EconS 424 - Strategy and Game Theory
Final Exam

Instructions:

- Write your answers to each exercise in a different page.
- Show all your work, and be as clear as possible in your answer. You can work in groups, but each student must submit his/her own exam.
- The due date of this take-home exam is Monday, May 4th, at 10:00am, in my office or mailbox. (You can also submit your exam in my office before this date.)
- I strongly recommend working on a few exercises every day, rather than trying to solve all exercises in one day. Late submission will be subject to grade reduction.

1. **Ultimatum bargaining game with altruistic players.** Consider the ultimatum bargaining game we discussed in class. If an agreement is reached, the responder earns the division that the proposer offered, d, while the proposer earns the remaining surplus, $1-d$. Assume, however, that the proposer's utility from this payoff distribution is

$$(1-d)+\alpha_P d,$$

where $\alpha_P \in [0,1]$ represents his degree of altruism. Similarly, the responder's utility from this payoff distribution is

$$d+\alpha_R(1-d),$$

where $\alpha_R \in [0,1]$ denotes his degree of altruism. Intuitively, every player cares about his own payoff (first component of every utility function) but also enjoys seeing the other player receive a higher payoff (second component of the utility function). When $\alpha_i = 0$, player $i = \{P, R\}$ does not exhibit altruistic preferences; when $\alpha_i \in (0,1)$ he cares more about his own payoff than his rival's; and when $\alpha_i = 1$, he assigns the same weight to his payoff and those of his rival. For generality, we allow for altruistic preferences to satisfy $\alpha_P > \alpha_R$, $\alpha_P < \alpha_R$, or $\alpha_P = \alpha_R$.

   (a) Find the responder's best response. How is it affected by his altruism parameter, α_R?

   (b) Find the proposer's offer in equilibrium. How is it affected by his altruism parameter, α_P? How is it affected by the responder's altruism parameter, α_R? Interpret.

   (c) How are your results affected if the proposer exhibits stronger altruistic preferences than the responder, $\alpha_P > \alpha_R$? What if, instead, the responder has stronger altruistic preferences?

   (d) Evaluate your equilibrium results at $\alpha_P = \alpha_R = 0$. Interpret.

   (e) Repeat parts (a)-(c) assuming that altruism parameters satisfy $\alpha_i \geq 0$ for every player i, thus not being restricted to $\alpha_i \in [0,1]$.

2. **Partial collusion in quantity competition.** Consider an industry with two firms competing in quantities, facing inverse demand function $p(Q) = 1 - Q$, where $Q = q_1 + q_2$ denotes aggregate output; and having the same marginal cost of production, c, which satisfies $1 > c \geq 0$.

   In this context, consider the GTS with partial collusion, as follows:

   - In period $t = 1$, every firm i chooses output $q_i = \alpha q_i^C + (1-\alpha) q_i^{NE}$, where $q_i^C = \frac{1-c}{4}$ denotes firm i's collusive output and $q_i^{NE} = \frac{1-c}{3}$ represents the NE output.
   - In all subsequent periods, $t > 1$, every firm i chooses output q_i if both firms produced q_i in every previous period. Otherwise, every firm i reverts to the NE of the stage game, choosing $q_i = \frac{1-c}{3}$ thereafter.

   Answer the following questions:

   (a) Find the minimal discount factor sustaining the above GTS (partial collusion) as a function of parameter α, $\underline{\delta}(\alpha)$.

(b) How is $\underline{\delta}(\alpha)$ affected by an increase in α? Interpret.

(c) Evaluate $\underline{\delta}(\alpha)$ at $\alpha = 0$ and at $\alpha = 1$. Interpret.

3. **Labor market signaling game when education is productivity enhancing - Separating equilibria.** Consider the labor market signaling game shown in figure 1. A worker privately observes whether he has High productivity or Low productivity with equal probability, and then decides whether to acquire some education that he will be able to use as a signal about his productivity level. The firm that is thinking of hiring him as a manager or a cashier without observing his productivity (only whether the worker acquired a college education or not). Additionally, there is no innative productivity differential between a low and a high type worker when they acquire no education but increases by α for the high type worker and by β for the low type worker.

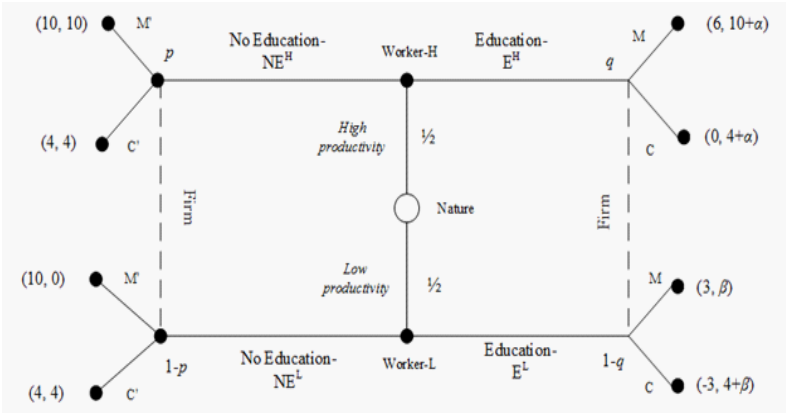

Figure 1 - Labor market signaling when education is productivity enhancing.

(a) *Separating* $NE^H E^L$. Can you sustain a separating equilibrium where only the low productivity worker acquires education?

(b) *Separating* $E^H NE^L$. Can you sustain a separating equilibrium where only the high productivity worker acquires education?

4. **Labor market signaling game when education is productivity enhancing - Pooling equilibria.** Consider the setting in exercise 3.

(a) *Pooling* $E^H E^H$. Can you sustain a pooling equilibrium where both worker types acquire education?

(b) *Pooling* $NE^H NE^L$. Can you sustain a pooling equilibrium where both worker types do not acquire education?

3. Exercises from Harrington (both editions should work)**:**

- Chapter 10: Exercise 19.
- Chapter 12: Exercise 6.