EconS 501 - Microeconomic Theory I
Homework #1 - Due date: Tuesday, September 8th, in class.

1. **Lexicographic preferences admit no utility representation.** Consider lexicographic preferences on $X = \mathbb{R}^2_+$: the bundle (x_1, x_2) is weakly preferred to (y_1, y_2) if either $x_1 > y_1$, or $x_1 = y_1$ and $x_2 \geq y_2$. (The consumer cares first about good 1, and uses good 2 only to break ties.)

   (a) Show that these preferences are complete and transitive.

   (b) Show that they are *not* continuous, by exhibiting a sequence of bundles $x^n \succsim y$ that converges to a bundle x with $x \prec y$.

   (c) Prove that no utility function represents these preferences. [*Hint:* suppose u represents $\succsim$. For each x_1, the two bundles $(x_1, 1)$ and $(x_1, 2)$ satisfy $(x_1, 2) \succ (x_1, 1)$, so one can choose a rational number $r(x_1) \in (u(x_1, 1), u(x_1, 2))$. Show that $x_1 \mapsto r(x_1)$ would be a one-to-one map from the reals into the rationals.]

2. **Bliss points and local non-satiation.** A consumer has preferences on $X = \mathbb{R}^2_+$ represented by the utility function

$$u(x_1, x_2) = -(x_1 - 4)^2 - (x_2 - 4)^2 ,$$

   so that satisfaction is highest at the "bliss point" $(4, 4)$ and falls as consumption moves away from it in either direction.

   (a) Show that $\succsim$ is complete, transitive, and continuous.

   (b) Are the preferences monotone? Identify the region of X in which "more is better" (both goods locally desirable) and the region in which increasing a good makes the consumer worse off.

   (c) Show that the preferences are *not* locally non-satiated, and identify the exact bundle at which local non-satiation fails.

   (d) Are the preferences convex? Explain the consequence of the local-non-satiation failure for the budget constraint: at prices $p \gg 0$ and wealth large enough to afford $(4, 4)$, will the consumer necessarily spend all of her wealth? Contrast with the monotone case.

3. **Intransitive indifference and just-noticeable differences.** A consumer evaluates amounts of money $x \in X = \mathbb{R}_+$, but perceives two amounts as equally good whenever they differ by less than one dollar (a "just-noticeable difference"). Formally, define $x \succsim y$ if and only if $x \geq y - 1$.

   (a) Show that $\succsim$ is complete. Show that the induced indifference relation $\sim$ (where $x \sim y$ iff $x \succsim y$ and $y \succsim x$, i.e., $|x - y| \leq 1$) is reflexive and symmetric but *not* transitive, by exhibiting three amounts x, y, z with $x \sim y$, $y \sim z$, but $x \succ z$.

(b) Show that the strict preference $\succ$ (where $x \succ y$ iff $x > y + 1$) *is* transitive. (A complete relation whose strict part is transitive in this way is called a *semiorder*.)

(c) Prove that no utility function can represent $\succsim$ in the standard sense $x \succsim y \Leftrightarrow u(x) \geq u(y)$. [*Hint*: such a representation would force $\sim$ to be transitive.]

(d) Verify that $u(x) = x$ delivers a *Scott-Suppes* representation, $x \succsim y \Leftrightarrow u(x) \geq u(y) - 1$, with a constant threshold of one dollar. Explain briefly why intransitive indifference is behaviorally plausible.

4. **CES preferences and the elasticity of substitution.** Consider the constant-elasticity-of-substitution utility function

$$u(x_1, x_2) = \left(\alpha x_1^{\rho} + (1-\alpha) x_2^{\rho}\right)^{1/\rho}, \quad \text{where } 0 < \alpha < 1,\ \rho < 1,\ \rho \neq 0.$$

(a) Find the marginal rate of substitution, and show that it depends only on the ratio x_2/x_1, so that the preferences are homothetic.

(b) Show that the elasticity of substitution, $\sigma \equiv \dfrac{d\ln(x_2/x_1)}{d\ln MRS}$, is constant and equal to $\dfrac{1}{1-\rho}$.

(c) Evaluate σ for $\rho = \frac{1}{2}$ and $\rho = -1$, and describe the limiting cases $\rho \to 1$ and $\rho \to -\infty$. Explain how σ relates the shape of the indifference curves to the ease of substituting one good for the other.