Chapter 1:
Introduction to Environmental Policy

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Environmental Regulation: A Game-Theoretic Approach

Overview of Chapter 1

- **Section 1.1.** Environmental externalities and regulation.
- **Section 1.2.** Policy instruments for environmental regulation.
- **Section 1.3.** Market structure and strategic behavior.
- **Section 1.4.** A short history of environmental regulation.
- **Book Overview.** Roadmap for Chapters 2–8.

Section 1.1

Environmental Externalities and Regulation

The Climate Crisis – Key Facts

- Global CO_2 emissions from energy-related activities reached a record **37.8 gigatonnes (Gt)** in 2024.
  - Atmospheric CO_2 concentration: 422.5 parts per million.
  - Over 50% higher than pre-industrial levels; see IEA (2025).
- **2024 was the hottest year on record**:
  - Global surface temperatures: 1.29°C above 20th-century baseline.
  - 1.5°C threshold exceeded for a full calendar year for the first time; see NASA (2025).
- Consequences: extreme weather, rising sea levels, ecosystem disruption.

The Economic Toll

- Natural catastrophes in 2024 inflicted **$417 billion** in global economic losses.
  - Only $154 billion covered by insurance.
- In the United States alone:
  - 27 separate billion-dollar weather disasters in 2024.
  - $182.7 billion in damages; over 560 lives lost; see NOAA (2025).
  - Hurricanes, wildfires, floods, and severe storms.
- These figures underscore the urgency of effective environmental policy.
  - Policies must mitigate emissions *and* build resilience.
  - A strategic, economically grounded approach is required.

What Is an Externality?

- An **externality** occurs when the actions of one party impose costs (or benefits) on others that are not reflected in market prices.
- **Negative externalities** from pollution:
  - Air pollution from manufacturing.
  - Water contamination from agriculture.
  - Greenhouse gas emissions from transportation.
- **Market outcome**: Too much pollution is produced, and too few resources are allocated to mitigation.
  - Firms equate private marginal cost to marginal revenue.
  - Social marginal cost = private marginal cost + marginal external damage.
  - Equilibrium quantity > socially optimal quantity.

Two Sources of Market Failure

- **Market failure 1: Negative externality.**
  - Pollution damages not internalized by the polluting firm.
  - Private optimum $\neq$ social optimum.
- **Market failure 2: Imperfect competition.**
  - Oligopolistic industries produce *less* output than the competitive benchmark.
  - Regulation interacts with market power: an emission fee may worsen the output distortion.
  - Subsidies may exacerbate overproduction under certain market structures.
- This book studies both market failures **simultaneously**:
  - Emission fees, permits, and abatement in imperfectly competitive markets.

Why Game Theory?

- In oligopolistic markets, firms **anticipate each other's reactions** and adjust behavior accordingly.
- Strategic interdependence affects:
  - Output levels and pricing.
  - Investment in abatement technology.
  - Responses to emission fees and permit systems.
- Game theory provides the tools to analyze these interactions:
  - **Nash Equilibrium**: each firm's strategy is a best response to others.
  - **Subgame Perfect Equilibrium**: backward induction removes non-credible threats.
- *Goal*: a unified framework for regulation in strategic environments.

Section 1.2

Policy Instruments for Environmental Regulation

Overview of Policy Instruments

- **Command-and-control (C&C) regulations**:
  - Technology standards and emission limits.
  - Direct mandates on what firms must do or use.
- **Market-based instruments**:
  - Emission taxes (Pigouvian fees): charge per unit of pollution.
  - Tradable permits (cap-and-trade): set a quantity ceiling.
  - Subsidies for clean technologies.
- **Information-based tools**:
  - Labeling and disclosure requirements.
- **Voluntary agreements**:
  - Firms commit to environmental targets in exchange for flexibility.

Emission Taxes (Pigouvian Fees)

- A tax t per unit of pollution sets the private cost of emitting equal to the social cost.
- **Optimal Pigouvian tax**: t^* = marginal external damage at the social optimum.
  - Firms reduce emissions until their marginal abatement cost = t^*.
  - Achieves the socially optimal emission level at least cost.
- **Strengths**:
  - Continuous incentive for abatement and innovation.
  - Revenue can fund environmental programs or reduce other taxes.
- **Weaknesses**:
  - Politically sensitive; uncertain abatement quantity.
  - Setting the right tax level requires information about damages.

Tradable Permits (Cap-and-Trade)

- Regulator sets a **cap** on total emissions and issues permits.
  - Firms buy and sell permits in a market.
  - Equilibrium permit price = marginal abatement cost across firms.
- **Strengths**:
  - Achieves cost-effective aggregate reductions (least-cost principle).
  - Certain quantity of aggregate emissions (unlike a tax).
- **Weaknesses**:
  - Requires robust market infrastructure and monitoring.
  - Permit price volatility can create investment uncertainty.
  - In oligopolistic markets: strategic permit trading may affect competitiveness.
- Studied extensively in Chapter 6 (tradable permits and market power).

Command-and-Control vs. Market-Based

- **C&C**: Technology standards, emission limits.
  - Easy to enforce and monitor.
  - May stifle innovation; firms meet the standard but no more.
  - Ignores heterogeneity in abatement costs.
- **Market-based**: Taxes and permits.
  - Achieve cost-efficiency by equating marginal abatement costs.
  - Encourage ongoing innovation beyond the standard.
- **Weitzman (1974) insight**: Under uncertainty, the instrument choice depends on the slope of marginal costs vs. marginal damages.
  - Steep marginal damage $\Rightarrow$ prefer quantity instrument (permits).
  - Steep marginal abatement cost $\Rightarrow$ prefer price instrument (tax).
- Analyzed rigorously in Chapters 3 and 4.

Equivalence and Non-Equivalence of Instruments

- Under **perfect competition** and certainty, an emission tax and a tradable permit system achieving the same aggregate quantity are equivalent.
  - The permit price equals the tax rate at equilibrium.
- Under **imperfect competition**, this equivalence breaks down:
  - Strategic output decisions interact differently with price vs. quantity instruments.
  - Chapter 2: Pigouvian tax vs. emission quota in a Cournot oligopoly.
  - Chapter 6: Tradable permits under market power.
- Asymmetric information also breaks equivalence:
  - Taxes fix price, permits fix quantity — but under uncertainty, the expected welfare differs.

Section 1.3
Market Structure and Strategic Behavior

Cournot Oligopoly

- n firms compete in quantities **simultaneously**.
  - Inverse demand: $p(Q) = a - bQ$, where $Q = \sum_{i=1}^{n} q_i$.
  - Firm i's profit: $\pi_i = p(Q) \cdot q_i - c \cdot q_i$.
- **Best response function (BRF)** of firm i:

$$q_i^*(q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}.$$

- **Symmetric Nash Equilibrium**: $q_i^* = \frac{a-c}{(n+1)b}$.
  - As $n \to \infty$, price $\to$ marginal cost (competitive outcome).
  - Monopoly when $n = 1$; duopoly when $n = 2$.

Stackelberg Model

- **Leader-follower** setting: firm 1 moves first, firm 2 responds.
- **Backward induction**:
  1. Firm 2 observes q_1 and best-responds: $q_2^*(q_1)$.
  2. Firm 1 anticipates $q_2^*(q_1)$ and maximizes its own profit.
- Equilibrium:

$$q_1^L = \frac{a-c}{2b}, \quad q_2^F = \frac{a-c}{4b}.$$

  - Leader produces more than under Cournot; follower less.
  - Total output > Cournot: $Q^S = \frac{3(a-c)}{4b}$.
- Relevant for regulatory settings: regulator as a Stackelberg "leader."

Nash Equilibrium

- A **Nash Equilibrium (NE)** is a strategy profile $(s_1^*, \ldots, s_n^*)$ such that no player can improve by deviating unilaterally:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \text{ for all } s_i \text{ and all } i.$$

- **Existence**: Every finite game has at least one NE (possibly in mixed strategies).
- **Key property**: mutual best responses – no firm regrets its choice given the others' choices.
- In a *static game*, NE is the standard solution concept.
  - Cournot duopoly: intersection of BRFs.

Subgame Perfect Equilibrium (SPE)

- In **sequential games**, a Nash Equilibrium may rely on non-credible threats.
- A **Subgame Perfect Equilibrium** requires Nash Equilibrium behavior in *every* subgame.
  - Eliminates non-credible threats via backward induction.
- **Backward induction procedure**:
  1. Solve the last stage first.
  2. Given last-stage solutions, solve the second-to-last stage.
  3. Continue until the first stage.
- Example: emission fee game.
  - Stage 1: regulator sets fee t.
  - Stage 2: firms choose output q_i.
  - SPE: solve firm decisions first, then regulator's optimal fee.

Heterogeneous Firms

- In practice, firms differ in:
  - Production costs: $c_1 \neq c_2$.
  - Pollution intensity: e_i / q_i differs across firms.
  - Abatement costs: marginal abatement cost curves differ.
- **Implications for regulation**:
  - A uniform emission fee may impose disproportionate burdens on high-cost abaters.
  - Differentiated regulation can be welfare-improving but harder to implement.
  - Permit trading allows abatement to occur where it is cheapest.
- Studied in Chapter 3 (asymmetric firms and second-best regulation).

Uncertainty and Instrument Choice

- Regulators typically face uncertainty about:
  - Marginal abatement costs (private information of firms).
  - Marginal damage from pollution.
- **Weitzman (1974)**: Under uncertainty, the choice between taxes (price instruments) and permits (quantity instruments) depends on the relative slopes of marginal cost and marginal damage:
  - If marginal damage is steep $\Rightarrow$ quantity instrument preferable.
  - If marginal abatement cost is steep $\Rightarrow$ price instrument preferable.
- **Hybrid instruments** (price-quantity combinations) can outperform either pure instrument in some settings.
- This book analyzes these trade-offs in strategic market environments.

Abatement Investment

- Firms can invest in **abatement technology** to reduce emissions per unit of output.
  - Emission: $e_i = q_i - z_i$, where z_i is abatement.
  - Abatement cost: $C(z_i)$ – increasing and convex.
- Multi-stage game:
  1. Firms invest in abatement (z_i).
  2. Regulator sets emission fee (t).
  3. Firms choose output (q_i).
- **Strategic incentive**: firms may invest in abatement to influence the regulator's future fee.
  - More abatement $\Rightarrow$ lower fee expected $\Rightarrow$ higher profits.
- Analyzed in Chapters 5 and 7.

Mergers, Cross-Ownership, and Regulation

- **Mergers** reduce the number of firms, affecting both market power and aggregate emissions.
  - Less output: lower emissions (good for environment).
  - But also less competition: higher prices (bad for consumers).
- Antitrust authority weighs both effects when evaluating mergers.
- **Cross-ownership**: firm i holds a partial stake α in firm j.
  - Softens competition: each firm internalizes a share of rivals' profits.
  - Environmental implications: cross-ownership may raise or lower industry emissions.
- **Environmental R&D cartels**: firms cooperate on abatement investment.
  - Internalize abatement spillovers; may raise social welfare.
- Studied in Chapters 7 and 8.

Section 1.4
A Short History of Environmental Regulation

Origins: London Smog and Early US Regulation

- Industrial pollution and urban smog became visible public health threats in the mid-20th century.
- **The Great London Smog of 1952**:
  - Five days of severe air pollution killed an estimated 4,000–12,000 people.
  - Led to the UK Clean Air Act of 1956.
- **US Clean Air Act of 1970**:
  - Established the Environmental Protection Agency (EPA).
  - Mandated large reductions in automobile emissions by 1975.
  - Led to catalytic converters and unleaded gasoline.
  - Significantly reduced lead and CO levels; see EPA (2011).

The 1990 Clean Air Act Amendments

- Significantly expanded regulatory authority in the US.
- Introduced **market-based mechanisms**:
  - Tradable permits for SO_2 under the Acid Rain Program.
  - Reformulated fuels to reduce urban smog.
- **Outcomes by early 2000s**:
  - NOx and particulate matter (PM) emissions reduced by over 90% in some sectors.
  - The SO_2 cap-and-trade program became a flagship example of successful environmental market design.
- **Lesson**: Well-designed market-based instruments can achieve large environmental gains at low cost.

Sweden's Carbon Tax and Chile's Carbon Price

- **Sweden (1991)**:
  - First country to introduce a comprehensive carbon tax.
  - Exceeded €115 per ton of CO_2 by the 2020s.
  - Contributed to a 27% reduction in greenhouse gas emissions between 1990 and 2018 – while maintaining GDP growth.
- **Chile (2017)**:
  - Carbon tax targeting large thermal power plants.
  - Prompted a shift toward cleaner energy sources; see United Nations (2021).
- **Canada – Output-Based Pricing System (OBPS)**:
  - Sector-specific emission fees to incentivize low-carbon production.
  - Allows output-based credits for efficiency improvements.
- Analyzed in Chapter 2: how emission fees align private incentives with social goals.

Developing Countries: Colombia and South Africa

- Adoption of emission fees in developing countries has been more gradual but increasingly significant.
- **Colombia (2017)**:
  - Modest carbon tax covering fossil fuel use.
  - Revenue directed toward environmental programs.
- **South Africa (2019)**:
  - National carbon tax initially set at R120 ($\approx$ \$8) per ton of CO_2.
  - Exemptions for energy-intensive sectors to ease transition.
- **Challenges in developing countries**:
  - Limited administrative capacity.
  - Political resistance and equity concerns.
  - Nonetheless: important steps toward internalizing externalities; see World Bank (2025).

EU ETS and US Cap-and-Trade

- **EU Emissions Trading System (EU ETS)** – launched 2005:
  - World's largest carbon market, covering $\sim$40% of EU greenhouse gas emissions.
  - Phase I–IV improvements: tighter caps, auctioning, carbon price $>$ €60/ton by 2022.
- **US Regional Greenhouse Gas Initiative (RGGI)**:
  - Cap-and-trade program covering power sector in 12 Northeast states.
  - Demonstrated feasibility of state-level carbon pricing.
- **California Cap-and-Trade (2013)**:
  - Linked with Quebec; covers $\sim$ 85% of California GHG emissions.
  - Allowance prices in the $25–35 range.
- Analyzed in Chapter 6: tradable permits in imperfectly competitive markets.

The Paris Agreement and Global Climate Governance

- **Paris Agreement (2015)**: 196 parties committed to limiting global warming to well below 2°C above pre-industrial levels.
  - Countries submit Nationally Determined Contributions (NDCs).
  - Voluntary pledges reviewed and ratcheted up every five years.
- **Economic challenges**:
  - Free-riding: countries benefit from others' abatement without contributing.
  - Verification and enforcement: no binding compliance mechanism.
  - Carbon leakage: production shifts to unregulated jurisdictions.
- **Market-based solutions**: Carbon border adjustments (EU CBAM), international carbon markets (Article 6 of Paris Agreement).
- These strategic dimensions are central to the models in this book.

The Coase Theorem

- **Coase (1960)**: In the absence of transaction costs and with well-defined property rights, private bargaining leads to an efficient outcome regardless of how rights are initially assigned.
- **Implication**: Government intervention may be unnecessary if:
  - Property rights are clearly defined.
  - Transaction costs are negligible.
  - Parties can negotiate freely.
- **Limitations in practice**:
  - Many polluters and victims $\Rightarrow$ high transaction costs.
  - Asymmetric information, hold-up problems.
  - Unclear property rights (who owns clean air?).
- **Empirical evidence**: Tradable permit systems partly reflect Coasian logic.
- Revisited in Chapter 2.

Book Overview

Roadmap for Chapters 2–8

Chapter 2: Regulating a Polluting Oligopoly

- Baseline model: n Cournot firms with emissions $e_i = q_i$.
- Regulator maximizes social welfare:
  $W = CS + PS + T - \frac{d}{2}E^2$, where $E = \sum e_i$.
- **Key results**:
  - Optimal emission fee and its relationship to market structure.
  - Comparison: fee vs. emission quota.
  - The Coase Theorem revisited in an oligopoly context.
- **Tools introduced**:
  - Best response functions (BRFs).
  - Two-stage Stackelberg game between regulator and firms.
  - Social marginal benefit / social marginal cost framework.

Chapters 3–4: Asymmetric Firms and Uncertainty

- **Chapter 3**: Regulating asymmetric Cournot oligopolists.
  - Firms differ in marginal costs or pollution intensity.
  - First-best vs. second-best (uniform fee) regulation.
  - Price vs. quantity instruments under asymmetric information.
- **Chapter 4**: Uncertainty and the Weitzman result.
  - Uncertainty about abatement costs and/or damages.
  - Expected welfare comparison: emission tax vs. quantity standard.
  - Extension to oligopolistic markets.
  - Hybrid instruments (taxes with a safety valve).

Chapters 5–6: Sequential Competition and Abatement Investment

- **Chapter 5**: Sequential competition and regulation.
  - Stackelberg (leader-follower) market structure.
  - How first-mover advantage interacts with emission fees.
  - Two-period model with pollution persistence; regulatory commitment.
- **Chapter 6**: Regulation when firms invest in abatement.
  - Firms choose output *and* abatement investment simultaneously.
  - Tax-saving effect (TSE) and emission-reduction effect (ERE).
  - Spillovers, free-riding, and environmental R&D joint ventures (ERJVs).

Chapters 7–8: Mergers and Managerial Incentives

- **Chapter 7**: Environmentally friendly mergers?
  - Effect of horizontal mergers on emissions and social welfare.
  - When do mergers reduce pollution? Antitrust vs. environmental goals.
  - Optimal emission fee before and after a merger.
- **Chapter 8**: Managerial incentives and environmental policy.
  - Owners delegate decisions to managers with profit-sales objectives.
  - How managerial contracts interact with emission regulation.
  - Implications for fee design when managers, not owners, choose output.
- **Unifying theme**: All chapters share the same baseline welfare function and oligopoly framework introduced in Chapter 2.

Analytical Framework and Notation

- **Baseline model** (used throughout):
  - n firms; inverse demand $p(Q) = 1 - Q$; marginal cost $c \in [0, 1]$.
  - Emissions: $e_i = q_i - z_i$ ($z_i \geq 0$: abatement).
  - Environmental damage: $ED = \frac{d}{2}E^2$, $d > 0$.
- **Social welfare**:

$$W = CS + PS + T - ED = \frac{Q^2}{2} + \sum_i \pi_i + tE - \frac{d}{2}E^2.$$

- **Key notation**:
  - t: emission fee per unit; $E = Q - Z$: aggregate net emissions.
  - $Z = \sum z_i$: aggregate abatement.
  - Superscripts: NE (Nash), SO (social optimum), NM (no merger), M (merger).

How Each Chapter Adds Complexity

- **Ch. 2** – Baseline: n symmetric Cournot firms, linear demand, per-unit emission fee.
- **Ch. 3** – Extension: asymmetric firms (different costs or pollution intensities); second-best uniform fees.
- **Ch. 4** – Extension: uncertainty about costs or damages; Weitzman price-vs.-quantity choice.
- **Ch. 5** – Extension: sequential (Stackelberg) competition; intertemporal pollution persistence.
- **Ch. 6** – Extension: endogenous abatement investment; spillovers; ERJVs.
- **Ch. 7** – Extension: horizontal mergers; antitrust-environment interaction.
- **Ch. 8** – Extension: managerial incentive contracts; owners delegate to profit-sales maximizing managers.

Prerequisites

- **Calculus**: Partial derivatives, constrained optimization, the envelope theorem.
- **Microeconomics**: Consumer surplus, producer surplus, welfare analysis; monopoly and oligopoly pricing.
- **Game theory** (covered in this chapter):
  - Normal-form and extensive-form games.
  - Nash Equilibrium and Subgame Perfect Equilibrium.
  - Backward induction; Stackelberg competition.
- **Not required**: Advanced econometrics, stochastic calculus, or industrial organization beyond intermediate microeconomics.
- Each chapter is self-contained and can be read independently after Chapter 2.

Chapter 1 – Key Takeaways

- **Environmental externalities** cause market failures: too much pollution, too little abatement.
- **Policy instruments** – taxes, permits, standards – differ in efficiency, enforceability, and political feasibility.
- **Market structure matters**: in oligopolies, regulation interacts with strategic behavior, making simple Pigouvian prescriptions incomplete.
- **Game theory** provides the tools to model these interactions: Nash Equilibrium, backward induction, and Subgame Perfect Equilibrium.
- **Historical evidence**: well-designed market-based instruments (US SO_2 trading, Sweden's carbon tax) can achieve large environmental gains at low cost.
- **This book's approach**: progressively add complexity to the baseline model, always solving for SPE via backward induction.

Thank you!

Chapter 2

Regulating a Polluting Oligopoly

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 2

- This chapter analyzes environmental regulation in oligopolistic industries using a game-theoretic framework.
- Sections:
  - 2.1 Introduction – motivation and guiding questions.
  - 2.2 Benchmark: unregulated Cournot oligopoly.
  - 2.3 Regulated oligopoly: optimal emission fee.
  - 2.4 Social marginal benefit and cost interpretation.
  - 2.5 Alternative instruments: quotas, tradable permits, Coase Theorem.
  - 2.6 Case studies. 2.7 Summary.

Section 2.1

Introduction

The Central Problem

- Firms in oligopolistic industries generate *emissions* as a by-product of production $\Rightarrow$ negative externalities reduce social welfare.
- How should policymakers regulate pollution while accounting for *strategic* firm behavior?
- Chapter 2 provides a game-theoretic answer:
  - Section 2.2: derive the unregulated benchmark (Cournot equilibrium).
  - Section 2.3: introduce emission fees and find the socially optimal fee.
  - Sections 2.4–2.5: interpret results and compare alternative instruments.
- Key insight: two market failures coexist in oligopoly — *market concentration* (underproduction) and *pollution* (overproduction). The optimal policy must balance both.

Two Competing Market Failures in Oligopoly

- **Market failure 1 – Pollution externality:** each firm ignores the damage its emissions impose on society $\Rightarrow$ firms produce *too much* output and generate excessive emissions.
  - Corrective instrument: **emission fee** (positive t).
- **Market failure 2 – Market concentration:** a duopoly produces *less* than the competitive level due to market power $\Rightarrow$ underproduction relative to the social optimum.
  - Corrective instrument: **output subsidy** (negative t).
- The optimal emission fee **balances** these two failures:

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

- When $d > \frac{1}{2}$: pollution failure dominates $\Rightarrow t^* > 0$.
- When $d < \frac{1}{2}$: concentration failure dominates $\Rightarrow t^* < 0$ (subsidy).

Guiding Questions

1. How does Cournot competition affect pollution and welfare in unregulated markets?
2. What are the marginal benefits and costs of optimal regulation?
3. What is the role of emission fees in aligning private and social incentives?
4. How does market concentration interact with the pollution externality to shape the optimal policy instrument?
5. Are alternative policy instruments—like quotas and tradable permits—as effective as emission fees at addressing pollution?
6. When can private bargaining (Coase Theorem) substitute for government regulation?

Real-World Context: Pollution in Concentrated Industries

- **Cement industry (global):** a few firms dominate each national market; production generates substantial CO_2 $\Rightarrow$ classic oligopoly-with-pollution setting.
- **Steel and aluminum:** energy-intensive, few large producers; subject to carbon pricing in EU, Canada, and emerging US proposals.
- **Electric power:** regional monopolies or tight oligopolies; largest single source of CO_2 in most economies.
- **Chemicals and petrochemicals:** concentrated global markets; generate toxic emissions regulated by emission fees and cap-and-trade.
- **Common pattern:** all these industries combine (i) market power and (ii) significant pollution externalities $\Rightarrow$ exactly the setting of Chapter 2.
- Chapter 2 provides the *baseline model* extended in all

Section 2.2

Benchmark: Unregulated Oligopoly

Model Setup

- **Industry**: two firms competing à la Cournot (simultaneous output choices). (Later extended to n firms.)
- **Inverse demand**: $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- **Marginal production cost**: c, symmetric, with $1 > c \geq 0$.
- **Emissions**: every unit of output generates one unit of emissions, so $e_i = q_i$ and aggregate emissions $E = Q$.
- **Environmental damage**: $Env = \frac{d}{2}E^2$, where $d \geq \frac{1}{2}$ measures pollution severity.
  - Damage is increasing and convex in emissions: marginal damage dE rises with E.
- **Social welfare**: $W = CS + PS - Env$, where $CS = \frac{Q^2}{2}$ and $PS = \pi_1 + \pi_2$.

Firm 1's Profit-Maximization Problem

Firm 1 chooses q_1 to solve:

$$\max_{q_1 \geq 0} \pi_1 = (1 - q_1 - q_2)q_1 - cq_1$$

- First term: total revenue $p(Q) \cdot q_1 = (1 - q_1 - q_2)q_1$.
- Second term: total production cost cq_1.
- Firm 1 takes q_2 as *given* (cannot control rival's output).

Taking the first-order condition (FOC) with respect to q_1:

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - c = 0$$

Interpretation: firm 1 increases q_1 until marginal revenue $1 - 2q_1 - q_2$ equals marginal cost c.

Firm 1's Best Response Function (Section 2.2.1)

Rearranging the FOC yields firm 1's **best response function (BRF)**:

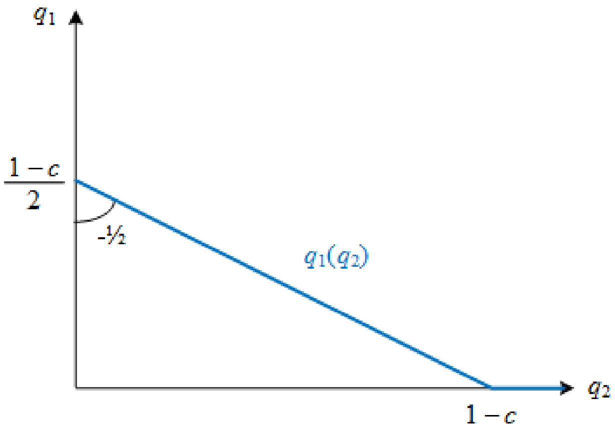
$$q_1(q_2) = \begin{cases} \dfrac{1-c}{2} - \dfrac{1}{2}q_2 & \text{if } q_2 \leq 1-c \\ 0 \quad \text{otherwise} \end{cases}$$

Key properties:

1. When $q_2 = 0$: firm 1 produces the *monopoly output* $\frac{1-c}{2}$.
2. As q_2 rises, q_1 *falls* — strategic substitutes: slope $= -\frac{1}{2}$.
3. When $q_2 = 1-c$: firm 1 shuts down $(q_1 = 0)$.

Interpretation: firm 1's optimal output is decreasing in its rival's output because rival production reduces the residual demand facing firm 1.

Figure 2.1 – Firm 1's Best Response Function



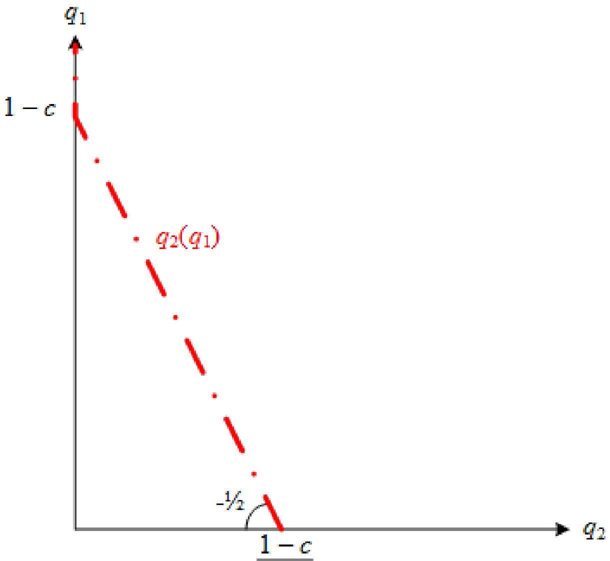
Firm 2's Best Response Function

By symmetry ($q_i = q_j$, same costs and demand), firm 2's BRF is:

$$q_2(q_1) = \begin{cases} \dfrac{1-c}{2} - \dfrac{1}{2}q_1 & \text{if } q_1 \le 1-c \\ 0 & \text{otherwise} \end{cases}$$

- Firm 2's BRF is plotted on the same axes as firm 1's, but with axes *reversed*: firm 2's output q_2 on the vertical axis, firm 1's output q_1 on the horizontal axis.
- Originates at $q_2 = 1 - c$ when $q_1 = 0$; decreases as q_1 rises; intercepts the horizontal axis at $q_1 = \frac{1-c}{2}$.
- Same slope $-\frac{1}{2}$ by symmetry.

Figure 2.2 – Firm 2's Best Response Function



Finding Equilibrium Output (Section 2.2.2)

Two methods to find the Nash equilibrium:

Method 1 — Invoke symmetry (same costs and demand):

- Impose $q_1 = q_2 = q$ in BRF$_1$: $q = \frac{1-c}{2} - \frac{1}{2}q$
- Solve: $\frac{3}{2}q = \frac{1-c}{2}$, so $q^{NR} = \dfrac{1-c}{3}$

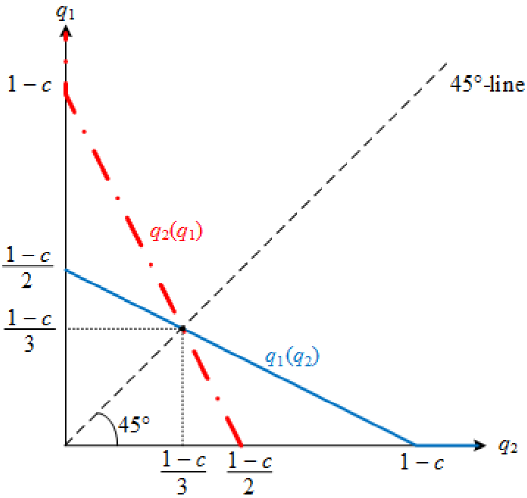
Method 2 — Substitute one BRF into the other:

- Insert $q_2(q_1)$ into $q_1(q_2)$: $q_1 = \frac{1-c}{2} - \frac{1}{2}\left(\frac{1-c}{2} - \frac{1}{2}q_1\right)$
- Simplify: $q_1 - \frac{1}{4}q_1 = \frac{1-c}{2} - \frac{1-c}{4}$, yielding $q_1 = \frac{1-c}{3}$. $\Rightarrow$ same result.

Equilibrium outcomes (superscript NR = no regulation):

$$q_1^{NR} = q_2^{NR} = \frac{1-c}{3}, \quad Q^{NR} = \frac{2(1-c)}{3}, \quad p^{NR} = \frac{1+2c}{3}$$

Figure 2.3 – Nash Equilibrium Output



Pollution and Welfare Without Regulation (Section 2.2.3)

Since $e_i = q_i$, aggregate emissions are $E^{NR} = Q^{NR} = \frac{2(1-c)}{3}$.

Welfare under no regulation:

$$\begin{aligned} W^{NR} &= CS^{NR} + PS^{NR} - Env^{NR} \\ &= \frac{(Q^{NR})^2}{2} + \left[(1 - Q^{NR})Q^{NR} - cQ^{NR}\right] - \frac{d}{2}(E^{NR})^2 \\ &= \frac{2(1-c)^2(2-d)}{9} \end{aligned}$$

Observations:

- W^{NR} is *decreasing* in pollution severity d.
- $W^{NR} < 0$ if $d > 2$ (environmental damage dominates all surplus).
- Firms ignore the externality $\Rightarrow$ socially *excessive* output and emissions.
- Regulation is needed to correct this market failure.

Extending to n Firms (Section 2.2.4)

With $n \geq 2$ symmetric firms, every firm i solves:

$$\max_{q_i \geq 0} \pi_i = (1 - q_i - Q_{-i})q_i - cq_i$$

where $Q_{-i} = \sum_{j \neq i} q_j$.

Invoking symmetry ($q_i = q$ for all i, so $Q_{-i} = (n-1)q$):

$$q^{NR}(n) = \frac{1-c}{n+1}, \qquad Q^{NR}(n) = \frac{n(1-c)}{n+1}$$

Key findings:

- *Individual* output *decreases* in n: each firm is smaller.
- *Aggregate* output and emissions *increase* in n: more competition $\Rightarrow$ more pollution.
- Welfare $W^{NR}(n) > 0$ iff $d < \overline{d}(n) \equiv \frac{n+2}{n}$.
- $\overline{d}(n)$ decreases in n: more firms $\Rightarrow$ welfare more likely negative.

Effect of More Firms: Intuition and Examples

- Each additional firm produces less individually (smaller residual demand).
- But the industry as a whole emits *more*: the entry effect dominates the composition effect.
- Welfare cutoff satisfies $\frac{\partial \bar{d}(n)}{\partial n} = -\frac{2}{n^2} < 0$.
- **Examples:**

n	$q^{NR}(n)$	$\bar{d}(n)$
2	$\frac{1-c}{3}$	2
3	$\frac{1-c}{4}$	$\frac{5}{3} \approx 1.67$
5	$\frac{1-c}{6}$	$\frac{7}{5} = 1.40$
∞	0	1

Policy implication: in more competitive industries, stricter regulation is needed to prevent welfare losses from rising aggregate pollution.

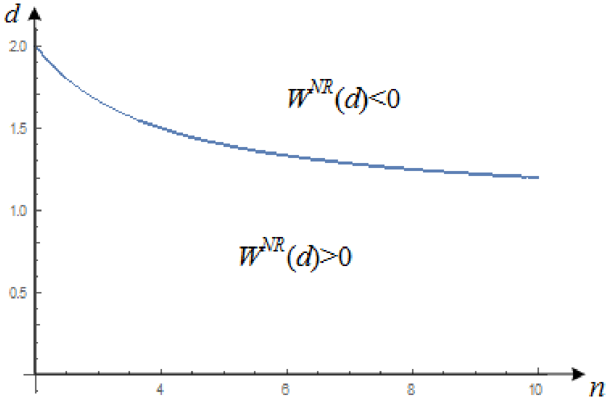
Welfare With n Firms: Conditions for Positive Welfare

- With n firms, social welfare under no regulation is:

$$W^{NR}(n) = \frac{n(1-c)^2[2+n(1-d)]}{2(n+1)^2}$$

- $W^{NR}(n) > 0$ if and only if $d < \overline{d}(n) \equiv \frac{n+2}{n}$.
- **Intuition for $\overline{d}(n)$ declining in n:**
  - More firms $\Rightarrow$ aggregate emissions rise $\Rightarrow$ environmental damage rises.
  - As $n \to \infty$: aggregate output approaches the competitive level $1-c$; environmental damage $\frac{d}{2}(1-c)^2$ can exceed total surplus when $d \geq 1$.
- **Policy implication**: highly competitive industries face a *more urgent* need for environmental regulation than concentrated ones — exactly because competition raises aggregate pollution.

Figure 2.4 – Welfare Cutoff $\overline{d}(n)$ as a Function of n



Section 2.3

Regulated Oligopoly: Optimal Emission Fee

Backward Induction and Subgame Perfect Equilibrium

- The two-stage game is solved by **backward induction**: start from Stage 2 (output) and work back to Stage 1 (fee-setting).
- **Why backward induction?** Each player's action at each stage must be optimal *given the strategies of all other players at all subsequent stages* — this is the definition of a **Subgame Perfect Equilibrium (SPE)**.
- **Stages of the game:**
  - *Stage 1*: EA commits to fee t knowing firms will respond optimally.
  - *Stage 2*: Firms take t as given and choose output simultaneously (Cournot).
- **Why not a simultaneous-move game?** The EA moves *first* (Stackelberg leader), committing to t before firms choose output. This commitment is central: firms know t is fixed when they choose q_i.
- Solving Stage 2 first gives us $q(t)$, which the EA anticipates in Stage 1.

The Two-Stage Regulatory Game

The regulator (Environmental Agency, EA) introduces a per-unit emission fee t. The game has two stages:

1. **Stage 1 (EA)**: The EA sets emission fee $t \geq 0$ paid by every firm per unit of emissions.
2. **Stage 2 (Firms)**: Each firm i observes t and chooses output q_i to maximize profits.

- Sequential-move game with *perfect information*.
- Solution concept: **Subgame Perfect Equilibrium (SPE)** via *backward induction*.
- We solve Stage 2 first, then Stage 1.
- Key: the EA anticipates firms' responses when setting the fee.

Stage 2: Firms' Output Decisions

Each firm i takes fee t as given and solves:

$$\max_{q_i \geq 0} \pi_i = (1 - q_i - q_j)q_i - (c + t)q_i$$

- The fee raises effective marginal cost from c to $c + t$.

FOC: $1 - 2q_i - q_j = c + t$. Invoking symmetry $(q_i = q_j = q)$:

$$q(t) = \frac{1 - c - t}{3}$$

Aggregate output and emissions as a function of the fee:

$$Q(t) = \frac{2(1 - c - t)}{3} = E(t)$$

- $Q(t)$ is *decreasing* in t.
- When $t = 0$: $Q(0) = Q^{NR} = \frac{2(1-c)}{3}$.
- When $t = 1 - c$: $Q = 0$ (firms shut down).

Stage 1: EA Sets the Optimal Fee — Direct Approach

Anticipating $Q(t)$, the EA maximizes social welfare:

$$\max_{t \geq 0} W = CS + PS + T - Env$$

where $T = tQ(t)$ is total tax revenue (returned to society as a lump-sum transfer). Note that $PS + T = (1-Q)Q - cQ$, so:

$$W = \frac{Q(t)^2}{2} + \left[(1 - Q(t))Q(t) - cQ(t)\right] - \frac{d}{2}Q(t)^2$$

FOC with respect to t:

$$\frac{\partial W}{\partial t} = \frac{2(1-c)(2d-1) - 4t(1+d)}{9} = 0$$

Optimal emission fee:

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Interpreting the Optimal Fee

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

When is $t^* > 0$? When $2d - 1 > 0$, i.e., $d > \frac{1}{2}$.

Two competing market failures in oligopoly:

1. **Pollution externality**: firms emit too much $\Rightarrow$ calls for a *fee* (positive t).
2. **Market concentration**: duopoly output is below the competitive level $\Rightarrow$ calls for a *subsidy* (negative t).

- If $d < \frac{1}{2}$: market power failure *dominates* $\Rightarrow$ $t^* < 0$ (subsidy).
- If $d = \frac{1}{2}$: failures exactly cancel $\Rightarrow$ $t^* = 0$.
- If $d > \frac{1}{2}$: pollution failure *dominates* $\Rightarrow$ $t^* > 0$ (fee). $\longleftarrow$ *our main case.*

Comparative Statics of t^*

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Effect of pollution severity d:

$$\frac{\partial t^*}{\partial d} = \frac{3(1-c)}{2(1+d)^2} > 0$$

More severe pollution $\Rightarrow$ *stricter* optimal fee.

Effect of marginal cost c:

$$\frac{\partial t^*}{\partial c} = -\frac{2d-1}{2(1+d)} < 0 \quad (\text{since } d > \tfrac{1}{2})$$

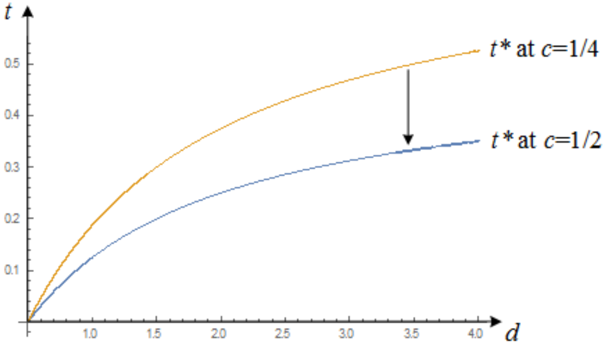
Higher production costs $\Rightarrow$ EA anticipates less output/emissions $\Rightarrow$ *less stringent* fee.

Summary: the optimal fee is jointly shaped by the severity of the pollution externality *and* the degree of market inefficiency.

Regulated Output and Fee with n Firms

- With n symmetric Cournot firms, equilibrium output in Stage 2:
$$q(t,n) = \frac{1-c-t}{n+1}, \qquad Q(t,n) = \frac{n(1-c-t)}{n+1}$$
- Stage 1 optimal fee (setting $Q(t,n) = Q^{SO} = \frac{1-c}{1+d}$):
$$t^*(n) = \frac{(1-c)(nd-1)}{n(1+d)}$$
- **Regulated output remains** $Q^*(n) = Q^{SO} = \frac{1-c}{1+d}$ — independent of n. The EA adjusts the fee to always achieve the same socially optimal aggregate output.
- **But per-firm output differs**: $q^*(n) = \frac{Q^{SO}}{n} = \frac{1-c}{n(1+d)}$ — decreasing in n.
- **Conclusion**: the optimal fee *increases* with n, while regulated output per firm *falls* with n, but total regulated output is constant at Q^{SO}

Figure 2.5 – Fee t^* Increases in d; Shifts Down in c



Fee as a Function of the Number of Firms n

- With n symmetric firms, the optimal fee generalizes to:

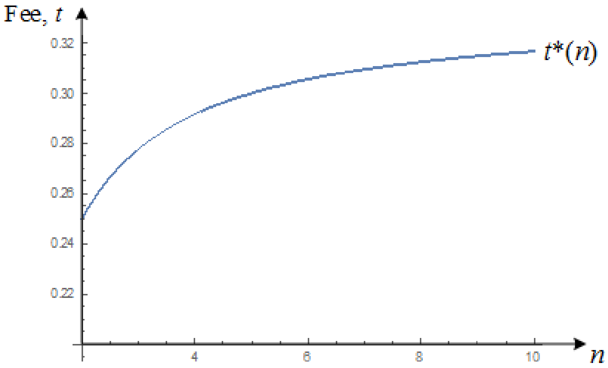
$$t^*(n) = \frac{(1-c)(nd-1)}{n(1+d)}$$

- **Effect of n:**

$$\frac{\partial t^*(n)}{\partial n} = \frac{(1-c)}{n^2(1+d)} > 0$$

- More firms $\Rightarrow$ more aggregate emissions $\Rightarrow$ EA sets a *higher* fee.
- **Special cases:**
  - $n = 2$ (duopoly): $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ (our main formula).
  - $n \to \infty$ (competitive): $t^* \to \frac{(1-c)d}{1+d}$ (approaches full Pigouvian tax).
- As competition increases, the concentration failure diminishes and the fee converges to the pure Pigouvian correction for the externality.

Figure 2.7 – Optimal Fee $t^*(n)$ as a Function of n



Fee as a Function of Emissions Intensity θ

- **Generalization**: each unit of output generates θ units of emissions, $e_i = \theta q_i$, so $E = \theta Q$.
- Welfare becomes $W = \frac{Q^2}{2} + (1-Q)Q - cQ - \frac{d}{2}\theta^2 Q^2$, and the socially optimal output is:

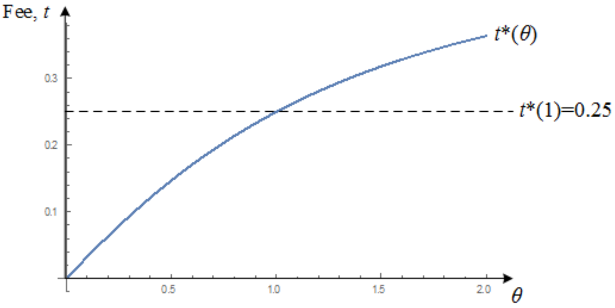
$$Q^{SO} = \frac{1-c}{1+d\theta^2}$$

- The optimal fee (per unit of emissions) that induces Q^{SO}:

$$t^*(\theta) = \frac{(1-c)(2d\theta^2 - 1)}{2\theta(1+d\theta^2)}$$

- **Key result**: $t^*(\theta)$ is increasing in θ for θ sufficiently large — higher emissions intensity requires a higher fee.
- **Baseline**: when $\theta = 1$, $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$, recovering our standard formula.

Figure 2.8 – Fee $t^*(\theta)$ as a Function of Emissions Intensity



Equilibrium Output Under the Optimal Fee

Inserting t^* into $q(t) = \frac{1-c-t}{3}$:

$$q(t^*) = \frac{1-c-t^*}{3} = \frac{1-c}{2(1+d)}$$

Aggregate regulated output:

$$Q(t^*) = \frac{2(1-c-t^*)}{3} = \frac{1-c}{1+d}$$

Comparison with unregulated output (since $d > \frac{1}{2} \Rightarrow 1+d > \frac{3}{2}$):

$$Q(t^*) = \frac{1-c}{1+d} < \frac{2(1-c)}{3} = Q^{NR}$$

Conclusion: the emission fee reduces output (and emissions) below the unregulated level, moving the market toward the social optimum.

Indirect Approach: Finding Socially Optimal Output

Step 1 — find Q^{SO} that maximizes welfare directly:

$$\max_{Q \geq 0} W = \frac{Q^2}{2} + \left[(1-Q)Q - cQ\right] - \frac{d}{2}Q^2$$

FOC: $Q + (1-2Q) - c - dQ = 0$, yielding:

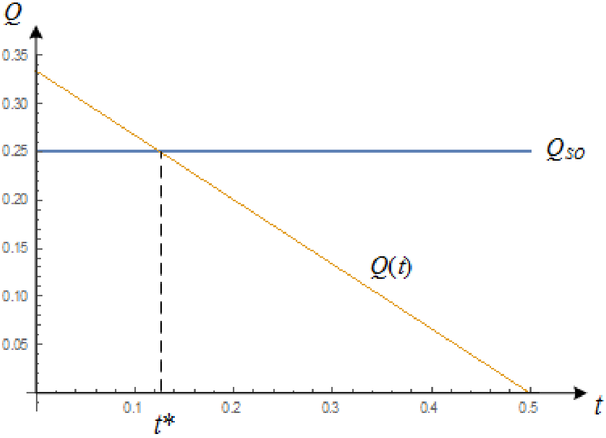
$$Q^{SO} = \frac{1-c}{1+d}$$

Step 2 — find fee t that induces $Q(t) = Q^{SO}$:

$$\frac{1-c}{1+d} = \frac{2(1-c-t)}{3} \quad \Rightarrow \quad t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

Key result: both approaches yield the same t^*.
The indirect approach separates two sub-problems: (i) what is the socially optimal outcome? and (ii) what fee implements it?

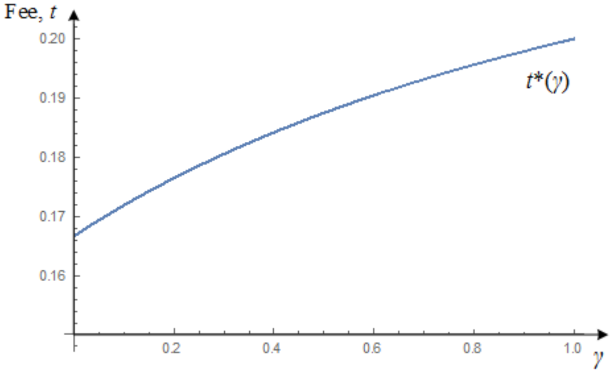
Figure 2.6 – $Q(t)$ and Q^{SO}: Finding the Optimal Fee



Sensitivity of t^* to a Cost Parameter (Low Pollution)

- The figures below illustrate how the optimal fee responds to changes in a **cost efficiency parameter** γ (where higher γ reflects a more productive industry), for two levels of pollution severity.
- **When pollution is mild** (low d): a more productive industry produces more output, increasing aggregate emissions and calling for a *higher* optimal fee.
- As γ rises from 0 to 1 with mild pollution, the pollution externality correction dominates, raising t^* moderately.
- The fee levels are relatively low because the pollution problem is not severe, so the EA requires only a modest correction to achieve the social optimum.

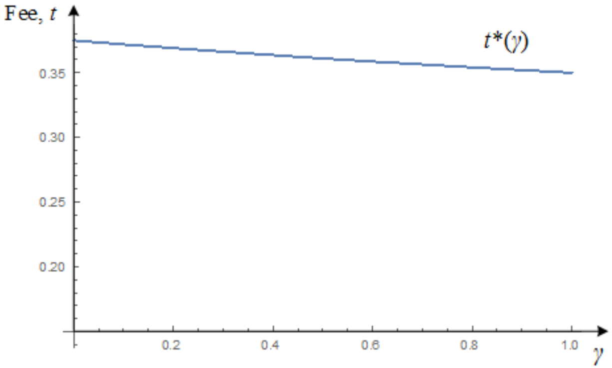
Figure 2.9 – Fee $t^*(\gamma)$ at Low Pollution Severity



Sensitivity of t^* to a Cost Parameter (High Pollution)

- **When pollution is severe** (high d): the pollution externality correction already dominates the market concentration failure — the fee is substantially higher.
- As γ rises with severe pollution, the *concentration correction* built into the fee becomes slightly less important relative to the pollution correction, causing t^* to decline *slightly*.
- Intuitively, when pollution is severe, the EA primarily targets emissions reduction. Higher productivity γ raises output but the fee's pollution-correcting role remains paramount; its concentration-reducing component becomes marginally weaker.
- This contrast — fee rising with γ under mild pollution, falling slightly under severe pollution — illustrates the **dual role of the emission fee** in correcting both externalities simultaneously.

Figure 2.10 – Fee $t^*(\gamma)$ at High Pollution Severity



Proposition: Sign of the Optimal Fee

- **Proposition 2.1.** In a symmetric Cournot duopoly with per-unit emission fee t, the socially optimal fee satisfies:

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)} \begin{cases} > 0 & \text{if } d > \frac{1}{2} \\ = 0 & \text{if } d = \frac{1}{2} \\ < 0 & \text{if } d < \frac{1}{2} \end{cases}$$

- **Interpretation:**
  - $d = \frac{1}{2}$: the pollution and concentration failures *exactly offset*; no policy intervention is needed.
  - $d > \frac{1}{2}$: pollution dominates; a per-unit *tax* reduces excessive output.
  - $d < \frac{1}{2}$: concentration dominates; a per-unit *subsidy* stimulates insufficient output.
- In the remainder of the book we maintain $d > \frac{1}{2}$, so $t^* > 0$.

Welfare Comparison: Regulated vs. Unregulated

- **Welfare without regulation**:

$$W^{NR} = \frac{2(1-c)^2(2-d)}{9}$$

- **Welfare under optimal fee**:

$$W^R = \frac{(1-c)^2(1+2d)}{4(1+d)^2}$$

- **Welfare gain from regulation**: $W^R - W^{NR} > 0$ iff $d > \frac{1}{2}$.
- **Key result**: the optimal fee unambiguously increases social welfare whenever pollution is severe enough to warrant a positive fee ($d > \frac{1}{2}$).
- **Components of welfare gain**:
  - Lower environmental damage (reduced output/emissions).
  - Tax revenue returned to society.
  - Reduced deadweight loss from market power correction.

Worked Example: Computing the Optimal Fee

- **Parameters**: $c = \frac{1}{2}$, $d = 1$.
  - **Unregulated equilibrium:**

$$q^{NR} = \frac{1 - \frac{1}{2}}{3} = \frac{1}{6}, \quad Q^{NR} = \frac{1}{3}, \quad p^{NR} = \frac{1+1}{3} = \frac{2}{3}$$

  - **Optimal fee:**

$$t^* = \frac{\frac{1}{2}(2 \cdot 1 - 1)}{2(1+1)} = \frac{\frac{1}{2}}{4} = \frac{1}{8}$$

  - **Regulated equilibrium:**

$$\begin{aligned} q(t^*) &= \frac{1 - \frac{1}{2} - \frac{1}{8}}{3} = \frac{\frac{3}{8}}{3} = \frac{1}{8}, \\ Q(t^*) &= \frac{1}{4} = Q^{SO} \end{aligned}$$

- **Comparison**: $Q^{NR} = \frac{1}{3} > \frac{1}{4} = Q^{SO}$ — regulation reduces output (and emissions) by 25% to reach the social optimum.

Graphical Interpretation: Three Regions of $Q(t)$

Plot $Q(t) = \frac{2(1-c-t)}{3}$ (downward sloping) and $Q^{SO} = \frac{1-c}{1+d}$ (horizontal) as functions of t:

- $Q(t)$ starts at $Q^{NR} = \frac{2(1-c)}{3}$ when $t = 0$.
- $Q(t)$ falls to zero at $t = 1 - c$.
- Q^{SO} is independent of t (a horizontal line).
- Their crossing point $Q(t^*) = Q^{SO}$ identifies the optimal fee t^*.

Graphical Interpretation: Three Regions of $Q(t)$

Three regions:

- $t < t^*$: $Q(t) > Q^{SO}$ — *socially excessive* output and emissions. The fee is too lax.
- $t = t^*$: $Q(t) = Q^{SO}$ — *social optimum*.
- $t > t^*$: $Q(t) < Q^{SO}$ — output and emissions *too low*. The fee is too stringent.

Section 2.4

Social Marginal Benefit and Cost Framework

Deriving SMB and SMC

From the welfare-maximization problem:

$$\max_{Q\geq 0} W = \frac{Q^2}{2} + \left[(1-Q)Q - cQ\right] - \frac{d}{2}Q^2$$

FOC and rearrangement:

$$\underbrace{1-Q}_{SMB} = \underbrace{c+dQ}_{SMC}$$

- **Social Marginal Benefit** $SMB = 1 - Q$: the net social gain from one more unit (higher consumer surplus and firm revenue).
- **Social Marginal Cost** $SMC = c + dQ = PMC + MEC$, where
  - $PMC = c$: private marginal production cost.
  - $MEC = dQ$: marginal external cost (environmental damage from that extra unit).

Solving $SMB = SMC$ gives $Q^{SO} = \frac{1-c}{1+d}$, confirming the result

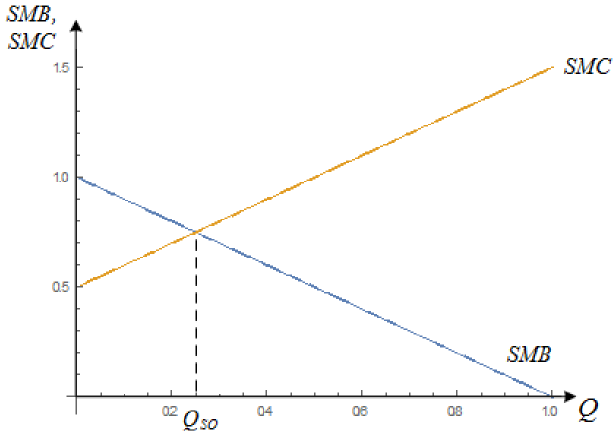
Private vs. Social Costs: PMC, MEC, and SMC

- **Private Marginal Cost** ($PMC = c$): what the firm pays per unit of output — production cost only.
- **Marginal External Cost** ($MEC = dQ$): the environmental damage per unit that the firm *ignores* — increasing in aggregate output (more pollution at the margin when total output is high).
- **Social Marginal Cost** ($SMC = PMC + MEC = c + dQ$): the true social cost of one more unit, accounting for both production and environmental damages.
- **The Pigouvian prescription**: set $t = MEC$ evaluated at Q^{SO}:

$$t^{Pigouvian} = d \cdot Q^{SO} = d \cdot \frac{1-c}{1+d} = \frac{d(1-c)}{1+d}$$

- **Key result**: in oligopoly, $t^* = \frac{(1-c)(2d-1)}{2(1+d)} < t^{Pigouvian}$ because the EA also accounts for the concentration failure

Figure 2.11 – Social Marginal Benefit and Cost



SMB = SMC: Three Cases

When $Q > Q^{SO}$: $SMC > SMB$

- Cost of the extra unit exceeds its benefit.
- *Reducing* output increases social welfare.

When $Q < Q^{SO}$: $SMC < SMB$

- One more unit brings more benefit than cost.
- *Increasing* output improves social welfare.

SMB = SMC: Three Cases

When $Q = Q^{SO}$: $SMB = SMC$

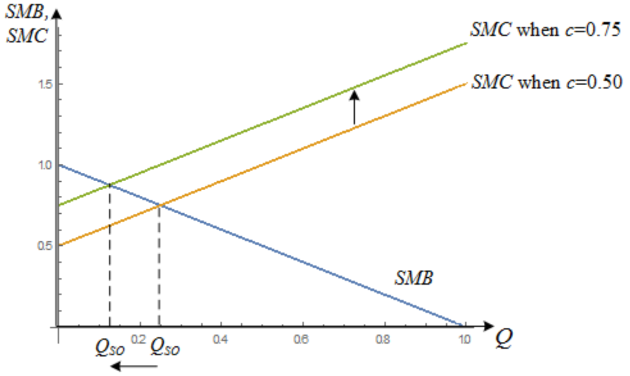
- No welfare gain from further adjustments.
- *Social optimum*.

Why does the unregulated market over-produce?
Under Cournot, firm i equates private MR to $PMC = c$, ignoring $MEC = dQ$. Hence $Q^{NR} > Q^{SO}$.

Comparative Statics: Effect of Higher Cost c

- $SMC = c + dQ$ shifts *upward* (parallel shift) when c rises.
- The *SMB–SMC* crossing point moves *leftward*: Q^{SO} decreases.
- Intuition: less efficient production $\Rightarrow$ it is socially optimal to produce fewer units.
- Formally: $\frac{\partial Q^{SO}}{\partial c} = -\frac{1}{1+d} < 0$.
- The optimal fee also decreases: $\frac{\partial t^*}{\partial c} = -\frac{2d-1}{2(1+d)} < 0$ since $d > \frac{1}{2}$.
- **Policy implication**: industries with higher production costs warrant *less stringent* emission fees, all else equal, because the market already produces closer to the social optimum.

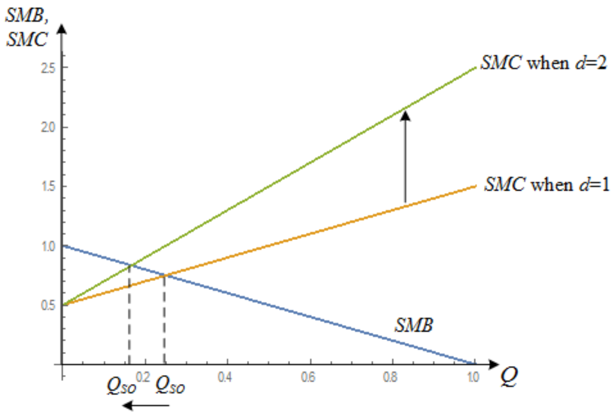
Figure 2.12 – Effect of Higher c: SMC Shifts Up



Comparative Statics: Effect of Higher Pollution Severity d

- $SMC = c + dQ$ becomes *steeper* (pivots upward) when d rises.
- The *SMB–SMC* crossing point moves *leftward*: Q^{SO} decreases.
  - Intuition: each unit causes more damage $\Rightarrow$ society wants less output.
  - Formally: $\frac{\partial Q^{SO}}{\partial d} = -\frac{1-c}{(1+d)^2} < 0$.
- The optimal fee increases: $\frac{\partial t^*}{\partial d} = \frac{3(1-c)}{2(1+d)^2} > 0$.
- **Policy implication**: more polluting industries (or industries with more severe health/environmental impacts) should face *stricter* emission fees.

Figure 2.13 – Effect of Higher *d*: SMC Becomes Steeper



Proposition: Pigouvian vs. Oligopoly Fee

- **Pigouvian tax** (perfect competition): set $t = MEC$ evaluated at the social optimum, yielding:

$$t^{Pigouvian} = d \cdot Q^{SO} = \frac{d(1-c)}{1+d}$$

- **Oligopoly fee** (Cournot duopoly):

$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

- **Comparison**: $t^* < t^{Pigouvian}$ since $t^{Pigouvian} - t^* = \frac{(1-c)}{2(1+d)} > 0$
- **Intuition**: in oligopoly, the EA sets a *lower* fee than under competition to avoid over-restricting output in an already concentrated market.
- The gap $\frac{(1-c)}{2(1+d)}$ shrinks as $n \to \infty$ (more competition) and disappears in the perfectly competitive limit.

Worked Example: SMB, SMC, and Social Optimum

Parameters: $c = 0.5$, $d = 1$.

SMB and SMC:

$$SMB = 1 - Q, \quad SMC = 0.5 + Q$$

Social optimum: $SMB = SMC \Rightarrow 1 - Q = 0.5 + Q \Rightarrow Q^{SO} = 0.25$.

Unregulated equilibrium: $Q^{NR} = \frac{2(1-0.5)}{3} = \frac{1}{3} \approx 0.333$.

Worked Example: SMB, SMC, and Social Optimum

At $Q^{NR} = \frac{1}{3}$: $SMC = 0.5 + \frac{1}{3} \approx 0.833$, $SMB = 1 - \frac{1}{3} \approx 0.667$.

- $SMC > SMB \Rightarrow$ unregulated output is socially excessive.
- *Reducing* output from $\frac{1}{3}$ to $\frac{1}{4}$ increases welfare by eliminating this deadweight loss.

Section 2.5

Alternative Regulatory Instruments

Emission Quotas (Section 2.5.1)

Quota system: each firm receives an individual output cap $\overline{q}_i$ (non-tradable).

Two-stage game under quotas:

1. **Stage 1**: regulator sets $\overline{q}_i$ for firm i.
2. **Stage 2**: firm i solves:
   $\max_{0 \leq q_i \leq \overline{q}_i} \pi_i = (1 - q_i - q_j)q_i - cq_i$.

Emission Quotas (Section 2.5.1)

Stage 2 outcome: quota is binding if $\overline{q}_i < \frac{1-c}{3}$ (the unregulated output). Firm then chooses $q_i = \overline{q}_i$.

Stage 1: regulator sets aggregate output equal to $Q^{SO} = \frac{1-c}{1+d}$, giving each firm:

$$\overline{q}_1 = \overline{q}_2 = \frac{Q^{SO}}{2} = \frac{1-c}{2(1+d)}$$

Since $d > \frac{1}{2}$, we have $\frac{1-c}{2(1+d)} < \frac{1-c}{3}$: quota is *binding*.

Policy Equivalence: Fees vs. Quotas

Result: When the regulator is *fully informed*, emission fees and quotas are *equally effective*:

- Both instruments induce aggregate output $Q^{SO} = \frac{1-c}{1+d}$.
- Both achieve the social optimum.

Policy Equivalence: Fees vs. Quotas

When does equivalence break down?

- If the regulator has *incomplete information* about costs or demand:
  - A fee chosen under wrong beliefs may induce $Q \neq Q^{SO}$.
  - A quota directly caps output — less sensitive to cost misspecification.
- **Weitzman (1974)**: under cost uncertainty, fees are preferred when the marginal benefit curve is relatively flat; quotas are preferred when it is steep.

Price vs. Quantity Instruments: The Weitzman Trade-off

- **Price instrument (emission fee)**: the regulator sets the marginal cost of emissions; quantity adjusts endogenously. Efficient if marginal benefits are predictable.
- **Quantity instrument (quota)**: the regulator directly caps total emissions; firms adjust behavior to meet the cap. Efficient if pollution threshold matters (e.g., irreversible tipping points).
- **Weitzman's insight**: the relative slope of *SMB* and *SMC* determines the preferred instrument:
  - Steep *SMB*, flat *SMC*: **quotas** preferred (small quantity errors are costly).
  - Flat *SMB*, steep *SMC*: **fees** preferred (fee errors are less harmful).
- **Practical example**: climate change often argued as a flat-SMB setting (feedback nonlinearities) $\Rightarrow$ case for carbon

Tradable Permits / Cap-and-Trade (Section 2.5.2)

Cap-and-trade: regulator sets an aggregate emissions cap, distributes permits as individual quotas, and *allows firms to trade* permits.

Key advantage over non-tradable quotas:

- Permits flow to firms with the highest abatement cost.
- Low-abatement-cost firms sell permits; high-abatement-cost firms buy.
- Result: *cost-effective* achievement of the aggregate target.

Tradable Permits / Cap-and-Trade (Section 2.5.2)

In equilibrium: the market permit price equals the marginal abatement cost, replicating the outcome under an optimal emission fee.

Real-world examples:

- EU Emissions Trading System (EU ETS, since 2005).
- California Cap-and-Trade Program (since 2013).
- Washington State Cap-and-Invest Program (since 2023).

The Coase Theorem (Section 2.5.3)

Coase Theorem (Coase, 1960): if property rights are *clearly defined* and transaction costs are *negligible*, private bargaining achieves an efficient allocation — regardless of who holds the initial rights.

Application to pollution:

- A polluting firm and affected residents negotiate over emissions.
- Bargaining drives the outcome to $SMB = SMC$ (social optimum).

The Coase Theorem (Section 2.5.3)

If residents hold clean-air rights:

- The firm *pays* residents to accept more pollution, up to $SMB = SMC$.

If the firm holds pollution rights:

- Residents *pay* the firm to reduce pollution, until $SMB = SMC$.

Both scenarios reach the same efficient outcome: Q^{SO}.

Limits of the Coase Theorem

Why private bargaining often fails in practice:

- **High transaction costs**: legal fees, negotiation time, coordination among many affected parties.
- **Ill-defined property rights**: who owns clean air? Clean water?
- **Asymmetric information**: firms may conceal true emissions; residents may exaggerate harm.
- **Public-good nature of pollution**: emissions affect many people simultaneously, creating free-rider problems.

Empirical evidence:

- Hoffman & Spitzer (1982, 1986): lab experiments show efficiency under low transaction costs.
- Deryugina, Moore & Tol (2021): bargaining works in small-scale, localized contexts but rarely scales up.

Conclusion: regulation is needed when bargaining breaks down.

Comparing the Three Instruments

	Emission fee	Quota	Trac
Achieves Q^{SO}?	Yes	Yes	
Cost-effective?	Yes	Only if symmetric	
Revenue for regulator?	Yes	No	Depe
Robust to asymmetric info?	Less	More	
Allows flexibility?	Yes	No	

- With full information and symmetric firms, all three instruments achieve the social optimum.
- Tradable permits combine quantity certainty (like quotas) with cost-effectiveness (like fees) — hence widely adopted in practice.

Section 2.6

Case Studies

Case Study 1: Canada's Cement Industry

- **Industry**: a few major firms (Lafarge, St. Marys Cement) — a classic oligopoly.
- **Policy**: Canada's federal Output-Based Pricing System (OBPS) imposes per-unit fees on facilities emitting above sector-specific benchmarks.
- **Mechanism**: firms emitting *above* benchmark pay a fee per tonne of CO_2; those *below* earn credits.
- **Strategic response**: St. Marys Cement adopted algae-based carbon capture to reduce emissions and avoid fees.
- **Theoretical link**: per-unit fee t raises effective marginal cost from c to $c + t$, inducing $q(t) = \frac{1-c-t}{3} < q^{NR}$.

Source: Cement Association of Canada (2023).

Case Study 2: Steel and Fertilizer Sectors (US)

- **Industry**: steel and fertilizer — energy-intensive sectors with few dominant firms (Stelco, Nucor, Nutrien).
- **Policy**: the Clean Competition Act (proposed 2023) introduces emission fees for firms exceeding sector-specific carbon intensity benchmarks.
- **Firm response**: investment in carbon capture and low-emission technologies to stay below the threshold.
- **Theoretical link**:
  - Fee raises effective marginal cost from c to $c + t$.
  - Firms reduce output and emissions, moving toward Q^{SO}.
  - The fee is calibrated to internalize $MEC = dQ$.

Case Study 3: Sweden's Carbon Tax

- One of the world's *highest* carbon taxes: over €115 per tonne of CO_2.
- **Industry**: cement sector dominated by a few firms (e.g., Cementa).
- **Firm behavior**: investment in alternative fuels and carbon capture; Cementa piloted a zero-emission plant in Slite, targeting full fossil-fuel elimination by 2030.
- **Theoretical interpretation**: a high pollution severity d justifies a high optimal fee $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$.
- **Key insight**: oligopolistic firms respond strategically to high fees — they invest in abatement rather than simply exit, consistent with $q(t) > 0$ as long as $t < 1 - c$.

Case Study 4: Chile's Electricity Sector

- **Industry**: dominated by a few large firms; carbon tax introduced in 2017 at \$5 per tonne of CO_2 for large thermal plants.
- **Reform**: a 2020 expansion broadened coverage to facilities emitting over 25,000 tonnes annually.
- **Outcomes**:
  - 94% of carbon tax revenue came from power generation.
  - Firms shifted toward natural gas and renewables to reduce tax exposure while preserving market share.
- **Theoretical link**: even a modest fee shifts effective marginal cost, reducing emissions. Firms facing $c + t$ optimize:

$$q_i(t) = \frac{1 - c - t}{3} < q^{NR} = \frac{1 - c}{3}$$

Source: Enel Chile (2022).

Case Study 5: Emission Trading Systems

EU ETS (since 2005):

- World's largest carbon market; covers power, industry, aviation.
- Allowances via auctions and free allocation.
- "Fit for 55" reforms tighten the cap; emissions in covered sectors cut by nearly half since inception.

Case Study 5: Emission Trading Systems

California Cap-and-Trade (since 2013):

- Covers 85% of state emissions; quarterly auctions, banking, offsets.
- Revenues fund climate initiatives via the Greenhouse Gas Reduction Fund.

Washington State Cap-and-Invest (since 2023):

- Covers 70% of state emissions; free allocation for trade-exposed sectors.
- Target: net-zero by 2050; exploring linkage with California and Québec.

Case Study 6: Testing the Coase Theorem

Laboratory experiments:

- Hoffman & Spitzer (1982, 1986): under complete information and low transaction costs, parties reach efficient agreements — confirming the Theorem.
- Efficiency deteriorates when information is asymmetric or bargaining costs rise.

Field studies:

- Cheung (1973): pollination contracts between orchard owners and beekeepers in California — private bargaining resolves the externality.
- Ellickson (1986): cattle trespass disputes in Shasta County, CA — informal norms substitute for legal enforcement.
- Deryugina, Moore & Tol (2021): bargaining works in small-scale contexts; fails to scale due to high transaction costs and diffuse property rights.

Overview: Chapter 2 Results at a Glance

Variable	No regulation	With optimal fee
Per-firm output	$\frac{1-c}{3}$	$\frac{1-c}{2(1+d)}$
Aggregate output	$\frac{2(1-c)}{3}$	$\frac{1-c}{1+d}$
Aggregate emissions	$\frac{2(1-c)}{3}$	$\frac{1-c}{1+d}$
Emission fee	0	$\frac{(1-c)(2d-1)}{2(1+d)}$
Welfare	$\frac{2(1-c)^2(2-d)}{9}$	$\frac{(1-c)^2(1+2d)}{4(1+d)^2}$

- Since $d > \frac{1}{2}$: regulated output $<$ unregulated output.
- Regulated welfare $>$ unregulated welfare.
- The optimal fee is *below* the Pigouvian level $\frac{d(1-c)}{1+d}$ by exactly $\frac{(1-c)}{2(1+d)}$.

Summary: Benchmark and Emission Fees

Benchmark (Unregulated Cournot Oligopoly):

- Firms compete à la Cournot; each unit of output generates one unit of emissions.
- Equilibrium: $q^{NR} = \frac{1-c}{3}$, $Q^{NR} = \frac{2(1-c)}{3}$.
- Firms produce *more* than the social optimum: they ignore $MEC = dQ$.
- More firms $\Rightarrow$ higher aggregate emissions; welfare cutoff $\overline{d}(n) = \frac{n+2}{n}$ decreases in n.

Optimal Emission Fee (Two-Stage SPE):

- $t^* = \frac{(1-c)(2d-1)}{2(1+d)} > 0$ iff $d > \frac{1}{2}$.
- Balances two market failures: pollution (overproduction) and market concentration (underproduction).
- Fee increases in d and in n; decreases in c.
- Regulated output: $Q(t^*) = Q^{SO} = \frac{1-c}{1+d} < Q^{NR}$.

Summary: SMB/SMC and Alternative Instruments

SMB/SMC Framework:

- $SMB = 1 - Q$; $SMC = PMC + MEC = c + dQ$.
- Social optimum: $SMB = SMC \Rightarrow Q^{SO} = \frac{1-c}{1+d}$.
- Higher c or higher d shifts SMC upward/steeper $\Rightarrow$ lower Q^{SO}.
- In oligopoly: $t^* < t^{Pigouvian}$ because of the concentration correction.

Alternative Instruments:

- **Emission quotas**: direct caps on output; achieve Q^{SO} when the regulator is fully informed. Quota is binding iff $d > \frac{1}{2}$.
- **Tradable permits**: cap-and-trade combines quantity certainty with cost-effectiveness; equilibrium permit price replicates t^*.
- **Coase Theorem**: private bargaining achieves Q^{SO} if property rights are well defined and transaction costs are negligible; in practice, these conditions often fail $\Rightarrow$ regulation is needed.

Chapter 2 as Baseline: Connection to Later Chapters

- **Chapter 2** establishes the *baseline model*: Cournot duopoly + emission fee; no abatement, no asymmetry, no entry/exit.
- **Chapters 3–8 extend Chapter 2:**
  - **Chapter 3**: asymmetric firms (different costs); Stackelberg competition; differentiated products.
  - **Chapter 4**: uncertainty about pollution severity d.
  - **Chapter 5**: Stackelberg output competition with regulation.
  - **Chapter 6**: firms invest in abatement technologies z_i before the fee is set (three-stage game).
  - **Chapter 7**: international trade and transboundary pollution.
  - **Chapter 8**: dynamic settings; repeated games.

Key Takeaways and Policy Implications

1. **Two failures, one instrument**: the emission fee simultaneously corrects the pollution externality and the oligopoly concentration distortion.
2. **Fee formula**: $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ is below the Pigouvian rate — setting the fee too high over-corrects for pollution while worsening the output distortion.
3. **Market structure matters**: with more firms $(n \uparrow)$, the concentration failure shrinks and the optimal fee rises toward the Pigouvian level.
4. **Emissions intensity matters**: industries generating more emissions per unit of output $(\theta \uparrow)$ require more stringent fees.
5. **Instrument equivalence**: fees, quotas, and tradable permits achieve the same outcome under full information — but differ in robustness to uncertainty and cost-effectiveness.
6. **Chapter 2 as baseline**: all extensions in subsequent chapters build on the model developed here.

Thank You

Environmental Regulation: A Game-Theoretic Approach

Chapter 2 — Regulating a Polluting Oligopoly

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 3

Extensions to the Benchmark Model

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

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Overview of Chapter 3

- Chapter 2 established the benchmark: two symmetric Cournot firms, linear costs, homogeneous product.
- Chapter 3 **relaxes** each assumption in turn and asks: how does the optimal emission fee change?
- Sections:
  - **3.1** Introduction and guiding questions.
  - **3.2** More firms (n symmetric firms).
  - **3.3** Pollution asymmetries (heterogeneous emission intensities).
  - **3.4** Cost asymmetries (different production costs).
  - **3.5** Nonlinear costs (diseconomies of scale).
  - **3.6** Product differentiation (imperfect substitutes).
  - **3.7** Cost externalities (rivals' output raises marginal cost).
  - **3.8–3.10** Case studies, related literature, summary.

Section 3.1
Introduction and Guiding Questions

Why Extend the Benchmark?

- The Chapter 2 model is a workhorse, but real polluting industries are far more heterogeneous.
- **Motivation for each extension:**
  - *More firms:* markets often have many competitors (e.g., cement, steel, paper).
  - *Pollution asymmetries:* different technologies produce very different emission intensities.
  - *Cost asymmetries:* incumbents and entrants face different production costs.
  - *Nonlinear costs:* diminishing returns are pervasive in manufacturing.
  - *Product differentiation:* most industries sell differentiated goods.
  - *Cost externalities:* rivals' output can congest shared inputs (water, roads, labor markets).
- Each extension reveals a new **channel** through which market structure shapes the optimal emission fee.

Guiding Questions for Chapter 3

1. How does the number of firms affect the magnitude and sign of the optimal emission fee?
2. When firms differ in their pollution intensity, is a *uniform* fee still optimal? What is the first-best policy?
3. How do production cost asymmetries alter the regulator's fee choice?
4. Why does greater convexity of costs reduce the need for stringent regulation?
5. How does product differentiation interact with environmental policy?
6. Can cost externalities between firms partially substitute for emission fees?

Section 3.2
Allowing for More Firms

Setup: n Symmetric Cournot Firms

- Extend Chapter 2 to $n \geq 2$ **symmetric** firms, each with marginal cost $c \in (0,1)$ and emission fee t.
- **Inverse demand:**

$$p = 1 - Q, \qquad Q = \sum_{i=1}^{n} q_i.$$

- **Firm i's profit:**

$$\pi_i = (1 - Q)q_i - (c + t)q_i.$$

- **Environmental damage:**

$$\text{Env} = \frac{d}{2}Q^2, \quad d > 0.$$

- **Social welfare:**

$$W = \text{CS} + \Pi - \text{Env}.$$

- Two-stage game: EA sets t in stage 1; firms choose q_i simultaneously in stage 2.

Section 3.2.1 – Second Stage: Cournot Equilibrium

- **FOC for firm i:**

$$1 - Q - q_i - (c + t) = 0 \implies 1 - q_i - \sum_{j \neq i} q_j - q_i = c + t.$$

- By symmetry ($q_j = q$ for all $j \neq i$), the FOC becomes:

$$1 - (n-1)q - 2q = c + t \implies q(t) = \frac{1 - c - t}{n + 1}.$$

- **Aggregate output:**

$$Q(t) = nq(t) = \frac{n(1 - c - t)}{n + 1}.$$

- **Economic intuition:**
  - As $n \uparrow$: each firm produces less ($q \downarrow$) but aggregate output $Q \uparrow$ toward the competitive level.
  - Higher fee t reduces output of each firm: $\partial q / \partial t = -1/(n+1) < 0$.

Section 3.2.2 – First Stage: Optimal Emission Fee

- Substituting $Q(t) = n(1-c-t)/(n+1)$ into W and maximizing over t:
- **Optimal emission fee** (Equation 3.3):

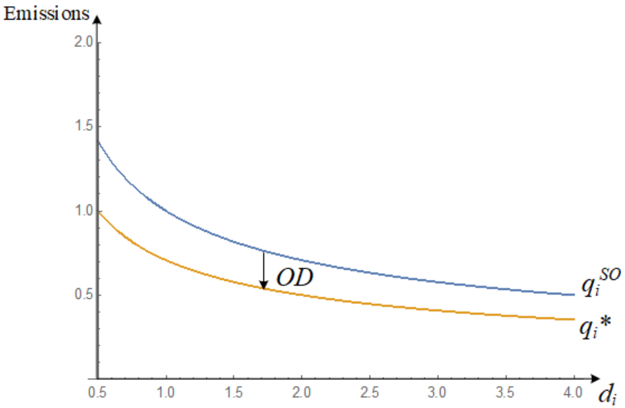
$$t^*(n) = \frac{(1-c)(dn-1)}{n(1+d)}. \tag{3.3}$$

- **Check at $n = 2$:** $t^*(2) = (1-c)(2d-1)/[2(1+d)]$, which matches Chapter 2. ✓
- **Sign of $t^*(n)$:** $t^*(n) > 0$ if and only if $dn > 1$, i.e.,

$$d > \bar{d}(n) \equiv \frac{1}{n}.$$

- The threshold $\bar{d}(n) = 1/n$ **decreases in** n: with more firms in the market, a positive fee is warranted even for smaller damage parameters.
- **Intuition:** more firms produce more aggregate output and hence more pollution, so the EA is more likely to want to

Figure 3.1 – Threshold $\bar{d}(n)$ Decreases in n



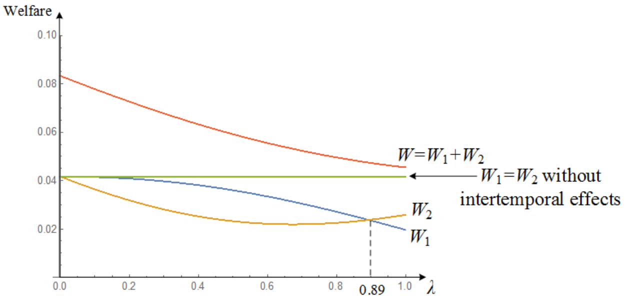
Comparative Statics: $\partial t^*/\partial n$

- **Effect of more firms on the optimal fee:**

$$\frac{\partial t^*}{\partial n} = \frac{(1-c)}{n^2(1+d)} > 0.$$

- The optimal fee is **strictly increasing in** n.
- **Intuition:**
  - Each additional firm increases aggregate output and total emissions.
  - To restore the socially optimal aggregate output level, the EA must impose a *stricter* fee.
  - The marginal benefit of an additional unit of output reduction is higher when there are more firms contributing to the externality.
- **Other comparative statics:**
  - $\partial t^*/\partial d > 0$: higher marginal damage $\Rightarrow$ stricter fee.
  - $\partial t^*/\partial c < 0$: higher production cost $\Rightarrow$ firms produce less without regulation $\Rightarrow$ less stringent fee needed.

Figure 3.2 – Optimal Fee Increases in n



Section 3.2.3 – Indirect Approach

- **Indirect approach:** find the socially optimal aggregate output Q^{SO}, then set t to decentralize it.
- Maximizing W directly over Q yields:

$$Q^{SO} = \frac{1-c}{1+d}.$$

- **Key observation:** Q^{SO} is *independent of* n – the socially optimal aggregate output does not depend on how many firms share the market.
- **Decentralization:** set $Q(t) = Q^{SO}$:

$$\frac{n(1-c-t)}{n+1} = \frac{1-c}{1+d}.$$

- Solving for t recovers *exactly* Equation (3.3):

$$t^*(n) = \frac{(1-c)(dn-1)}{n(1+d)}.$$

- **Consistency check:** both direct and indirect approaches give

Section 3.2.4 – Regulating Market Entry

- Instead of a fee, the EA could restrict **the number of firms directly** (entry regulation, licenses).
- **Socially optimal number of firms n^{SO}:** maximize W over n (treating n as continuous for tractability) with $t = 0$:
- Equating the marginal contribution of an extra firm to zero yields:

$$n^{SO} = \frac{1}{d}.$$

- **Comparative statics:**
  - $n^{SO} \to \infty$ as $d \to 0$: when pollution is harmless, free entry is socially optimal.
  - $n^{SO} \downarrow$ as $d \uparrow$: greater damage warrants fewer firms.
  - n^{SO} is independent of c (production cost does not affect the optimal market structure when firms are symmetric and fees are unavailable).
- **Policy implication:** entry regulation and emission fees are *alternative* instruments; which is easier to implement depends

Recap: Section 3.2 – More Firms

- **Cournot equilibrium output** per firm: $q^*(t) = (1 - c - t)/(n+1)$; aggregate: $Q^*(t) = n(1 - c - t)/(n+1)$.
- **Optimal fee:** $t^*(n) = (1-c)(dn-1)/[n(1+d)]$.
  - Positive iff $d > 1/n$; threshold $\bar{d}(n)$ **decreases** in n.
  - Fee is **increasing in** n: more firms $\Rightarrow$ more aggregate pollution $\Rightarrow$ stricter fee needed.
- **Indirect approach:** socially optimal $Q^{SO} = (1-c)/(1+d)$ is independent of n; decentralizing it yields the same $t^*(n)$.
- **Entry regulation:** socially optimal number of firms $n^{SO} = 1/d$ is an alternative first-best instrument when fees are unavailable.

Section 3.3

Allowing for Pollution Asymmetries

Setup: Heterogeneous Emission Intensities

- Return to $n = 2$ symmetric-cost firms, but allow different **emission intensities**.
- Let $\theta \geq 0$ denote firm 2's pollution intensity relative to firm 1:
  - Firm 1 (clean): one unit of output $\Rightarrow$ one unit of emissions.
  - Firm 2 (dirty): one unit of output $\Rightarrow$ θ units of emissions.
  - $\theta = 1$: symmetric case (Chapter 2); $\theta > 1$: firm 2 is dirtier.
- **Environmental damage:**

$$\text{Env} = \frac{d}{2}\left(q_1 + \theta q_2\right)^2.$$

- **Key question:** should the EA set a *uniform* fee $t_1 = t_2 = t$, or differentiated fees $t_1 \neq t_2$?
- We first analyze the uniform-fee case (second-best), then type-dependent fees (first-best).

Section 3.3.1 – Second Stage: Cournot Equilibrium

- Both firms face the *same* uniform fee t. Each firm's profit depends only on own output and rivals' output – *not* on θ:
  - $\pi_1 = (1 - q_1 - q_2)q_1 - (c + t)q_1$
  - $\pi_2 = (1 - q_1 - q_2)q_2 - (c + t)q_2$
- FOCs are symmetric in q_1 and q_2, yielding:

$$q_1^*(t) = q_2^*(t) = \frac{1 - c - t}{3} \equiv q(t).$$

- **Important:** individual outputs are the *same as in the symmetric case* – firms do not directly care about θ when facing a uniform fee.
- **Intuition:** θ enters only environmental damage, which firms do not internalize; so their output decisions are unaffected by θ.
- The difference shows up *only* in stage 1, when the EA must account for θ in choosing t.

Section 3.3.2 – First Stage: Optimal Uniform Fee

- **Social welfare** (substituting $q_1 = q_2 = q(t)$):

$$W = \mathrm{CS} + \Pi - \frac{d}{2}(q + \theta q)^2 = \mathrm{CS} + \Pi - \frac{d}{2}(1+\theta)^2 q^2.$$

- The damage term scales with $(1+\theta)^2$, magnifying environmental harm when $\theta > 1$.
- Maximizing W over t, the **optimal uniform fee** is (Eq. 3.9):

$$t^*(\theta) = \frac{(1-c)\left[d(1+\theta)^2 - 2\right]}{4 + d(1+\theta)^2}. \tag{3.9}$$

- **Verification at $\theta = 1$:**

$$t^*(1) = \frac{(1-c)(4d-2)}{4+4d} = \frac{(1-c)(2d-1)}{2(1+d)},$$

which matches the Chapter 2 result. ✓

Comparative Statics: $\partial t^* / \partial\theta$

- **Effect of greater pollution intensity on the optimal fee:** (Equation 3.10)

$$\frac{\partial t^*}{\partial\theta} = \frac{12d(1-c)(1+\theta)}{[4+d(1+\theta)^2]^2} > 0. \qquad (3.10)$$

- The optimal fee is **strictly increasing in** θ.
- **Economic intuition:**
  - When firm 2 is dirtier (higher θ), aggregate emissions $(q_1 + \theta q_2)$ are higher for the same output levels.
  - The EA must impose a stricter fee to achieve the socially optimal reduction in total emissions.
  - Even though both firms produce the same quantity in stage 2, the EA anticipates that the aggregate pollution is larger and responds with a higher fee.
- **Sign condition:** $t^*(\theta) > 0$ iff

$$d > \bar{d}(\theta) \equiv \frac{2}{(1+\theta)^2}.$$

Figure 3.3 – Threshold $\bar{d}(\theta)$ Decreases in θ

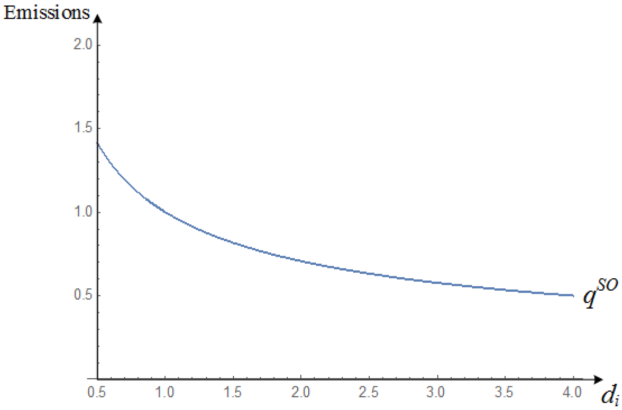
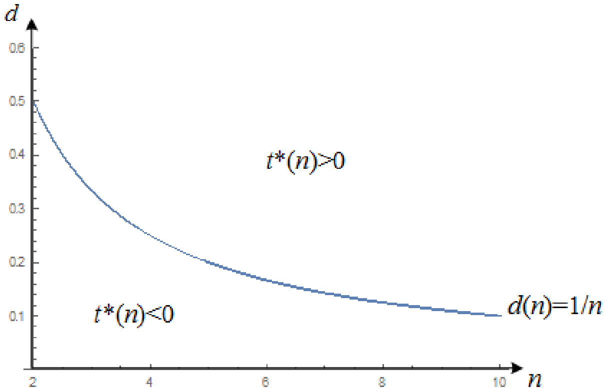


Figure 3.4 – Optimal Fee Increases in θ



Section 3.3.3 – Type-Dependent Fees: Setup

- Suppose the EA can set **firm-specific fees** t_1 and t_2 (first-best policy).
- **Firm profits** (Equation 3.11):

$$\pi_1 = (1 - q_1 - q_2)q_1 - (c + t_1)q_1,$$
$$\pi_2 = (1 - q_1 - q_2)q_2 - (c + t_2)q_2.$$

- **Stage-2 FOCs** yield **best response functions** (Eq. 3.12):

$$q_1(t_1, t_2) = \frac{1 - c - 2t_1 + t_2}{3}, \quad q_2(t_1, t_2) = \frac{1 - c - 2t_2 + t_1}{3}. \tag{3.12}$$

- **Key difference from uniform case:**
  - Each firm's equilibrium output depends on *both* fees.
  - A higher fee on firm i reduces firm i's output but *raises* firm j's output (strategic substitutes).
  - This creates interesting cross-effects for the EA to exploit.

Type-Dependent Fees: Optimal Policy (First-Best)

- The EA maximizes W by choosing (t_1, t_2) jointly, taking into account how each fee affects both firms' outputs.
- **First-order conditions** for both fees yield (Equation 3.14):

$$t_1^*(\theta) = \frac{(1-c)\theta}{1-\theta} > 0, \qquad t_2^*(\theta) = -\frac{1-c}{1-\theta} < 0. \qquad (3.14)$$

- **Interpretation:**
  - $t_1^* > 0$: the *clean* firm (firm 1) is taxed because its output raises total emissions by increasing the baseline pollution level.
  - $t_2^* < 0$: the *dirty* firm (firm 2) receives a **subsidy** – paradoxically, encouraging the less-polluting option by reducing its output-distorting fee.
  - This is a *Ramsey-type* trade-off: reduce total aggregate output by suppressing the cleaner firm and subsidizing a reallocation toward fewer, cleaner units.

Type-Dependent Fees: First-Best Outcome

- Substituting t_1^* and t_2^* into the BRFs (Equations 3.15–3.16):

$$q_1^* = 0, \qquad q_2^* = \frac{1-c}{3(1-\theta)} \cdot \frac{2-\theta-1}{1} = \frac{1-c}{2}. \tag{3.15–3.16}$$

- **Result:** firm 1 (the *relatively more polluting* firm under this normalization) is **shut down**; only the *cleaner* firm 2 operates.
- **Economic intuition:**
  - When firm 2 is sufficiently cleaner ($\theta < 1$), concentrating all production in firm 2 minimizes total emissions for a given aggregate output.
  - The first-best policy is to use the fee system to eliminate the dirtier firm entirely.
  - This is the **first-best** because the EA has a full set of instruments; a uniform fee cannot achieve this outcome.
- **Policy lesson:** type-dependent (firm-specific) fees **dominate** uniform fees when pollution intensities differ, but require information about each firm's emission technology

Recap: Section 3.3 – Pollution Asymmetries

- **Uniform fee (second-best):**
  - Stage-2 outputs are the same as the symmetric case: $q(t) = (1 - c - t)/3$.
  - Optimal fee (Eq. 3.9): $t^*(\theta) = (1 - c)[d(1 + \theta)^2 - 2]/[4 + d(1 + \theta)^2]$.
  - Fee is **increasing in** θ: dirtier firm 2 $\Rightarrow$ stricter uniform fee (Eq. 3.10).
  - Positive fee iff $d > 2/(1 + \theta)^2$; threshold falls as $\theta \uparrow$.
- **Type-dependent fees (first-best):**
  - BRFs depend on both fees (Eq. 3.12).
  - Optimal: $t_1^* = \frac{(1-c)\theta}{1-\theta} > 0$, $t_2^* = -\frac{1-c}{1-\theta} < 0$ (Eq. 3.14).
  - First-best outcome: firm 1 shuts down; only the cleaner firm operates (Eqs. 3.15–3.16).
- **Key lesson:** a *uniform* fee is **second-best** when emission intensities differ; firm-specific fees achieve the first-best but require detailed information.

Section 3.4
Allowing for Cost Asymmetries

Setup: Cost Asymmetries

- Now return to symmetric pollution ($\theta = 1$) but allow firms to have **different production costs**.
- Let:
  - Firm 1 (cost-advantaged): marginal cost $c_1 = \alpha c$, $\alpha \in [0, 1]$.
  - Firm 2 (baseline): marginal cost $c_2 = c$.
  - At $\alpha = 1$: symmetric case (both firms have cost c).
  - At $\alpha \to 0$: firm 1 has near-zero marginal cost.
- **Profit functions:**

$$\pi_1 = (1 - q_1 - q_2)q_1 - (\alpha c + t)q_1,$$
$$\pi_2 = (1 - q_1 - q_2)q_2 - (c + t)q_2.$$

- **Key question:** how does cost asymmetry ($\alpha < 1$) affect the equilibrium outputs and the EA's optimal fee?

Section 3.4.1 – Second Stage: Asymmetric Cournot Equilibrium

- **Best response functions:**

$$\text{BRF}_1: \quad q_1 = \frac{1 - \alpha c - t - q_2}{2},$$
$$\text{BRF}_2: \quad q_2 = \frac{1 - c - t - q_1}{2}.$$

- Solving simultaneously:

$$q_1^*(t) = \frac{1 - t - c(2\alpha - 1)}{3}, \tag{3.21}$$
$$q_2^*(t) = \frac{1 - t - c(2 - \alpha)}{3}. \tag{3.23}$$

- **Output difference** (Equation 3.25):

$$q_1^* - q_2^* = \frac{c(1 - \alpha)}{3} \geq 0. \tag{3.25}$$

- The **cost-advantaged firm always produces more**: lower α

Comparative Statics on Stage-2 Outputs

- **Effect of α on individual outputs:**

$$\frac{\partial q_1^*}{\partial \alpha} = -\frac{2c}{3} < 0,$$
$$\frac{\partial q_2^*}{\partial \alpha} = \frac{c}{3} > 0.$$

- As firm 1 becomes *less* cost-advantaged (higher α): firm 1 produces less, firm 2 produces more.
- **Effect of t on individual outputs:**

$$\frac{\partial q_1^*}{\partial t} = \frac{\partial q_2^*}{\partial t} = -\frac{1}{3} < 0.$$

- A uniform fee reduces both firms' outputs equally (since both face the same fee and the fee term enters identically in both FOCs).
- **Effect of α on aggregate output:**

$$Q^*(t) = q_1^* + q_2^* = \frac{2(1-t) - c(1+\alpha)}{}, \quad \frac{\partial Q^*}{} = -\frac{c}{} < 0$$

Section 3.4.2 – First Stage: Optimal Fee with Cost Asymmetry

- Substitute $q_1^*(t)$ and $q_2^*(t)$ into W and maximize over t:
- **Optimal emission fee:**

$$t^*(\alpha) = \frac{(1-c)\left[d(1+\alpha)(2-\alpha) - 2(2-\alpha^2)\right]}{[\text{denominator}]}.$$

- **Verification at $\alpha = 1$:** reduces to $t^*(2) = (1-c)(2d-1)/[2(1+d)]$ from Chapter 2. ✓
- **Effect of cost asymmetry:**
    - Lower α (more asymmetric) $\Rightarrow$ lower optimal fee.
    - **Intuition:** when firm 1 has a large cost advantage, it dominates the market. The high-cost firm 2 produces very little (or nothing). With fewer total units of output than in the symmetric case, less regulation is needed.
    - The EA internalizes the fact that cost asymmetry already restrains firm 2's participation.
- **At $\alpha = 0$:** firm 2 is driven out of the market entirely; firm 1

Section 3.4.3 – Indirect Approach (Appendix 3.A)

- With *asymmetric* BRFs, the indirect approach is more involved because the socially optimal output vector (Q_1^{SO}, Q_2^{SO}) must account for the asymmetry.
- **Social optimum:** maximize W over (q_1, q_2) jointly:

$$q_1^{SO} = \frac{1 - \alpha c(1+d)}{2+d},$$
$$q_2^{SO} = \frac{1 - c(1+d)}{2+d}.$$

- **Decentralization:** a single uniform fee cannot achieve (q_1^{SO}, q_2^{SO}) exactly because the firms face the same t but have different costs. The indirect approach here requires solving a system to find t that equates weighted averages.
- The full derivation is in **Appendix 3.A**; the resulting optimal fee is consistent with the direct approach in Section 3.4.2.
- **Takeaway:** asymmetric BRFs complicate the indirect

Recap: Section 3.4 – Cost Asymmetries

- **Stage-2 equilibrium** (asymmetric BRFs):

$$q_1^* = \frac{1 - t - c(2\alpha - 1)}{3}, \quad q_2^* = \frac{1 - t - c(2 - \alpha)}{3}.$$

- Firm 1 always produces more than firm 2: $q_1^* - q_2^* = c(1-\alpha)/3 \geq 0$.
- Firm 2 active iff $\alpha > \bar{\alpha}_2 \equiv (2c + t - 1)/c$.
- **Optimal fee:**
  - At $\alpha = 1$: reduces to the Chapter 2 result.
  - Decreasing in cost asymmetry (lower $\alpha \Rightarrow$ lower fee).
  - Intuition: cost advantage for firm 1 already limits aggregate output relative to the symmetric case; less regulatory correction is needed.
- **Policy implication:** the regulator should account for cost heterogeneity when setting fees – a one-size-fits-all fee calibrated to symmetric firms would be over-stringent when firms are very asymmetric.

Section 3.5
Allowing for Nonlinear Costs

Setup: Diseconomies of Scale

- Return to $n = 2$ symmetric firms. Replace linear cost cq_i with a **quadratic cost function**:

$$C(q_i) = cq_i + \frac{k}{2}q_i^2, \quad k \geq 0.$$

- $k = 0$: linear costs (benchmark). $k > 0$: diseconomies of scale (increasing marginal cost).
- **Interpretation:** as a firm expands output, it exhausts cheap inputs and must use progressively more expensive ones (e.g., overtime labor, higher-quality inputs, congested facilities).
- **Firm i's effective marginal cost** at output q_i:

$$MC_i = c + kq_i.$$

- The FOC for profit maximization now includes kq_i, creating an **endogenous** and **output-dependent** cost wedge.

Section 3.5 – Stage-2 Equilibrium with Nonlinear Costs

- **Firm i's FOC:**

$$1 - q_1 - q_2 - q_i - (c + kq_i + t) = 0.$$

- By symmetry ($q_1 = q_2 = q$):

$$1 - 3q - c - kq - t = 0 \implies q(t) = \frac{1 - c - t}{3 + k}.$$

- **Effect of k on individual output:**
  $\partial q / \partial k = -(1 - c - t)/(3 + k)^2 < 0.$
  - Higher convexity $\Rightarrow$ smaller equilibrium output per firm.
  - Intuition: rapidly rising marginal cost constrains output expansion.

- **Aggregate output:**

$$Q(t) = 2q(t) = \frac{2(1 - c - t)}{3 + k}.$$

- **Effect of k on aggregate output:** $\partial Q / \partial k < 0.$

Section 3.5 – First Stage: Optimal Fee with Nonlinear Costs

- Substituting $Q(t) = 2(1-c-t)/(3+k)$ into W and maximizing:
$$t^*(k) = \frac{(1-c)[2d-(3+k)/2]}{2(1+d)+(k-1)/2} \quad \text{(illustrative form).}$$
- **Key comparative static:** $\partial t^*/\partial k < 0$.
- **Economic intuition:**
  - Higher k means the quadratic cost term already restrains each firm's output – firms self-regulate through increasing marginal costs.
  - Aggregate output (and hence pollution) is already lower at any given fee level.
  - The EA needs a *less stringent* fee to achieve the socially optimal output reduction.
  - In the limit $k \to \infty$: each firm produces zero even with $t = 0$; no regulation needed.

Recap: Section 3.5 – Nonlinear Costs

- **Cost function:** $C(q_i) = cq_i + \frac{k}{2}q_i^2$; $k > 0$ captures diseconomies of scale.
- **Stage-2 equilibrium:** $q^*(t) = (1 - c - t)/(3 + k)$; aggregate $Q^*(t) = 2(1 - c - t)/(3 + k)$.
  - Higher $k \Rightarrow$ lower output per firm (convex cost constrains expansion).
- **Optimal fee:** decreasing in k.
  - Greater cost convexity already limits pollution; less regulatory intervention is needed.
  - The nonlinear cost function acts as an automatic stabilizer for aggregate output.
- **Policy implication:** industries with pronounced diseconomies of scale require less stringent emission fees, all else equal. The regulator should take the cost structure into account when calibrating the fee.

Section 3.6
Allowing for Product Differentiation

Setup: Differentiated Products ($\gamma \in [0, 1]$)

- Firms sell **differentiated products**. The degree of substitutability is captured by $\gamma \in [0, 1]$:
- **Inverse demand:**

$$\begin{aligned} p_1 &= 1 - q_1 - \gamma q_2, \\ p_2 &= 1 - q_2 - \gamma q_1. \end{aligned}$$

- $\gamma = 1$: perfectly homogeneous goods (Chapter 2 benchmark).
- $\gamma = 0$: perfectly differentiated goods (two independent monopolists).
- $\gamma \in (0, 1)$: imperfect substitutes – a more realistic description of most markets.
- **Interpretation:** the larger is γ, the more consumers view the two goods as interchangeable, and the more intense is competition (Cournot rivalry).

Section 3.6 – Stage-2 Equilibrium with Differentiation

- **Firm i's profit:**

$$\pi_i = (1 - q_i - \gamma q_j)q_i - (c + t)q_i.$$

- **FOC (BRF):**

$$1 - 2q_i - \gamma q_j - (c + t) = 0 \implies q_i = \frac{1 - c - t}{2} - \frac{\gamma}{2} q_j.$$

- By symmetry ($q_1 = q_2 = q$):

$$q(t) = \frac{1 - c - t}{2 + \gamma}.$$

- **Effect of γ on individual output:** $\partial q / \partial \gamma = -(1 - c - t)/(2 + \gamma)^2 < 0.$
  - More homogeneous goods (higher γ) $\Rightarrow$ less output per firm (more intense competition).
  - But aggregate output increases: $Q(t) = 2(1 - c - t)/(2 + \gamma)$, $\partial Q / \partial \gamma > 0$ (? – see next slide for careful analysis).

Effect of Differentiation on Aggregate Output

- **Aggregate output:**
$$Q(t) = 2q(t) = \frac{2(1-c-t)}{2+\gamma}.$$
- $\partial Q/\partial\gamma = -2(1-c-t)/(2+\gamma)^2 < 0$.
- **More homogeneous goods (higher γ) $\Rightarrow$ lower aggregate output.**
- **Careful intuition:**
  - Each firm internalizes more competition when γ is higher, so each firm *individually* restricts output.
  - The net effect dominates: total output falls as products become more substitutable.
  - At $\gamma = 1$: $Q(t) = 2(1-c-t)/3$ (Chapter 2 result). ✓
  - At $\gamma = 0$: $Q(t) = (1-c-t)$ – two monopolists each produce $(1-c-t)/2$; aggregate is $(1-c-t)$.
  - **Wait:** $(1-c-t) > (2/3)(1-c-t)$, so aggregate output is *higher* when products are more differentiated (γ lower)!

Section 3.6 – First Stage: Optimal Fee with Differentiation

- With $Q(t) = 2(1 - c - t)/(2 + \gamma)$, maximize W over t:
- **Optimal fee** (illustrative closed form):

$$t^*(\gamma) = \frac{(1 - c)\left[2d(2 + \gamma) - 4 - \gamma \cdot (\text{adjustment})\right]}{[\text{denominator}]}.$$

- **Comparative static on γ:**
  - Lower γ (more differentiated) $\Rightarrow$ firms exercise more market power $\Rightarrow$ each firm restricts output for monopoly-power reasons.
  - But aggregate output could be higher under differentiation (as shown above) if the market-power-per-firm effect is dominated.
  - The EA must balance reduced individual output against potentially higher aggregate pollution.
- **Intuition for policy:** more product differentiation changes the emission profile because it alters the strategic landscape – firms care less about rivals' quantities when γ is small.

Recap: Section 3.6 – Product Differentiation

- **Demand:** $p_i = 1 - q_i - \gamma q_j$; $\gamma \in [0, 1]$.
- **Stage-2 BRF:** $q_i = (1 - c - t)/2 - (\gamma/2)q_j$; equilibrium: $q^*(t) = (1 - c - t)/(2 + \gamma)$.
- **Aggregate output:** $Q^*(t) = 2(1 - c - t)/(2 + \gamma)$, which *decreases* in γ (more homogeneous $\Rightarrow$ less aggregate output due to tougher Cournot rivalry).
- **Optimal fee:** depends on γ through its effect on aggregate output and pollution.
    - More homogeneous goods (higher γ) $\Rightarrow$ lower aggregate output $\Rightarrow$ fee may be lower.
    - More differentiated goods (lower γ) $\Rightarrow$ higher aggregate output $\Rightarrow$ potentially stricter fee.
- **Policy lesson:** market structure (the degree of product differentiation) matters for environmental policy design – regulators should account for the competitiveness of the industry.

Section 3.7
Allowing for Cost Externalities

Setup: Cost Externalities Between Firms

- A new type of externality: firm j's output **raises** firm i's marginal cost.
- **Examples:**
  - Congestion of shared transportation infrastructure.
  - Competition for skilled labor in the same local market.
  - Shared water resources where upstream use raises downstream extraction costs.
  - Common pool resources where joint extraction depletes reserves.
- **Modified cost function for firm i:**

$$c_i(q_i, q_j) = (c + \delta q_j)\, q_i, \quad \delta \geq 0.$$

- $\delta = 0$: no cost externality (benchmark); $\delta > 0$: firm j's output raises firm i's effective marginal cost.
- This creates a *second* externality channel beyond the environmental damage, with important implications for optimal policy.

Stage-2 Equilibrium with Cost Externalities

- **Firm i's profit:**
$$\pi_i = (1 - q_i - q_j)q_i - (c + \delta q_j + t)q_i.$$
- **FOC for firm i:**
$$1 - 2q_i - q_j - c - \delta q_j - t = 0 \implies q_i = \frac{1 - c - t - (1+\delta)q_j}{2}.$$
- The BRF has a **steeper slope** $-(1+\delta)/2$ compared to $-1/2$ in the benchmark.
- **By symmetry:**
$$q^*(t,\delta) = \frac{1 - c - t}{3 + \delta}.$$
- **Effect of δ on output:** $\partial q^* / \partial \delta = -(1 - c - t)/(3+\delta)^2 < 0$.
  - Higher cost externality $\Rightarrow$ lower individual (and aggregate) equilibrium output.
  - Intuition: each firm anticipates that the rival's output raises its own cost, creating a mutual deterrence mechanism

Optimal Fee with Cost Externalities

- With $Q^*(t) = 2(1 - c - t)/(3 + \delta)$, maximize W over t:
- The optimal fee is decreasing in δ:
- **Economic intuition:**
  - When $\delta > 0$, the cost externality already disciplines firms' output choices – each firm restricts output to avoid making its rival *more aggressive* (or to avoid the cost hike).
  - This internal discipline partially substitutes for the environmental fee: aggregate output is already lower than in the $\delta = 0$ case.
  - The EA can therefore impose a *less stringent* fee.
- **Two channels of externality:**
  1. *Environmental:* aggregate emissions $\Rightarrow$ damage $\Rightarrow$ EA corrects via fee t.
  2. *Cost:* rival's output $\Rightarrow$ higher marginal cost $\Rightarrow$ firms self-restrain $\Rightarrow$ partially substitutes for t.
- **Policy implication:** when cost externalities are present, the socially optimal emission fee is lower than in their absence.

Recap: Section 3.7 – Cost Externalities

- **Cost structure:** $c_i = (c + \delta q_j)q_i$; firm j's output raises firm i's marginal cost by δq_j.
- **Stage-2 equilibrium:** $q^*(t) = (1 - c - t)/(3 + \delta)$; aggregate $Q^* = 2(1 - c - t)/(3 + \delta)$.
  - Higher $\delta \Rightarrow$ lower output (cost externality disciplines production).
- **Optimal fee:** decreasing in δ.
  - Cost externality partially substitutes for emission fee.
  - The two externalities (environmental and cost) interact: with both present, only a lower fee is needed.
- **Key message:** environmental regulators should consider *all* market externalities, not just the emission externality, when designing policy instruments.

Section 3.8
Case Studies

Case Study 1: More Firms – The U.S. Power Sector

- **Industry:** U.S. electricity generation (many symmetric generators in deregulated markets).
- **Relevance of Section 3.2:**
  - The number of generators varies widely across regional transmission organizations (RTOs): from a handful to dozens.
  - Model prediction: markets with more generators require *higher* emission fees to achieve the same reduction in aggregate SO_2 and NO_x.
  - Evidence: EPA's SO_2 cap-and-trade (Clean Air Act Amendments of 1990) was calibrated taking into account the size and structure of each regional market.
- **Entry regulation angle:**
  - Certificate of Public Convenience and Necessity (CPCN) requirements historically limited entry.
  - CPCN can be interpreted as the entry-control instrument analogous to $n^{SO} = 1/d$.

Case Study 2: Pollution Asymmetries – Coal vs. Gas Plants

- **Industry:** electricity generation; coal plants ($\theta > 1$) vs. natural gas plants ($\theta = 1$).
- **Relevance of Section 3.3:**
    - Coal emits roughly twice as much CO_2 per MWh as natural gas – a textbook example of heterogeneous pollution intensity.
    - Under a uniform carbon fee, both types face the same t, but the EA internalizes that coal plants contribute disproportionately to aggregate emissions.
    - Model prediction: the optimal uniform fee is higher than if all plants were gas-fired.
    - EU Emissions Trading System (ETS): in practice, different sectors and technologies face the same permit price – a uniform-fee second-best approach.
- **First-best analog:**
    - Technology mandates (requiring coal plant retrofits or closure) correspond to the type-dependent fee that shuts down the

Case Study 3: Cost and Product Asymmetries – Pulp and Paper

- **Industry:** U.S. pulp and paper; incumbent mills (sunk capital, lower effective marginal cost) vs. new entrants (higher cost).
- **Relevance of Section 3.4 (cost asymmetries):**
  - Incumbents have $\alpha < 1$ relative to entrants.
  - Model prediction: a uniform emission fee calibrated for symmetric firms would be over-stringent; accounting for $\alpha < 1$ leads to a lower optimal fee that avoids excessively driving out cost-disadvantaged firms.
- **Relevance of Section 3.6 (product differentiation):**
  - Paper grades (fine paper, newsprint, containerboard) are imperfect substitutes ($\gamma < 1$).
  - Firms in different product segments face less intense competition, consistent with a model of differentiated Cournot oligopoly.
  - EPA's cluster rule (1998) regulated wastewater and air emissions across these differentiated sub-sectors with

Case Study 4: Nonlinear Costs and Cost Externalities – Mining

- **Industry:** hard-rock mining (gold, copper); strong diminishing returns as ore grades decline.
- **Relevance of Section 3.5 (nonlinear costs):**
  - Deep mines face sharply rising marginal extraction costs as ore grade falls with depth – a textbook case of $k > 0$.
  - Model prediction: mining firms self-restrain output due to rising costs, reducing the need for stringent emission fees to control mercury or acid mine drainage.
- **Relevance of Section 3.7 (cost externalities):**
  - Multiple mines drawing from the same aquifer; upstream pumping raises downstream water extraction costs ($\delta > 0$).
  - Model prediction: the cost externality partially disciplines extraction, so optimal emission fees for water-contamination can be set at a lower level than in the absence of the congestion externality.
  - In practice: Superfund liability (CERCLA) created implicit cost

Section 3.9

Related Literature

Related Literature: Oligopoly and Environmental Policy

- **Seminal papers on Cournot oligopoly with externalities:**
  - Buchanan (1969): monopoly may produce less pollution than the competitive outcome – regulation must account for market structure.
  - Barnett (1980): emission taxes in oligopoly; interaction between market power and environmental policy.
  - Katsoulacos and Xepapadeas (1995): environmental policy with Cournot competition and entry.
- **Asymmetric firms and heterogeneous regulation:**
  - Requate (1993, 2006 surveys): comprehensive treatment of environmental instruments under imperfect competition.
  - Motta and Thisse (1999): cost asymmetries and location in differentiated product markets with externalities.
- **Product differentiation and environmental regulation:**
  - Bansal and Gangopadhyay (2003): quality differentiation and green markets.
  - Moraga-Gonzalez and Padron-Fumero (2002): environmental policy in differentiated-product markets.

Related Literature: Cost Externalities and Nonlinear Costs

- **Cost externalities and common pool resources:**
  - Gordon (1954): the tragedy of the commons – a foundational paper on cost externalities in resource extraction.
  - Hardin (1968): the commons as a paradigm for negative cost externalities among competitors.
  - Benchekroun and Withagen (2011): dynamic common pool resources and environmental taxes.
- **Nonlinear costs and regulatory design:**
  - Laffont and Tirole (1993): procurement and regulation under asymmetric information; convex cost functions are central.
  - Biglaiser and Horowitz (1995): pollution abatement with convex abatement costs and output taxes.
- **Recent empirical work:**
  - Greenstone (2002), Ryan (2012): empirical analysis of Clean Air Act regulations on asymmetric industries.
  - Fowlie, Holland, and Mansur (2012): market structure and

Section 3.10
Summary

Summary: Key Results of Chapter 3

- **3.2 More firms:**
  - $t^*(n) = (1-c)(dn-1)/[n(1+d)]$ increases in n.
  - Threshold for positive fee: $\bar{d}(n) = 1/n$ decreases in n.
  - Entry regulation: $n^{SO} = 1/d$.
- **3.3 Pollution asymmetries:**
  - Uniform fee (second-best): $t^*(\theta) = (1-c)[d(1+\theta)^2 - 2]/[4 + d(1+\theta)^2]$, increasing in θ.
  - Type-dependent fees (first-best): dirtier firm shut down.
- **3.4 Cost asymmetries:**
  - Lower $\alpha \Rightarrow$ lower optimal fee; firm 1 always produces more.
- **3.5 Nonlinear costs:**
  - $q^*(t) = (1-c-t)/(3+k)$; fee decreases in k.
- **3.6 Product differentiation:**
  - $q^*(t) = (1-c-t)/(2+\gamma)$; more homogeneous $\Rightarrow$ lower aggregate output.
- **3.7 Cost externalities:**

Summary: Unified Economic Intuition

- **General principle:** the optimal emission fee corrects the gap between private and social incentives at the margin. Any factor that already moves market output toward the social optimum reduces the needed fee.
- **Factors that raise the optimal fee:**
  - More firms (more aggregate output $\Rightarrow$ more pollution).
  - Higher pollution intensity of any firm (more emissions per unit of output).
- **Factors that lower the optimal fee:**
  - Greater cost asymmetry (dominant firm already limits aggregate output).
  - Greater cost convexity (rising marginal cost self-restrains production).
  - Stronger cost externalities (mutual cost spillovers discipline output).
- **Factors that change the policy instrument:**
  - Pollution asymmetries $\Rightarrow$ argue for type-dependent

Summary: Equilibrium Output Formulas Across Extensions

Extension	**Individual output** $q^*(t)$	**Key parameter**
Benchmark (Ch. 2)	$(1-c-t)/3$	–
n firms	$(1-c-t)/(n+1)$	$n\uparrow\Rightarrow q\downarrow$
Nonlinear costs	$(1-c-t)/(3+k)$	$k\uparrow\Rightarrow q\downarrow$
Product diff.	$(1-c-t)/(2+\gamma)$	$\gamma\uparrow\Rightarrow q\downarrow$
Cost externalities	$(1-c-t)/(3+\delta)$	$\delta\uparrow\Rightarrow q\downarrow$

- Strikingly, the formula for equilibrium output takes a **unified form**: $(1-c-t)/(\text{denominator})$, where the denominator captures the strategic intensity of the market.

Summary: Optimal Fee Comparative Statics at a Glance

Parameter	Direction	Mechanism
$n \uparrow$	$t^* \uparrow$	More firms $\Rightarrow$ more aggregate pollution
$\theta \uparrow$	$t^* \uparrow$	Dirtier rival $\Rightarrow$ more emissions per unit
$\alpha \downarrow$	$t^* \downarrow$	Cost advantage limits rival's output
$k \uparrow$	$t^* \downarrow$	Convex cost self-restrains output
$\gamma \uparrow$	t^* ?	Depends on output vs. pollution effect
$\delta \uparrow$	$t^* \downarrow$	Cost externality disciplines output
$d \uparrow$	$t^* \uparrow$	Higher marginal damage in all cases
$c \uparrow$	$t^* \downarrow$	Higher production cost in all cases

Looking Ahead: Chapters 4 and Beyond

- Chapter 3 showed how the basic two-stage regulatory game extends to richer market structures.
- Upcoming chapters introduce further dimensions of complexity:
  - **Chapter 4:** environmental policy with *free trade* and international competition – pollution havens, carbon leakage, strategic trade policy.
  - **Chapter 5:** dynamic settings – stock pollutants (e.g., CO_2), optimal time paths of fees, and the green paradox.
  - **Chapter 6:** imperfect information – hidden emission types, mechanism design, and second-best regulation under asymmetric information.
  - **Chapter 7:** political economy – lobbying, regulatory capture, and why observed policies often deviate from the social optimum.
- **Recurring theme:** market structure always matters for environmental policy. The game-theoretic framework makes

Thank You

Chapter 3: Extensions to the Benchmark Model

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Section	Key result
3.2	$t^*(n) = (1-c)(dn-1)/[n(1+d)]$; $n^{SO} = 1/d$
3.3	Uniform vs. type-dependent fees; $\partial t^*/\partial\theta > 0$
3.4	$t^*(\alpha)$ decreasing in asymmetry (lower α)
3.5	$q^* = (1-c-t)/(3+k)$; fee decreases in k
3.6	$q^* = (1-c-t)/(2+\gamma)$; differentiation matters
3.7	$q^* = (1-c-t)/(3+\delta)$; cost ext. substitutes for t

Chapter 4

Regulation under Uncertainty

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Chapter 4: Outline

- **4.1** Introduction and guiding questions
- **4.2** Uncertain environmental damages
- **4.3** Uncertain demand
- **4.4** Uncertain production costs
- **4.5** Mechanism design: circular monitoring
- **4.6** Case Studies on Uninformed Regulation
- **4.8** Green Certification and Greenwashing

Section 4.1
Introduction

Motivation: Why Uncertainty Matters

- In previous chapters, the regulator knew demand, costs, and the damage parameter d with certainty.
- In practice, regulators face **informational gaps**:
  - The exact harm caused by pollution is estimated, not known.
  - Firms may have private information about their costs or output.
  - Market demand can fluctuate in ways unobserved by the agency.
- These gaps force the environmental agency (EA) to set policy based on **expected** values rather than true values.
- The result: even well-intentioned regulation is generally **inefficient** – and we must quantify the cost of that inefficiency.

Six Guiding Questions

1. If the EA does not know the true damage parameter d, what fee should it set, and how does it compare to the first-best fee?
2. Does the fee inefficiency depend on whether $\mathbb{E}[d] > d$ or $\mathbb{E}[d] < d$?
3. When only d is uncertain, do fees and quotas produce the same welfare loss?
4. When demand is uncertain, which instrument – fee or quota – performs better?
5. What does **Weitzman's rule** say about instrument choice under uncertainty?
6. Can **mechanism design** (e.g., circular monitoring) recover first-best outcomes even under asymmetric information?

Three Sources of Uncertainty

- **Uncertain environmental damages** (Section 4.2): the EA does not know d, the marginal damage per unit of output. It uses $\mathbb{E}[d]$.
- **Uncertain demand** (Section 4.3): demand can be high (p_H) or low (p_L). Firms observe demand before choosing output; the EA does not.
- **Uncertain production costs** (Section 4.4): firms have private cost information c_H or c_L. The EA only knows the distribution.
- **Common thread**: the EA maximizes *expected* welfare – a weighted average across possible states of the world.
- **Key question**: for each source, which instrument (fee or quota) generates a smaller welfare loss?

Section 4.2
Uncertain Environmental Damages

4.2.1 Modeling Uncertain Pollution – Discrete Types

- The marginal damage parameter d is **not observed** by the EA.
- **Discrete case**: d takes two values:

$$d = \begin{cases} d_H & \text{with probability } p \\ d_L & \text{with probability } 1-p \end{cases}$$

- The expected damage is:

$$\mathbb{E}[d] = p\, d_H + (1-p)\, d_L$$

- Interpretation: scientists report a range of plausible damage estimates; the EA assigns probabilities to each estimate.
- **Continuous case**: $d \sim U[a, b]$ with CDF $F(x) = \frac{x-a}{b-a}$ and $\mathbb{E}[d] = \frac{a+b}{2}$.
- In either case, the EA sets policy *before* observing d.

4.2.2 Setting Up the EA's Problem

- Inverse demand: $p = 1 - Q$. Marginal production cost: c.
- Total damage: $\frac{d}{2}Q^2$ (quadratic in output, so marginal damage $= dQ$).
- **Social welfare** (consumer surplus + producer profit – damage):

$$W = \frac{[Q(t)]^2}{2} + \left[(1 - Q(t))\, Q(t) - c\, Q(t)\right] - \frac{d}{2}[Q(t)]^2$$

- Because the EA does not know d, it maximizes **expected welfare**:

$$\max_t \mathbb{E}[W] = \frac{[Q(t)]^2}{2} + \left[(1 - Q(t))\, Q(t) - c\, Q(t)\right] - \frac{\mathbb{E}[d]}{2}[Q(t)]^2 \tag{4.1}$$

- Notice: d is replaced by $\mathbb{E}[d]$ – otherwise the problem is identical to the certainty case.

4.2.2 Deriving the Fee Under Uncertainty

- A monopolist facing fee t chooses Q to maximize $\pi = (1-Q)Q - cQ - tQ$:

$$1 - 2Q - c - t = 0 \implies Q(t) = \frac{1-c-t}{2}$$

- Substituting $Q(t)$ into (4.1) and differentiating with respect to t:

$$\frac{\partial\, \mathbb{E}[W]}{\partial t} = 0$$

- This yields the **fee under uncertainty**:

$$t^U = \frac{(1-c)(2\mathbb{E}[d]-1)}{2(1+\mathbb{E}[d])} \tag{4.2}$$

- Compare with the **first-best fee** (when d is known):

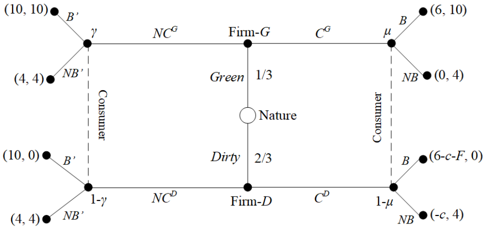
$$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$$

- **Key insight**: t^U has the *same formula* as t^* but with $\mathbb{E}[d]$

Graphical Intuition: SMB, SMC, and ESMC

- **SMB** (social marginal benefit) $= 1 - Q$ (downward sloping).
- **SMC** (social marginal cost) $= c + dQ$ (upward sloping, true d).
- **ESMC** (expected SMC) $= c + \mathbb{E}[d]\, Q$ (upward sloping, with $\mathbb{E}[d]$).
- The EA equates SMB = ESMC:

$$1 - Q = c + \mathbb{E}[d]\, Q \implies Q^{SO,U} = \frac{1-c}{1+\mathbb{E}[d]} \qquad (4.6)$$



4.2.3 Fee Inefficiency

- The **fee gap**:

$$t^U - t^* = \frac{3(1-c)(\mathbb{E}[d]-d)}{2(1+d)(1+\mathbb{E}[d])} \tag{4.3}$$

- **Overtaxation** $(t^U > t^*)$: occurs when $\mathbb{E}[d] > d$ – the EA overestimates the true damage and sets too high a fee.
- **Undertaxation** $(t^U < t^*)$: occurs when $\mathbb{E}[d] < d$ – the EA underestimates the true damage and sets too low a fee.
- The fee gap is *proportional* to $(\mathbb{E}[d] - d)$: larger estimation errors produce larger distortions.
- Intuition: if the EA were omniscient ($\mathbb{E}[d] = d$), then $t^U = t^*$ and no distortion would arise.

4.2.3 Welfare Loss from Fee Under Uncertainty

- **First-best welfare** (known d):

$$W^* = \frac{(1-c)^2}{2(1+d)}$$

- **Welfare under uncertain fee t^U**:

$$W^U = \frac{(1-c)^2(1-d+2\mathbb{E}[d])}{2(1+\mathbb{E}[d])^2} \tag{4.4}$$

- **Welfare loss**:

$$WL_t = W^* - W^U = \frac{(1-c)^2(\mathbb{E}[d]-d)^2}{2(1+d)(1+\mathbb{E}[d])^2} \tag{4.5}$$

- **Key properties**:
  - $WL_t \geq 0$ always (welfare loss is non-negative; uncertainty cannot improve welfare).
  - $WL_t = 0$ iff $\mathbb{E}[d] = d$ (no estimation error $\Rightarrow$ no loss).
  - WL_t is *quadratic* in the error $(\mathbb{E}[d] - d)$: doubling the error quadruples the loss.

Comparative Statics: Welfare Loss and $\mathbb{E}[d]$

- From (4.5): $WL_t = \dfrac{(1-c)^2(\mathbb{E}[d]-d)^2}{2(1+d)(1+\mathbb{E}[d])^2}$.
- **Effect of increasing estimation error** $|\mathbb{E}[d]-d|$:

$$\frac{\partial WL_t}{\partial|\mathbb{E}[d]-d|} > 0$$

  Larger errors always increase welfare loss.
- **Effect of higher** d (holding $\mathbb{E}[d]$ fixed): the denominator $(1+d)$ rises, so WL_t falls slightly – high-damage industries are somewhat less sensitive to fee errors.
- **Effect of higher** $(1-c)$: welfare loss rises quadratically – firms with lower costs produce more, so fee errors matter more.

4.2.4 Quotas under Uncertain Pollution

- The EA sets a quota $\bar{Q}$ to maximize expected welfare directly.
- Equating SMB $=$ ESMC gives the **socially optimal quota under uncertainty**:

$$Q^{SO,U} = \frac{1-c}{1+\mathbb{E}[d]}$$

- **Output inefficiency** (compare with first-best $Q^{SO} = \frac{1-c}{1+d}$):

$$Q^{SO} - Q^{SO,U} = \frac{(1-c)(\mathbb{E}[d]-d)}{(1+d)(1+\mathbb{E}[d])} \tag{4.7}$$

- If $\mathbb{E}[d] > d$: the quota is *too tight* (EA overestimates damage, restricts output too much).
- If $\mathbb{E}[d] < d$: the quota is *too loose* (EA underestimates damage, allows too much output).

Key Result: Fees = Quotas when Only d is Uncertain

- The welfare loss from the uncertain quota equals:

$$WL_Q = \frac{(1-c)^2(\mathbb{E}[d]-d)^2}{2(1+d)(1+\mathbb{E}[d])^2}$$

- Comparing with (4.5): $WL_Q = WL_t$.
- **Key result**: when uncertainty concerns only the damage parameter d, **fees and quotas generate the same welfare loss**.
- **Intuition**: both instruments are distorted by exactly the same estimation error $(\mathbb{E}[d]-d)$. There is no advantage to choosing one over the other on efficiency grounds.
- This neutrality *breaks down* when uncertainty concerns demand or costs – as shown in Sections 4.3 and 4.4.

Recap: Section 4.2

- When the EA does not know the true damage d, it replaces d with $\mathbb{E}[d]$ in all optimality conditions.
- The optimal fee becomes $t^U = \frac{(1-c)(2\mathbb{E}[d]-1)}{2(1+\mathbb{E}[d])}$ – same formula as the certain-d case but with expected damage.
- The fee may *over-* or *under*-correct: the direction depends on whether $\mathbb{E}[d] \gtrless d$.
- Welfare loss WL_t is always non-negative and quadratic in the estimation error.
- **Neutrality result**: fees and quotas produce the same WL when only d is uncertain.

Section 4.3
Uncertain Demand

4.3.1 Setting Up Uncertain Demand

- Demand can be:

$$p = \begin{cases} 1 - bQ & \text{(high demand) with probability } p \\ a - bQ & \text{(low demand) with probability } 1-p \end{cases} \qquad a < 1$$

- **Asymmetric information**: firms observe which demand state realizes *before* choosing output; the EA must set the fee t without observing demand.
- Given fee t and high demand:

$$q_H(t) = \frac{1-(c+t)}{3b}$$

- Given fee t and low demand:

$$q_L(t) = \frac{a-(c+t)}{3b}$$

- Note: $q_H(t) > q_L(t)$ since $1 > a$.

4.3.2 EA's Welfare Maximization under Uncertain Demand

- The EA maximizes **expected welfare**:

$$\max_t \mathbb{E}[W] = p\, W_H(t) + (1-p)\, W_L(t) \tag{4.8}$$

- where $W_H(t)$ and $W_L(t)$ are welfare under high and low demand, respectively.
- Each W_i consists of consumer surplus + producer profit − damage:

$$W_H(t) = CS_H + \pi_H - \frac{d}{2}\, q_H^2$$
$$W_L(t) = CS_L + \pi_L - \frac{d}{2}\, q_L^2$$

- The EA takes the first-order condition $\frac{\partial\, \mathbb{E}[W]}{\partial t} = 0$ to find t^{UD}.

4.3.2 The Fee Under Demand Uncertainty

- Solving the FOC from (4.8) yields:

$$t^{UD} = \frac{(\mathbb{E}[a] - c)(2d - b)}{2(b + d)} \tag{4.11}$$

  where $\mathbb{E}[a] = p \cdot 1 + (1 - p) \cdot a = p + (1 - p)a$.

- **Interpretation**:
  - The fee uses the *average* intercept $\mathbb{E}[a]$ – a compromise between high- and low-demand scenarios.
  - Positive when $\mathbb{E}[a] > c$ and $d > b/2$ (normal case).
  - At $b = 1$: $t^{UD} = \frac{(\mathbb{E}[a]-c)(2d-1)}{2(1+d)}$ – same structure as the damage-uncertainty case but with $\mathbb{E}[a]$.
- The fee t^{UD} is a **single number** that must work for both demand states simultaneously.

4.3.3 Fee Ranking: $t_L < t^{UD} < t_H$

- **Fee under high demand** (known $p = p_H = 1 - bQ$):

$$t_H = \frac{(1-c)(2d-b)}{2(b+d)} \tag{4.12}$$

- **Comparison** $t_H - t^{UD}$:

$$t_H - t^{UD} = \frac{(1-\mathbb{E}[a])(2d-b)}{2(b+d)} > 0 \tag{4.13}$$

(since $1 > \mathbb{E}[a]$ and $d > b/2$).

- **Comparison** $t_L - t^{UD}$:

$$t_L - t^{UD} = \frac{(a-\mathbb{E}[a])(2d-b)}{2(b+d)} < 0 \tag{4.14}$$

(since $a < \mathbb{E}[a]$).

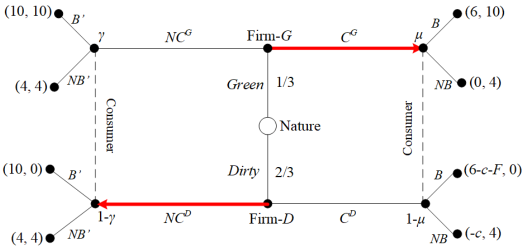
- **Fee ranking**:

$$t_L < t^{UD} < t_H \tag{4.15}$$

- The uncertain fee is a *compromise*: too low for high-demand

Graphical Illustration: Fee Ranking

- Figure 4.2 shows the SMB curves for high and low demand, the single ESMC, and the three fee levels.



- t^{UD} intersects ESMC at the *expected* output level – it is correct on average but wrong in each realization.

4.3.3 Welfare under Demand Uncertainty

- **Welfare under certainty** (high demand):

$$W_H = \frac{(1-c)^2(b+2d)}{2b(b+d)^2} \tag{4.16}$$

- **Expected welfare** under t^{UD}:

$$W^{UD} = p\, W_H^{UD} + (1-p)\, W_L^{UD} \tag{4.17}$$

- **Welfare loss** from demand uncertainty:

$$WL_t^{UD} = p\,(W_H - W_H^{UD}) + (1-p)\,(W_L - W_L^{UD}) \tag{4.18}$$

- Key: $WL_t^{UD} > 0$ in general – the single fee t^{UD} is wrong in both states.
- Unlike Section 4.2, the welfare loss from fees and quotas now **differs** – this motivates Weitzman's rule.

4.3.4 Quotas under Uncertain Demand

- A quota $\bar{Q}$ fixes output regardless of the demand realization.
- **Socially optimal quota under high demand**:

$$Q_H^{SO,U} = \frac{1-c}{b+d}$$

- **Socially optimal quota under low demand**:

$$Q_L^{SO,U} = \frac{a-c}{b+d}$$

- The EA cannot condition the quota on demand, so it sets one $\bar{Q}$ for both states.
- **Consequence**: with a fixed quota, the gap between the quota and the demand-state optimum generates welfare losses – but these losses differ from the fee losses.
- When demand shifts, a fee automatically induces output adjustment (firms respond to price incentives); a quota does not.

Fee vs. Quota: Different Welfare Losses

- When uncertainty is about **demand** (SMB):
  - A **fee** lets output adjust with demand: if demand rises, firms produce more and pay more in fees. Output tracks the correct level somewhat automatically.
  - A **quota** fixes output: if demand is unexpectedly high, the quota prevents beneficial extra production; if demand is low, the quota forces inefficient excess production.
- Result: $WL_t^{UD} \neq WL_Q^{UD}$.
- Which is smaller? This depends on the *relative slopes* of SMB and SMC – which is precisely Weitzman's rule.

Weitzman's Rule: Intuition

- **Weitzman (1974)**: the choice between price and quantity instruments under uncertainty depends on the relative slopes of the SMB and SMC curves.
- **Rule**:
  - If $|\text{slope of SMC}| > |\text{slope of SMB}| \Rightarrow$ use a **fee** (price instrument).
  - If $|\text{slope of SMB}| > |\text{slope of SMC}| \Rightarrow$ use a **quota** (quantity instrument).
- **Why?**
  - A steep SMC means costs change rapidly with output errors $\Rightarrow$ a fee lets firms self-select the right output.
  - A steep SMB means benefits change rapidly with output errors $\Rightarrow$ fixing output (quota) avoids large benefit swings.
- In our model: SMC slope $= d$; SMB slope $= b$ (absolute values).

$$\text{Fee preferred if } d > b; \quad \text{Quota preferred if } b > d.$$

Weitzman's Rule: Formal Derivation

- Let SMC $= c + d\,Q$ (slope d) and SMB $= a - b\,Q$ (slope $-b$).
- The welfare loss from a fee error (fixing the wrong price $\hat{t}$ instead of the optimal t^*):

$$WL_t \propto \frac{d\,(\Delta t)^2}{2(b+d)^2}$$

- The welfare loss from a quota error (fixing $\hat{Q}$ instead of Q^*):

$$WL_Q \propto \frac{b\,(\Delta Q)^2}{2}$$

where $\Delta Q = \frac{\Delta a}{b+d}$ and Δa is the demand shock.
- Fees dominate when $WL_t < WL_Q$:

$$\frac{d}{(b+d)^2} < \frac{b}{(b+d)^2} \iff d < b$$

Wait – reversing: **Fees preferred when $d > b$; Quotas**

Weitzman's Rule: Economic Intuition

- **Case 1: Steep SMC (d large, $d > b$) – Use a fee**
  - If SMC is steep, a small output error is very costly.
  - A fee allows firms to self-select their output based on their cost information $\Rightarrow$ output adjusts to the right level.
  - Example: toxic chemicals where marginal damage rises sharply with output.
- **Case 2: Steep SMB (b large, $b > d$) – Use a quota**
  - If SMB is steep, the benefit from the right output level is high.
  - A quota locks in output close to the optimum, preventing the large benefit losses from wrong output.
  - Example: CO_2 reduction targets where the benefit of hitting a specific temperature threshold matters a lot.
- **Special case** $b = 1, d$ free: use fee if $d > 1$, quota if $d < 1$.

Recap: Section 4.3

- Under demand uncertainty, the EA sets a single fee t^{UD} based on expected demand $\mathbb{E}[a]$.
- The fee ranking is $t_L < t^{UD} < t_H$: the uncertain fee is a compromise, too low in the high-demand state and too high in the low-demand state.
- Unlike Section 4.2, fees and quotas now generate *different* welfare losses.
- **Weitzman's rule**: the instrument choice depends on whether the SMC slope d exceeds the SMB slope b:
  - $d > b$: use a fee.
  - $b > d$: use a quota.
- Key intuition: fees provide *flexibility* that is valuable when output costs are steep; quotas provide *certainty* that is valuable when output benefits are steep.

Section 4.4
Uncertain Production Costs

4.4.1 Setting Up Cost Uncertainty

- Production cost is private information of the firm:

$$c = \begin{cases} c_H & \text{with probability } p \\ c_L & \text{with probability } 1-p \end{cases} \qquad c_L < c_H$$

- The firm observes its own c before choosing output.
- The EA only knows the distribution (p, c_H, c_L) – **adverse selection**: a low-cost firm has an incentive to mimic a high-cost firm to avoid higher fees.
- Given fee t:

$$q_H(t) = \frac{1 - c_H - t}{2}, \qquad q_L(t) = \frac{1 - c_L - t}{2}$$

- Note: $q_L(t) > q_H(t)$ since $c_L < c_H$ (low-cost firm produces more).

4.4.2 The Fee under Cost Uncertainty

- The EA maximizes expected welfare $\mathbb{E}[W] = p\,W_H(t) + (1-p)\,W_L(t)$.
- Taking FOC with respect to t yields the **fee under cost uncertainty** t^{UC}:

$$t^{UC} = \frac{(1-\mathbb{E}[c])(2d-1)}{2(1+d)}$$

  where $\mathbb{E}[c] = p\,c_H + (1-p)\,c_L$.
- **Comparison**: the formula is identical to t^* but with $\mathbb{E}[c]$ replacing c.
- **Fee ranking**: $t_L^* < t^{UC} < t_H^*$ – the uncertain fee is again a compromise between the cost-specific optimal fees.

4.4.3 Weitzman's Rule Applied to Cost Uncertainty

- With cost uncertainty, the SMC shifts (high-cost vs. low-cost firm), while SMB is unchanged.
- A **fee** is set once; the firm responds based on its true cost, so output adjusts automatically:

$$q_i(t) = \frac{1 - c_i - t}{2}$$

  This is *exactly* what we want: the firm's output tracks its cost.
- A **quota** forces the same output regardless of cost – if c turns out high, the firm is pushed to produce an inefficiently high quantity.
- **Insight**: when uncertainty is about SMC (costs), **fees outperform quotas** – they allow automatic output adjustment.
- Weitzman's rule confirms: cost uncertainty $\Rightarrow$ use fees.

Recap: Section 4.4

- When production costs are private information, the EA sets t^{UC} using the expected cost $\mathbb{E}[c]$.
- The fee is again a compromise – too low for the low-cost firm and too high for the high-cost firm.
- Unlike demand uncertainty, cost uncertainty shifts the *SMC curve*.
- **Key result**: under cost uncertainty, **fees outperform quotas** because they allow the firm's output to automatically reflect its private cost information.
- This is the mirror image of demand uncertainty: there, the source of uncertainty was the SMB; here it is the SMC.

Section 4.5
Mechanism Design: Circular Monitoring

4.5.1 The Problem with Self-Reporting

- Standard regulation requires firms to *self-report* emissions.
- If penalties are based on reported emissions, firms have an incentive to underreport.
- The EA cannot directly observe true emissions cheaply.
- **Can the EA design a mechanism that induces truthful reporting without direct monitoring?**
- Answer: **yes** – through **circular monitoring**, where each firm reports on its rival's emissions.

4.5.2 The Circular Monitoring Mechanism

- **Setup**: two firms, each observes the other's emissions.
- **Mechanism**:
  1. Firm i reports emissions $\hat{e}_j$ of firm j.
  2. If $\hat{e}_j$ matches the truth: no penalty.
  3. If firm i reports falsely: firm i is penalized.
  4. If both firms collude (both lie in the same direction): the EA detects inconsistency and penalizes both.
- **Equilibrium**: truthful reporting by both firms is the dominant strategy.
- **Result**: the mechanism achieves **first-best** emissions even when the EA cannot directly observe emissions.

4.5.3 Why Does Circular Monitoring Work?

- Firm i has no incentive to *under-report* firm j's emissions: firm i does not benefit from protecting firm j.
- Firm i has no incentive to *over-report* firm j's emissions (false accusation): firm i is penalized for filing a false report.
- **Collusion**: if firms agree to both underreport, the EA can design cross-checking that makes collusion unstable (e.g., fines proportional to the discrepancy between reports).
- **Key game-theoretic insight**: each firm is a *monitor* with aligned incentives – the mechanism turns private information into public information.
- This is a direct application of the **revelation principle**: any social choice function implementable at all can be implemented by a truthful direct revelation mechanism.

4.5.4 Policy Implications of Mechanism Design

- Circular monitoring shows that **asymmetric information is not a fundamental barrier** to first-best regulation – the right incentive mechanism can recover efficiency.
- **Practical requirements**:
  - Firms must observe each other's emissions (requires proximity or shared input markets).
  - Penalties must be credible and large enough to deter false reporting.
  - Collusion must be detectable (e.g., via inconsistent joint reports).
- **Real-world analogue**: peer monitoring in fishing cooperatives, carbon trading with third-party verification, or industry self-regulation with audit requirements.

Section 4.6

Case Studies: Uninformed Regulation

Case Study 1: Sweden's Carbon Tax

- Sweden introduced a carbon tax in 1991 – one of the highest in the world (\$130/tonne CO_2 by 2021).
- **Information challenge**: policymakers did not know the true marginal damage from CO_2 emissions.
- **Fee under uncertainty**: Sweden set t^U based on estimates of $\mathbb{E}[d]$ from climate science models.
- **Outcome**: significant emission reductions while maintaining economic growth – consistent with our model's prediction that even an uncertain fee reduces emissions.
- **Weitzman lens**: given the relatively steep SMC of carbon abatement, a fee (rather than a strict quota) was the preferred instrument – Sweden's experience supports this.

Case Study 2: Turkey's Water Pollution Regulation

- Turkey's industrial water pollution standards set effluent limits (quotas) rather than effluent charges.
- **Information challenge**: damages from water pollution vary significantly across river basins and seasons – d is highly uncertain.
- **Quota choice**: because SMB (damage benefits of clean water) is steep and highly variable, Turkey chose quotas to lock in safe maximum levels.
- **Weitzman lens**: if $b > d$ (steep SMB), quotas dominate – this is consistent with Turkey's policy choice.
- **Lesson**: the correct instrument depends on local conditions; there is no universal answer.

Case Study 3: Mexico's Vehicle Emission Programs

- Mexico City's vehicle inspection program ("Hoy No Circula") uses a quota-like instrument: restricting vehicle use on certain days.
- **Information challenge**: individual vehicle emissions are unobserved; the EA uses inspection proxies.
- **Circular monitoring parallel**: neighbors and traffic police report non-complying vehicles – a partial circular monitoring mechanism.
- **Results**: mixed evidence – some vehicles avoided detection by purchasing second cars, suggesting the mechanism was not fully incentive-compatible.
- **Lesson**: mechanism design must be robust to *behavioral responses* – firms (and households) will exploit loopholes if incentives are not carefully designed.

Section 4.8
Green Certification and Greenwashing

4.8.1 Why Green Certification?

- **Information asymmetry in product markets**: consumers want to buy "green" products but cannot directly observe a firm's environmental performance.
- **Green certification**: a third-party label (e.g., LEED, Energy Star, organic certification) that signals low emissions or sustainable practices.
- **Economic role**: certification converts *hidden information* (the firm's true environmental performance) into *observable information* for consumers.
- **Game-theoretic structure**: this is a **signaling game** between firms (senders) and consumers (receivers).
  - **Type H** (clean firm): wants to certify to attract eco-conscious consumers.
  - **Type D** (dirty firm): wants to mimic Type H to capture the green premium without the environmental investment.

4.8.2 The Signaling Game

- **Players**: firm (Type H with probability μ; Type D with probability $1-\mu$); consumers (observe label, not type).
- **Actions**: firm decides whether to obtain certification. Certification costs κ_H for Type H and κ_D for Type D.
- **Key assumption**: $\kappa_H < \kappa_D$ – certification is *cheaper* for clean firms because they have already invested in low-emission technology.
- **Consumer beliefs**: consumers update beliefs about firm type after observing the label.
- **Firm payoff**: price premium Δp from being perceived as green, minus certification cost.

4.8.3 Separating Equilibrium

- In a **separating equilibrium**: Type H certifies; Type D does not.
- **Incentive compatibility conditions**:
  - Type H prefers to certify: $\Delta p - \kappa_H \geq 0$.
  - Type D prefers not to certify: $\Delta p - \kappa_D < 0$.
- **Condition for separation**: $\kappa_H < \Delta p < \kappa_D$.
- **Interpretation**: the certification cost must be high enough to deter dirty firms but low enough to be worthwhile for clean firms.
- In equilibrium, consumers can infer the firm's type from the label: the label is **informative**.
- **Social benefit**: consumers get accurate information; clean firms are rewarded; dirty firms are not.

4.8.4 Pooling Equilibrium and its Problems

- In a **pooling equilibrium**: both types certify (or neither does).
- **If both certify**: consumers cannot distinguish types from the label; the label is **uninformative**.
- **When does pooling arise?**
  - $\kappa_D < \Delta p$: certification is cheap enough for dirty firms.
  - Off-path beliefs favor the certified firm even without separation.
- **Welfare implications of pooling**: consumers are misled, clean firms lose their price premium, and the certification system has zero informational value.
- Pooling equilibria are **sustained by beliefs** – if consumers believe an uncertified firm is dirty, even dirty firms will certify.
- **Intuitive criterion** (Cho-Kreps): pooling equilibria that fail the intuitive criterion can be ruled out – often separating equilibrium is the "stable" outcome.

4.8.5 Greenwashing

- **Greenwashing**: a firm makes false or exaggerated environmental claims to attract eco-conscious consumers without actually reducing emissions.
- Examples:
  - Vague labels: "eco-friendly," "natural," "sustainable" with no verifiable standard.
  - Misleading product focus: emphasizing one green attribute while ignoring larger harms (e.g., "made with recycled packaging" for a polluting product).
  - False certifications: using unofficial or self-created "green" logos.
- **Game-theoretic view**: greenwashing is a *babbling equilibrium* outcome – the signal (the green claim) conveys no information because all types mimic it.
- **Policy response**: third-party verification, legal liability for false environmental claims, standardized certification criteria.

4.8.6 Certification Cost as a Signal

- **Key economic insight**: the certification cost itself plays an important role as a *signal*.
- If the cost to certify is *too low*: both types certify $\Rightarrow$ pooling $\Rightarrow$ uninformative labels.
- If the cost to certify is *too high*: clean firms may not certify even though they qualify $\Rightarrow$ good firms are unrecognized.
- **Optimal certification cost**: $\Delta p - \kappa_D < 0 < \Delta p - \kappa_H$, i.e., $\kappa_H < \Delta p < \kappa_D$.
- **Design implication**: certification bodies should set standards (and hence costs) at a level that separates clean from dirty firms – not so low that dirty firms can freely obtain the label, not so high that clean firms are excluded.
- This is an application of **screening by costs** – widely used in labor economics, insurance, and finance.

4.8.7 Welfare Analysis of Certification

- **With effective certification** (separating equilibrium):
  - Clean firms earn price premium Δp, offsetting certification cost κ_H.
  - Dirty firms cannot mimic $\Rightarrow$ no pollution premium.
  - Consumers get accurate information $\Rightarrow$ can support clean production.
  - **Net welfare effect**: positive if $\Delta p > \kappa_H$ and the green premium stimulates enough additional clean production.
- **With greenwashing** (pooling equilibrium):
  - Certification is worthless as an information device.
  - Dirty firms capture the premium without reducing emissions.
  - **Net welfare effect**: may be negative if certification costs are wasted and consumers are systematically misled.
- **Policy implication**: the value of a certification system depends entirely on whether it achieves separation.

Recap: Section 4.8

- Green certification is a **signaling mechanism** that addresses the information asymmetry between firms and consumers about environmental performance.
- **Separating equilibrium**: clean firms certify; dirty firms do not. The label is informative. Requires $\kappa_H < \Delta p < \kappa_D$.
- **Pooling equilibrium**: both types certify (or neither does). The label conveys no information.
- **Greenwashing**: dirty firms make false green claims, destroying the informational value of eco-labels.
- **Optimal design**: certification costs should be calibrated to screen out dirty firms while remaining attractive to clean firms.

Section 4.10

Case Studies: Green Certification

Case Study: LEED Certification in the US

- **LEED** (Leadership in Energy and Environmental Design): the dominant green building certification in the US.
- **Signaling game perspective**:
  - Clean (energy-efficient) buildings can obtain LEED at lower net cost because they already invest in efficient systems.
  - Conventional buildings face higher costs to retrofit to LEED standards.
- **Evidence of separation**: LEED-certified buildings command rental premiums of 5–10%, consistent with a separating equilibrium where the label is informative.
- **Greenwashing concern**: the "LEED Silver" level has been criticized as too easy to achieve – some argue the standard should be raised to maintain separating equilibrium properties.

Case Study: Organic Certification and Greenwashing

- **USDA Organic** label: requires third-party verification of farming practices; costly to obtain for conventional farms.
- **Separation**: the certification cost structure ($\kappa_H < \Delta p < \kappa_D$) is approximately satisfied – organic farms find certification cost-effective; conventional farms generally do not.
- **Greenwashing examples**:
  - "Natural" label on food products has no legal definition in the US – creates a pooling equilibrium where the label is uninformative.
  - Products marketed as "eco-friendly" or "green" without third-party verification: consumers cannot distinguish genuinely green products from impostors.
- **Policy response**: FTC Green Guides regulate environmental marketing claims; EU Green Claims Directive (2023) requires substantiation.

Section 4.11
Summary

Summary: Key Results of Chapter 4

- **Uncertain damages** (Section 4.2): the EA uses $\mathbb{E}[d]$ in place of d. The resulting fee t^U deviates from t^* in proportion to the estimation error. *Fees and quotas produce the same welfare loss.*
- **Uncertain demand** (Section 4.3): the uncertain fee t^{UD} is a compromise with $t_L < t^{UD} < t_H$. Fees and quotas generate *different* welfare losses. Weitzman's rule applies.
- **Uncertain costs** (Section 4.4): firms hold private cost information. Fees dominate quotas because firms' output automatically adjusts to their true cost.
- **Mechanism design** (Section 4.5): circular monitoring can achieve first-best outcomes even under asymmetric information.
- **Green certification** (Section 4.8): a well-designed certification system achieves a separating equilibrium where labels are informative; poorly designed systems invite

Summary: The Instrument Choice Matrix

Source of uncertainty	**Fee**	**Quota**
Uncertain damage (d)	WL_t	$WL_Q = WL_t$
Uncertain demand (SMB steep, $b > d$)	Higher WL	**Lower** WL
Uncertain demand (SMC steep, $d > b$)	**Lower** WL	Higher WL
Uncertain costs	**Lower** WL	Higher WL

- Weitzman's rule summarizes rows 2–4: use fees when SMC is steeper; use quotas when SMB is steeper.
- Row 1 (uncertain d) is the special case where the neutrality result holds.

Summary: Key Equations

- Fee under uncertain damage (4.2): $t^U = \dfrac{(1-c)(2\mathbb{E}[d]-1)}{2(1+\mathbb{E}[d])}$
- Fee inefficiency (4.3): $t^U - t^* = \dfrac{3(1-c)(\mathbb{E}[d]-d)}{2(1+d)(1+\mathbb{E}[d])}$
- Welfare loss – damage uncertainty (4.5):
  $$WL_t = \frac{(1-c)^2(\mathbb{E}[d]-d)^2}{2(1+d)(1+\mathbb{E}[d])^2}$$
- Output inefficiency – quota (4.7):
  $$Q^{SO} - Q^{SO,U} = \frac{(1-c)(\mathbb{E}[d]-d)}{(1+d)(1+\mathbb{E}[d])}$$
- Fee under demand uncertainty (4.11):
  $$t^{UD} = \frac{(\mathbb{E}[a]-c)(2d-b)}{2(b+d)}$$
- Fee ranking (4.15): $t_L < t^{UD} < t_H$

Summary: Weitzman's Rule

- **Weitzman (1974)**: under uncertainty, the relative slopes of SMB and SMC determine the preferred regulatory instrument.
- **Formal condition**: let $\alpha = |\text{slope of SMC}|$ and $\beta = |\text{slope of SMB}|$.
  - If $\alpha > \beta$: **fee preferred** (price instrument).
  - If $\beta > \alpha$: **quota preferred** (quantity instrument).
- **Intuition**: use the instrument that mimics the side of the market that is more "rigid" – fees when costs are steep (firms need flexibility); quotas when benefits are steep (locking in output is valuable).
- This rule is one of the most important results in environmental economics and applies broadly to climate policy, pollution control, and fisheries management.

Summary: Green Certification

- Green certification addresses information asymmetry in product markets.
- The signaling game structure:
  - **Separating equilibrium** (informative label): requires $\kappa_H < \Delta p < \kappa_D$.
  - **Pooling equilibrium** (uninformative label): occurs when certification is too cheap or the premium too large.
- **Greenwashing** undermines the informational value of eco-labels and can lead to worse welfare outcomes than no certification at all.
- **Policy design**: certification standards should be set to ensure separation; third-party verification and legal liability reduce greenwashing.
- The same logic applies to any credence good – a product whose quality cannot be verified even after consumption (e.g., organic food, fair trade, sustainable timber).

Looking Ahead

- Chapter 4 examined how *uninformed* regulation performs under various types of uncertainty.
- **Chapter 5** takes a different approach: instead of a single firm, we consider **oligopoly** – how does strategic interaction among firms affect the efficiency of environmental policy?
- Key questions for Chapter 5:
  - Does market structure (monopoly vs. duopoly vs. oligopoly) affect the optimal fee?
  - How does strategic output competition interact with environmental externalities?
  - Can we still achieve the social optimum, and at what cost?

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-garcia

Chapter 4: Regulation under Uncertainty

Chapter 5

Sequential Competition and Regulation

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 5

- This chapter extends the Chapter 2 framework to *sequential* (Stackelberg) competition, where one firm acts as the market leader.
- Sections:
  - 5.1 Introduction and guiding questions.
  - 5.2 Sequential competition: Stackelberg model without and with regulation.
  - 5.2.3 Fee comparison: sequential vs. simultaneous competition.
  - 5.3 Two time periods: pollution persistence and intertemporal regulation.
  - 5.4 Regulatory commitment.
  - 5.6–5.8 Case studies, related literature, and summary.

Section 5.1

Introduction

Why Sequential Competition?

- Chapter 2 assumed *simultaneous* Cournot competition: firms choose output without observing each other's decision.
- Many real industries feature *sequential* decisions: a dominant firm (leader) commits to its output first; smaller rivals (followers) then optimize their response.
- Sequential structure creates strategic asymmetries:
  - The leader gains a *first-mover advantage*.
  - Higher unregulated aggregate output $\Rightarrow$ more pollution.
  - The EA must set *more stringent* fees under Stackelberg than under Cournot.
- We also extend to *two time periods* with pollution persistence, examining intertemporal fee design.

Real-World Motivation

- **Steel industry**: POSCO (South Korea) and NSSMC (Japan) invest sequentially in capacity, with the leader's commitment shaping the follower's response.
- **Aviation**: Lufthansa expands hub capacity before Ryanair decides on new European routes – a sequential game with carbon costs.
- **Electricity generation**: incumbent utilities choose generation capacity before new entrants; carbon pricing then applies to aggregate output.
- **Key insight**: when firms move sequentially, the market leader produces *more* than in a Cournot equilibrium. This has direct implications for emission fees.
- This chapter provides the theoretical framework to analyze these industries rigorously.

Guiding Questions

1. How does Stackelberg competition differ from Cournot in terms of strategic behavior?
2. What is the impact of sequential decision-making on emissions and welfare?
3. Why might emission fees be more stringent under sequential competition?
4. How does pollution persistence across time periods affect regulatory design?
5. What are the trade-offs between regulatory commitment and flexibility?
6. How do dynamic models enhance our understanding of intertemporal regulation?

Section 5.2

Sequential Competition (Stackelberg Model)

Setup: Stackelberg Duopoly

- Two firms face inverse demand $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- Marginal costs: c_1 (leader) and c_2 (follower) with $c_1 \leq c_2$.
- Normalize: set $c_1 = 0$ and $c_2 \equiv c \geq 0$. Thus c measures the leader's *cost advantage* over the follower.
- Assumption: $c < \frac{1}{3}$ ensures both firms remain active in equilibrium (positive output for both).
- When $c = 0$: symmetric costs; the leader benefits only from its timing advantage.
- When $c > 0$: leader benefits from *both* first-mover and cost advantages.

Section 5.2.1 – Game Without Regulation: Timing

- **Two-stage game (no regulation):**
  1. **Stage 1**: Leader (firm 1) observes nothing and chooses $q_1 \geq 0$.
  2. **Stage 2**: Follower (firm 2) observes q_1 and chooses $q_2 \geq 0$.
- Solution concept: **Subgame Perfect Nash Equilibrium** (SPNE).
- We solve by **backward induction**: start from Stage 2 (follower), then move back to Stage 1 (leader).
- The follower's best response derived in Stage 2 is inserted into the leader's problem in Stage 1.
- Timing structure means the leader can *credibly commit* to its output – this is the source of the first-mover advantage.

Stage 2 (No Regulation): Follower's Problem

- The follower takes the leader's output q_1 as given and maximizes profit:

$$\max_{q_2 \geq 0} \pi_2 = (1 - q_1 - q_2)q_2 - cq_2. \tag{5.1}$$

- Expanding:

$$\pi_2 = q_2 - q_1 q_2 - q_2^2 - cq_2.$$

- First-order condition (FOC):

$$\frac{\partial \pi_2}{\partial q_2} = 1 - q_1 - 2q_2 - c = 0.$$

- Solving for q_2:

$$q_2(q_1) = \frac{1 - c}{2} - \frac{1}{2} q_1. \tag{5.2}$$

Stage 2 (No Regulation): Interpreting the Follower's BRF

- Best response function (5.2):

$$q_2(q_1) = \frac{1-c}{2} - \frac{1}{2}q_1.$$

- **Vertical intercept** $\frac{1-c}{2}$: follower's monopoly output on the residual demand. Decreases in c – higher cost $\Rightarrow$ less residual profit.
- **Slope** $-\frac{1}{2}$: for each additional unit the leader produces, the follower reduces its output by $\frac{1}{2}$.
- Outputs are *strategic substitutes*: follower shrinks when the leader expands.
- Note: this is the same slope as in Cournot BRF, but the follower *observes* q_1 before choosing q_2 – this changes how the equilibrium is computed.

Stage 1 (No Regulation): Leader's Problem

- The leader inserts the follower's BRF (5.2) into its own profit function. With $c_1 = 0$:

$$\pi_1 = \big(1 - q_1 - q_2(q_1)\big)\, q_1. \tag{5.3}$$

- Substituting $q_2(q_1) = \frac{1-c}{2} - \frac{q_1}{2}$:

$$1 - q_1 - q_2(q_1) = 1 - q_1 - \frac{1-c}{2} + \frac{q_1}{2} = \frac{1+c}{2} - \frac{q_1}{2}. \tag{5.4}$$

- Leader's profit simplifies to:

$$\pi_1 = \left(\frac{1+c}{2} - \frac{q_1}{2}\right) q_1. \tag{5.5}$$

Stage 1 (No Regulation): Solving for Leader's Output

- Leader maximizes $\pi_1 = \left(\frac{1+c}{2} - \frac{q_1}{2}\right) q_1$:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1+c}{2} - q_1 = 0.$$

- Leader's equilibrium output:

$$q_1^{NR} = \frac{1+c}{2}. \tag{5.6}$$

- Follower's equilibrium output (substituting q_1^{NR} into BRF):

$$q_2^{NR} = \frac{1-c}{2} - \frac{1}{2} \cdot \frac{1+c}{2} = \frac{1-c}{2} - \frac{1+c}{4} = \frac{1-3c}{4}.$$

- $q_2^{NR} > 0$ iff $c < \frac{1}{3}$ – confirmed by our assumption.
- NR = "no regulation."

Equilibrium Outputs: Stackelberg vs. Cournot

- **Stackelberg equilibrium (no regulation):**

$$q_1^{NR} = \frac{1+c}{2}, \qquad q_2^{NR} = \frac{1-3c}{4}, \qquad Q^{NR} = \frac{3-c}{4}.$$

- **Cournot equilibrium (Chapter 2, $c_1 = 0$, $c_2 = c$):**

$$q_1^{C} = \frac{1+c}{3}, \qquad q_2^{C} = \frac{1-2c}{3}, \qquad Q^{C} = \frac{2-c}{3}.$$

- **Comparison:**
  - Leader: $q_1^{NR} = \frac{1+c}{2} > \frac{1+c}{3} = q_1^C$ – leader produces *more* under Stackelberg.
  - Follower: $q_2^{NR} < q_2^C$ – follower produces *less*.
  - Aggregate: $Q^{NR} = \frac{3-c}{4} > \frac{2-c}{3} = Q^C$ for $c < 1/3$.

Leader's Output Advantage (No Regulation)

- Define the **output advantage** as: $\Delta q^{NR} \equiv q_1^{NR} - q_2^{NR}$.
- Computing:

$$\Delta q^{NR} = \frac{1+c}{2} - \frac{1-3c}{4} = \frac{2(1+c)-(1-3c)}{4} = \frac{1+5c}{4}. \tag{5.7}$$

- **Decomposition** into first-mover and cost advantages:

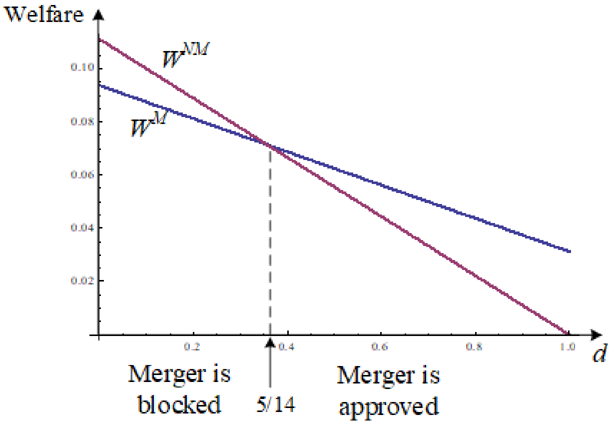
$$\Delta q^{NR} = \underbrace{\frac{1}{4}}_{FMA^{NR}} + \underbrace{\frac{5}{4}c}_{CA^{NR}}. \tag{5.8}$$

- When $c = 0$: only first-mover advantage, $\Delta q^{NR} = \frac{1}{4}$.
- When $c > 0$: both advantages reinforce each other; Δq^{NR} increases linearly in c.

Decomposing the Output Advantage

- $FMA^{NR} = \frac{1}{4}$: **first-mover advantage**.
  - Exists even with symmetric costs ($c = 0$).
  - Arises because the leader *commits* to a large output, forcing the follower to accommodate.
- $CA^{NR} = \frac{5}{4}c$: **cost advantage**.
  - Proportional to c; larger when the leader's cost edge is greater.
  - Coefficient $\frac{5}{4} > 1$: the strategic position amplifies the direct cost advantage.
- Key question: How does regulation affect each component?
- Preview: fees *reduce* the first-mover advantage but may *amplify* the cost advantage – a nuanced distributional effect.

Figure 5.1 – Output Advantage Without Regulation



Section 5.2.2

Sequential Competition with Regulation

Regulated Stackelberg: 3-Stage Game

- When the Environmental Authority (EA) introduces a per-unit emission fee t, the game expands to three stages:
  1. **Stage 1**: EA sets emission fee $t \geq 0$ to maximize social welfare $W = CS + PS + T - Env$.
  2. **Stage 2**: Leader (firm 1) observes t and chooses $q_1 \geq 0$.
  3. **Stage 3**: Follower (firm 2) observes t and q_1, then chooses $q_2 \geq 0$.
- **Solution concept**: Subgame Perfect Nash Equilibrium.
- **Method**: backward induction – Stage 3 $\rightarrow$ Stage 2 $\rightarrow$ Stage 1.
- Welfare components: $CS = \frac{Q^2}{2}$, $Env = \frac{d}{2}Q^2$, $T = tQ$, where $d > 0$ measures pollution severity.

Stage 3 (With Regulation): Follower's Problem

- Follower pays the emission fee t on each unit produced. Follower's profit:

$$\pi_2 = (1 - q_1 - q_2)q_2 - (c + t)q_2. \tag{5.10}$$

- FOC: $1 - q_1 - 2q_2 - (c + t) = 0$.
- Best response function:

$$q_2(q_1, t) = \frac{1 - (c + t)}{2} - \frac{1}{2}q_1. \tag{5.11}$$

- Same structure as without regulation, but shifted *downward* by the fee t.
- $\frac{\partial q_2}{\partial t} = -\frac{1}{2} < 0$: higher fee reduces follower's output at every q_1.

Stage 2 (With Regulation): Leader's Problem

- Leader (with $c_1 = 0$) inserts follower's BRF (5.11) into its profit:

$$\pi_1 = \big(1 - q_1 - q_2(q_1, t)\big)\, q_1 - t\, q_1. \tag{5.12}$$

- Substituting $q_2(q_1, t)$:

$$1 - q_1 - q_2(q_1, t) = \frac{1 + c + t}{2} - \frac{q_1}{2} - t = \frac{1 + c - t}{2} - \frac{q_1}{2} + \frac{t}{2}.$$

- After simplification, leader's profit:

$$\pi_1 = \left(\frac{1 + c - t}{2} - \frac{q_1}{2}\right) q_1. \tag{5.13}$$

Stage 2 (With Regulation): Leader's Equilibrium Output

- Maximizing $\pi_1 = \left(\frac{1+c-t}{2} - \frac{q_1}{2}\right) q_1$:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1+c-t}{2} - q_1 = 0.$$

- Leader's output as a function of t:

$$q_1(t) = \frac{1+c-t}{2}. \tag{5.14}$$

- Inserting $q_1(t)$ into follower's BRF (5.11):

$$q_2(t) = \frac{1-(c+t)}{2} - \frac{1}{2} \cdot \frac{1+c-t}{2} = \frac{1-3c-t}{4}. \tag{5.15}$$

- Both outputs are decreasing in t (fee reduces both firms' output).

Differential Effect of the Fee on Outputs

- From (5.14) and (5.15):

$$\frac{\partial q_1}{\partial t} = -\frac{1}{2}, \qquad \frac{\partial q_2}{\partial t} = -\frac{1}{4}. \tag{5.16}$$

- **Key result**: $\left|\frac{\partial q_1}{\partial t}\right| = \frac{1}{2} > \frac{1}{4} = \left|\frac{\partial q_2}{\partial t}\right|$.
- The fee reduces the leader's output by *twice as much* as the follower's output.
- Intuition: The leader internalizes that reducing q_1 also leads the follower to produce more (via BRF). But the fee still penalizes the leader more in equilibrium because the leader sets its output *before* the follower adjusts.
- Implication: the fee partially *levels the playing field* between leader and follower.

Effect of the Fee on the Output Advantage

- Regulated output advantage:

$$\Delta q^R(t) = q_1(t) - q_2(t) = \frac{1+c-t}{2} - \frac{1-3c-t}{4} = \frac{1+5c-t}{4}. \tag{5.17}$$

- Compare to unregulated advantage:

$$\Delta q^{NR} = \frac{1+5c}{4}. \tag{5.7}$$

- Difference: $\Delta q^{NR} - \Delta q^R(t) = \frac{t}{4} > 0$.
- For any $t > 0$: $\Delta q^R < \Delta q^{NR}$. (5.18)
- The fee **reduces the leader's output advantage**. The reduction equals $\frac{t}{4}$: one quarter of the fee per unit.

Stage 1 (With Regulation): EA's Welfare

- Aggregate output: $Q(t) = q_1(t) + q_2(t) = \frac{3-c-3t}{4}$.
- Welfare components:

$$\begin{aligned} CS &= \frac{Q(t)^2}{2} = \frac{(3-c-3t)^2}{32}. \\ T &= t \cdot Q(t) = \frac{t(3-c-3t)}{4}. \\ Env &= \frac{d}{2}Q(t)^2 = \frac{d(3-c-3t)^2}{32}. \\ PS &= \pi_1(t) + \pi_2(t) = \frac{(1+c-t)^2}{4} + \frac{(1-3c-t)^2}{8}. \end{aligned}$$

- EA maximizes $W = CS + PS + T - Env$. (5.19)

Stage 1: Deriving the Optimal Fee

- Differentiating W with respect to t and setting equal to zero:

$$\frac{\partial W}{\partial t} = 0. \tag{5.20}$$

- After computing and simplifying, the FOC yields:

$$t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}. \tag{5.21}$$

- **Sign**: $t^{Seq} > 0$ since $c < \frac{1}{3}$ (so $3 - c > 0$) and we require $d > \frac{1}{2}$ (so $3d - 1 > 0$; also $d + 1 > 0$).
- **Effect of pollution severity**: $\frac{\partial t^{Seq}}{\partial d} = \frac{3(3-c)\cdot 2}{9(1+d)^2} > 0$ – stricter fee when pollution is more damaging.
- **Effect of cost asymmetry**: $\frac{\partial t^{Seq}}{\partial c} = -\frac{3d-1}{9(1+d)} < 0$ – less stringent fee when the leader has a larger cost advantage.

Interpreting the Optimal Fee t^{Seq}

- $t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}$ reflects:
    - **Numerator**: $(3 - c)$ – aggregate output capacity (decreases in c, since a more asymmetric market has lower total output in the no-regulation equilibrium); $(3d - 1)$ – net marginal environmental damage above the tax's opportunity cost.
    - **Denominator**: $9(1 + d)$ – normalizes for market structure and pollution weight.
- **Why does t^{Seq} decrease in c?** Higher c means the follower is relatively weaker. Aggregate unregulated output $Q^{NR} = \frac{3-c}{4}$ falls in c. Less output $\Rightarrow$ less pollution $\Rightarrow$ less need for intervention.
- **Limiting case**: when $c \to \frac{1}{3}$, $t^{Seq} \to 0$ (follower is about to exit; fee would eliminate it).

Section 5.2.3

Fee Comparison: Sequential vs. Simultaneous

Cournot Fee (Recall from Chapter 2)

- In Chapter 2, under Cournot competition with asymmetric costs $c_1 = 0$, $c_2 = c$, the optimal fee was:

$$t^C = \frac{(1-c)(2d-1)}{2(1+d)}.$$

- **Interpretation**: $(1-c)$ measures the effective market size available to the higher-cost firm; $(2d-1)$ reflects net marginal damage.
- Also $t^C > 0$ iff $d > \frac{1}{2}$.
- The question: Is t^{Seq} greater or less than t^C?

Fee Differential: $t^{Seq} - t^{C}$

- Subtracting t^C from t^{Seq}:

$$\begin{aligned} \Delta t &= t^{Seq} - t^{C} \\ &= \frac{(3-c)(3d-1)}{9(1+d)} - \frac{(1-c)(2d-1)}{2(1+d)} \\ &= \frac{2(3-c)(3d-1) - 9(1-c)(2d-1)}{18(1+d)}. \end{aligned}$$

- Expanding the numerator:

$$\begin{aligned} & 2(9d-3-3cd+c) - 9(2d-1-2cd+c) \\ = \; & 18d - 6 - 6cd + 2c - 18d + 9 + 18cd - 9c \\ = \; & 3 + 12cd - 7c = 3 + c(12d-7). \end{aligned}$$

- Result:

$$\Delta t = \frac{3 + c(12d-7)}{18(1+d)}. \tag{5.22}$$

Is the Stackelberg Fee Always More Stringent?

- Fee differential: $\Delta t = \frac{3+c(12d-7)}{18(1+d)}$.
- **Sign**: Is $\Delta t > 0$ for all $c \in [0, \frac{1}{3})$ and $d > \frac{1}{2}$?
- When $d > \frac{7}{12} \approx 0.583$: $12d - 7 > 0$, so numerator $3 + c(12d - 7) > 0$ for all $c \geq 0$. $\Rightarrow \Delta t > 0$.
- When $\frac{1}{2} < d \leq \frac{7}{12}$: $12d - 7 \leq 0$. Worst case at $c = \frac{1}{3}$: numerator $= 3 + \frac{1}{3}(12d - 7) = 3 + 4d - \frac{7}{3} = \frac{2}{3} + 4d$. Since $d > \frac{1}{2}$, this exceeds $\frac{2}{3} + 2 > 0$.
- **Conclusion**: $\Delta t > 0$ for *all admissible* c and d. (5.22)
- The Stackelberg fee is **always more stringent** than the Cournot fee.

Why Is the Stackelberg Fee More Stringent?

- **Root cause**: Under Stackelberg, unregulated aggregate output exceeds Cournot:

$$Q^{NR}(S) = \frac{3-c}{4} > \frac{2-c}{3} = Q^{NR}(C). \tag{5.24}$$

- **Proof**: $\frac{3-c}{4} - \frac{2-c}{3} = \frac{3(3-c)-4(2-c)}{12} = \frac{9-3c-8+4c}{12} = \frac{1+c}{12} > 0.$
- More output under Stackelberg $\Rightarrow$ more pollution $\Rightarrow$ greater social cost of unregulated production.
- To internalize this larger externality, the EA must set a *higher* fee under Stackelberg.
- **Policy implication**: misclassifying a Stackelberg market as Cournot would lead the EA to set an *insufficiently stringent* fee, resulting in excess pollution and welfare loss.

Comparative Statics of Δt with Respect to c

- How does the fee differential change as cost asymmetry c increases?

$$\frac{\partial \Delta t}{\partial c} = \frac{12d - 7}{18(1+d)}. \tag{5.23}$$

- Sign depends on d:
  - If $d > \frac{7}{12}$: $\frac{\partial \Delta t}{\partial c} > 0$ – fee gap *widens* as cost asymmetry grows.
  - If $d < \frac{7}{12}$: $\frac{\partial \Delta t}{\partial c} < 0$ – fee gap *narrows*.
- Intuition: greater c increases $\Delta Q^{NR} = \frac{1+c}{12}$ (Stackelberg vs. Cournot output gap). When pollution is severe (d large), the EA responds more aggressively to this gap – Δt widens.

Comparative Statics of Δt with Respect to d

- How does the fee differential change as pollution severity d increases?

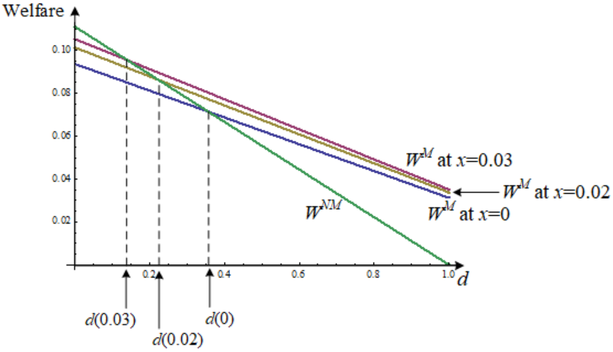
$$\frac{\partial \Delta t}{\partial d} = \frac{3(3+c) - [3 + c(12d-7)] \cdot 0}{18(1+d)^2} \cdot \ldots$$

After full computation:

$$\frac{\partial \Delta t}{\partial d} = \frac{3-19c}{18(1+d)^2}.$$

- Sign depends on c:
  - If $c < \frac{3}{19} \approx 0.158$: $\frac{\partial \Delta t}{\partial d} > 0$ – fee gap *widens* as d rises (Stackelberg fee grows faster).
  - If $c > \frac{3}{19}$: $\frac{\partial \Delta t}{\partial d} < 0$ – fee gap *narrows* as d rises.
- Regardless, $\Delta t > 0$ for all admissible values.

Figure 5.2 – Fees Under Sequential and Simultaneous Competition



Sections 5.2.4–5.2.5

Equilibrium Outputs and Output Advantage Under Regulation

Equilibrium Outputs at t^{Seq}

- Inserting $t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}$ into $q_1(t)$ and $q_2(t)$:

$$q_1^R = \frac{1+c-t^{Seq}}{2} = \frac{2[3+c(2+3d)]}{9(1+d)} > 0.$$

$$q_2^R = \frac{1-3c-t^{Seq}}{4} = \frac{3-c(7+6d)}{9(1+d)}.$$

- $q_2^R > 0$ iff $c < \frac{3}{7+6d} \equiv c_A$.
- Cutoff $c_A = \frac{3}{7+6d}$ is decreasing in d: stricter pollution regulation makes it harder for the follower to remain active.
- At $d = \frac{1}{2}$: $c_A = \frac{3}{10}$; recall $c < \frac{1}{3}$, so for some parameter values the fee may drive the follower out.

Leader's Output Advantage Under Regulation

- Regulated output advantage:

$$\Delta q^R = q_1^R - q_2^R = \underbrace{\frac{1}{3(1+d)}}_{FMA^R} + \underbrace{\frac{11+12d}{9(1+d)}c}_{CA^R}. \tag{5.28}$$

- **First-mover advantage** comparison:

$$FMA^{NR} = \frac{1}{4} \quad \text{vs.} \quad FMA^R = \frac{1}{3(1+d)}.$$

  $FMA^{NR} > FMA^R$ iff $\frac{1}{4} > \frac{1}{3(1+d)}$ iff $d > \frac{1}{12}$ (satisfied for all $d > \frac{1}{2}$). Fees **reduce** first-mover advantage.

- **Cost advantage** comparison: $CA^R = \frac{11+12d}{9(1+d)}c$ vs. $CA^{NR} = \frac{5}{4}c$. $CA^R > CA^{NR}$ iff $d > \frac{1}{6}$ (satisfied). Fees **amplify** cost advantage.

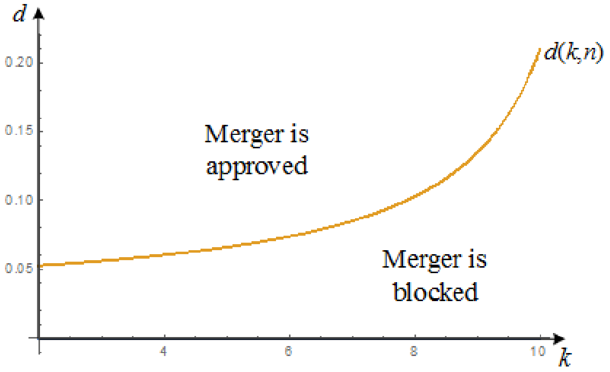
Net Effect: Regulation Reduces Total Output Advantage

- Although the *cost advantage* component rises with regulation, the *first-mover advantage* component falls by more. Net effect:

$$\Delta q^{NR} - \Delta q^{R} = \frac{(3-c)(3d-1)}{36(1+d)} = \frac{t^{Seq}}{4} > 0. \qquad (5.32)$$

- The reduction in output advantage exactly equals $\frac{t^{Seq}}{4}$: one-quarter of the optimal fee.
- **More severe pollution** ($\uparrow d$): t^{Seq} rises $\Rightarrow$ larger reduction in Δq.
- **More cost asymmetry** ($\uparrow c$): t^{Seq} falls $\Rightarrow$ smaller reduction in Δq.
- The fee partially levels the playing field – but the leader always retains an advantage.

Figure 5.3 – Output Advantage With and Without Regulation



Recap: Section 5.2 Key Results

- **No regulation**: Leader produces $q_1^{NR} = \frac{1+c}{2}$, follower $q_2^{NR} = \frac{1-3c}{4}$; aggregate output $Q^{NR} = \frac{3-c}{4}$.
- **Output advantage**: $\Delta q^{NR} = FMA^{NR} + CA^{NR} = \frac{1}{4} + \frac{5c}{4}$.
- **With regulation**: Optimal fee $t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)} > 0$.
- **Fee comparison**: $t^{Seq} > t^C$ for all admissible parameters – Stackelberg fee is always more stringent than Cournot fee.
- **Why**: $Q^{NR}(S) > Q^{NR}(C) \Rightarrow$ EA must be more aggressive to control excess pollution from sequential competition.
- **Output advantage**: Regulation reduces Δq by $t^{Seq}/4$ – fee disproportionately penalizes the leader.

Section 5.3

Two Time Periods with Pollution Persistence

Motivation: Why a Two-Period Model?

- The static model treats each period in isolation. But many pollutants accumulate over time: greenhouse gases, heavy metals, persistent organic compounds.
- **Pollution persistence**: emissions in period 1 partially survive into period 2, adding to the stock of pollution.
- This creates *intertemporal linkages*:
  - Period-1 pollution increases period-2 environmental damage.
  - Optimal period-2 fee depends on how much first-period pollution persisted.
  - Firms in period 1 anticipate stricter future fees – this changes their first-period behavior.
- The EA must *jointly* design fees across both periods.
- Key questions: How do fees compare across periods? Does commitment help?

Two-Period Game: Setup

- We return to *symmetric Cournot* competition (two firms) but extend to two periods.
- Notation: $X = x_1 + x_2$ (period-1 aggregate output); $Q = q_1 + q_2$ (period-2 aggregate output).
- Inverse demands: $p_1(X) = 1 - X$ and $p_2(Q) = 1 - Q$.
- Symmetric marginal cost c in both periods.
- **Persistence parameter** $\lambda \in [0, 1]$:
  - $\lambda = 0$: no persistence; periods are independent.
  - $\lambda = 1$: full persistence; period-1 emissions fully add to period-2 pollution stock.
- **Environmental damage in period 2:** $Env_2 = \frac{d}{2}(Q + \lambda X)^2$. (5.35)

Two-Period Game: Timing (No Commitment)

- Under **no commitment**, the EA sets fees sequentially:
  1. **Stage 1**: EA sets first-period fee t_1.
  2. **Stage 2**: Firms simultaneously choose first-period outputs x_1, x_2.
  3. **Stage 3**: EA observes $X = x_1 + x_2$ and sets second-period fee $t_2(X)$.
  4. **Stage 4**: Firms simultaneously choose second-period outputs q_1, q_2.
- The EA adjusts t_2 after observing how much pollution was generated in period 1. This is "time-consistent" but potentially suboptimal (ratchet effect).
- We solve by backward induction: Stage 4 $\rightarrow$ Stage 3 $\rightarrow$ Stage 2 $\rightarrow$ Stage 1.

Stage 4: Second-Period Cournot Equilibrium

- In Stage 4, each firm i maximizes: $\pi_{i,2} = (1 - Q)q_i - (c + t_2)q_i$.
- Symmetric Cournot FOC: $1 - Q - q_i - (c + t_2) = 0$.
- Symmetric equilibrium:

$$q(t_2) = \frac{1 - c - t_2}{3}, \qquad Q(t_2) = \frac{2(1 - c - t_2)}{3}. \tag{5.38}$$

- Period-2 welfare (in terms of t_2 and X):

$$\begin{aligned} W_2(t_2, X) &= CS_2 + PS_2 + T_2 - Env_2 \\ &= \frac{Q^2}{2} + 2\pi_{i,2} + t_2 Q - \frac{d}{2}(Q + \lambda X)^2. \end{aligned}$$

Stage 3: EA Sets Second-Period Fee

- EA observes X and maximizes W_2 with respect to t_2.
- FOC for t_2, after substituting $Q(t_2)$:
- Optimal second-period fee:

$$t_2(X) = \underbrace{\frac{(1-c)(2d-1)}{2(1+d)}}_{t^*} + \frac{3d\lambda}{2(1+d)} X. \tag{5.40}$$

- **Two components**:
  - t^*: static optimal fee (Chapter 2 result).
  - $\frac{3d\lambda}{2(1+d)} X$: additional term reflecting the pollution stock carried over from period 1.
- When $\lambda = 0$: $t_2 = t^*$ (no spillover from period 1).
- When $\lambda > 0$: $t_2 > t^*$ – EA must be stricter in period 2 because of accumulated period-1 pollution.

Stage 2: First-Period Firm Behavior

- In Stage 2, firm i anticipates that a larger x_i will trigger a stricter $t_2(X)$ in Stage 3, reducing period-2 profits.
- Intertemporal trade-off for firm i:
  - $(+)$ Higher $x_i \Rightarrow$ more first-period profit.
  - $(-)$ Higher $x_i \Rightarrow$ more persistent pollution $\Rightarrow$ higher $t_2 \Rightarrow$ lower period-2 profit.
- Solving the symmetric Cournot equilibrium in period 1:

$$x(t_1) = \frac{2(1-c-t_1)(1+d)^2 - (1-c)d\lambda}{6(1+d)^2 - 2d^2\lambda^2}. \qquad (5.45)$$

- When $\lambda = 0$: $x(t_1) = \frac{1-c-t_1}{3}$ (same as static Cournot – no intertemporal effect).

Intertemporal Effect on First-Period Output

- Comparing with the no-persistence case ($\lambda = 0$):

$$x^{\lambda>0}(t_1) < \frac{1 - c - t_1}{3} = x^{\lambda=0}(t_1).$$

- When $\lambda > 0$, firms *voluntarily* reduce first-period output below the static Cournot level, because they internalize the future penalty from accumulated pollution.
- This is a **self-disciplining effect**: firms restrain themselves even before the EA adjusts the second-period fee.
- The EA exploits this dynamic: a lower t_1 now suffices to achieve the same first-period output as the static case, because firms already have incentives to reduce output.
- The key question is whether the EA should also tighten t_1 above t^*.

Stage 1: Optimal First-Period Fee

- EA maximizes total welfare $W_1 + W_2$ choosing t_1:

$$t_1 = \underbrace{\frac{(1-c)(2d-1)}{2(1+d)}}_{t^*} + \frac{(1-c)d\lambda\left[2+3\lambda+d(2-\lambda)(1+\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}. \tag{5.47}$$

- When $\lambda = 0$: $t_1 = t^*$ (no intertemporal spillover).
- When $\lambda > 0$: the second term is strictly positive, so $t_1 > t^*$.
- **Intuition**: The EA preemptively raises t_1 to curb first-period pollution before it persists into period 2 and causes additional damage.
- Both fees are above t^*: $t_1 > t^*$ and $t_2 > t^*$ when $\lambda > 0$.

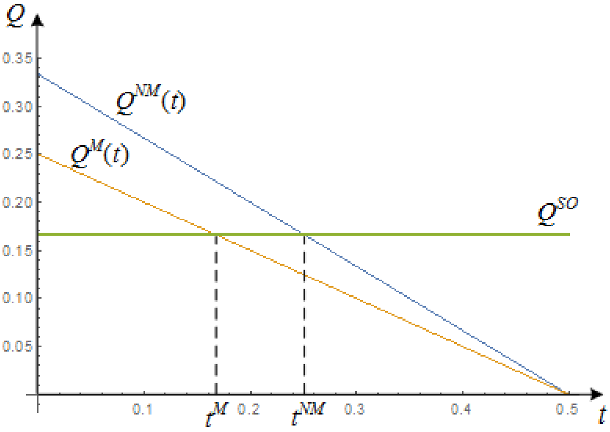
Fee Ranking Across Periods

- The second-period fee at equilibrium:

$$t_2 = t^* + \frac{3(1-c)d\lambda\left[1+d(1-\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]} > t^*. \qquad (5.50)$$

- **Fee ranking** depends on λ:
  - When $\lambda < \bar{\lambda}$: $t_2 > t_1$ – second-period fee is more stringent (pollution stock from period 1 hits harder in period 2).
  - When $\lambda > \bar{\lambda}$: $t_1 > t_2$ – first-period fee is more stringent (EA acts preemptively to prevent accumulation).
- Cutoff $\bar{\lambda}$ decreases in d: with more severe damage, preemptive action becomes optimal at lower persistence levels.
- Both regimes: $t_1, t_2 > t^*$ whenever $\lambda > 0$.

Figure 5.4 – Fees t_1 and t_2 as Functions of λ



Welfare in the Two-Period Model

- Total welfare:

$$W = \frac{(1-c)^2}{1+d} - \frac{(1-c)^2 d\lambda\left[2+\lambda+d(2-\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]} < \frac{(1-c)^2}{1+d}. \quad (5.54)$$

- Welfare decreases in λ: more persistence is always *harmful* even when the EA optimally adjusts both fees.
- **Intuition**: Higher $\lambda \Rightarrow$ more accumulated damage in period 2. Even optimally set fees cannot eliminate the harm from the persistent stock.
- Welfare when $\lambda = 0$: $W = \frac{(1-c)^2}{1+d} = 2 \cdot \frac{(1-c)^2}{2(1+d)} = 2 \cdot W^*$ (sum of two independent static optima – confirms consistency).

Recap: Section 5.3 Key Results

- When $\lambda = 0$: both periods are independent; $t_1 = t_2 = t^*$.
- When $\lambda > 0$: both fees rise above t^*:
  - $t_2(X) = t^* + \frac{3d\lambda}{2(1+d)}X$ – period-2 fee responds to first-period emissions.
  - $t_1 > t^*$ – first-period fee is preemptively tightened.
- **Fee ranking**: $t_2 > t_1$ for low persistence; $t_1 > t_2$ for high persistence.
- **Welfare**: Total welfare decreases in λ – more persistent pollution is always harmful.
- **Key insight**: Intertemporal externalities require intertemporal regulation. A static fee applied separately in each period would be suboptimal.

Section 5.4

Regulatory Commitment

The Commitment Problem

- Under **no commitment**, the EA sets t_2 after observing X. This is *time-consistent*: given X, it is optimal to set $t_2(X)$.
- But firms know the EA will react to X. This creates a **dynamic moral hazard**: firms may overproduce in period 1, expecting the EA to clean up in period 2.
- This is the **ratchet effect**: forward-looking regulation may be undermined if firms anticipate future tightening.
- Under **commitment**, the EA announces (t_1, t_2) at the start and is *bound* by this announcement, regardless of what happens in period 1.
- The trade-off: commitment may deliver better outcomes by changing firms' first-period incentives – but at the cost of flexibility.

Commitment: Game Structure

- Under **commitment**, the game has three stages:
  1. **Stage 1**: EA announces and commits to (t_1^{Com}, t_2^{Com}).
  2. **Stage 2**: Firms simultaneously choose first-period outputs x_1, x_2.
  3. **Stage 3**: Firms simultaneously choose second-period outputs q_1, q_2.
- Since t_2^{Com} is known from Stage 1, second-period profits are fully determined at Stage 2. Firms in Stage 2 do not need to consider how their output affects t_2 – it is fixed.
- First-period output under commitment:

$$x^{Com}(t_1) = \frac{1 - c - t_1}{3}. \tag{5.57}$$

- Same form as static Cournot – the intertemporal self-disciplining effect is *absent*.

Equilibrium Fees Under Commitment

- EA chooses $\left(t_1^{Com}, t_2^{Com}\right)$ jointly to maximize $W_1 + W_2$:

$$
\begin{aligned}
t_1^{Com} &= t^* + \frac{3d\lambda(1-c)(1+d+\lambda)}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}. \\
t_2^{Com} &= t^* + \frac{3d\lambda(1-c)\left[1+d(1-\lambda)\right]}{2(1+d)\left[1+d(2+d+\lambda^2)\right]}.
\end{aligned}
$$

- **Fee ranking under commitment**:

$$
t_1^{Com} \geq t_2^{Com} \geq t^* \quad \text{for all } \lambda \in [0,1]. \tag{5.61}
$$

- When $\lambda = 0$: $t_1^{Com} = t_2^{Com} = t^*$.
- Under commitment, the first-period fee is *always* at least as stringent as the second-period fee – opposite to the no-commitment case for small λ.

Comparing Fees: Commitment vs. No Commitment

- **First-period fee**:

$$t_1^{Com} - t_1 = \frac{d\lambda(1-c)\left[1 + d(1-\lambda(1-\lambda))\right]}{2(1+d)\left[1 + d(2 + d + \lambda^2)\right]} > 0. \quad (5.62)$$

- **Result**: $t_1^{Com} > t_1$ – commitment leads to a *more stringent* first-period fee.
- **Second-period fee**: $t_2^{Com} = t_2$ – both regimes set the same second-period fee.
- Intuition for $t_1^{Com} > t_1$: Under no commitment, firms reduce first-period output voluntarily (anticipating stricter t_2). With commitment, this self-discipline disappears – so the EA must be more aggressive with t_1 to compensate.

The Welfare Equivalence Result

- Period-by-period welfare comparison:

$$W_1^{Com} = W_1, \qquad W_2^{Com} = W_2.$$

- Total welfare: $W^{Com} = W_1^{Com} + W_2^{Com} = W.$
- **Main Result**: *Commitment does not change total welfare.* (5.64)
- **Proof sketch**: Under commitment, firms lose their self-disciplining incentive, so first-period output is higher than under no-commitment (given the same t_1). But the EA responds with a higher t_1^{Com} that exactly restores the optimal first-period output. The two channels precisely offset.
- **Policy implication**: A regulator's ability to commit to future fees does *not* translate into welfare gains – as long as the regulator optimizes in both cases.

Intuition for Welfare Equivalence

- Under **no commitment**:
  - Firms self-discipline in period 1 (reduce output) $\Rightarrow$ less pollution.
  - EA adjusts t_2 upward based on observed X.
  - Net effect: first-period production is below static optimum; second-period fee overshoots.
- Under **commitment**:
  - Firms do not self-discipline (fixed t_2 removes incentive).
  - EA preemptively raises t_1 to correct for lack of self-discipline.
  - Net effect: first-period production is pushed to the same optimum via a different mechanism.
- Both mechanisms achieve the same allocation – **different instruments, same outcome**.
- The result holds only under *optimal* policy in each regime. Suboptimal commitment can be harmful.

Recap: Section 5.4 Key Results

- **No commitment**: EA observes period-1 pollution and adjusts t_2. Firms self-discipline in period 1.
- **Commitment**: EA announces both fees in advance. Firms lose self-disciplining incentive; EA must compensate with higher t_1.
- **Fee comparison**: $t_1^{Com} > t_1$; $t_2^{Com} = t_2$.
- **Fee ranking under commitment**: $t_1^{Com} \geq t_2^{Com}$ always (front-loaded). Under no commitment, fee ranking depends on λ.
- **Welfare equivalence**: $W^{Com} = W$ – commitment does not change welfare. Two instruments (self-discipline vs. preemptive fee) achieve the same social outcome.
- **Caveat**: Equivalence requires the EA to optimize in both cases. Commitment may matter when the EA is subject to political constraints.

Section 5.6

Case Studies

Case Study 1: Steel Industry – POSCO vs. NSSMC

- POSCO (South Korea) and Nippon Steel & Sumitomo Metal Corporation (NSSMC, Japan) compete globally in crude steel.
- POSCO historically acts as the Stackelberg leader in Asian markets: it commits to large-scale production capacity first; NSSMC then adjusts.
- Under emissions trading schemes in both countries, POSCO's production decisions shape both market price and aggregate pollution.
- **Model prediction**: Carbon pricing must be *more stringent* to achieve a given pollution target than if the two firms competed simultaneously.
- Steel production involves long investment lags – reinforcing the sequential structure: capacity decisions precede actual production and fee-setting has to be forward-looking.

Case Study 2: Aviation – Lufthansa vs. Ryanair

- Lufthansa expands hub-and-spoke capacity at major European airports before low-cost carriers such as Ryanair decide on route expansion.
- Sequential entry: Lufthansa commits to slots and terminal capacity first; Ryanair observes these decisions and responds.
- Under the EU Emissions Trading System (ETS), aviation emissions are capped and traded. Lufthansa's large initial output stake influences the equilibrium allowance price.
- **Model prediction**: EA must account for sequential structure when setting the cap – the first-mover (Lufthansa) produces more than it would under Cournot, so the cap must be tighter.
- Output advantage of the leader generates excess emissions that a Cournot-calibrated cap would miss.

Case Study 3: US Oil Production (ExxonMobil)

- ExxonMobil historically sets production targets first in US oil markets, with Chevron and other firms responding.
- Under California's cap-and-trade system and the EPA's greenhouse gas rules, ExxonMobil's output decisions directly shape aggregate emissions and allowance prices.
- Follows the Stackelberg pattern: leader's early commitment to output levels defines the residual demand for followers who then optimize given the emission fee and the leader's decision.
- **Model prediction**: Emission fees must be *more stringent* in this sequential setting than they would be under Cournot to achieve the same pollution target.
- Misclassifying this market as Cournot would systematically underestimate required fee levels.

Case Study 4: PJM Electricity Market

- In the PJM Interconnection (US), a dominant producer acted as Stackelberg leader under the EPA's NOx Budget Program.
- The leader *strategically overproduced*, consuming more NOx allowances than necessary, driving up allowance prices for follower firms.
- This increased followers' costs, suppressed their output, and expanded the leader's market share.
- Illustrates how sequential output decisions under emission fees can be used to *exercise market power*, beyond simply polluting more.
- **Model prediction**: the more the leader exploits its first-mover position, the wider Δq^R – regulation reduces but cannot eliminate the output advantage. Antitrust monitoring is complementary to environmental policy.

Case Study 5: Spanish Electricity (Iberdrola vs. Endesa)

- In Spain's liberalized electricity market, Iberdrola commits to generation capacity before Endesa and Naturgy decide on their own output.
- Under Spanish carbon taxation, Iberdrola's early generation commitments influence both market prices and competitors' responses.
- Sequential structure: the leader internalizes the impact of its production on emissions and pricing while followers optimize accordingly.
- **Policy relevance**: assuming simultaneous competition when firms actually compete sequentially leads to *insufficiently stringent* carbon fees and excess emissions.
- Similar patterns appear in cement (Holcim vs. followers) and refining industries globally.

Section 5.7

Related Literature

Related Literature: Stackelberg and Regulation

- **Stackelberg (1934)**: Original framework for sequential oligopoly with a dominant leader and a quantity-setting follower.
- **Downing and White (1986)**: Early analysis of environmental regulation under market leadership; showed that ignoring sequential structure biases fee estimates.
- **Ulph (1996)**: Strategic environmental policy when firms can choose whether to act as leaders or followers; endogenous market structure.
- **Requate (2005)**: Survey of environmental regulation under imperfect competition; discusses sequential models and fee design.
- **Espinola-Arredondo and Muñoz-Garcia (2013)**: Analysis linking output advantage decomposition to optimal fee design in sequential duopolies.

Related Literature: Dynamic and Two-Period Models

- **Kydland and Prescott (1977)**: Seminal paper on time consistency in economic policy; introduced the commitment vs. discretion trade-off.
- **Spulber (1985)**: Dynamic regulation under asymmetric information; showed that commitment can improve welfare when firms' behavior is strategic.
- **Biglaiser and Horowitz (1995)**: Two-period emission fee model with persistent pollutants; characterized how fees should be front- or back-loaded.
- **Petrakis and Xepapadeas (2001)**: Dynamic games of pollution control; analyzed the value of commitment in sequential environmental policy.
- **Benchekroun and Long (2002)**: Differential game approach to pollution control; showed welfare equivalence between open-loop and feedback solutions in some settings.

Section 5.8

Summary

Summary I: Sequential Competition

- **Stackelberg vs. Cournot**: Leader commits to output first; follower observes and responds. Leader gains first-mover advantage ($FMA^{NR} = \frac{1}{4}$) plus cost advantage ($CA^{NR} = \frac{5c}{4}$).
- **Aggregate output**: $Q^{NR}(S) = \frac{3-c}{4} > \frac{2-c}{3} = Q^{NR}(C)$ – Stackelberg generates more total output (and pollution).
- **With regulation**: EA sets optimal fee before firms compete. Fee $t^{Seq} = \frac{(3-c)(3d-1)}{9(1+d)}$ is always *more stringent* than t^C.
- **Differential fee response**: Leader's output is twice as responsive to the fee as the follower's ($|\partial q_1/\partial t| = \frac{1}{2} > |\partial q_2/\partial t| = \frac{1}{4}$).
- **Output advantage**: Regulation reduces the leader's output advantage by $t^{Seq}/4$ – but cannot eliminate it entirely.

Summary II: Two-Period Model and Commitment

- **Two-period model**: Pollution from period 1 persists into period 2 with intensity $\lambda \in [0, 1]$.
- **No commitment**: Both fees exceed t^* when $\lambda > 0$. Fee ranking: $t_2 > t_1$ for low λ; $t_1 > t_2$ for high λ. Welfare decreases in λ.
- **Commitment**: EA announces both fees in advance. $t_1^{Com} > t_1$ (more stringent first-period fee); $t_2^{Com} = t_2$. Fee ranking always: $t_1^{Com} \geq t_2^{Com}$ (front-loaded).
- **Welfare equivalence**: $W^{Com} = W$ – commitment does not improve (or worsen) total welfare when the EA optimizes in both cases.
- **Policy implication**: Regulatory commitment changes the *distribution* of fees across time, but not the overall welfare outcome under optimal policy.

Summary III: Policy Implications

- **Know your market structure**: Sequential and simultaneous competition require different fee levels. Assuming Cournot when the market is Stackelberg leads to *underprovision* of environmental regulation.
- **Fee stringency**: $t^{Seq} > t^{C}$ for all parameter values. The gap widens with cost asymmetry when pollution is severe ($d > \frac{7}{12}$).
- **Distributional effects**: Higher fees disproportionately shrink the leader's output advantage. Environmental regulation has unintended competition effects.
- **Intertemporal regulation**: Persistent pollutants require fees above the static optimum in *both* periods. Fee front- or back-loading depends on persistence λ and pollution severity d.
- **Commitment**: Changes the fee path but not welfare – as long as the EA is welfare-maximizing in either case.

Thank You

Questions and Discussion

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-garcia

Chapter 6

Regulation when Firms Invest in Abatement

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 6

- This chapter extends the baseline model by allowing firms to invest in abatement technologies that reduce emissions.
- Sections:
  - 6.1 Introduction – motivation and guiding questions.
  - 6.2 Modeling assumptions – three-stage game, abatement cost, spillovers.
  - 6.3 Non-cooperative abatement – no spillovers and with spillovers.
  - 6.4 Cooperative abatement – environmental R&D joint ventures (ERJVs).
  - 6.5 Sequential abatement decisions.
  - 6.6 Different regulatory timing – regulator moves first.
  - 6.7 Extensions. 6.8 Case studies. 6.9 Summary.

Section 6.1

Introduction

What this chapter studies

- Previous chapters examined how firms adjust *output* in response to emission fees, taking abatement technology as fixed.
- This chapter introduces a richer environment: firms choose both their output levels **and** how much to invest in abatement technologies.
- Examples of abatement investment: smokestack scrubbers, wastewater treatment systems, advanced filtration, carbon capture equipment.
- Key question: how do abatement investments interact with the regulator's emission fee? Does one "crowd out" the other?
- The baseline model from Chapter 2 is recovered as a special case when abatement is prohibitively expensive ($\gamma \to +\infty$).

Two new strategic layers

- **Firm–regulator interaction:** each firm's abatement investment lowers net emissions, inducing the regulator to set a *less stringent* fee – the **tax-saving effect (TSE)**.
- **Firm–firm interaction:** when abatement generates spillovers, one firm's investment also reduces its rival's net emissions – the **emission-reduction effect (ERE)**.
- These two externalities create **free-riding incentives** and determine whether firms should coordinate their abatement choices.
- This chapter systematically studies TSE and ERE under non-cooperative and cooperative abatement.

Chapter 6 in Context: Extensions of Chapter 2

- **Chapter 2:** Cournot duopoly + emission fee; no abatement investment by firms.
- **Chapter 6 adds:**
  - Abatement investment stage (before fee-setting).
  - Spillovers in abatement (β).
  - Cooperative abatement (ERC/ERJV).
  - Sequential timing in abatement.
  - Alternative regulatory timing (fee before abatement).
- **What changes:**
  - The optimal emission fee is lower (abatement partially corrects externality).
  - Firms' strategies now involve both output and abatement.
  - Free-riding incentives are richer: TSE and ERE.
- Chapter 6 provides a more complete picture of how firms respond to environmental regulation in the presence of clean technology investment.

Guiding Questions

1. What motivates firms to invest in abatement technologies under regulation?
2. How do spillovers in abatement affect strategic and cooperative behavior?
3. What are the welfare implications of environmental research joint ventures (ERJVs)?
4. How does the timing of abatement decisions influence equilibrium outcomes?
5. What is the role of regulatory commitment in shaping investment strategies?
6. How do different regulatory designs affect incentives for green innovation?

Real-World Examples of Abatement Investment

- **Smokestack scrubbers:** power plants install flue-gas desulfurization systems to capture SO_2 before it is emitted – high upfront cost, ongoing operational savings from lower fees.
- **Wastewater treatment:** chemical plants invest in treatment facilities to reduce toxic discharges – cost is sunk after installation (irreversible).
- **Carbon capture:** cement and steel producers pilot CCS (carbon capture and storage) technologies; rivals can observe and learn (spillovers).
- **Key regulatory interaction:** in each case, the regulator observes total industry abatement and adjusts the emission fee accordingly.
- This strategic interaction between abatement investment and fee-setting is the core mechanism analyzed in Chapter 6.

Section 6.2

Modeling Assumptions

Basic Setup

- Duopoly with inverse demand $p(Q) = 1 - Q$ and symmetric marginal cost c, where $1 > c \geq 0$.
- Firms can invest in **abatement technologies** $z_i \geq 0$ to reduce their net emissions.
- Without abatement: $e_i = q_i$ (emissions equal output).
- With abatement and **spillover parameter** $\beta \in [0, 1]$:

$$e_i = q_i - z_i - \beta z_j$$

- Three cases for β:
  - $\beta = 0$: no spillovers – firm i's emissions reduced only by its own investment.
  - $\beta \in (0, 1)$: partial spillovers – firm i benefits from a share β of rival's abatement.
  - $\beta = 1$: full spillovers – $e_i = q_i - z_i - z_j$.

Abatement Cost and Investment Efficiency

- Total cost of investing in abatement:

$$C(z_i) = \frac{\gamma(z_i)^2}{2}$$

  increasing and *convex* in abatement z_i.
- Parameter $\gamma > 0$ denotes **investment efficiency**:
  - Lower $\gamma \Rightarrow$ higher efficiency (abatement cheaper).
  - $\gamma \to 0$: abatement is essentially free.
  - $\gamma \to +\infty$: abatement is prohibitively costly; $z_i = 0$ and results collapse to Chapter 2.
- Marginal cost of abatement: γz_i – positive and increasing in z_i.
- Firm pays emission fee $te_i = t(q_i - z_i - \beta z_j)$ in stage 2.

Three-Stage Game (Poyago-Theotoky, 2007)

- **Stage 1 (Abatement):** Every firm i independently and simultaneously invests in abatement $z_i \geq 0$.
- **Stage 2 (Fee-setting):** The regulator observes investment decisions $z = (z_1, z_2)$ and sets per-unit emission fee $t \geq 0$.
- **Stage 3 (Output):** Every firm i observes z and t, then chooses output q_i (Cournot competition).
- Key feature: firms invest in abatement *before* the regulator sets the fee – strategic anticipation of regulatory response.
- Stages 2–3 coincide with Chapter 2; Stage 1 is new.
- Section 6.6 reverses this timing: regulator moves first.

Welfare Function with Abatement

- Welfare function retains the same structure as in previous chapters:

$$W = CS + PS + T - Env$$

- Aggregate abatement: $Z = z_1 + z_2$; aggregate emissions: $E = Q - Z$.
- Tax revenue: $T = tE = t(Q - Z)$.
- Environmental damage:

$$Env = \frac{d}{2}E^2 = \frac{d}{2}(Q - Z)^2$$

  where $d \geq \frac{1}{2}$ measures **pollution severity**.
- Two channels through which abatement improves welfare:
  - *Direct*: reduces environmental damage for given output.
  - *Indirect*: lowers fee stringency, boosting output, CS and PS.

Backward Induction: Solving the Three-Stage Game

- We solve the game by **backward induction**: start from Stage 3 and work back to Stage 1.
- **Stage 3** $\rightarrow$ output as a function of (t, z_i, z_j).
- **Stage 2** $\rightarrow$ optimal fee as a function of (z_i, z_j).
- **Stage 1** $\rightarrow$ optimal abatement given anticipated fee and output.
- This approach ensures *subgame perfect* equilibrium: each firm's strategy is optimal at every stage, given the strategies of all other players.
- Key insight: firms in Stage 1 anticipate how their abatement affects the regulator's fee in Stage 2, which in turn affects output in Stage 3. This is the strategic "abatement for tax relief" mechanism.

Welfare Components with Abatement

- Welfare $W = CS + PS + T - Env$ with abatement:
  - $CS = \frac{Q^2}{2}$ with $Q = Q(t)$.
  - $PS = \sum_i \pi_i = \sum_i \left[q^2 + tz_i - \frac{\gamma z_i^2}{2}\right]$.
  - $T = tE = t(Q - Z)$ – tax revenue net of abatement.
  - $Env = \frac{d}{2}(Q - Z)^2$ – environmental damage on net emissions.
- Note: abatement cost $\frac{\gamma Z^2}{2}$ is *subtracted* from profits and thus from welfare – abatement is costly but improves the environment.
- Regulator maximizes W choosing t in Stage 2, taking $Z = (z_1 + z_2)$ as given (already chosen in Stage 1).
- The regulator does *not* control abatement directly; only the emission fee.

Section 6.3

Non-Cooperative Abatement

Stage 3: Output Competition (No Spillovers, $\beta = 0$)

- Each firm i chooses q_i to maximize:

$$(1 - q_i - q_j)q_i - cq_i - t(q_i - z_i)$$

- First-order condition yields best response: $q_i(q_j) = \frac{1-c-t}{2} - \frac{1}{2}q_j$.
- Symmetric equilibrium: each firm's output $q = \frac{1-c-t}{3}$, aggregate output $Q(t) = \frac{2(1-c-t)}{3}$.
- Equilibrium profit:

$$\pi_i = \frac{(1 - c - t)^2}{9} + tz_i = q^2 + tz_i$$

- Profit π_i increases in abatement z_i because abatement lowers the fee the firm effectively pays.

Stage 2: Optimal Emission Fee

- Regulator sets t to maximize $W = CS + PS + T - Env$, anticipating equilibrium output $Q(t) = \frac{2(1-c-t)}{3}$.
- First-order condition (differentiating W w.r.t. t) yields:

$$t(Z) = \frac{(2d-1)(1-c) - 3dZ}{2(1+d)}$$

- Fee properties:
  - *Increases* with pollution severity d: $\frac{\partial t(Z)}{\partial d} > 0$ when $Z < 1 - c$.
  - *Decreases* with marginal cost c: $\frac{\partial t(Z)}{\partial c} < 0$.
  - **Decreases with aggregate abatement** Z: $\frac{\partial t(Z)}{\partial z_i} = -\frac{3d}{2(1+d)} < 0$.
- Key insight: when either firm invests more in abatement, the regulator anticipates fewer net emissions and *reduces* the fee.

Free-Riding via the Tax-Saving Effect

- Fee sensitivity is *symmetric*: $\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d}{2(1+d)} < 0$.
- This symmetry allows firm i to **free-ride** on its rival's investment z_j:
    - Firm j invests more in abatement.
    - Regulator reduces emission fee for everyone.
    - Firm i benefits from lower fee *without* investing.
- This **tax-saving effect (TSE)** occurs *with and without* spillovers ($\beta \geq 0$).
- Free-riding arises because every firm's investment reduces *total* emissions, inducing a less stringent fee that benefits all firms.

Stage 1: Abatement Best Response Function

- Each firm i chooses z_i to maximize profit, anticipating equilibrium fee $t(Z)$:

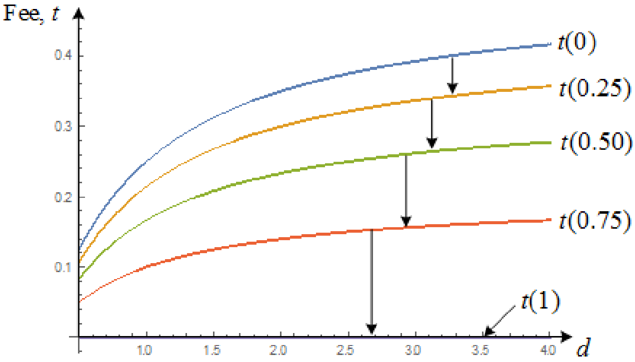
$$\max_{z_i \geq 0} \pi_i = q^2 + t(Z)z_i - \frac{\gamma(z_i)^2}{2}$$

- First-order condition yields best response function:

$$z_i(z_j) = \underbrace{\frac{[2d(1+d)-1](1-c)}{d(6+5d)+2\gamma(1+d)^2}}_{\text{vertical intercept}>0} - \underbrace{\frac{d(3+2d)}{d(6+5d)+2\gamma(1+d)^2}}_{\text{slope}>0} z_j$$

- **Negative slope**: abatement levels are *strategic substitutes*.
- Intuition: rival's abatement $\uparrow \Rightarrow$ fee $\downarrow \Rightarrow$ less incentive for firm i to invest.

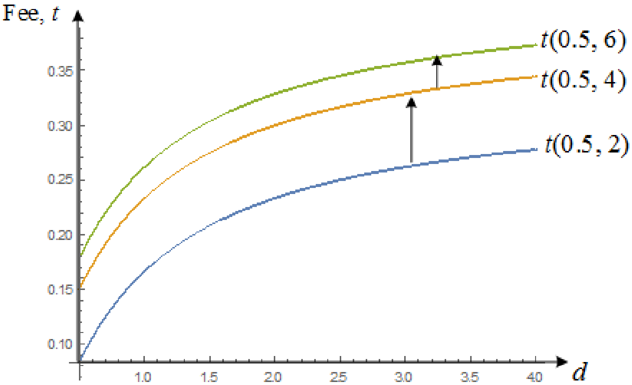
Figure 6.1 – Best Response Function $z_i(z_j)$



Effect of Pollution Severity on Best Response

- The **vertical intercept** shifts *upward* when pollution is more severe (higher d): $\frac{\partial z_i^{Vert.}}{\partial d} > 0$.
- Intuition: higher $d \Rightarrow$ more stringent fee $\Rightarrow$ stronger incentive to invest in abatement to reduce tax burden.
- The **slope** of the best response function becomes steeper (in absolute value) when $\gamma > \bar{\gamma} \equiv \frac{3d^2}{2(1+d)(3+d)}$.
- When abatement is costly (γ large), firms are more sensitive to rival's investment – free-riding incentives are *stronger*.
- Cutoff $\bar{\gamma}$ ranges between $\frac{1}{14}$ (at $d = \frac{1}{2}$) and $\frac{3}{2}$ (as $d \to \infty$).
- When $\gamma > \frac{3}{2}$, condition $\gamma > \bar{\gamma}$ holds for *all* values of d.

Figure 6.2 – Cutoff $\overline{\gamma}$ as a Function of d



Equilibrium Abatement (No Spillovers)

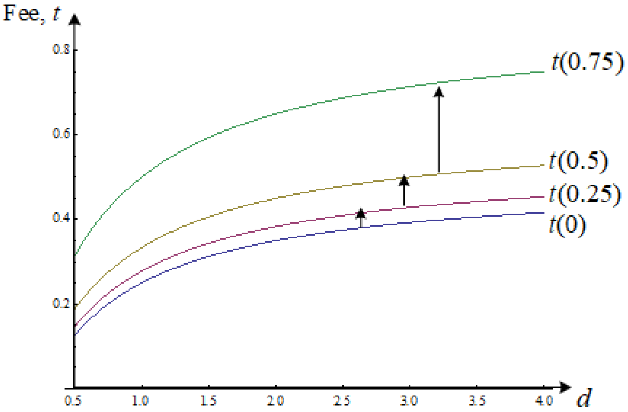
- In symmetric equilibrium ($z_i = z_j$), equilibrium abatement is:

$$z_i^* = \frac{(1-c)[2d(1+d)-1]}{2\gamma(1+d)^2 + d(9+7d)} > 0$$

(positive since $d > \frac{1}{2}$ by assumption).

- **Comparative statics:**
  - $\frac{\partial z_i^*}{\partial d} > 0$: higher pollution severity $\Rightarrow$ more abatement investment.
  - $\frac{\partial z_i^*}{\partial \gamma} < 0$: more expensive abatement $\Rightarrow$ less investment.
  - $\frac{\partial z_i^*}{\partial c} < 0$: higher marginal cost $\Rightarrow$ less output, fewer emissions, less stringent fee, less investment.

Figure 6.3 – Equilibrium Abatement z_i^*



Equilibrium Emission Fee (No Spillovers)

- Substituting $Z^* = 2z_i^*$ into $t(Z)$, equilibrium fee:

$$t(Z^*) = \frac{(1-c)\left[d(2\gamma - 3 + 2d(1+2\gamma)) - 2\gamma\right]}{4\gamma(1+d)^2 + 2d(9+7d)}$$

- Fee is **positive** if and only if:

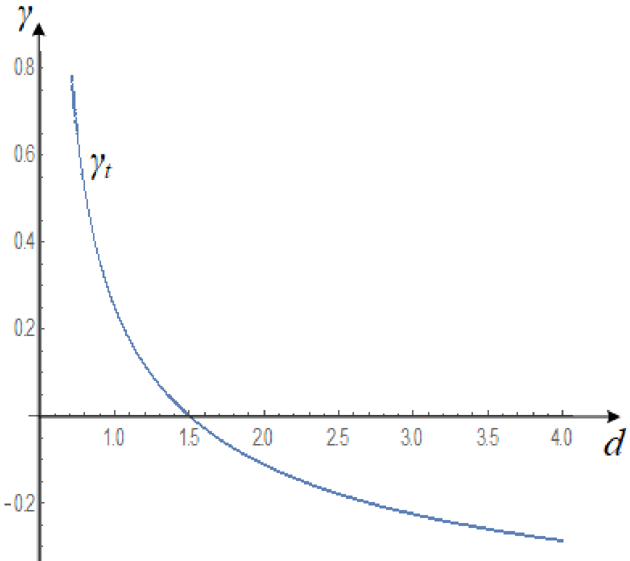
$$\gamma > \gamma_t \equiv \frac{(3-2d)d}{2(2d^2+d-1)}$$

- When $d > \frac{3}{2}$: cutoff $\gamma_t < 0$, so fee is always positive.
- When $d \leq \frac{3}{2}$: abatement must be sufficiently costly ($\gamma > \gamma_t$) for a positive fee.
- When $\gamma \leq \gamma_t$: firms invest heavily in abatement $\Rightarrow$ per-unit *subsidy* is socially optimal (not a fee).

Intuition: Fee vs. Subsidy

- **Why can the optimal instrument be a subsidy?**
    - When abatement is cheap (γ small), firms invest heavily, reducing net emissions substantially.
    - When pollution is also mild ($d \leq \frac{3}{2}$), environmental externality is small relative to the oligopoly distortion (underproduction).
    - Boosting output via a *subsidy* can dominate using an emission *fee*.
- **Key contrast with Chapter 2 (no abatement):**
    - Without abatement: the fee is always positive (emission fee needed to correct the externality).
    - With abatement: abatement itself partially corrects the externality, so the optimal fee can be zero or negative.

Figure 6.4 – Cutoff γ_t as a Function of d



Equilibrium Output (No Spillovers)

- Substituting $t(Z^*)$ into $q(t) = \frac{1-c-t}{3}$:

$$q(t(Z^*)) = \frac{(1-c)\left[d(7+4d)+2\gamma(1+d)\right]}{4\gamma(1+d)^2+2d(9+7d)} > 0$$

- Output is *lower* when pollution is more severe (higher d) or abatement is more expensive (higher γ): the regulator sets a more stringent fee in both cases.
- Comparison with Chapter 2: when $\gamma \to +\infty$, abatement investment is zero and equilibrium output converges to the Chapter 2 result.
- When γ is finite, abatement lowers net emissions $\Rightarrow$ regulator reduces fee $\Rightarrow$ output is *higher* than in Chapter 2.

Section 6.3.2 – Spillovers in Abatement ($\beta > 0$)

- With spillovers, firm i's emissions become:

$$e_i = q_i - z_i - \beta z_j$$

- Sources of spillovers: non-patentable innovations, worker mobility (know-how transfer), geographical proximity (measurement errors).
- **Symmetric spillovers**: model assumes firm i equally benefits from any rival's investment (parameter β is identical for all pairs).
- **Asymmetric spillovers** (richer model): $e_1 = q_1 - z_1 - \beta_1 z_2$ and $e_2 = q_2 - z_2 - \beta_2 z_1$ with $\beta_1 \neq \beta_2$.
- We focus on symmetric case; equilibrium output in Stage 3 is unchanged: $q = \frac{1-c-t}{3}$ (spillovers affect output only indirectly via the fee).

Stage 2: Optimal Fee with Spillovers

- Regulator's optimal fee with spillovers:

$$t(Z) = \frac{(2d-1)(1-c) - 3d(1+\beta)Z}{2(1+d)}$$

- Coincides with equation (6.9) when $\beta = 0$.
- Fee sensitivity to abatement now *amplified* by spillovers:

$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d(1+\beta)}{2(1+d)} < 0$$

- As $\beta \uparrow$: each unit of abatement reduces emissions more (both firms benefit) $\Rightarrow$ regulator reduces fee more aggressively.
- Equilibrium output: $q = \frac{1-c+d(1+\beta)Z}{2(1+d)}$, increasing in aggregate abatement Z.

TSE and ERE: Two Externalities from Rivals' Abatement

- When firm j invests z_j, firm i benefits through two channels:
  1. **Tax-saving effect (TSE):** firm j's investment reduces aggregate emissions $\Rightarrow$ regulator lowers fee $t \Rightarrow$ firm i pays less per unit of net emissions.
  2. **Emission-reduction effect (ERE):** spillovers ($\beta > 0$) directly reduce firm i's per-unit emissions, lowering its tax bill even further.
- TSE is present *regardless* of spillovers ($\beta \geq 0$).
- ERE is present *only when* $\beta > 0$.
- Both effects create incentives for firm i to **free-ride** on firm j's abatement.
- With spillovers, the slope of the best response function is steeper $\Rightarrow$ stronger strategic substitutability.

Equilibrium Abatement with Spillovers (z^{NC})

- In symmetric equilibrium, non-cooperative abatement:

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$$

- Superscript NC = non-cooperative (firms do not coordinate).
- **Effect of spillovers (β):** Stronger spillovers $\uparrow \Rightarrow$ ERE becomes larger $\Rightarrow$ each firm free-rides more $\Rightarrow z^{NC}$ *decreases*.
- **Effect of pollution severity (d):** Higher $d \Rightarrow$ more stringent fee $\Rightarrow$ stronger incentive to invest $\Rightarrow z^{NC}$ *increases* (upward shift).
- **Effect of production cost (c):** Higher $c \Rightarrow$ less output, less stringent fee $\Rightarrow z^{NC}$ *decreases* (downward shift).

Intuition: When Does a Subsidy Replace the Fee?

- The optimal policy instrument is a **per-unit subsidy** (negative fee, $t < 0$) when:

$$\gamma \leq \gamma_t \equiv \frac{(3-2d)d}{2(2d^2+d-1)}$$

- This requires two conditions simultaneously:
  1. Low abatement cost (γ small): firms invest heavily, reducing net emissions substantially.
  2. Mild pollution ($d \leq \frac{3}{2}$): environmental damage is limited, so the externality is small.
- When net emissions are very low, the social cost of the oligopoly output distortion (underproduction) *dominates* the environmental externality.
- A subsidy corrects the oligopoly distortion by boosting output.
- As γ rises: firms invest less $\Rightarrow$ more net emissions $\Rightarrow$ fee becomes positive.

Spillovers: Symmetric vs. Asymmetric

- **Symmetric spillovers** (this chapter): firm i benefits equally from any rival's investment – parameter β is the same for all pairs.

$$e_i = q_i - z_i - \beta z_j$$

- **Asymmetric spillovers** (extended model):

$$e_1 = q_1 - z_1 - \beta_1 z_2, \quad e_2 = q_2 - z_2 - \beta_2 z_1$$

  where $\beta_1 \neq \beta_2$ – firm 1 and firm 2 absorb spillovers differently.
- Possible sources of asymmetry: different geographic proximity, different absorptive capacity, or directional knowledge flows (large firm to small firm).
- Asymmetric spillovers create heterogeneous free-riding incentives – the firm with the higher β free-rides more.
- This chapter maintains the symmetric case for tractability.

Section 6.4

Cooperative Abatement – ERJVs

Motivation for Cooperation in Abatement

- Under non-cooperative abatement, firms ignore positive externalities from rivals' investment $\Rightarrow$ *underinvestment* in abatement.
- Two positive externalities each firm's investment creates for its rival:
  - TSE: lower emission fee benefits all firms.
  - ERE: spillovers directly reduce rival's emissions.
- **Environmental Research Cartel (ERC):** firms coordinate z_i and z_j to maximize *joint* profit, internalizing both externalities.
- **Environmental R&D Joint Venture (ERJV):** ERC with full spillovers ($\beta = 1$), so innovations are fully shared and duplication is avoided.
- Stages 2–3 are unaffected by cooperation: equilibrium output and fee formula remain the same.

Cooperative Abatement: ERC Equilibrium

- In Stage 1, firms jointly choose (z_i, z_j) to maximize $\Pi_i + \Pi_j$.
- First-order conditions internalize the cross-firm effects; ERC equilibrium abatement:

$$z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2 + 2\gamma(1+d)^2} > 0$$

  (positive since $d > 1/2$).
- **Investment comparison** (z^{ERC} vs. z^{NC}):
  - Not always the case that $z^{ERC} > z^{NC}$.
  - The ranking depends on pollution severity d and spillover strength β.
- Full information sharing ($\beta = 1$) characterizes the ERJV.

Investment Comparison: z^{ERC} vs. z^{NC} – No Spillovers

- When $\beta = 0$, only TSE is present (no ERE).
- **TSE can be positive or negative:**
  - When d is small (mild pollution): fees are less stringent, aggregate output is high (inelastic demand region). Firm j's investment $\uparrow \Rightarrow$ lower fee $\Rightarrow$ firm i's output and profit $\uparrow$ (*positive* externality). ERC internalizes this $\Rightarrow z^{ERC} > z^{NC}$.
  - When d is large (severe pollution): fees are stringent, output in elastic region. Lower fee $\Rightarrow$ price falls more than proportionally $\Rightarrow$ rival's profit $\downarrow$ (*negative* externality). ERC internalizes this $\Rightarrow z^{ERC} < z^{NC}$.
- At $d = 2$ (illustrative example): $z^{ERC} = z^{NC}$.

Investment Comparison: With Spillovers ($\beta > 0$)

- When $\beta > 0$, the ERC internalizes *both* TSE and ERE.
- ERE is unambiguously *positive*: rival's abatement directly reduces firm i's emissions $\Rightarrow$ lowers firm i's tax bill.
- When spillovers are *sufficiently strong* ($\beta > 1/3$): positive ERE dominates TSE regardless of $d \Rightarrow z^{ERC} > z^{NC}$ for all values of pollution severity.
- When firms *fully share* innovations ($\beta = 1$, ERJV): $z^{ERC} - z^{NC} > 0$ for all d and is *unaffected* by d.
- Summary of investment ranking:
    - Low β and high d: $z^{ERC} < z^{NC}$ possible.
    - $\beta > 1/3$: $z^{ERC} > z^{NC}$ unambiguously.
    - $\beta = 1$ (ERJV): $z^{ERC} > z^{NC}$ always.

Alternative Cooperative Regimes (Ouchida & Goto, 2016)

- Four institutional regimes (two binary policy choices):

	ERJVs banned ($\beta \neq 1$)	*ERJVs allowed* (
Coordination banned	Model (i): NC	Model (iii): ERJV
Coordination allowed	Model (ii): ERC	Model (iv): ERJV

- **Model (i):** Non-cooperative R&D with generic β – as in Section 6.3.
- **Model (iv):** ERJV cartelization – ERC with $\beta = 1$.
- Welfare ranking depends on pollution severity d and R&D cost γ.

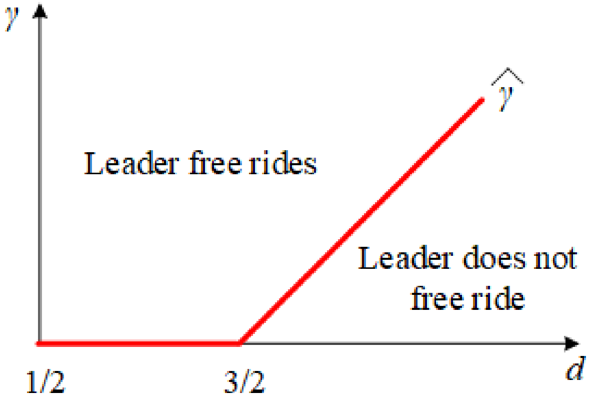
Welfare Ranking of Cooperative Regimes

- **When pollution is mild and R&D costs are moderate:**
  - Independent competition (Model i) is socially optimal.
  - Allowing coordination or full spillovers may cause excessive investment or duplication $\Rightarrow$ lower welfare.
  - Optimal policy: *prohibit* both coordination and ERJVs.
- **When pollution is severe or R&D costs are high:**
  - ERJV cartelization (Model iv) is both privately and socially preferred.
  - Coordinated investment under full spillovers internalizes positive externalities $\Rightarrow$ higher investment and better environmental outcomes.
  - Optimal policy: *allow* ERJV cartelization.
- Firms have strong private incentives to form ERJV cartels when duplication is costly and spillovers are large.

Proposition: Investment Ranking When $\beta = 0$

- When there are no spillovers ($\beta = 0$), only TSE is present.
- **Proposition 6.6.** (ERC vs. NC, no spillovers)
  - When d is small (mild pollution): $z^{ERC} > z^{NC}$ (ERC internalizes a positive externality).
  - When d is large (severe pollution): $z^{ERC} < z^{NC}$ (ERC internalizes a negative externality).
  - At an intermediate value of d: $z^{ERC} = z^{NC}$.
- **Economic intuition (large d case):** with severe pollution, the emission fee is very stringent $\Rightarrow$ output lies in the elastic portion of demand $\Rightarrow$ lower fee causes a large drop in price $\Rightarrow$ rival's profit falls (negative externality of TSE).
- ERC internalizes this *negative* externality by *reducing* investment below the NC level.

Figure 6.5 – z^{ERC} vs. z^{NC} as a Function of β



Welfare Under ERC/ERJV vs. Non-Cooperative Benchmark

- **When $z^{ERC} > z^{NC}$:** cooperation raises abatement.
  - Lower net emissions $\Rightarrow$ reduced environmental damage.
  - Lower fee $\Rightarrow$ higher output and surplus.
  - But higher abatement costs – welfare comparison not trivial.
- **When $z^{ERC} < z^{NC}$:** cooperation reduces abatement.
  - Higher net emissions $\Rightarrow$ more environmental damage.
  - Higher fee $\Rightarrow$ lower output and surplus.
  - Lower abatement cost – ambiguous welfare effect.
- **Ouchida & Goto (2016):** welfare ranking depends on d and γ. ERJV cartelization (Model iv) is welfare-superior when pollution is severe or R&D is costly.
- Regulators should tailor their policy stance (allow/ban ERJVs and coordination) to the specific industry parameters.

Section 6.5

Sequential Abatement Decisions

Motivation: Why Sequential Abatement?

- Baseline model assumed *simultaneous* abatement decisions.
- Many real industries feature **sequential** abatement: firms observe rivals' investment choices before committing their own.
- Relevant for large, capital-intensive environmental investments: new machinery, facility retrofitting, post-disaster clean-up.
- **Real-world example:** Intel (May 2020) pledged a 10% reduction in greenhouse gas emissions by 2030 – first such commitment in chipset industry. AMD responded the following year with a 50% reduction target.
- Sequential structure introduces Stackelberg-like dynamics in abatement: leader invests first; follower best-responds.
- Strandholm et al. (2023) analyze this setting, abstracting from spillovers to isolate the timing effect.

Stages 2–3: Unaffected by Timing

- Equilibrium output in Stage 3:
$$q(t) = \frac{1 - c - t}{3}$$
unchanged from simultaneous setting.
- Regulator's optimal fee in Stage 2:
$$t(Z) = \frac{(2d - 1)(1 - c) - 3d(1 + \beta)Z}{2(1 + d)}$$
also unchanged (coincides with equation 6.24).
- Substituting into $q(t(Z))$:
$$q(t(Z)) = \frac{1 - c + d(1 + \beta)Z}{2(1 + d)},$$
increasing in aggregate abatement Z.
- The key divergence from the simultaneous setting arises in **Stage 1**, where the leader internalizes the follower's reaction.

Follower's Best Response in Sequential Abatement

- Follower (firm 2) observes z_1 and chooses z_2 to maximize:

$$\max_{z_2 \geq 0} \ \pi_2 = \frac{[1 - c - t(Z)]^2}{9} + t(Z)z_2 - \frac{\gamma}{2}(z_2)^2$$

- Follower's best response function:

$$z_2(z_1) = \frac{(1-c)[d(2+2d+\beta)-1]}{2\gamma + dB} - \frac{d[3+d(2-\beta)](1+\beta)}{2\gamma+dB} z_1$$

where $B \equiv 6 + 4\gamma + 6\beta + d[2\gamma + (5-\beta)(1+\beta)]$.

- Slope is **negative**: abatement levels remain *strategic substitutes* under sequential timing.
- Leader anticipates this reaction and chooses z_1^{Seq} accordingly.

Free-Riding by the Leader

- A surprising result: the leader may invest *less* than the follower, $z_1^{Seq} < z_2^{Seq}$, when:

$$\gamma > \hat{\gamma} \equiv \max\left\{0, \frac{2d(1-2\beta)-3}{2}\right\}$$

- When $\gamma < \hat{\gamma}$: abatement is cheap; leader invests aggressively: $z_1^{Seq} > z_2^{Seq}$.
- When $\gamma > \hat{\gamma}$: abatement costs outweigh benefits; leader **free-rides** on the follower: $z_1^{Seq} < z_2^{Seq}$.
- Follower responds by *increasing* its abatement (strategic substitutes).
- Analogy: sequential public good games where early contributors reduce effort, expecting later contributors to compensate.

Role of Pollution Severity in Sequential Abatement

- Higher pollution severity (higher d) $\Rightarrow$ more stringent fee $\Rightarrow$ higher marginal benefit of abatement investment.
- As d increases, the threshold $\hat{\gamma}$ rises: the leader must bear higher costs before free-riding becomes optimal.
- Therefore, **severe pollution reduces free-riding incentives** for the leader.
- Conversely, when pollution is mild, regulatory response is weak $\Rightarrow$ benefit of abatement is low $\Rightarrow$ leader more likely to rely on the follower's investment.
- This underscores the importance of environmental context for determining strategic behavior in sequential investment settings.

Sequential vs. Simultaneous Abatement: Comparison

- **Result** (Strandholm et al., 2023): equilibrium investment in the sequential setting exceeds that in the simultaneous setting:

$$z_i^{Seq} > z_i^{NC}$$

- The difference $z_i^{Seq} - z_i^{NC}$:
  - *Increases* with pollution severity d: stringent fees make abatement more valuable in both settings, but sequential timing amplifies this.
  - *Decreases* with abatement cost γ: costly abatement reduces investment in both settings, compressing the difference.
- First-mover advantage in abatement is strongest when pollution is severe (high d) or abatement is cheap (low γ).
- In the sequential game, the follower invests *less* than in the simultaneous game, compensated by the leader's higher investment.

Dominant Strategies under Pre-Commitment Regulation

- When the regulator sets the fee *before* firms invest in abatement (regulatory commitment), strategic interaction in abatement *disappears*.
- With a fixed fee t, each firm i chooses z_i to solve:

$$\max_{z_i \geq 0} \ \pi_i = q^2 + tz_i - \frac{\gamma}{2}(z_i)^2$$

- First-order condition: $t = \gamma z_i$, yielding:

$$z_i(t) = \frac{t}{\gamma}$$

- This optimal abatement is *independent* of rival's investment $z_j \Rightarrow$ abatement is a **strictly dominant strategy**.
- Sequential vs. simultaneous timing no longer matters when the fee is pre-committed.

Cutoff $\hat{\gamma}$: When Does the Leader Free-Ride?

- The leader free-rides ($z_1^{Seq} < z_2^{Seq}$) when:

$$\gamma > \hat{\gamma} \equiv \max\left\{0, \frac{2d(1-2\beta)-3}{2}\right\}$$

- **Special case $\beta = 0$:** $\hat{\gamma} = \max\left\{0, \frac{2d-3}{2}\right\}$.
  - $\hat{\gamma} = 0$ when $d \leq \frac{3}{2}$.
  - $\hat{\gamma} > 0$ when $d > \frac{3}{2}$ (more severe pollution).
- **Effect of spillovers (β):** higher β lowers the threshold $\hat{\gamma}$ – ERE makes it *easier* for the leader to free-ride (more benefit from rival's investment without own cost).
- At $\beta = \frac{1}{2}$: $\hat{\gamma} = \max\{0, -3/2\} = 0$, so the leader *always* free-rides regardless of d.
- At $\beta \geq \frac{1}{2}$: $\hat{\gamma} = 0$ for all d, meaning free-riding is a dominant strategy for the leader.

Worked Example: Sequential vs. Simultaneous ($\beta = 0$, $d = 2$)

- Parameters: $c = \frac{1}{2}$, $d = 2$, $\gamma = 1$, $\beta = 0$.
- **Simultaneous (NC):** Equilibrium abatement:

$$z^{NC} = \frac{(1 - \frac{1}{2})[2 \cdot 2 \cdot 3 - 1]}{2 \cdot 1 \cdot 9 + 2(18 + 14)} = \frac{\frac{1}{2} \cdot 11}{18 + 64} \approx \frac{5.5}{82} \approx 0.067$$

- **Sequential:** Leader anticipates follower's reaction; total abatement $Z^{Seq} > Z^{NC}$.
- Intuition: leader commits to investment, follower reduces investment in response, but total investment rises because leader's commitment is larger than the follower's reduction.
- This mirrors the Stackelberg output model, but for abatement instead of production quantities.
- Both firms invest more than under NC: $z_i^{Seq} > z_i^{NC}$ for each firm i.

Section 6.6

Different Regulatory Timing

Regulatory Commitment: Motivation

- Lambertini et al. (2017) consider a reversed timing:
  1. **Stage 1**: Regulator commits to an emission fee t.
  2. **Stage 2**: Firms independently choose abatement investments z_i.
  3. **Stage 3**: Firms compete à la Cournot in output markets.
- **Relevant when:** firms can easily revise abatement (scrubbers, pipe filters), but regulatory parameters are fixed by legislative or institutional constraints.
- **Contrast with Poyago-Theotoky (2007):** appropriate when investments are large-scale and irreversible (e.g., factory retrofits), while regulators can adjust policy in response to observed behavior.
- Lambertini et al. also allow for $n \geq 2$ firms to study how market structure affects aggregate abatement.

Stage 3: Output and Stage 2: Dominant-Strategy Abatement

- **Stage 3 (output, n firms):** Equilibrium output per firm:
$$q_i(t) = \frac{1-c-t}{n+1}$$
Aggregate output $Q(t) = \frac{n(1-c-t)}{n+1}$, approaching $1-c-t$ as $n \to \infty$ (perfect competition).
- **Stage 2 (abatement):** With fixed fee t, first-order condition: $t = \gamma z_i$, yielding:
$$z_i(t) = \frac{t}{\gamma}$$
a *strictly dominant strategy* (independent of rivals' choices).
- Aggregate abatement: $Z(t) = \frac{nt}{\gamma}$ – increases with n and t; decreases with γ.
- Arrowian result: with exogenous t, increased competition $\uparrow n$ raises aggregate R&D unambiguously.

Stage 1: Regulator Sets the Optimal Fee

- Regulator maximizes social welfare $W(t)$ (with $d = 2$):

$$W(t) = \frac{1}{2}[Q(t)]^2 + n\pi_i(t) - n^2[e_i(t)]^2$$

- Optimal emission fee (function of n and β):

$$t(n) = \frac{\gamma[2n(n+1)(1+\beta(n-1)) + \gamma(n-2)](1-c)}{4\gamma n(n+1)[1+\beta(n-1)] + \gamma^2(n-2) - A(\gamma)}$$

  where
  $A(\gamma) \equiv (n+1)^2[\gamma(1+2\beta(n-1)) - 2n(1+\beta(n-1))^2]$.

- Equilibrium abatement: $z_i(n) = \frac{t(n)}{\gamma}$.
- When the fee is endogenous, the relationship between n and aggregate abatement becomes non-trivial – depends on spillovers.

Market Concentration and Abatement – No Spillovers ($\beta = 0$)

- With $\beta = 0$, $c = 0$, $\gamma = 5$, aggregate abatement evaluates to:

$$Z(n) = \frac{n(2n^2 + 7n - 10)}{2n^3 + 19n^2 + 37n - 55}$$

- Derivative $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$: aggregate abatement **increases monotonically** with the number of firms.
- Intuition: without spillovers (no ERE), each firm's abatement reduces *only its own* tax burden $\Rightarrow$ each new entrant independently invests to reduce its tax $\Rightarrow$ total investment grows.
- Consistent with the **Arrowian prediction**: increased competition fosters more innovation when spillovers are absent.
- Note: TSE is also absent here (fee is pre-committed, not responsive to abatement).

Market Concentration and Abatement – With Spillovers ($\beta = 1/2$)

- With $\beta = 1/2$, aggregate abatement:

$$Z(n) = \frac{2n^4 + 4n^3 + 12n^2 - 20n}{n^5 + 4n^4 + 16n^3 + 24n^2 + 61n - 100}$$

- Derivative $\frac{\partial Z(n)}{\partial n}$ is positive for $n < \bar{n} = 3.432$ but negative for $n > \bar{n}$.
- **Inverted U-shape**: aggregate abatement first rises, reaches a peak near $n \approx 3$, then falls with additional firms.
- Intuition: with spillovers, firms free-ride on rivals' abatement via ERE. As competition intensifies, free-riding dominates $\Rightarrow$ individual investment falls $\Rightarrow$ total abatement eventually declines.
- Policy implication: when spillovers are present, *more competition is not always better* for environmental outcomes.

Comparing Poyago-Theotoky vs. Lambertini: TSE Absent

- **Key difference from Poyago-Theotoky (2007):**
  - In Poyago-Theotoky, the regulator *responds* to observed abatement $\Rightarrow$ investing more lowers the fee (TSE is present).
  - In Lambertini et al., the fee is *pre-committed* $\Rightarrow$ abatement does not affect the fee (TSE is absent).
- With regulatory commitment: firms' abatement decisions are purely driven by saving taxes (at the given fixed t), not by anticipating a softer regulatory response.
- ERE still arises when $\beta > 0$: rivals' abatement reduces firm i's emissions $\Rightarrow$ free-riding via ERE.
- An ERC could still help internalize ERE, but the urgency is reduced relative to the Poyago-Theotoky setting (since TSE is absent).

Poyago-Theotoky (2007) vs. Lambertini et al. (2017): A Comparison

- **Poyago-Theotoky (2007):**
  - Timing: Abatement $\rightarrow$ Fee $\rightarrow$ Output.
  - Fee responds to observed abatement (TSE present).
  - Strategic abatement decisions: firms anticipate regulatory response.
  - Best for: irreversible capital investments; flexible regulation.
- **Lambertini et al. (2017):**
  - Timing: Fee $\rightarrow$ Abatement $\rightarrow$ Output.
  - Fee is pre-committed (TSE absent).
  - Abatement is a dominant strategy: $z_i = t/\gamma$.
  - Best for: modular/reversible abatement; rigid legislative fee.
- **Common finding:** when spillovers are present ($\beta > 0$), free-riding via ERE reduces individual abatement in both frameworks.
- **Key difference:** strategic interaction in abatement only arises in Poyago-Theotoky's framework.

Worked Example: Dominant-Strategy Abatement

- Parameters: $n = 2$, $c = 0$, $\gamma = 5$, $\beta = 0$, $d = 2$.
- Regulator sets $t(n = 2)$ using equation (6.47).
- Given t, each firm independently chooses:

$$z_i = \frac{t}{\gamma} = \frac{t}{5}$$

- Firm i does not consider firm j's abatement when making this decision – it is a dominant strategy.
- Aggregate abatement: $Z = 2 \times \frac{t}{5} = \frac{2t}{5}$.
- As n increases (more firms), $t(n)$ adjusts; aggregate abatement $Z(n) = \frac{n \cdot t(n)}{5}$ first rises then falls (inverted U-shape when $\beta > 0$).
- This illustrates how the Arrowian result holds or fails depending on spillovers.

Section 6.7

Extensions

Extension 1: Endogenous Entry (Katsoulacos & Xepapadeas, 1995)

- While most models assume an exogenous number of firms n, some settings feature **endogenous entry**.
- Framework: regulator sets fee first; firms decide whether to enter; then choose output and abatement.
- Key finding: equilibrium emission fee may *exceed* marginal environmental damage – **"overinternalization"** of externalities.
- Mechanism: a higher fee deters entry, which can be socially beneficial when entry would be excessive.
- Optimal two-instrument policy:
  - An **entry fee** to limit socially excessive entry.
  - An **emission fee set below marginal damages** to avoid overinternalization.
- This combination yields higher welfare than a single emission fee alone.

Extension 2: Subsidies and Regulatory Design (Ouchida & Goto, 2014)

- Extends Poyago-Theotoky (2007) to analyze the role of **output subsidies**.
- Subsidies can be *welfare-improving* when pollution is mild (low d): the oligopoly distortion (underproduction) dominates the environmental externality $\Rightarrow$ optimal to subsidize output ($t < 0$).
- When R&D costs are high (γ large):
    - Firms invest minimally in abatement.
    - Regulation may paradoxically yield *higher* pollution than laissez-faire (firms respond to fee by cutting output more than the abatement saves).
    - Nonetheless, regulation can still improve welfare.
- When R&D is inexpensive (low γ): firms invest heavily $\Rightarrow$ lower emissions than under laissez-faire.

Extension 3: Patenting vs. Licensing Green Technologies

- **Biglaiser & Horowitz (1995):** four-stage model – R&D investment, fee determination, technology adoption (patent or license), output choice.
- Firms choose between *exclusive use* of an innovation (patent) and *licensing* it to competitors.
- Trade-off: exclusive use provides competitive advantage; licensing generates revenue and may raise rivals' abatement.
- **Denicolo (1999):** single innovating firm decides whether to retain or license technology.
  - Compares two regulatory regimes: fee set before vs. after R&D.
  - Timing of fee-setting significantly affects licensing incentives and welfare.
- **Key insight**: the timing of regulatory intervention shapes both the direction and magnitude of technology diffusion.

Extension 4: Product Differentiation and Mixed Oligopolies

- **Poyago-Theotoky & Teerasuwannajak (2002):** horizontal product differentiation alters regulatory outcomes.
  - High differentiation: pre-commitment fee is more stringent; abatement is higher under time-consistent regulation.
  - Homogeneous products: pre-commitment regulation yields higher welfare.
- **Mixed oligopolies** (Bárcena-Ruiz & Garzón, 2006; Haruna & Goel, 2009):
  - A public firm competes with private firms, maximizing a weighted combination of profit and social welfare (including environmental damages).
  - Public firm partially internalizes externalities $\Rightarrow$ improves overall welfare.
  - Privatization can be welfare-reducing when emission taxes are present.
  - Pal & Saha (2014): first-best outcomes achievable via

Extension 5: Two-Period Models with Stock Pollution

- Some pollutants accumulate over time (stock pollution), so abatement in period 1 affects environmental damage in period 2.
- In two-period models with abatement investment:
  - Period 1 abatement reduces both current and future emissions.
  - Firms can invest in durable abatement capital that depreciates over time.
  - Regulator must set fees in both periods, potentially adjusting for the stock of pollution.
- **Intertemporal trade-off:** early abatement investment is more valuable when the pollution stock is large or when abatement capital is long-lived.
- Dynamic versions of ERJVs: firms may form longer-term cooperation agreements that govern both abatement investment and spillover sharing over multiple periods.
- This connects Chapter 6 to the dynamic models covered in Chapter 5.

Summary of Extensions (6.7)

- **6.7.1 Endogenous entry** (Katsoulacos & Xepapadeas, 1995): fee may exceed marginal damage; two-instrument policy (entry fee + emission fee) is optimal.
- **6.7.2 Subsidies** (Ouchida & Goto, 2014): subsidies welfare-improving when pollution is mild; costly R&D may paradoxically worsen outcomes.
- **6.7.3 Patents vs. licensing** (Biglaiser & Horowitz, 1995; Denicolo, 1999): timing of regulatory commitment affects technology diffusion and licensing incentives.
- **6.7.4 Product differentiation** (Poyago-Theotoky & Teerasuwannajak, 2002): differentiation reverses the ranking of commitment vs. time-consistent regulation.
- **6.7.5 Mixed oligopolies** (Bárcena-Ruiz & Garzón, 2006): privatization can reduce welfare; first-best may require two instruments.

Section 6.8

Case Studies

Case Study 1: Simultaneous Abatement – HP and Dell

- **HP and Dell** both invested heavily in reducing emissions from manufacturing processes and product lifecycles.
- **HP (2007):** among the first to publicly report greenhouse gas emissions; launched recycling and toxic substance reduction programs.
- **Dell:** followed with closed-loop recycling and energy-efficient product design – simultaneous and parallel efforts.
- Investments driven by emission-related compliance costs and competitive sustainability benchmarks.
- **European automakers** (Volkswagen, BMW, Stellantis): simultaneously committed billions to electrification and battery innovation under tightening EU emission standards.
  - Volkswagen: freed €15 bn annually for EV R&D (Dec. 2024).
  - Stellantis: pledged €30 bn for electrification by 2025.

Case Study 2: ERJVs – Canada's Oil Sands Innovation Alliance

- **Canada's Oil Sands Innovation Alliance (COSIA):** formed by major oil sands producers (Suncor, Cenovus, Canadian Natural Resources) to *jointly invest* in environmental R&D.
- Focus: reducing GHG emissions, improving water use, land reclamation.
- Firms pool resources and share technology – a real-world ERJV: coordinates abatement to meet regulatory requirements and lower per-unit emission costs.
- **Japanese refining sector:** JXTG Nippon Oil & Energy and Idemitsu coordinate green R&D for emission reductions in refining operations (scrubbers, catalytic converters).
- These cases illustrate how ERJVs internalize spillovers and avoid duplicative investment.

Case Study 3: Sequential Abatement – Manufacturing and US Refining

- **Cross-country manufacturing study (Brown et al., 2022):** 33,500 firms; high-emission sectors with strong knowledge spillovers (cement, mineral products).
  - Early adopters invested in cleaner production technologies after introduction of emission taxes.
  - Leaders used R&D to retool existing processes; followers later adopted similar technologies to remain cost-competitive.
- **US refining industry:** Chevron and ExxonMobil act as leaders in green investment.
  - Under California's carbon pricing, they made high-profile investments in carbon capture and low-carbon fuels.
  - Smaller refiners followed when strategies proved cost-effective under the emission fee.
- Consistent with the sequential abatement model: leaders invest first, followers respond.

Case Studies: Connecting Theory to Practice

- **Three case types in Chapter 6:**
  1. **Simultaneous abatement** (Section 6.3): HP, Dell, European automakers – firms independently invest in parallel.
  2. **ERJVs** (Section 6.4): Canada's COSIA, Japanese refining associations – firms pool investment and share technology.
  3. **Sequential abatement** (Section 6.5): Intel/AMD in chipsets, Chevron/ExxonMobil in US refining – early movers signal, followers adapt.
- **Common theme:** in all cases, abatement investment is triggered by regulatory pressure (emission fees, carbon pricing, EU standards) and shaped by competitive and spillover dynamics.
- The theoretical predictions of Chapter 6 – free-riding, strategic substitutability, ERJVs internalizing spillovers – are all present in these real-world settings.

Section 6.9

Summary

Looking Back: Connecting Chapter 6 to Chapter 2

- **Special case:** when $\gamma \to +\infty$, abatement is prohibitively expensive; all Chapter 6 results reduce to Chapter 2.
- **Emission fee:**
  - Chapter 2: $t^{Ch2} = \frac{(2d-1)(1-c)}{2(1+d)}$.
  - Chapter 6: $t(Z^*) < t^{Ch2}$ when γ is finite.
- **Output:**
  - Chapter 6 output is higher than Chapter 2 (lower fee).
- **Net emissions:**
  - Can be lower in Chapter 6 if abatement is large enough.
- **Welfare:**
  - Chapter 6 welfare $\geq$ Chapter 2 welfare: firms have an additional tool (abatement) to improve outcomes.
- Adding abatement expands the policy-relevant parameter space significantly.

Chapter 6 Road Map: How Sections Connect

- **Section 6.2:** lays the foundations – game structure, emission function, welfare.
- **Section 6.3:** non-cooperative abatement as the baseline for comparison.
  - 6.3.1: no spillovers (identifies TSE).
  - 6.3.2: with spillovers (adds ERE, strengthens free-riding).
- **Section 6.4:** cooperative abatement (ERC/ERJV) internalizes TSE and ERE.
- **Section 6.5:** sequential timing raises total abatement; free-riding by leader possible.
- **Section 6.6:** regulatory commitment – TSE disappears; dominant-strategy abatement; inverted U-shape in n.
- **Section 6.7:** extensions (entry, subsidies, patents, differentiation, mixed oligopoly).
- **Sections 6.8–6.9:** case studies and summary.

Summary: Modeling Framework

- Chapter 6 introduces a **three-stage game**: firms invest in abatement (Stage 1), regulator sets emission fee (Stage 2), firms choose output (Stage 3).
- Abatement reduces net emissions and influences regulatory behavior, creating strategic incentives for investment.
- Two key parameters:
  - γ: investment efficiency (lower $\gamma \Rightarrow$ more abatement).
  - β: spillover strength (higher $\beta \Rightarrow$ more free-riding).
- Chapter 2 is a special case of this model when $\gamma \to +\infty$ (no abatement).
- Solving by backward induction yields equilibrium abatement, fee, output, and welfare.

Proposition: Fee Decreases with Aggregate Abatement (General β)

- **Proposition 6.7.** With spillovers ($\beta \geq 0$), the optimal emission fee satisfies:
$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d(1+\beta)}{2(1+d)} < 0$$
- The magnitude of the fee reduction per unit of abatement is *increasing* in β: when spillovers are stronger, a given investment has a greater emission-reduction effect $\Rightarrow$ regulator reduces fee more.
- This reinforces free-riding: both TSE and ERE provide incentives to let the rival invest while benefiting from their abatement.
- **Welfare implication:** the regulator accommodates firms' abatement by setting a softer fee, which boosts consumer surplus and producer surplus at the cost of potentially

Summary: Non-Cooperative Abatement

- Firms invest independently, anticipating that higher investment leads to lower emission fees (**TSE**).
- Best response functions for abatement are *downward-sloping* $\Rightarrow$ abatement is a **strategic substitute**.
- Equilibrium abatement:
  - *Increases* with pollution severity d.
  - *Decreases* with abatement cost γ and production cost c.
- Optimal emission fee *decreases* with aggregate abatement.
- Fee can be negative (subsidy optimal) when abatement is cheap and pollution is mild.
- With spillovers ($\beta > 0$): ERE amplifies free-riding, reducing z^{NC}.

Key Formula Reference: Chapter 6 Equilibrium Objects

- **Emission function:** $e_i = q_i - z_i - \beta z_j, \quad \beta \in [0, 1]$.
- **Abatement cost:** $C(z_i) = \frac{\gamma z_i^2}{2}$.
- **Stage 3 output:** $q = \frac{1-c-t}{3}$.
- **Stage 2 fee:** $t(Z) = \frac{(2d-1)(1-c)-3d(1+\beta)Z}{2(1+d)}$.
- **NC abatement:** $z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$.
- **ERC abatement:** $z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2+2\gamma(1+d)^2}$.
- **Commitment abatement:** $z_i(t) = \frac{t}{\gamma}$.
- **Aggregate abatement (n firms, commitment):** $Z(t) = \frac{nt}{\gamma}$.

Summary: Cooperative Abatement and ERJVs

- Firms coordinate abatement via ERC (generic β) or ERJV ($\beta = 1$, full information sharing).
- Cooperation internalizes positive externalities from spillovers (ERE) and tax savings (TSE).
- Investment ranking:
  - $z^{ERC} > z^{NC}$ when spillovers are strong ($\beta > 1/3$) or pollution is mild.
  - $z^{ERC} < z^{NC}$ when pollution is severe and spillovers are weak (β small).
  - $\beta = 1$ (ERJV): $z^{ERC} > z^{NC}$ always.
- Welfare ranking of cooperative regimes depends on d and γ: ERJV cartelization is preferred when pollution is severe or R&D is costly.

Looking Ahead: Chapters 7 and Beyond

- Chapter 6 established that abatement investment is a strategic tool: firms invest to reduce their tax burden (TSE) and benefit from rivals' abatement (ERE).
- **Future topics extend these insights:**
  - **Transboundary pollution:** when pollution crosses borders, national regulators must coordinate fee-setting. Abatement investment becomes a strategic tool in international environmental agreements.
  - **International trade:** firms competing in export markets face regulatory differences across countries; abatement investment affects competitiveness.
  - **Dynamic models:** firms invest in abatement capital over multiple periods; long-term incentives differ from static analysis.
- The three-stage game structure of Chapter 6 – investment, fee-setting, output – is a building block for these richer models.

Summary: Sequential and Regulatory-Commitment Models

- **Sequential abatement** (Strandholm et al., 2023):
  - $z_i^{Seq} > z_i^{NC}$: sequential timing raises total abatement.
  - Leader may free-ride when $\gamma > \hat{\gamma}$.
  - Severe pollution reduces free-riding incentives for the leader.
- **Regulatory commitment** (Lambertini et al., 2017):
  - Abatement is a strictly dominant strategy: $z_i(t) = t/\gamma$.
  - TSE is absent (fee is pre-committed).
  - Without spillovers: aggregate abatement increases monotonically in n (Arrowian result).
  - With spillovers ($\beta > 0$): inverted U-shape in n; free-riding via ERE eventually dominates.
- **Timing matters**: whether the regulator commits before or after abatement investment fundamentally changes firms' strategic incentives.

Worked Example: Equilibrium with $c = \frac{1}{2}$, $d = 1$, $\gamma = 2$

- Parameter values: $c = \frac{1}{2}$, $d = 1$, $\gamma = 2$, $\beta = 0$.
- Best response function: $z_i(z_j) = \frac{1}{18} - \frac{5}{27} z_j$.
- Symmetric equilibrium abatement:

$$z_i^* = \frac{(1 - \frac{1}{2})[2 \cdot 1 \cdot 2 - 1]}{2 \cdot 2 \cdot 4 + 1 \cdot 16} = \frac{\frac{1}{2} \cdot 3}{16 + 16} = \frac{3/2}{32} = \frac{3}{64}$$

- Aggregate abatement: $Z^* = 2 \cdot \frac{3}{64} = \frac{3}{32}$.
- Equilibrium fee: $t(Z^*) = \frac{(2 \cdot 1 - 1)(1 - \frac{1}{2}) - 3 \cdot 1 \cdot \frac{3}{32}}{2(1+1)} = \frac{\frac{1}{2} - \frac{9}{32}}{4} = \frac{\frac{7}{32}}{4} = \frac{7}{128}$.
- Compared to Chapter 2 (no abatement): fee is *lower*, output is *higher*.

Proposition: Abatement Reduces the Emission Fee

- **Proposition 6.1.** In the three-stage game without spillovers ($\beta = 0$), the optimal emission fee $t(Z)$ satisfies:

$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d}{2(1+d)} < 0$$

- That is, **any increase in abatement – by either firm – reduces the emission fee**.
- **Intuition:** higher abatement $\Rightarrow$ lower net emissions $\Rightarrow$ less need for a stringent fee to correct the environmental externality.
- **Corollary:** firms can lower their tax burden by investing in abatement – this is the tax-saving effect (TSE).
- The TSE gives firms a positive private return to abatement, even without environmental altruism.

Proposition: Equilibrium Abatement Increases in d

- **Proposition 6.2.** In the non-cooperative equilibrium without spillovers, $\frac{\partial z_i^*}{\partial d} > 0$.
- **Proof sketch:**
  - Higher $d \Rightarrow$ higher fee $t(Z)$ for any given Z.
  - Higher fee increases marginal benefit of abatement investment (saves more taxes per unit).
  - Firms respond by investing more in abatement.
- **Proposition 6.3.** $\frac{\partial z_i^*}{\partial \gamma} < 0$ and $\frac{\partial z_i^*}{\partial c} < 0$.
  - Higher abatement cost (γ): direct reduction in investment.
  - Higher marginal cost (c): less output $\Rightarrow$ fewer emissions $\Rightarrow$ less stringent fee $\Rightarrow$ lower marginal benefit of abatement.

Intuition: Strategic Substitutability in Abatement

- **Why are abatement decisions strategic substitutes?**
- When firm j invests more: $z_j \uparrow$
  1. Total abatement $Z = z_i + z_j$ increases.
  2. Regulator anticipates lower net emissions $\Rightarrow$ reduces fee $t(Z)$.
  3. Lower fee reduces marginal benefit of firm i's abatement (tax saved per unit of abatement falls).
  4. Firm i responds by investing less: $z_i \downarrow$.
- **Contrast with R&D in cost-reduction models:** in those models, rival's cost-reducing R&D can make your own investment more or less valuable depending on spillovers. Here, the fee mechanism creates a strong substitutability channel even without direct spillovers.
- The negative slope of $z_i(z_j)$ is the graphical representation of this substitutability.

Comparison: With vs. Without Abatement Investment

- **Chapter 2 (no abatement, $\gamma \to \infty$):**
  - Fee: $t^{Ch2} = \frac{(2d-1)(1-c)}{2(1+d)}$ (does not depend on any abatement).
  - Output: $q^{Ch2} = \frac{1-c-t^{Ch2}}{3}$.
- **Chapter 6 (with abatement, finite γ):**
  - Fee is lower: $t(Z^*) < t^{Ch2}$ (abatement reduces net emissions, reducing fee stringency).
  - Output is higher: $q(t(Z^*)) > q^{Ch2}$ (lower fee boosts output).
  - Net emissions can be lower or higher depending on relative magnitude of output increase vs. abatement.
- **Key insight:** abatement investment creates a "double dividend" – lower fees *and* lower net emissions – when abatement is sufficiently efficient (low γ).

Section 6.3.2 – Effect of Spillovers on Equilibrium Abatement

- Non-cooperative equilibrium abatement with spillovers:

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$$

- **Numerator analysis:** $d(2+\beta+2d)-1$ is increasing in $\beta \Rightarrow$ numerator rises with β.
- **Denominator analysis:** $d(1+\beta)[9+7d+\beta(3+d)]$ also increases in β, and by more.
- Net effect: z^{NC} **decreases** in β – stronger spillovers $\Rightarrow$ more free-riding $\Rightarrow$ less investment.
- Graphically: z^{NC} as a function of β slopes downward; the curve shifts up when d increases (pollution severity raises incentives to invest).

Worked Example: Spillovers and Free-Riding

- Parameters: $c = 1/2$, $\gamma = 1/4$, $d = 1$, and $\beta = 0$ vs. $\beta = 1$.
- **With no spillovers** ($\beta = 0$): only TSE is present.
  - Non-cooperative abatement: $z^{NC}(\beta = 0) > 0$.
  - Each firm invests to save on the emission fee.
- **With full spillovers** ($\beta = 1$): both TSE and ERE present.
  - ERE: each unit of firm j's abatement directly reduces firm i's emissions.
  - Firm i can reduce its tax bill *without investing* by benefiting from firm j's abatement.
  - Result: $z^{NC}(\beta = 1) < z^{NC}(\beta = 0)$ – free-riding is stronger.
- **Policy lesson:** when spillovers are strong, non-cooperative abatement significantly undershoots the social optimum.

ERC Joint Profit Maximization: First-Order Conditions

- In the ERC, firms choose (z_i, z_j) to maximize:

$$\max_{z_i, z_j \geq 0} \ \Pi_i + \Pi_j$$

- Differentiating w.r.t. z_i yields the first-order condition – now accounting for the effect of z_i on *both* firm i's and firm j's profit.
- Specifically, the ERC's FOC includes two extra terms relative to the NC problem:
  1. Effect of z_i on firm j's profit via TSE: $\frac{\partial \Pi_j}{\partial t} \cdot \frac{\partial t}{\partial z_i} > 0$ (less stringent fee raises rival's profit too).
  2. Effect of z_i on firm j's profit via ERE (if $\beta > 0$): direct emission reduction for firm j.
- Internalizing these effects raises the ERC's optimal abatement above the NC level (when externalities are positive).

Proposition: Investment Ranking under ERJV ($\beta = 1$)

- **Proposition 6.4.** When $\beta = 1$ (full spillovers, ERJV): $z^{ERC} > z^{NC}$ for all values of pollution severity d and abatement cost γ.
- **Proof sketch:**
  - With $\beta = 1$, ERE is maximal: each unit of rival's investment fully reduces firm i's emissions.
  - The positive ERE externality dominates any negative TSE effect.
  - ERC internalizes both externalities and always chooses higher investment than the NC benchmark.
- **Implication:** when innovation is fully shared, coordination is always socially beneficial in terms of abatement investment.
- At $\beta = 1$, the difference $z^{ERC} - z^{NC}$ is *independent* of d (all pollution severity curves coincide at the right-hand side of Figure 6.8).

Section 6.5 – Intuition: Leader's Investment Decision

- In the sequential setting, the leader (firm 1) solves a more complex problem than in the NC case.
- Leader's optimization incorporates the follower's best response $z_2(z_1)$:

$$\max_{z_1 \geq 0} \ \pi_1(z_1, z_2(z_1))$$

- The leader accounts for two effects of increasing z_1:
  1. *Direct effect*: lower fee $\Rightarrow$ lower tax burden (TSE, positive).
  2. *Strategic effect*: follower reduces z_2 in response (strategic substitutes). This lowers Z slightly, potentially softening the fee reduction.
- The net incentive depends on the cost of investment γ: when γ is low, direct benefit dominates and leader invests heavily; when γ is high, free-riding off the follower is preferred.

Proposition: Sequential vs. Simultaneous Abatement

- **Proposition 6.5** (Strandholm et al., 2023). In equilibrium, each firm's abatement in the sequential setting exceeds that in the simultaneous setting:

$$z_i^{Seq} > z_i^{NC}$$

- Mechanism: the leader, by committing to an investment level, moves the equilibrium to a point where total abatement is higher.
- When $\gamma < \hat{\gamma}$: leader invests more ($z_1^{Seq} > z_2^{Seq}$); both firms invest more than in NC.
- When $\gamma > \hat{\gamma}$: leader free-rides, but follower compensates by investing more; total abatement still exceeds NC level.
- The efficiency gain from sequential timing is largest when d is high (stringent fees make abatement very valuable) or γ is low (cheap to invest).

Section 6.6 – Equilibrium Emissions under Regulatory Commitment

- Each firm's net emissions under regulatory commitment (with n firms):

$$e_i(t) = \frac{1-c-t}{n+1} - \frac{t[1+\beta(n-1)]}{\gamma}$$

- First term: output net of fee reduction.
- Second term: abatement by firm i plus spillovers from rivals.
- Equilibrium profits:

$$\pi_i(t) = \left(\frac{1-c-t}{n+1}\right)^2 + \frac{t^2[1+2\beta(n-1)]}{2\gamma}$$

- Profit increases in the emission fee t! (through the abatement savings term – firms benefit from the "forced" tax-saving through optimal abatement $z_i = t/\gamma$).
- Higher n: more competition lowers the output component,

Proposition: Arrowian Result and Its Limits

- **Arrowian result (no spillovers):** $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$ when $\beta = 0$.
- With exogenous fee: more competition $\Rightarrow$ more firms each independently investing $\Rightarrow$ aggregate abatement rises monotonically.
- **Limit: spillovers break the Arrowian result.**
- With $\beta > 0$: each firm benefits from rivals' abatement via ERE. As n grows, free-riding incentives intensify; eventually each firm's individual investment falls faster than the number of firms grows.
- **Inverted U-shape:** $Z(n)$ peaks near $\bar{n} \approx 3.4$ when $\beta = 1/2$.
- **Policy implication:** with spillovers, regulators should not simply promote competition to boost green innovation – the relationship is non-monotone.

Comparison: Key Results Across Sections 6.3–6.6

- **Abatement levels:**
  - Non-cooperative (6.3): z^{NC} – lowest with high β.
  - Cooperative ERC (6.4): $z^{ERC} > z^{NC}$ when $\beta > 1/3$.
  - Sequential (6.5): $z^{Seq} > z^{NC}$ always.
  - Regulatory commitment (6.6): $z_i = t/\gamma$, dominant strategy.
- **Role of spillovers:**
  - In Sections 6.3–6.5 (regulator responds): spillovers reduce z^{NC} but can raise z^{ERC}.
  - In Section 6.6 (commitment): spillovers reduce individual investment via ERE free-riding; no TSE to counteract.
- **Market structure (Section 6.6):** more firms raise aggregate abatement without spillovers but reduce it with strong spillovers.

Abatement, Fees, and Welfare: Putting It Together

- Abatement investment improves welfare through two channels:
  1. **Direct channel:** lower net emissions reduce environmental damage $Env = \frac{d}{2}(Q - Z)^2$.
  2. **Indirect channel:** lower fee $\Rightarrow$ higher output $\Rightarrow$ higher consumer and producer surplus.
- The **optimal emission fee** balances:
  - Correcting the remaining emissions externality.
  - Not discouraging output excessively given the oligopoly distortion.
- When abatement is cheap and pollution mild: fee may be negative (output subsidy optimal).
- When abatement is costly and pollution severe: positive fee is always optimal, and its level is higher than in the no-abatement case.
- Cooperation (ERJV) can raise welfare when it leads to socially beneficial increases in abatement investment.

Policy Implications of Chapter 6

- **Emission fees alone are not enough:** when firms can invest in abatement, the regulator must account for the feedback between investment incentives and the optimal fee.
- **Supporting ERJVs:** policymakers should allow or encourage ERJVs when pollution is severe or R&D costs are high – coordination raises abatement and can improve welfare.
- **Regulatory timing:** committing to a fee before firms invest removes strategic distortions (no free-riding via TSE) but also removes beneficial flexibility.
- **Market structure:** promoting competition is not always pro-environmental – spillovers can generate an inverted U-shape in aggregate abatement as the number of firms increases.
- **Two-instrument policies:** in settings with endogenous entry, combining an entry fee with an emission fee dominates a single instrument.

Summary: Extensions and Case Studies

- **Extensions** broaden the Chapter 6 framework in several directions:
  - Endogenous entry, subsidies, patents, product differentiation, and mixed oligopolies all modify the basic free-riding and cooperation results.
  - Key message: a single emission fee is often insufficient; multiple instruments are needed for first-best outcomes.
- **Case studies** illustrate that:
  - HP, Dell, and European automakers demonstrate simultaneous abatement with competitive benchmarking.
  - COSIA and Japanese refining associations are real-world ERJVs.
  - Intel/AMD and US refiners show sequential adoption of clean technologies.
- Theory and evidence are complementary: the models of Chapter 6 help interpret strategic patterns observed in regulated industries.

Glossary of Key Terms – Chapter 6

- **Abatement** (z_i): investment reducing a firm's net emissions.
- **Spillover** (β): fraction of rival's abatement that reduces a firm's own emissions.
- **TSE** (tax-saving effect): reduction in emission fee caused by increased aggregate abatement.
- **ERE** (emission-reduction effect): direct emission reduction for firm i due to rival's abatement when $\beta > 0$.
- **Strategic substitutes:** abatement levels where each firm invests less when rivals invest more (negative-slope best response).
- **ERC** (environmental research cartel): firms coordinate abatement to maximize joint profit; generic β.
- **ERJV** (environmental R&D joint venture): ERC with $\beta = 1$; full information sharing.
- **Regulatory commitment:** fee is set before abatement investment; abatement is a dominant strategy.

Key Takeaways – Chapter 6

- Abatement investment creates two strategic externalities: **TSE** (always present) and **ERE** (present when $\beta > 0$).
- Both externalities incentivize firms to *free-ride* on rivals' abatement, leading to underinvestment.
- Cooperation (ERC/ERJV) can internalize these externalities, but its welfare impact depends on d, β, and γ.
- Sequential timing can increase total abatement relative to simultaneous, but the leader may strategically free-ride when abatement is costly.
- When the regulator pre-commits to a fee, strategic interaction in abatement disappears; individual abatement becomes a dominant strategy.
- The relationship between market concentration and aggregate abatement depends critically on spillovers: monotone without, inverted U-shape with.

Chapter 7

Environmentally Friendly Mergers?

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 7

- **Section 7.1.** Introduction and six guiding questions.
- **Section 7.2.** Merger-to-monopoly: duopoly $\rightarrow$ monopoly.
    - 3-stage game; no-merger and merger equilibria; welfare comparison.
- **Section 7.3.** Cost-reducing mergers (synergies).
- **Section 7.4.** Nonlinear costs and merger incentives.
- **Section 7.5.** Multi-firm mergers: the 80% rule.
- **Section 7.6.** Mergers with environmental regulation: the neutrality result.
- **Section 7.7.** Case studies.
- **Section 7.8.** Merger review in practice: market definition, HHI, effects, efficiencies, remedies.
- **Section 7.9.** Related literature.
- **Section 7.10.** Cartels in polluting industries.

Section 7.1

Introduction

Mergers and Environmental Policy

- Competition authorities worldwide are under growing pressure to explicitly incorporate **environmental criteria** in merger reviews.
- **Examples:**
  - *European Commission:* environmental considerations in Norsk Hydro/Alumetal and Sika/MBCC approvals.
  - *UK Competition and Markets Authority (CMA):* explicit sustainability guidelines since 2021.
  - *Japan Fair Trade Commission* and *US FTC*: ongoing debates on including environmental factors.
- **Core question:** can a merger reduce pollution enough to offset the consumer surplus loss from greater market power?
- This chapter provides a **game-theoretic framework** to answer this question and related policy questions.

Why Mergers Can Be Environmentally Friendly

- A merger reduces the number of competing firms and lowers aggregate output.
- **Lower output** $\Rightarrow$ **lower aggregate emissions** $\Rightarrow$ **lower environmental damage**.
- Traditional welfare analysis of mergers:
  - Consumers lose (prices rise, CS falls).
  - Producers gain (market power increases, PS rises).
  - Net effect on $W = CS + PS$ typically negative.
- With environmental damage $D = d \cdot Q$:
  - A third effect appears: **reduced environmental damage**.
  - If pollution is sufficiently severe (d large), the environmental gain can **dominate** the consumer surplus loss.
- Result: merger can be **welfare-improving** even without efficiency gains.

Guiding Questions (1–3)

1. **How can mergers affect pollution levels and total welfare?**
   - Mergers reduce output and emissions; net welfare depends on the damage parameter d.
2. **Why should competition authorities consider environmental criteria?**
   - Ignoring environmental damage causes CAs to block welfare-improving mergers when d is large.
3. **Does the CA's approval criterion matter?**
   - Consumer-surplus (CS) criterion always blocks mergers (CS always falls). Total-welfare criterion approves when $d > 5/7$.

Guiding Questions (4–6)

1.

4. **Do cost-reducing synergies change the analysis?**
   - Synergies lower marginal cost, expand output, reduce price; even the CS criterion may approve highly synergistic mergers.
5. **Does emission-fee regulation make the merger decision irrelevant?**
   - Yes: if the environmental authority (EA) adjusts the fee optimally after the merger decision, social welfare is the same whether or not the CA approves.
6. **Are emission fees and competition policy complements in fighting cartels?**
   - Yes: higher emission fees reduce cartel profits, making it easier for competition policy to deter collusion.

Section 7.2

Merger-to-Monopoly

Model Setup

- Two identical firms ($i = 1, 2$) producing a homogeneous good.
- **Inverse demand:** $p = 1 - Q$, where $Q = q_1 + q_2$.
- **Marginal cost:** $c \in (0, 1)$ per unit.
- **Environmental damage:** $D = d \cdot Q$, where $d \geq 0$ is the per-unit damage parameter.
- **Social welfare:**
$$W = CS + PS - D$$
- **Three-stage game:**
  1. *Stage 1:* Firms simultaneously decide whether to submit a merger request.
  2. *Stage 2:* Competition Authority (CA) approves or blocks the merger.
  3. *Stage 3:* Firms choose output (joint monopoly if approved; Cournot duopoly if blocked).
- Solved by **backward induction**.

Section 7.2.1

Third Stage: Output Equilibrium

Stage 3 – No Merger: Cournot Duopoly

- Each firm maximizes its own profit taking the rival's output as given.
- **Firm i's problem** (equation 7.1):

$$\max_{q_i} \ \pi_i = (1 - q_i - q_j)\, q_i - c\, q_i \tag{7.1}$$

- **First-order condition:**

$$1 - 2q_i - q_j - c = 0$$

- **Best-response function** (equation 7.2):

$$q_i(q_j) = \frac{1-c}{2} - \frac{1}{2}\, q_j \tag{7.2}$$

- Slopes downward in q_j: firms are **strategic substitutes**.

Stage 3 – No Merger: Cournot Equilibrium

- Impose symmetry $q_1 = q_2 = q^{NM}$ in the best-response function:
- **Equilibrium output per firm** (equation 7.3):

$$q_i^{NM} = \frac{1-c}{3} \tag{7.3}$$

- **Aggregate output** (equation 7.4):

$$Q^{NM} = \frac{2(1-c)}{3} \tag{7.4}$$

- **Equilibrium price:** $p^{NM} = \dfrac{1+2c}{3}$.
- **Profit per firm** (equation 7.5):

$$\pi_i^{NM} = \frac{(1-c)^2}{9} \tag{7.5}$$

Stage 3 – No Merger: Welfare

- **Consumer surplus:**

$$CS^{NM} = \frac{1}{2}\left(Q^{NM}\right)^2 = \frac{2(1-c)^2}{9}$$

- **Producer surplus:** $PS^{NM} = 2\pi_i^{NM} = \dfrac{2(1-c)^2}{9}$.
- **Environmental damage:** $D^{NM} = d \cdot Q^{NM} = \dfrac{2d(1-c)}{3}$.
- **Social welfare** (equation 7.6):

$$W^{NM} = CS^{NM} + PS^{NM} - D^{NM} = \frac{2(1-c)^2(2-d)}{9} \quad (7.6)$$

- Note: W^{NM} decreases in the damage parameter d (higher pollution $\Rightarrow$ lower welfare, ceteris paribus).

Stage 3 – Merger: Joint Profit Maximization

- The merged entity maximizes combined profit (equations 7.7–7.9):
- Let $Q = q_1 + q_2$ be total output. Problem:

$$\max_{Q} \ \Pi^M = (1 - Q)\, Q - c\, Q \tag{7.7}$$

- **First-order condition:**

$$1 - 2Q - c = 0 \tag{7.8}$$

- **Monopoly output:**

$$Q^M = \frac{1-c}{2}, \qquad q_i^M = \frac{Q^M}{2} = \frac{1-c}{4} \tag{7.9–7.10}$$

- **Monopoly price:** $p^M = \dfrac{1+c}{2} > p^{NM}$.
- **Joint profit:** $\Pi^M = \dfrac{(1-c)^2}{4}$; profit per firm: $\pi^M = \dfrac{(1-c)^2}{}$

Stage 3 – Merger: Welfare

- **Consumer surplus:**

$$CS^M = \frac{1}{2}\left(Q^M\right)^2 = \frac{(1-c)^2}{8}$$

- **Producer surplus:** $PS^M = \dfrac{(1-c)^2}{4}$.
- **Environmental damage:** $D^M = d \cdot Q^M = \dfrac{d(1-c)}{2}$.
- **Social welfare** (equation 7.11):

$$W^M = \frac{(1-c)^2(3-d)}{8} \tag{7.11}$$

- W^M also decreases in d, but the **rate of decrease** is smaller than for W^{NM} because output (and hence emissions) is lower.

Comparison: Merger vs. No Merger (Table 7.1)

	No Merger	Merger	Direction
Output Q	$\frac{2(1-c)}{3}$	$\frac{1-c}{2}$	$\downarrow$
Price p	$\frac{1+2c}{3}$	$\frac{1+c}{2}$	$\uparrow$
CS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c)^2}{8}$	$\downarrow$
PS	$\frac{2(1-c)^2}{9}$	$\frac{(1-c)^2}{4}$	$\uparrow$
Emissions D/d	$\frac{2(1-c)}{3}$	$\frac{1-c}{2}$	$\downarrow$
W	$\frac{2(1-c)^2(2-d)}{9}$	$\frac{(1-c)^2(3-d)}{8}$	**?**

- Output (and emissions) **always** fall after a merger.
- CS **always** falls; PS **always** rises.

Welfare Comparison: The Threshold Condition

- Compute the welfare difference (equation 7.12):

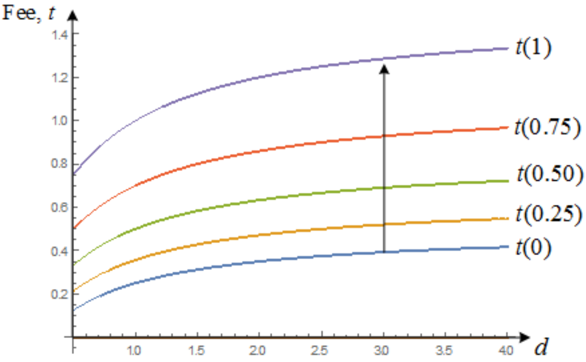
$$\Delta W \equiv W^M - W^{NM} = \frac{(1-c)^2(7d-5)}{72} \tag{7.12}$$

- **Sign:**

$$\Delta W \gtrless 0 \quad \iff \quad 7d - 5 \gtrless 0 \quad \iff \quad d \gtrless \frac{5}{7} \approx 0.71$$

- **Interpretation:**
  - If pollution is **severe** ($d > 5/7$): merger improves welfare (emission reduction dominates).
  - If pollution is **mild** ($d < 5/7$): merger reduces welfare (CS loss dominates).
  - $d = 5/7$: welfare-neutral merger.

Figure 7.1: Welfare as a Function of d



- W^M and W^{NM} are both **decreasing** in d.
- W^M decreases **more slowly** (lower output $\Rightarrow$ lower marginal damage from d).
- The two curves **cross** at $d^* = 5/7$.
- For $d > 5/7$: $W^M > W^{NM}$ — merger is socially desirable.

Section 7.2.2

Second Stage: CA Approval Decision

Stage 2 – CS Criterion

- Under the **consumer-surplus (CS) criterion**, the CA approves the merger if and only if $CS^M \geq CS^{NM}$.
- Check:

$$CS^M = \frac{(1-c)^2}{8} < \frac{2(1-c)^2}{9} = CS^{NM}$$

- *CS* **always** falls after a merger (output falls, price rises).
- **Conclusion:** under the CS criterion, the CA **always blocks** the merger — regardless of the value of d.
- This criterion **ignores** the environmental benefit, leading to excessive blocking.

Stage 2 – Total Welfare Criterion

- Under the **total welfare criterion**, the CA approves if and only if $W^M \geq W^{NM}$.
- From equation (7.12): $\Delta W = \dfrac{(1-c)^2(7d-5)}{72} \geq 0$ iff $d \geq \dfrac{5}{7}$.
- **CA decision rule:**

$$\text{Approve if } d \geq \frac{5}{7}; \qquad \text{Block if } d < \frac{5}{7}.$$

- **Policy implication:** a CA that uses the total-welfare criterion must assess the **severity of pollution** (damage parameter d) to make the correct decision.
- When d is large (dirty industry), approving a merger may actually **improve** total welfare.

Section 7.2.3

First Stage: Firms' Merger Submission Decision

Stage 1 – Do Firms Want to Merge?

- Firms submit a merger request if the merger (if approved) yields higher profit than the Cournot duopoly:

$$\pi_i^M > \pi_i^{NM}$$

- Substitute equilibrium values:

$$\frac{(1-c)^2}{8} > \frac{(1-c)^2}{9}$$

- This inequality **always holds** (since $8 < 9$, we have $1/8 > 1/9$).
- **Conclusion:** Firms **always** prefer to submit the merger request.
- Intuition: internalizing the rival's output reaction raises joint profit above the Cournot duopoly profit.
- The CA is the **only binding constraint**: firms want to merge; whether they succeed depends on the CA's criterion and d.

Section 7.2 – Recap

- **Stage 3:** Merger reduces output ($Q^M < Q^{NM}$), raises price, increases PS, decreases CS and emissions.
- **Stage 2 (CS criterion):** CA **always blocks** — CS always falls.
- **Stage 2 (W criterion):** CA approves iff $d > 5/7 \approx 0.71$ — dirty industries justify merger.
- **Stage 1:** Firms **always** want to merge ($\pi_i^M > \pi_i^{NM}$ always).
- **Key tension:** Firms' private incentives (always merge) vs. social desirability (merge only if $d > 5/7$).
- **Policy lesson:** the choice of welfare criterion for the CA matters enormously — the CS criterion is too restrictive in polluting industries.

Section 7.3

Cost-Reducing Mergers

Synergies: Motivation

- In practice, mergers often generate **cost efficiencies** (synergies): elimination of duplication, economies of scale, technology transfer.
- Standard result (Williamson 1968): even a small cost reduction can outweigh the deadweight loss from market power.
- We now extend the baseline model to allow the merger to **reduce marginal cost**.
- This introduces a **counter-force**: lower cost $\Rightarrow$ lower price $\Rightarrow$ higher output $\Rightarrow$ higher emissions.
- The environmental and efficiency effects now pull in opposite directions for output, creating a richer welfare tradeoff.

Cost-Reducing Mergers: Setup

- After the merger, the merged entity's **marginal cost** falls to:
$$c^M = (1-s)\,c, \quad s \in (0,1)$$
where s is the **synergy parameter** (higher s = larger cost reduction).
- **Merged entity's problem:**
$$\max_Q \,(1-Q)\,Q - (1-s)\,c\,Q$$
- **First-order condition:**
$$1 - 2Q - (1-s)c = 0$$
- **Merger output with synergies:**
$$Q^M(s) = \frac{1-(1-s)c}{2} = \frac{1-c+sc}{2}$$
- Note: $Q^M(s) > Q^M(0) = \dfrac{1-c}{2}$ for $s > 0$.

Output and Welfare with Synergies

- Synergies **expand output** relative to the no-synergy merger: $Q^M(s) > Q^M$.
- If s is large enough, $Q^M(s)$ may **exceed** $Q^{NM} = \dfrac{2(1-c)}{3}$.
- **Consumer surplus** rises relative to no-synergy merger (but may still be below CS^{NM} for low s).
- **Environmental damage** rises relative to no-synergy merger (more output $\Rightarrow$ more emissions).
- **Net welfare:** threshold $d^*(s)$ such that merger welfare-improving iff $d > d^*(s)$.
  - $d^*(s)$ is **decreasing in** s: higher synergies lower the damage threshold needed for approval.
  - With very high s: even the **CS criterion** may approve.

Synergies and the CS Criterion

- Recall: under the CS criterion, the CA approves iff $CS^M(s) \geq CS^{NM}$.
- $CS^M(s) = \dfrac{1}{2}\left(Q^M(s)\right)^2 = \dfrac{(1-c+sc)^2}{8}$.
- Condition $CS^M(s) \geq CS^{NM}$:

$$\frac{(1-c+sc)^2}{8} \geq \frac{2(1-c)^2}{9}$$

- Solving for s: merger is approved under the CS criterion iff $s \geq s^*$, where

$$s^* = \frac{1}{c}\left(\frac{4(1-c)\sqrt{2}}{3} - (1-c)\right) \cdot \frac{1}{\sqrt{9/8}}$$

  (explicit threshold depends on c).
- **Key message:** sufficiently large synergies can make a merger welfare-improving for **consumers** even without considering the environmental benefit.

Section 7.3 – Recap

- Cost-reducing synergies ($s > 0$) expand merger output relative to the no-synergy case.
- Synergies **improve CS** (good for CS criterion) but **raise emissions** (bad for the environment).
- The welfare threshold $d^*(s)$ **falls** in s: more synergistic mergers require *less* severe pollution to be approved.
- For high enough s: even the strict CS criterion may approve.
- **Takeaway:** efficiency gains and environmental benefits are *both* reasons for a CA to be more lenient toward mergers in polluting industries.

Section 7.4

Mergers with Nonlinear Costs

Convex Costs: Setup

- Extend the baseline model so that each firm i faces a **convex cost function**:

$$C_i(q_i) = c\,q_i + \frac{k}{2}\,q_i^2, \quad k > 0$$

- The parameter k measures **cost convexity** (diminishing returns to production).
- **Marginal cost** is $c + k\,q_i$, which **increases** with output.
- Cost convexity gives each firm an incentive to avoid large output (rising marginal costs).
- Mergers now have an additional benefit: the merged entity can **balance output across plants** to minimize convex costs.

No-Merger Equilibrium with Convex Costs

- Each firm solves:

$$\max_{q_i} \, (1 - q_i - q_j)\, q_i - c\, q_i - \frac{k}{2} q_i^2$$

- **Best-response function:**

$$q_i(q_j) = \frac{1 - c - q_j}{2 + k}$$

- **Symmetric equilibrium:**

$$q_i^{NM}(k) = \frac{1 - c}{3 + k}, \qquad Q^{NM}(k) = \frac{2(1 - c)}{3 + k}$$

- As $k \uparrow$: output falls (higher marginal cost discourages production).
- Recovers the linear case at $k = 0$: $q_i^{NM}(0) = (1 - c)/3$.

Merger Equilibrium with Convex Costs

- The merged entity minimizes joint cost by equalizing marginal costs across plants: $q_1^M = q_2^M = Q^M/2$.
- **Joint cost:** $C^M(Q) = cQ + \frac{k}{4}Q^2$ (merger halves the convexity penalty per unit of output).
- **Merged entity's problem:**

$$\max_Q (1 - Q)\, Q - cQ - \frac{k}{4}Q^2$$

- **Monopoly output with convex costs:**

$$Q^M(k) = \frac{1 - c}{2 + k/2} = \frac{2(1 - c)}{4 + k}$$

- Note: $Q^M(k) < Q^{NM}(k)$ always (monopoly produces less than duopoly even with convex costs).

Welfare Effects with Convex Costs

- Welfare comparison is analogous to the linear case, but the threshold $d^*(k)$ changes.
- **Key result:** with convex costs, the **welfare threshold is higher** than in the linear case:

$$d^*(k) > \frac{5}{7} \quad \text{for } k > 0$$

- **Intuition:**
  - Convex costs already limit output in the duopoly; the merger's output-reducing effect is smaller relative to the baseline.
  - The CS loss from the merger is larger relative to the emission reduction.
  - Therefore, pollution must be **more severe** to justify the merger under the total welfare criterion.
- **Profit incentive:** cost convexity gives firms *more* incentive to merge (avoid high marginal costs), but the CA must set a stricter threshold.

Section 7.4 – Recap

- Convex costs ($k > 0$) reduce equilibrium output in both the duopoly and merged monopoly.
- The merger enables cost-balancing across plants, reducing the effective cost convexity.
- Firms have **stronger incentives to merge** with convex costs (avoid high marginal costs).
- But the CA must apply a **stricter welfare standard**: the damage threshold $d^*(k)$ exceeds $5/7$ and increases in k.
- **Policy lesson:** in industries with strongly convex costs, the CA should be *more*, not less, cautious about approving mergers on environmental grounds.

Section 7.5

Mergers Between Several Firms

The Salant–Switzer–Reynolds (1983) Puzzle

- **Salant, Switzer, and Reynolds (1983)** showed that in a standard Cournot oligopoly:
  - A merger among m of n firms is **profitable** only if $m/n \geq 0.8$ (approximately).
  - Otherwise, non-merging firms **free-ride**: they expand output to fill the void left by the merging firms.
  - The merging firms lose market share and earn *lower* profits than before.
- **The "80% rule":** a merger is privately profitable only if the merging coalition controls at least 80% of firms.
- In large industries, most mergers are **unprofitable** for the insiders — yet they happen. Why?

Multi-firm Mergers: Setup

- n symmetric firms in the industry; m firms ($2 \leq m \leq n$) propose to merge.
- After the merger: $n - m + 1$ entities compete in Cournot — the merged entity plus $n - m$ outsiders.
- **No merger:** all n firms compete; individual output $q^{NM} = \dfrac{1-c}{n+1}$.
- **Merger:** merged entity and $n - m$ outsiders compete; the merged entity acts as a single player.
- **Free-rider effect:** outsider firms expand output post-merger (best-response to merged entity's reduced output).
- **Net industry output**: may *rise or fall* depending on m/n.

Multi-firm Mergers: Environmental Dimension

- **Emission reduction:** a larger merger (m higher) reduces aggregate output more, which is environmentally beneficial.
- **But:** if $m/n < 0.8$, the merger is **not profitable** for the insiders (outsiders free-ride and take market share).
- A conflict arises:
  - **Privately unprofitable** but **socially beneficial** if d is large (emission reduction dominates).
  - A CA using the total-welfare criterion might *want* to approve a merger that the firms themselves would not find profitable.
- **Instrument:** the CA could *mandate* or *subsidize* the merger if it is socially desirable but privately unattractive.
- Environmental regulation (emission fee) helps: by reducing individual output incentives, the fee partially offsets the free-rider expansion.

Section 7.5 – Recap

- The **80% rule** (Salant et al. 1983): mergers are privately profitable only if $m/n \geq 0.8$ in a symmetric Cournot oligopoly.
- Environmental dimension: larger mergers reduce aggregate emissions more, increasing welfare when d is large.
- A **conflict** can arise: the merger is welfare-improving but not profit-improving for the insiders.
- The free-rider problem **undermines** the environmental benefit of partial mergers: outsiders expand output and offset emission reductions.
- **Policy implication:** in polluting industries, the CA may need to actively *facilitate* mergers that are socially but not privately desirable, or use emission fees to limit outsider expansion.

Section 7.6

Merger with Environmental Regulation

The Neutrality Result

Adding an Environmental Authority (EA)

- Now suppose an **Environmental Authority (EA)** sets an **emission fee** t per unit of output.
- **Extended game sequence:**
  1. *Stage 1:* Firms decide whether to submit a merger request.
  2. *Stage 2:* CA approves or blocks.
  3. *Stage 3:* EA sets the optimal emission fee t.
  4. *Stage 4:* Firms (or merged entity) choose output given t.
- The EA **observes** the merger outcome and adjusts t optimally in each case.
- **Key question:** does the EA's fee adjustment make the CA's merger decision irrelevant for total welfare?

Optimal Fee: No Merger Case

- Without a merger ($n = 2$ Cournot firms), each firm faces effective marginal cost $c + t$.
- **Equilibrium output per firm:**

$$q_i^{NM}(t) = \frac{1-c-t}{3}, \qquad Q^{NM}(t) = \frac{2(1-c-t)}{3}$$

- **EA maximizes** $W^{NM}(t) = CS^{NM}(t) + PS^{NM}(t) - d \cdot Q^{NM}(t)$:
- **Optimal fee (no merger):**

$$t^{NM*} = \frac{d(1-c)}{1+d} \cdot \frac{3}{2+d} \quad \text{(derived from FOC)}$$

- The EA internalizes the external damage; the fee equals **marginal external damage** at the social optimum.

Optimal Fee: Merger Case

- With a merger (monopoly), the merged entity faces effective marginal cost $c + t$.
- **Monopoly output:**

$$Q^M(t) = \frac{1 - c - t}{2}$$

- **EA maximizes** $W^M(t) = CS^M(t) + PS^M(t) - d \cdot Q^M(t)$:
- **Optimal fee (merger):**

$$t^{M*} = \frac{d(1-c)}{1+d} \cdot \frac{2}{1+d} \quad \text{(derived from FOC)}$$

- Note: $t^{M*} < t^{NM*}$ — the EA sets a **lower fee** under monopoly (the monopoly already restricts output; less additional correction needed).

The Social Optimum: Same in Both Cases

- The EA's goal is to achieve the **socially optimal output**: Q^{SO} that maximizes total welfare.
- **Social optimum:** set $p = c + d$ (price equals full social marginal cost):

$$1 - Q^{SO} = c + d \quad \Longrightarrow \quad Q^{SO} = \frac{1-c}{1+d}$$

- **Key result:** The EA can achieve Q^{SO} *regardless* of whether a merger occurs:
  - Under **no merger**: EA sets t^{NM*} so that $Q^{NM}(t^{NM*}) = Q^{SO}$.
  - Under **merger**: EA sets t^{M*} so that $Q^{M}(t^{M*}) = Q^{SO}$.
- **Same** $Q^{SO} \Rightarrow$ **same emissions, same welfare** in both cases.

The Neutrality Result

Neutrality Theorem: When the Environmental Authority sets the emission fee optimally (after observing the merger outcome), social welfare is the **same** whether or not the CA approves the merger. The CA's merger decision is **inconsequential** for aggregate output, emissions, and total welfare.

- Both market structures deliver $Q^{SO} = \dfrac{1-c}{1+d}$ under optimal regulation.
- What **differs** between the two cases:
  - **Tax revenue:** $T^M = t^{M*} \cdot Q^{SO} < t^{NM*} \cdot Q^{SO} = T^{NM}$ (merger generates less revenue).
  - **Distribution:** consumer and producer surplus differ.
- Welfare is the same; **distributional** effects differ.

Intuition for the Neutrality Result

- The emission fee is a **flexible instrument**: it can be adjusted to correct any level of market power.
- Under **monopoly**: large market distortion $\Rightarrow$ EA needs a *lower* fee (monopoly already undersupplies).
- Under **duopoly**: smaller market distortion $\Rightarrow$ EA needs a *higher* fee (more competitive output to be corrected).
- In both cases the fee brings output to Q^{SO} — the EA perfectly substitutes for the market power effect.
- **Implication:** the CA need not worry about the environmental consequences of its merger decision if the EA can optimally adjust the emission fee.
- **Caveat:** neutrality requires the EA to have full flexibility; if the fee is constrained (political limits, asymmetric information), merger decisions do matter.

Section 7.6 – Recap

- When an EA can adjust the emission fee **after** the merger decision, both market structures achieve the same social optimum Q^{SO}.
- **The CA's merger decision is welfare-neutral** when the EA operates optimally.
- **Fees differ:** $t^{M*} < t^{NM*}$ — the merger lowers the required fee but does not change total welfare.
- **Tax revenue differs:** merger generates less fee revenue, which matters for the government's budget.
- **Caveat:** neutrality breaks down if the fee is constrained, the EA is not fully informed, or if the fee affects other margins (e.g., entry, innovation).

Section 7.7

Case Studies

Case Study 1: Norsk Hydro / Alumetal (EU, 2022)

- **Transaction:** Norsk Hydro (Norway) acquires Alumetal (Poland), creating a major European aluminum recycler.
- **Environmental dimension:** aluminum recycling is far less energy-intensive than primary production ($\sim$95% less energy use).
- **EC's approach:**
  - Assessed whether the merger would strengthen or weaken recycling incentives.
  - Considered whether combined entity would shift sourcing toward recycled (low-emission) aluminum.
  - Concluded: merger *supports* the EU Green Deal objectives.
- **Outcome:** Approved with commitments on recycling capacity.
- **Link to theory:** large emission reduction (high effective d) justifies merger despite market concentration.

Case Study 2: Sika / MBCC Group (EU, 2022)

- **Transaction:** Sika (Switzerland) acquires MBCC Group (specialty construction chemicals), creating a global leader.
- **Environmental dimension:** construction chemicals (concrete admixtures) directly affect buildings' CO_2 footprint:
  - Better admixtures $\Rightarrow$ less cement needed $\Rightarrow$ lower CO_2 per building.
- **EC's approach:**
  - Weighed whether merger would accelerate R&D in low-carbon construction solutions.
  - Also assessed whether concentration would raise prices and deter adoption of green products.
- **Outcome:** Approved with divestiture remedies.
- **Link to theory:** synergies ($s > 0$) in green innovation reduce the welfare cost; high d from climate damage supports approval.

Case Study 3: UK CMA and Sustainability Mergers

- The UK **Competition and Markets Authority (CMA)** revised its **Merger Assessment Guidelines** in **March 2021**, listing **environmental sustainability** and the transition to a low-carbon economy among the benefits that may offset a loss of competition.
- **Key provisions:**
  - Environmental benefits count as **relevant customer benefits** under the Enterprise Act, capable of outweighing a competitive harm.
  - Benefits must be **merger-specific** (would not occur without the merger) and **verifiable**.
  - The CMA may therefore clear a merger that reduces CS if the environmental benefit is sufficiently large.
  - **Caveat:** no published CMA decision has yet turned on this provision.
- **Do not confuse** this with the CMA's **Green Agreements Guidance** (October 2023), which concerns *agreements*

Case Study 4: Cement Industry Mergers

- **Context:** cement production accounts for $\sim$8% of global CO_2 emissions.
- **Typical pattern:** mergers in cement are scrutinized for regional market power (high transport costs segment markets locally).
- **Environmental angle:**
  - Merged entities can spread carbon capture and storage (CCS) investments over larger production volumes.
  - Consolidation may accelerate transition to lower-emission kilns.
  - But: monopoly cement producers may delay green investment if not profitable in the short run.
- **Link to theory:**
  - High d (cement is a major emitter): welfare criterion may favor merger.
  - Synergies ($s > 0$) from shared green technology: lowers cost threshold.
  - But free-rider effect in local markets: outsider cement firms

Section 7.8

Merger Review in Practice

From Models to Decisions

- Our models ask whether a merger raises **total welfare**.
- A competition authority answers a narrower question, **in advance**: will *this* transaction harm the customers of *this* market?
- Between 2013 and 2023 the European Commission examined roughly **3,500 mergers** and prohibited about **eleven**.
- **Question:** how does an authority sort the eleven from the rest?
  - Define the market $\rightarrow$ measure concentration $\rightarrow$ identify the theory of harm $\rightarrow$ weigh efficiencies $\rightarrow$ choose a remedy.
- These tools are less demanding mathematically than Sections 7.2–7.6, but they are the language in which real decisions are written.

7.8.1 Defining the Relevant Market

- Sections 7.2–7.6 write a single inverse demand $p(Q)$, which **presumes the market is already defined**.
- **Hypothetical monopolist (SSNIP) test:** start from a narrow candidate market. Could a single owner profitably raise price by 5–10%?
  - If too many buyers switch away, the market is **too narrow**: add the next substitute and repeat.
- Two dimensions: the **product market** (substitutes) and the **geographic market** (similar competitive conditions).
  - Cement is costly to transport, so its geographic market is local (radius $\approx$ 200 km). This is why Cemex/Argos is best read region by region.
- **Why it matters:** market definition determines shares, and shares drive everything that follows.
- EC revised its Market Definition Notice in **February 2024** (first change since 1997): more weight on quality, innovation and sustainability; adapted to zero-price digital markets

7.8.2 Measuring Concentration: the HHI

- **Herfindahl-Hirschman Index:** sum of squared market shares (in percentage points).

	Shares before	Shares after
Firm 1	40	40
Firm 2	30	30
Firms 3 and 4	20 and 10	30
HHI	3,000	3,400

- **Shortcut.** A merger of firms with shares s_1, s_2 raises the index by exactly

$$(s_1 + s_2)^2 - s_1^2 - s_2^2 = 2s_1 s_2,$$

here $2 \times 20 \times 10 = 400$.

- **Benchmark.** With n equal firms, $\text{HHI} = \dfrac{10{,}000}{n}$.

7.8.2 Thresholds and the Link to Section 7.5

- **2023 U.S. Merger Guidelines** (DOJ and FTC, December 2023):
  - Market is **highly concentrated** if HHI $> 1{,}800$.
  - An increase of more than **100 points** is significant.
  - Both together $\Rightarrow$ merger is **presumed** to lessen competition; burden shifts to the parties.
- The **2010** guidelines they replaced used 2,500 and 200: the revision **lowered both thresholds**.
- Our example (HHI 3,400, increase 400) is presumed anticompetitive under either standard.
- **Link to Section 7.5.** There we showed a merger must cover a large share before it is *profitable* (the 80% rule, Salant et al. 1983). The HHI is how an authority asks the same question **from outside**, without observing costs.

7.8.3 Unilateral and Coordinated Effects

- **Unilateral effects.** The merged firm *alone* finds it profitable to raise price, because sales it loses now partly return to it.
  - No agreement with rivals required. This is exactly the mechanism of **Section 7.2**: one fewer independent competitor $\Rightarrow$ output falls, price rises.
- **Coordinated effects.** The merger makes collusion among the *remaining* firms easier to sustain.
  - Fewer firms $\Rightarrow$ deviations easier to detect, punishments easier to coordinate. This is the logic of **Section 7.10**.
- The two are **complements**, not substitutes: a merger may be challenged on either ground, and real decisions address both.

7.8.4 Weighing Cost Savings Against Market Power

- **Williamson (1968) trade-off:** deadweight loss from a higher price vs. cost saving on units still produced.
- Example: p rises from 10 to 12, Q falls from 100 to 90, and MC falls from 10 to 8.

Deadweight loss, $\frac{1}{2} \times 2 \times 10$	10
Cost saving, 2×90	180
Net change in total welfare	$+170$

- **Intuition:** the saving is a **rectangle** over every unit still produced; the deadweight loss is a **triangle** over units lost. Even modest efficiencies offset substantial market power.

7.8.4 Two Qualifications

- **Efficiencies must be merger-specific and verifiable** (Farrell and Shapiro, 1990). Savings the firms could achieve separately do not count.
- **The criterion differs from ours.** Sections 7.2–7.6 use **total welfare**, $CS + PS - D$. The EC and the U.S. agencies apply a **consumer welfare standard**.
  - In the table above, total welfare rises by 170, but consumers pay 12 instead of 10.
  - Under a consumer welfare standard the merger is **still challenged**.
- **This gap explains many cases** in which an economist judges a merger beneficial and an authority blocks it.
- It is also why the environmental term D matters so much in our model: it enters *total* welfare, not consumer surplus.

7.8.5 Remedies

- Most problems are resolved by **changing the transaction**, not by prohibition.
- **Structural remedies:** sell assets (a plant, a brand, a subsidiary) to a buyer who will compete.
- **Behavioral remedies:** keep the structure, constrain conduct (supply rivals, license technology, no bundling).
- Authorities **prefer structural** remedies: a divestiture needs no ongoing supervision, a behavioral commitment must be monitored for years.
    - Often an **up-front buyer** condition: no closing until a purchaser is approved.
- **Sika/MBCC** (Section 7.7.2) is structural: divestiture of the admixtures business, then the deal proceeded.
- **Booking/eTraveli (2023)** is the counter-example: a rival choice screen was rejected (run by Booking's own subsidiary, too narrow, algorithm not monitorable). Merger **prohibited**.

7.8.6 Mergers That Remove a Future Competitor

- Our models treat the parties as firms competing **today**. Some cases involve buying a firm that is *not yet* a competitor.
- **Cunningham, Ederer and Ma (2021):** between **5.3% and 7.4%** of pharmaceutical acquisitions were **killer acquisitions** — the acquirer discontinued the project it had just bought.
  - They cluster **just below** the size that triggers review.
- **Why this matters here.** A polluting incumbent may buy a firm developing a **cleaner substitute** and shelve it.
  - The loss is not today's price: it is **abatement that never happens**.
  - Section 7.6 showed a well-set fee can make merger policy inconsequential. **That result fails here**: no emission fee delivers abatement from a withdrawn technology.
- **Illumina/GRAIL:** too small to notify; prohibited 2022; EUR 432m fine; in 2024 the ECJ held the EC could not accept the referrals. Several states added **call-in powers** (Denmark, July 2024)

Policy Outlook: the 2026 Draft EU Merger Guidelines

- On **30 April 2026** the European Commission published for consultation a **draft set of new Merger Guidelines**, merging and replacing the horizontal (2004) and non-horizontal (2008) guidelines.
- **What the draft would change:**
  - Sustainability treated as a **parameter of competition**.
  - Explicit protection of competition in **green innovation**.
  - Efficiency claims may take the form of **sustainability or resilience benefits**, over a more flexible time horizon.
- Consultation closed **26 June 2026**; adoption expected before the end of 2026.
- **Careful:** much “sustainability in competition law” concerns *agreements*, not mergers (EC Horizontal Guidelines 2023; Dutch ACM 2023; UK Green Agreements Guidance 2023, which excludes mergers).
- The clearest merger precedent is the **UK Merger Assessment Guidelines (2021)**, which list environmental

Section 7.8 – Recap

- **Market definition** comes first and drives everything: shares, concentration, and the theory of harm.
- **HHI** summarizes concentration; a merger raises it by $2s_1s_2$. Presumption of harm above 1,800 with a $+100$ increase.
- **Unilateral** effects (Section 7.2) and **coordinated** effects (Section 7.10) are complementary theories of harm.
- **Efficiencies** can outweigh deadweight loss because they apply to every unit produced — but they must be merger-specific and verifiable, and a **consumer welfare standard** may still block the merger.
- **Remedies**, mostly structural, resolve more cases than prohibitions do.
- **Killer acquisitions** are the case where our neutrality result of Section 7.6 breaks down: no fee restores a technology that was shelved.

Section 7.9

Related Literature

Key References

- **Salant, Switzer, and Reynolds (1983):** "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." *Quarterly Journal of Economics*.
  - Established the 80% rule; showed most mergers are privately unprofitable in Cournot.
- **Farrell and Shapiro (1990):** "Horizontal Mergers: An Equilibrium Analysis." *American Economic Review*.
  - Characterized when mergers raise consumer surplus; proposed the GUPPI (Gross Upward Pricing Pressure Index).
- **Gelves and Philippi (2022):** "Mergers in Polluting Industries." *Journal of Environmental Economics and Management*.
  - Extended merger analysis to polluting industries; derived welfare thresholds; analyzed interaction with emission regulation.

Additional References

- **Williamson (1968):** "Economies as an Antitrust Defense." *American Economic Review*.
  - Classic paper showing that small cost reductions can justify mergers that raise price.
- **Fikru and Gautier (2016):** "Mergers in Markets with Environmental Externalities." *Environmental and Resource Economics*.
  - Showed that emission fees and merger policy interact; partial neutrality results.
- **Espinola-Arredondo and Muñoz-Garcia (2013):** "Can Environmental Taxes Deter Polluting Mergers?"
  - Analyzed cartels and emission fees as complements; showed that higher fees reduce cartel incentives in polluting industries.
- **European Commission Merger Guidelines (2022 update):** Sustainability and Green Deal considerations in merger assessments.

Section 7.10

Cartels in Polluting Industries

Cartels vs. Mergers

- A **cartel** is an *illegal* agreement among competing firms to restrict output and raise prices — a "secret merger."
- Unlike a merger, a cartel:
  - Is not submitted to the CA for approval.
  - Is subject to detection and fines by competition authorities.
  - May be unstable (each member has an incentive to cheat).
- In the **absence** of competition policy: cartel members set output as if they were a monopoly:

$$Q^{\text{cartel}} = Q^M = \frac{1-c}{2}$$

- **Environmental effect:** cartel reduces output and emissions — same as a merger.
- **But:** cartel is illegal; the CA wants to deter it.

Collusion Incentives with Competition Policy

- With competition policy: firms face a **probability of detection** ρ and a **fine** F if caught.
- **Expected gain from collusion per firm:**

$$\pi_i^{\text{cartel}} - \pi_i^{NM} = \frac{(1-c)^2}{8} - \frac{(1-c)^2}{9} = \frac{(1-c)^2}{72}$$

- **Expected cost:** $\rho \cdot F$.
- Firms collude iff: $\pi_i^{\text{cartel}} - \pi_i^{NM} > \rho F$.
- **CA's deterrence condition:** set $\rho F > \dfrac{(1-c)^2}{72}$ to prevent collusion.
- **Higher emission fee** t: reduces both cartel and Cournot profits, but reduces *cartel gain* by more (since cartel output is more sensitive to cost increases at monopoly level).

Emission Fees Without Competition Policy

- **Without competition policy:** the CA does not fine cartels.
- An emission fee t is imposed.
- **Cartel output with fee:** $Q^{\text{cartel}}(t) = \dfrac{1-c-t}{2}$.
- **Cournot output with fee:** $Q^{NM}(t) = \dfrac{2(1-c-t)}{3}$.
- The fee **reduces output equally proportionally** for both market structures (both respond to $c + t$).
- The **cartel gain** $\pi_i^{\text{cartel}}(t) - \pi_i^{NM}(t)$:
  - Both profits fall with t.
  - The *gain from collusion* is proportional to $(1-c-t)^2$.
  - Fee reduces both profits equally in percentage terms.
- **Conclusion (no competition policy):** emission fee does **not** break the cartel — it reduces profits but preserves collusion incentives.

Emission Fees With Competition Policy: Complements

- **With competition policy:** the CA levies a fine F on detected cartels.
- **Key insight:** a higher emission fee t makes it **easier** for the CA to deter collusion:
  1. Higher $t \Rightarrow$ lower cartel profits $\Rightarrow$ smaller incentive to collude.
  2. For the same probability of detection ρ, the required fine F to deter collusion is **lower** when t is higher.
  3. Alternatively: for the same fine, the CA can deter collusion with **lower detection probability** ρ.
- **Formal result:** $\partial(\pi_i^{\text{cartel}} - \pi_i^{NM})/\partial t < 0$ under competition policy — the emission fee **reduces the attractiveness of collusion**.
- **Emission fees and competition policy are COMPLEMENTS** in reducing collusion.

Complementarity: Intuition

- Why does competition policy *change* the role of the emission fee?
- **Without competition policy:**
  - Firms take cartel as given; fee reduces both cartel and Cournot profits proportionally.
  - No impact on collusion margin.
- **With competition policy:**
  - Firms compare collusion gain vs. expected fine.
  - Fee reduces the *absolute* gain from collusion.
  - A *smaller* gain means the *same* expected fine is more effective as deterrence.
  - The CA's task becomes easier as t increases.
- **Policy implication:** in polluting industries, a higher emission fee acts as a *force multiplier* for competition policy, reducing the resources needed for cartel detection and enforcement.

Section 7.10 – Recap: Cartels

- Cartels in polluting industries reduce output and emissions (same as mergers) but are illegal.
- **Without competition policy:** emission fee reduces profits but does *not* break cartel incentives.
- **With competition policy:** emission fee *reduces the gain from collusion*, making fines more effective as deterrence.
- **Emission fees and competition policy are complements**: each policy instrument amplifies the effectiveness of the other.
- **Practical implication:** environmental regulators and competition authorities should **coordinate** their policies in polluting industries — not operate in silos.

Section 7.11

Summary

Summary: Main Results (I)

1. **Merger-to-monopoly (Section 7.2):**
   - Merger always reduces output, CS, and emissions; always increases PS.
   - Net welfare effect: $\Delta W > 0$ iff $d > 5/7 \approx 0.71$.
   - Firms always want to merge; CA's decision depends on criterion and d.
   - CS criterion: CA always blocks. Total welfare criterion: CA approves iff $d > 5/7$.
2. **Cost-reducing mergers (Section 7.3):**
   - Synergies ($s > 0$) expand merger output, reduce price, raise CS.
   - Welfare threshold $d^*(s)$ decreases in s: larger synergies $\Rightarrow$ merger needs less pollution severity to be approved.
   - High synergies may enable approval even under the CS criterion.

Summary: Main Results (II)

1.

3. **Nonlinear costs (Section 7.4):**
   - Convex costs give firms stronger merger incentives (avoid rising marginal costs).
   - But CA must apply a *stricter* standard: welfare threshold $d^*(k) > 5/7$ and increases in k.
4. **Multi-firm mergers (Section 7.5):**
   - 80% rule: merger profitable only if $m/n \geq 0.8$.
   - Conflict: merger may be welfare-improving (high d) but not privately profitable (free-rider problem).
5. **Environmental regulation (Section 7.6):**
   - **Neutrality theorem:** when EA adjusts the fee optimally, the CA's merger decision does not affect total welfare.
   - Both market structures achieve the same $Q^{SO} = (1-c)/(1+d)$.

Summary: Main Results (III)

1.

6. **Cartels (Section 7.10):**
   - Without competition policy: emission fee does not deter cartels.
   - With competition policy: emission fee reduces cartel gains $\Rightarrow$ easier to deter collusion.
   - **Emission fees and competition policy are complements** in polluting industries.
7. **Case studies (Section 7.7):** EC and CMA are moving toward total-welfare criteria that incorporate environmental sustainability in merger reviews (Norsk Hydro/Alumetal, Sika/MBCC, cement industry).
8. **Overall policy message:** competition authorities in polluting industries should adopt total welfare criteria, coordinate with environmental regulators, and adjust approval thresholds to reflect pollution severity.

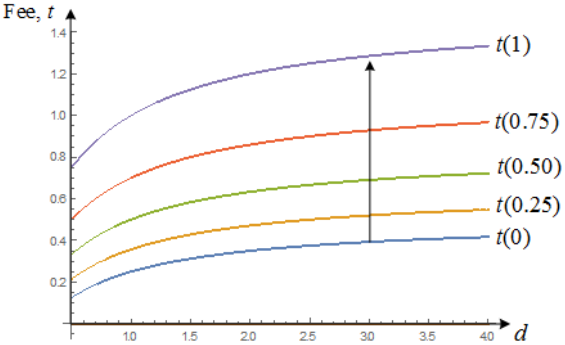
Key Equations at a Glance

Quantity	Formula
q_i^{NM}	$(1-c)/3$
Q^{NM}	$2(1-c)/3$
π_i^{NM}	$(1-c)^2/9$
W^{NM}	$2(1-c)^2(2-d)/9$
Q^{M}	$(1-c)/2$
π_i^{M}	$(1-c)^2/8$
W^{M}	$(1-c)^2(3-d)/8$
ΔW	$(1-c)^2(7d-5)/72$
d^*	$5/7 \approx 0.71$
Q^{SO}	$(1-c)/(1+d)$

Policy Decision Tree for a Competition Authority

- **Step 1:** Identify the pollution severity in the industry. Is d above or below $5/7$?
- **Step 2:** Are there cost-reducing synergies ($s > 0$)?
  - If yes: adjust threshold downward ($d^*(s) < 5/7$).
- **Step 3:** Is there an Environmental Authority that can adjust the emission fee?
  - If yes: *neutrality theorem applies* — welfare is independent of merger decision; CA may focus on distributional concerns.
- **Step 4:** Are there cartel risks?
  - If yes: ensure coordination with the EA; a higher emission fee helps deter collusion.
- **Step 5:** Apply the appropriate welfare criterion (CS vs. total welfare) given the jurisdiction's mandate.

Figure 7.1 (Revisited): The Core Result



- **Below** $d^* = 5/7$: no-merger welfare exceeds merger welfare $\Rightarrow$ CA blocks.
- **Above** $d^* = 5/7$: merger welfare exceeds no-merger welfare $\Rightarrow$ CA approves.
- The environmental damage parameter d is the **single**

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Questions? Comments?

Chapter 8

Managerial Incentives and Environmental Regulation

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

Outline of Chapter 8

- This chapter examines environmental regulation when firms pursue objectives *beyond* standard profit maximization.
- Three managerial incentive environments:
  - **Section 8.2:** Mixed oligopolies — a public firm maximizes a weighted combination of social welfare and profits alongside a private rival.
  - **Section 8.4:** Sales-based incentives — managers maximize a combination of revenue (sales) and profits.
  - **Section 8.6:** Cross-ownership — firms hold partial equity stakes in rivals' profits.
- Case studies: 8.3 (mixed oligopolies), 8.5 (sales incentives), 8.7 (cross-ownership).
- Section 8.8: Summary and comparison.

Introduction: Why Managerial Incentives Matter

- Standard Cournot analysis (Chapters 2–7) assumes **profit maximization**.
- In practice, firms often pursue *other* objectives:
  - State-owned enterprises must balance social welfare and profitability.
  - Executives compensated partly on *sales volume* tend to overproduce.
  - Firms holding stakes in rivals internalize part of those rivals' profits.
- Each distortion shifts best-response functions $\Rightarrow$ changes equilibrium output and emissions $\Rightarrow$ changes the optimal emission fee set by the environmental authority (EA).
- **Key question:** Does each incentive distortion make the EA's optimal fee *stricter* or *more lenient*?

Chapter 8 at a Glance: Road Map

Section	Model	Main result
8.2	Mixed oligopoly	$t^*(\alpha)$ independent of α (neutrality)
8.3	Case studies	EDF, Baowu, Delhi DTC
8.4	Sales incentives	$t^*(\mu)$ increasing in μ
8.5	Case studies	Autos, airlines, fast food
8.6	Cross-ownership	$t^*(\delta)$ decreasing in δ
8.7	Case studies	Delta-Virgin, energy, Toyota-Subaru
8.8	Summary	Comparison table

Profit Maximization: The Benchmark

Before studying distortions, recall the Chapter 2 benchmark:

- Both firms are **private profit maximizers**: max $\pi_i = (1 - q_i - q_j)q_i - (c + t)q_i$.
- Symmetric Cournot–Nash equilibrium:

$$q^* = \frac{1 - c - t}{3}, \quad Q^* = \frac{2(1 - c - t)}{3}, \quad p^* = \frac{1 + 2(c + t)}{3}$$

- Optimal emission fee (EA maximizes W):

$$t^* = \frac{(1 - c)(2d - 1)}{2(1 + d)}$$

- This benchmark is the **reference point** for all three managerial incentive models in Chapter 8: each model reduces to it when the distortion parameter $= 0$.

Three Sources of Distorted Incentives

Type	Who decides?	What objective?
Mixed oligopoly	Public firm management	$\alpha W + (1-\alpha)\pi_2$
Sales incentives	Private managers	$(1-\mu)\pi + \mu R$
Cross-ownership	Shareholder-owners	$\pi_i + \delta\pi_j$

- All three share the same inverse demand and cost structure.
- Differences lie entirely in the **objective function**.
- Each distortion shifts or rotates best-response functions, producing different output and fee predictions.
- Comparing across the three cases reveals the general principle: what matters for environmental policy is *what firms maximize*, not just their market structure.

Section 8.2

Mixed Oligopolies

Mixed Oligopolies: Setup

- **Duopoly:** Firm 1 is *private* (profit-maximizer); Firm 2 is *public* (welfare-weighted objective).
- Inverse demand: $p(Q) = 1 - Q$, where $Q = q_1 + q_2$.
- Common unit cost $c \geq 0$; emission damage parameter $d \geq \frac{1}{2}$.
- EA levies a per-unit emission fee $t \geq 0$ (one unit of output = one unit of emissions for simplicity).
- Timing:
  1. EA sets t.
  2. Firms compete in quantities (Cournot).
- **Degree of nationalization** $\alpha \in [0, 1]$: $\alpha = 0$ means fully private; $\alpha = 1$ means fully nationalized.

Firm Objectives

Private firm 1 maximizes profit:

$$\pi_1 = (1 - Q)q_1 - (c + t)q_1 \tag{8.1}$$

Public firm 2 maximizes the weighted objective:

$$V = \alpha W + (1 - \alpha)\pi_2 \tag{8.2}$$

where

$$W = CS + PS + T - \text{Env}$$

$$\pi_2 = (1 - Q)q_2 - (c + t)q_2$$

- W = social welfare = consumer surplus + producer surplus + tax revenue − environmental damage.
- At $\alpha = 0$: Firm 2 is fully private $\Rightarrow$ standard Cournot.
- At $\alpha = 1$: Firm 2 is a pure welfare maximizer.

Section 8.2.1 — Best-Response Functions

Private firm 1 (standard Cournot FOC: $\partial \pi_1 / \partial q_1 = 0$):

$$1 - 2q_1 - q_2 - (c + t) = 0$$

$$q_1(q_2) = \frac{1 - c - t}{2} - \frac{1}{2} q_2 \tag{8.3}$$

- Slope $= -1/2$ (standard strategic substitutes).
- Unaffected by α or d: public firm's objective does not enter firm 1's FOC.

Public Firm 2: FOC

Social welfare components:

$$CS = \frac{1}{2}Q^2, \quad T = tQ, \quad \text{Env} = \frac{d}{2}Q^2, \quad PS = \pi_1 + \pi_2$$

Differentiate $V = \alpha W + (1-\alpha)\pi_2$ with respect to q_2:

$$\alpha \frac{\partial W}{\partial q_2} + (1-\alpha)\frac{\partial \pi_2}{\partial q_2} = 0$$

$$\alpha\left[1 - c - (1+d)(q_1+q_2)\right] + (1-\alpha)(1 - 2q_2 - q_1 - c - t) = 0$$

- The first bracket is the *social marginal value* of q_2 (includes damage d).
- The second bracket is the *private marginal profit* of firm 2.
- When $\alpha = 0$: reduces to standard Cournot FOC for firm 2.

Best-Response Function of the Public Firm

Solving the FOC for q_2:

$$q_2(q_1) = \frac{1 - c - t(1-\alpha)}{2 - \alpha(1-d)} - \frac{1 + \alpha d}{2 - \alpha(1-d)} q_1 \qquad (8.4)$$

Slope of BRF$_2$:

$$s(\alpha, d) = -\frac{1 + \alpha d}{2 - \alpha(1-d)}$$

Comparative statics of the slope:

$$\frac{\partial s}{\partial \alpha} = -\frac{1 + d}{[2 - \alpha(1-d)]^2} < 0, \qquad \frac{\partial s}{\partial d} = -\frac{\alpha(1-\alpha)}{[2 - \alpha(1-d)]^2} < 0 \qquad (8.5)$$

- Higher α (more nationalized) $\Rightarrow$ steeper BRF $\Rightarrow$ **stronger strategic substitutes**.
- Higher d (more damage) $\Rightarrow$ also steepens BRF.
- Intuition: a more nationalized firm internalizes the social cost of rivals' output and cuts back more aggressively when the

Table 8.1: BRF Slopes at Special Values of (α, d)

α	d	Slope $s(\alpha, d)$	Interpretation
0	any	$-1/2$	Both firms are private (standard Cournot)
$1/2$	$1/2$	$-5/8$	Partial nationalization, low damage
$1/2$	1	$-3/4$	Partial nationalization, high damage
1	$1/2$	$-3/2$	Fully nationalized, low damage
1	1	-2	Fully nationalized, high damage

- As $\alpha \to 1$ and/or d increases, the slope steepens substantially.
- A fully nationalized firm with high damage responds very aggressively to rivals' output expansions.

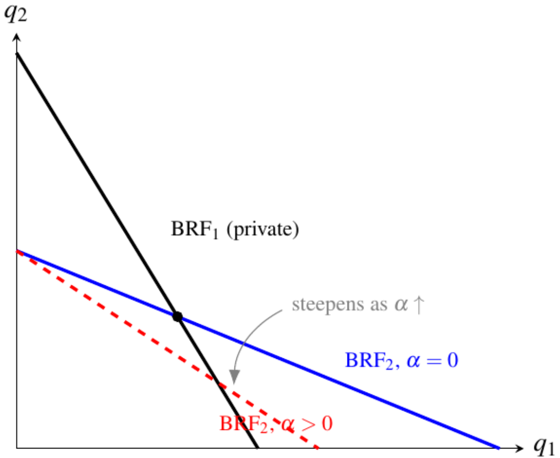
Section 8.2.1 — Interpreting the Slope Comparative Statics

Recall eq. (8.5):

$$\frac{\partial s}{\partial \alpha} = -\frac{1+d}{[2-\alpha(1-d)]^2} < 0, \qquad \frac{\partial s}{\partial d} = -\frac{\alpha(1-\alpha)}{[2-\alpha(1-d)]^2} < 0$$

- $\partial s/\partial \alpha < 0$: **More nationalization steepens BRF$_2$.**
  - A more nationalized firm places greater weight on welfare.
  - Welfare includes damage $\frac{d}{2}(q_1+q_2)^2$: the public firm internalizes more of the social cost of aggregate output.
  - When q_1 rises, public firm cuts q_2 more to limit total damage $\Rightarrow$ steeper BRF.
- $\partial s/\partial d < 0$: **Higher damage steepens BRF$_2$.**
  - Higher d magnifies the welfare cost of output $\Rightarrow$ public firm responds more aggressively to rival's expansion.
  - Note: at $\alpha = 0$, $\partial s/\partial d = 0$ (fully private firm ignores d). ✓

BRF Diagram: Mixed Duopoly



- BRF_1 has slope $-1/2$ (unchanged from standard Cournot).
- BRF_2 rotates *clockwise* (steeper) as α increases.

Section 8.2.2 — Finding Equilibrium Output

Key: We *cannot* invoke symmetry — firms have different objectives. Substitute BRF_1 into BRF_2:

$$q_2(t) = \frac{(1-c-t)(1-\alpha d) + 2\alpha t}{3-\alpha(2-d)} \tag{8.9}$$

$$q_1(t) = \frac{(1-c-t)(1+\alpha d) - \alpha(1-c)}{3-\alpha(2-d)} \tag{8.10}$$

Verification at $\alpha = 0$:

$$q_2(t)\Big|_{\alpha=0} = \frac{1-c-t}{3} = q_1(t)\Big|_{\alpha=0}$$

$\Rightarrow$ reduces to symmetric Cournot. ✓

Equilibrium Output: Derivation Steps

Step 1. Write BRF_2 (eq. 8.4):

$$q_2 = \frac{1-c-t(1-\alpha)}{2-\alpha(1-d)} - \frac{1+\alpha d}{2-\alpha(1-d)}\,q_1$$

Step 2. Substitute BRF_1: $q_1 = \frac{1-c-t}{2} - \frac{1}{2}q_2$ into BRF_2 and solve for q_2:

$$q_2 = \frac{1-c-t(1-\alpha)}{2-\alpha(1-d)} - \frac{1+\alpha d}{2-\alpha(1-d)}\left(\frac{1-c-t}{2} - \frac{1}{2}q_2\right)$$

Step 3. Collect q_2 terms, simplify denominator $3-\alpha(2-d) \Rightarrow$ obtain eq. (8.9).
Step 4. Substitute eq. (8.9) back into $\text{BRF}_1 \Rightarrow$ obtain eq. (8.10).

Aggregate Output and Emissions

Aggregate output:

$$Q(t, \alpha) = q_1(t) + q_2(t) = \frac{2(1-c-t) - \alpha(1-c)}{3 - \alpha(2-d)}$$

- Standard result: $\partial Q / \partial t < 0$ (fee reduces output).
- Effect of nationalization:

$$\frac{\partial Q}{\partial \alpha} > 0 \quad \text{(for sufficiently small } t\text{)}$$

  $\Rightarrow$ More nationalization $\Rightarrow$ higher aggregate output $\Rightarrow$ **more emissions**.
- Intuition: the public firm expands output to fight market power (enhance consumer surplus), which more than offsets the private firm's contraction.
- Yet the *optimal* fee turns out to be **independent** of α: at the welfare-maximizing fee the private firm's contraction exactly offsets the public firm's expansion (neutrality, Section 8.2.3).

Output Asymmetry: Public vs. Private Firm

From eqs. (8.9)–(8.10), at a given fee t:

$$q_2(t) - q_1(t) = \frac{\alpha\left[(1-c-t)d + (1-c)\right]}{3-\alpha(2-d)} \geq 0$$

- At $\alpha = 0$: $q_1 = q_2$ (symmetric Cournot). ✓
- For $\alpha > 0$: $q_2 > q_1$ — the public firm **produces more** than the private rival.
- The gap widens with α and d.
- Public firm overproduces to raise consumer surplus, but this also **raises aggregate emissions**.
- EA must account for this asymmetry when setting t^*.

Effect of Nationalization on Equilibrium: Numerical Example

Using eqs. (8.9)–(8.10) with $c = 0$, $d = 1$, $t = 0$:

$$q_2(\alpha) = \frac{(1)(1-\alpha)}{3-\alpha(1)} = \frac{1-\alpha}{3-\alpha}, \qquad q_1(\alpha) = \frac{(1)(1+\alpha)-\alpha}{3-\alpha} = \frac{1}{3-\alpha}$$

α	q_1	q_2	Q
0	1/3	1/3	2/3
1/2	2/5	1/5	3/5
1	1/2	0	1/2

- Higher α: public firm retreats, private firm expands; aggregate output *falls* (at $t = 0$) because of welfare internalization.
- At the *optimal* fee, aggregate output is held at Q^{SO} for every α, and the fee itself does not vary with α—the neutrality

Social Welfare Decomposition: Mixed Duopoly

Social welfare:

$$W = \underbrace{\frac{Q^2}{2}}_{CS} + \underbrace{\pi_1 + \pi_2}_{PS} + \underbrace{tQ}_{T} - \underbrace{\tfrac{d}{2}Q^2}_{\text{Env}}$$

Substituting equilibrium output $Q^*(t, \alpha)$:

- **CS** is increasing in Q (consumers benefit from higher output).
- **PS** depends on the output split between firms 1 and 2.
- **T** $=$ emission fee revenue $= tQ^*$ (transferred lump-sum to consumers in this model).
- **Env** $=$ total damage $\frac{d}{2}Q^2$ — quadratic in aggregate output.

EA chooses t to maximize W. Since W depends only on *aggregate* output Q, the EA targets Q^{SO} for every α, and the optimal fee is the same regardless of nationalization—a **neutrality** result.

Section 8.2.3 — Optimal Emission Fee

The EA maximizes social welfare $W = CS + PS + T - \text{Env}$. Because welfare depends only on *aggregate* output Q, the EA targets the socially optimal $Q^{SO} = \frac{1-c}{1+d}$ regardless of the degree of nationalization.

Optimal fee (any α):

$$t^*(\alpha) = \frac{(1-c)(2d-1)}{2(1+d)} \quad \text{—independent of } \alpha$$

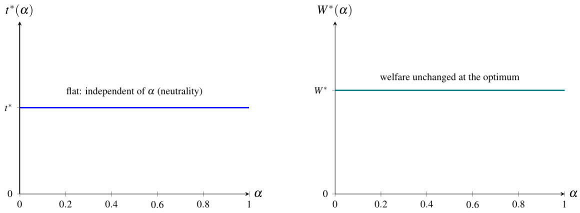
- The fee coincides with the purely private (Chapter 2) benchmark for *every* α: a **neutrality result**.
- As α rises the public firm expands but the private firm contracts by the same amount, leaving aggregate output—and hence the optimal fee—unchanged.

Optimal Fee: Intuition

- **Why is $t^*(\alpha)$ independent of α?**
  - Welfare depends only on *aggregate* output Q, so the EA's target is Q^{SO} for every α.
  - As α rises the public firm expands, but the private firm crowds back by the same amount, so aggregate output at the benchmark fee stays at Q^{SO}.
  - The EA therefore needs no adjustment: the same fee implements Q^{SO} regardless of ownership.
- **Comparison with Chapter 2:**

Setting	Optimal fee
Both firms private ($\alpha = 0$)	$t^* = \frac{(1-c)(2d-1)}{2(1+d)}$
Mixed duopoly ($\alpha > 0$)	$t^*(\alpha) = t^*\|_{\alpha=0}$

Fee and Welfare Under Mixed Duopoly

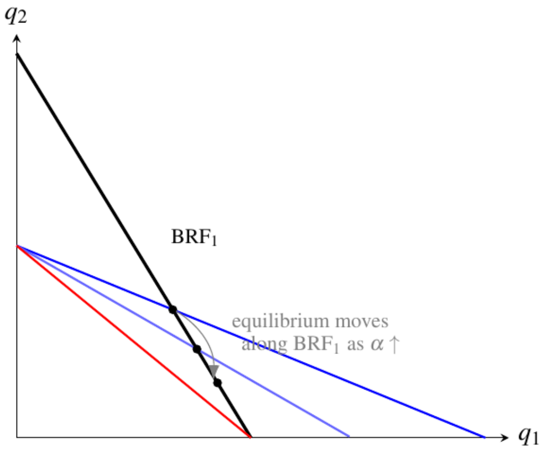


- Left panel: $t^*(\alpha)$ is **flat** in α (neutrality)—the optimal fee equals the Chapter 2 benchmark for every α.
- Right panel: Social welfare may rise or fall depending on the trade-off between higher consumer surplus and higher environmental damage.

Recap: Mixed Oligopolies (Section 8.2)

- **Setup:** Duopoly with one private firm (profit max) and one public firm (weighted welfare–profit objective parameterized by α).
- **BRF effects:**
  - BRF_1 unchanged (slope $-1/2$).
  - BRF_2 steepens as $\alpha \uparrow$ or $d \uparrow$.
- **Equilibrium:**
  - Public firm overproduces relative to private rival.
  - At the optimal fee, the private firm's contraction offsets this, holding aggregate output at Q^{SO}.
- **Optimal fee:**
  - $t^*(\alpha)$ is **independent** of α (neutrality).
  - The optimal fee equals the purely private Chapter 2 benchmark regardless of the degree of nationalization.

Mixed Oligopoly: Graphical Summary



- As α increases from 0 to 1:
  - Equilibrium point moves along BRF_1 (private firm adjusts).

Section 8.3

Case Studies: Mixed Oligopolies

Case Study 1: EDF (France) — Electricity Market

- **Setting:** Électricité de France (EDF) is a majority state-owned utility competing with private generators across Europe.
- **Objective mix:** EDF operates under an explicit public service mandate (energy access, security of supply) alongside profitability targets $\Rightarrow$ directly reflects eq. (8.2) with $\alpha > 0$.
- **Observed behavior:** EDF historically priced below the level that maximizes profit, expanding output and grid coverage.
- **Environmental implication:** Higher electricity output $\Rightarrow$ higher emissions from thermal generation $\Rightarrow$ French regulators set stricter carbon pricing for the electricity sector.
- **Model connection:** A mixed duopoly with EDF internalizing welfare ($\alpha > 0$); by the neutrality result the socially optimal fee equals the purely private benchmark, so regulation targets aggregate output rather than ownership itself.

Case Study 2: Baowu Steel (China) — Steel Industry

- **Setting:** China Baowu Steel Group is the world's largest state-owned steelmaker, competing with both domestic private mills and international producers.
- **Objective mix:** Government mandates employment targets, strategic production volumes, and regional development goals alongside financial returns $\Rightarrow \alpha$ close to 1 in our model.
- **Observed behavior:** Baowu consistently maintains high capacity utilization, even during downturns when private mills cut back.
- **Environmental implication:** Steel is highly emissions-intensive. Overproduction by the state firm raises aggregate steel-sector emissions.
- **Regulatory response:** China's Ministry of Ecology and Environment imposes differentiated emission standards, with Baowu often producing cleaner steel—regulators use the public firm to balance output and environmental goals rather

Case Study 3: Delhi Transport Corporation — Urban Transit

- **Setting:** Delhi Transport Corporation (DTC) operates public buses alongside private operators in India's capital.
- **Objective mix:** DTC balances ridership (social access), fare affordability, and fiscal targets $\Rightarrow$ strong public-welfare component (α high).
- **Observed behavior:** DTC fields a larger fleet and lower fares than cost-recovery would justify; private operators follow suit to retain market share.
- **Environmental implication:** More buses $\Rightarrow$ more vehicle emissions in an already polluted airshed.
- **Policy response:** Delhi's Supreme Court mandated CNG conversion of the entire bus fleet — a technology standard equivalent to raising t — precisely because market incentives alone were insufficient given the public firm's expansion.

Mixed Oligopoly: Endogenous vs. Exogenous α

- In this chapter α is treated as **exogenous** — it reflects the government's chosen degree of nationalization.
- **Extension:** If α is chosen by the government *optimally* (simultaneously with or before t), the problem becomes a two-instrument design problem:

$$\max_{\alpha,\, t} W(\alpha, t)$$

- This raises the question: is there an optimal α^* that maximizes welfare, and how does it relate to t^*?
- In general, $\alpha^* > 0$ is possible if the welfare gain from expanding consumer surplus exceeds the environmental cost of higher emissions (when d is small).
- For high d: the government should choose $\alpha^* = 0$ (fully private) and use only the fee.
- This two-instrument extension is a natural direction for further research.

Section 8.4

Sales-Based Managerial Incentives

Sales Incentives: Motivation

- Owners often delegate output decisions to managers whose compensation is tied partly to **sales revenue**, not just profits.
- Why? Revenue targets are observable and contractible; they may also signal market dominance.
- **Key parameter:** $\mu \in [0, 1]$ = weight on revenue relative to profit in managerial bonus.
  - $\mu = 0$: pure profit maximizer (Chapter 2).
  - $\mu = 1$: pure revenue maximizer.
  - $\mu \in (0, 1)$: typical managerial contract.
- **Strategic effect:** Sales-oriented managers act more aggressively — they expand output beyond the profit-maximizing level.
- **Environmental consequence:** Higher output $\Rightarrow$ more emissions $\Rightarrow$ stricter optimal fee.

Firm Objective with Sales Incentives

Each firm i maximizes:

$$U_i = (1-\mu)\pi_i + \mu R_i$$

where $R_i = p \cdot q_i = (1-Q)q_i$ is *revenue* and
$\pi_i = (1-Q)q_i - (c+t)q_i$ is profit.

Rewrite:

$$U_i = \pi_i + \mu(c+t)q_i = \left[(1-Q) - (c+t)\right] q_i + \mu(c+t)q_i$$

- The revenue component adds $\mu(c+t)q_i$ — a subsidy-like term that lowers the effective marginal cost perceived by the manager.
- Manager acts as if cost is $(1-\mu)(c+t)$ instead of $(c+t)$.

FOC and Best-Response Function with Sales Incentives

FOC for firm i ($\partial U_i / \partial q_i = 0$):

$$(1-\mu)(1-2q_i-q_j-(c+t))+\mu(1-2q_i-q_j)=0$$

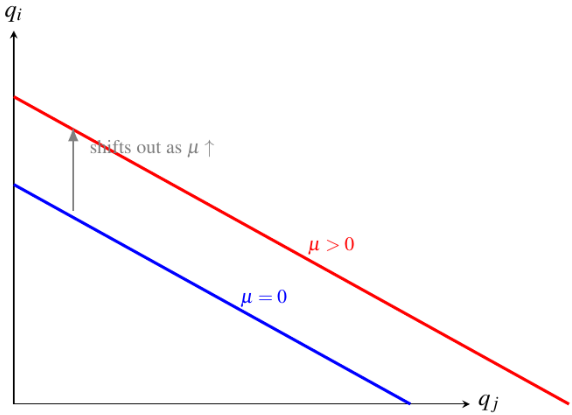
$$1-2q_i-q_j-(1-\mu)(c+t)=0$$

Best-response function:

$$q_i(q_j)=\frac{1-(1-\mu)(c+t)}{2}-\frac{1}{2}q_j$$

- **Slope unchanged** at $-1/2$: strategic substitutes as before.
- **Intercept increases** with μ: BRF shifts *outward* as μ rises.
- Intuition: the manager perceives a lower effective cost and therefore wants to produce more for any given rival output.

BRF Diagram: Sales Incentives



- Both BRFs shift *outward* symmetrically as μ increases.
- Equilibrium output rises: $q^*(t, \mu) > \frac{1-c-t}{3}$ for $\mu > 0$.
- Slope $-1/2$ is unchanged $\Rightarrow$ the equilibrium shift is driven

Equilibrium Output with Sales Incentives

Using symmetry (both firms face the same μ):

$$q^*(t,\mu) = \frac{1-(1-\mu)(c+t)}{3}$$

$$Q^*(t,\mu) = \frac{2\left[1-(1-\mu)(c+t)\right]}{3}$$

Effect of μ:

$$\frac{\partial q^*}{\partial \mu} = \frac{c+t}{3} > 0$$

- Higher sales pressure $\Rightarrow$ **more output per firm and in aggregate**.
- Aggregate emissions $e = Q^*$ rise with μ.
- At $\mu = 0$: standard Cournot output $\frac{1-c-t}{3}$. ✓

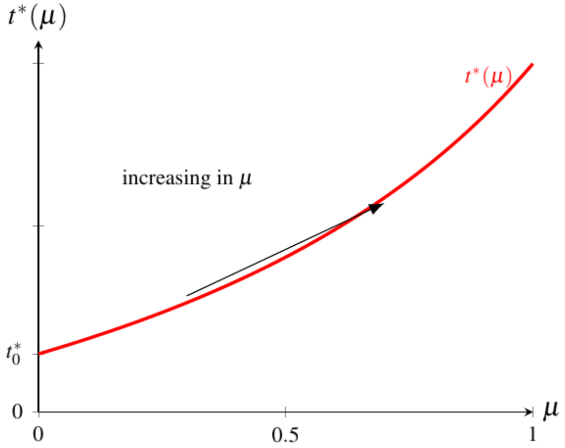
Optimal Fee with Sales Incentives

EA maximizes $W = CS + PS + T - \text{Env}$ taking μ as given. After substituting equilibrium outputs and differentiating:

$$t^*(\mu) > \left. t^* \right|_{\mu=0} \quad \text{for all } \mu > 0$$

- The optimal fee **increases** in μ.
- At $\mu = 0$: $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ (Chapter 2 benchmark). ✓
- **Intuition:** Managers perceive a lower effective cost and over-expand relative to the profit-maximizing level. EA compensates by raising t to force internalization of the externality.
- **Contrast with mixed oligopoly:** Both settings increase t^*, but for different reasons — public ownership vs. managerial compensation structure.

Fee Stringency vs. Sales Weight μ



- $t^*(\mu)$ is monotonically increasing in μ.
- Even moderate sales incentives ($\mu \approx 0.3$–0.5) require

Sales Incentives: Deriving Equilibrium — Step by Step

Step 1. BRF for each firm:

$$q_i(q_j) = \frac{1-(1-\mu)(c+t)}{2} - \frac{1}{2}q_j$$

Step 2. Invoke symmetry $q_1^* = q_2^* = q^*$:

$$q^* = \frac{1-(1-\mu)(c+t)}{2} - \frac{1}{2}q^*$$

Step 3. Solve:

$$\frac{3}{2}q^* = \frac{1-(1-\mu)(c+t)}{2}$$

$$q^*(t,\mu) = \frac{1-(1-\mu)(c+t)}{3}$$

Check at $\mu = 0$:

$$q^*(t,0) = \frac{1-c-t}{3} = \text{standard Cournot.}\checkmark$$

Sales Incentives: Numerical Example

Let $c = 0$, $d = 1$, and vary μ and t:

μ	$t = 0$	$t = 0.10$	$t = 0.20$
0	$1/3 \approx 0.333$	0.300	0.267
0.25	0.375	0.342	0.308
0.50	0.417	0.383	0.350
0.75	0.458	0.425	0.392

(Table entries: per-firm equilibrium output $q^*(t, \mu)$.)

- Output rises markedly with μ for any given fee level.
- A fee sufficient to reach the social optimum at $\mu = 0$ is *insufficient* when $\mu > 0$.
- The EA must raise t to compensate for the revenue-motivated overproduction.

Sales Incentives: Effective Cost Interpretation

Alternative way to see the μ effect:

- Rewrite the FOC:

$$1 - 2q_i - q_j = \underbrace{(1-\mu)(c+t)}_{\text{effective marginal cost}}$$

- The manager perceives a marginal cost of $(1-\mu)(c+t)$ instead of $c+t$.
- For $\mu > 0$: effective cost is **lower** $\Rightarrow$ manager produces more.
- For the firm to produce the *same* output as the $\mu = 0$ case, the fee must be higher:

$$(1-\mu)(c + t^{\text{new}}) = c + t^{\text{old}} \implies t^{\text{new}} = \frac{c + t^{\text{old}}}{1-\mu} - c > t^{\text{old}}$$

- This is precisely why $t^*(\mu) > t^*|_{\mu=0}$.

Sales Incentives: Welfare Analysis

With $Q^*(t,\mu) = \frac{2[1-(1-\mu)(c+t)]}{3}$, welfare is:

$$W(t,\mu) = \frac{[Q^*]^2}{2} + tQ^* - dQ^{*2} + \underbrace{\text{PS}}_{>0}$$

- Higher μ raises Q^* $\Rightarrow$ higher CS but also higher environmental damage.
- EA maximizes W with respect to t:

$$\frac{\partial W}{\partial t} = Q^* \frac{\partial Q^*}{\partial t} + Q^* + t\frac{\partial Q^*}{\partial t} - 2dQ^* \frac{\partial Q^*}{\partial t} = 0$$

- Solving gives $t^*(\mu)$, which is an increasing function of μ.
- **Firms' owners** are worse off under the higher fee (profit falls); **society** is indifferent (fee is optimally set); but the *managerial* incentive structure forces the EA to intervene more aggressively.

Recap: Sales-Based Incentives (Section 8.4)

- **Setup:** Both firms' managers maximize $(1-\mu)\pi + \mu R$ with $\mu \in [0,1]$.
- **BRF effects:**
  - Slope unchanged $(-1/2)$.
  - Intercept increases in $\mu \Rightarrow$ outward shift.
- **Equilibrium:**
  - Each firm produces $q^*(t,\mu) = \frac{1-(1-\mu)(c+t)}{3}$.
  - Aggregate output and emissions rise with μ.
- **Optimal fee:**
  - $t^*(\mu)$ **increases** in μ.
  - More sales pressure $\Rightarrow$ stricter environmental regulation.

Section 8.5

Case Studies: Sales-Based Incentives

Case Study 4: Automobile Industry — Sales Volume Bonuses

- **Setting:** Major automakers (GM, Ford, Toyota) compensate executives partly on **unit sales** and market-share metrics.
- **Incentive distortion:** Sales-tied bonuses induce managers to price more aggressively and expand production — exactly the $\mu > 0$ mechanism.
- **Observed behavior:** OEM production consistently exceeds what profit maximization alone would predict; dealer incentive programs push additional units.
- **Environmental implication:** More vehicles on the road $\Rightarrow$ higher tailpipe emissions; more manufacturing $\Rightarrow$ higher supply-chain emissions.
- **Regulatory response:** EPA CAFE standards and Euro 7 norms are set more stringently precisely to offset this output-expanding pressure — consistent with $t^*(\mu) > t^*|_{\mu=0}$.

Case Study 5: Airlines — Frequent Flier Programs

- **Setting:** Airlines compensate managers partly on passenger revenue and load-factor targets; frequent flier programs reward *flight frequency* rather than efficiency.
- **Incentive distortion:** Revenue and passenger targets push airlines to add routes and frequencies beyond the profit-maximizing level ($\mu > 0$ in revenue).
- **Environmental implication:** More flights $\Rightarrow$ more aviation CO_2 and NO_x.
- **Regulatory challenge:** Aviation has historically been exempt from carbon taxes. Model predicts that the *absence* of the correct $t^*(\mu)$ leads to socially excessive flight volumes.
- **Policy implication:** EU Emissions Trading System (EU ETS) extension to aviation is consistent with the need for a higher effective t when $\mu > 0$.

Case Study 6: Fast Food — Growth Targets and Packaging Waste

- **Setting:** Fast-food chains (McDonald's, Yum! Brands) set corporate growth targets measured by **number of outlets and total sales revenue**.
- **Incentive distortion:** Franchise and corporate managers are rewarded for opening new outlets and maximizing throughput — a clear $\mu > 0$ sales-driven objective.
- **Environmental implication:** More outlets $\Rightarrow$ more single-use packaging waste, food waste, and supply-chain emissions.
- **Regulatory response:** Several jurisdictions have imposed packaging levies, mandatory compostability standards, and extended producer responsibility — instruments that effectively raise t to counteract the output-expanding incentive.
- **Model prediction confirmed:** The need for *stricter* packaging fees arises precisely from the sales-driven incentive

Sales Incentives: Comparison with Mixed Oligopoly

Both mixed oligopoly and sales incentives raise the optimal fee.
But they work through *different* BRF mechanisms:

	Mixed oligopoly	**Sales incentives**
BRF affected	BRF_2 only	Both BRFs
Mechanism	Slope steepens	Intercept shifts out
Source of distortion	Public ownership	Managerial contract
Asymmetric equilibrium?	Yes	No (symmetric)
Firms over-produce?	Firm 2 only	Both firms
Fee direction	$t^* \uparrow$	$t^* \uparrow$

- Key difference: sales incentives preserve the *symmetry* of the

Sales Incentives: Endogenous μ?

- In this chapter μ is taken as **exogenous** (determined by labor market conditions, corporate culture, or owner–manager contracts).
- **Two-stage extension:** Owners choose μ in stage 1 (before output competition) to maximize *profit* (not utility U):
  - If rivals use $\mu > 0$, best response is to also use $\mu > 0$ (commit to aggressive behavior).
  - Result: all owners choose $\mu > 0$ in equilibrium even though joint profits are lower — a classic Prisoner's Dilemma in contracting.
- **Implication for environmental policy:**
  - The EA cannot simply recommend that firms reduce μ — the incentive to use sales contracts is robust to competitive pressures.
  - The correct policy response is to set a *higher* t to correct the resulting externality, not to regulate managerial contracts directly.

Section 8.6

Cross-Ownership

Cross-Ownership: Motivation and Setup

- Firms in concentrated industries frequently hold **partial equity stakes** in their rivals.
- Examples: airline alliances, energy generators, automotive joint ventures.
- **Key parameter:** $\delta \in [0, \frac{1}{2}]$ = fraction of rival's profits internalized.
  - $\delta = 0$: no cross-ownership (standard Cournot).
  - $\delta = \frac{1}{2}$: symmetric cross-ownership (intermediate between competition and monopoly).
- Each firm i maximizes:

$$\Pi_i = \pi_i + \delta\pi_j$$

- **Strategic effect:** Firm i internalizes part of rival j's profit — competing less aggressively benefits j and therefore i through the cross-ownership link.

Cross-Ownership: FOC and Best-Response Function

Firm i maximizes $\Pi_i = \pi_i + \delta\pi_j$:

$\pi_i = (1 - q_i - q_j)q_i - (c+t)q_i, \quad \pi_j = (1 - q_i - q_j)q_j - (c+t)q_j$

FOC with respect to q_i:

$$\frac{\partial \Pi_i}{\partial q_i} = (1 - 2q_i - q_j - c - t) + \delta(-q_j) = 0$$

$$1 - 2q_i - q_j - c - t - \delta q_j = 0$$

Best-response function:

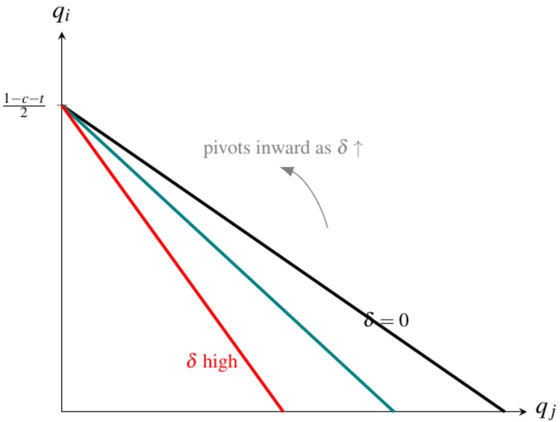
$$q_i(q_j) = \frac{1-c-t}{2} - \frac{1+\delta}{2} q_j$$

- **Slope steepens** to $-\frac{1+\delta}{2} < -\frac{1}{2}$ for $\delta > 0$.
- Intercept unchanged: $\frac{1-c-t}{2}$.
- Cross-ownership *rotates* BRF clockwise — stronger strategic substitutes.

Intuition: Why Does Cross-Ownership Steepen BRF?

- When firm i increases q_i, it hurts rival j's profit (market price falls).
- But firm i *owns* a fraction δ of j's profits — so hurting j hurts i through the ownership link.
- Therefore, firm i wants to **cut back more** when j expands, to avoid harming j's (and thus its own) profits.
- Mathematically: the cross-partial term $\delta(-q_j)$ in the FOC adds an extra incentive to reduce q_i when q_j is large $\Rightarrow$ BRF slope is $-\frac{1+\delta}{2}$ (steeper).
- **Compared to mixed oligopoly:** Both steepen BRF_2, but cross-ownership steepens *both* BRFs symmetrically.

BRF Diagram: Cross-Ownership



- Both BRFs rotate *clockwise* (steepen) symmetrically as δ increases.
- Equilibrium shifts *inward*: each firm produces less.

Equilibrium Output with Cross-Ownership

Using symmetry ($q_1^* = q_2^* = q^*$):
Substitute $q_j = q_i$ into BRF:

$$q^* = \frac{1-c-t}{2} - \frac{1+\delta}{2} q^*$$

$$q^* \left(1 + \frac{1+\delta}{2}\right) = \frac{1-c-t}{2}$$

$$q^*(t, \delta) = \frac{1-c-t}{3+\delta}$$

$$Q^*(t, \delta) = \frac{2(1-c-t)}{3+\delta}$$

Effect of δ:

$$\frac{\partial q^*}{\partial \delta} = -\frac{1-c-t}{(3+\delta)^2} < 0$$

$\Rightarrow$ More cross-ownership $\Rightarrow$ **less output and fewer emissions**.

Special Cases of Equilibrium Output

δ	$q^*(t, \delta)$	Interpretation
0	$\dfrac{1-c-t}{3}$	Standard Cournot (no cross-ownership)
1/4	$\dfrac{1-c-t}{3.25}$	Moderate cross-ownership
1/2	$\dfrac{1-c-t}{3.5}$	Maximum symmetric cross-ownership
1	$\dfrac{1-c-t}{4}$	Merger equivalent (= duopoly $\to$ monopoly halfway)

- Output falls monotonically from Cournot toward the monopoly level as δ rises.
- At $\delta = 1$: firms fully internalize each other's profits $\Rightarrow$ joint profit maximization (monopoly output per firm).

Optimal Fee with Cross-Ownership

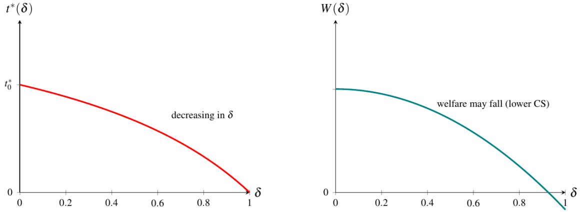
EA maximizes W substituting $Q^*(t,\delta) = \frac{2(1-c-t)}{3+\delta}$.

Key result:

$$t^*(\delta) < t^*\Big|_{\delta=0} \quad \text{for all } \delta > 0$$

- The optimal fee **decreases** in δ.
- **Intuition:** Cross-ownership already softens competition — aggregate output and emissions are lower even without the fee. The EA does not need to work as hard.
- At $\delta = 0$: $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ (Chapter 2 benchmark). ✓
- **Contrast:**
  - Mixed oligopolies and sales incentives $\Rightarrow$ t^* **rises**.
  - Cross-ownership $\Rightarrow$ t^* **falls**.
- Note: lower t^* does *not* mean better welfare — consumer surplus also falls due to reduced competition.

Fee and Welfare Under Cross-Ownership



- Left: $t^*(\delta)$ decreasing in δ.
- Right: Welfare may *decrease* even though emissions fall — the anticompetitive effect (lower CS) can dominate.
- Policy dilemma: antitrust regulators and environmental regulators may have conflicting interests.

Antitrust vs. Environmental Policy Under Cross-Ownership

- Cross-ownership creates a **policy tension**:
  - *Antitrust perspective:* Cross-ownership softens competition $\Rightarrow$ raises prices $\Rightarrow$ reduces consumer welfare $\Rightarrow$ regulators want to **restrict** δ.
  - *Environmental perspective:* Less output $\Rightarrow$ fewer emissions $\Rightarrow$ environmental agencies may **welcome** $\delta > 0$.
- The model clarifies the **exact trade-off**: the EA can set a lower t^* when $\delta > 0$, but the net welfare effect depends on the relative magnitude of d (damage parameter) vs. the loss in CS.
- For large d: the emission reduction benefit can dominate; some cross-ownership may be *socially desirable*.
- For small d: the anticompetitive effect dominates; antitrust concerns take precedence.

Cross-Ownership: Deriving Equilibrium — Step by Step

Step 1. BRF for firm i:

$$q_i(q_j) = \frac{1-c-t}{2} - \frac{1+\delta}{2} q_j$$

Step 2. Invoke symmetry $q_1^* = q_2^* = q^*$:

$$q^* = \frac{1-c-t}{2} - \frac{1+\delta}{2} q^*$$

Step 3. Collect:

$$q^* \left(1 + \frac{1+\delta}{2}\right) = \frac{1-c-t}{2}$$

$$q^* \cdot \frac{3+\delta}{2} = \frac{1-c-t}{2} \implies q^*(t,\delta) = \frac{1-c-t}{3+\delta}$$

Check at $\delta = 0$:

$$q^*(t,0) = \frac{1-c-t}{3} = \text{standard Cournot.}\checkmark$$

Cross-Ownership: Numerical Example

Let $c = 0$, $d = 1$, vary δ and t:

δ	$t = 0$	$t = 0.10$	$t = 0.20$
0	0.333	0.300	0.267
0.25	0.308	0.277	0.246
0.50	0.286	0.257	0.229

(Table entries: per-firm equilibrium output $q^*(t, \delta)$.)

- Output falls significantly with δ, for any fee level.
- The fee is less effective at reducing output because output is already low due to cross-ownership softening.
- EA needs only a *lower* t^* to achieve the social optimum.

Cross-Ownership: Welfare Implications

- **Environmental gain:** $Q^*(\delta) = \frac{2(1-c-t)}{3+\delta}$ is decreasing in $\delta \Rightarrow$ environmental damage $\frac{d}{2}Q^2$ is lower.
- **Consumer surplus loss:** $CS = \frac{Q^2}{2}$ is also lower $\Rightarrow$ consumers are harmed.
- **Producer surplus gain:** Higher price $p = 1 - Q$ and less aggressive competition raise profits.
- Net welfare effect depends on relative magnitudes:

$$\frac{dW}{d\delta} = \underbrace{(Q - 2dQ)\frac{\partial Q}{\partial \delta}}_{\text{output effect}} + \underbrace{\text{profit changes}}_{\text{ownership effect}}$$

- For large d: net welfare effect of cross-ownership can be *positive* (emission reduction dominates).
- For small d: net welfare effect is *negative* (consumer surplus loss dominates).

Cross-Ownership vs. Standard Cournot: Price Comparison

With $Q^*(t,\delta) = \frac{2(1-c-t)}{3+\delta}$, the equilibrium price is:

$$p^*(t,\delta) = 1 - Q^* = 1 - \frac{2(1-c-t)}{3+\delta} = \frac{(1+c+t)(3+\delta) + 2t - 2(1 -}{3+\delta}$$

Simplified:

$$p^*(t,\delta) = \frac{(3+\delta)(c+t) + 1 + \delta}{3+\delta}$$

- **Price rises** with δ: less competition $\Rightarrow$ higher market price $\Rightarrow$ consumers pay more.
- At $\delta = 0$: standard Cournot price.
- The cross-ownership arrangement benefits *producers* and *the environment*, but harms *consumers*.
- This three-way trade-off is the key insight of Section 8.6.

Recap: Cross-Ownership (Section 8.6)

- **Setup:** Each firm i maximizes $\pi_i + \delta\pi_j$ with $\delta \in [0, \frac{1}{2}]$.
- **BRF effects:**
  - Intercept unchanged: $\frac{1-c-t}{2}$.
  - Slope steepens to $-\frac{1+\delta}{2}$ $\Rightarrow$ inward rotation.
- **Equilibrium:**
  - $q^*(t,\delta) = \frac{1-c-t}{3+\delta}$ — decreases in δ.
  - Aggregate output and emissions **fall** with δ.
- **Optimal fee:**
  - $t^*(\delta)$ **decreases** in δ.
  - More cross-ownership $\Rightarrow$ more lenient fee.
  - But antitrust concerns may counteract environmental benefits.

Cross-Ownership: Comparison with Standard Cournot

Why does cross-ownership steepen BRF while sales incentives shift the intercept outward?

- **Sales incentives:** the distortion enters as an *additive* term in the perceived marginal cost. This shifts the BRF's vertical intercept but does not change how aggressively firm i responds to q_j.
- **Cross-ownership:** the distortion enters as a *multiplicative* interaction with the rival's output ($-\delta q_j$ in the FOC). This changes the slope of the BRF — firm i cuts output more when q_j rises (because expanding hurts the rival it co-owns).
- **Geometric summary:**
  - Sales: BRF slides up/right (outward shift).
  - Cross-ownership: BRF rotates clockwise around the vertical axis (steepens, intercept fixed).
- This distinction has empirical implications: one can distinguish the two mechanisms by estimating whether BRF

Cross-Ownership: Limit Cases

- $\delta = 0$ **(no cross-ownership):** Standard Cournot.

$$q^* = \frac{1 - c - t}{3}, \quad p^* = \frac{1 + 2(c + t)}{3}$$

- $\delta = 1$ **(full internalization):** Joint profit maximization.

$$q^* = \frac{1 - c - t}{4}, \quad p^* = \frac{1 + 3(c + t)}{4}$$

  (Same as a monopolist splitting production equally between two plants.)

- $\delta = 1/2$ **(symmetric cross-ownership):**

$$q^* = \frac{1 - c - t}{3.5} = \frac{2(1 - c - t)}{7}$$

  Output is between Cournot and monopoly — "softened duopoly."

- As δ rises from 0 to 1: output falls, price rises, emissions fall, and the optimal fee falls.

Section 8.7

Case Studies: Cross-Ownership

Case Study 7: Airlines — Delta and Virgin Atlantic

- **Setting:** Delta Air Lines holds a significant equity stake in Virgin Atlantic ($\sim 49\%$) — a direct instance of cross-ownership with δ close to 0.49.
- **Theoretical prediction:** Both carriers should internalize a large fraction of the other's profit $\Rightarrow$ less aggressive capacity competition $\Rightarrow$ fewer flights at higher prices.
- **Observed behavior:** Studies of transatlantic routes where Delta and Virgin overlap show higher fares and reduced frequency compared to fully competitive segments.
- **Environmental implication:** Fewer flights $\Rightarrow$ lower aviation CO_2 on the affected routes; EA could set a lower carbon levy for these routes relative to fully competitive ones.
- **Antitrust tension:** DOT granted antitrust immunity to the Delta–Virgin Atlantic joint venture — regulators accepted the trade-off.

Case Study 8: Energy Sector — Overlapping Ownership

- **Setting:** Institutional investors (BlackRock, Vanguard) hold large positions across multiple competing electricity generators.
- **Cross-ownership channel:** The same shareholders benefit from all generators' profits $\Rightarrow$ effectively high δ across the sector.
- **Observed behavior:** Azar, Schmalz, and Tecu (2018) find that common ownership in airlines significantly raises ticket prices; similar results arise in energy markets.
- **Environmental implication:** Less aggressive generation competition $\Rightarrow$ lower aggregate electricity output $\Rightarrow$ lower emissions. EA may set a less stringent carbon price in such markets.
- **Policy debate:** The Federal Trade Commission and DOJ are scrutinizing common ownership precisely because of antitrust concerns — the antitrust–environment trade-off from the

Case Study 9: Automotive — Toyota-Subaru Cross-Ownership

- **Setting:** Toyota holds approximately 20% equity in Subaru (Fuji Heavy Industries). Both compete in overlapping vehicle segments.
- **Theoretical prediction:** With $\delta \approx 0.20$: $q^*(t, 0.20) = \frac{1-c-t}{3.20}$ — output is lower than standard Cournot but far above monopoly.
- **Observed behavior:** Toyota and Subaru collaborate on joint production platforms (e.g., 86/BRZ sports cars), reducing duplicate R&D and manufacturing — consistent with softened competition.
- **Environmental implication:** Shared platforms and less aggressive market competition may reduce total fleet size marginally, lowering emissions; but the effect is small for $\delta = 0.20$.
- **Regulatory insight:** Emission standards (CAFE in the US

Section 8.8

Comparative Analysis and Summary

How Do Managerial Incentives Change the Optimal Fee?

Incentive type	**BRF effect**	**Output/emissions**	**Op**
Benchmark (Ch. 2)	—	$\frac{1-c-t}{3}$	t^*
Mixed oligopoly ($\alpha\uparrow$)	BRF_2 steepens	Q neutral	t^*
Sales incentives ($\mu\uparrow$)	Both BRF shift out	$Q\uparrow$	
Cross-ownership ($\delta\uparrow$)	Both BRF steepen	$Q\downarrow$	

Key insight: *Sales incentives* push firms to *overproduce* relative to the profit-maximizing benchmark $\Rightarrow$ EA needs a *stricter* fee. *Cross-ownership* induces *underproduction* relative to competition $\Rightarrow$ EA can afford a *more lenient* fee. *Mixed ownership* is *neutral*:

Mechanism Comparison: BRF Effects

Incentive	Slope of BRF	Intercept of BI
Benchmark	$-1/2$	$(1 - c - t)/2$
Mixed oligopoly ($\alpha > 0$)	$< -1/2$ (BRF_2 only)	Changes (BRF_2 o
Sales incentives ($\mu > 0$)	$-1/2$ (unchanged)	$\uparrow$ (both BRFs
Cross-ownership ($\delta > 0$)	$< -1/2$ (both BRFs)	$(1 - c - t)/2$ (unch

- **Sales incentives:** intercept shift only (output level up, no change in strategic substitutability).
- **Cross-ownership and mixed oligopoly:** slope change (strategic interaction intensifies).

Equilibrium Outputs: Side-by-Side Comparison

Setting	Per-firm equilibrium output
Benchmark	$\dfrac{1-c-t}{3}$
Sales incentives ($\mu > 0$)	$\dfrac{1-(1-\mu)(c+t)}{3} > \dfrac{1-c-t}{3}$
Cross-ownership ($\delta > 0$)	$\dfrac{1-c-t}{3+\delta} < \dfrac{1-c-t}{3}$

- Both deviations from standard Cournot are analytically clean:
  - Sales incentives: effective cost $(1-\mu)(c+t)$ replaces $c+t$ in numerator.
  - Cross-ownership: δ is added to the denominator.
- The public-firm case (mixed oligopoly) is asymmetric and requires separate formulas (8.9)–(8.10).

Policy Implications

1. **Mixed oligopolies:** When public firms compete with private rivals, the public firm's expansion is exactly offset by the private firm's contraction. By this *neutrality* result the socially optimal fee is *independent* of the degree of nationalization α—it equals the purely private benchmark.
2. **Sales-based compensation:** Industries where executives are rewarded on sales volumes warrant *stricter* regulation. The EA should monitor managerial compensation structures as an input to environmental policy design.
3. **Cross-ownership:** Industries with significant cross-holdings (airlines, energy, automotive) may need *less stringent* emission fees, but policymakers must weigh environmental gains against antitrust costs (lower consumer surplus).
4. **General lesson:** The structure of firm objectives — not just industry concentration — determines the appropriate stringency of environmental regulation.

Key Equations: Quick Reference

Equation	Description
(8.1)	Private firm 1 profit: $\pi_1 = (1 - Q)q_1 - (c + t)q_1$
(8.2)	Public firm 2 objective: $V = \alpha W + (1 - \alpha)\pi_2$
(8.3)	BRF_1: $q_1 = \frac{1-c-t}{2} - \frac{1}{2}q_2$
(8.4)	BRF_2: $q_2 = \frac{1-c-t(1-\alpha)}{2-\alpha(1-d)} - \frac{1+\alpha d}{2-\alpha(1-d)}q_1$
(8.5)	Slope: $\frac{\partial s}{\partial \alpha} < 0$, $\frac{\partial s}{\partial d} < 0$
(8.9)	$q_2(t) = \frac{(1-c-t)(1-\alpha d)+2\alpha t}{3-\alpha(2-d)}$
(8.10)	$q_1(t) = \frac{(1-c-t)(1+\alpha d)-\alpha(1-c)}{3-\alpha(2-d)}$

Key Equations: Sales Incentives and Cross-Ownership

Setting	**Key formula**
Sales BRF	$q_i(q_j) = \frac{1-(1-\mu)(c+t)}{2} - \frac{1}{2}q_j$
Sales equilibrium	$q^* = \frac{1-(1-\mu)(c+t)}{3}$
Sales fee direction	$t^*(\mu)$ increasing in μ
Cross-ownership BRF	$q_i(q_j) = \frac{1-c-t}{2} - \frac{1+\delta}{2}q_j$
Cross-ownership equilibrium	$q^* = \frac{1-c-t}{3+\delta}$
Cross-ownership fee direction	$t^*(\delta)$ decreasing in δ

EA's Information Requirements Under Each Incentive Type

- To set the optimal fee, the EA must know more than just industry concentration. It needs information about:
  1. **Mixed oligopoly:** The degree of nationalization α of the public firm (publicly observable in principle).
  2. **Sales incentives:** The managerial compensation parameter μ (often not disclosed; may require auditing executive contracts or inferred from output data).
  3. **Cross-ownership:** The cross-holding fraction δ (partly observable via equity filings, but beneficial ownership may be hidden).
- **Information rents:** If firms can choose μ or δ without the EA's knowledge, they may exploit information asymmetry to avoid strict regulation.
- **Connection to Chapter 7:** Asymmetric information about firm objectives makes optimal fee design even harder.

Broader Context: Managerial Incentives in Policy Design

- **Standard regulatory theory** assumes profit maximization. Chapter 8 shows this assumption matters.
- **Practical policy relevance:**
  - Many sectors combine state and private ownership (utilities, banking, transport, telecoms).
  - Executive pay tied to sales or market share is pervasive in consumer goods, technology, and auto industries.
  - Institutional cross-ownership is a growing phenomenon documented in finance research (Azar et al.; He and Huang).
- **A unified message:** Environmental policy should be *context-specific*. A "one-size-fits-all" emission fee calibrated on the profit-maximization benchmark will be:
  - *Too lenient* under sales incentives.
  - *Too strict* (or unnecessary) under cross-ownership.
  - *Exactly right* in mixed oligopolies (the neutrality result).

Modeling Summary: Common Structure Across Sections

All three models in Chapter 8 share a common structure:

1. **Stage 1:** EA sets emission fee t.
2. **Stage 2:** Firms compete in quantities given t and their distorted objectives.

The EA solves backward induction:

$$\max_t W\left(q_1^*(t,\theta), q_2^*(t,\theta)\right)$$

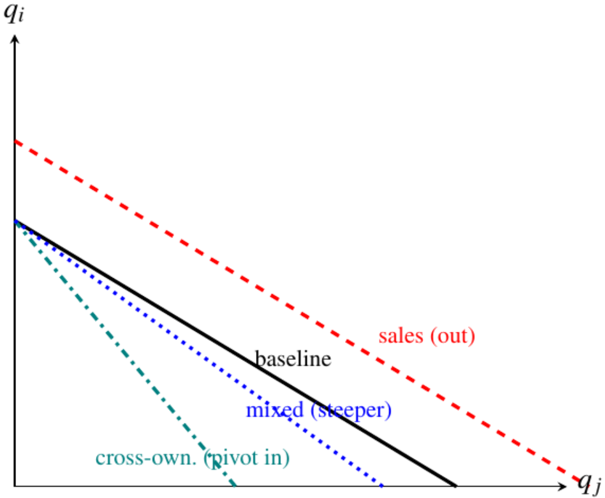
where $\theta \in \{\alpha, \mu, \delta\}$ is the distortion parameter.

- The key comparative static is $\partial t^*/\partial\theta$.
- Mixed oligopoly ($\theta = \alpha$) and sales incentives ($\theta = \mu$): $\partial t^*/\partial\theta > 0$.
- Cross-ownership ($\theta = \delta$): $\partial t^*/\partial\theta < 0$.

Chapter 8: Summary

- **Mixed oligopolies** (Section 8.2):
  - Public firm expands, but the private firm contracts by the same amount.
  - **Neutrality:** t^* is independent of α (equals the Chapter 2 benchmark).
- **Sales-based incentives** (Section 8.4):
  - Revenue weighting in managerial pay leads to *overproduction*.
  - EA needs a **stricter** fee; t^* rises with μ.
- **Cross-ownership** (Section 8.6):
  - Equity stakes in rivals soften competition, leading to *underproduction*.
  - EA can set a **more lenient** fee; t^* falls with δ.
  - But antitrust concerns may dominate for welfare.
- **Common thread:** The EA must understand how firm-level incentives shape market outcomes before designing the optimal environmental instrument.

Comparing BRF Diagrams: All Three Cases



- Left: Mixed oligopoly — one BRF (BRF_2) steepens; BRF_1

Welfare Rankings Under Each Incentive Type

Setting	CS	Emissions	Net W
Benchmark ($\alpha = \mu = \delta = 0$)	W^0_{CS}	E^0	W^0
Mixed oligopoly ($\alpha > 0$)	–	–	– (neutral)
Sales incentives ($\mu > 0$)	$\uparrow$	$\uparrow$	ambiguous
Cross-ownership ($\delta > 0$)	$\downarrow$	$\downarrow$	ambiguous

- In all cases, *under the optimal fee*, net welfare W^* is restored to the socially optimal level. For sales incentives and cross-ownership the EA adjusts t^*; for mixed ownership no adjustment is needed (neutrality). The arrows above are measured at the benchmark fee.
- **Caveat:** If the EA observes α, μ, or δ imperfectly and uses the wrong t, welfare can be higher or lower than the benchmark.

Connections to Other Chapters

- **Chapter 2 benchmark:** All three sections reduce to the standard Cournot–Nash result with $t^* = \frac{(1-c)(2d-1)}{2(1+d)}$ when the distortion parameter is zero ($\alpha = 0$, $\mu = 0$, or $\delta = 0$).
- **Chapters 3–4 (R&D and abatement):** Managerial incentives can similarly distort firms' abatement choices; future extensions could combine μ or δ with the abatement technology of Chapter 3.
- **Chapter 6 (international trade):** Cross-ownership is especially prevalent across borders; the fee analysis of Chapter 8 would interact with trade policy instruments studied in Chapter 6.
- **Chapter 7 (asymmetric information):** Regulators may face additional information problems when managerial objective functions are unobservable — combining Sections 8.4 and Chapter 7 is a natural extension.

Comparative Statics: Three Incentive Types at a Glance

- Each panel shows equilibrium output and optimal fee as the distortion parameter (α, μ, or δ) varies from 0 to its maximum.
- Sales incentives: fee rises in μ. Mixed oligopoly: fee is **flat** in α (neutrality).
- Cross-ownership: fee falls; output falls even faster.

Combining Incentive Distortions

- What if a mixed duopoly *also* has sales-based managerial incentives?
- Or if cross-owning firms are sales-oriented?
- Qualitative principles carry over, but magnitudes change:
  - Mixed oligopoly + sales incentives: mixed ownership is fee-neutral, so the sales force alone drives a *stricter* fee.
  - Cross-ownership + sales incentives: opposing forces; net effect on t^* depends on relative magnitudes of δ and μ.
- This chapter analyzes each distortion *in isolation* as a building block; real-world analysis requires combining them.
- **General principle:** Any factor that increases equilibrium output (at the benchmark fee) calls for a stricter fee; any factor that reduces output calls for a more lenient fee.

Design of Environmental Policy: Lessons from Chapter 8

1. **Observe firm ownership structure.** State ownership does *not* change the optimal fee—the public firm's expansion is offset by the private rival's contraction (neutrality). Sales incentives and cross-ownership, in contrast, do move the fee.
2. **Examine executive compensation contracts.** Sales-weighted bonuses are a signal that the EA should set stricter standards.
3. **Map equity cross-holdings.** Industries with significant cross-ownership may be self-regulating their output; the EA can be more lenient but must watch for antitrust effects.
4. **Update policy dynamically.** As ownership structures or compensation schemes change, the optimal fee should be recalibrated.
5. **Coordinate with antitrust authorities.** Cross-ownership creates shared jurisdiction between competition and

Review Questions

1. In the mixed duopoly, why is BRF_1 unaffected by α, while BRF_2 depends on both α and d?
2. Show that eqs. (8.9) and (8.10) reduce to the standard Cournot output $\frac{1-c-t}{3}$ when $\alpha = 0$.
3. Explain intuitively why the optimal fee $t^*(\alpha)$ is *independent* of α (the neutrality result). What happens to consumer surplus as α rises?
4. In the sales-incentives model, the BRF slope is unchanged at $-1/2$. Why does equilibrium output still rise with μ?
5. Show that equilibrium output under cross-ownership reduces to the standard Cournot level when $\delta = 0$ and to the monopoly output level when $\delta = 1$.
6. Under what condition on d (the damage parameter) could cross-ownership be *welfare-improving* despite reducing consumer surplus?

Thank You

Environmental Regulation: A Game-Theoretic Approach

Ana Espinola-Arredondo and Felix Muñoz-Garcia

End of Chapter 8 Slides