Chapter 6

Regulation when Firms Invest in Abatement

Instructor Slides

Environmental Regulation: A Game-Theoretic Approach

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Outline of Chapter 6

- This chapter extends the baseline model by allowing firms to invest in abatement technologies that reduce emissions.
- Sections:
  - 6.1 Introduction – motivation and guiding questions.
  - 6.2 Modeling assumptions – three-stage game, abatement cost, spillovers.
  - 6.3 Non-cooperative abatement – no spillovers and with spillovers.
  - 6.4 Cooperative abatement – environmental R&D joint ventures (ERJVs).
  - 6.5 Sequential abatement decisions.
  - 6.6 Different regulatory timing – regulator moves first.
  - 6.7 Extensions. 6.8 Case studies. 6.9 Summary.

Section 6.1

Introduction

What this chapter studies

- Previous chapters examined how firms adjust *output* in response to emission fees, taking abatement technology as fixed.
- This chapter introduces a richer environment: firms choose both their output levels **and** how much to invest in abatement technologies.
- Examples of abatement investment: smokestack scrubbers, wastewater treatment systems, advanced filtration, carbon capture equipment.
- Key question: how do abatement investments interact with the regulator's emission fee? Does one "crowd out" the other?
- The baseline model from Chapter 2 is recovered as a special case when abatement is prohibitively expensive ($\gamma \to +\infty$).

Two new strategic layers

- **Firm–regulator interaction:** each firm's abatement investment lowers net emissions, inducing the regulator to set a *less stringent* fee – the **tax-saving effect (TSE)**.
- **Firm–firm interaction:** when abatement generates spillovers, one firm's investment also reduces its rival's net emissions – the **emission-reduction effect (ERE)**.
- These two externalities create **free-riding incentives** and determine whether firms should coordinate their abatement choices.
- This chapter systematically studies TSE and ERE under non-cooperative and cooperative abatement.

Chapter 6 in Context: Extensions of Chapter 2

- **Chapter 2:** Cournot duopoly + emission fee; no abatement investment by firms.
- **Chapter 6 adds:**
  - Abatement investment stage (before fee-setting).
  - Spillovers in abatement (β).
  - Cooperative abatement (ERC/ERJV).
  - Sequential timing in abatement.
  - Alternative regulatory timing (fee before abatement).
- **What changes:**
  - The optimal emission fee is lower (abatement partially corrects externality).
  - Firms' strategies now involve both output and abatement.
  - Free-riding incentives are richer: TSE and ERE.
- Chapter 6 provides a more complete picture of how firms respond to environmental regulation in the presence of clean technology investment.

Guiding Questions

1. What motivates firms to invest in abatement technologies under regulation?
2. How do spillovers in abatement affect strategic and cooperative behavior?
3. What are the welfare implications of environmental research joint ventures (ERJVs)?
4. How does the timing of abatement decisions influence equilibrium outcomes?
5. What is the role of regulatory commitment in shaping investment strategies?
6. How do different regulatory designs affect incentives for green innovation?

Real-World Examples of Abatement Investment

- **Smokestack scrubbers:** power plants install flue-gas desulfurization systems to capture SO_2 before it is emitted – high upfront cost, ongoing operational savings from lower fees.
- **Wastewater treatment:** chemical plants invest in treatment facilities to reduce toxic discharges – cost is sunk after installation (irreversible).
- **Carbon capture:** cement and steel producers pilot CCS (carbon capture and storage) technologies; rivals can observe and learn (spillovers).
- **Key regulatory interaction:** in each case, the regulator observes total industry abatement and adjusts the emission fee accordingly.
- This strategic interaction between abatement investment and fee-setting is the core mechanism analyzed in Chapter 6.

Section 6.2

Modeling Assumptions

Basic Setup

- Duopoly with inverse demand $p(Q) = 1 - Q$ and symmetric marginal cost c, where $1 > c \geq 0$.
- Firms can invest in **abatement technologies** $z_i \geq 0$ to reduce their net emissions.
- Without abatement: $e_i = q_i$ (emissions equal output).
- With abatement and **spillover parameter** $\beta \in [0, 1]$:

$$e_i = q_i - z_i - \beta z_j$$

- Three cases for β:
  - $\beta = 0$: no spillovers – firm i's emissions reduced only by its own investment.
  - $\beta \in (0, 1)$: partial spillovers – firm i benefits from a share β of rival's abatement.
  - $\beta = 1$: full spillovers – $e_i = q_i - z_i - z_j$.

Abatement Cost and Investment Efficiency

- Total cost of investing in abatement:

$$C(z_i) = \frac{\gamma(z_i)^2}{2}$$

  increasing and *convex* in abatement z_i.
- Parameter $\gamma > 0$ denotes **investment efficiency**:
  - Lower $\gamma \Rightarrow$ higher efficiency (abatement cheaper).
  - $\gamma \to 0$: abatement is essentially free.
  - $\gamma \to +\infty$: abatement is prohibitively costly; $z_i = 0$ and results collapse to Chapter 2.
- Marginal cost of abatement: γz_i – positive and increasing in z_i.
- Firm pays emission fee $te_i = t(q_i - z_i - \beta z_j)$ in stage 2.

Three-Stage Game (Poyago-Theotoky, 2007)

- **Stage 1 (Abatement):** Every firm i independently and simultaneously invests in abatement $z_i \geq 0$.
- **Stage 2 (Fee-setting):** The regulator observes investment decisions $z = (z_1, z_2)$ and sets per-unit emission fee $t \geq 0$.
- **Stage 3 (Output):** Every firm i observes z and t, then chooses output q_i (Cournot competition).
- Key feature: firms invest in abatement *before* the regulator sets the fee – strategic anticipation of regulatory response.
- Stages 2–3 coincide with Chapter 2; Stage 1 is new.
- Section 6.6 reverses this timing: regulator moves first.

Welfare Function with Abatement

- Welfare function retains the same structure as in previous chapters:
$$W = CS + PS + T - Env$$
- Aggregate abatement: $Z = z_1 + z_2$; aggregate emissions: $E = Q - Z$.
- Tax revenue: $T = tE = t(Q - Z)$.
- Environmental damage:
$$Env = \frac{d}{2}E^2 = \frac{d}{2}(Q - Z)^2$$
where $d \geq \frac{1}{2}$ measures **pollution severity**.
- Two channels through which abatement improves welfare:
  - *Direct*: reduces environmental damage for given output.
  - *Indirect*: lowers fee stringency, boosting output, CS and PS.

Backward Induction: Solving the Three-Stage Game

- We solve the game by **backward induction**: start from Stage 3 and work back to Stage 1.
- **Stage 3** $\rightarrow$ output as a function of (t, z_i, z_j).
- **Stage 2** $\rightarrow$ optimal fee as a function of (z_i, z_j).
- **Stage 1** $\rightarrow$ optimal abatement given anticipated fee and output.
- This approach ensures *subgame perfect* equilibrium: each firm's strategy is optimal at every stage, given the strategies of all other players.
- Key insight: firms in Stage 1 anticipate how their abatement affects the regulator's fee in Stage 2, which in turn affects output in Stage 3. This is the strategic "abatement for tax relief" mechanism.

Welfare Components with Abatement

- Welfare $W = CS + PS + T - Env$ with abatement:
  - $CS = \frac{Q^2}{2}$ with $Q = Q(t)$.
  - $PS = \sum_i \pi_i = \sum_i \left[q^2 + tz_i - \frac{\gamma z_i^2}{2}\right]$.
  - $T = tE = t(Q - Z)$ – tax revenue net of abatement.
  - $Env = \frac{d}{2}(Q - Z)^2$ – environmental damage on net emissions.
- Note: abatement cost $\frac{\gamma Z^2}{2}$ is *subtracted* from profits and thus from welfare – abatement is costly but improves the environment.
- Regulator maximizes W choosing t in Stage 2, taking $Z = (z_1 + z_2)$ as given (already chosen in Stage 1).
- The regulator does *not* control abatement directly; only the emission fee.

Section 6.3

Non-Cooperative Abatement

Stage 3: Output Competition (No Spillovers, $\beta = 0$)

- Each firm i chooses q_i to maximize:

$$(1 - q_i - q_j)q_i - cq_i - t(q_i - z_i)$$

- First-order condition yields best response: $q_i(q_j) = \frac{1-c-t}{2} - \frac{1}{2}q_j$.
- Symmetric equilibrium: each firm's output $q = \frac{1-c-t}{3}$, aggregate output $Q(t) = \frac{2(1-c-t)}{3}$.
- Equilibrium profit:

$$\pi_i = \frac{(1-c-t)^2}{9} + tz_i = q^2 + tz_i$$

- Profit π_i increases in abatement z_i because abatement lowers the fee the firm effectively pays.

Stage 2: Optimal Emission Fee

- Regulator sets t to maximize $W = CS + PS + T - Env$, anticipating equilibrium output $Q(t) = \frac{2(1-c-t)}{3}$.
- First-order condition (differentiating W w.r.t. t) yields:

$$t(Z) = \frac{(2d-1)(1-c) - 3dZ}{2(1+d)}$$

- Fee properties:
  - *Increases* with pollution severity d: $\frac{\partial t(Z)}{\partial d} > 0$ when $Z < 1 - c$.
  - *Decreases* with marginal cost c: $\frac{\partial t(Z)}{\partial c} < 0$.
  - **Decreases with aggregate abatement** Z: $\frac{\partial t(Z)}{\partial z_i} = -\frac{3d}{2(1+d)} < 0$.
- Key insight: when either firm invests more in abatement, the regulator anticipates fewer net emissions and *reduces* the fee.

Free-Riding via the Tax-Saving Effect

- Fee sensitivity is *symmetric*: $\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d}{2(1+d)} < 0$.
- This symmetry allows firm i to **free-ride** on its rival's investment z_j:
  - Firm j invests more in abatement.
  - Regulator reduces emission fee for everyone.
  - Firm i benefits from lower fee *without* investing.
- This **tax-saving effect (TSE)** occurs *with and without* spillovers ($\beta \geq 0$).
- Free-riding arises because every firm's investment reduces *total* emissions, inducing a less stringent fee that benefits all firms.

Stage 1: Abatement Best Response Function

- Each firm i chooses z_i to maximize profit, anticipating equilibrium fee $t(Z)$:

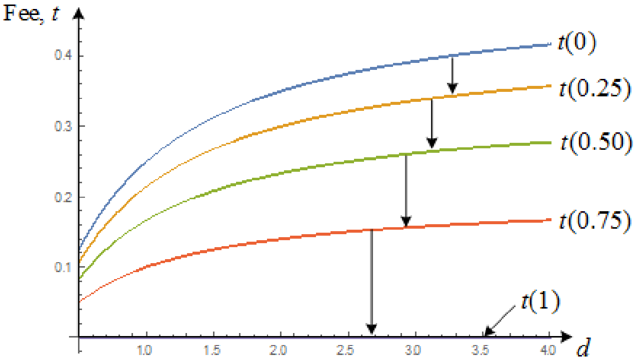
$$\max_{z_i \geq 0} \pi_i = q^2 + t(Z)z_i - \frac{\gamma(z_i)^2}{2}$$

- First-order condition yields best response function:

$$z_i(z_j) = \underbrace{\frac{[2d(1+d)-1](1-c)}{d(6+5d)+2\gamma(1+d)^2}}_{\text{vertical intercept}>0} - \underbrace{\frac{d(3+2d)}{d(6+5d)+2\gamma(1+d)^2}}_{\text{slope}>0} z_j$$

- **Negative slope**: abatement levels are *strategic substitutes*.
- Intuition: rival's abatement $\uparrow \Rightarrow$ fee $\downarrow \Rightarrow$ less incentive for firm i to invest.

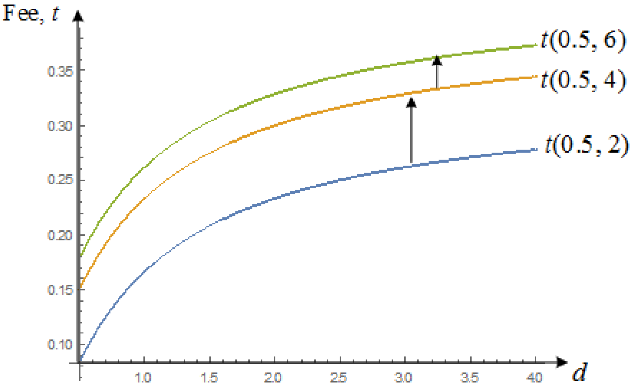
Figure 6.1 – Best Response Function $z_i(z_j)$



Effect of Pollution Severity on Best Response

- The **vertical intercept** shifts *upward* when pollution is more severe (higher d): $\frac{\partial z_i^{Vert.}}{\partial d} > 0$.
- Intuition: higher $d \Rightarrow$ more stringent fee $\Rightarrow$ stronger incentive to invest in abatement to reduce tax burden.
- The **slope** of the best response function becomes steeper (in absolute value) when $\gamma > \bar{\gamma} \equiv \frac{3d^2}{2(1+d)(3+d)}$.
- When abatement is costly (γ large), firms are more sensitive to rival's investment – free-riding incentives are *stronger*.
- Cutoff $\bar{\gamma}$ ranges between $\frac{1}{14}$ (at $d = \frac{1}{2}$) and $\frac{3}{2}$ (as $d \to \infty$).
- When $\gamma > \frac{3}{2}$, condition $\gamma > \bar{\gamma}$ holds for *all* values of d.

Figure 6.2 – Cutoff $\overline{\gamma}$ as a Function of d



Equilibrium Abatement (No Spillovers)

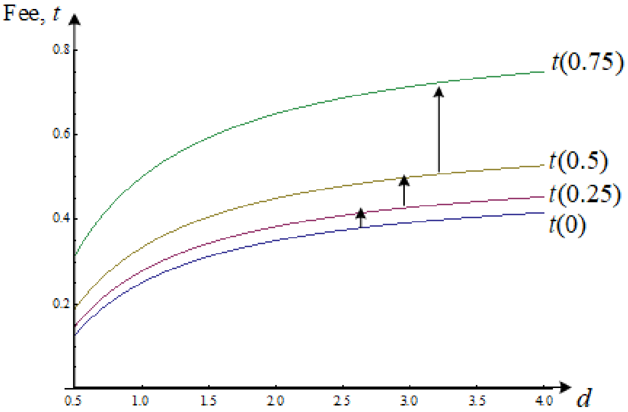
- In symmetric equilibrium ($z_i = z_j$), equilibrium abatement is:

$$z_i^* = \frac{(1-c)[2d(1+d)-1]}{2\gamma(1+d)^2 + d(9+7d)} > 0$$

(positive since $d > \frac{1}{2}$ by assumption).

- **Comparative statics:**
  - $\frac{\partial z_i^*}{\partial d} > 0$: higher pollution severity $\Rightarrow$ more abatement investment.
  - $\frac{\partial z_i^*}{\partial \gamma} < 0$: more expensive abatement $\Rightarrow$ less investment.
  - $\frac{\partial z_i^*}{\partial c} < 0$: higher marginal cost $\Rightarrow$ less output, fewer emissions, less stringent fee, less investment.

Figure 6.3 – Equilibrium Abatement z_i^*



Equilibrium Emission Fee (No Spillovers)

- Substituting $Z^* = 2z_i^*$ into $t(Z)$, equilibrium fee:

$$t(Z^*) = \frac{(1-c)[d(2\gamma - 3 + 2d(1+2\gamma)) - 2\gamma]}{4\gamma(1+d)^2 + 2d(9+7d)}$$

- Fee is **positive** if and only if:

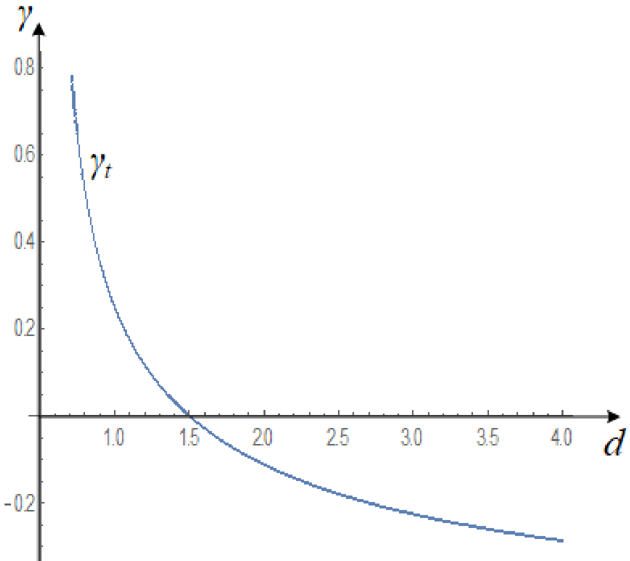
$$\gamma > \gamma_t \equiv \frac{(3-2d)d}{2(2d^2+d-1)}$$

- When $d > \frac{3}{2}$: cutoff $\gamma_t < 0$, so fee is always positive.
- When $d \leq \frac{3}{2}$: abatement must be sufficiently costly ($\gamma > \gamma_t$) for a positive fee.
- When $\gamma \leq \gamma_t$: firms invest heavily in abatement $\Rightarrow$ per-unit *subsidy* is socially optimal (not a fee).

Intuition: Fee vs. Subsidy

- **Why can the optimal instrument be a subsidy?**
  - When abatement is cheap (γ small), firms invest heavily, reducing net emissions substantially.
  - When pollution is also mild ($d \leq \frac{3}{2}$), environmental externality is small relative to the oligopoly distortion (underproduction).
  - Boosting output via a *subsidy* can dominate using an emission *fee*.
- **Key contrast with Chapter 2 (no abatement):**
  - Without abatement: the fee is always positive (emission fee needed to correct the externality).
  - With abatement: abatement itself partially corrects the externality, so the optimal fee can be zero or negative.

Figure 6.4 – Cutoff γ_t as a Function of d



Equilibrium Output (No Spillovers)

- Substituting $t(Z^*)$ into $q(t) = \frac{1-c-t}{3}$:

$$q(t(Z^*)) = \frac{(1-c)\left[d(7+4d) + 2\gamma(1+d)\right]}{4\gamma(1+d)^2 + 2d(9+7d)} > 0$$

- Output is *lower* when pollution is more severe (higher d) or abatement is more expensive (higher γ): the regulator sets a more stringent fee in both cases.
- Comparison with Chapter 2: when $\gamma \to +\infty$, abatement investment is zero and equilibrium output converges to the Chapter 2 result.
- When γ is finite, abatement lowers net emissions $\Rightarrow$ regulator reduces fee $\Rightarrow$ output is *higher* than in Chapter 2.

Section 6.3.2 – Spillovers in Abatement ($\beta > 0$)

- With spillovers, firm i's emissions become:

$$e_i = q_i - z_i - \beta z_j$$

- Sources of spillovers: non-patentable innovations, worker mobility (know-how transfer), geographical proximity (measurement errors).
- **Symmetric spillovers**: model assumes firm i equally benefits from any rival's investment (parameter β is identical for all pairs).
- **Asymmetric spillovers** (richer model): $e_1 = q_1 - z_1 - \beta_1 z_2$ and $e_2 = q_2 - z_2 - \beta_2 z_1$ with $\beta_1 \neq \beta_2$.
- We focus on symmetric case; equilibrium output in Stage 3 is unchanged: $q = \frac{1-c-t}{3}$ (spillovers affect output only indirectly via the fee).

Stage 2: Optimal Fee with Spillovers

- Regulator's optimal fee with spillovers:

$$t(Z) = \frac{(2d-1)(1-c) - 3d(1+\beta)Z}{2(1+d)}$$

- Coincides with equation (6.9) when $\beta = 0$.
- Fee sensitivity to abatement now *amplified* by spillovers:

$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d(1+\beta)}{2(1+d)} < 0$$

- As $\beta \uparrow$: each unit of abatement reduces emissions more (both firms benefit) $\Rightarrow$ regulator reduces fee more aggressively.
- Equilibrium output: $q = \frac{1-c+d(1+\beta)Z}{2(1+d)}$, increasing in aggregate abatement Z.

TSE and ERE: Two Externalities from Rivals' Abatement

- When firm j invests z_j, firm i benefits through two channels:
  1. **Tax-saving effect (TSE):** firm j's investment reduces aggregate emissions $\Rightarrow$ regulator lowers fee t $\Rightarrow$ firm i pays less per unit of net emissions.
  2. **Emission-reduction effect (ERE):** spillovers ($\beta > 0$) directly reduce firm i's per-unit emissions, lowering its tax bill even further.
- TSE is present *regardless* of spillovers ($\beta \geq 0$).
- ERE is present *only when* $\beta > 0$.
- Both effects create incentives for firm i to **free-ride** on firm j's abatement.
- With spillovers, the slope of the best response function is steeper $\Rightarrow$ stronger strategic substitutability.

Equilibrium Abatement with Spillovers (z^{NC})

- In symmetric equilibrium, non-cooperative abatement:

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$$

- Superscript NC = non-cooperative (firms do not coordinate).
- **Effect of spillovers (β):** Stronger spillovers $\uparrow \Rightarrow$ ERE becomes larger $\Rightarrow$ each firm free-rides more $\Rightarrow z^{NC}$ *decreases*.
- **Effect of pollution severity (d):** Higher $d \Rightarrow$ more stringent fee $\Rightarrow$ stronger incentive to invest $\Rightarrow z^{NC}$ *increases* (upward shift).
- **Effect of production cost (c):** Higher $c \Rightarrow$ less output, less stringent fee $\Rightarrow z^{NC}$ *decreases* (downward shift).

Intuition: When Does a Subsidy Replace the Fee?

- The optimal policy instrument is a **per-unit subsidy** (negative fee, $t < 0$) when:

$$\gamma \leq \gamma_t \equiv \frac{(3-2d)d}{2(2d^2+d-1)}$$

- This requires two conditions simultaneously:
    1. Low abatement cost (γ small): firms invest heavily, reducing net emissions substantially.
    2. Mild pollution ($d \leq \frac{3}{2}$): environmental damage is limited, so the externality is small.
- When net emissions are very low, the social cost of the oligopoly output distortion (underproduction) *dominates* the environmental externality.
- A subsidy corrects the oligopoly distortion by boosting output.
- As γ rises: firms invest less $\Rightarrow$ more net emissions $\Rightarrow$ fee becomes positive.

Spillovers: Symmetric vs. Asymmetric

- **Symmetric spillovers** (this chapter): firm i benefits equally from any rival's investment – parameter β is the same for all pairs.

$$e_i = q_i - z_i - \beta z_j$$

- **Asymmetric spillovers** (extended model):

$$e_1 = q_1 - z_1 - \beta_1 z_2, \quad e_2 = q_2 - z_2 - \beta_2 z_1$$

  where $\beta_1 \neq \beta_2$ – firm 1 and firm 2 absorb spillovers differently.
- Possible sources of asymmetry: different geographic proximity, different absorptive capacity, or directional knowledge flows (large firm to small firm).
- Asymmetric spillovers create heterogeneous free-riding incentives – the firm with the higher β free-rides more.
- This chapter maintains the symmetric case for tractability.

Section 6.4

Cooperative Abatement – ERJVs

Motivation for Cooperation in Abatement

- Under non-cooperative abatement, firms ignore positive externalities from rivals' investment $\Rightarrow$ *underinvestment* in abatement.
- Two positive externalities each firm's investment creates for its rival:
  - TSE: lower emission fee benefits all firms.
  - ERE: spillovers directly reduce rival's emissions.
- **Environmental Research Cartel (ERC):** firms coordinate z_i and z_j to maximize *joint* profit, internalizing both externalities.
- **Environmental R&D Joint Venture (ERJV):** ERC with full spillovers ($\beta = 1$), so innovations are fully shared and duplication is avoided.
- Stages 2–3 are unaffected by cooperation: equilibrium output and fee formula remain the same.

Cooperative Abatement: ERC Equilibrium

- In Stage 1, firms jointly choose (z_i, z_j) to maximize $\Pi_i + \Pi_j$.
- First-order conditions internalize the cross-firm effects; ERC equilibrium abatement:

$$z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2 + 2\gamma(1+d)^2} > 0$$

  (positive since $d > 1/2$).
- **Investment comparison** (z^{ERC} vs. z^{NC}):
  - Not always the case that $z^{ERC} > z^{NC}$.
  - The ranking depends on pollution severity d and spillover strength β.
- Full information sharing ($\beta = 1$) characterizes the ERJV.

Investment Comparison: z^{ERC} vs. z^{NC} – No Spillovers

- When $\beta = 0$, only TSE is present (no ERE).
- **TSE can be positive or negative:**
  - When d is small (mild pollution): fees are less stringent, aggregate output is high (inelastic demand region). Firm j's investment $\uparrow \Rightarrow$ lower fee $\Rightarrow$ firm i's output and profit $\uparrow$ (*positive* externality). ERC internalizes this $\Rightarrow z^{ERC} > z^{NC}$.
  - When d is large (severe pollution): fees are stringent, output in elastic region. Lower fee $\Rightarrow$ price falls more than proportionally $\Rightarrow$ rival's profit $\downarrow$ (*negative* externality). ERC internalizes this $\Rightarrow z^{ERC} < z^{NC}$.
- At $d = 2$ (illustrative example): $z^{ERC} = z^{NC}$.

Investment Comparison: With Spillovers ($\beta > 0$)

- When $\beta > 0$, the ERC internalizes *both* TSE and ERE.
- ERE is unambiguously *positive*: rival's abatement directly reduces firm i's emissions $\Rightarrow$ lowers firm i's tax bill.
- When spillovers are *sufficiently strong* ($\beta > 1/3$): positive ERE dominates TSE regardless of $d \Rightarrow z^{ERC} > z^{NC}$ for all values of pollution severity.
- When firms *fully share* innovations ($\beta = 1$, ERJV): $z^{ERC} - z^{NC} > 0$ for all d and is *unaffected* by d.
- Summary of investment ranking:
  - Low β and high d: $z^{ERC} < z^{NC}$ possible.
  - $\beta > 1/3$: $z^{ERC} > z^{NC}$ unambiguously.
  - $\beta = 1$ (ERJV): $z^{ERC} > z^{NC}$ always.

Alternative Cooperative Regimes (Ouchida & Goto, 2016)

- Four institutional regimes (two binary policy choices):

	ERJVs banned ($\beta \neq 1$)	*ERJVs allowed* (
Coordination banned	Model (i): NC	Model (iii): ERJV
Coordination allowed	Model (ii): ERC	Model (iv): ERJV

- **Model (i):** Non-cooperative R&D with generic β – as in Section 6.3.
- **Model (iv):** ERJV cartelization – ERC with $\beta = 1$.
- Welfare ranking depends on pollution severity d and R&D cost γ.

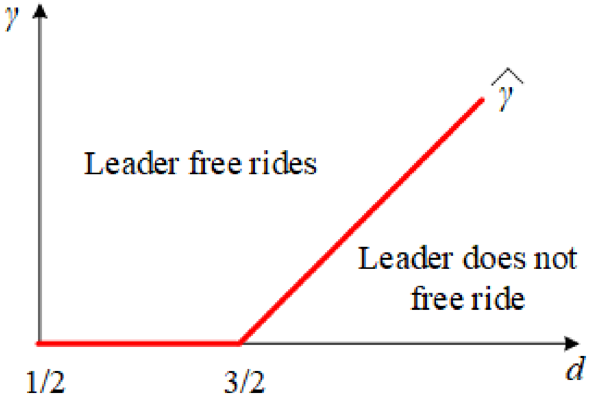
Welfare Ranking of Cooperative Regimes

- **When pollution is mild and R&D costs are moderate:**
  - Independent competition (Model i) is socially optimal.
  - Allowing coordination or full spillovers may cause excessive investment or duplication $\Rightarrow$ lower welfare.
  - Optimal policy: *prohibit* both coordination and ERJVs.
- **When pollution is severe or R&D costs are high:**
  - ERJV cartelization (Model iv) is both privately and socially preferred.
  - Coordinated investment under full spillovers internalizes positive externalities $\Rightarrow$ higher investment and better environmental outcomes.
  - Optimal policy: *allow* ERJV cartelization.
- Firms have strong private incentives to form ERJV cartels when duplication is costly and spillovers are large.

Proposition: Investment Ranking When $\beta = 0$

- When there are no spillovers ($\beta = 0$), only TSE is present.
- **Proposition 6.6.** (ERC vs. NC, no spillovers)
  - When d is small (mild pollution): $z^{ERC} > z^{NC}$ (ERC internalizes a positive externality).
  - When d is large (severe pollution): $z^{ERC} < z^{NC}$ (ERC internalizes a negative externality).
  - At an intermediate value of d: $z^{ERC} = z^{NC}$.
- **Economic intuition (large d case):** with severe pollution, the emission fee is very stringent $\Rightarrow$ output lies in the elastic portion of demand $\Rightarrow$ lower fee causes a large drop in price $\Rightarrow$ rival's profit falls (negative externality of TSE).
- ERC internalizes this *negative* externality by *reducing* investment below the NC level.

Figure 6.5 – z^{ERC} vs. z^{NC} as a Function of β



Welfare Under ERC/ERJV vs. Non-Cooperative Benchmark

- **When $z^{ERC} > z^{NC}$:** cooperation raises abatement.
  - Lower net emissions $\Rightarrow$ reduced environmental damage.
  - Lower fee $\Rightarrow$ higher output and surplus.
  - But higher abatement costs – welfare comparison not trivial.
- **When $z^{ERC} < z^{NC}$:** cooperation reduces abatement.
  - Higher net emissions $\Rightarrow$ more environmental damage.
  - Higher fee $\Rightarrow$ lower output and surplus.
  - Lower abatement cost – ambiguous welfare effect.
- **Ouchida & Goto (2016):** welfare ranking depends on d and γ. ERJV cartelization (Model iv) is welfare-superior when pollution is severe or R&D is costly.
- Regulators should tailor their policy stance (allow/ban ERJVs and coordination) to the specific industry parameters.

Section 6.5

Sequential Abatement Decisions

Motivation: Why Sequential Abatement?

- Baseline model assumed *simultaneous* abatement decisions.
- Many real industries feature **sequential** abatement: firms observe rivals' investment choices before committing their own.
- Relevant for large, capital-intensive environmental investments: new machinery, facility retrofitting, post-disaster clean-up.
- **Real-world example:** Intel (May 2020) pledged a 10% reduction in greenhouse gas emissions by 2030 – first such commitment in chipset industry. AMD responded the following year with a 50% reduction target.
- Sequential structure introduces Stackelberg-like dynamics in abatement: leader invests first; follower best-responds.
- Strandholm et al. (2023) analyze this setting, abstracting from spillovers to isolate the timing effect.

Stages 2–3: Unaffected by Timing

- Equilibrium output in Stage 3:
$$q(t) = \frac{1 - c - t}{3}$$
unchanged from simultaneous setting.
- Regulator's optimal fee in Stage 2:
$$t(Z) = \frac{(2d - 1)(1 - c) - 3d(1 + \beta)Z}{2(1 + d)}$$
also unchanged (coincides with equation 6.24).
- Substituting into $q(t(Z))$:
$$q(t(Z)) = \frac{1 - c + d(1 + \beta)Z}{2(1 + d)},$$
increasing in aggregate abatement Z.
- The key divergence from the simultaneous setting arises in **Stage 1**, where the leader internalizes the follower's reaction.

Follower's Best Response in Sequential Abatement

- Follower (firm 2) observes z_1 and chooses z_2 to maximize:

$$\max_{z_2 \geq 0} \ \pi_2 = \frac{[1 - c - t(Z)]^2}{9} + t(Z) z_2 - \frac{\gamma}{2}(z_2)^2$$

- Follower's best response function:

$$z_2(z_1) = \frac{(1-c)[d(2+2d+\beta)-1]}{2\gamma + dB} - \frac{d[3+d(2-\beta)](1+\beta)}{2\gamma + dB} z_1$$

where $B \equiv 6 + 4\gamma + 6\beta + d[2\gamma + (5-\beta)(1+\beta)]$.

- Slope is **negative**: abatement levels remain *strategic substitutes* under sequential timing.
- Leader anticipates this reaction and chooses z_1^{Seq} accordingly.

Free-Riding by the Leader

- A surprising result: the leader may invest *less* than the follower, $z_1^{Seq} < z_2^{Seq}$, when:

$$\gamma > \hat{\gamma} \equiv \max\left\{0, \frac{2d(1-2\beta)-3}{2}\right\}$$

- When $\gamma < \hat{\gamma}$: abatement is cheap; leader invests aggressively: $z_1^{Seq} > z_2^{Seq}$.
- When $\gamma > \hat{\gamma}$: abatement costs outweigh benefits; leader **free-rides** on the follower: $z_1^{Seq} < z_2^{Seq}$.
- Follower responds by *increasing* its abatement (strategic substitutes).
- Analogy: sequential public good games where early contributors reduce effort, expecting later contributors to compensate.

Role of Pollution Severity in Sequential Abatement

- Higher pollution severity (higher d) $\Rightarrow$ more stringent fee $\Rightarrow$ higher marginal benefit of abatement investment.
- As d increases, the threshold $\hat{\gamma}$ rises: the leader must bear higher costs before free-riding becomes optimal.
- Therefore, **severe pollution reduces free-riding incentives** for the leader.
- Conversely, when pollution is mild, regulatory response is weak $\Rightarrow$ benefit of abatement is low $\Rightarrow$ leader more likely to rely on the follower's investment.
- This underscores the importance of environmental context for determining strategic behavior in sequential investment settings.

Sequential vs. Simultaneous Abatement: Comparison

- **Result** (Strandholm et al., 2023): equilibrium investment in the sequential setting exceeds that in the simultaneous setting:

$$z_i^{Seq} > z_i^{NC}$$

- The difference $z_i^{Seq} - z_i^{NC}$:
  - *Increases* with pollution severity d: stringent fees make abatement more valuable in both settings, but sequential timing amplifies this.
  - *Decreases* with abatement cost γ: costly abatement reduces investment in both settings, compressing the difference.
- First-mover advantage in abatement is strongest when pollution is severe (high d) or abatement is cheap (low γ).
- In the sequential game, the follower invests *less* than in the simultaneous game, compensated by the leader's higher investment.

Dominant Strategies under Pre-Commitment Regulation

- When the regulator sets the fee *before* firms invest in abatement (regulatory commitment), strategic interaction in abatement *disappears*.
- With a fixed fee t, each firm i chooses z_i to solve:

$$\max_{z_i \geq 0} \ \pi_i = q^2 + t z_i - \frac{\gamma}{2}(z_i)^2$$

- First-order condition: $t = \gamma z_i$, yielding:

$$z_i(t) = \frac{t}{\gamma}$$

- This optimal abatement is *independent* of rival's investment $z_j \Rightarrow$ abatement is a **strictly dominant strategy**.
- Sequential vs. simultaneous timing no longer matters when the fee is pre-committed.

Cutoff $\hat{\gamma}$: When Does the Leader Free-Ride?

- The leader free-rides ($z_1^{Seq} < z_2^{Seq}$) when:

$$\gamma > \hat{\gamma} \equiv \max\left\{0, \frac{2d(1-2\beta)-3}{2}\right\}$$

- **Special case $\beta = 0$:** $\hat{\gamma} = \max\left\{0, \frac{2d-3}{2}\right\}$.
  - $\hat{\gamma} = 0$ when $d \leq \frac{3}{2}$.
  - $\hat{\gamma} > 0$ when $d > \frac{3}{2}$ (more severe pollution).
- **Effect of spillovers (β):** higher β lowers the threshold $\hat{\gamma}$ – ERE makes it *easier* for the leader to free-ride (more benefit from rival's investment without own cost).
- At $\beta = \frac{1}{2}$: $\hat{\gamma} = \max\{0, -3/2\} = 0$, so the leader *always* free-rides regardless of d.
- At $\beta \geq \frac{1}{2}$: $\hat{\gamma} = 0$ for all d, meaning free-riding is a dominant strategy for the leader.

Worked Example: Sequential vs. Simultaneous ($\beta = 0$, $d = 2$)

- Parameters: $c = \frac{1}{2}$, $d = 2$, $\gamma = 1$, $\beta = 0$.
- **Simultaneous (NC):** Equilibrium abatement:

$$z^{NC} = \frac{(1 - \frac{1}{2})[2 \cdot 2 \cdot 3 - 1]}{2 \cdot 1 \cdot 9 + 2(18 + 14)} = \frac{\frac{1}{2} \cdot 11}{18 + 64} \approx \frac{5.5}{82} \approx 0.067$$

- **Sequential:** Leader anticipates follower's reaction; total abatement $Z^{Seq} > Z^{NC}$.
- Intuition: leader commits to investment, follower reduces investment in response, but total investment rises because leader's commitment is larger than the follower's reduction.
- This mirrors the Stackelberg output model, but for abatement instead of production quantities.
- Both firms invest more than under NC: $z_i^{Seq} > z_i^{NC}$ for each firm i.

Section 6.6

Different Regulatory Timing

Regulatory Commitment: Motivation

- Lambertini et al. (2017) consider a reversed timing:
  1. **Stage 1**: Regulator commits to an emission fee t.
  2. **Stage 2**: Firms independently choose abatement investments z_i.
  3. **Stage 3**: Firms compete à la Cournot in output markets.
- **Relevant when:** firms can easily revise abatement (scrubbers, pipe filters), but regulatory parameters are fixed by legislative or institutional constraints.
- **Contrast with Poyago-Theotoky (2007):** appropriate when investments are large-scale and irreversible (e.g., factory retrofits), while regulators can adjust policy in response to observed behavior.
- Lambertini et al. also allow for $n \geq 2$ firms to study how market structure affects aggregate abatement.

Stage 3: Output and Stage 2: Dominant-Strategy Abatement

- **Stage 3 (output, n firms):** Equilibrium output per firm:

$$q_i(t) = \frac{1 - c - t}{n + 1}$$

  Aggregate output $Q(t) = \frac{n(1-c-t)}{n+1}$, approaching $1 - c - t$ as $n \to \infty$ (perfect competition).
- **Stage 2 (abatement):** With fixed fee t, first-order condition: $t = \gamma z_i$, yielding:

$$z_i(t) = \frac{t}{\gamma}$$

  a *strictly dominant strategy* (independent of rivals' choices).
- Aggregate abatement: $Z(t) = \frac{nt}{\gamma}$ – increases with n and t; decreases with γ.
- Arrowian result: with exogenous t, increased competition $\uparrow n$ raises aggregate R&D unambiguously.

Stage 1: Regulator Sets the Optimal Fee

- Regulator maximizes social welfare $W(t)$ (with $d = 2$):

$$W(t) = \frac{1}{2}[Q(t)]^2 + n\pi_i(t) - n^2[e_i(t)]^2$$

- Optimal emission fee (function of n and β):

$$t(n) = \frac{\gamma[2n(n+1)(1+\beta(n-1)) + \gamma(n-2)](1-c)}{4\gamma n(n+1)[1+\beta(n-1)] + \gamma^2(n-2) - A(\gamma)}$$

where
$A(\gamma) \equiv (n+1)^2[\gamma(1+2\beta(n-1)) - 2n(1+\beta(n-1))^2]$.

- Equilibrium abatement: $z_i(n) = \frac{t(n)}{\gamma}$.
- When the fee is endogenous, the relationship between n and aggregate abatement becomes non-trivial – depends on spillovers.

Market Concentration and Abatement – No Spillovers ($\beta = 0$)

- With $\beta = 0$, $c = 0$, $\gamma = 5$, aggregate abatement evaluates to:

$$Z(n) = \frac{n(2n^2 + 7n - 10)}{2n^3 + 19n^2 + 37n - 55}$$

- Derivative $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$: aggregate abatement **increases monotonically** with the number of firms.
- Intuition: without spillovers (no ERE), each firm's abatement reduces *only its own* tax burden $\Rightarrow$ each new entrant independently invests to reduce its tax $\Rightarrow$ total investment grows.
- Consistent with the **Arrowian prediction**: increased competition fosters more innovation when spillovers are absent.
- Note: TSE is also absent here (fee is pre-committed, not responsive to abatement).

Market Concentration and Abatement – With Spillovers ($\beta = 1/2$)

- With $\beta = 1/2$, aggregate abatement:

$$Z(n) = \frac{2n^4 + 4n^3 + 12n^2 - 20n}{n^5 + 4n^4 + 16n^3 + 24n^2 + 61n - 100}$$

- Derivative $\frac{\partial Z(n)}{\partial n}$ is positive for $n < \bar{n} = 3.432$ but negative for $n > \bar{n}$.
- **Inverted U-shape**: aggregate abatement first rises, reaches a peak near $n \approx 3$, then falls with additional firms.
- Intuition: with spillovers, firms free-ride on rivals' abatement via ERE. As competition intensifies, free-riding dominates $\Rightarrow$ individual investment falls $\Rightarrow$ total abatement eventually declines.
- Policy implication: when spillovers are present, *more competition is not always better* for environmental outcomes.

Comparing Poyago-Theotoky vs. Lambertini: TSE Absent

- **Key difference from Poyago-Theotoky (2007):**
  - In Poyago-Theotoky, the regulator *responds* to observed abatement $\Rightarrow$ investing more lowers the fee (TSE is present).
  - In Lambertini et al., the fee is *pre-committed* $\Rightarrow$ abatement does not affect the fee (TSE is absent).
- With regulatory commitment: firms' abatement decisions are purely driven by saving taxes (at the given fixed t), not by anticipating a softer regulatory response.
- ERE still arises when $\beta > 0$: rivals' abatement reduces firm i's emissions $\Rightarrow$ free-riding via ERE.
- An ERC could still help internalize ERE, but the urgency is reduced relative to the Poyago-Theotoky setting (since TSE is absent).

Poyago-Theotoky (2007) vs. Lambertini et al. (2017): A Comparison

- **Poyago-Theotoky (2007):**
  - Timing: Abatement $\rightarrow$ Fee $\rightarrow$ Output.
  - Fee responds to observed abatement (TSE present).
  - Strategic abatement decisions: firms anticipate regulatory response.
  - Best for: irreversible capital investments; flexible regulation.
- **Lambertini et al. (2017):**
  - Timing: Fee $\rightarrow$ Abatement $\rightarrow$ Output.
  - Fee is pre-committed (TSE absent).
  - Abatement is a dominant strategy: $z_i = t/\gamma$.
  - Best for: modular/reversible abatement; rigid legislative fee.
- **Common finding:** when spillovers are present ($\beta > 0$), free-riding via ERE reduces individual abatement in both frameworks.
- **Key difference:** strategic interaction in abatement only arises in Poyago-Theotoky's framework.

Worked Example: Dominant-Strategy Abatement

- Parameters: $n = 2$, $c = 0$, $\gamma = 5$, $\beta = 0$, $d = 2$.
- Regulator sets $t(n = 2)$ using equation (6.47).
- Given t, each firm independently chooses:

$$z_i = \frac{t}{\gamma} = \frac{t}{5}$$

- Firm i does not consider firm j's abatement when making this decision – it is a dominant strategy.
- Aggregate abatement: $Z = 2 \times \frac{t}{5} = \frac{2t}{5}$.
- As n increases (more firms), $t(n)$ adjusts; aggregate abatement $Z(n) = \frac{n \cdot t(n)}{5}$ first rises then falls (inverted U-shape when $\beta > 0$).
- This illustrates how the Arrowian result holds or fails depending on spillovers.

Section 6.7

Extensions

Extension 1: Endogenous Entry (Katsoulacos & Xepapadeas, 1995)

- While most models assume an exogenous number of firms n, some settings feature **endogenous entry**.
- Framework: regulator sets fee first; firms decide whether to enter; then choose output and abatement.
- Key finding: equilibrium emission fee may *exceed* marginal environmental damage – **"overinternalization"** of externalities.
- Mechanism: a higher fee deters entry, which can be socially beneficial when entry would be excessive.
- Optimal two-instrument policy:
    - An **entry fee** to limit socially excessive entry.
    - An **emission fee set below marginal damages** to avoid overinternalization.
- This combination yields higher welfare than a single emission fee alone.

Extension 2: Subsidies and Regulatory Design (Ouchida & Goto, 2014)

- Extends Poyago-Theotoky (2007) to analyze the role of **output subsidies**.
- Subsidies can be *welfare-improving* when pollution is mild (low d): the oligopoly distortion (underproduction) dominates the environmental externality $\Rightarrow$ optimal to subsidize output ($t < 0$).
- When R&D costs are high (γ large):
  - Firms invest minimally in abatement.
  - Regulation may paradoxically yield *higher* pollution than laissez-faire (firms respond to fee by cutting output more than the abatement saves).
  - Nonetheless, regulation can still improve welfare.
- When R&D is inexpensive (low γ): firms invest heavily $\Rightarrow$ lower emissions than under laissez-faire.

Extension 3: Patenting vs. Licensing Green Technologies

- **Biglaiser & Horowitz (1995):** four-stage model – R&D investment, fee determination, technology adoption (patent or license), output choice.
- Firms choose between *exclusive use* of an innovation (patent) and *licensing* it to competitors.
- Trade-off: exclusive use provides competitive advantage; licensing generates revenue and may raise rivals' abatement.
- **Denicolo (1999):** single innovating firm decides whether to retain or license technology.
  - Compares two regulatory regimes: fee set before vs. after R&D.
  - Timing of fee-setting significantly affects licensing incentives and welfare.
- **Key insight**: the timing of regulatory intervention shapes both the direction and magnitude of technology diffusion.

Extension 4: Product Differentiation and Mixed Oligopolies

- **Poyago-Theotoky & Teerasuwannajak (2002):** horizontal product differentiation alters regulatory outcomes.
  - High differentiation: pre-commitment fee is more stringent; abatement is higher under time-consistent regulation.
  - Homogeneous products: pre-commitment regulation yields higher welfare.
- **Mixed oligopolies** (Bárcena-Ruiz & Garzón, 2006; Haruna & Goel, 2009):
  - A public firm competes with private firms, maximizing a weighted combination of profit and social welfare (including environmental damages).
  - Public firm partially internalizes externalities $\Rightarrow$ improves overall welfare.
  - Privatization can be welfare-reducing when emission taxes are present.
  - Pal & Saha (2014): first-best outcomes achievable via

Extension 5: Two-Period Models with Stock Pollution

- Some pollutants accumulate over time (stock pollution), so abatement in period 1 affects environmental damage in period 2.
- In two-period models with abatement investment:
  - Period 1 abatement reduces both current and future emissions.
  - Firms can invest in durable abatement capital that depreciates over time.
  - Regulator must set fees in both periods, potentially adjusting for the stock of pollution.
- **Intertemporal trade-off:** early abatement investment is more valuable when the pollution stock is large or when abatement capital is long-lived.
- Dynamic versions of ERJVs: firms may form longer-term cooperation agreements that govern both abatement investment and spillover sharing over multiple periods.
- This connects Chapter 6 to the dynamic models covered in Chapter 5.

Summary of Extensions (6.7)

- **6.7.1 Endogenous entry** (Katsoulacos & Xepapadeas, 1995): fee may exceed marginal damage; two-instrument policy (entry fee + emission fee) is optimal.
- **6.7.2 Subsidies** (Ouchida & Goto, 2014): subsidies welfare-improving when pollution is mild; costly R&D may paradoxically worsen outcomes.
- **6.7.3 Patents vs. licensing** (Biglaiser & Horowitz, 1995; Denicolo, 1999): timing of regulatory commitment affects technology diffusion and licensing incentives.
- **6.7.4 Product differentiation** (Poyago-Theotoky & Teerasuwannajak, 2002): differentiation reverses the ranking of commitment vs. time-consistent regulation.
- **6.7.5 Mixed oligopolies** (Bárcena-Ruiz & Garzón, 2006): privatization can reduce welfare; first-best may require two instruments.

Section 6.8

Case Studies

Case Study 1: Simultaneous Abatement – HP and Dell

- **HP and Dell** both invested heavily in reducing emissions from manufacturing processes and product lifecycles.
- **HP (2007):** among the first to publicly report greenhouse gas emissions; launched recycling and toxic substance reduction programs.
- **Dell:** followed with closed-loop recycling and energy-efficient product design – simultaneous and parallel efforts.
- Investments driven by emission-related compliance costs and competitive sustainability benchmarks.
- **European automakers** (Volkswagen, BMW, Stellantis): simultaneously committed billions to electrification and battery innovation under tightening EU emission standards.
  - Volkswagen: freed €15 bn annually for EV R&D (Dec. 2024).
  - Stellantis: pledged €30 bn for electrification by 2025.

Case Study 2: ERJVs – Canada's Oil Sands Innovation Alliance

- **Canada's Oil Sands Innovation Alliance (COSIA):** formed by major oil sands producers (Suncor, Cenovus, Canadian Natural Resources) to *jointly invest* in environmental R&D.
- Focus: reducing GHG emissions, improving water use, land reclamation.
- Firms pool resources and share technology – a real-world ERJV: coordinates abatement to meet regulatory requirements and lower per-unit emission costs.
- **Japanese refining sector:** JXTG Nippon Oil & Energy and Idemitsu coordinate green R&D for emission reductions in refining operations (scrubbers, catalytic converters).
- These cases illustrate how ERJVs internalize spillovers and avoid duplicative investment.

Case Study 3: Sequential Abatement – Manufacturing and US Refining

- **Cross-country manufacturing study (Brown et al., 2022):** 33,500 firms; high-emission sectors with strong knowledge spillovers (cement, mineral products).
  - Early adopters invested in cleaner production technologies after introduction of emission taxes.
  - Leaders used R&D to retool existing processes; followers later adopted similar technologies to remain cost-competitive.
- **US refining industry:** Chevron and ExxonMobil act as leaders in green investment.
  - Under California's carbon pricing, they made high-profile investments in carbon capture and low-carbon fuels.
  - Smaller refiners followed when strategies proved cost-effective under the emission fee.
- Consistent with the sequential abatement model: leaders invest first, followers respond.

Case Studies: Connecting Theory to Practice

- **Three case types in Chapter 6:**
  1. **Simultaneous abatement** (Section 6.3): HP, Dell, European automakers – firms independently invest in parallel.
  2. **ERJVs** (Section 6.4): Canada's COSIA, Japanese refining associations – firms pool investment and share technology.
  3. **Sequential abatement** (Section 6.5): Intel/AMD in chipsets, Chevron/ExxonMobil in US refining – early movers signal, followers adapt.
- **Common theme:** in all cases, abatement investment is triggered by regulatory pressure (emission fees, carbon pricing, EU standards) and shaped by competitive and spillover dynamics.
- The theoretical predictions of Chapter 6 – free-riding, strategic substitutability, ERJVs internalizing spillovers – are all present in these real-world settings.

Section 6.9

Summary

Looking Back: Connecting Chapter 6 to Chapter 2

- **Special case:** when $\gamma \to +\infty$, abatement is prohibitively expensive; all Chapter 6 results reduce to Chapter 2.
- **Emission fee:**
  - Chapter 2: $t^{Ch2} = \frac{(2d-1)(1-c)}{2(1+d)}$.
  - Chapter 6: $t(Z^*) < t^{Ch2}$ when γ is finite.
- **Output:**
  - Chapter 6 output is higher than Chapter 2 (lower fee).
- **Net emissions:**
  - Can be lower in Chapter 6 if abatement is large enough.
- **Welfare:**
  - Chapter 6 welfare $\geq$ Chapter 2 welfare: firms have an additional tool (abatement) to improve outcomes.
- Adding abatement expands the policy-relevant parameter space significantly.

Chapter 6 Road Map: How Sections Connect

- **Section 6.2:** lays the foundations – game structure, emission function, welfare.
- **Section 6.3:** non-cooperative abatement as the baseline for comparison.
  - 6.3.1: no spillovers (identifies TSE).
  - 6.3.2: with spillovers (adds ERE, strengthens free-riding).
- **Section 6.4:** cooperative abatement (ERC/ERJV) internalizes TSE and ERE.
- **Section 6.5:** sequential timing raises total abatement; free-riding by leader possible.
- **Section 6.6:** regulatory commitment – TSE disappears; dominant-strategy abatement; inverted U-shape in n.
- **Section 6.7:** extensions (entry, subsidies, patents, differentiation, mixed oligopoly).
- **Sections 6.8–6.9:** case studies and summary.

Summary: Modeling Framework

- Chapter 6 introduces a **three-stage game**: firms invest in abatement (Stage 1), regulator sets emission fee (Stage 2), firms choose output (Stage 3).
- Abatement reduces net emissions and influences regulatory behavior, creating strategic incentives for investment.
- Two key parameters:
  - γ: investment efficiency (lower $\gamma \Rightarrow$ more abatement).
  - β: spillover strength (higher $\beta \Rightarrow$ more free-riding).
- Chapter 2 is a special case of this model when $\gamma \to +\infty$ (no abatement).
- Solving by backward induction yields equilibrium abatement, fee, output, and welfare.

Proposition: Fee Decreases with Aggregate Abatement (General β)

- **Proposition 6.7.** With spillovers ($\beta \geq 0$), the optimal emission fee satisfies:
$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d(1+\beta)}{2(1+d)} < 0$$
- The magnitude of the fee reduction per unit of abatement is *increasing* in β: when spillovers are stronger, a given investment has a greater emission-reduction effect $\Rightarrow$ regulator reduces fee more.
- This reinforces free-riding: both TSE and ERE provide incentives to let the rival invest while benefiting from their abatement.
- **Welfare implication:** the regulator accommodates firms' abatement by setting a softer fee, which boosts consumer surplus and producer surplus at the cost of potentially

Summary: Non-Cooperative Abatement

- Firms invest independently, anticipating that higher investment leads to lower emission fees (**TSE**).
- Best response functions for abatement are *downward-sloping* $\Rightarrow$ abatement is a **strategic substitute**.
- Equilibrium abatement:
  - *Increases* with pollution severity d.
  - *Decreases* with abatement cost γ and production cost c.
- Optimal emission fee *decreases* with aggregate abatement.
- Fee can be negative (subsidy optimal) when abatement is cheap and pollution is mild.
- With spillovers ($\beta > 0$): ERE amplifies free-riding, reducing z^{NC}.

Key Formula Reference: Chapter 6 Equilibrium Objects

- **Emission function:** $e_i = q_i - z_i - \beta z_j, \quad \beta \in [0, 1]$.
- **Abatement cost:** $C(z_i) = \frac{\gamma z_i^2}{2}$.
- **Stage 3 output:** $q = \frac{1-c-t}{3}$.
- **Stage 2 fee:** $t(Z) = \frac{(2d-1)(1-c)-3d(1+\beta)Z}{2(1+d)}$.
- **NC abatement:** $z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$.
- **ERC abatement:** $z^{ERC} = \frac{(1-c)[d(3+2d)-1](1+\beta)}{4d(3+2d)(1+\beta)^2+2\gamma(1+d)^2}$.
- **Commitment abatement:** $z_i(t) = \frac{t}{\gamma}$.
- **Aggregate abatement (*n* firms, commitment):** $Z(t) = \frac{nt}{\gamma}$.

Summary: Cooperative Abatement and ERJVs

- Firms coordinate abatement via ERC (generic β) or ERJV ($\beta = 1$, full information sharing).
- Cooperation internalizes positive externalities from spillovers (ERE) and tax savings (TSE).
- Investment ranking:
  - $z^{ERC} > z^{NC}$ when spillovers are strong ($\beta > 1/3$) or pollution is mild.
  - $z^{ERC} < z^{NC}$ when pollution is severe and spillovers are weak (β small).
  - $\beta = 1$ (ERJV): $z^{ERC} > z^{NC}$ always.
- Welfare ranking of cooperative regimes depends on d and γ: ERJV cartelization is preferred when pollution is severe or R&D is costly.

Looking Ahead: Chapters 7 and Beyond

- Chapter 6 established that abatement investment is a strategic tool: firms invest to reduce their tax burden (TSE) and benefit from rivals' abatement (ERE).
- **Future topics extend these insights:**
  - **Transboundary pollution:** when pollution crosses borders, national regulators must coordinate fee-setting. Abatement investment becomes a strategic tool in international environmental agreements.
  - **International trade:** firms competing in export markets face regulatory differences across countries; abatement investment affects competitiveness.
  - **Dynamic models:** firms invest in abatement capital over multiple periods; long-term incentives differ from static analysis.
- The three-stage game structure of Chapter 6 – investment, fee-setting, output – is a building block for these richer models.

Summary: Sequential and Regulatory-Commitment Models

- **Sequential abatement** (Strandholm et al., 2023):
  - $z_i^{Seq} > z_i^{NC}$: sequential timing raises total abatement.
  - Leader may free-ride when $\gamma > \hat{\gamma}$.
  - Severe pollution reduces free-riding incentives for the leader.
- **Regulatory commitment** (Lambertini et al., 2017):
  - Abatement is a strictly dominant strategy: $z_i(t) = t/\gamma$.
  - TSE is absent (fee is pre-committed).
  - Without spillovers: aggregate abatement increases monotonically in n (Arrowian result).
  - With spillovers ($\beta > 0$): inverted U-shape in n; free-riding via ERE eventually dominates.
- **Timing matters**: whether the regulator commits before or after abatement investment fundamentally changes firms' strategic incentives.

Worked Example: Equilibrium with $c = \frac{1}{2}$, $d = 1$, $\gamma = 2$

- Parameter values: $c = \frac{1}{2}$, $d = 1$, $\gamma = 2$, $\beta = 0$.
- Best response function: $z_i(z_j) = \frac{1}{18} - \frac{5}{27} z_j$.
- Symmetric equilibrium abatement:

$$z_i^* = \frac{(1 - \frac{1}{2})[2 \cdot 1 \cdot 2 - 1]}{2 \cdot 2 \cdot 4 + 1 \cdot 16} = \frac{\frac{1}{2} \cdot 3}{16 + 16} = \frac{3/2}{32} = \frac{3}{64}$$

- Aggregate abatement: $Z^* = 2 \cdot \frac{3}{64} = \frac{3}{32}$.
- Equilibrium fee:
  $t(Z^*) = \frac{(2 \cdot 1 - 1)(1 - \frac{1}{2}) - 3 \cdot 1 \cdot \frac{3}{32}}{2(1+1)} = \frac{\frac{1}{2} - \frac{9}{32}}{4} = \frac{\frac{7}{32}}{4} = \frac{7}{128}$.
- Compared to Chapter 2 (no abatement): fee is *lower*, output is *higher*.

Proposition: Abatement Reduces the Emission Fee

- **Proposition 6.1.** In the three-stage game without spillovers ($\beta = 0$), the optimal emission fee $t(Z)$ satisfies:

$$\frac{\partial t(Z)}{\partial z_i} = \frac{\partial t(Z)}{\partial z_j} = -\frac{3d}{2(1+d)} < 0$$

- That is, **any increase in abatement – by either firm – reduces the emission fee**.
- **Intuition:** higher abatement $\Rightarrow$ lower net emissions $\Rightarrow$ less need for a stringent fee to correct the environmental externality.
- **Corollary:** firms can lower their tax burden by investing in abatement – this is the tax-saving effect (TSE).
- The TSE gives firms a positive private return to abatement, even without environmental altruism.

Proposition: Equilibrium Abatement Increases in d

- **Proposition 6.2.** In the non-cooperative equilibrium without spillovers, $\frac{\partial z_i^*}{\partial d} > 0$.
- **Proof sketch:**
  - Higher $d \Rightarrow$ higher fee $t(Z)$ for any given Z.
  - Higher fee increases marginal benefit of abatement investment (saves more taxes per unit).
  - Firms respond by investing more in abatement.
- **Proposition 6.3.** $\frac{\partial z_i^*}{\partial \gamma} < 0$ and $\frac{\partial z_i^*}{\partial c} < 0$.
  - Higher abatement cost (γ): direct reduction in investment.
  - Higher marginal cost (c): less output $\Rightarrow$ fewer emissions $\Rightarrow$ less stringent fee $\Rightarrow$ lower marginal benefit of abatement.

Intuition: Strategic Substitutability in Abatement

- **Why are abatement decisions strategic substitutes?**
- When firm j invests more: $z_j \uparrow$
  1. Total abatement $Z = z_i + z_j$ increases.
  2. Regulator anticipates lower net emissions $\Rightarrow$ reduces fee $t(Z)$.
  3. Lower fee reduces marginal benefit of firm i's abatement (tax saved per unit of abatement falls).
  4. Firm i responds by investing less: $z_i \downarrow$.
- **Contrast with R&D in cost-reduction models:** in those models, rival's cost-reducing R&D can make your own investment more or less valuable depending on spillovers. Here, the fee mechanism creates a strong substitutability channel even without direct spillovers.
- The negative slope of $z_i(z_j)$ is the graphical representation of this substitutability.

Comparison: With vs. Without Abatement Investment

- **Chapter 2 (no abatement, $\gamma \to \infty$):**
  - Fee: $t^{Ch2} = \frac{(2d-1)(1-c)}{2(1+d)}$ (does not depend on any abatement).
  - Output: $q^{Ch2} = \frac{1-c-t^{Ch2}}{3}$.
- **Chapter 6 (with abatement, finite γ):**
  - Fee is lower: $t(Z^*) < t^{Ch2}$ (abatement reduces net emissions, reducing fee stringency).
  - Output is higher: $q(t(Z^*)) > q^{Ch2}$ (lower fee boosts output).
  - Net emissions can be lower or higher depending on relative magnitude of output increase vs. abatement.
- **Key insight:** abatement investment creates a "double dividend" – lower fees *and* lower net emissions – when abatement is sufficiently efficient (low γ).

Section 6.3.2 – Effect of Spillovers on Equilibrium Abatement

- Non-cooperative equilibrium abatement with spillovers:

$$z^{NC} = \frac{[d(2+\beta+2d)-1](1-c)}{d(1+\beta)[9+7d+\beta(3+d)]+2\gamma(1+d)^2}$$

- **Numerator analysis:** $d(2+\beta+2d)-1$ is increasing in $\beta \Rightarrow$ numerator rises with β.
- **Denominator analysis:** $d(1+\beta)[9+7d+\beta(3+d)]$ also increases in β, and by more.
- Net effect: z^{NC} **decreases** in β – stronger spillovers $\Rightarrow$ more free-riding $\Rightarrow$ less investment.
- Graphically: z^{NC} as a function of β slopes downward; the curve shifts up when d increases (pollution severity raises incentives to invest).

Worked Example: Spillovers and Free-Riding

- Parameters: $c = 1/2$, $\gamma = 1/4$, $d = 1$, and $\beta = 0$ vs. $\beta = 1$.
- **With no spillovers** ($\beta = 0$): only TSE is present.
    - Non-cooperative abatement: $z^{NC}(\beta = 0) > 0$.
    - Each firm invests to save on the emission fee.
- **With full spillovers** ($\beta = 1$): both TSE and ERE present.
    - ERE: each unit of firm j's abatement directly reduces firm i's emissions.
    - Firm i can reduce its tax bill *without investing* by benefiting from firm j's abatement.
    - Result: $z^{NC}(\beta = 1) < z^{NC}(\beta = 0)$ – free-riding is stronger.
- **Policy lesson:** when spillovers are strong, non-cooperative abatement significantly undershoots the social optimum.

ERC Joint Profit Maximization: First-Order Conditions

- In the ERC, firms choose (z_i, z_j) to maximize:

$$\max_{z_i, z_j \geq 0} \ \Pi_i + \Pi_j$$

- Differentiating w.r.t. z_i yields the first-order condition – now accounting for the effect of z_i on *both* firm i's and firm j's profit.
- Specifically, the ERC's FOC includes two extra terms relative to the NC problem:
  1. Effect of z_i on firm j's profit via TSE: $\frac{\partial \Pi_j}{\partial t} \cdot \frac{\partial t}{\partial z_i} > 0$ (less stringent fee raises rival's profit too).
  2. Effect of z_i on firm j's profit via ERE (if $\beta > 0$): direct emission reduction for firm j.
- Internalizing these effects raises the ERC's optimal abatement above the NC level (when externalities are positive).

Proposition: Investment Ranking under ERJV ($\beta = 1$)

- **Proposition 6.4.** When $\beta = 1$ (full spillovers, ERJV): $z^{ERC} > z^{NC}$ for all values of pollution severity d and abatement cost γ.
- **Proof sketch:**
  - With $\beta = 1$, ERE is maximal: each unit of rival's investment fully reduces firm i's emissions.
  - The positive ERE externality dominates any negative TSE effect.
  - ERC internalizes both externalities and always chooses higher investment than the NC benchmark.
- **Implication:** when innovation is fully shared, coordination is always socially beneficial in terms of abatement investment.
- At $\beta = 1$, the difference $z^{ERC} - z^{NC}$ is *independent* of d (all pollution severity curves coincide at the right-hand side of Figure 6.8).

Section 6.5 – Intuition: Leader's Investment Decision

- In the sequential setting, the leader (firm 1) solves a more complex problem than in the NC case.
- Leader's optimization incorporates the follower's best response $z_2(z_1)$:

$$\max_{z_1 \geq 0} \ \pi_1(z_1, z_2(z_1))$$

- The leader accounts for two effects of increasing z_1:
  1. *Direct effect*: lower fee $\Rightarrow$ lower tax burden (TSE, positive).
  2. *Strategic effect*: follower reduces z_2 in response (strategic substitutes). This lowers Z slightly, potentially softening the fee reduction.
- The net incentive depends on the cost of investment γ: when γ is low, direct benefit dominates and leader invests heavily; when γ is high, free-riding off the follower is preferred.

Proposition: Sequential vs. Simultaneous Abatement

- **Proposition 6.5** (Strandholm et al., 2023). In equilibrium, each firm's abatement in the sequential setting exceeds that in the simultaneous setting:

$$z_i^{Seq} > z_i^{NC}$$

- Mechanism: the leader, by committing to an investment level, moves the equilibrium to a point where total abatement is higher.
- When $\gamma < \hat{\gamma}$: leader invests more ($z_1^{Seq} > z_2^{Seq}$); both firms invest more than in NC.
- When $\gamma > \hat{\gamma}$: leader free-rides, but follower compensates by investing more; total abatement still exceeds NC level.
- The efficiency gain from sequential timing is largest when d is high (stringent fees make abatement very valuable) or γ is low (cheap to invest).

Section 6.6 – Equilibrium Emissions under Regulatory Commitment

- Each firm's net emissions under regulatory commitment (with n firms):

$$e_i(t) = \frac{1-c-t}{n+1} - \frac{t[1+\beta(n-1)]}{\gamma}$$

- First term: output net of fee reduction.
- Second term: abatement by firm i plus spillovers from rivals.
- Equilibrium profits:

$$\pi_i(t) = \left(\frac{1-c-t}{n+1}\right)^2 + \frac{t^2[1+2\beta(n-1)]}{2\gamma}$$

- Profit increases in the emission fee t! (through the abatement savings term – firms benefit from the "forced" tax-saving through optimal abatement $z_i = t/\gamma$).
- Higher n: more competition lowers the output component,

Proposition: Arrowian Result and Its Limits

- **Arrowian result (no spillovers):** $\frac{\partial Z(n)}{\partial n} > 0$ for all $n \geq 2$ when $\beta = 0$.
- With exogenous fee: more competition $\Rightarrow$ more firms each independently investing $\Rightarrow$ aggregate abatement rises monotonically.
- **Limit: spillovers break the Arrowian result.**
- With $\beta > 0$: each firm benefits from rivals' abatement via ERE. As n grows, free-riding incentives intensify; eventually each firm's individual investment falls faster than the number of firms grows.
- **Inverted U-shape:** $Z(n)$ peaks near $\bar{n} \approx 3.4$ when $\beta = 1/2$.
- **Policy implication:** with spillovers, regulators should not simply promote competition to boost green innovation – the relationship is non-monotone.

Comparison: Key Results Across Sections 6.3–6.6

- **Abatement levels:**
  - Non-cooperative (6.3): z^{NC} – lowest with high β.
  - Cooperative ERC (6.4): $z^{ERC} > z^{NC}$ when $\beta > 1/3$.
  - Sequential (6.5): $z^{Seq} > z^{NC}$ always.
  - Regulatory commitment (6.6): $z_i = t/\gamma$, dominant strategy.
- **Role of spillovers:**
  - In Sections 6.3–6.5 (regulator responds): spillovers reduce z^{NC} but can raise z^{ERC}.
  - In Section 6.6 (commitment): spillovers reduce individual investment via ERE free-riding; no TSE to counteract.
- **Market structure (Section 6.6):** more firms raise aggregate abatement without spillovers but reduce it with strong spillovers.

Abatement, Fees, and Welfare: Putting It Together

- Abatement investment improves welfare through two channels:
  1. **Direct channel:** lower net emissions reduce environmental damage $Env = \frac{d}{2}(Q - Z)^2$.
  2. **Indirect channel:** lower fee $\Rightarrow$ higher output $\Rightarrow$ higher consumer and producer surplus.
- The **optimal emission fee** balances:
  - Correcting the remaining emissions externality.
  - Not discouraging output excessively given the oligopoly distortion.
- When abatement is cheap and pollution mild: fee may be negative (output subsidy optimal).
- When abatement is costly and pollution severe: positive fee is always optimal, and its level is higher than in the no-abatement case.
- Cooperation (ERJV) can raise welfare when it leads to socially beneficial increases in abatement investment.

Policy Implications of Chapter 6

- **Emission fees alone are not enough:** when firms can invest in abatement, the regulator must account for the feedback between investment incentives and the optimal fee.
- **Supporting ERJVs:** policymakers should allow or encourage ERJVs when pollution is severe or R&D costs are high – coordination raises abatement and can improve welfare.
- **Regulatory timing:** committing to a fee before firms invest removes strategic distortions (no free-riding via TSE) but also removes beneficial flexibility.
- **Market structure:** promoting competition is not always pro-environmental – spillovers can generate an inverted U-shape in aggregate abatement as the number of firms increases.
- **Two-instrument policies:** in settings with endogenous entry, combining an entry fee with an emission fee dominates a single instrument.

Summary: Extensions and Case Studies

- **Extensions** broaden the Chapter 6 framework in several directions:
  - Endogenous entry, subsidies, patents, product differentiation, and mixed oligopolies all modify the basic free-riding and cooperation results.
  - Key message: a single emission fee is often insufficient; multiple instruments are needed for first-best outcomes.
- **Case studies** illustrate that:
  - HP, Dell, and European automakers demonstrate simultaneous abatement with competitive benchmarking.
  - COSIA and Japanese refining associations are real-world ERJVs.
  - Intel/AMD and US refiners show sequential adoption of clean technologies.
- Theory and evidence are complementary: the models of Chapter 6 help interpret strategic patterns observed in regulated industries.

Glossary of Key Terms – Chapter 6

- **Abatement** (z_i): investment reducing a firm's net emissions.
- **Spillover** (β): fraction of rival's abatement that reduces a firm's own emissions.
- **TSE** (tax-saving effect): reduction in emission fee caused by increased aggregate abatement.
- **ERE** (emission-reduction effect): direct emission reduction for firm i due to rival's abatement when $\beta > 0$.
- **Strategic substitutes:** abatement levels where each firm invests less when rivals invest more (negative-slope best response).
- **ERC** (environmental research cartel): firms coordinate abatement to maximize joint profit; generic β.
- **ERJV** (environmental R&D joint venture): ERC with $\beta = 1$; full information sharing.
- **Regulatory commitment:** fee is set before abatement investment; abatement is a dominant strategy.

Key Takeaways – Chapter 6

- Abatement investment creates two strategic externalities: **TSE** (always present) and **ERE** (present when $\beta > 0$).
- Both externalities incentivize firms to *free-ride* on rivals' abatement, leading to underinvestment.
- Cooperation (ERC/ERJV) can internalize these externalities, but its welfare impact depends on d, β, and γ.
- Sequential timing can increase total abatement relative to simultaneous, but the leader may strategically free-ride when abatement is costly.
- When the regulator pre-commits to a fee, strategic interaction in abatement disappears; individual abatement becomes a dominant strategy.
- The relationship between market concentration and aggregate abatement depends critically on spillovers: monotone without, inverted U-shape with.