

Eco-Labeling with Noisy Certification Tests

A Toy-Model Walkthrough of Mason (2011)

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Roadmap

- **Motivation.** Eco-labels and the market failure they try to fix.
- **The toy model.** Single-unit sellers, one precision dial τ .
- **No-information benchmark.** A pooling price $P_0 = \lambda$.
- **The noisy test.** Type I and Type II errors, controlled by τ .
- **Bayesian updating.** $P_c = \mu$, $P_u = \nu$.
- **Application payoffs.** $W_G > W_B$.
- **Four equilibrium regions.** Pooling, partial, separating.
- **A numerical example.**

Primary source: Mason, C. F. (2011), “Eco-Labeling and Market Equilibria with Noisy Certification Tests,” *Environmental and Resource Economics* 48(4): 537–560.

Eco-Labels Are Everywhere

- An *eco-label* is a third-party certification that a product meets an environmental standard.
- **Blue Angel (Germany)**. Since 1978; today over 12,000 products.
- **USDA Organic (US)**. Since 2000; a \$63B market in 2022.
- **Fair Trade, MSC, FSC, LEED, Energy Star, EU Ecolabel, Nordic Swan.**
- In every case, consumers cannot verify the environmental attribute on their own, so they rely on the label.

The Market Failure, and a Noisy Fix

- Some consumers pay a premium for green products, but the technology is *not observable* at the point of sale.
- Absent certification, green and brown pool at one price: a lemons problem.
- A label is a third-party signal that tries to solve it, but certification is *imperfect*:
 - some green products *fail* (Type I error);
 - some brown products *slip through* (Type II error).

Given a noisy test, what happens to prices, sorting, and who applies?

Preview of the Main Results

- **Sorting.** Labeled goods are strictly greener than the population; unlabeled goods strictly browner.
- **Ordered incentives.** Green sellers have a strictly stronger incentive to certify than brown sellers.
- **Four equilibrium regions** as the fee rises: pooling (both apply), partially separating, separating, and pooling (no one applies).
- Prices equal beliefs, and every condition is linear in the belief gap $\mu - \nu$.

Section 1

The Toy Model

Consumers: Prices Are Beliefs

- Consumers value a green unit more than a brown one but cannot tell them apart. Demand is perfectly elastic.
- **Normalize** the willingness to pay:

$$v_G = 1, \quad v_B = 0.$$

- A consumer who believes a unit is green with probability μ pays

$$P(\mu) = \mu v_G + (1 - \mu) v_B = \mu.$$

Prices are beliefs. This is the identification the whole analysis rests on.

Sellers: One Unit Each

- A unit mass of sellers, each offering *one unit*.
- A fraction $\lambda \in (0, 1)$ are green, $1 - \lambda$ brown; each seller privately knows its type.
- Because each seller brings one unit, its payoff is simply the price it receives, net of any certification fee.

The single-unit structure is what keeps every equilibrium condition linear in the beliefs (no output choice, no convex costs).

Section 2

The No-Information Benchmark

The Pooling Price without a Label

- With no label, the whole market pools. Consumers pay the expected willingness to pay for a random unit:

$$P_0 = \lambda v_G + (1 - \lambda) v_B = \lambda.$$

- Every seller, green or brown, receives $P_0 = \lambda$.

This is the **reference price** the label is measured against, and the fallback whenever the fee is too high for anyone to apply.

Section 3

The Noisy Certification Test

One Precision Dial τ

- A seller that applies pays a fee $A \geq 0$ and gets a public verdict. Index the test by a single *precision* $\tau \in [0, 1]$:

$$\phi_G = \Pr(\text{pass} \mid G) = \frac{1 + \tau}{2}, \quad \phi_B = \Pr(\text{pass} \mid B) = \frac{1 - \tau}{2}.$$

- **Type I error** $= 1 - \phi_G = \frac{1 - \tau}{2}$ (green fails).
- **Type II error** $= \phi_B = \frac{1 - \tau}{2}$ (brown passes).
- Equal error rates, so informativeness is $\phi_G - \phi_B = \tau$.

The Confusion Matrix

	Labeled (pass)	Unlabeled (fail)
Green seller	$\frac{1 + \tau}{2}$	$\frac{1 - \tau}{2}$ (type I)
Brown seller	$\frac{1 - \tau}{2}$ (type II)	$\frac{1 + \tau}{2}$

The diagonal (correct classification) carries mass $\frac{1 + \tau}{2}$; each off-diagonal error carries mass $\frac{1 - \tau}{2}$. A more precise test moves mass onto the diagonal.

The Two Extremes

$\tau = 0$	Lab.	Unlab.
Green	$\frac{1}{2}$	$\frac{1}{2}$
Brown	$\frac{1}{2}$	$\frac{1}{2}$

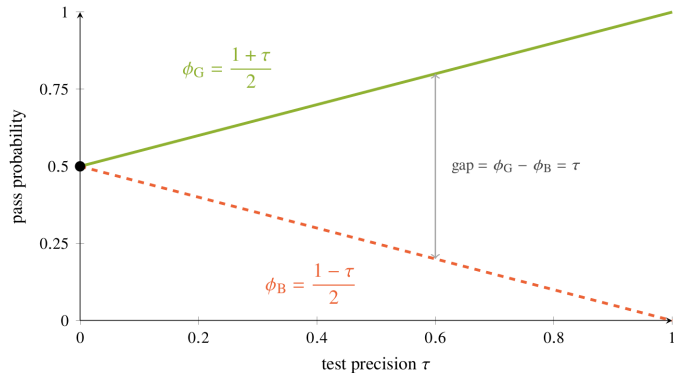
(a) Uninformative.

$\tau = 1$	Lab.	Unlab.
Green	1	0
Brown	0	1

(b) Perfect.

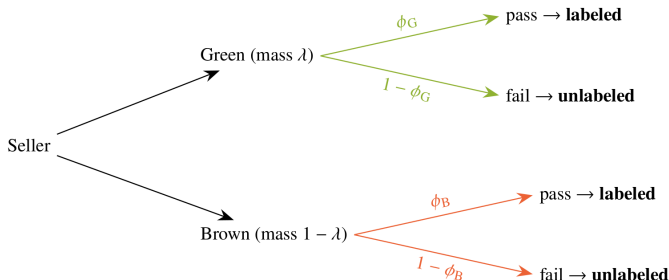
At $\tau = 0$ the rows are identical: a pass says nothing. At $\tau = 1$ the errors vanish: the label separates perfectly.

Pass Probabilities against τ



The two lines start together at $\frac{1}{2}$ ($\tau = 0$) and separate to 1 and 0 ($\tau = 1$). The vertical gap is the informativeness τ .

The Four Flows through the Test



Failures and non-applicants pool together in the *unlabeled* segment.

Section 4

Posterior Beliefs and Prices

Posteriors (Bayes' Rule)

Suppose every seller applies. Since each brings one unit, the beliefs are plain mass ratios:

$$\mu = \Pr(G \mid \text{labeled}) = \frac{\lambda\phi_G}{\lambda\phi_G + (1-\lambda)\phi_B},$$

$$\nu = \Pr(G \mid \text{unlabeled}) = \frac{\lambda(1-\phi_G)}{\lambda(1-\phi_G) + (1-\lambda)(1-\phi_B)}.$$

At $\lambda = \frac{1}{2}$ these collapse to $\mu = \frac{1+\tau}{2}$ and $\nu = \frac{1-\tau}{2}$: the posteriors equal the pass probabilities.

Prices Equal Posteriors; Sorting

- Competition sets each segment's price to the expected WTP, i.e. the posterior:

$$P_c = \mu, \quad P_u = \nu, \quad P_0 = \lambda.$$

- **Result 1 (sorting).** Any $\tau > 0$ gives $\phi_G > \phi_B$, hence

$$\nu < \lambda < \mu.$$

The label makes the labeled segment greener and the unlabeled segment browner, with a gap proportional to the precision τ and to $\lambda(1 - \lambda)$.

Section 5

Application Payoffs and Ordered Incentives

The Gain from Applying

- Applying: labeled (price μ) with prob ϕ_k , unlabeled (price ν) with prob $1 - \phi_k$, less the fee A .
- Not applying: unlabeled for sure, price ν .
- Net gain, **linear** in the price gap:

$$W_k = [\phi_k \mu + (1 - \phi_k) \nu - A] - \nu = \phi_k (\mu - \nu) - A.$$

No squares. This is the payoff to compare across types.

Ordered Incentives: $W_G > W_B$

Subtracting, and using $\phi_G - \phi_B = \tau$,

$$W_G - W_B = (\phi_G - \phi_B)(\mu - \nu) = \tau(\mu - \nu) > 0.$$

Intuition. The two types face the same price gap; the green seller simply multiplies it by a larger pass probability.

Corollary. No equilibrium has some brown sellers applying while no green sellers do: if brown are in, greens are in too.

Section 6

The Equilibrium Classes

Four Regions, Three PBE Types

As the fee A rises, the equilibrium is one of:

- **Pooling PBE (both apply)** — low fee.
- **Partially separating PBE** — brown mix — intermediate fee.
- **Separating PBE** — only green apply — high fee.
- **Pooling PBE (no seller applies)** — prohibitive fee, back to no information.

Both “both apply” and “no one applies” are *pooling* (both types take the same action). The cutoffs are all in closed form.

Pooling and Separating Cutoffs

- **Pooling PBE (both apply)** requires $W_B \geq 0$:

$$A \leq \phi_B(\mu - \nu) \equiv \bar{A}_{\text{pool}}.$$

- **Separating PBE:** only green apply, so $\hat{\mu} = 1$,

$$\hat{\nu} = \frac{\lambda(1 - \phi_G)}{\lambda(1 - \phi_G) + (1 - \lambda)}, \text{ and the fee must satisfy}$$

$$\underbrace{\phi_B(1 - \hat{\nu})}_{\text{brown: do not apply}} \leq A \leq \underbrace{\phi_G(1 - \hat{\nu})}_{\text{green: do apply}}.$$

Partially Separating PBE

For fees in the gap $\bar{A}_{\text{pool}} < A < \phi_B(1 - \hat{v})$, brown sellers mix with probability $\lambda_B \in (0, 1)$ solving the indifference condition $\phi_B(\mu^* - \nu^*) = A$, where

$$\mu^* = \frac{\lambda\phi_G}{\lambda\phi_G + (1 - \lambda)\lambda_B\phi_B}, \quad \nu^* = \frac{\lambda(1 - \phi_G)}{\lambda(1 - \phi_G) + (1 - \lambda)(1 - \lambda_B\phi_B)}.$$

These interpolate: $\lambda_B = 1$ gives the pooling posteriors, $\lambda_B = 0$ the separating ones.

The Mixing Probability Is Interior

Write the brown seller's gain from applying as $g(\lambda_B) = \phi_B(\mu^* - \nu^*)$.

- More brown apply $\Rightarrow \mu^*$ falls, ν^* rises, so g is *strictly decreasing*.
- Endpoints: $g(0) = \underline{A}_{\text{sep}}$ and $g(1) = \overline{A}_{\text{pool}}$, with $g(0) > g(1)$.

A partially separating fee lies strictly between the two cutoffs, so $g(\lambda_B) = A$ has a unique solution $\lambda_B \in (0, 1)$.

Comparative Static: Higher Fee, Fewer Browns

Since g is decreasing, $\frac{\partial \lambda_B}{\partial A} = \frac{1}{g'(\lambda_B)} < 0$.

Dilution feedback. A higher fee deters brown applicants; fewer brown make the label cleaner (higher μ^* , wider gap), restoring indifference.

At $\lambda = \frac{1}{2}$, $\tau = \frac{4}{5}$:

Fee A	λ_B
$2/25 = 0.080$	1 (pooling)
0.084	0.61
0.088	0.25
$1/11 \approx 0.091$	0 (separating)

Precision, and the Green Seller

- **Effect of τ on λ_B is non-monotone:** higher τ lowers ϕ_B (deters) but widens $\mu^* - \nu^*$ (attracts). Either can dominate (as in Mason, 2011).
- **The green seller applies for sure.** At brown's indifference, $\mu^* - \nu^* = A/\phi_B$, so its net gain is

$$\phi_G(\mu^* - \nu^*) - A = A \frac{\phi_G - \phi_B}{\phi_B} = \frac{2A\tau}{1 - \tau} > 0.$$

Brown's mixing pins the label's value; the green seller still strictly profits, and its advantage grows with τ .

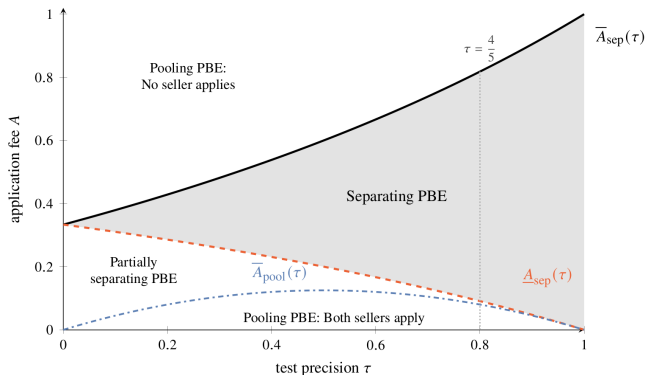
The Equilibrium Map (at $\lambda = \frac{1}{2}$)

At $\lambda = \frac{1}{2}$ the three cutoffs become

$$\bar{A}_{\text{pool}}(\tau) = \frac{\tau(1-\tau)}{2}, \quad \underline{A}_{\text{sep}}(\tau) = \frac{1-\tau}{3-\tau}, \quad \bar{A}_{\text{sep}}(\tau) = \frac{1+\tau}{3-\tau}.$$

These are exactly the cutoffs Mason (2011) obtains. As A rises the market moves: both apply $\rightarrow$ partially separating $\rightarrow$ separating $\rightarrow$ no one applies.

The Equilibrium Map: Figure



The separating region widens with precision; the both-apply pooling region is a thin sliver, largest at intermediate τ .

Section 7

A Numerical Example

Parameters and Prices

Take $\lambda = \frac{1}{2}$, $\tau = \frac{4}{5}$, $A = \frac{1}{20}$, so $\phi_G = \frac{9}{10}$, $\phi_B = \frac{1}{10}$.

- No-label price: $P_0 = \lambda = \frac{1}{2}$.
- Posteriors = prices:

$$\mu = \frac{1 + \tau}{2} = \frac{9}{10}, \quad \nu = \frac{1 - \tau}{2} = \frac{1}{10}, \quad \mu - \nu = \frac{4}{5}.$$

- $P_c = \frac{9}{10}$ and $P_u = \frac{1}{10}$ straddle $P_0 = \frac{1}{2}$.

Which Equilibrium? Gains

Pooling cutoff:

$$\bar{A}_{\text{pool}} = \phi_B(\mu - \nu) = \frac{1}{10} \cdot \frac{4}{5} = \frac{2}{25} = 0.08.$$

Since $A = \frac{1}{20} = 0.05 < 0.08$, we are in the **pooling PBE: both apply**.

Gains $W_k = \phi_k \cdot \frac{4}{5} - \frac{1}{20}$:

$$W_G = \frac{9}{10} \cdot \frac{4}{5} - \frac{1}{20} = \frac{67}{100}, \quad W_B = \frac{1}{10} \cdot \frac{4}{5} - \frac{1}{20} = \frac{3}{100}.$$

Both positive, $W_G > W_B$. Raising A above $\frac{2}{25}$ enters partial separation; above $\frac{1}{11}$, full separation.

Section 8

Wrap-up

What the Toy Model Keeps

- ① **Bayesian sorting**, $v < \lambda < \mu$, gap proportional to τ .
- ② **Ordered incentives**, $W_G > W_B$.
- ③ **Four equilibrium regions** with the same cutoffs Mason obtains, $\underline{A}_{\text{sep}} = \frac{1 - \tau}{3 - \tau}$, $\bar{A}_{\text{sep}} = \frac{1 + \tau}{3 - \tau}$.
- ④ **An interior, well-behaved mixing probability**: unique, decreasing in the fee, non-monotone in precision, leaving the green seller a gain $\frac{2A\tau}{1 - \tau}$.

What It Drops, and a Reference

Relative to Mason (2011) and the certifier-design literature, the toy model drops three things:

- single-unit sellers (no output choice, no producer-surplus welfare);
- the symmetric test ties the two error rates together at $\frac{1 - \tau}{2}$;
- precision τ is fixed, not chosen by a certifier or regulator.

Each can be reinstated one at a time.

Mason, C. F. (2011). "Eco-Labeling and Market Equilibria with Noisy Certification Tests." *Environ. Resource Econ.* 48(4): 537–560.

Questions?