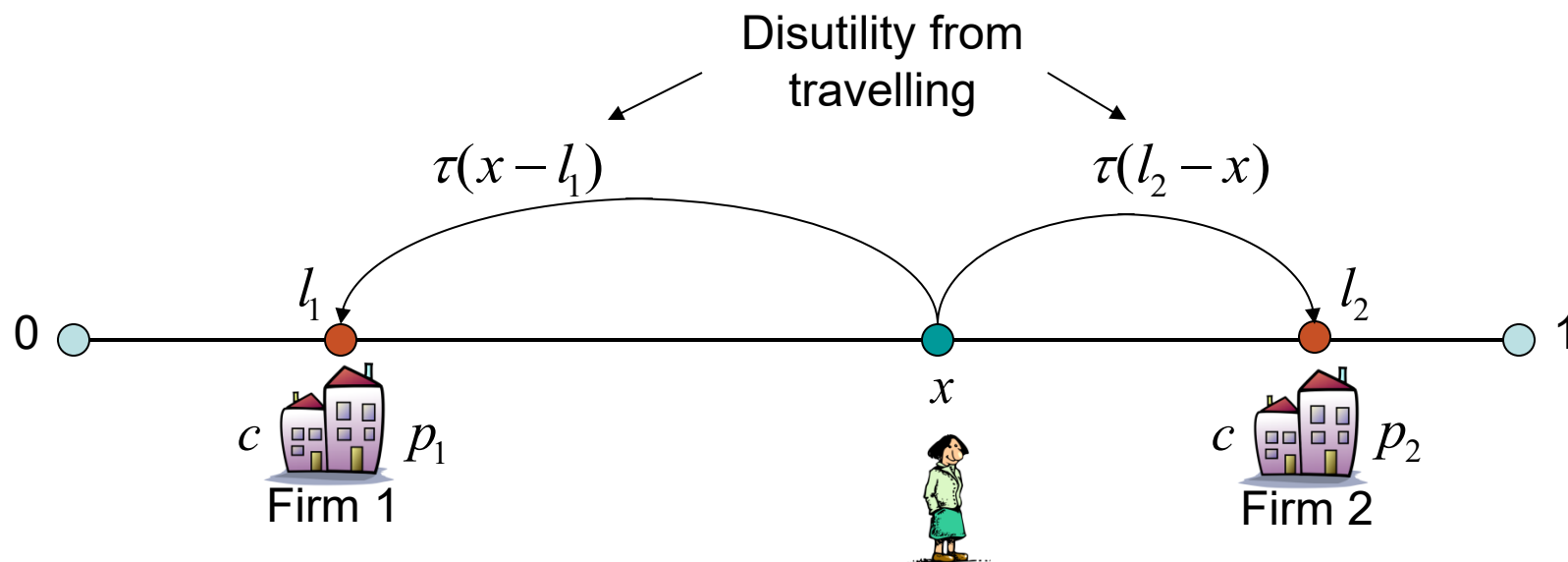
- Example:  $Q(p) = 10 - p$ ,  $c_1 = 2$ , and  $c_2 = 4$ . Then  $p^* = 4$  and  $\pi_1^* = (4 - 2) \times 6 = 12$ .
- Intuitively, the more efficient firm earns a profit equal to its cost advantage, while the less efficient firm constrains its price.

## Bertrand competition with uncertain costs

- Each firm has private information about its costs
  - Trade-off between margins and likelihood of winning the competition
  - See particular model in the book.
- **Lesson:** In the price competition model with homogeneous products and private information about marginal costs, at equilibrium,
  - firms set price above marginal costs;
  - firms make strictly positive expected profits;
  - more firms  $\rightarrow$  price-cost margins $\downarrow$ , output $\uparrow$ , profits $\downarrow$ ;
  - number of firms explodes  $\rightarrow$  competitive limit.

## Price competition with differentiated products

- Firms may avoid intense competition by offering products that are imperfect substitutes.
- Hotelling model (1929)**



Mass 1 of consumers, uniformly distributed

## Hotelling model (cont'd)

- **Step 1. Indifferent consumer.** Firms are located at the extremes,  $l_1 = 0$  and  $l_2 = 1$ . A consumer at  $x$  obtains utility  $r - \tau|l_i - x| - p_i$  when buying from firm  $i$ .
- The indifferent consumer  $\hat{x}$  obtains the same utility from both firms:

$$r - \tau\hat{x} - p_1 = r - \tau(1 - \hat{x}) - p_2 \quad \Leftrightarrow \quad \hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$$

- Consumers to the left of  $\hat{x}$  buy from firm 1, and those to the right buy from firm 2. We assume that  $r$  is high enough for all consumers to buy.

## Hotelling model (cont'd)

- **Step 2. Demands.** Since consumers are uniformly distributed on  $[0,1]$  with mass 1,

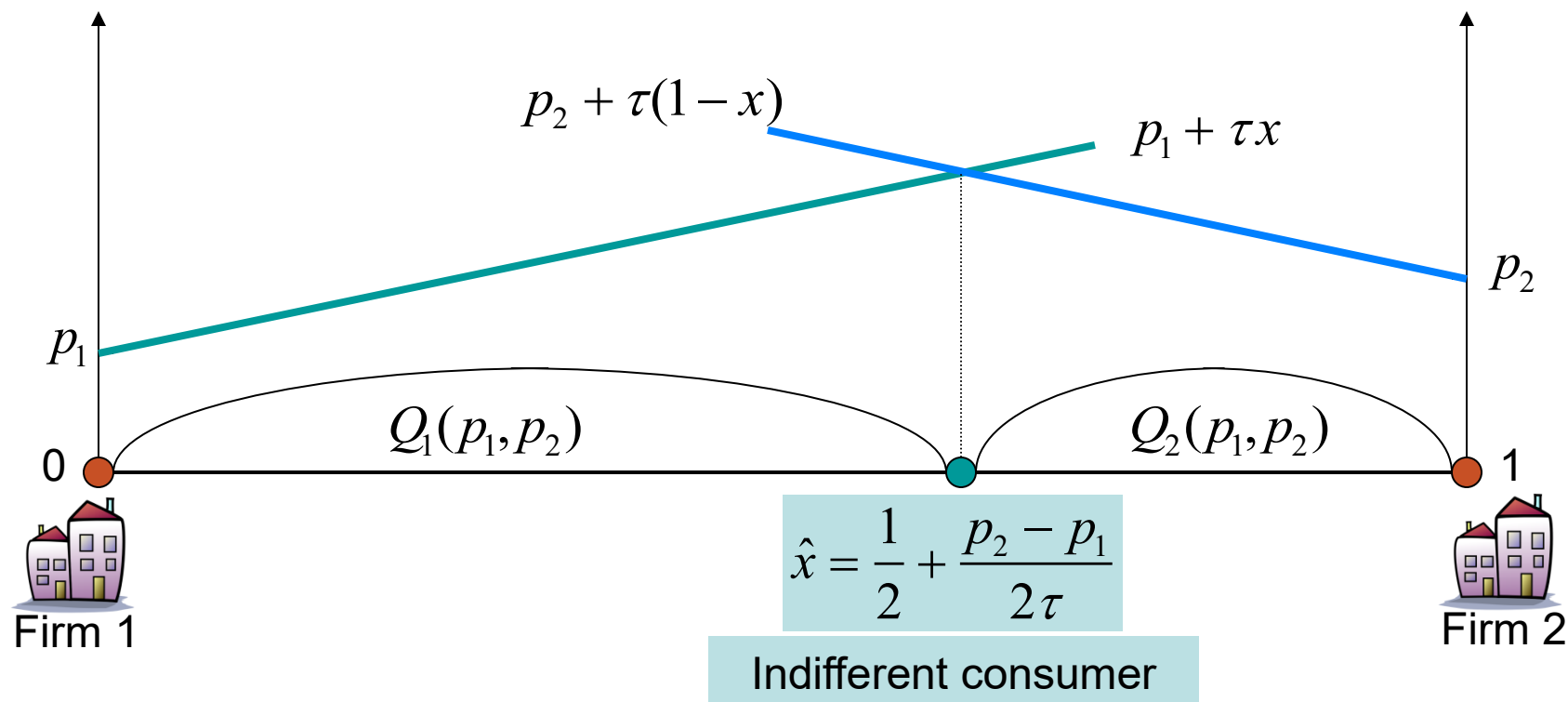
$$Q_1(p_1, p_2) = \hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2\tau}$$

$$Q_2(p_1, p_2) = 1 - \hat{x} = \frac{1}{2} + \frac{p_1 - p_2}{2\tau}$$

- A firm that cuts its price below its rival's does not attract all consumers: those located far away still prefer the rival. Demand responds gradually to prices.

## Hotelling model (cont'd)

- Suppose location at the extreme points



## Hotelling model (cont'd)

- **Step 3. Firm  $i$ 's problem.** Firm  $i$  chooses  $p_i$  to solve

$$\max_{p_i} (p_i - c) \left[ \frac{1}{2} + \frac{p_j - p_i}{2\tau} \right]$$

- The first-order condition is

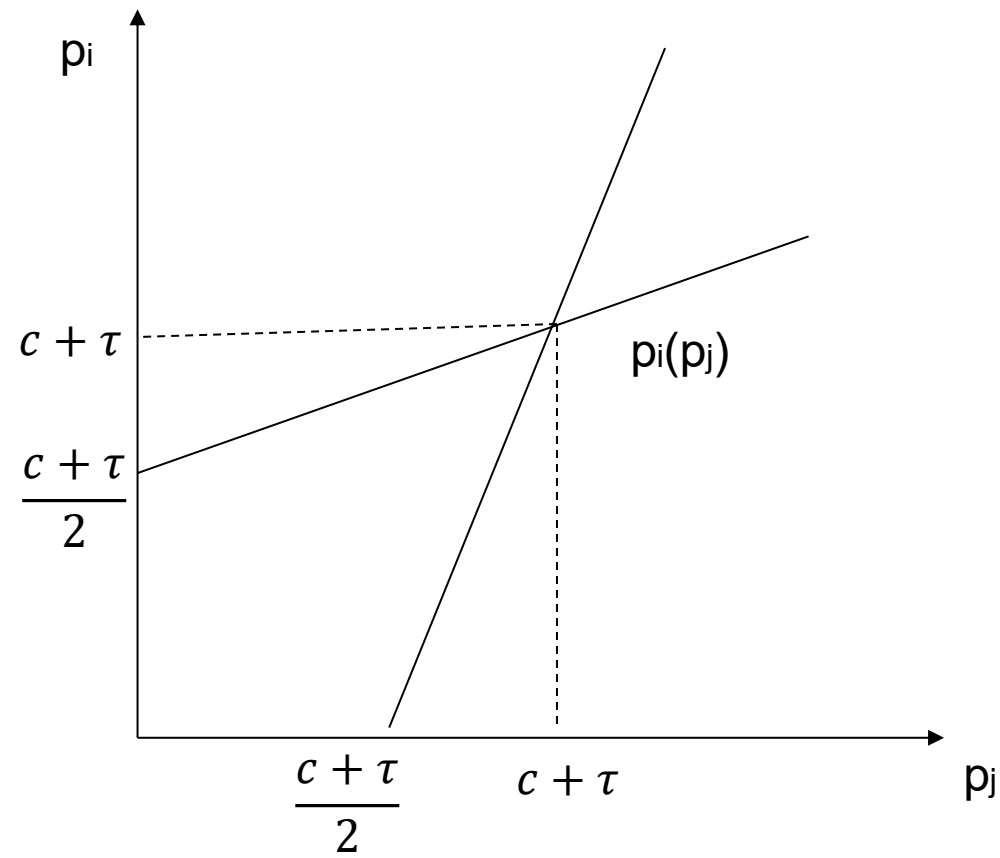
$$\frac{1}{2} + \frac{p_j - p_i}{2\tau} - \frac{p_i - c}{2\tau} = 0 \quad \Leftrightarrow \quad p_j - 2p_i + c + \tau = 0$$

- **Step 4. Best response.** Solving for  $p_i$ ,

$$p_i(p_j) = \frac{c + \tau}{2} + \frac{1}{2}p_j$$

- The best response increases in  $p_j$ : prices are strategic complements.

# Hotelling model (cont'd)



## Hotelling model (cont'd)

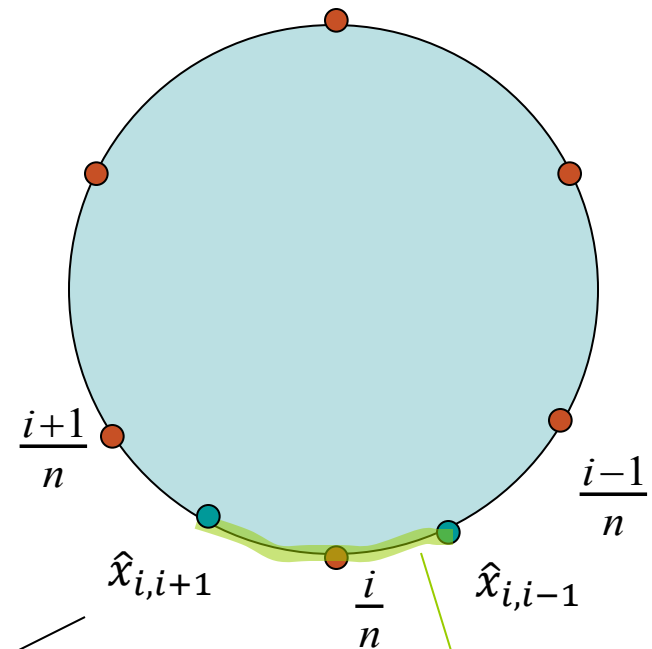
- **Step 5. Equilibrium.** In a symmetric equilibrium,  $p_1 = p_2 = p^*$ . Substituting into the best response,  $p^* = \frac{c+\tau}{2} + \frac{1}{2}p^*$ , so
 
$$p^* = c + \tau, \quad \hat{x} = \frac{1}{2}, \quad \text{and} \quad \pi^* = (p^* - c) \frac{1}{2} = \frac{\tau}{2}$$
  - Example:  $c = 10$  and  $\tau = 4$ . Then  $p^* = 14$ , each firm serves half of the market, and  $\pi^* = 2$ .
- **Extensions** (next slides):  $n$  firms on a circle (Salop model), and asymmetric competition.

- **Lesson:** If products are more differentiated (higher  $\tau$ ), firms enjoy more market power: prices and profits increase in  $\tau$ . As  $\tau$  approaches zero, we return to the Bertrand outcome,  $p^* = c$ .

## Extension 1: Salop model

- Setting

- Firms equidistantly located on circle with circumference 1
- Consumers uniformly distributed on circle
- They buy at most one unit, from firm with lowest 'generalized price'
- Unit transportation cost,  $\tau$



$$\begin{aligned} r - \tau \left( \hat{x}_{i,i+1} - \frac{i}{n} \right) - p_i &= r - \tau \left( \frac{i+1}{n} - \hat{x}_{i,i+1} \right) - p_{i+1} \\ \Leftrightarrow \hat{x}_{i,i+1} &= \frac{2i+1}{2n} + \frac{p_{i+1} - p_i}{2\tau} \end{aligned}$$

Firm i's demand

## Extension 1: Salop model (cont'd)

- **Step 1. Indifferent consumers.** Firm  $i$  is located at  $\frac{i}{n}$ . The consumer between firms  $i - 1$  and  $i$  who is indifferent between them solves

$$r - \tau \left( \hat{x}_{i-1,i} - \frac{i-1}{n} \right) - p_{i-1} = r - \tau \left( \frac{i}{n} - \hat{x}_{i-1,i} \right) - p_i$$

$$\Leftrightarrow \hat{x}_{i-1,i} = \frac{2i-1}{2n} + \frac{p_i - p_{i-1}}{2\tau}$$

- **Step 2. Demand.** Firm  $i$  serves the consumers between  $\hat{x}_{i-1,i}$  and  $\hat{x}_{i,i+1}$ . If its neighbors charge  $p_{i-1} = p_{i+1} = p$ ,

$$Q_i(p_i, p) = \hat{x}_{i,i+1} - \hat{x}_{i-1,i} = \frac{1}{n} + \frac{p - p_i}{\tau}$$

## Extension 1: Salop model (cont'd)

- **Step 3. Firm  $i$ 's problem.** Firm  $i$  solves  $\max_{p_i} (p_i - c) \left[ \frac{1}{n} + \frac{p - p_i}{\tau} \right]$ , with first-order condition

$$\frac{1}{n} + \frac{p - 2p_i + c}{\tau} = 0$$

- **Step 4. Symmetric equilibrium.** Setting  $p_i = p$ , we obtain  $\frac{1}{n} + \frac{c - p}{\tau} = 0$ , so

$$p^* = c + \frac{\tau}{n} \quad \text{and} \quad \pi^* = \frac{\tau}{n} \times \frac{1}{n} = \frac{\tau}{n^2}$$

- As  $n$  increases, neighboring products become closer substitutes, competition intensifies, and  $p^*$  decreases.
- As  $n \rightarrow \infty$ ,  $p^* \rightarrow c$ : perfect competition.

## Extension 2: Asymmetric competition with differentiated products

- Same setting as Hotelling model
- Only difference: product 1 is of superior quality
  - Consumer's indirect utility:

$$\begin{cases} r_1 - \tau x - p_1 & \text{if buy 1} \\ r_2 - \tau(1-x) - p_2 & \text{if buy 2} \end{cases} \quad \text{with } r_1 > r_2$$

- Assume:  $r_2 + \tau > r_1 \rightarrow$  product 2 more attractive for some consumers
- Indifferent consumer

$$\hat{x} = \frac{1}{2} + \frac{(r_1 - r_2) - (p_1 - p_2)}{2\tau} = Q_1(p_1, p_2)$$

## Extension 2: Asymmetric competition (cont'd)

- **Step 1. Firm  $i$ 's problem.** From the indifferent consumer, firm  $i$ 's demand is  $Q_i = \frac{1}{2} + \frac{(r_i - r_j) - (p_i - p_j)}{2\tau}$ . Firm  $i$  solves

$$\max_{p_i} (p_i - c) \left[ \frac{1}{2} + \frac{(r_i - r_j) - (p_i - p_j)}{2\tau} \right]$$

- The first-order condition is

$$\frac{1}{2\tau} [p_j - 2p_i + c + \tau + (r_i - r_j)] = 0$$

- **Step 2. Best response.**

$$p_i(p_j) = \frac{c + \tau + (r_i - r_j)}{2} + \frac{1}{2}p_j$$

## Extension 2: Asymmetric competition (cont'd)

- **Step 3. Equilibrium.** Let  $\Delta \equiv r_1 - r_2 > 0$ . Solving the two best responses simultaneously,

$$p_1^* = c + \tau + \frac{\Delta}{3} \quad \text{and} \quad p_2^* = c + \tau - \frac{\Delta}{3}$$

- **Step 4. Demand.** Since  $p_1^* - p_2^* = \frac{2\Delta}{3}$ ,

$$Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{\Delta - \frac{2\Delta}{3}}{2\tau} = \frac{1}{2} + \frac{\Delta}{6\tau}$$

- The high-quality firm sets a higher price and sells more.
  - Example:  $c = 0$ ,  $\tau = 3$ , and  $\Delta = 3$ . Then  $p_1^* = 4$ ,  $p_2^* = 2$ , and  $Q_1 = \frac{2}{3}$ .

## Extension 2: Asymmetric competition with differentiated products (cont'd)

- Welfare maximization → sell at marginal cost

$$Q_1(c, c) = \frac{1}{2} + \frac{r_1 - r_2}{2\tau} > Q_1(p_1^*, p_2^*) = \frac{1}{2} + \frac{r_1 - r_2}{6\tau}$$

- Firm 1's equilibrium demand is too low from a social point of view.
- Same analysis if  $r_1 = r_2 = r$ , but  $c_1 < c_2$

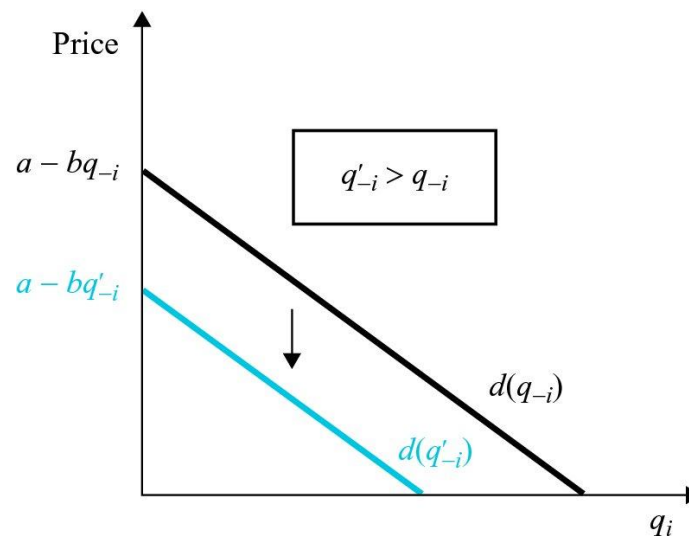
- Lesson:** Under imperfect competition, the firm with higher quality or lower marginal cost sells too few units from a welfare perspective.

# The linear Cournot model

- Model
  - Homogeneous product market with  $n$  firms
  - Firm  $i$  sets quantity  $q_i$
  - Total output:  $q = q_1 + q_2 + \dots + q_n$
  - Market price given by  $P(q) = a - bq$
  - Linear cost functions:  $C_i(q_i) = c_i q_i$
  - Notation:  $q_{-i} = q - q_i$
- Residual demand

$$P(q) = (a - bq_{-i}) - bq_i$$

$$\equiv d(q_i; q_{-i})$$



## The linear Cournot model (cont'd)

- **Step 1. Firm  $i$ 's problem.** Taking the rivals' output  $q_{-i}$  as given (Cournot conjecture), firm  $i$  acts as a monopolist on its residual demand:

$$\max_{q_i} (a - bq_i - bq_{-i} - c_i)q_i$$

- The first-order condition is

$$a - c_i - 2bq_i - bq_{-i} = 0$$

- **Step 2. Best response.** Solving for  $q_i$ ,

$$q_i(q_{-i}) = \frac{1}{2b} (a - c_i - bq_{-i})$$

- The best response decreases in  $q_{-i}$ : quantities are strategic substitutes.

## The linear Cournot model (cont'd)

- **Step 3. Duopoly equilibrium.** Let  $c_1 \leq c_2$ . Solving the two best responses simultaneously,

$$q_1^* = \frac{a - 2c_1 + c_2}{3b} \quad \text{and} \quad q_2^* = \frac{a - 2c_2 + c_1}{3b}$$

- Firm 2 produces a positive output if  $c_2 \leq \frac{a+c_1}{2}$ .
- **Step 4. Profits.** From the first-order condition, the price margin equals  $p^* - c_i = bq_i^*$ , so

$$\pi_i^* = (p^* - c_i)q_i^* = b(q_i^*)^2$$

- Since  $c_1 \leq c_2$ , then  $q_1^* \geq q_2^*$  and  $\pi_1^* \geq \pi_2^*$ .

## Worked example: Cournot duopoly

- **Parameters.**  $P(q) = 10 - q$  ( $a = 10$ ,  $b = 1$ ),  $c_1 = 1$ , and  $c_2 = 2$ .
- **Outputs.**

$$q_1^* = \frac{10 - 2 + 2}{3} = \frac{10}{3} \quad \text{and} \quad q_2^* = \frac{10 - 4 + 1}{3} = \frac{7}{3}$$

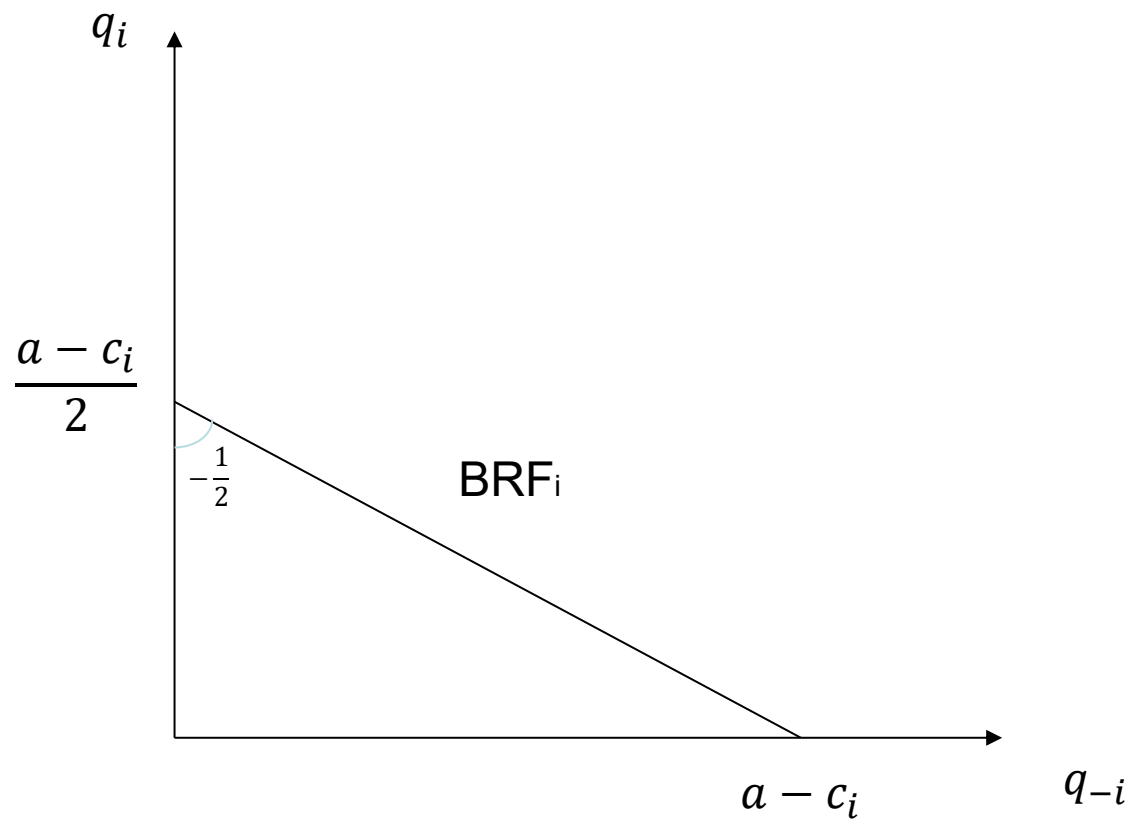
- **Price.**  $p^* = 10 - \frac{10}{3} - \frac{7}{3} = \frac{13}{3} \approx 4.33$ .

- **Profits.**

$$\pi_1^* = \left(\frac{10}{3}\right)^2 = \frac{100}{9} \approx 11.1 \quad \text{and} \quad \pi_2^* = \left(\frac{7}{3}\right)^2 = \frac{49}{9} \approx 5.4$$

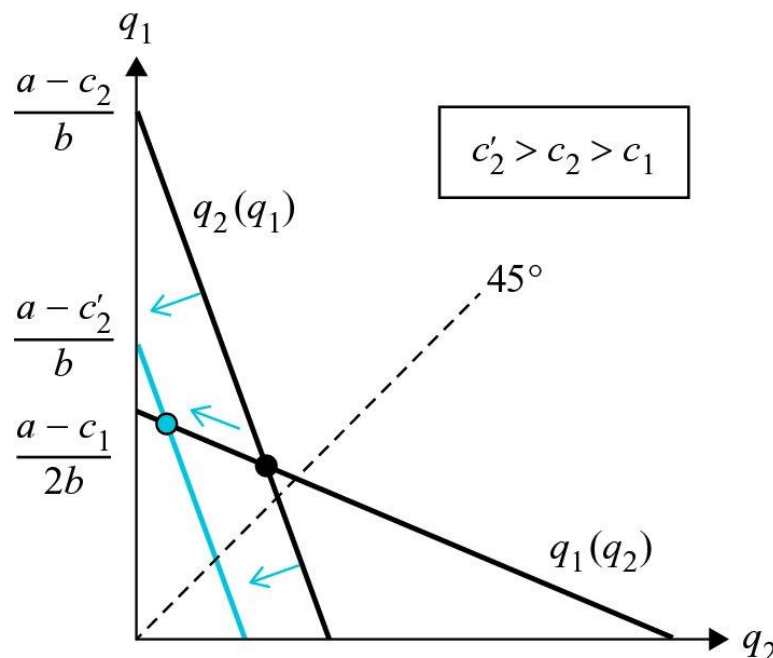
- Check for firm 1:  $(p^* - c_1)q_1^* = \left(\frac{13}{3} - 1\right)\frac{10}{3} = \frac{100}{9}$ .

## The linear Cournot model (cont'd)



## The linear Cournot model (cont'd)

- Duopoly



- **Lesson:** In the linear Cournot model with homogeneous products, a firm's equilibrium profit increases when the firm becomes relatively more efficient than its rivals.

## Symmetric Cournot oligopoly

- **Step 1. Equilibrium output.** With  $c_i = c$ , set  $q_i = q$  and  $q_{-i} = (n - 1)q$  in the first-order condition:

$$a - c - 2bq - b(n - 1)q = 0 \quad \Rightarrow \quad q^*(n) = \frac{a - c}{b(n + 1)}$$

- **Step 2. Price and markup.** Total output is  $nq^*(n)$ , so

$$p^*(n) = a - bnq^*(n) = \frac{a + nc}{n + 1} \quad \text{and} \quad L(n) = \frac{p^*(n) - c}{p^*(n)} = \frac{a - c}{a + nc}$$

- As  $n$  increases, individual output decreases, while total output increases; price and markup decrease. As  $n \rightarrow \infty$ ,  $p^* \rightarrow c$ .

- **Lesson:** The (symmetric linear) Cournot model converges to perfect competition as the number of firms increases.

## Implications of Cournot competition

- General demand and cost functions
- Cournot pricing formula (details see next slide)

$$\frac{P(q) - C'_i(q_i)}{P(q)} = \frac{\alpha_i}{\eta} \text{ with } \alpha_i = q_i / q$$

- **Lesson:** In the Cournot model, the markup of firm  $i$  is larger the larger is the market share of firm  $i$  and the less elastic is market demand.

- If marginal costs are constant

$$\frac{p - \sum_{i=1}^n \alpha_i c_i}{p} = \frac{I_H}{\eta} \text{ with } I_H = \sum_{i=1}^n \alpha_i^2, \text{ Herfindahl index}$$

↖  
Average Lerner index

## Details: Cournot pricing formula

- **Step 1. First-order condition.** Firm  $i$  solves  $\max_{q_i} P(q)q_i - C_i(q_i)$ ,  
so

$$P'(q)q_i + P(q) - C'_i(q_i) = 0$$

- Rearranging and dividing by  $P(q)$ ,

$$\frac{P(q) - C'_i(q_i)}{P(q)} = \underbrace{-\frac{P'(q)q}{P(q)}}_{\text{inverse elasticity, } 1/\eta} \times \underbrace{\frac{q_i}{q}}_{\text{market share, } \alpha_i} = \frac{\alpha_i}{\eta}$$

- Firm  $i$ 's markup increases in its market share and decreases in the demand elasticity.

## Details: Cournot pricing formula (cont'd)

- **Step 2. Constant marginal costs.** With  $C_i(q_i) = c_i q_i$ , Step 1 becomes  $\frac{p - c_i}{p} = \frac{\alpha_i}{\eta}$ .
- Multiplying by  $\alpha_i$  and summing over all firms, using  $\sum_i \alpha_i = 1$ ,
 
$$\frac{p - \sum_{i=1}^n \alpha_i c_i}{p} = \frac{\sum_{i=1}^n \alpha_i^2}{\eta} = \frac{I_H}{\eta}$$
- The average Lerner index (weighted by market shares) is proportional to the Herfindahl index  $I_H$ .
  - Example: with  $n$  symmetric firms,  $\alpha_i = \frac{1}{n}$ , so  $I_H = \frac{1}{n}$  and  $L = \frac{1}{n\eta}$ .

## Price versus quantity competition

- Comparison of previous results
  - Let  $Q(p)=a-p$ ,  $c_1=c_2=c$
  - Bertrand:  $p_1=p_2=c$ ,  $q_1=q_2=(a-c)/2$ ,  $\pi_1=\pi_2=0$
  - Cournot:  $q_1=q_2=(a-c)/3$ ,  $p=(a+2c)/3$ ,  $\pi_1=\pi_2=(a-c)^2/9$
- **Lesson:** Homogeneous product case → higher price, lower quantity, higher profits under quantity than under price competition.
- To refine the comparison
  - Limited capacities of production
  - Direct comparison within a unified model
  - Identify characteristics of price or quantity competition

## Limited capacity and price competition

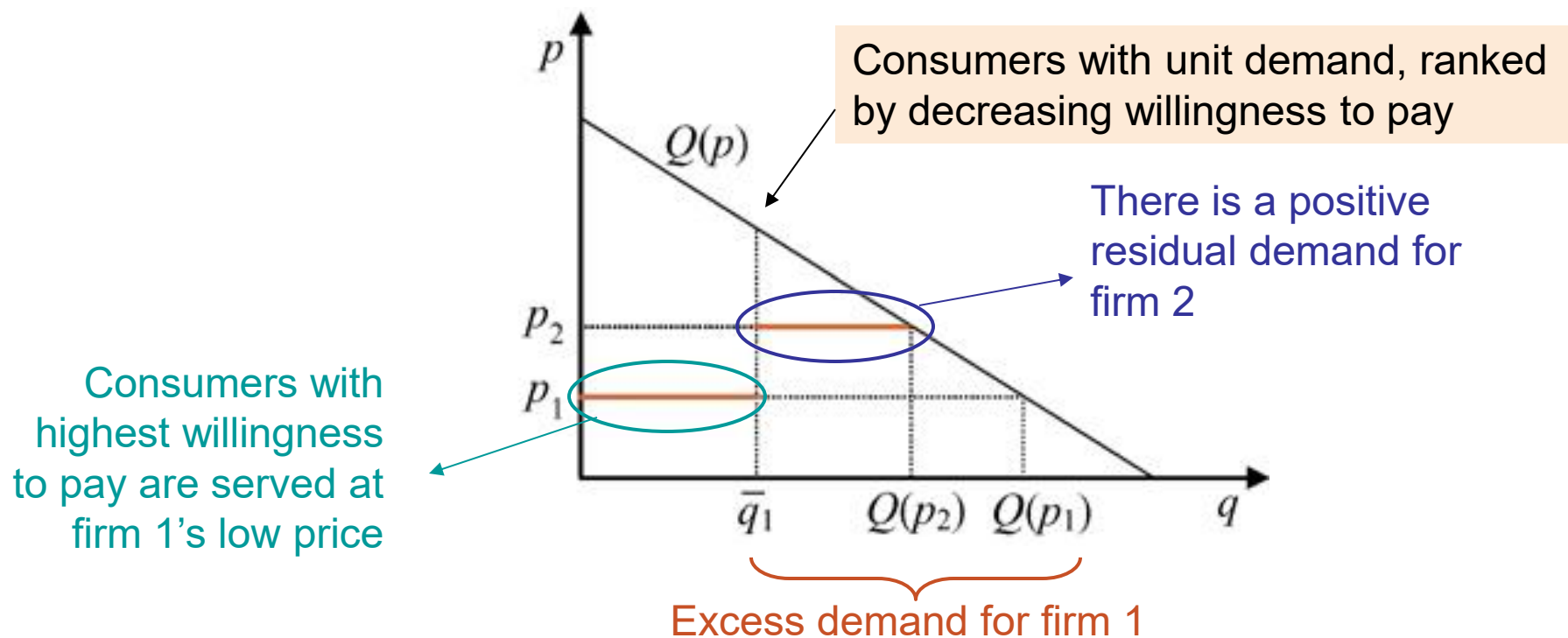
- Edgeworth's critique (1897)
  - Bertrand model: no capacity constraint
  - But capacity may be limited in the short run.
- Examples
  - Retailers order supplies well in advance
  - DVD-by-mail industry
    - Larger demand for latest movies → need to hold extra stock of copies → higher costs and stock may well be insufficient
  - Flights more expensive around Xmas
- To account for this: **two-stage model**
  1. Firms precommit to capacity of production
  2. Price competition

## Capacity-then-price model (Kreps & Scheinkman)

- Setting
  - Stage 1: firms set capacities  $\bar{q}_i$  and incur cost of capacity,  $c$
  - Stage 2: firms set prices  $p_i$ ; cost of production is 0 up to capacity (and infinite beyond capacity); demand is  $Q(p) = a - p$ .
  - Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices
- Rationing
  - If quantity demanded to firm  $i$  exceeds its supply...
  - ... some consumers have to be rationed...
  - ... and possibly buy from more expensive firm  $j$ .
  - Crucial question: Who will be served at the low price?

## Capacity-then-price model (cont'd)

- Efficient rationing
  - First served: consumers with higher willingness to pay.
  - Justification: queuing system, secondary markets



## Capacity-then-price model (cont'd)

- Equilibrium (details next slides)
  - Stage 2. If  $p_1 < p_2$  and excess demand for firm 1, then demand for 2 is:

$$\hat{Q}(p_2) = \begin{cases} Q(p_2) - \bar{q}_1 & \text{if } Q(p_2) - \bar{q}_1 \geq 0 \\ 0 & \text{else} \end{cases}$$

Claim: if  $c < a < (4/3)c$ , then both firms set the market-clearing price:  $p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2$

- Stage 1. Same reduced profit functions as in Cournot:

$$\bar{\pi}_1(\bar{q}_1, \bar{q}_2) = (a - \bar{q}_1 - \bar{q}_2)\bar{q}_1 - c\bar{q}_1$$

- **Lesson**: In the capacity-then-price game with efficient consumer rationing (and with linear demand and constant marginal costs), the chosen capacities are equal to those in a standard Cournot market.

## Details: Capacity-then-price model

- Setting
  - Stage 1: firms set capacities  $\bar{q}_i$  and incur cost of capacity,  $c$
  - Stage 2: firms set prices  $p_i$ ; cost of production is 0 up to capacity (and infinite beyond capacity); demand is  $Q(p) = a - p$ .
  - Subgame-perfect equilibrium: firms know that capacity choices may affect equilibrium prices
  - Efficient rationing
- Upper bound on capacity at stage 1

$$c\bar{q}_i \leq \max_q (a - q)q = a^2 / 4 \Leftrightarrow \bar{q}_i \leq a^2 / (4c)$$

## Details: Capacity-then-price model (cont'd)

- **Claim.** If  $c < a < \frac{4}{3}c$ , both firms set the market-clearing price  $p_1 = p_2 = p^* = a - \bar{q}_1 - \bar{q}_2$ .
- **Proof.** Let  $p_1 = p^*$ . We show that  $p_2 = p^*$  is firm 2's best response.
- **Step 1. A lower price.** If  $p_2 < p^*$ , firm 2 still sells its capacity  $\bar{q}_2$ , but at a lower price: not profitable.
- **Step 2. A higher price.** If  $p_2 > p^*$ , firm 1 is capacity-constrained, and firm 2 serves the residual demand. Its revenue is

$$p_2(a - p_2 - \bar{q}_1), \quad \text{which is maximized at } \tilde{p}_2 = \frac{a - \bar{q}_1}{2}$$

- Raising  $p_2$  above  $p^*$  is not profitable if revenue decreases at  $p^*$ , that is, if  $p^* > \tilde{p}_2$  (next slide).

## Details: Capacity-then-price model (cont'd)

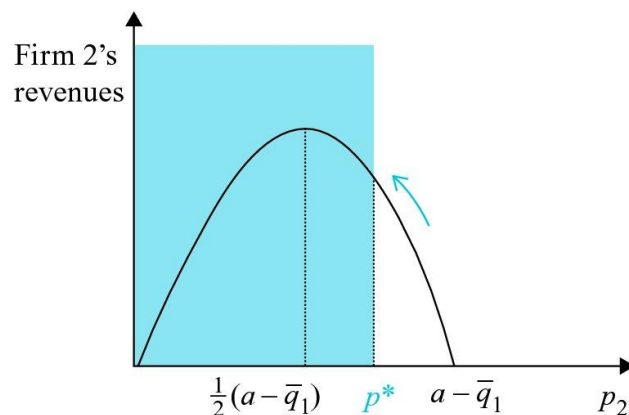
- Proof (cont'd)
  - Max reached at  $\bar{p}_2 = (a - \bar{q}_1) / 2$
  - Revenues are decreasing at  $p^*$  if

$$p^* > \bar{p}_2 \Leftrightarrow a - \bar{q}_1 - \bar{q}_2 > \frac{a - \bar{q}_1}{2} \Leftrightarrow a > \bar{q}_1 + 2\bar{q}_2$$

Since  $\bar{q}_1, \bar{q}_2 \leq a^2/(4c)$ ,  $\bar{q}_1 + 2\bar{q}_2 \leq (3/4)(a^2/c)$

Assumption  $a < (4/3)c \Leftrightarrow (3/4)(a/c) < 1$

- Hence, not profitable to set  $p_2 > p^*$ . QED



## Capacity-then-price model: stage 1

- **Stage 1.** Anticipating the market-clearing price  $p^* = a - \bar{q}_1 - \bar{q}_2$  in stage 2, firm 1 chooses its capacity to solve

$$\max_{\bar{q}_1} (a - \bar{q}_1 - \bar{q}_2) \bar{q}_1 - c \bar{q}_1$$

- This is the Cournot problem with marginal cost  $c$ , so

$$\bar{q}_1 = \bar{q}_2 = \frac{a - c}{3} \quad \text{and} \quad p^* = \frac{a + 2c}{3}$$

- Example:  $a = 11$  and  $c = 9$ , which satisfies  $c < a < \frac{4}{3}c = 12$ . Then  $\bar{q}_i = \frac{2}{3}$  and  $p^* = \frac{29}{3} \approx 9.67$ .

## Capacity Constraints

- Go to EconS 503's website, Chapter 8, slides 178-192

## Cournot Model of Quantity Competition

- Go to EconS 503's website, Chapter 8, slides 102-141

## Product Differentiation

- Go to EconS 503's website, Chapter 8, slides 142-153

## Differentiated products: Cournot vs. Bertrand

- **Setting.** Duopoly with substitutable products ( $b > d > 0$ ). Consumers maximize

$$U(q_0, q_1, q_2) = aq_1 + aq_2 - \frac{1}{2}(bq_1^2 + 2dq_1q_2 + bq_2^2) + q_0$$

- subject to the budget constraint  $y = q_0 + p_1q_1 + p_2q_2$ .
- **Inverse demands:**  $P_i(q_i, q_j) = a - bq_i - dq_j$ .
- **Demands:**  $Q_i(p_i, p_j) = \bar{a} - \bar{b}p_i + \bar{d}p_j$ , with

$$\bar{a} = \frac{a}{b+d}, \quad \bar{b} = \frac{b}{b^2-d^2}, \quad \text{and} \quad \bar{d} = \frac{d}{b^2-d^2}$$

# Differentiated products: Cournot vs. Bertrand

(cont'd)

- **Cournot.** Firm  $i$  solves  $\max_{q_i} (a - bq_i - dq_j - c_i)q_i$ , with best response

$$q_i(q_j) = \frac{a - c_i - dq_j}{2b}$$

- Downward-sloping: quantities are **strategic substitutes**.
- **Bertrand.** Firm  $i$  solves  $\max_{p_i} (p_i - c_i)(\bar{a} - \bar{b}p_i + \bar{d}p_j)$ , with best response

$$p_i(p_j) = \frac{\bar{a} + \bar{b}c_i + \bar{d}p_j}{2\bar{b}}$$

- Upward-sloping: prices are **strategic complements**.

# Differentiated products: Cournot vs. Bertrand

(cont'd)

- **Comparison of equilibria** with symmetric costs,  $c_1 = c_2 = c$ . Solving each pair of best responses,

$$p^C - c = \frac{b(a - c)}{2b + d} \quad \text{and} \quad p^B - c = \frac{(b - d)(a - c)}{2b - d}$$

- The difference is

$$p^C - p^B = \frac{d^2(a - c)}{(2b - d)(2b + d)} > 0$$

- The gap vanishes as  $d \rightarrow 0$  (independent products), where both firms behave as monopolists.

- **Lesson:** Price as the strategic variable gives rise to a more competitive outcome than quantity as the strategic variable.

## Appropriate modelling choice: price or quantity?

- Monopoly: it doesn't matter.
- Oligopoly: price and quantity competitions lead to different residual demands
  - Price competition
    - $p_j$  fixed  $\rightarrow$  rival willing to serve any demand at  $p_j$
    - $i$ 's residual demand: market demand at  $p_i < p_j$ ; zero at  $p_i > p_j$
    - So, residual demand is very sensitive to price changes.
  - Quantity competition
    - $q_j$  fixed  $\rightarrow$  irrespective of price obtained, rival sells  $q_j$
    - $i$ 's residual demand: "what's left" (i.e., market demand  $- q_j$ )
    - So, residual demand is less sensitive to price changes.

## Appropriate modelling choice (cont'd)

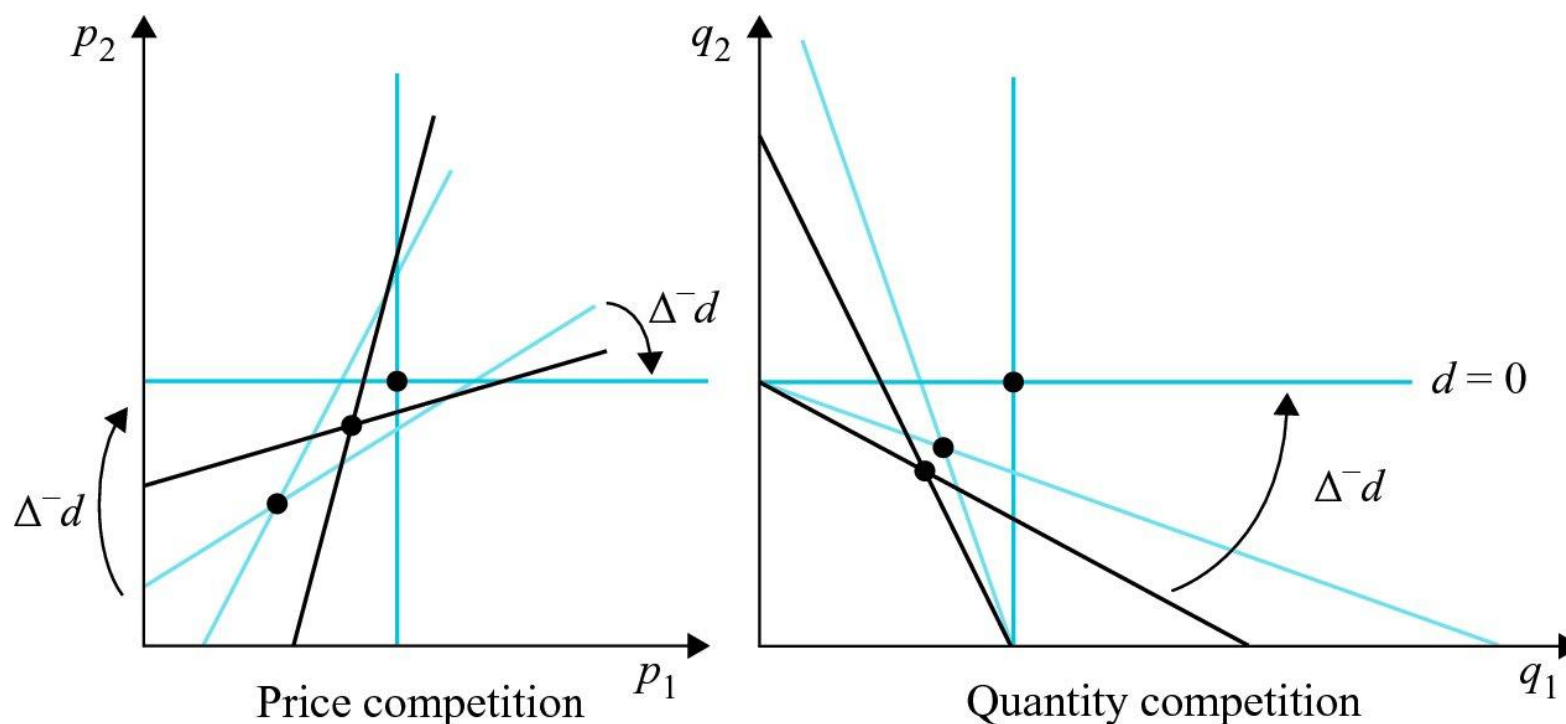
- How do firms behave in the marketplace?
  - Stick to a price and sell any quantity at this price?
    - **price competition**
    - appropriate choice when
      - Unlimited capacity
      - Prices more difficult to adjust in the short run than quantities
      - Example: mail-order business
  - Stick to a quantity and sell this quantity at any price?
    - **quantity competition**
    - appropriate choice when
      - Limited capacity (even if firms are price-setters)
      - Quantities more difficult to adjust in the short run than prices
      - Example: package holiday industry
- Influence of technology (e.g. Print-on-demand vs. batch printing)

## Strategic substitutes and complements

- How does a firm react to the rivals' actions?
- Look at the slope of reaction functions.
  - Upward sloping: competitor  $\uparrow$  its action  $\rightarrow$  marginal profitability of my own action  $\uparrow$   
 $\rightarrow$  variables are strategic **complements**
    - Example: price competition (with substitutable products);  
See Bertrand and Hotelling models
  - Downward sloping: competitor  $\uparrow$  its action  $\rightarrow$  marginal profitability of my own action  $\downarrow$   
 $\rightarrow$  variables are strategic **substitutes**
    - Example: quantity competition (with substitutable products);  
see Cournot model

## Strategic substitutes and complements (cont'd)

- Linear demand model of product differentiation  
(with  $d$  measuring the degree of product substitutability)



# Estimating market power

- Setting
  - Symmetric firms producing homogeneous product
  - Demand equation:  $p = P(q, x)$  (1)
    - $q$ : total quantity in the market
    - $x$ : vector of exogenous variables affecting demand (not cost)
  - Marginal costs:  $c(q, w)$ 
    - $w$ : vector of exogenous variables affecting (variable) costs
- Interpretation 1. Nest various market structures in a single model

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q$$

$\lambda = 0$  competitive market  
 $\lambda = 1$  monopoly  
 $\lambda = 1 / n$   $n$ -firm Cournot

Firm's *conjecture* as to how strongly price reacts to its change in output

## Estimating market power (cont'd)

- Interpretation 1 (cont'd)
  - Basic model to be estimated non-parametrically:  
demand equation (1) + equilibrium condition (2)

$$MR(\lambda) = p + \lambda \frac{\partial P(q, x)}{\partial q} q = c(q, w)$$

- Interpretation 2. Be agnostic about precise game being played
  - From equilibrium condition (2), Lerner index is

$$L = \frac{p - c(q, w)}{p} = -\lambda \frac{\partial P(q, x)}{\partial q} \frac{q}{p} = \frac{\lambda}{\eta}$$

- (2) is identified if single  $c(q, w)$  and single  $\lambda$  satisfy it

## Review questions

- How does product differentiation relax price competition? Illustrate with examples.
- How does the number of firms in the industry affect the equilibrium of quantity competition?
- When firms choose first their capacity of production and next, the price of their product, this two-stage competition sometimes looks like (one-stage) Cournot competition. Under which conditions?
- Using a unified model of horizontal product differentiation, one comes to the conclusion that price competition is fiercer than quantity competition. Explain the intuition behind this result.

## Review questions (cont'd)

- Define the concepts of strategic complements and strategic substitutes. Illustrate with examples.
- What characteristics of a specific industry will you look for to determine whether this industry is better represented by price competition or by quantity competition? Discuss.