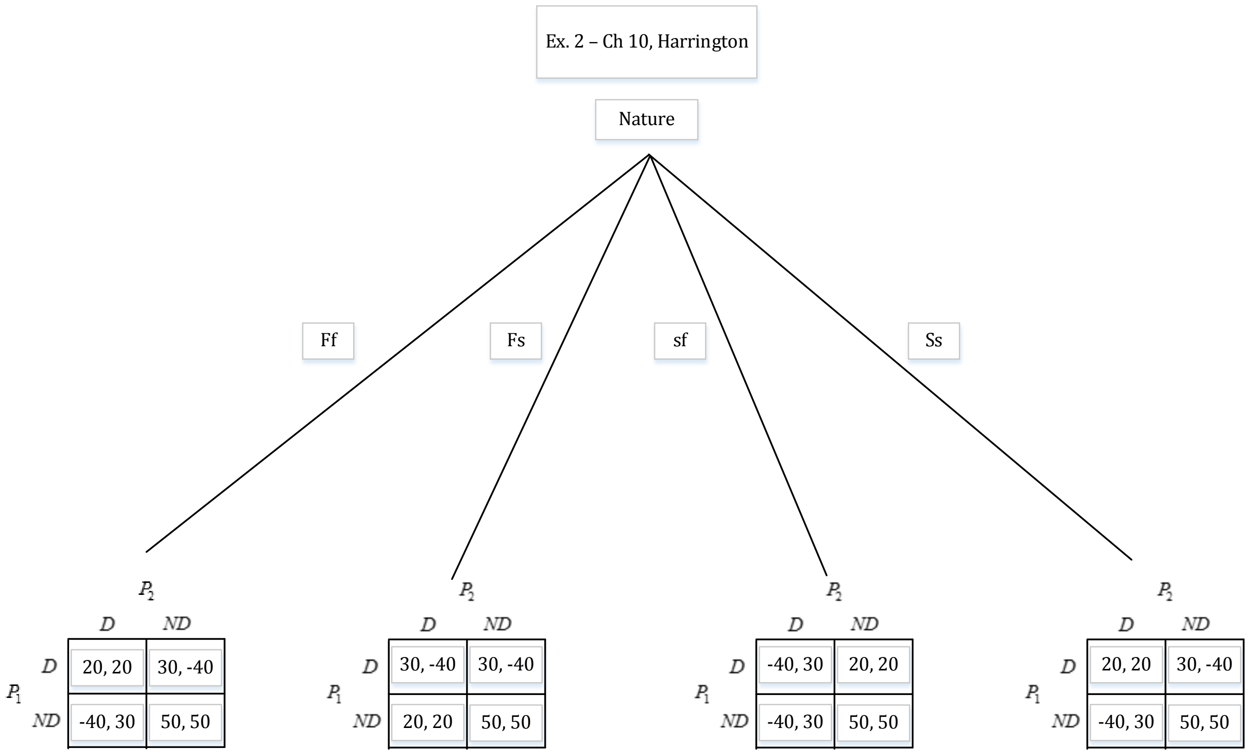
ECONS 424 – STRATEGY AND GAME THEORY

HOMEWORK #5 – ANSWER KEY

HARRINGTON CHAPTER 10 –EXERCISE 2



Where F/S denotes Fast/Slow for player 1 (Bat)

f/s denotes fast/slow for player 2 (Curly Bill)

Where D represents Draw

ND represents Not Draw

fast, then each has a payoff of 20. If at least one chooses draw, then there is a gunfight.

a. Is it consistent with Bayes–Nash equilibrium for there to be a gunfight for sure? (That is, both gunfighters draw, regardless of their type.)

ANSWER: Yes, as it is an equilibrium for each gunfighter to use a strategy that has him draw regardless of his type. The equilibrium conditions for Bat are

Fast Type (Draw): $.6 \times 20 + .4 \times 30 \geq .6 \times (-40) + .4 \times 20 \Rightarrow 24 \geq -16$

Slow Type (Draw): $.6 \times (-40) + .4 \times 20 \geq .6 \times (-40) + .4 \times (-40) \Rightarrow -32 \geq -40.$

The equilibrium conditions for Curly Bill are

Fast Type (Draw): $.65 \times 20 + .35 \times 30 \geq .65 \times (-40) + .35 \times 20 \Rightarrow 23.5 \geq -19$

Slow Type (Draw): $.65 \times (-40) + .35 \times 20 \geq .65 \times (-40) + .35 \times (-40) \Rightarrow -19 \geq -40.$

b. Is it consistent with Bayes–Nash equilibrium for there to be no gunfight for sure? (That is, both gunfighters wait, regardless of their type.)

ANSWER: Yes, as it is an equilibrium for each gunfighter to use a strategy that has him wait regardless of his type. Doing so realizes a payoff of 50—as a gunfight is avoided—and all other outcomes yield a lower payoff, so the expected payoff from any drawing must be less. More explicitly, the equilibrium conditions for Bat are

Fast Type (Wait): $.6 \times 50 + .4 \times 50 \geq .6 \times 30 + .4 \times 30 \Rightarrow 50 \geq 30$

Slow Type (Wait): $.6 \times 50 + .4 \times 50 \geq .6 \times 20 + .4 \times 30 \Rightarrow 50 \geq 24.$

The equilibrium conditions for Curly Bill are

Fast Type (Draw): $.65 \times 50 + .35 \times 50 \geq .65 \times 30 + .35 \times 30 \Rightarrow 50 \geq 30$

Slow Type (Draw): $.65 \times 50 + .35 \times 50 \geq .65 \times 20 + .35 \times 30 \Rightarrow 50 \geq 23.5.$

c. Is it consistent with Bayes–Nash equilibrium for a gunfighter to draw only if he is slow?

ANSWER: Yes. Consider a strategy profile in which each draws when slow and waits when fast. The equilibrium conditions for Bat are

Fast Type (Wait): $.6 \times 50 + .4 \times 20 \geq .6 \times 30 + .4 \times 30 \Rightarrow 38 \geq 30$

Slow Type (Draw): $.6 \times 20 + .4 \times 20 \geq .6 \times 50 + .4 \times (-40) \Rightarrow 20 \geq 14.$

The equilibrium conditions for Curly Bill are

Fast Type (Wait): $.65 \times 50 + .35 \times 20 \geq .65 \times 30 + .35 \times 30 \Leftrightarrow 39.5 \geq 30$

Slow Type (Draw): $.65 \times 20 + .35 \times 20 \geq .65 \times 50 + .35 \times (-40) \Leftrightarrow 20 \geq 18.5.$

HARRINGTON CHAPTER 10 – EXERCISE 5

Player 2 has only two strategies $S_2 = \{a, b\}$

Player 1 has four strategies $S_1 = \{xx', xy', yx', yy'\}$ where the first component of every strategy pair denotes what player 1 chooses when his type is H and the second component is what he selects when his type is L.

Here, the Bayesian normal form representation of the game is:

Player 2

Player 1

	a	b
xx’		
xy’		
yx’		
yy’		

Let's find the EU from strategy profile (xx', a),

$$EU_1 = p * 3 + (1 - p) * 2 = 2 + p$$

$$EU_2 = p * 1 + (1 - p) * 3 = 3 - 2 * p$$

$$\rightarrow (2 + p, 3 - 2p)$$

Similarly, for strategy profile (xy', a),

$$EU_1 = p * 3 + (1 - p) * 3 = 3$$

$$EU_2 = p * 1 + (1 - p) * 1 = 1$$

$$\rightarrow (3, 1)$$

Proceeding in this fashion, we find the complete normal form game to be:

Player 2

Player 1

	a	b
xx'	2+p, 3-2p	1, 2+p
xy'	3, 1	4-3p, 3p
yx'	2, 3-2p	1+4p, 2
yy'	3-p, 1	4+p, 2p

A) When p= 0.75 the above matrix becomes:

Player 2

Player 1

	a	b
xx'	2.75, 1.5	1, 2.75
xy'	3, 1	1.75, 2.25
yx'	2, 1.5	4, 2
yy'	2.25, 1	4.75, 1.5

Doing the usual underlining to find best responses for each player, we find that there is a unique BNE: (yy', b).

B) Player 1:

When player 2 selects a, he prefers: $3>2+p$ for all p

$3>2$

$3>3-p$ Hence, he selects xx'

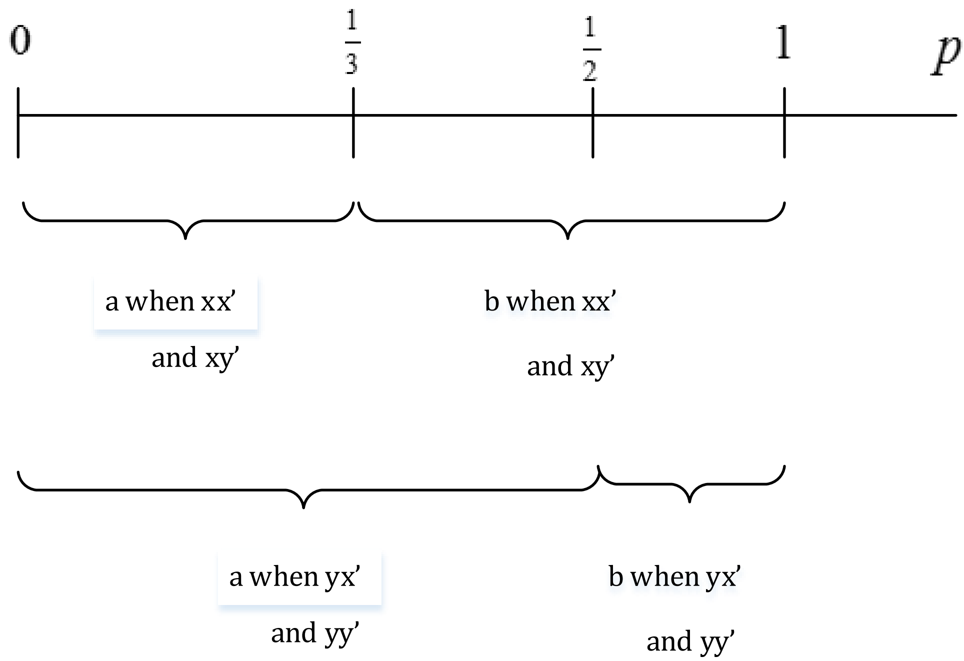
Player 2:

When player 1 selects xx', he prefers a if $3-2p>2+p \rightarrow 1 > 3p \rightarrow \frac{1}{3} > p$

When player 1 selects xy', he prefers a if $1 > 3p \rightarrow \frac{1}{3} > p$ (otherwise he prefers b)

When player 1 selects yx', he prefers a if $3 - 2p > 2 \rightarrow 1 > 2p \rightarrow \frac{1}{2} > p$

When player 1 selects yy', he prefers a if $1 > 2p \rightarrow \frac{1}{2} > p$ (otherwise he prefers b)



We can then divide our analysis into three different matrices

- One matrix for $p < \frac{1}{3}$
- Another matrix for $p \in \left[\frac{1}{3}, \frac{1}{2}\right]$
- Another matrix for $p > \frac{1}{2}$

First Case: $p < \frac{1}{3}$

Player 2

Player 1		a	b
	xx'	2+p, 3-2p	1, 2+p
	xy'	3, 1	4-3p, 3p
	yx'	2, 3-2p	1+4p, 2
	yy'	3-p, 1	4+p, 2p

Unique BNE: (xy', a)

Second Case: $p \in \left[\frac{1}{3}, \frac{1}{2}\right]$

		Player 2	
		a	b
	xx'	2+p, 3-2p	1, $\underline{2+p}$
Player 1	xy'	$\underline{3}$, 1	4-3p, $\underline{3p}$
	yx’	2, $\underline{3-2p}$	1+4p, 2
	yy'	3-p, $\underline{1}$	$\underline{4+p}$, 2p

Third Case: $p > \frac{1}{2}$

Player 2

Player 1		a	b
	xx'	2+p, 3-2p	1, 2+p
	xy'	3, 1	4-3p, 3p
	yx'	2, 3-2p	1+4p, 2
	yy'	3-p, 1	4+p, 2p

Unique BNE: (yy', b)

(This BNE is consistent with part (a) of this exercise, where $= 0.75 > \frac{1}{2}$).