Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 1: Introduction

Outline

- What are Common Pool Resources (CPRs)?
- Differences between CPRs and other goods
- Overexploiting the commons
- The tragedy of the commons- Static and dynamic components
- The tragedy of the commons under incomplete information

What are
Common Pool Resources?

If we ask you to find examples of Common pool resources (CPRs), you may think in:

- Fishing grounds
- hunting grounds
- Forests, along with oil fields, pastures, irrigation systems, and aquifers

➢ More examples:

- The use of a computer facility
- Wi-Fi internet connections that require no password

But the question is……..

What are the distinctive features these examples, and generally CPRs, exhibit ?

- ✓ **The are two important properties** to qualify a good or service as a CPR;

1. **It must be** rival

2. **It must be** non-excludable

1. It must be rival

- Its consumption by one individual reduces the amount of the good available to other individuals

- This property holds in all our examples. ***Why*?**

I. A larger fishing catch by one fisherman reduces the available stock that other fisherman can catch. (e.g. Fishing grounds)

II. The internet browsing by one more individual reduces the Wi-Fi speed other individuals can enjoy (e.g. Wi-Fi internet)

2. **It must be** non-excludable

- It means that preventing an individual to not enjoy the good is costly, or impossible

- Our examples satisfy this property. ***Why*?**

I. Since preventing that a new fisherman accesses a fishing ground is relatively costly.

Differences between CPRs and other goods

1.2 Differences between CPRs and other goods

- We classify different types of goods **<u>into 4 types</u>**

  - It depends on **<u>whether</u>** these goods satisfy the two properties: rival & non-excludable

- From the section before we know that;
  To qualify a good or service as CPR, it needs to satisfy two properties; rival & non-excludable

  - That leaving us with three other types of goods
    (Describe in table 1.I)

Summary the different types of goods

	Excludable	**Non-Excludable**
Rival	Private goods (e.g. Apple)	Common Pool Resources (e.g. Fishing ground)
Non- rival	Club goods (e.g. Gym)	Public goods (e.g. National defense)

Table 1.1. Different types of goods.

❖ *The discussion of three types of goods is on the next slide.*

I. Common Pool Resources CPRs;

- CPRs are rival in consumption but non-excludable.
- e.g. fishing grounds, hunting grounds

II. Private goods;

- It is consumption is rival and excludable.
- e.g. Apple

(if you eat it, I cannot enjoy the same apple) and (if you don't pay for an apple, you cannot eat it)

III. Club goods;

- Club goods are non-rival and excludable.
- e.g. Gym membership, Satellite TV, or pay TV channels

- ***Non-rival:*** The good can be enjoyed by several members without affecting each other's utility.

- ***Excludable:*** Gym owners can easily prevent non-members from accessing the center.

IV. Public goods;

- Public goods are non-rival and non-excludable

- E.g. national defense, clean air, public fireworks, official statistics, and publicly available inventions through unpatented R&D

  - ***Non-rival:***

  Its consumption by one individual does not reduce the amount of the good available to other individuals.

  - ***Non-excludable:***

  Preventing an individual to not enjoy the good is extremely expensive, or impossible

Overexploiting the commons

- CPRs share a key feature with public goods.
  - Both are non-excludable ➡ making it difficult preventing individuals or firms from enjoying the good.
- ***Unlike*** public goods, CPRs are rival in consumption which makes things worse. ***Why*?**
  - Consider a fishing ground. As a rival good, each fisherman's appropriation (e.g., 10 tons of fish) cannot be appropriated by other fishermen;
  - A feature that does not apply to public goods where all agents can benefit from the public good without affecting each others' utility.

- The rivalry feature of CPRs can be understood as a ***negative externality***. ***Why*?**

  - When a fisherman appropriates one more ton of fish, this ton is not available to other fishermen which increases their appropriation costs if they seek to maintain their appropriation level unaltered.

  - After fisherman i increased her appropriation forcing all other fishermen $j \neq i$ to spend more time or resources to catch the same amount of fish than before.

  - When fisherman i chooses her appropriation level, she considers her private benefits and costs from appropriation, but ignores the external effects on other fishermen.

- If, instead, all fishermen coordinated their appropriation decisions, or

- If a regulator sets appropriation decisions to each fisherman using policy instruments like fishing quotas

they would consider the joint profits of all fishermen, internalizing the external effects that each of their appropriations impose on other fishermen's costs

The tragedy of the commons - Static and dynamic components

1.4 The tragedy of the commons- Static and dynamic components

- *Equilibrium appropriation:* The appropriation that each fisherman chooses when left unregulated exceeds the socially optimal level.

- *Socially optimal appropriation:* the appropriation level that they would choose if they coordinated their decisions (as it maximizes welfare for all agents in the society).

This means that equilibrium appropriation is socially excessive or, more compactly, that the resource is overexploited

(tragedy of the commons)

1.4 The tragedy of the commons- Static and dynamic components

- *Tragedy of the commons:*

Equilibrium appropriation exceeds the socially optimal appropriation.

- Tragedy of the commons arises even in static settings where:

  - Fishermen exploit the commons during only one period (Chapter 2)
  - Firms interact during several periods in a dynamic setting (Chapter 3)
  - Firms face entry threats in future periods and use their current appropriation to deter entry of potential competitors. (Chapter 4)

1.4 The tragedy of the commons- Static and dynamic components

- This can inform regulators about the relative size of the tragedy of the commons in different CPRs, being:

  - nil in those resources where a single firm operates during all periods, as this firm fully considers the effect of its appropriation decisions both in its current and future profits

  - larger in commons where more than a firm operate since they ignore the external effects that their individual appropriation impose on other firms' costs or profits;

1.4 The tragedy of the commons- Static and dynamic components

- larger in CPRs where a single firm expects (with certainty) that other firm/s will enter in future periods

- even larger in those commons facing entry threats where the incumbent can use her current appropriation to deter the potential entrant from joining the CPR

The tragedy of the commons under incomplete information

1.5 The tragedy of the commons under incomplete information

- In Chapters 5-7, the standard CPR problem inserts in a different setting;

- Chapter 5

  - It considers contexts in which firms interact repeatedly, either for a finite or infinite rounds.

  - It identifies under which conditions firms have incentives to cooperate, decreasing their appropriation, and thus protecting the commons.

➢ Chapter 6

- Inserts the CPR problem in a setting where firms interact under incomplete information.
- Considers environments in which all firms face a common source of uncertainty;
  - such as what the available stock is, or how they will be affected by each other's appropriation decisions.
- Seeks to evaluate whether firms' appropriation is lower when they operate under certainty than under uncertainty.

➢ Chapter 7

- Inserts the CPR problem in a setting where firms interact under incomplete information.

- Considers a context in which the incumbent is better informed than the potential entrant about the initial stock,

- which could happen when the incumbent operated in the CPR for a long time thus accumulating detailed information about the stock.

- Continue chapter 7

  - The potential entrant observes the incumbent's appropriation, using it as a signal of the (unobserved) initial stock.
  - This signal helps the entrant decide whether the stock is sufficiently abundant to merit entry, or scarce enough to remain outside the CPR.
  - Investigate under which conditions the incumbent has incentives to decrease its appropriation of the resource enough to signal the stock is low, thus deterring entry.

➢ Continue Chapter 7

- This type of behavior will actually protect the commons, and arises because of the incomplete information setting in which firms interact.

- Incomplete information can serve to reduce the above inefficiencies, so prevalent in the commons if they induce the incumbent to reduce its appropriation to deter future entry.

➢ Continue Chapter 7

- Identify conditions for which the incumbent chooses to decrease its appropriation to deter entry.

- We then evaluate under which conditions this appropriation reduction is welfare improving,

- besides in which contexts incomplete information becomes welfare reducing.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 2: Common Pool Resources in a Static Setting

Outline

- Modeling the Common Pool Resources CPR
- Finding equilibrium appropriation
- Common pool resources – Socially optimal appropriation
- Facing our first inefficiency
- Inefficient exploitation with more general functions
- Policy instruments

Modeling the Common Pool Resources CPR

Assumption

- Assume that N firms (or individuals) have free access to the resource.

- Perfect competition (Every unit of appropriation is sold in the international market)
  - Every fisherman's appropriation represents a small share of industry catches, thus not affecting market prices for this variety of fish

- Every firm takes the market price p as given (normalize to $p = \$1$)

2.2 Modeling the Common Pool Resources CPR

- Every firm faces the following cost function;

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

➢ Where;

- $S > 0$ denotes the stock of the resource, which reduces fisherman *i*'s cost when the resource becomes more abundant.

- q_i represents fisherman *i*'s appropriation.

- $Q_{-i} = \sum_{i \neq j} q_j$ reflects aggregate appropriations by individuals other than i.

➢ ***Case 1:*** Having only two fishermen exploit the resource.

- The total cost function simplifies to

$$C(q_1, q_2) = \frac{q_1(q_1 + q_2)}{S} \qquad \textit{for fisherman 1}$$

$$C(q_2, q_1) = \frac{q_2(q_2 + q_1)}{S} \qquad \textit{for fisherman 2}$$

➢ ***Propositions on Cost Functions:***

- The cost function is increasing in fisherman i's own appropriation, q_i, and in his rival's appropriations, Q_{-i}

- Intuitively, the fishing ground becomes more depleted as other firms appropriate fish, making fisherman i more difficult to catch fish.

➢ ***Case 2:*** What if we have three fishermen exploiting the resource?

- The same principle applies, as seen from the following derivatives.

$$\frac{\partial C(q_i, Q_{-i})}{\partial q_i} = \frac{2q_i + Q_{-i}}{S} > 0$$

$$\frac{\partial C(q_i, Q_{-i})}{\partial Q_{-i}} = \frac{q_i}{S} > 0$$

- Agent *i*'s profit-maximization problem
  - Every fisherman chooses its appropriation level q_i to maximize its profits as follows;

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S}$$

  - The first term represents the fisherman's revenue from additional units of appropriation (recalling that p_i = \$1).
  - The second term indicates the total cost that the fisherman incurs when appropriating q_i units of fish while his rivals appropriate Q_i units.

Finding equilibrium appropriation

2.3.1 Comparative statics

2.3.2 Extension - What if fishermen have some market power?

2.3 Finding equilibrium appropriation

Goal: Find the appropriation that each fisherman chooses in equilibrium.

- Every agent chooses its appropriation level simultaneously.
- The information about the stock and agents' cost functions is common knowledge (complete information).

Cournot game of simultaneous quantity competition

How to solve this game?

Step 1: Solve each player's profit maximization problem which provides us with the players best response function

Step 2: Use the best response function of all players (step 1) to identify the Nash equilibrium of the game.

2.3 Finding equilibrium appropriation

➤ ***Step 1:*** Find fisherman i's best response function.

- Differentiating with respect to q_i in the above maximization problem for fisherman i we obtain;

$$\underbrace{1}_{\text{MR}} - \underbrace{\frac{2q_i + Q_{-i}}{S}}_{\text{MC}} = 0$$

- The first term captures the marginal revenue from catching additional units of fish.

- The second term indicates the marginal cost that the firm experiences from these additional catches.

- That is, the fisherman increases appropriation until the marginal revenue and marginal cost exactly offset each other.
  - Rearranging the expression yields

$$S = 2q_i + Q_{-i}$$

- Fisherman *i*'s best response function is

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i} \qquad (BRF_i)$$

- It describes how many units to appropriate, q_i, as a response to how many units his rivals appropriate, Q_{-i}.

2.3 Finding equilibrium appropriation

- She appropriates half of the available stock, $\frac{S}{2}$, when his rivals do not appropriate any units, $Q_{-i} = 0$

- But his appropriation decreases as his rivals appropriate positive amounts, $Q_{-i} > 0$

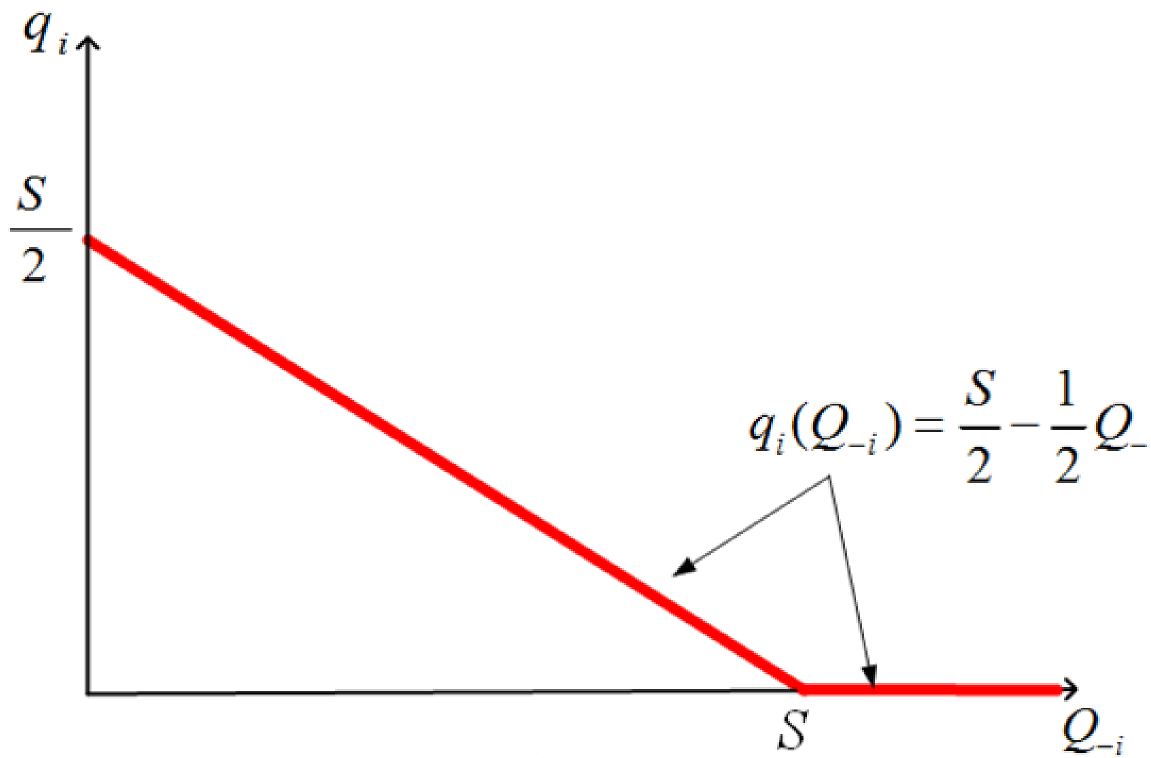


Figure 2.1.

2.3 Finding equilibrium appropriation

- Fisherman *j*'s best response function

  - Since firms face the same price for each unit of fish ($1)

  - And they face the same cost function (symmetric)

  - The best response function of any other firm *j* (where $j \neq i$) is symmetric to the best response function of firm *i*;

$$q_j(Q_{-j}) = \frac{S}{2} - \frac{1}{2}Q_{-j} \qquad (BRF_j)$$

➢ ***Step 2:*** Using best response functions to find the Nash equilibrium.

- In a symmetric equilibrium; each fisherman appropriates the same amount of fish

implying that $q_1^* = q_2^* = \cdots = \mathrm{q}_N^* = q^*$ *All firms' catches coincide*

Q_{-i} becomes;

$$Q_{-i}^* = \sum_{j \neq i} q^* = (N-1)q^*$$

2.3 Finding equilibrium appropriation

- Inserting this result in the best response function yields

$$q^* = \frac{S}{2} - \frac{1}{2}(N-1)q^*$$

- S is the stock
- N is the number of fishermen
- Rearranging the above expression yields;

$$\frac{S}{2} = \frac{2q^* + (N-1)q^*}{2} \quad \text{or} \quad S = (N+1)q^*$$

2.3 Finding equilibrium appropriation

- The equilibrium appropriation becomes ;

$$q^* = \frac{S}{N+1}$$

- Numerical example
  - Assume that the stock is $S = 100$ tons of fish
  - The number of fishermen is $N = 9$

✓ The equilibrium and the aggregate appropriations become

$$q^* = 10\ tons \qquad Q^* = Nq^* = \frac{NS}{N+1} = 90\ tons$$

- ***Case*:** Having two firms ($N = 2$) , i *and* j.

The aggregate appropriation by i's rivals simplifies to $Q_{-i} = q_j$, implying that the best response function of firm i, *and* j is;

$$q_i(q_j) = \frac{S}{2} - \frac{1}{2} q_j \qquad (BRF_i)$$

$$q_j(q_i) = \frac{S}{2} - \frac{1}{2} q_i \qquad (BRF_j)$$

- Figure 2.2 depicted the Nash equilibrium where both firms' best response functions cross each other….. *(Next slide)*

2.3 Finding equilibrium appropriation

- Since we have two firms $N = 2$;

  - The equilibrium appropriation becomes;

$$q^* = \frac{S}{N+1} = \frac{S}{2+1} = \frac{S}{3}$$

  - The aggregate appropriation becomes

$$Q^* = \frac{2S}{3}$$

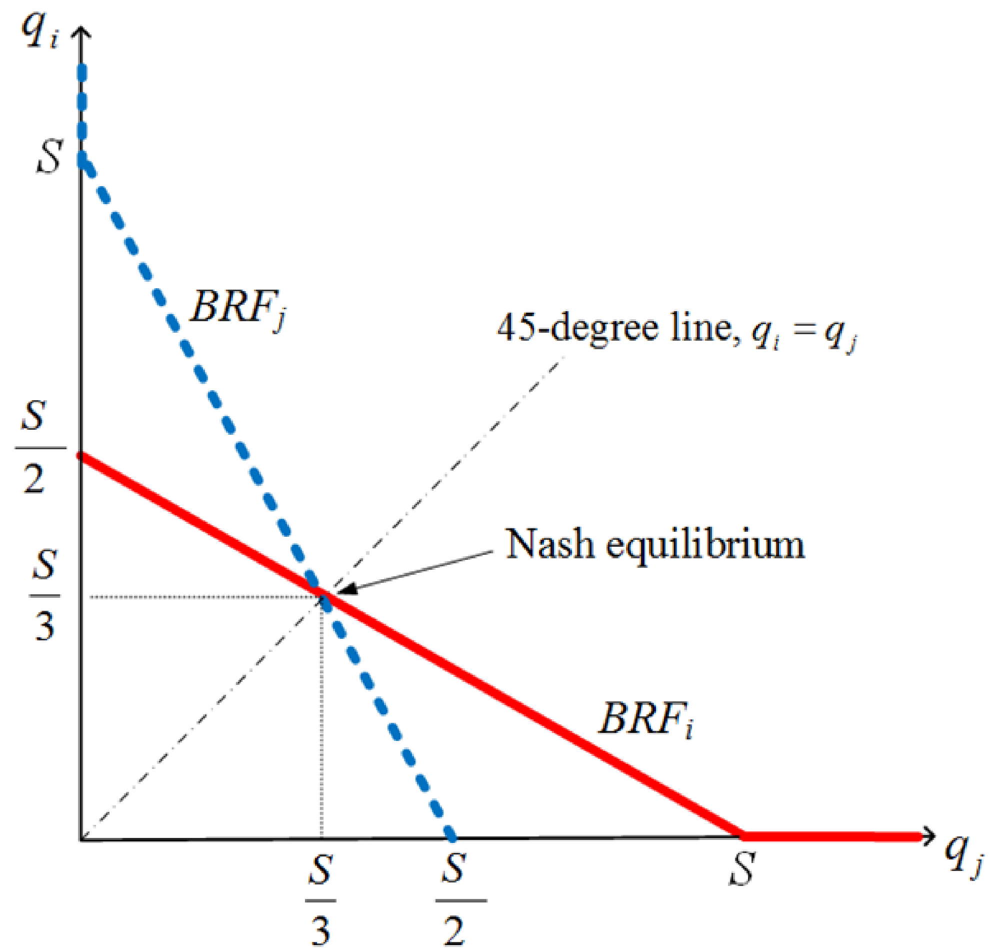


Figure 2.2

➢ *Comparative statics*

- Discuss how the result is affected by changes in one of the parameters.

➢ Example 1

- The equilibrium appropriation;

$$q^* = \frac{S}{N+1}$$

- It only depends on the stock of the resource, S, and the number of firms competing for it, N.

2.3.1 Finding equilibrium appropriation - Comparative static-

$$\frac{dq^*}{dS} = \frac{1}{N+1}$$

- ✓ We can observe the equilibrium appropriation q^* **increases** in S

$$\frac{dq^*}{dN} = -\frac{S}{(N+1)^2}$$

- ✓ We can observe the equilibrium appropriation q^* **decreases** in N

  - ➢ *Intuitively*

    Every fisherman increase his catches as the resource becomes more abundant (higher S) but decreases them as competition becomes fiercer (higher N).

➢ Example 2

- The aggregate appropriation is

$$Q^* = \frac{NS}{N+1}$$

$$\frac{dQ^*}{dN} = \frac{(N+1-N)S}{(N+1)^2} = \frac{S}{(N+1)^2} > 0$$

✓ We can observe the aggregate appropriation Q^* **increases** in N

➢ Figure 2.3a.

- Depicts the equilibrium appropriation q^* as a function of the number of firms exploiting the commons.

$$q^* = \frac{S}{(N+1)} = \frac{100}{N+1}$$

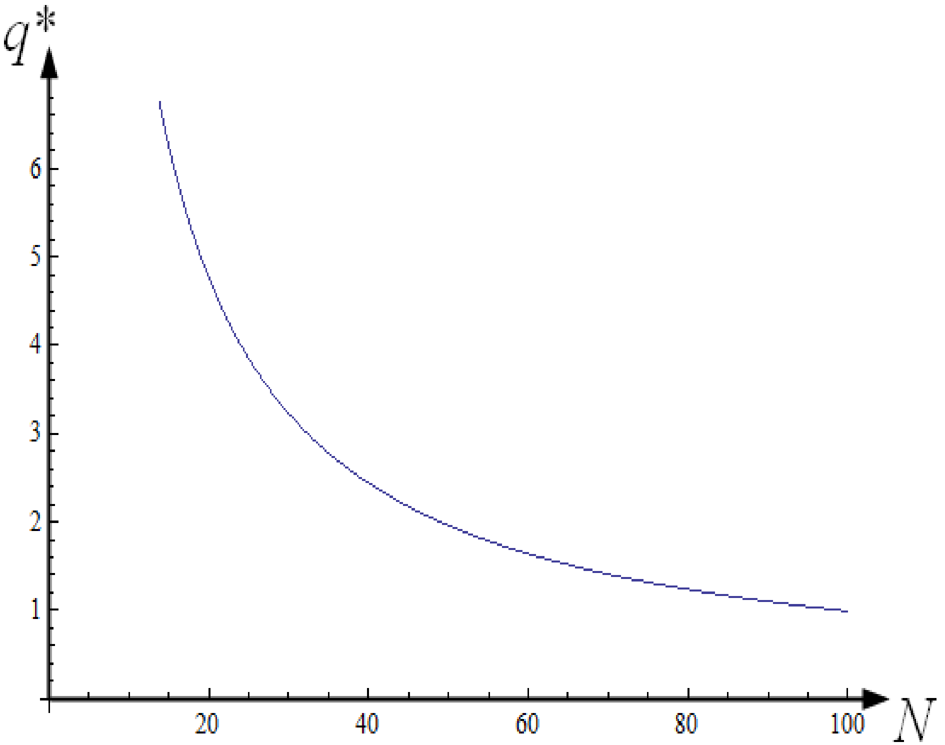


Figure 2.3a. Equilibrium appropriation q^* as a function of N.

➢ Figure 2.3b.

- Illustrates the aggregate equilibrium appropriation.

- $S = 100$

$$Q^* = \frac{NS}{N+1} = \frac{100N}{N+1}$$

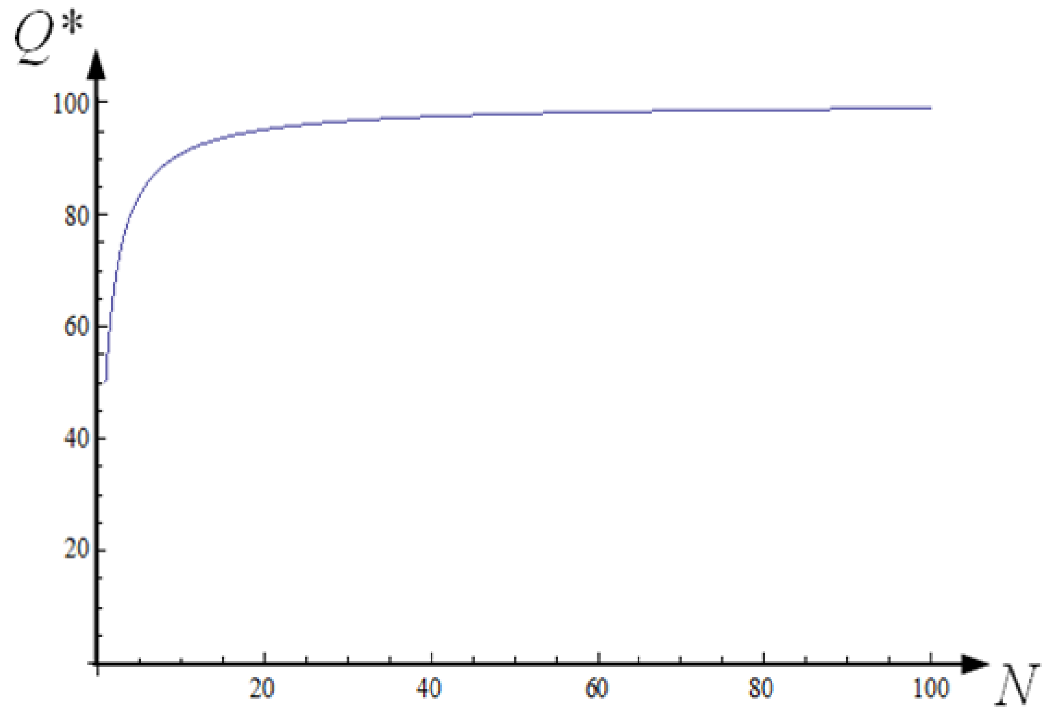


Figure 2.3b. Aggregate equilibrium appropriation Q^* as a function of N.

Figure 2.4

- Depicts q^* as a function of the available stock, S
- $N = 2$

$$q^* = \frac{S}{N+1} = \frac{S}{3}$$

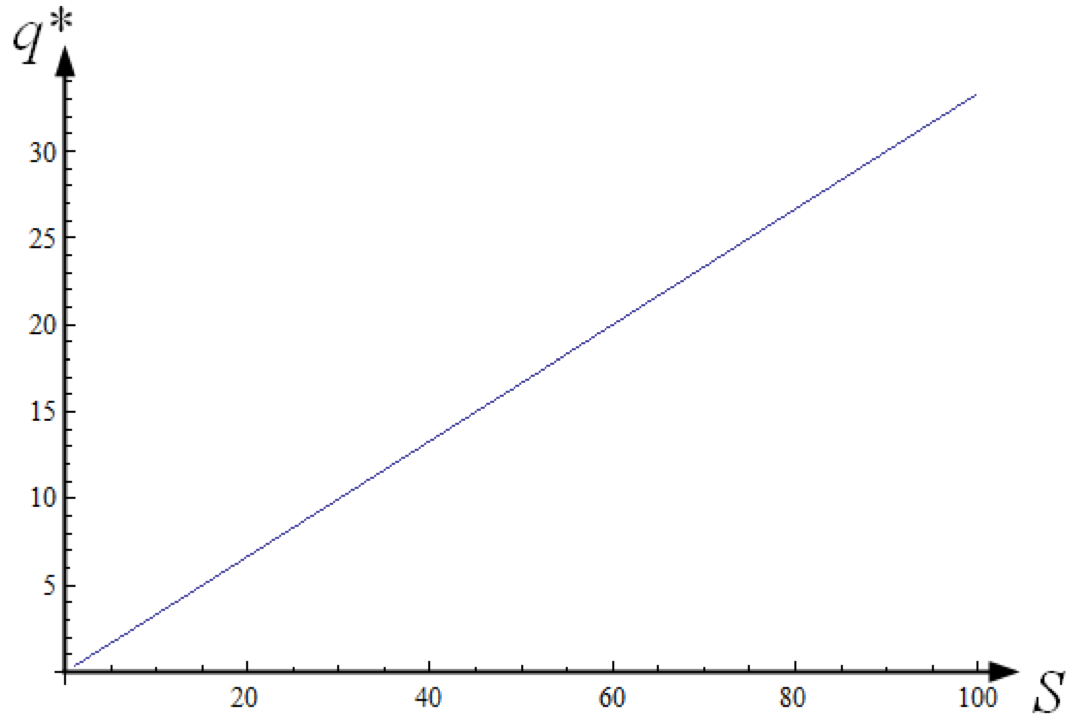


Figure 2.4. Equilibrium appropriation q^* as a function of S.

2.3.2 Extension - What if fishermen have some market power?

- In this stetting, we assume that;

  - Finite number of firms selling homogeneous products.

  - Other CPRs can be characterized by a few firms.

  - Each selling a relatively large share of total appropriations
    (Ex. North Sea, the Bering Sea, and the Western Pacific)

- In this setting, <u>**we can no longer**</u> assume that fishermen take prices as given.

2.3.2 Extension - What if fishermen have some market power?

➢ Modelling the CPRs

- The market demand $p(Q) = a - bQ$
- Q denotes the aggregate appropriation
- *where* $Q = q_i + Q_{-i}$ is the sum of fisherman *i*'s and those of all his rivals
- $a \geq 1$ and $b \geq 0$ are both positive parameters
- $b \geq 0$ indicates a larger appropriation decreases the market price at which all fishermen sell their product.

➢ Continue modelling the CPRs

- Every firm faces the following cost function;

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

- The market demand can be expressed as

$$p(Q) = a - bq_i - bQ_{-i}$$

- Fisherman *i*'s profit maximization problem

$$\max_{q_i \geq 0} \ \pi_i = \underbrace{(a - bq_i - bQ_{-i})q_i}_{\text{Total revenue}} - \underbrace{\frac{q_i(q_i + Q_{-i})}{S}}_{\text{Total costs}}$$

- ***How to solve this game?***

Step 1: Finding fisherman *i*'s best response function.

Step 2: Using best response functions to find the Nash equilibrium.

➤ ***Step 1:*** Finding fisherman *i*'s best response function.

- Differentiating the profit function with respect to q_i yields

$$a - 2bq_i - bQ_{-i} = \frac{2q_i + Q_{-i}}{S}$$

- Solving for q_i;

$$q_i(Q_{-i}) = \frac{aS}{2(1 + bS)} - \frac{1}{2}Q_{-i} \qquad (BRF_i)$$

➢ Numerical example

- When $a = 1$ and $b = 0$

I. The market price collapses to $p(Q) = \$1$,

II. The best response function simplifies to

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2} Q_{-i}$$

- The market prices are ***insensitive to sales*** (due to $b = 0$)

Numerical example

- When $N > 1$ and $b > 0$

I. The best response function simplifies to

$$q_i(Q_{-i}) = \frac{aS}{2(1+bS)} - \frac{1}{2}Q_{-i}$$

II. When b increases, the vertical intercept of the best response function ***decreases.***

- producing a downward shift without affecting its slope, $-\frac{1}{2}$

- Intuitively,

For a given appropriation by *i*'s rivals, and by treating Q_{-i} as given ;

the appropriation by fisherman i ***decreases*** *when the market price becomes more sensitive to aggregate appropriation (when parameter b increases)*

- **<u>The opposite effect</u>** arises when demand increases (as captured by an increase in a), as the vertical intercept of the best response function, $\frac{aS}{2(1+bS)}$ now increases, shifting the function upwards.

➢ ***Step 2:*** Using best response functions to find the Nash equilibrium.

- In a symmetric equilibrium; each fisherman appropriates the same amount of fish

implying that $q_1^* = q_2^* = \cdots = q_N^* = q^*$ All firms' catches coincide

so Q_{-i} becomes;

$$Q_{-i}^* = \sum_{j \neq i} q^* = (N-1)q^*$$

- Inserting this result in the best response function yields

$$q^* = \frac{aS}{2(1+bS)} - \frac{1}{2}(N-1)q^*$$

➢ The equilibrium appropriation becomes

$$q^* = \frac{aS}{(N+1)(1+bS)}$$

Numerical example

- $When\ a = 1 \quad and \quad b = 0$

I. The equilibrium appropriation simplifies to

$$q^* = \frac{S}{N+1}$$

➢ Numerical example

- $When\ N > 1 \quad and \quad b > 0$

I. The equilibrium appropriation simplifies to

$$q^* = \frac{aS}{(N+1)(1+bS)}$$

✓ II. When *b* increases every firm decreases its equilibrium appropriation q^* because consumers are more price sensitive.

✓ III. Its sales create now a negative effect on the market price which did not exist when such a price was given.

✓ IV. Its sales decrease in *N* when the market becomes more competitive but increase in *a* for a larger market size.

- Intuitively, the firm anticipates that selling more units will reduce market prices, so that it does not appropriate as much fish as when prices are insensitive to its catches.

- The aggregate equilibrium appropriation is

$$Q^* = Nq^* = \frac{aNS}{(N+1)(1+bS)}$$

which is increasing in N and S but decreasing in b because

$$\frac{\partial Q^*}{\partial b} = -\frac{aNS^2}{(N+1)(1+bS)^2} < 0$$

$$\frac{\partial Q^*}{\partial N} = \frac{aS}{(N+1)^2(1+bS)} > 0$$

$$\frac{\partial Q^*}{\partial S} = \frac{aN}{(N+1)(1+bS)^2} > 0$$

2.3.2 Extension - What if fishermen have some market power?

- The aggregate appropriation can be supported if

$$\frac{aNS}{(N+1)(1+bS)} \leq S$$

that we rearrange to yield

$$S \geq \underline{S} \equiv \frac{N(a-1)-1}{b(N+1)}$$

which is increasing in a and N but decreasing in b because

$$\frac{\partial \underline{S}}{\partial N} = \frac{a-1}{b(N+1)^2} > 0$$

- Intuitively, more resources are needed to support more firms.

- When $N \to \infty$, the aggregate appropriation becomes

$$\lim_{N\to\infty} Q^* = \lim_{N\to\infty} \frac{aNS}{(N+1)(1+bS)} = \frac{aS}{1+bS}$$

which is admissible if it falls below S*, that is*

$$\frac{aS}{1+bS} \leq S$$

that we rearrange to yield

$$S \geq \frac{a-1}{b}$$

Common pool resources Socially optimal appropriation

Question….!

Is equilibrium appropriation excessive from a social point of view?

- To answer that question, we start by defining the socially optimal appropriation;

➢ *Definition 1*:

The socially optimal appropriation is the one maximizing the fishermen's joint profits

$$W = PS$$

where $PS = \sum_{i=1}^{N} \pi_i$ denotes the sum of all firms' profits

- In the case of only two fishermen (a CPR cartel)

$$W \text{ } collapses \text{ } to \quad W = \pi_1 + \pi_2$$

- *Definition 2*:

General welfare function is the sum of consumer and producer surplus;

$$W = CS + PS$$

where $CS = \int_0^Q p(Q)dQ - p(Q)Q$ denotes consumer surplus

Continue definition 2:

Welfare function in definition 2 is more common in CPRs where catches are sold in the domestic market, thus affecting domestic consumers.

Definition 3:

Welfare function can be further generalized to

$$W = (1 - \lambda)\, CS + \lambda PS$$

where;

λ : the weight that the social planner assigns to producer surplus

$(1 - \lambda)$ captures the weight that she assigns to consumer surplus

➢ ***Special cases on λ;***

- When $\lambda = 1$

  ✓ The welfare function collapses to;

$$W = PS$$

- Indicating that the social planner does not care about consumer surplus.

- ***This case happened*** when all appropriation is sold overseas so domestic consumers are not affected by the price of the good as, in short, they do not buy the product

2.4 Common pool resources Socially optimal appropriation

- When $\lambda = \frac{1}{2}$

  ✓ The welfare function becomes

$$W = \frac{CS + PS}{2}$$

- Since $\frac{1}{2}$ enters as a constant, it can be graphically understood as a vertical shifter of $CS + PS$, and as a result;

$W = \frac{CS + PS}{2}$ coincides with that maximizing $W = CS + PS$

- When $\lambda = 0$

  ✓ The welfare function collapses to

$$W = CS$$

- Indicating that the social planner does not assign any weight to fishermen's profits

- ***This case happened*** if they are all foreign firms operating at a CPR overseas which does not have effects on domestic welfare, other than those channeled through the demand function and CS

- Find the socially optimal appropriation that maximizes welfare

  - In the next slides we will discuss how to find the socially optimal appropriation *__under special cases__* when;

    - Only profits matter (In section 2.4.1)

    - Consumers and profits matter (In section 2.4.2)

- We focus on the case in which;

  - Fishermen take prices as given $p = \$1$

  - There are two fishermen $N = 2$

2.4.1 Socially optimal appropriation when only profits matter

- When $\lambda = 1$

The social planner considers the welfare function $W = PS$

$$\max_{q_1, q_2 \geq 0} W = PS = \pi_1 + \pi_2$$

which can be rewritten as

$$\max_{q_1, q_2 \geq 0} \pi_1 + \pi_2 = \left(q_1 - \frac{q_1(q_1 + q_2)}{S}\right) + \left(q_2 - \frac{q_2(q_2 + q_1)}{S}\right)$$

- This problem is equivalent to that of a fishermen cartel where fishermen 1 and 2 coordinate their catches, q_1 and q_2, to maximize their joint profits

- Differentiating with respect to q_1, q_2;

$$1 - \frac{2(q_1 + q_2)}{S} = 0$$

➢ Intuitively

- The first term represents the marginal revenue (MR) from additional catches
- The second term captures fisherman *i*'s marginal cost (MC)
- Increasing catches produces twice as much marginal costs. ***Why*?**

✓ Since every fisherman takes into account not only the increase in his own costs but also the increase in his rival's cost

2.4.1 Socially optimal appropriation when only profits matter

- *In brief;*

Every fisherman internalizes the cost externality that his appropriation generates on other fishermen, as a larger q_i increases the cost of fisherman *j*.

$$S = 2(q_1 + q_2)$$

- Solving for q_1;

$$q_1(q_2) = \frac{S}{2} - q_2 \qquad \text{for fisherman 1}$$

$$q_2(q_1) = \frac{S}{2} - q_1 \qquad \text{for fisherman 2}$$

2.4.1 Socially optimal appropriation when only profits matter

❖ ***The discussion in the next slide.***

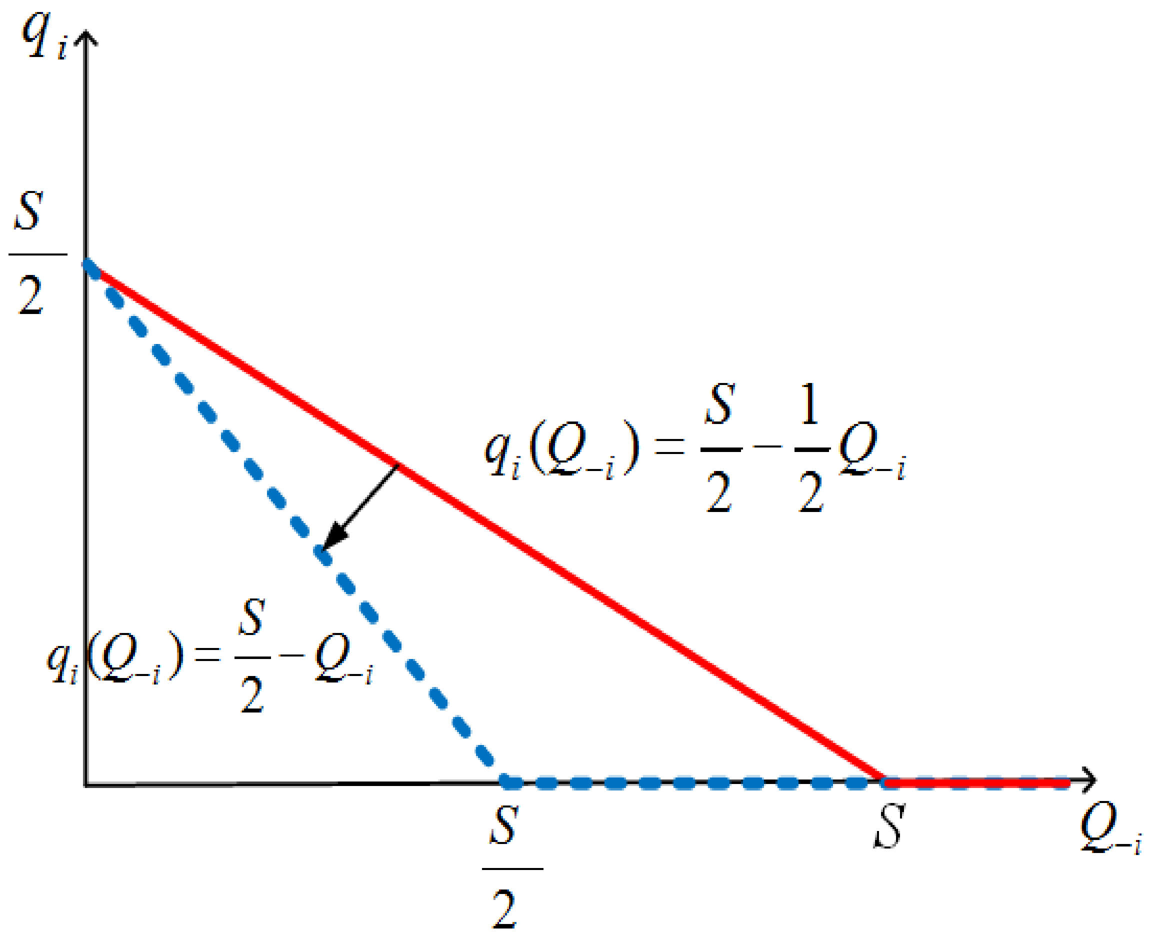


Figure 2.5. Equilibrium vs. joint profit maximization in the commons

- Figure 2.5 indicates the following:

- For a given amount of appropriation from firm 2 (q_2), firm 1 chooses to appropriate fewer units when firms coordinate their exploitation of the resource (jointly maximizing profits) than when every firm independently selects its own appropriation

- If fisherman 2 appropriates half of the available stock, $q_2 = \frac{S}{2}$, fisherman 1 responds by not appropriating anything, $q_1 = 0$

❖ *Question….!*

How to find the horizontal intercept of expression $q_1(q_2) = \frac{S}{2} - q_2$*?*

➢ *Confirm the finding;*

I. Let us simultaneously solve for appropriation levels q_1 and q_2

$$q_1(q_2) = \frac{S}{2} - q_2 \qquad \text{for fisherman } 1$$

$$q_2(q_1) = \frac{S}{2} - q_1 \qquad \text{for fisherman } 2$$

II. We consider that, among all optimal pairs, a natural equilibrium is that in which both firms appropriate the same amount.

- Since firms are symmetric, the socially optimal output, q^{SO}, becomes

$$q_1^{SO} = q_2^{SO} = q^{SO}$$

III. Inserting $q_1^{SO} = q_2^{SO} = q^{SO}$ in the equation for fisherman 1

$$q^{SO} = \frac{S}{2} - q^{SO}$$

and solving for q^{SO};

$$q^{SO} = \frac{S}{4}$$

2.4.1 Socially optimal appropriation when only profits matter

IV. When agents are independent,

$$q^* = \frac{S}{N+1}$$

Evaluating at the case of $N = 2$ fishermen;

$$q^* = \frac{S}{2+1} = \frac{S}{3}$$

2.4.1 Socially optimal appropriation when only profits matter

- Comparing the results

$$q^* > q^{SO}$$

$$\frac{S}{3} > \frac{S}{4}$$

- *In words,*

The agents exploit the resource less intensively when they coordinate their appropriation decisions (and thus internalize the cost externalities their appropriation generates on others) than when they do not coordinate their exploitation.

"The tragedy of the commons"

2.4.2 Socially optimal appropriation with consumers and profits matter

- When the social planner considers welfare function

$$W = (1-\lambda)CS + \lambda PS$$

- She chooses the level of catches q_1 and q_2 to solve

$$\max_{q_1, q_2 \geq 0} W = (1-\lambda)CS + \lambda PS$$

where

$$CS = \int_0^Q p(Q)dQ - p(Q)Q$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- The inverse demand function;

$p(Q) = 1 - Q$ is linear in the aggregate appropriation

- Consumer surplus can be expressed as the area of the triangle below the demand function;

$$CS = \frac{1}{2}[1 - (1 - Q)](Q - 0) = \frac{1}{2}Q^2$$

- The aggregate appropriation can be expanded as;

$$Q = q_1 + q_2$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- The social welfare can be rewritten as;

$$\max_{q_1,q_2\geq 0} W = (1-\lambda)\frac{1}{2}(q_1+q_2)^2 + \lambda(\pi_1+\pi_2)$$

- Differentiating with respect to q_1 and q_2

$$\frac{\partial W}{\partial q_1} = (1-\lambda)(q_1+q_2) + \lambda\left(1-2(q_1+q_2) - \frac{2(q_1+q_2)}{S}\right) = 0$$

$$\frac{\partial W}{\partial q_2} = (1-\lambda)(q_1+q_2) + \lambda\left(1-2(q_1+q_2) - \frac{2(q_1+q_2)}{S}\right) = 0$$

2.4.2 Socially optimal appropriation with consumers and profits matter

➢ In a symmetric social optimum, firms exploit the CPR at the same rate;

$$q_1^{SO} = q_2^{SO} = q^{SO}$$

$$2(1-\lambda)q^{SO} + \lambda\left(1 - 4q^{SO} - \frac{4q^{SO}}{S}\right) = 0$$

- Solving for q^{SO}, we obtain the socially optimal appropriation,

$$q^{SO}(\lambda) = \frac{\lambda S}{2(3\lambda S + 2\lambda - S)}$$

- which is positive if and only if $3\lambda S + 2\lambda - S > 0$, solving

$$\lambda > \bar{\lambda} \equiv \frac{S}{2 + 3S}$$

2.4.2 Socially optimal appropriation with consumers and profits matter

- The socially optimal aggregate appropriation is

$$Q^{SO}(\lambda) = \frac{\lambda S}{3\lambda S + 2\lambda - S}$$

which is admissible if it falls below S, that is

$$\frac{\lambda S}{3\lambda S + 2\lambda - S} \leq S$$

that we rearrange to yield

$$\lambda \geq \underline{\lambda} \equiv \frac{S}{1 + 3S}$$

where $\bar{\lambda} > \underline{\lambda}$ suggests that whenever firms produce positive socially optimal units, the resource can be supported.

2.4.2 Socially optimal appropriation with consumers and profits matter

- The socially optimal aggregate appropriation is increasing in S since

$$\frac{\partial Q^{SO}(\lambda)}{\partial S} = \frac{2\lambda^2}{(3\lambda S + 2\lambda - S)^2} > 0$$

- The cutoff for positive extraction, $\bar{\lambda}$, increases in S beause

$$\frac{\partial \bar{\lambda}}{\partial S} \equiv \frac{2}{(2 + 3S)^2} > 0$$

entailing that when the resource becomes more abundant, the social planner has to pay more heed to the fishermen's interests.

- Taking $S \to \infty$, we find $\lim_{S\to\infty} \bar{\lambda} = \lim_{S\to\infty} \frac{1}{\frac{2}{S}+3} = \frac{1}{3}$, so that positive units are always solicited when policy weight satisfies $\lambda \geq 1/3$.

2.4.2 Socially optimal appropriation with consumers and profits matter

- Case: when $\lambda = 1$

  - ✓ The socially optimal appropriation simplifies to

$$q^{SO}(1) = \frac{S}{4(1+S)}$$

  - ✓ The social planner only considered producer surplus $(\lambda = 1)$

Question..!

What is the impact of change in the weight on producer surplus on the socially optimal appropriation level?

2.4.2 Socially optimal appropriation with consumers and profits matter

- General case

- Differentiating $q^{SO}(\lambda)$ with respect to λ

$$\frac{\partial q^{SO}(\lambda)}{\partial \lambda} = -\frac{S^2}{2(3\lambda S + 2\lambda - S)^2}$$ (which is negative)

- Intuitively,

The regulator decreases the socially optimal appropriation when she assigns a larger weight to producer surplus as a means to mitigate negative externality each firm inflicts on one another.

Facing our first inefficiency

2.5 Facing our first inefficiency

- *From previous section,*

  - Our results help us to identify the first inefficiency in the exploitation of the commons by individual firms.

- Firms' equilibrium appropriation is larger than that a social planner would select. This happens regardless of the welfare function that she considers, that is, both when;

  I. she only seeks to maximize firms' joint profits

$$W = \pi_1 + \pi_2$$

II. her objective is to maximize a weighted sum of consumer and producer surplus

$$W = (1 - \lambda)CS + \lambda PS$$

- Intuitively;

- Every individual fisherman ignores the negative cost externality that his appropriation produces on the other fishermen, and thus exploits the resource above the socially optimal level.

  - ***Ex.*** The Chilean jack mackerel in the Southeast Pacific, and the Peruvian anchovy in the Southeast Pacific.

2.5 Facing our first inefficiency

- Our result is analogous to that in the standard Cournot model of quantity competition, where firms tend to produce too much, relative to the output that would maximize their joint profits in a cartel,

    - Since they ignore the negative effect that their sales generate on their rivals' revenues

      (as these sales decrease the market price which, in turn, reduce the total revenue of all firms in the industry)

- This negative effect is, however, internalized when firms coordinate their production decisions to maximize their joint profits or, more generally, when a social planner determines individual output levels

Inefficient exploitation with more general functions

2.6 Inefficient exploitation with more general functions

- In this section, we want to show that;
  - The appropriation is excessive relative to the social optimum,
  - Or more compactly, the equilibrium appropriation is socially excessive

$$q^* > q^{SO}$$

- We show this result **without** assuming a specific cost function
  - Our previous analysis considered a specific cost function for every firm i

$$C_i(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

2.6 Inefficient exploitation with more general functions

➢ We only assume that firm *i*'s marginal cost $MC_i = \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i}$ satisfies the following properties:

- ***Assumption 1*:**
  Positive, $MC_i > 0$, and increasing in firm *i*'s own appropriation, $\frac{\partial MC_i}{\partial q_i} > 0$;

- ***Assumption 2*:**
  Decreasing in the available stock, $\frac{\partial MC_i}{\partial S} < 0$;

- ***Assumption 3*:**
  Increasing in the appropriation of any rival firm j, $\frac{\partial MC_i}{\partial q_j} > 0$, where $j \neq i$.

➢ Intuitively;

- Assumption 1 says that every fisherman i faces a positive and increasing cost for every additional unit the firm appropriates.

- Assumption 2 suggests that fisherman i can capture q_i tons of fish more easily when the stock becomes more abundant.

- Assumption 3 indicates that, when other fishermen increase their appropriation Q_{-i}, the resource becomes more scarce, increasing the time and effort that fisherman i needs to spend in order to appropriate a given amount.

2.6 Inefficient exploitation with more general functions

Given that $$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

we have $$\frac{\partial C(q_i, Q_{-i})}{\partial q_i} = \frac{2q_i + Q_{-i}}{S} = MC_i$$

which is

✓ Positive and increasing in q_i (as required by Assumption 1)

✓ Decreasing in the stock S (as required by Assumption 2)

✓ Increasing in the appropriation by firm *i*'s rivals, Q_{-i} (as required by Assumption 3)

2.6 Inefficient exploitation with more general functions

➢ Equilibrium appropriation

$$\max_{q_i \geq 0} \pi_i = q_i - C_i(q_i, Q_{-i})$$

$$\frac{\partial \pi_i}{\partial q_i} = 1 - \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i} = 0$$

- We can express the above result more compactly as

$$MC_i = 1$$

➢ In words,

every fisherman i increases his individual appropriation until the point where his marginal revenue from additional sales coincides with the marginal cost of this additional appropriation.

❖ *The discussion in the next slide….!*

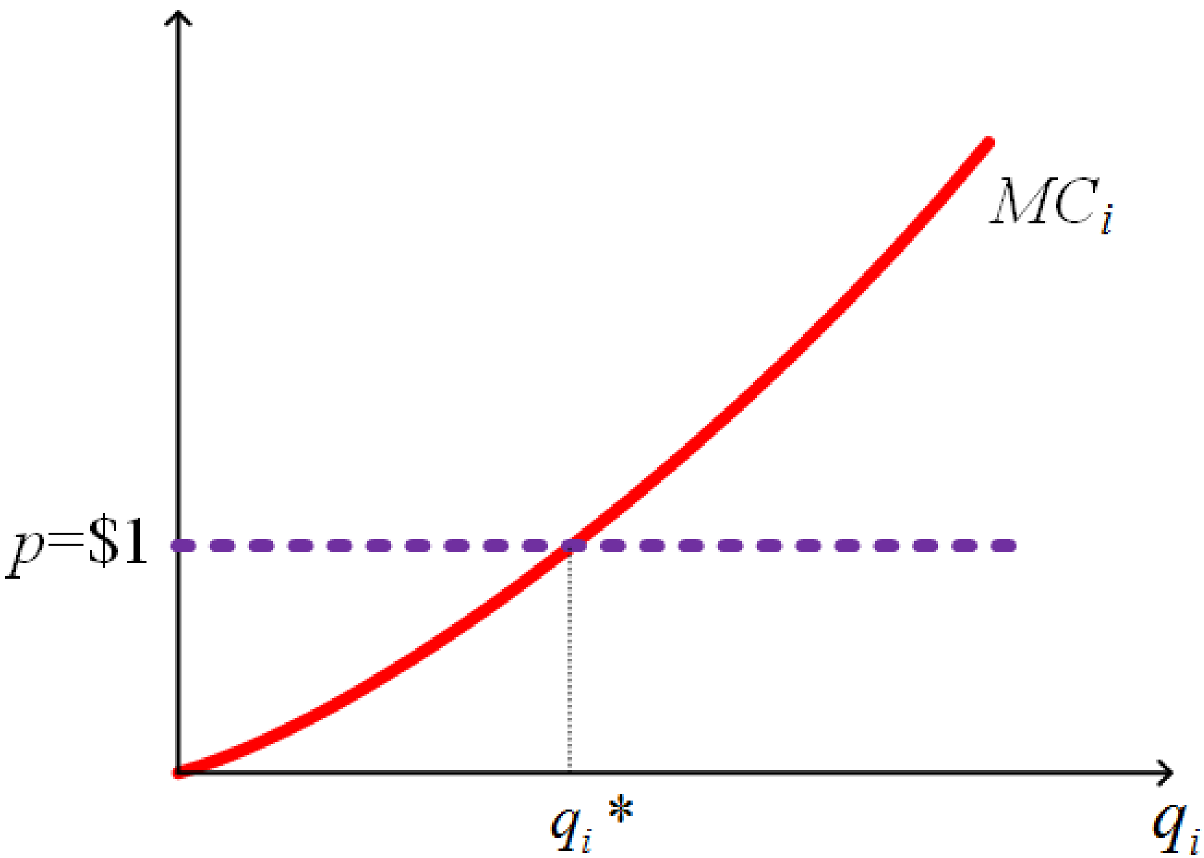


Figure 2.6. Equilibrium appropriation q_i^*

2.6 Inefficient exploitation with more general functions

- Figure 2.6 depicts condition $MC_i = 1$, by separately plotting the price $p = \$1$ and the marginal cost MC_i. This marginal cost is increasing in firm *i*'s appropriation q_i since, ***by Assumption 1***, $\frac{\partial MC_i}{\partial q_i} > 0$.

- When firm j increases its individual appropriation q_j, firm *i*'s marginal cost MC_i increases, since $\frac{\partial MC_i}{\partial q_j} > 0$ by ***Assumption 3***; whereas the marginal revenue in the right-hand side of $MC_i = 1$ is unaffected.

- In Figure 2.6, curve MC_i shifts upward, entailing that the crossing point between MC_i and \$1 moves to the left, reducing firm *i*'s equilibrium appropriation q_i.

➢ The socially optimal appropriation

- Assuming the welfare function considers only joint profits, the social planner solves a problem, that is

$$\max_{q_1,\ldots,q_N \geq 0} W = PS = \sum_{i=1}^{N} \pi_i = \sum_{i=1}^{N} [q_i - C_i(q_i, Q_{-i})]$$

2.6 Inefficient exploitation with more general functions

- Which can be expanded as the sum of firm *i*'s profits plus the profits of all its rivals $\pi_i + \sum_{j \neq i} \pi_j$, as follows

$$\max_{q_i, \ldots, q_N \geq 0} W = [q_i - C_i(q_i, Q_{-i})] + \sum_{j \neq i} [q_j - C_j(q_j, Q_{-j})]$$

Differentiating with respect to every q_i, we find

$$1 - \frac{\partial C_i(q_i, Q_{-i})}{\partial q_i} - \sum_{j \neq i} \frac{\partial C_j(q_j, Q_{-j})}{\partial q_i} = 0$$

2.6 Inefficient exploitation with more general functions

- Since Q_{-j} includes q_i as one of its components,

  - we can rearrange the expression as;

$$MC_i + \sum_{j \neq i} \frac{\partial C_j(q_j, Q_{-j})}{\partial q_i} = 1$$

- ✓ Our result then coincides with equilibrium condition $MC_i = 1$, except for the new term $\sum_{j \neq i} \frac{\partial C_j(q_j, Q_{-j})}{\partial q_i}$.

- Intuitively,

Every firm i increases its individual appropriation until the point where its marginal revenue from appropriating one more unit $(p = \$1)$ coincides with the sum of its own additional cost, MC_i, and the additional cost that its appropriation generates on all other firms, $\sum_{j \neq i} \frac{\partial C_j(q_j, Q_{-j})}{\partial q_i}$.

2.6 Inefficient exploitation with more general functions

- Relative to the equilibrium condition $MC_i = 1$, every firm now internalizes the negative cost externality that its individual appropriation q_i produces on its rivals.

  - As a result of this additional cost, firm i chooses a lower exploitation in the social optimum than in equilibrium, $q_i^{SO} < q_i^*$

- Figure 2.7 illustrates this result and compares it against that emerging from equilibrium condition, $MC_i = 1$.

2.6 Inefficient exploitation with more general functions

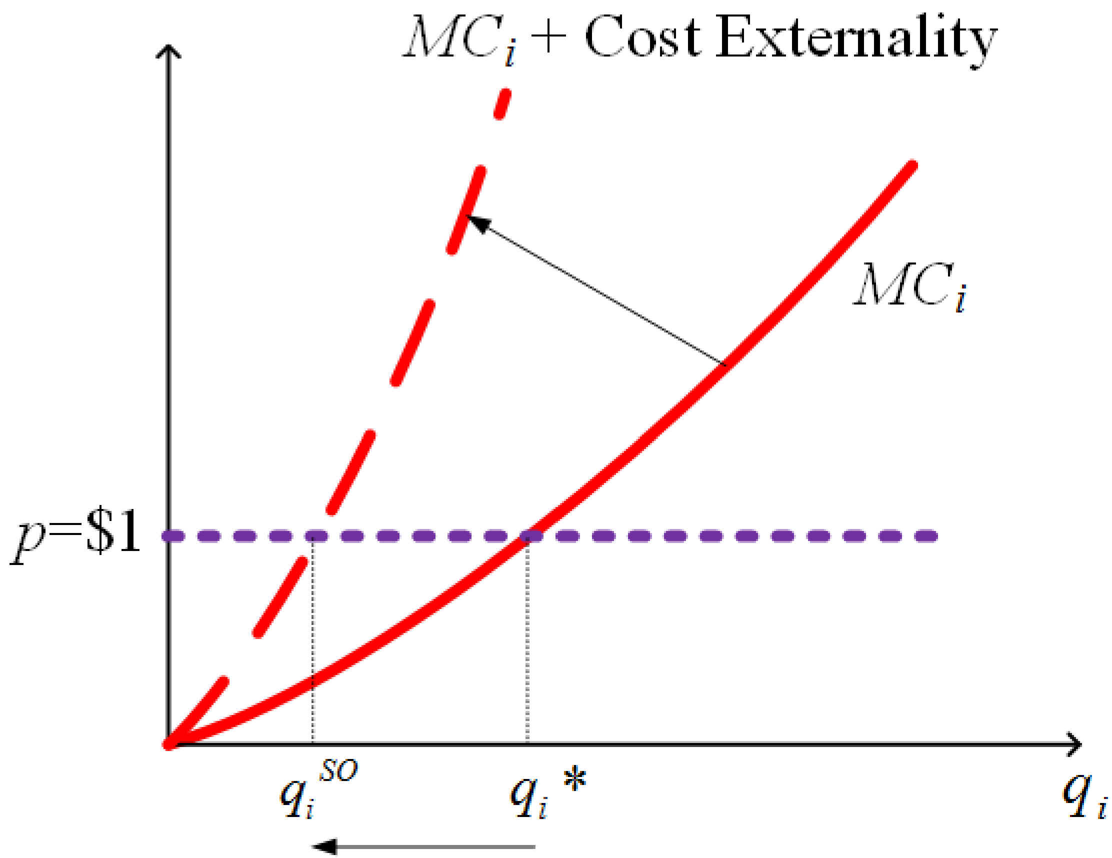


Figure 2.7. Equilibrium and socially optimal appropriation

2.7 Policy instruments

➢ In this section,

- We discuss some policy instruments to correct the socially excessive exploitation that we identified in our previous results.

➢ Two policy instruments

I. Quotas
II. Appropriation fees

2.7.1. Policy instruments-Quotas-

- The regulator can set a quota that lets fisherman i catch as much fish as the socially optimal level, q_i^{SO}, facing stringent penalties if it exceeds this allowance.

- Quotas are rather common in several CPRs such as;

  - ***Common Fisheries Policy*** in the European Union, which sets quotas on which types of fish each member state can fish.

  - ***Individual transferable quotas*** assigned to each fisherman in the U.S. or New Zealand.

*These quotas are also known as **Catch Share***

2.7.1. Policy instruments-Quotas-

- How these quotas work?

  - The regulator starts by setting a total allowable catch for each species of fish and for a given time period;

  - and then a dedicated portion is assigned to individual fishermen in the form of quotas, which are transferable, and thus can be bought, sold, and leased to other fishermen.

- *Example;*

- In 2008, 148 major fisheries and 100 smaller fisheries around the world had adopted some from of individual transferable quota.

- *How do these quotas assign?*

- Quotas are often initially assigned according to the recent catch history of the fishermen, implying that those who more intensively appropriate the resource receive larger quotas.

- This assignment rule can, then, induce fishermen to increase their relative appropriation of the resource to receive a larger transferable quota, which they can keep or sell in future periods.

2.7.1. Policy instruments-Quotas-

➢ Quota auctions

- Quota auctions have been proposed as an alternative allocation mechanism, which may prevent the previous perverse incentives to increase appropriation before the quota is allocated and, in addition, raises public funds for access to fisheries.

➢ Quotas in aquifers

- Quotas in aquifers are less common, but countries such as Mexico and Spain set limits on private use; otherwise, the farmer can lose his water permit.

➢ Other command-and-control regulations

- Other command-and-control regulations include restrictions on the boat size, fishing gear (such as mesh or net size), limits on the days certain boats can fish, or prohibiting the catch of juvenile fish; among others.

➢ Appropriation fees

- The regulator can set an appropriation fee to fisherman i, t_i, that induces this fisherman appropriate the socially optimal level q_i^{SO}.

➢ In this setting,

- Every fisherman i solves a problem analogous to

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S}$$

- but with marginal costs increased by t_i.

- Fisherman *i*'s objective function now becomes

$$\max_{q_i \geq 0} \pi_i = q_i - \frac{q_i(q_i + Q_{-i})}{S} - q_i t_i$$

- First-order condition with respect to q_i

$$1 - \frac{2q_i + Q_{-i}}{S} - t_i = 0$$

- Solving for appropriation q_i, we find best response function

$$q_i(Q_{-i}) = \frac{S(1 - t_i)}{2} - \frac{1}{2} Q_{-i}$$

➢ *When the appropriation fee is absent,* $t_i = 0$

$$q_i(Q_{-i}) = \frac{S}{2} - \frac{1}{2}Q_{-i} \qquad (BRF_i)$$

✓ It coincides with that in section 2.3

➢ *When the appropriation fee is present* t_i

- A more stringent fee decreases the vertical intercept of the best response function, $\frac{S(1-t_i)}{2}$, without affecting its slope.

- Graphically;
  - We can imply a parallel downward shift of fisherman i's best response function.

(Try to draw it on Figure 2.1)

- Intuitively,
  - For a given aggregate appropriation from his rivals Q_{-i}, fisherman i decreases his individual appropriation when facing a more stringent fee.
  - This comes as no surprise since this fee increases the fisherman's marginal cost of additional appropriation, reducing his incentives to exploit the resource.

2.7.2 Policy instruments-Appropriation fees

- In a symmetric equilibrium,

  - $q_i^* = q_j^* = q^*$, which entails that $Q_{-i}^* = (N-1)q_i^*$.

  - Inserting this property in the above best response function;

$$q^* = \frac{S(1-t_i)}{2} - \frac{1}{2}(N-1)q^*$$

Rearranging yields $q^*(N+1) = S(1-t_i)$

2.7.2 Policy instruments-Appropriation fees

Solving for q^*;

$$q^*(t_i) = \frac{S(1-t_i)}{N+1}$$

"The equilibrium appropriation"

- *Case 1: When the appropriation fee is absent,* $t_i = 0$

$$q^*(0) = \frac{S}{N+1}$$

- *Case 2: When the appropriation fee is present,* $t_i > 0$

Nonetheless, equilibrium appropriation is lower

➢ Questions…!

- How can the regulator find the appropriation fee t_i that induces fisherman *i* exploit the resource at the socially optimal level q_i^{SO}?

- What appropriation fee t_i, inserted in fisherman *i*'s equilibrium appropriation $q^*(t_i)$, induces this fisherman to appropriate q_i^{SO}?

2.7.2 Policy instruments-Appropriation fees

- The regulator seeks to achieve $q^*(t_i) = q_i^{SO}$;

$$q^*(t_i) = \frac{S(1 - t_i)}{N + 1} \quad \text{and} \quad q_i^{SO} = \frac{S}{2N}$$

- Setting $q^*(t_i) = q_i^{SO}$, yields

$$\frac{S(1 - t_i)}{N + 1} = \frac{S}{2N}$$

- Solving for t_i^*, we obtain

$$t_i^* = \frac{N - 1}{2N}$$

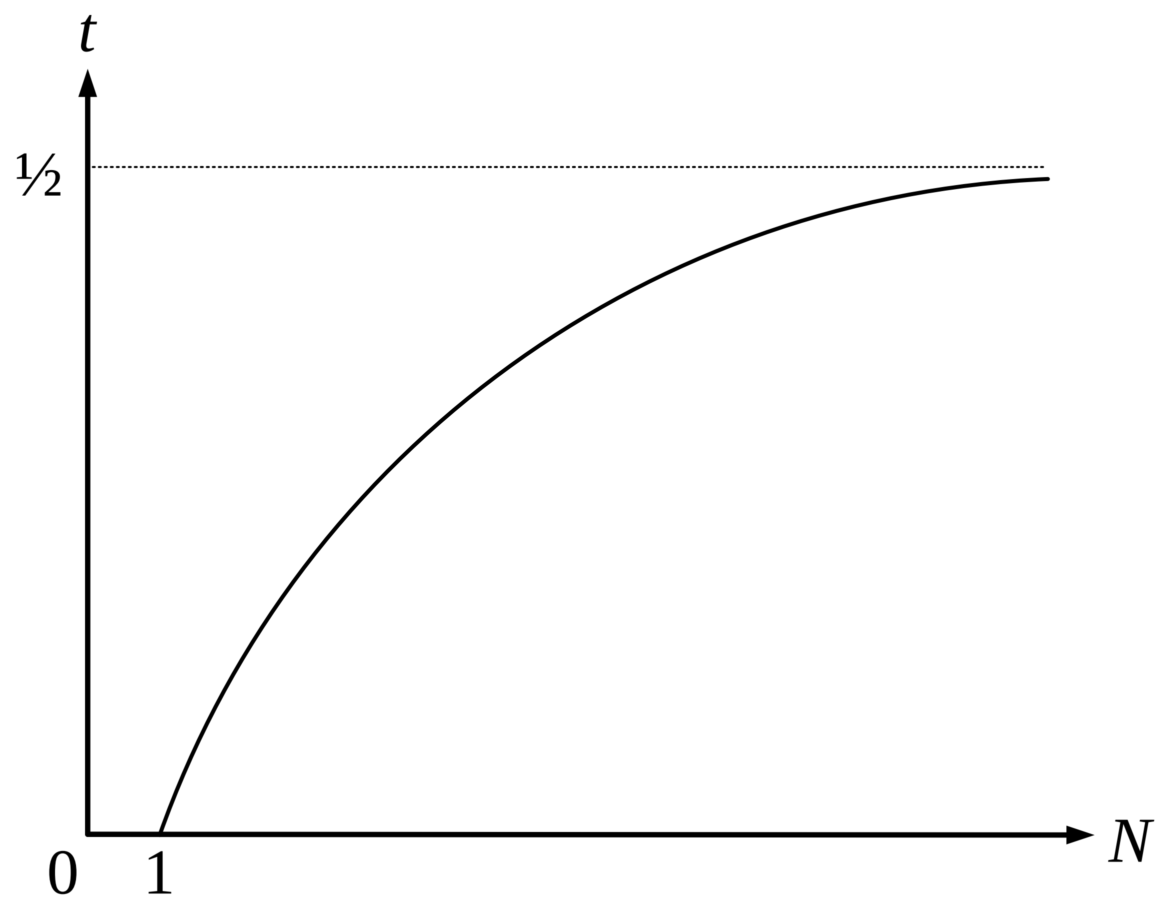
- Note that t_i^* is increasing in the number of firms since

$$\frac{\partial t_i^*}{\partial N} = \frac{1}{2N^2} > 0$$

- Intuitively, the regulator seeks to induce the same socially optimal aggregate output, $Q^{SO} = \frac{S}{2}$, regardless of the number of firms.
- When $N = 1$, the above fee reduces to $t_i^* = \frac{1-1}{2\times 1} = 0$, since the monopoly chooses socially optimal appropriation, and thus, does not require appropriation fee to adjust its production behavior.

2.7.2 Policy instruments-Appropriation fees

- When $N \to \infty$, the above fee becomes $t_i^* = \lim_{N\to\infty} \frac{1-\frac{1}{N}}{2} = \frac{1}{2}$.



2.7.2 Policy instruments-Appropriation fees

- What is the impact of the number of firms on the equilibrium appropriation $q^*(t_i)$?

  - When few firms operate in the commons, the equilibrium exploitation of each firm, $q^*(t_i)$, exceeds q_i^{SO}, requiring a positive fee to reduce exploitation. When several firms compete, the equilibrium appropriation of each firm, $q^*(t_i)$, is relatively low, while $q_i^{SO} = \frac{S}{2N}$ decreases more rapidly in N, leading the regulator to set a more stringent appropriation fee.
  - While appropriation fees are less common in fisheries, they are relatively frequent in groundwater agricultural use.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 3: Common Pool Resources in a Dynamic Setting

Outline

- Modeling CPRs in a dynamic setting
- Finding equilibrium appropriation
- Socially optimal appropriation
- Static and dynamic inefficiencies
- Equilibrium vs. socially optimal number of firms

Modeling CPRs in a dynamic setting

- In this chapter:
  - We examine another form of inefficiency in CPRs which is *dynamic inefficiency*.
- *Dynamic inefficiency (DI)*:
  - DI arises from firms ignoring the negative externality that their current appropriation generates on other firms' future costs.
- In this setting, we consider a sequential-move game where:
  - *In the first period;*
    only one firm (incumbent) operates in the CPR.
  - *In the second period;*
    two firms compete for the resource (the incumbent and an entrant).

- Unlike *static* inefficiency model,
  - In a two-period sequential-move model, every firm *i* chooses its individual appropriation x_i facing a cost function similar to the *static* inefficiency model;

$$C(x_i, X_{-i}) = \frac{x_i(x_i + X_{-i})}{S} \qquad \textit{(First-period cost)}$$

- Where:
  - x_i represents individual *i*'s appropriation.
  - $X_{-i} = \sum_{i \neq j} x_j$ reflects aggregate appropriations by firm *i*'s rival.
  - $S \geq 0$ denotes the stock of the resource.

➢ ***Case:*** Having only one firm operates in the first period, $X_{-i} = 0$.

- The cost function simplifies to

$$C(x_i, X_{-i}) = \frac{x_i^2}{S} \qquad \textit{(First-period cost)}$$

➢ *In the second period:*

- Every firm faces a slightly different cost function, which accounts for the resource exploitation, as follows:

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S - (1 - r)x}$$ *(Second-period cost)*

➢ Where:

- q_i represents firm *i*'s second period appropriation.
- $Q_{-i} = \sum_{i \neq j} q_j$ reflects aggregate appropriations by firm *i*'s rival.
- $r \in [0,1]$ represents the regeneration rate of the resource.

➢ ***Role on second-period-costs:***

❖ ***Case 1:*** When the resource fully regenerates, $r = 1$.

1. The second-period cost function simplifies to

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S}$$

2. Second period cost becomes symmetric to that in the first period.

3. First-period aggregate appropriation, X, ***does not affect the firm's second-period costs,*** since the stock fully regenerates across periods.

➢ ***Continue role on second-period-costs:***

❖ ***Case 2:*** When the regeneration rate is nil, $r = 0$.

1. The second-period cost function simplifies to

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S - x}$$

2. This suggests that the stock available at the beginning of the second period is $S - x$.

3. Namely, the initial stock S diminished exactly by every unit of the first-period appropriation, x.

Given the second-period cost function of

$$C(q_i, Q_{-i}) = \frac{q_i(q_i + Q_{-i})}{S - (1-r)x} \qquad \textit{(Second-period cost)}$$

The marginal cost function becomes

$$MC_i(q_i, Q_{-i}) = \frac{2q_i + Q_{-i}}{S - (1-r)x}$$

➢ ***Proof***: *The* MC_i *satisfies the* ***three assumptions*** *in* ***Chapter 2****.*
(You may check this as a practice)

➢ *Intuitively,*

- Marginal cost is decreasing in the regeneration rate, *r*, indicating that exploiting the resource becomes easier for the fisherman as a larger proportion of first-period appropriation is replenished with new fish before the beginning of the second period. Check that

$$\frac{\partial C(q_i, Q_{-i})}{\partial r} = -\frac{q_i(q_i + Q_{-i})x}{[S-(1-r)x]^2} < 0$$

- ***How to solve the game?***
  - This game is a sequential-move game of complete information.
    - ✓ We apply backward induction to find the Subgame Perfect Equilibrium (SPE):

also known as rollback equilibrium

- ***In the second period:***
  - ✓ We analyze firms' behavior by taking the first period appropriation as given.

- ***In the first period:***
  - ✓ We examine first-period appropriation where firms perfectly anticipate their profits in the second period of the game.

Finding equilibrium appropriation

3.3.1 Equilibrium appropriation in the second period.

3.3.2 Equilibrium appropriation in the first period.

- Remark:
  - We examine dynamic inefficiency *(DI)* in CPRs.

- We consider a sequential-move game where:
  - ***<u>In the first period</u>*:**

    only one firm "incumbent" operates in the CPR.
    - In this period, the incumbent chooses its first-period appropriation x
  - ***<u>In the second period</u>*:**

    two firms select for the resource (the incumbent and an entrant).
    - In this period, the incumbent and an entrant choose their second-period appropriation, q_1 and q_2, respectively.

3.3.1 Equilibrium appropriation in the second period

➢ ***Operating by backward induction***

- In the second period, the incumbent and an entrant choose their second-period appropriation, q_1 and q_2, respectively

$$\max_{q_i \geq 0} \pi_i^{2nd} = q_i - \frac{q_i(q_i + q_j)}{S - (1-r)\, x}$$

➢ Differentiating with respect to q_i, yields:

$$\underbrace{1}_{MR_i} - \underbrace{\frac{2q_i + q_j}{S - (1-r)x}}_{MC_i} = 0$$

3.3.1 Equilibrium appropriation in the second period

- Solving for q_i, we obtain firm *i*'s best response function:

$$q_i(q_j) = \frac{S-(1-r)x}{2} - \frac{1}{2}q_j \qquad (BRF_i)$$

- And similarly firm *j*'s best response function:

$$q_j(q_i) = \frac{S-(1-r)x}{2} - \frac{1}{2}q_i \qquad (BRF_j)$$

3.3.1 Equilibrium appropriation in the second period

➢ *Case 1: When the stock fully regenerates across period,* $r = 1$.

$$q_i(q_j) = \frac{S}{2} - \frac{1}{2} q_j \qquad (BRF_i)$$

- Thus becoming analogous to that in the static model of Chapter 2.

- In this context, first-period aggregate appropriation x plays ***no role*** in firm *i*'s second-period decisions.

3.3.1 Equilibrium appropriation in the second period

➢ *Case 2: When the stock partially regenerates across period, $r < 1$.*

$$q_i(q_j) = \frac{S - (1 - r)x}{2} - \frac{1}{2} q_j \qquad (BRF_i)$$

- First-period aggregate appropriation x decreases the vertical intercept of the best response function.

➢ Graphically:

- A downward parallel shift of the best response function.

- **Intuitively,**
  - The resource did not fully regenerate across periods, and then firms find a more depleted CPR at the beginning of the second period, making their appropriation more difficult.

- A similar argument applies when
  - For a given regeneration rate $r < 1$, first-period appropriation increases, as that decreases the stock available in the second period.

3.3.1 Equilibrium appropriation in the second period

- Firms are symmetric in their production costs.
  - In a symmetric equilibrium, they all extract the same second-period appropriation, $q_i{}^* = q_j^* = q^*$.

$$q^* = \frac{S-(1-r)x}{2} - \frac{1}{2}q^* \qquad (BRF_i)$$

- Rearranging, we find

$$\frac{3}{2}q^* = \frac{S-(1-r)x}{2}$$

3.3.1 Equilibrium appropriation in the second period

- Solving for q^*, we get
  - The second-period appropriation as a function of first-period appropriation;

$$q^*(x) = \frac{S - (1-r)x}{3}$$

- *Comparative statics:*

$$\frac{\partial q^*(x)}{\partial S} = \frac{1}{3} > 0$$

$$\frac{\partial q^*(x)}{\partial r} = \frac{x}{3} > 0$$

$$\frac{\partial q^*(x)}{\partial x} = -\frac{1-r}{3} < 0$$

3.3.1 Equilibrium appropriation in the second period

➢ *In this context,*

- the second-period appropriation is increasing in the initial stock S and in the regeneration rate r, *but* decreasing in the first-period aggregate appropriation x.

➢ *Overall,*

- comparative statics can be understood as that second-period appropriation is increasing in the net stock available at the beginning of the second period, $S-(1-r)x$.

- ***The second-period profits, π_i^{2nd}, is***

$$\Pi_i^{2nd} = q^*(x) - \frac{q^*(x)\left(q^*(x) + q^*(x)\right)}{S - (1-r)\,x}$$

- Substituting equilibrium extraction into the profit function, we find

$$\Pi_i^{2nd} = \frac{S-(1-r)x}{3}\left[1 - \frac{\frac{S-(1-r)x}{3} + \frac{S-(1-r)x}{3}}{S-(1-r)\,x}\right]$$

$$= \frac{S-(1-r)x}{3}\left(1 - \frac{2}{3}\right)$$

$$= \frac{S-(1-r)x}{9}$$

3.3.1 Equilibrium appropriation in the second period

$$\Pi_i^{2nd} = \frac{S - (1 - r)x}{9}$$

✓ The second period profits are increasing in the net stock available at the beginning of the second period, $S - (1 - r)x$.

➢ ***Note on the notation:***

- Π_i^{2nd} denotes profits evaluated in second-period equilibrium appropriation, $q^*(x)$.

- π_i^{2nd} evaluated at any second-period appropriation.

- ***Moving to the first period***
  - The incumbent is the only firm operating and chooses its appropriation x to maximize the sum of first- and second-period profits.
- The incumbent's first-period problem can express as:

$$\max_{x\geq 0}\ \pi^{1st}+\delta\Pi_i^{2nd}=\underbrace{\left[x-\frac{x^2}{S}\right]}_{\pi^{1st}}+\delta\underbrace{\left[\frac{S-(1-r)x}{9}\right]}_{\Pi_i^{2nd}}$$

where

$\delta\in[0,1]$ denotes the firm's discount factor.

- ***Case 1:*** When $\delta = 0$,
  - The second-period profits are irrelevant for the firm.

- ***Case 2:*** When $\delta = 1$,
  - The second-period profits receive the same weight as first-period profits.

- **In words,**
  - The incumbent chooses its first-period appropriation, x, seeking to maximize the sum of first-period profits and the (discounted value) of second-period profits.

3.3.2 Equilibrium appropriation in the first period

- Notice that, profits in both periods are affected by the firm's catches today, x:

  - First period profits are a direct function of first-period appropriation.

  - Second-period profits depend on the net stock available at the beginning of the second period, $S - (1 - r)x$, which decreases in the first-period appropriation.

3.3.2 Equilibrium appropriation in the first period

- Let us recall the intertemporal profit function:

$$\max_{x\geq 0}\ \pi^{1st}+\delta\Pi_i^{2nd}=\left[x-\frac{x^2}{S}\right]+\delta\left[\frac{S-(1-r)x}{9}\right]$$

- Differentiating with respect to x:

$$1-\frac{2x}{S}-\frac{\delta(1-r)}{9}=0$$

- Solving for x, yields first-period equilibrium propitiation:

$$x^*=\frac{S[9-\delta(1-r)]}{18}$$

➢ Case 1. $\delta = 0$

- When the incumbent ***does not assign*** any value to future payoffs, first-period equilibrium appropriation collapses to:

$$x^* = \frac{S}{2}$$

➢ **Intuitively,**

- First- and second-period appropriation decisions become independent in this case.

- Case 2. $r = 1$
  - When the stock ***fully regenerates*** across periods, the equilibrium appropriation also collapses to:

$$x^* = \frac{S}{2}$$

- ***Intuitively,***
  - The available stock completely regenerates across periods, letting the incumbent treat each period appropriation as independent decisions, since in both periods the initial stock, S, is fully available.

3.3.2 Equilibrium appropriation in the first period

- Case 3. $\delta > 0$

  - When the incumbent ***assigns a positive*** value to future payoffs, first-period equilibrium appropriation collapses to:

$$x^* < \frac{S}{2}$$

- Case 4. $r < 1$

  - When the stock ***does not*** fully regenerate across periods, the equilibrium appropriation also collapses to:

$$x^* < \frac{S}{2}$$

- **Intuitively,**

  - The incumbent anticipates its first-period appropriation depletes part of the resource, ***which will not be fully regenerated***, and the firm cares about its future profits.

    - ✓ As a consequence, the incumbent reduces its appropriation x^* to balance its profits across periods.

➢ Case 5. $\delta = 1$ and $r = 0$

- When the incumbent ***assigns the same weight*** to both periods, and the stock ***does not*** regenerate at all across periods, the first-period equilibrium appropriation becomes to:

$$x^* = \frac{S[9 - 1(1 - 0)]}{18} = \frac{8S}{18} \simeq 0.44\ S$$

To sum up.

- Subgame Perfect Equilibrium (SPE) of this game.

- The **incumbent's** *first-period* appropriation: $x^* = \frac{S[9 - \delta(1 - r)]}{18}$

- The **incumbent's** *second-period* appropriation: $q^*(x) = \frac{S - (1 - r)x}{3}$

- The **entrant** responds to any first-period appropriation x from the incumbent choosing second-period appropriation:

$$q^*(x) = \frac{S - (1 - r)x}{3}$$

- ***Important note:***
  - ***<u>We do not report</u>*** second-period appropriation evaluated at the equilibrium first period appropriation x^*, that is, we do not report:

$$q^*(x^*) = \frac{S-(1-r)x^*}{3}$$
$$= \frac{S[18-9(1-r)+\delta(1-r)^2]}{54}$$
$$= \frac{S\big[9+\delta+r[9-\delta(2-r)]\big]}{54}$$

  - ***<u>We report</u>*** each firm's second-period appropriation as a function of any first-period appropriation x, $q^*(x)$, which lets firms respond to both equilibrium first-period appropriation and to off-the-equilibrium appropriation levels $x \neq x^*$.

- ***Nevertheless,***
  - We report total appropriation across both periods in equilibrium, noting that $(1-r)$ of first-period appropriation is not recovered:

$$T^* = Q^* + (1-r)x^*$$

$$= \frac{2S\left[9+\delta+r[9-\delta(2-r)]\right]+3S(1-r)[9-\delta(1-r)]}{54}$$

$$= \frac{S[36+9(1-r)-\delta(1-r)^2]}{54}$$

which can be supported if it does not exceed S, that is,

$$\frac{S[36+9(1-r)-\delta(1-r)^2]}{54} \leq S$$

3.3.2 Equilibrium appropriation in the first period

- Simplifying, we obtain

$$18 - 9(1 - r) + \delta(1 - r)^2 \geq 0$$

that can be supported for any positive values of $\delta, r \in [0,1]$.

- Total appropriation T^* increases in S but decreases in δ and r since

$$\frac{\partial T^*}{\partial \delta} = -\frac{S(1 - r)^2}{54} < 0$$

$$\frac{\partial T^*}{\partial r} = -\frac{S[9 - 2\delta(1 - r)]}{54} < 0$$

$$\frac{\partial T^*}{\partial S} = \frac{36 + 9(1 - r) - \delta(1 - r)^2}{54} > 0$$

3.3.2 Equilibrium appropriation in the first period

- When the incumbent assigns a higher weight to future payoff (δ increases), it reduces first-period appropriation, which more than offsets the increase in second-period aggregate appropriation since

$$\frac{\partial x^*}{\partial \delta} = -\frac{S(1-r)}{18} < 0$$

$$\frac{\partial Q^*}{\partial \delta} = \frac{S(1-r)^2}{27} > 0$$

$$\frac{\partial T^*}{\partial \delta} = \frac{\partial Q^*}{\partial \delta} + (1-r)\frac{\partial x^*}{\partial \delta} = -\frac{S(1-r)^2}{54} < 0$$

3.3.2 Equilibrium appropriation in the first period

- When the resource exhibits a higher regeneration rate (r increases), both first- and second-period appropriation increase, but overall appropriation decreases.
- This happens because the indirect effect from regeneration of the stock (negative 3rd term) more than compensates the direct effect from appropriation (positive 1st and 2nd terms).

$$\frac{\partial T^*}{\partial r} = \frac{\partial Q^*}{\partial r} + (1-r)\frac{\partial x^*}{\partial r} - x^* =$$

$$\frac{\overbrace{S[9-2\delta(1-r)]}^{\frac{\partial Q^*}{\partial r}>0}}{27} + \frac{\overbrace{\delta(1-r)S}^{(1-r)\frac{\partial x^*}{\partial r}>0}}{18} - \frac{\overbrace{S[9-\delta(1-r)]}^{x^*}}{18} < 0$$

Socially optimal appropriation

- ***Goal:***
  - In this section, we evaluate the socially optimal appropriation.
- **For simplicity,**
  - we assume that the social planner seeks to maximize welfare function, $W = PS$.
- **How to evaluate the social optimal in the dynamic setting?**
  - We operate by backward induction as well:
    - ✓ ***In the first period:*** we select first-period appropriation x.
    - ✓ ***In the second period:*** the social planner takes the first-period appropriation x as given and solves the second-period appropriation levels q_i and q_j.

➢ *In the second-period:*

- Taking the first-period aggregate appropriation x as given, the social planner solves

$$\max_{q_i, q_j} W^{2nd} = \pi_i^{2nd} + \pi_j^{2nd} = \underbrace{\left[q_i - \frac{q_i(q_i + q_j)}{S - (1-r)x}\right]}_{\pi_i^{2nd}} + \underbrace{\left[q_j - \frac{q_j(q_i + q_j)}{S - (1-r)x}\right]}_{\pi_j^{2nd}}$$

- Differentiating with respect to q_i yields

$$\frac{\partial W^{2nd}}{\partial q_i} = 1 - \frac{2q_i + q_j}{S - (1-r)x} - \underbrace{\frac{q_j}{S - (1-r)x}}_{New\ term} = 0$$

- The new term captures the cost externality that a larger second-period appropriation by firm *i* generates on firm *j*.

  - Differentiating with respect to q_j yields

$$\frac{\partial W^{2nd}}{\partial q_j} = 1 - \frac{q_i}{S-(1-r)x} - \frac{2q_j + q_i}{S-(1-r)x} = 0$$

- Solving for q_i, we obtain the best response function of firm *i*;

$$q_i(q_j) = \frac{S-(1-r)x}{2} - q_j$$

3.4 Socially optimal appropriation

- In a symmetric appropriation profile $q_i^{SO} = q_j^{SO} = q^{SO}$

$$q^{SO} = \frac{S-(1-r)x}{2} - q^{SO}$$

$$2q^{SO} = \frac{S-(1-r)x}{2}$$

- The socially optimal second-period appropriation, as a function of first-period appropriation, becomes

$$q^{SO}(x) = \frac{S-(1-r)x}{4}$$

- which is increasing in the net stock available at the beginning of the second period, $S-(1-r)x$.

- Second-period welfare becomes:

$$W^{2nd}(q^{SO},q^{SO}) = \left[q^{SO} - \frac{q^{SO}(q^{SO}+q^{SO})}{S-(1-r)x}\right] + \left[q^{SO} - \frac{q^{SO}(q^{SO}+q^{SO})}{S-(1-r)x}\right]$$

$$= \frac{S-(1-r)x}{4}\left[1 - \frac{\frac{S-(1-r)x}{2}}{S-(1-r)x}\right] + \frac{S-(1-r)x}{4}\left[1 - \frac{\frac{S-(1-r)x}{2}}{S-(1-r)x}\right]$$

$$= \frac{S-(1-r)x}{4}$$

➢ *Moving to the first-period*

- The social planner anticipates the second-period welfare, and how it depends on the first-period appropriation, solving:

$$\max_{x\geq 0} \pi^{1st} + \delta\, W^{2nd}(q^{SO}, q^{SO}) = \left[x - \frac{x^2}{S}\right] + \delta\left[\frac{S-(1-r)x}{4}\right]$$

➢ **For simplicity,**

- we assume that the social planner's discount factor δ coincides with that of the incumbent.

- Differentiating with respect to x, we obtain:

$$1 - \frac{2x}{S} - \frac{\delta(1-r)}{4} = 0$$

- Comparing it against the FOC in the expression for firm i:

$$1 - \frac{2x}{S} - \frac{\delta(1-r)}{9} = 0$$

- which we can further rearrange to

$$1 - \frac{2x}{S} - \frac{\delta(1-r)}{9} - \underbrace{\frac{5\delta(1-r)}{36}}_{\textit{New term}} = 0$$

3.4 Socially optimal appropriation

- ***Intuitively,***
  - The new term captures the negative effect that an increase in first-period appropriation causes on the entrant's second period profits.
  - The incumbent only considered the effect of first-period appropriation on its own second-period profits but overlooked the effect on the entrant's.
  - The social planner internalizes that dynamic external effect.

➢ Solving for x:

$$\frac{2x}{S} = \frac{4-\delta(1-r)}{4}$$

- We find the first-period equilibrium appropriation, as follows

$$x^{SO} = \frac{S[4-\delta(1-r)]}{8}$$

- which is lower than that in equilibrium, $x^* = \frac{S[9-\delta(1-r)]}{18}$, since

$$x^* - x^{SO} = \frac{S[9-\delta(1-r)]}{18} - \frac{S[4-\delta(1-r)]}{8}$$

$= \frac{5S\delta(1-r)}{72}$ that is clearly positive.

- **The second-period socially optimal equilibrium appropriation is**

$$q^{SO} = \frac{S[8 - 4(1-r) + \delta(1-r)^2]}{32}$$

- Total appropriation across both periods becomes

$$Q^{SO} + (1-r)x^{SO}$$
$$= \frac{S[8 - 4(1-r) + \delta(1-r)^2] + 2S(1-r)[4 - \delta(1-r)]}{16}$$

- which we simplify to

$$T^{SO} = \frac{S[8 + 4(1-r) - \delta(1-r)^2]}{16}$$

3.3.2 Equilibrium appropriation in the first period

- It can be supported if total appropriation does not exceed S, that is,

$$\frac{S[8+4(1-r)-\delta(1-r)^2]}{16} \leq S$$

- Simplifying, we obtain

$$8-4(1-r)+\delta(1-r)^2 \geq 0$$

that can be supported for any positive values of $\delta, r \in [0,1]$.

- Similar to T^*, T^{SO} increases in S but decreases in δ and r since

$$\frac{\partial T^{SO}}{\partial r} = -\frac{S[2-\delta(1-r)]}{8} < 0$$

3.3.2 Equilibrium appropriation in the first period

- When the resource experiences a higher regeneration rate r, both first- and second-period socially optimal appropriation increase.
- However, since the resource has a smaller base to grow the stock, overall appropriation decreases, which, technically speaking, the negative 3rd term more than offsets the positive 1st and 2nd terms.

$$\frac{\partial T^{SO}}{\partial r} = \frac{\partial q^{SO}}{\partial r} + (1-r)\frac{\partial x^{SO}}{\partial r} - x^* =$$

$$\frac{\overbrace{S[2-\delta(1-r)]}^{\frac{\partial q^{SO}}{\partial r}>0}}{16} + \frac{\overbrace{\delta(1-r)S}^{(1-r)\frac{\partial x^{SO}}{\partial r}>0}}{8} - \frac{\overbrace{S[4-\delta(1-r)]}^{x^*}}{8} < 0$$

Static and dynamic inefficiencies

- ***In this section:***

  - We evaluate the inefficiencies that arise in two settings (static & dynamic) by comparing equilibrium behavior against the socially optimal appropriation.

➢ Static Inefficiency (*SI*)

- The social planner chooses a lower second-period appropriation to correct the static inefficiency *(i.e., firms exploit the resource at socially excessive levels)*

➢ Static inefficiency can be measured by the difference:

$$SI = q^*(x) - q^{SO}(x) = \frac{S-(1-r)x}{3} - \frac{S-(1-r)x}{4} = \frac{S-(1-r)x}{12}$$

- The static inefficiency SI is increasing in the net stock available at the beginning of the second period $S - (1 - r)x$, entailing that SI expands as the initial stock and regeneration rate increase, but shrinks as first-period appropriation increases.

- **Figure 3.1 (*next slide*):**

  - Illustrates the static inefficiency as occurring because two firms simultaneously choose their second-period appropriation without considering the cost externalities that their actions generate on their rivals.

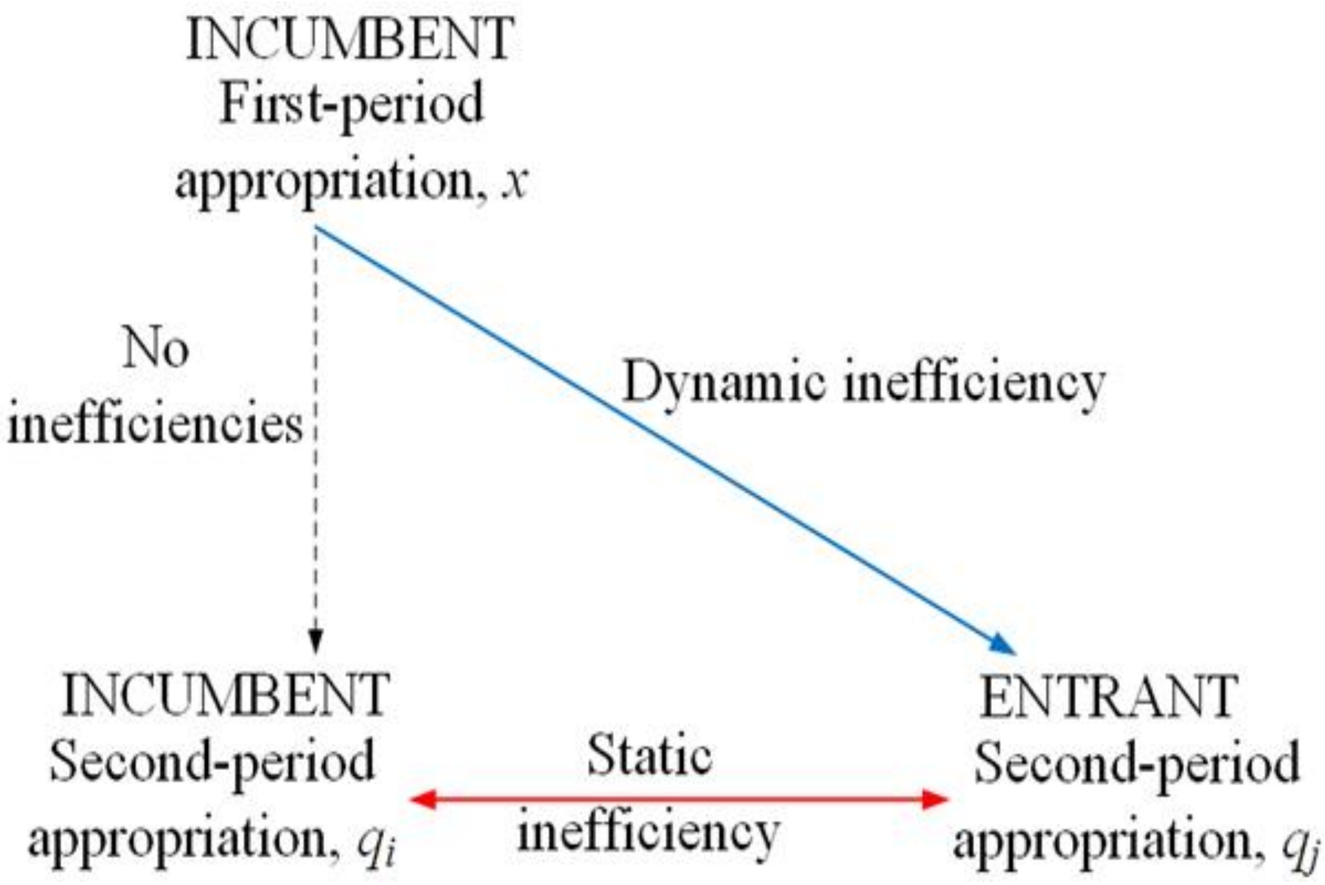


Figure 3.1 Static and dynamic inefficiencies

- Dynamic Inefficiency (DI)
  - The social planner also selects a lower first-period appropriation to correct the dynamic inefficiency, namely, ***that the incumbent ignores*** how a more depleted resource impacts the entrant's profits.
- Dynamic inefficiency can be measured by the difference:

$$DI = x^* - x^{SO} = \frac{S[9-\delta(1-r)]}{18} - \frac{S[4-\delta(1-r)]}{8} = \frac{5S\delta(1-r)}{72}$$

  - When $\delta = 0$, the dynamic inefficiency DI collapses to zero.

Dynamic Inefficiency (DI)

- It increases in δ because when the discount rate increases, both x^* and x^{SO} decrease, but the unregulated x^* decreases slower than x^{SO}, that is,

$$\frac{\partial x^*}{\partial \delta} = -\frac{S(1-r)}{18} > -\frac{S(1-r)}{8} = \frac{\partial x^{SO}}{\partial \delta}$$

- Therefore, dynamic inefficiency increases in δ where

$$\frac{\partial DI}{\partial \delta} = \frac{5S(1-r)}{72} > 0$$

➢ Dynamic Inefficiency (DI)

- It increases in r because when the regeneration rate increases, both x^* and x^{SO} increase, but the unregulated x^* increases less significantly than x^{SO}, that is,

$$\frac{\partial x^*}{\partial r} = \frac{\delta S}{18} < \frac{\delta S}{8} = \frac{\partial x^{SO}}{\partial r}$$

- Therefore, dynamic inefficiency decreases in r where

$$\frac{\partial DI}{\partial r} = -\frac{5\delta S}{72} < 0$$

➢ Dynamic Inefficiency (DI)

- It increases in S because when the stock becomes more abundant, both x^* and x^{SO} increase, but the unregulated x^* increases faster than x^{SO}, that is,

$$\frac{\partial x^*}{\partial S} = \frac{9 - \delta(1-r)}{18} > \frac{4 - \delta(1-r)}{8} = \frac{\partial x^{SO}}{\partial S}$$

- Therefore, dynamic inefficiency increases in S where

$$\frac{\partial DI}{\partial S} = \frac{5\delta(1-r)}{72} > 0$$

➢ Intuitively,

- Regulator and incumbent ***do not assign any*** value to future payoffs, making appropriation decisions in each period independent, thus eliminating the potential for dynamic inefficiencies.

➢ How about static inefficiency?

- The static inefficiency is still arising in this context, **since *SI* is not** a function of discount factor δ.

- **When $r = 1$,**

  - $DI = 0$, since in this setting the resources fully regenerates, so that appropriation decisions in each period become independent of each other.

- **For a given $\delta \neq 0$ and $r \neq 1$,**

  - The dynamic inefficiency becomes more severe as the initial stock S expands.

Equilibrium vs. socially optimal number of firms

3.6.1 Equilibrium entry

3.6.2 Socially optimal entry

3.6.3 No entry costs

- ***In this section:***

  - ✓ We study the equilibrium number of firms that enter a CPR.
  - ✓ Then we find the socially optimal number of firms, that is, the number of firms that maximizes social welfare.
  - ✓ Finally, we compare whether the equilibrium entry is socially excessive.

➢ ***Regulators can use***

- licenses to induce the socially optimal number of firms or, alternatively, set quotas so that each firm catches the socially optimal appropriation, q^{SO} (where exceeding it entails large monetary fines), or appropriation fees that also induce every firm to catch q^{SO}.
- While licenses are more common in settings where monitoring catches is relatively difficult or costly, appropriation quotas and fees are more typical otherwise.

- Consider a firm evaluating whether or not to operate in a CPR.
  - where $N-1$ firms already operate.
- If it enters, the resource is exploited by *N* firms facing an inverse demand function $p(Q)=1-Q$.
  - That is, every firm *i*'s equilibrium appropriation is $q^{*}=\frac{S}{(N+1)(S+1)}$, entailing aggregate output of

$$Q^{*}=\frac{NS}{(N+1)(S+1)}$$

- Entry profits become
  - where $N - 1$ firms already operate.

$$\pi_i^{Entry} = (1 - Q^*)q^* - \frac{q^* Q^*}{S} = \frac{S}{(N+1)^2(S+1)}$$

  - which is increasing in the available stock, S, since

$$\frac{\partial \pi_i^{Entry}}{\partial S} = \frac{S}{(N+1)^2(S+1)^2} > 0$$

  - but decreasing in the number of firms, N, because

$$\frac{\partial \pi_i^{Entry}}{\partial N} = -\frac{2S}{(N+1)^3(S+1)} < 0$$

- We assume that the potential entrant must pay a fixed entry cost $F \geq 0$:
  - which allows for the CPR to be open access ($F = 0$) or to require some fixed entry costs, such as licensing, capital and technology investment.
- The potential entrant then joins the commons if and only if $\pi_i^{Entry} \geq F$, or

$$\frac{S}{(N+1)^2(S+1)} \geq F$$

3.6.1 Equilibrium entry

- Applying square roots on both sides, we obtain

$$\sqrt{\frac{S}{(N+1)^2(S+1)}} \geq \sqrt{F}$$

- which is rearranged to yield

$$\sqrt{\frac{S}{S+1}} \geq \sqrt{F}(N+1)$$

- Solving for N, we find

$$N \leq N^* \equiv \sqrt{\frac{S}{F(S+1)}} - 1$$

that is positive **as long as** $F < \frac{S}{S+1}$.

3.6.1 Equilibrium entry

➢ ***Intuitively,***

- The fixed entry cost is relatively low, which is a reasonable assumption in most CPRs.

❖ Figure 3.2 depicts the entry profit, $\pi_i^{Entry} = \frac{S}{(N+1)^2(S+1)}$.

➢ The equilibrium number of firms N^* is

- Increasing in the available stock S, as the CPR becomes more attractive and thus more firms enter.

$$\frac{\partial N^*}{\partial S} = \frac{(S+1)^{-\frac{3}{2}}}{2\sqrt{FS}} > 0 \qquad \frac{\partial N^*}{\partial F} = -\frac{F^{-\frac{3}{2}}}{2}\sqrt{\frac{S}{S+1}} < 0$$

- Decreasing in the entry cost F, as more firms are deterred from joining the commons.

3.6.1 Equilibrium entry

- An increase in S shifts the entry profit π_i^{Entry} upward, thus moving the crossing point rightward towards more firms entering the CPR.

- An increase in F shifts the horizontal line upwards, implying that the crossing point moves leftward towards fewer firms joining the commons.

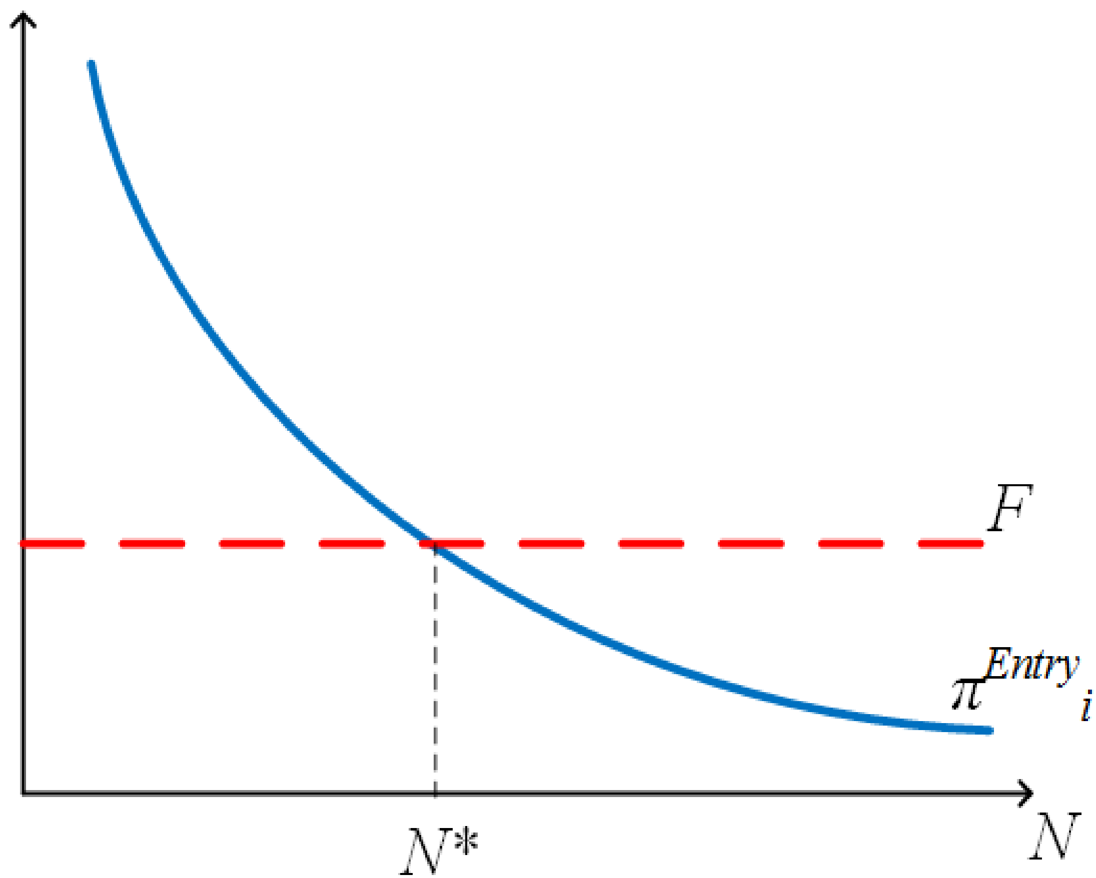


Figure 3.2. Equilibrium number of firms

- Consider the welfare function $W = CS + PS - (N \times F)$
  - where the last term denotes aggregate entry costs; and thus producer surplus ***PS does not*** include entry costs.
- Assuming a linear demand function $p(Q) = 1 - Q$.
- The social planner chooses the number of firms, *N*, to solve

$$\max_{N \geq 1} W = \underbrace{\int_0^{Q^*} (1 - s)ds - N \times C(q_i^*, Q_{-i}^*)}_{CS + PS} - \underbrace{(N \times F)}_{\text{Entry Cost}}$$

3.6.2 Socially optimal entry

➢ which we can rewrite as

$$\int_0^{Q^*} (1-s)ds - \frac{Nq^*[q^* + (N-1)q^*]}{S} - (N \times F)$$

➢ where

- $Q^* = Nq^*$ is the aggregate equilibrium appropriation.
- $C(q_i^*, Q_{-i}^*) = \frac{q^*[q^*+(N-1)q^*]}{S}$ is the total cost for each firm.

3.6.2 Socially optimal entry

- Consider the consumer surplus

$$\int_0^{Q^*}(1-s)ds = \left[s-\frac{s^2}{2}\right]_0^{Q^*} = Q^* - \frac{1}{2}(Q^*)^2$$

- Substituting $Q^* = \frac{NS}{(N+1)(S+1)}$ into the above expression yields

$$CS = \frac{NS}{(N+1)(S+1)} - \frac{N^2S^2}{2(N+1)^2(S+1)^2}$$

$$= \frac{NS[NS+2(N+S+1)]}{2(N+1)^2(S+1)^2}$$

- The social planner's problem can simplify to

$$\max_{N\geq 1} W = \frac{NS[NS + 2(N + S + 1)]}{2(N+1)^2(S+1)^2} - \frac{N^2 S}{(N+1)^2(S+1)^2} - NF$$

- which we simplify to yield

$$\max_{N\geq 1} W = \frac{NS[NS + 2(S + 1)]}{2(N+1)^2(S+1)^2} - NF$$

- Differentiating with respect to N, we find

$$\frac{\partial W}{\partial N} = \frac{S(S + 1 - N)}{(N+1)^3(S+1)^2} - F$$

- which is a highly nonlinear expression in N.

- Applying Implicit Function Theorem, we find

$$\frac{\partial N^{SO}}{\partial F} = -\frac{\frac{\partial^2 W}{\partial N \partial F}}{\frac{\partial^2 W}{\partial N^2}} = -\frac{1}{\frac{S[N+1+3(S+1-N)]}{(N+1)^4(S+1)^2}}$$

- Since $S + 1 - N > 0$ for $\partial W/\partial N = 0$ to hold, we have

$$\frac{\partial N^{SO}}{\partial F} = -\frac{(N+1)^4(S+1)^2}{S[N+1+3(S+1-N)]} < 0$$

- so that when entry becomes more costly (higher F), fewer firms can be supported from a social welfare perspective.

- Applying Implicit Function Theorem, we find

$$\frac{\partial N^{SO}}{\partial S} = -\frac{\frac{\partial^2 W}{\partial N \partial S}}{\frac{\partial^2 W}{\partial N^2}} = \frac{\frac{SN + (S+1-N)}{(N+1)^3 (S+1)^3}}{\frac{S[N+1+3(S+1-N)]}{(N+1)^4 (S+1)^2}}$$

- Since $S + 1 - N > 0$ for $\partial W / \partial N = 0$ to hold, we have

$$\frac{\partial N^{SO}}{\partial S} = \frac{(N+1)[SN + (S+1-N)]}{S(S+1)[N+1+3(S+1-N)]} > 0$$

- so that when resource becomes more abundant (higher S), more firms can be supported from a social welfare perspective.

3.6.2 Socially optimal entry

- For example, when $S = 9$ and $F = 0.1$, we have

$$N^* = \sqrt{\frac{10 \times 9}{(9+1)}} - 1 = 2$$

- Whereas, the social planner's first order condition solves

$$\frac{9(9+1-N)}{(N+1)^3(9+1)^2} = \frac{1}{10}$$

- which simplifying yields $N \approx 1.008$, so $N^* = 2 > 1 = N^{SO}$.
- Thus, the market has 2 firms in an unregulated equilibrium, but the social planner intends only for 1 firm operating in the CPR.

➢ *Figure 3.3:*

- Depicts N^{SO} as a function of the entry cost, F, as well as the equilibrium number of firms, N^*.

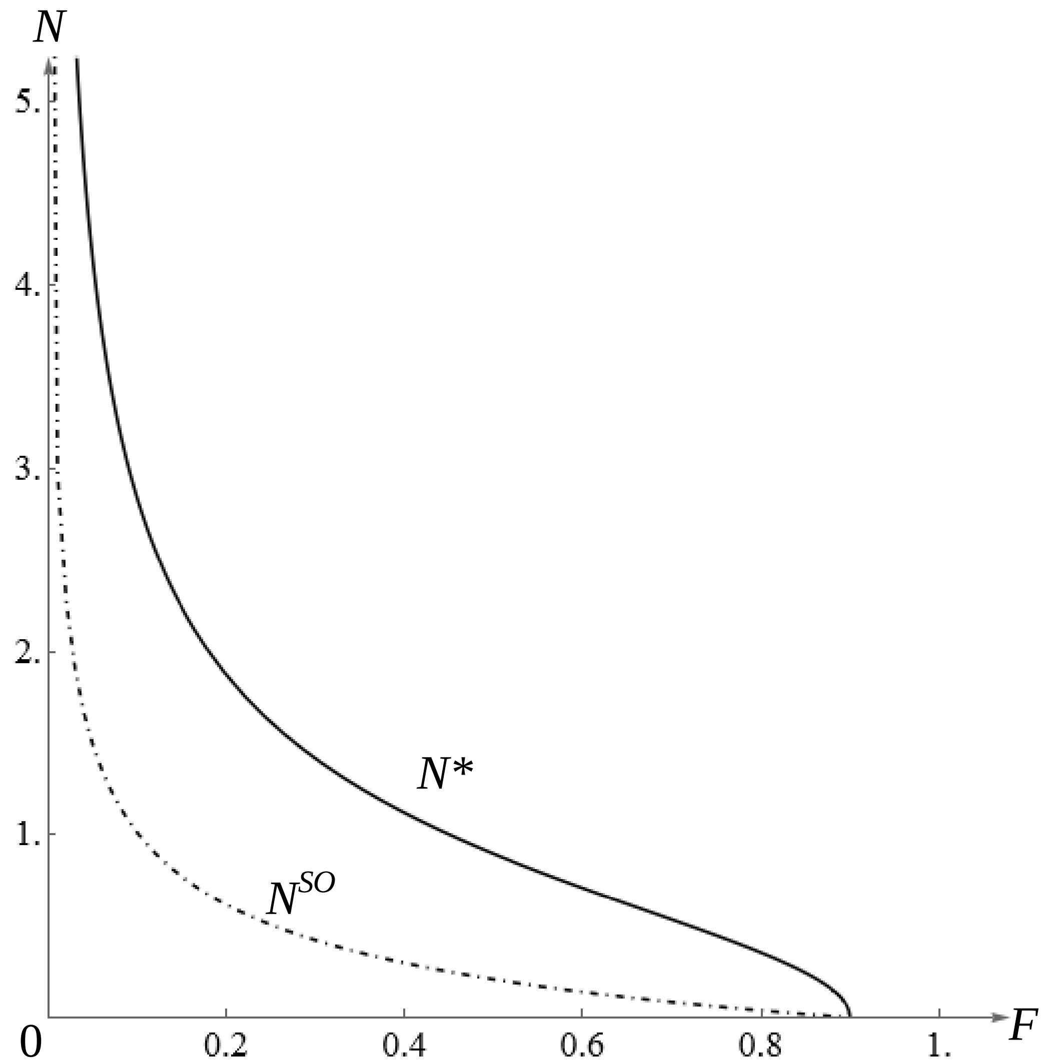


Figure 3.3. Equilibrium and socially optimal number of firms.

3.6.2 Socially optimal entry

➢ ***In general,***

- The equilibrium number of firms entering the CPR is socially excessive, $N^* > N^{SO}$, which occurs for all values of $F \geq 0$.

➢ ***Intuitively,***

- When a firm enters into the CPR, it only considers its own profit from doing so, π_i^{Entry}, but ignores the business-stealing effect that its entry implies on all existing firms, i.e., the individual appropriation of each firm $q^* = \frac{S}{(N+1)(S+1)}$ decreases in N^*.

3.6.2 Socially optimal entry

- The social planner, in contrast, considers both the additional profit that the firm brings to total welfare and the business-stealing effect.

- This yields the socially optimal number of firms, N^{so}, that lies below the equilibrium number of firms if entry is unregulated, N^*.

3.6.3 No entry costs

- In the special case in which entry cost is zero, $F = 0$
  - The equilibrium number of firms solves

$$N^* = \sqrt{\frac{S}{0(S+1)}} - 1 = \infty$$

  - Intuitively, firms need not pay entry cost, so that as many firms can enter into the market to appropriate the resource.
  - The market is perfectly competitive. However, it is socially optimal for fewer firms operating in the CPR.

3.6.3 No entry costs

- In the special case in which entry cost is zero, $F = 0$
  - The equilibrium number of firms becomes $N^* = \infty$.

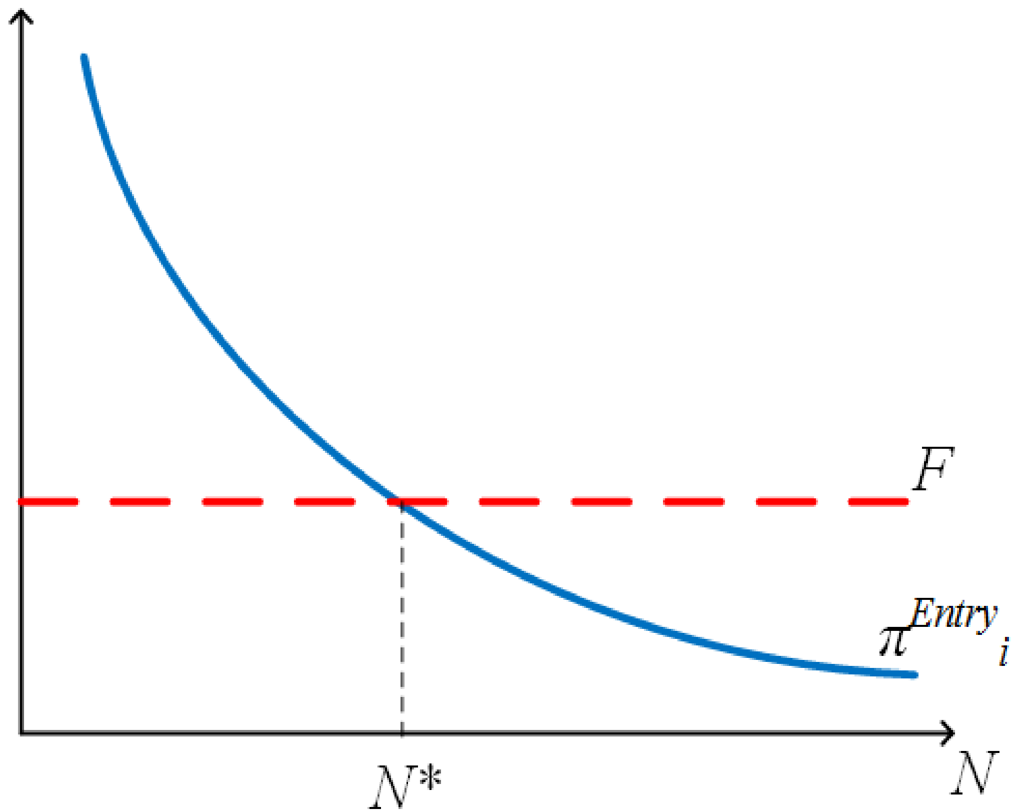


Figure 3.2. Equilibrium number of firms

➢ *Discussion on Figure 3.2*

- If the horizontal line representing the fixed entry cost F decreases until overlapping the horizontal axis, $F = 0$, the individual profit π_i^{Entry} ***does not cross*** F at any value of N, meaning that more firms keep entering the CPR since their net profits from doing so are still positive.

➢ *In contrast,*

- the socially optimal number of firms is not necessarily infinite even with no entry cost, where $\frac{\partial W}{\partial N} = 0$ entails $N = S + 1$, so a more abundant resource supports higher N^{SO}.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 4: Entry deterrence in the commons

Outline

- Modeling entry deterrence
- A larger dynamic inefficiency

In this chapter:

➢ ***We relax one of our assumption in Ch 3***

- that the entrant joins the CPR in the second period.
  - We considered that the entrant exploits the CPR regardless of how depleted the resource becomes after the incumbent’s first-period appropriation.
  - We examine the incumbent’s strategic exploitation.

Modeling entry deterrence

This section covers the following:

4.2.1 Second period appropriation - No entry

4.2.2 Second period appropriation - Entry

4.2.3 Second period appropriation - Enter or not?

4.2.4 First period appropriation - Entry deterrence

➢ Consider a CPR with the following time structure:

1. ***In the first stage***, the incumbent chooses its appropriation level x.
2. ***In the second stage***, after observing x, the potential entrant chooses whether to enter or not.
   - **If entry does not occur:**
     - ✓ The incumbent selects its second-period appropriation q, while the entrant's profits from staying out are normalized to zero.
   - **If entry ensues:**
     - ✓ The incumbent and the entrant compete for the CPR, simultaneously and independently selecting their second-period appropriation q_1 and q_2, respectively.

➢ Assumptions

- Firms take price as given, and normalized to one:

$$p = \$1$$

- The incumbent's first-period cost function is:

$$C(x) = \frac{x^2}{S}$$

- The incumbent's and entrant's second-period cost function is symmetric and given by:

$$C(q_i, q_j, x) = \frac{q_i(q_i + q_j)}{S - (1 - r)x}$$

- Where
  - S denotes the initial stock.
  - $r \in [0,1]$ represents the CPR's regeneration rate.
- Consider that the potential entrant faces a fixed entry cost $F > 0$.
  - This assumption
    - ✓ makes entry endogenous rather than exogenous to the model.
    - ✓ helps us predict under which parameter conditions the potential entrant chooses to enter.

- ***How to solve the model?***
  - Since this is a sequential-move game of complete information,
    - ✓ We seek to find its Subgame Perfect Equilibrium (SPE).

- We next find the SPE of this game applying backward induction (two stage game).
  - We start analyzing firms' decisions ***in the second period.***
  - Then, we move on to study the ***first period***.

4.2.1 Second period appropriation – No Entry

➢ **If entry does not occur, $q_j = 0$.**

- The cost function, $C(q_i, q_j, x) = \frac{q_i(q_i+q_j)}{S-(1-r)x}$, simplifies to

$$C(q, 0, x) = \frac{q^2}{S-(1-r)x}$$

✓ The incumbent is ***the only firm*** exploiting the resource in the second period.

4.2.1 Second period appropriation – No Entry

- The incumbent chooses its second-period appropriation q to solve

$$\max_{q\geq 0}\ \pi^{2nd} = q - \frac{q^2}{S-(1-r)x}$$

- Differentiating with respect to q yields

$$1 - \frac{2q}{S-(1-r)x} = 0$$

- Solving for q, we find the incumbent's second-period appropriation under no entry *(where superscript NE denotes "no entry").*

$$q^{NE}(x) = \frac{S-(1-r)x}{2}$$

- The incumbent's second-period profits when entry does not occur are

$$\Pi_{NE}^{2nd} = q^{NE}(x) - \frac{\left(q^{NE}(x)\right)^2}{S-(1-r)x}$$

$$= \frac{S-(1-r)x}{2} - \frac{[S-(1-r)x]^2}{4[S-(1-r)x]}$$

$$= \frac{S-(1-r)x}{4}$$

✓ which is decreasing in the net stock available at the beginning of the second period, $S-(1-r)x$.

➢ If entry ensues, $q_j \neq 0$.

- The incumbent and entrant *simultaneously and independently* choose their second-period appropriation q_1 *and* q_2, respectively.

- Every firm $i = \{1,2\}$ chooses q_i to solve

$$\max_{q_i \geq 0} \pi_i^{2nd} = q_i - \frac{q_i(q_i + q_j)}{S - (1-r)x}$$

4.2.2 Second period appropriation - Entry

- Differentiating with respect to q_i

$$1 - \frac{2q_i + q_j}{S - (1-r)x} = 0$$

- Solving for q_i yields

$$q_i(q_j) = \frac{S - (1-r)x}{2} - \frac{1}{2} q_j \qquad (BRF_i)$$

- In a symmetric equilibrium, all firms choose the same second-period appropriation:

$$q_i^* = q_j^* = q^*$$

- So, the best response function reduces to

$$q^* = \frac{S-(1-r)x}{2} - \frac{1}{2}q^*$$

- Rearranging the above equation, we obtain

$$\frac{3}{2}q^* = \frac{S-(1-r)x}{2}$$

4.2.2 Second period appropriation - Entry

- Solving for q^*, the second-period appropriation under entry is

$$q^E(x) = \frac{S - (1 - r)x}{3}$$

- where superscript E denotes "entry", and **this appropriation is lower than that when the incumbent chooses no entry**, since

$$q^E(x) = \frac{S - (1 - r)x}{3} < \frac{S - (1 - r)x}{2} = q^{NE}(x)$$

- However, aggregate second-period appropriation is larger with than without entry because

$$Q^E(x) = 2q^E(x) = \frac{2[S-(1-r)x]}{3} > \frac{S-(1-r)x}{2} = q^{NE}(x) = Q^{NE}(x)$$

  - ✓ where we use $N = 2$ because there are only two firms (the incumbent and the entrant) in the industry **<u>if entry occurs</u>**.

- Inserting $q^E(x)$ into second-period profits, π_i^{2nd}, we find every firm *i*'s second-period equilibrium profits under entry, as follows:

$$\Pi_E^{2nd} = q^E(x) - \frac{q^E(x)[q^E(x)+q^E(x)]}{S-(1-r)x}$$

$$= \frac{S-(1-r)x}{3}\left[1 - \frac{2[S-(1-r)x]}{3[S-(1-r)x]}\right] = \frac{S-(1-r)x}{9}$$

➢ Comparison:

- The incumbent's second-period profits are higher under no entry than under entry, since

$$\underbrace{\frac{S-(1-r)x}{4}}_{\Pi_{NE}^{2nd}} > \underbrace{\frac{S-(1-r)x}{9}}_{\Pi_{E}^{2nd}}$$

✓ A condition that holds regardless of the net stock at the beginning of the second period, $S-(1-r)x$.

4.2.3. Second period appropriation - Enter or not?

- Profits under entry

$$\Pi_E^{2nd} = \frac{S-(1-r)x}{9}$$

- captures the profits of every firm, the incumbent and the entrant.
- helps us understand the entrant's entry decision in the second stage.

- The entrant compares its profits from entering against its profits from staying out, which is zero.
- It chooses to enter if and only if $\Pi_E^{2nd} - F \geq 0$, solving

$$\frac{S-(1-r)x}{9} \geq F$$

4.2.3. Second period appropriation - Enter or not?

- Solving for x,

$$x \leq \frac{S - 9F}{1 - r} \equiv x_{ED}$$

  - where *ED* denotes entry deterrence.

- Figure 4.1: *(Next slide)*

  - Separately plots the entrant's profits from joining the CPR, Π_E^{2nd}, against its fixed entry cost, F.

4.2.3. Second period appropriation - Enter or not?

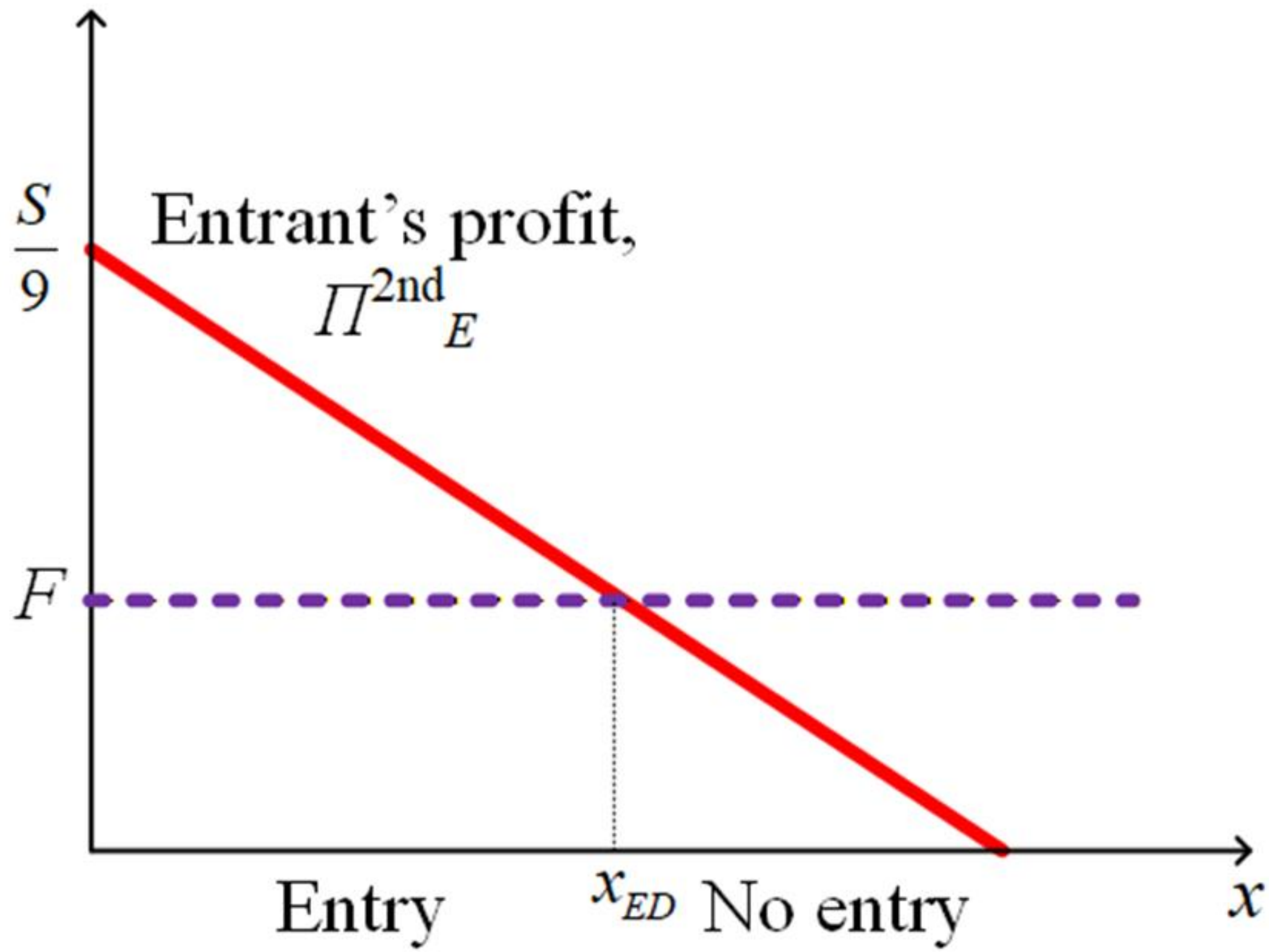


Figure 4.1. The entrant's decision: To enter or not to enter?

4.2.3. Second period appropriation - Enter or not?

- For all values of first-period appropriation to **the right-hand side** of x_{ED}:
    - The resource becomes so depleted that the potential entrant chooses to stay out.
    - In this case, the incumbent's first-period appropriation successfully deters entry.
- In contrast, for values of x to **the left-hand side** of x_{ED}:
    - The CPR is still sufficiently attractive for the entrant to join.

4.2.3. Second period appropriation - Enter or not?

- When entry does not occur, total appropriation becomes

$$(1-r)x + q^{NE}(x) = \frac{S + (1-r)x}{2}$$

- Substituting x_{ED} into the above expression, yields

$$\frac{S + (1-r) \times \frac{S - 9F}{1-r}}{2} = S - \frac{9F}{2}$$

which can be supported by the resource S because

$$S - \frac{9F}{2} \leq S$$

reduces to $F \geq 0$ that holds by definition.

- The minimal first-period appropriation that deters entry, x_{ED}, **is decreasing in** ***F*.**
  - Since a higher entry cost F shifts the horizontal line upwards, moving the crossing point x_{ED} leftward.

- ***Intuitively,***
  - As entry becomes more costly, the incumbent needs to exploit the resource less intensively during the first period if it seeks to deter entry.

4.2.3. Second period appropriation - Enter or not?

- The minimal first-period appropriation that deters entry, x_{ED}, **is increasing** when:
  - The initial stock *S* becomes more plentiful, and/or
  - the resource regeneration rate *r* increases.

- In this context,
  - The CPR becomes more attractive for the entrant, requiring the incumbent to increase its first-period appropriation if it seeks to deter entry.

➢ *Assumption*

- We assume that entry costs are not extreme, $F < \overline{F} \equiv \frac{S}{9}$.

➢ *This assumption:*

- Guarantees that the expression of x_{ED} is positive.

- Can be rationalized by looking at the entrant's second-period profits, $\Pi_E^{2nd} = \frac{S-(1-r)x}{9}$, again, and evaluating it at the point where the incumbent does not appropriate any catches during the first period, $x = 0$, which yields $\frac{S}{9}$.

4.2.3. Second period appropriation - Enter or not?

- The previous condition $F < \overline{F}$ says that:
  - The entrant has incentives to enter the CPR, at least, when the incumbent did not exploit the resource at all!

- Graphically: *(Refer to slide 21)*
  - The above condition says that the horizontal line depicting the entry cost *F* always originates below the vertical intercept of the entrant's profit function, Π_E^{2nd}, which starts at $\frac{S}{9}$.

4.2.4 First period appropriation - Entry deterrence

- In this subsection,
  - We want to know when the incumbent practices entry deterrence. ***How?***
- We compare the profits where:
  - The incumbent earns from allowing entry and from practicing entry deterrence.
- The incumbent chooses the first-period appropriation x that maximizes the sum of its first and second-period profits separately considering the two scenarios in which:
  - Its current appropriation deters entry ($x \geq x_{ED}$) ***and*** when it does not deter entry ($x < x_{ED}$).

➢ First scenario: Allowing entry

- When the incumbent chooses a first-period appropriation to the ***left-hand side of x_{ED}*** in Figure 4.1, it allows entry,
  - ✓ thus making profit Π_E^{2nd} in the second period.

➢ The incumbent then solves:

$$\max_{x<x_{ED}} \pi^{1st} + \delta\Pi_E^{2nd} = \underbrace{\left[x - \frac{x^2}{S}\right]}_{\pi^{1st}} + \delta \underbrace{\left[\frac{S-(1-r)x}{9}\right]}_{\Pi_E^{2nd}}$$

✓ where $\delta \in [0, 1]$ denotes the firm's discount factor.

4.2.4 First period appropriation - Entry deterrence

- Differentiating with respect to x:

$$1 - \frac{2x}{S} - \frac{\delta(1-r)}{9} = 0$$

- Solving for x, yields:

$$x_E = \frac{S[9 - \delta(1-r)]}{18}$$

- where the subscript E indicates that this is the profit-maximizing first-period appropriation under entry.

4.2.3. Second period appropriation - Enter or not?

- Inserting x_E into the entrant's profit function:

$$\Pi_E^{2nd} = \frac{S-(1-r)x_E}{9} - F$$

$$= \frac{S}{9}\left[1 - \frac{(1-r)[9-\delta(1-r)]}{18}\right] - F$$

$$= \frac{S[9(1+r)+\delta(1-r)^2]}{162} - F$$

so entry is admissible when the fixed cost F is not too high, where

$$F < \underline{F} \equiv \frac{S[9(1+r)+\delta(1-r)^2]}{162}$$

4.2.4 First period appropriation - Entry deterrence

➢ Inserting x_E into the incumbent's objective function:

$$\Pi^{AE} = \left[x_E - \frac{x_E^2}{S}\right] + \delta\left[\frac{S-(1-r)x_E}{9}\right]$$

$$= \frac{S[9-\delta(1-r)]}{18}\left[1 - \frac{9-\delta(1-r)}{18}\right] + \frac{\delta S}{9}\left[1 - \frac{9(1-r)-\delta(1-r)^2}{18}\right]$$

$$= \frac{S[81-\delta^2(1-r)^2+36\delta-18\delta(1-r)+2\delta^2(1-r)^2]}{324}$$

$$= \frac{S[81+18\delta(1+r)+\delta^2(1-r)^2]}{324}$$

4.2.3. Second period appropriation - Enter or not?

- When entry occurs, total appropriation becomes

$$(1-r)x + Q^E(x) = \frac{2S + (1-r)x}{3}$$

  - Substituting x_E into the above expression, yields

$$\frac{2S + (1-r) \times \frac{S[9-\delta(1-r)]}{18}}{3} = \frac{[36 + 9(1-r) - \delta(1-r)^2]S}{54}$$

which can be supported by the resource S because

$$\frac{[36 + 9(1-r) - \delta(1-r)^2]S}{54} \leq S$$

reduces to $18 - 9(1-r) + \delta(1-r)^2 \geq 0$ that holds for all admissible values of $\delta, r \in [0,1]$.

➢ Second scenario: Entry deterrence

- When the incumbent chooses a first-period appropriation to the ***right-hand side of*** $\boldsymbol{x_{ED}}$ in Figure 4.1.
- It deters entry, thus making profit Π_{NE}^{2nd} in the second period.

➢ The incumbent then solves:

$$\max_{x \geq x_{ED}} \pi^{1st} + \delta \Pi_{NE}^{2nd} = \underbrace{\left[x - \frac{x^2}{S}\right]}_{\pi^{1st}} + \delta \underbrace{\left[\frac{S - (1-r)x}{4}\right]}_{\Pi_{NE}^{2nd}}$$

- Differentiating with respect to x:

$$\frac{\partial\left[\pi^{1st} + \delta\Pi_{NE}^{2nd}\right]}{\partial x} = 1 - \frac{2x}{S} - \frac{\delta(1-r)}{4}$$

- which is negative if and only if

$$x > \frac{S[4-\delta(1-r)]}{8} = x^{SO}$$

- where x^{SO} is the socially optimal extraction in the first period (as shown in Section 3.4) that satisfies $x_E > x^{SO}$.
- Intuitively, the incumbent's profits decrease in x, so that when this firm seeks to deter entry, it chooses the minimal first-period appropriation that achieves this objective, x_{ED}.

➢ ***The incumbent appropriates above the socially optimal level if***

$$x_{ED} = \frac{S - 9F}{1 - r} > \frac{S[4 - \delta(1 - r)]}{8} = x^{SO}$$

which we rearrange to yield

$$4S(1 + r) + \delta S(1 - r)^2 - 72F > 0$$

- The fixed cost is low enough to cause over-exploitation, where

$$F < \hat{F} \equiv \frac{S[4(1 + r) + \delta(1 - r)^2]}{72}$$

- Intuitively, the fixed cost must be sufficiently low to cause over-exploitation; otherwise over-exploitation above the socially optimal level x^{SO} cannot be sustained in equilibrium.

4.2.4 First period appropriation - Entry deterrence

- Cutoff $\hat{F}$ is increasing in δ where

$$\frac{\partial \hat{F}}{\partial \delta} = \frac{S(1-r)^2}{72} > 0$$

because when the incumbent assigns more weights to future payoffs, deterring entry becomes more costly, and a higher fixed cost is needed to extract above the socially optimal level.

- Cutoff $\hat{F}$ is increasing in r where

$$\frac{\partial \hat{F}}{\partial r} = \frac{S[2-\delta(1-r)]}{36} > 0$$

as when the stock regenerates more rapidly, the entrant has more incentives to enter, and can only deterred with a higher fixed cost.

4.2.4 First period appropriation - Entry deterrence

- Cutoff $\hat{F}$ is increasing in S where

$$\frac{\partial \hat{F}}{\partial S} = \frac{4(1+r) + \delta(1-r)^2}{72} > 0$$

as when the resource becomes more abundant, the entrant finds the CPR more profitable to enter, so the incumbent has to appropriate more intensively to raise the bar in deterring entry.

- We find a more demanding condition on the fixed cost F when

$$\overline{F} = \frac{S}{9} > \frac{S[4(1+r) + \delta(1-r)^2]}{72} = \hat{F}$$

which we rearrange to yield

$$(1-r)[4 - \delta(1-r)] > 0$$

- ***The incumbent appropriates above the competitive level if***

$$x_{ED} = \frac{S - 9F}{1 - r} > \frac{S[9 - \delta(1 - r)]}{18} = x_E$$

which we rearrange to yield

$$9S(1 + r) + \delta S(1 - r)^2 - 162F > 0$$

- Rearranging and solving for F, the fixed cost becomes

$$F < \underline{F} \equiv \frac{S[9(1 + r) + \delta(1 - r)^2]}{162}$$

which is more demanding than that to cause over-exploitation, since

$$\hat{F} = \frac{S[4(1 + r) + \delta(1 - r)^2]}{72} > \frac{S[9(1 + r) + \delta(1 - r)^2]}{162} = \underline{F}$$

reduces to $5\delta(1 - r)^2 > 0$ that holds for all values of $\delta, r \in [0,1]$.

➢ ***Depending on the incumbent's appropriation, we have four regions.***

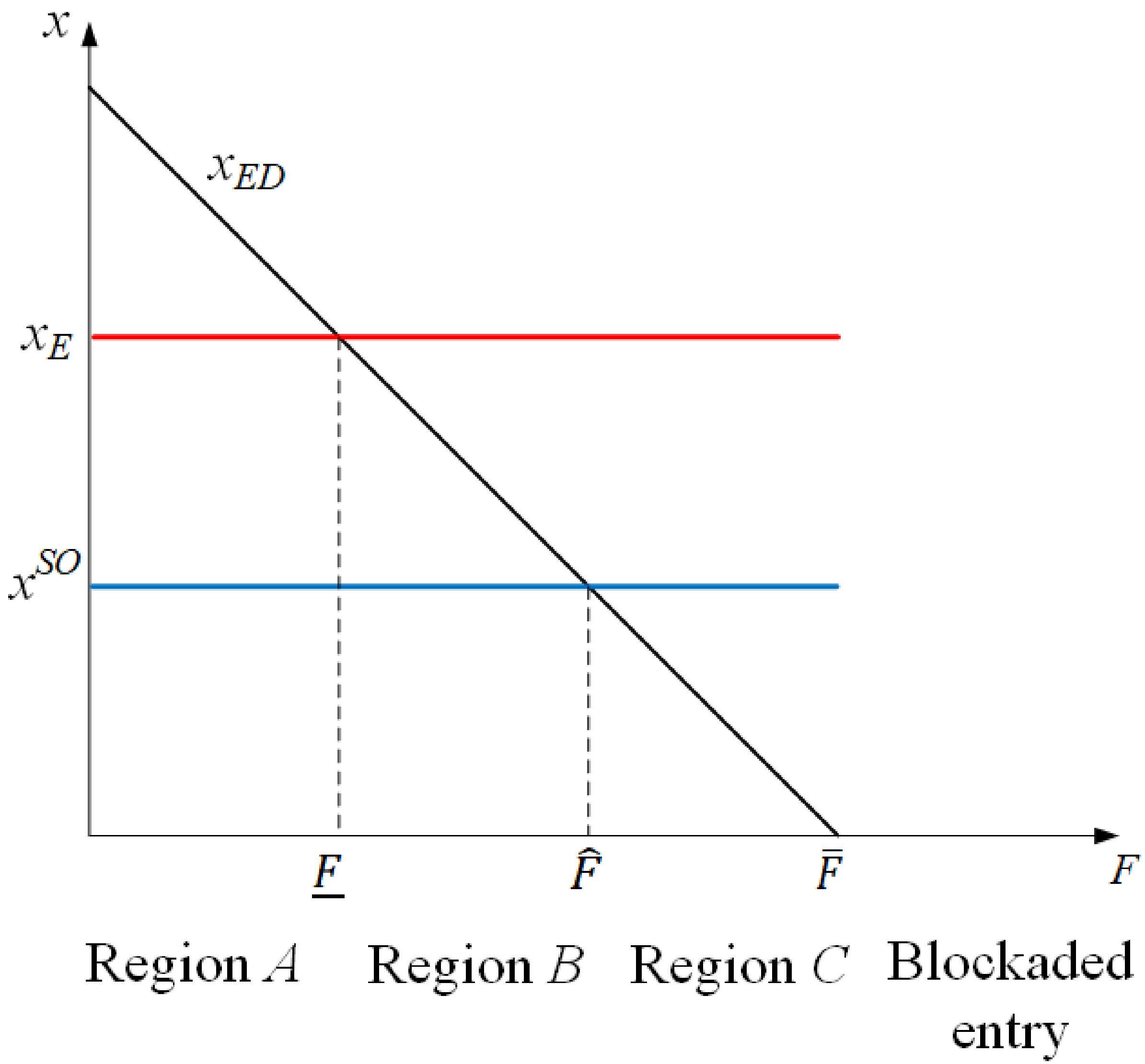
- **Region *A***: When entry cost F is relatively low, $F \leq \underline{F}$, x_{ED} is higher than x^E and x^{SO}, that is, $x_{ED} \geq x^E > x^{SO}$. In this case, the incumbent can deter entry with x_{ED}, which is socially excessive, $x_{ED} > x^{SO}$, and also exceeds that were entry accommodated, $x_{ED} \geq x^E$.

- **Region *B***: When entry cost F is intermediate, $\hat{F} \geq F > \underline{F}$, x_{ED} is also intermediate, lying between x^E and x^{SO}, that is, $x^E > x_{ED} \geq x^{SO}$. In this setting, the incumbent can deter entry with x^{SO} since entry is blockaded. In other words, since the entrant finds entry unprofitable, the incumbent does not choose x_{ED} which results in lower profits.

- **Region *C***: When entry cost F is relatively high, $\overline{F} \geq F > \widehat{F}$, x_{ED} is lower than x^E and x^{SO}, that is, $x^E > x^{SO} > x_{ED}$. In this context, the incumbent can deter entry with an appropriation level x^{SO}, which lies below the appropriation level that accommodates entry, x^E, and that renders x_{ED} slack. Informally, the incumbent "protects" the resource, exploiting exactly the socially optimal level x^{SO} while deterring entry.

- **Blockaded entry**: Finally, when entry cost is extremely high, $F > \overline{F}$, the entrant is barred from entry for all levels of $x_{ED} \geq 0$. Therefore, even if the incumbent remains dormant, the entrant does not find entry profitable. In this context, entry is blockaded for all levels of appropriation.

4.2.4 First period appropriation - Entry deterrence



4.2.4 First period appropriation - Entry deterrence

- Inserting x_{ED}, overall profits from practicing entry deterrence are

$$\Pi^{ED} = \left[x_{ED} - \frac{x_{ED}^2}{S}\right] + \delta\left[\frac{S-(1-r)x_{ED}}{4}\right]$$

$$= \frac{S-9F}{1-r}\left[1-\frac{S-9F}{S(1-r)}\right] + \frac{\delta}{4}[S-(S-9F)]$$

$$= \frac{(S-9F)(9F-rS)}{S(1-r)^2} + \frac{9\delta F}{4}$$

4.2.4 First period appropriation - Entry deterrence

➢ In this context, the incumbent deters entry if $\Pi^{ED} \geq \Pi^{AE}$, solving

$$\frac{(S-9F)(9F-rS)}{S(1-r)^2}+\frac{9\delta F}{4} \geq \frac{S[81+18\delta(1+r)+\delta^2(1-r)^2]}{324}$$

- which we reduce to the following quadratic inequality,

$$\{(1-r)^2[81+18\delta(1+r)+\delta^2(1-r)^2]+324r\}S^2$$
$$-729F[4(1+r)+\delta(1-r)^2]S+26244F^2 \leq 0$$

- Applying the quadratic formula, we find

$$S \geq \frac{81F\left\{9[4(1+r)+\delta(1-r)^2]-(1-r)\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right\}}{2\{(1-r)^2[81+18\delta(1+r)+\delta^2(1-r)^2]+324r\}}$$

or $$S \leq \frac{81F\left\{9[4(1+r)+\delta(1-r)^2]+(1-r)\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right\}}{2\{(1-r)^2[81+18\delta(1+r)+\delta^2(1-r)^2]+324r\}}$$

4.2.4 First period appropriation - Entry deterrence

- For simplicity, we consider $\delta = 1$, $r = \frac{1}{4}$, and $F = \frac{1}{100}$.

$$\Pi^{ED} = \frac{\left(S - \frac{9}{100}\right)\left(\frac{9}{100} - \frac{S}{4}\right)}{S\left(1 - \frac{1}{4}\right)^2} + \frac{9}{400} = \frac{89}{400} - \frac{9}{625S} - \frac{4}{9}S$$

$$\Pi^{AE} = \frac{S\left[81 + 18\left(1 + \frac{1}{4}\right) + \left(1 - \frac{1}{4}\right)^2\right]}{324} = \frac{185}{576}S$$

4.2.4 First period appropriation - Entry deterrence

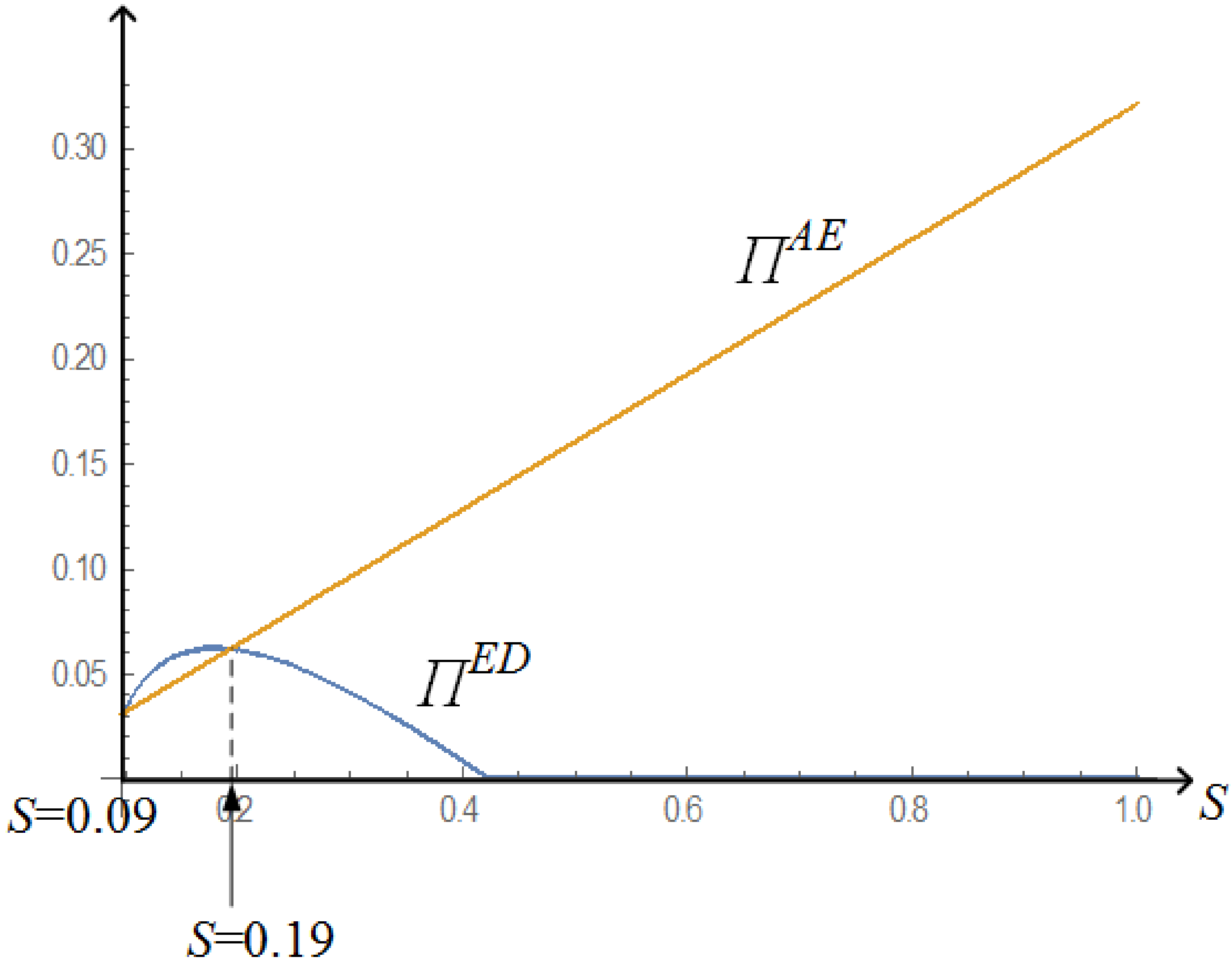


Figure 4.2. Entry deterrence/allowance regions.

4.2.4 First period appropriation - Entry deterrence

- The figure considers that the initial stock S satisfies condition $F < S/9$, which can be rearranged as $S > 9F$.

- Since the figure takes into account $F = \frac{1}{100}$, this condition becomes $S > \frac{9}{100} = 0.09$.

  - This explains why the horizontal axis starts at $S = 0.09$ rather than at zero.

4.2.4 First period appropriation - Entry deterrence

- Solving for the entry deterrence condition, $\Pi^{ED} \geq \Pi^{AE}$,

$$\frac{89}{400} - \frac{9}{625S} - \frac{4}{9}S \geq \frac{185}{576}S$$

- which we rearrange to yield

$$30625S^2 - 8900S + 576 \leq 0$$

- Applying the quadratic formula, we have

$$S \geq \frac{8900 - \sqrt{(8900)^2 - 4 \times 30625 \times 576}}{2 \times 30625} = \frac{178 - 2\sqrt{865}}{1225}$$

$$S \leq \frac{8900 + \sqrt{(8900)^2 - 4 \times 30625 \times 576}}{2 \times 30625} = \frac{178 + 2\sqrt{865}}{1225}$$

- Setting the inequality equal to zero, we find two roots,

$$S_1 = 0.097 \text{ and } S_2 = 0.193$$

- Since $S_1 = 0.097 > 0.09$, we claim that the incumbent firm has incentives to practice entry deterrence, as depicted to the right of the first crossing point between Π^{ED} and Π^{AE}.
- When $S < S_2 = 0.193$, the initial stock is not too abundant for the incumbent to find entry deterrence more profitable than accommodation, to the left of the second crossing point between Π^{ED} and Π^{AE} as depicted in Figure 4.2.

4.2.4 First period appropriation - Entry deterrence

- When $S_2 \leq 0.19$,
  - Entry-deterring profit Π^{ED} **lies above** that of allowing entry, Π^{AE}, implying that the incumbent deters entry.
    - ✓ The resource is relatively scarce, so the incumbent can easily exploit the resource to make it sufficiently unattractive for the potential competitor.
- When $S_2 > 0.19$,
  - Entry-deterring profit Π^{ED} **lies below** that of allowing entry, Π^{AE}, implying that the incumbent allows entry.
    - ✓ Intuitively, the stock is relatively abundant, making it unprofitable for the incumbent to deplete the CPR enough to prevent entry, leading the incumbent to allow the potential entrant to participate in the CPR.

- Adding up Π_E^{2nd} and Π^{AE}, welfare under entry accommodation is

$$W^{AE} = \Pi^{AE} + \delta\Pi_E^{2nd}$$

$$= \frac{S[81 + 18\delta(1+r) + \delta^2(1-r)^2]}{324} + \frac{\delta S[9(1+r) + \delta(1-r)^2]}{162} - \delta F$$

$$= \frac{S[27 + 12\delta(1+r) + \delta^2(1-r)^2]}{108} - \delta F$$

- When entry is deterred, welfare coincides with the incumbent's Π^{ED}:

$$W^{ED} = \frac{(S-9F)(9F-rS)}{S(1-r)^2} + \frac{9\delta F}{4}$$

4.2.4 First period appropriation - Entry deterrence

➢ It is socially desirable to deter entry if $W^{ED} \geq W^{AE}$, solving

$$\frac{(S-9F)(9F-rS)}{S(1-r)^2} + \frac{9\delta F}{4} \geq \frac{S[27+12\delta(1+r)+\delta^2(1-r)^2]}{108} - \delta F$$

- which we reduce to the following quadratic inequality,

$$\{(1-r)^2[27+12\delta(1+r)+\delta^2(1-r)^2]+108r\}S^2$$
$$-27F[36(1+r)+13\delta(1-r)^2]S+8748F^2 \leq 0$$

- Applying the quadratic formula, we find

$$S \geq \frac{27F\left[36(1+r)+13\delta(1-r)^2-(1-r)\sqrt{\delta[360(1+r)+121\delta(1-r)^2]}\right]}{2\{(1-r)^2[27+12\delta(1+r)+\delta^2(1-r)^2]+108r\}}$$

or $$S \leq \frac{27F\left[36(1+r)+13\delta(1-r)^2+(1-r)\sqrt{\delta[360(1+r)+121\delta(1-r)^2]}\right]}{2\{(1-r)^2[27+12\delta(1+r)+\delta^2(1-r)^2]+108r\}}$$

4.2.3. Second period appropriation - Enter or not?

- Plugging $\delta = 1$, $r = \frac{1}{4}$, and $F = \frac{1}{100}$, welfare under accommodation is

$$W^{AE} = \frac{S\left[27 + 12\left(1 + \frac{1}{4}\right) + \left(1 - \frac{1}{4}\right)^2\right]}{108} - \frac{1}{100} = \frac{227}{576}S - \frac{1}{100}$$

- Operating similarly, welfare under entry deterrence becomes

$$W^{ED} = \Pi^{ED} = \frac{89}{400} - \frac{9}{625S} - \frac{4}{9}S$$

- Thus, deterrence is socially preferred to accommodation if

$$W^{ED} = \frac{89}{400} - \frac{9}{625S} - \frac{4}{9}S \geq \frac{227}{576}S - \frac{1}{100} = W^{AE}$$

- Simplifying, we solve the quadratic inequality in S, as follows:

$$100625S^2 - 27900S + 1728 \leq 0$$

- Applying the quadratic formula, we have

$$S \geq \underline{S} \equiv \frac{558 - 6\sqrt{921}}{4025} \approx 0.093$$

or

$$S \leq \overline{S} \equiv \frac{558 + 6\sqrt{921}}{4025} \approx 0.184$$

4.2.3. Second period appropriation - Enter or not?

➢ In summary, we have five ranges of values:

- When $9F < S \leq \underline{S}$, the incumbent accommodates entry being socially optimal.
- When $\underline{S} < S \leq S_1$, the incumbent accommodates entry, but since the resource is so scarce, opening up to the entrant hurts social welfare.
- When $S_1 < S \leq \overline{S}$, the incumbent deters entry, and given intermediate resource, deterring is better off than accommodating entry.
- When, $\overline{S} < S \leq S_2$, the incumbent deters entry, but since the resource is more plentiful, deterring is socially worse off than accommodating entry.
- When, $S > S_2$, the incumbent accommodates entry, and thanks for the abounding resource, accommodating is superior to deterring entry.

A larger dynamic inefficiency

4.3 A larger dynamic inefficiency

- The strategic exploitation of the resource that the incumbent carries out to deter entry leads to a larger dynamic inefficiency when
  - The incumbent finds it optimal to practice entry deterrence, $\Pi^{ED} \geq \Pi^{AE}$, choosing a first-period appropriation, $x_{ED} = \frac{S-9F}{1-r}$.
- In this context, the dynamic inefficiency becomes

$$DI = x_{ED} - x^{SO} = \frac{S-9F}{1-r} - \frac{S[4-\delta(1-r)]}{8}$$

- Comparing dynamic inefficiency:
  - Dynamic inefficiency from Chapter 3

$$DI = x^* - x^{SO} = \frac{S[9-\delta(1-r)]}{18} - \frac{S[4-\delta(1-r)]}{8} = \frac{5S\delta(1-r)}{72}$$

  - Dynamic inefficiency from Chapter 4

$$DI = x_{ED} - x^{SO} = \frac{S-9F}{1-r} - \frac{S[4-\delta(1-r)]}{8}$$

- We see that the dynamic inefficiency is larger when the incumbent faces entry threats since $x_{ED} > x^*$ for $F \leq \underline{F}$.

4.3 A larger dynamic inefficiency

- That is, this firm needs to increase its first-period appropriation if it seeks to deter entry, whereas the socially optimal first-period appropriation x^{SO} is the same in both contexts.
  - Formally, $x_{ED} - x^{SO} > x^* - x^{SO}$ since $x_{ED} > x^*$.

- This inefficiency increase can be attributed to the incumbent's strategic increase of its first-period appropriation to prevent entry,
  - since in the setting we study in this chapter entry is endogenous while in Chapter 3 entry was exogenous, i.e., assumed to happen with certainty in the second period.

4.3 A larger dynamic inefficiency

- When the incumbent **does not have** incentives to practice entry deterrence, $\Pi^{ED} < \Pi^{AE}$,
  - It chooses the same first-period appropriation as when entry happens with certainty, $x^* = \frac{S[9-\delta(1-r)]}{18}$.
- Therefore, dynamic inefficiency DI coincides both when entry is endogenous and exogenous,
  - thus coinciding with our results in Chapter 3.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 5: Repeated interaction in the commons

Outline

- Modeling repeated interaction
- Finite repetitions
- Infinite repetitions
- Experimental studies of repeated interaction in the commons

In this chapter:

- We discuss games where firms **interact several times**, facing the same game repeatedly, also known as *repeated games*.

- **The feature of repeated games:**
  - They can help us rationalize players' cooperation, even when such cooperation could not be sustained in the unrepeated version of the game.

Modeling repeated interaction

In this section:

- We present a simple model of a CPR game, highlighting its similarities with the canonical prisoner's dilemma game.

➢ Consider the CPR game in Matrix 5.1:

- Where every firm **simultaneously and independently** chooses between a high and a low appropriation level.
- Firm 1 selects a row, while firm 2 chooses a column.
- The first payoff in every cell corresponds to firm 1 and the second payoff to firm 2.

		Firm 2	
		High approp.	Low approp.
Firm 1	High approp.	*d*, *d*	*b*, *c*
	Low approp.	*c*, *b*	*a*, *a*

Matrix 5.1. CPR game as a prisoner's dilemma

- When **both firms choose Low appropriation**, at the bottom-right hand corner of the matrix, both earn a payoff of **$*a***.
- When **either firm** unilaterally chooses a High appropriation, its payoff increases from a to b (where $b > a$), while that of its rival decreases from a to c (where $a > c$).
- When **both firms choose a High appropriation,** at the top-hand corner of the matrix, they both earn a payoff of **$*d***. where $a > d > c$.
- In summary, payoffs satisfy $\boldsymbol{b > a > d > c}$, for example, b = $7, a = $5, d = $1, and c = $0 satisfy this payoff ranking.

➢ *How to solve the game?*

- The game in Matrix 5.1 can solve in **two ways.**

I. Finding strictly dominated strategies

- The game is strategically analogous to the common “prisoner’s dilemma” game since every firm finds High appropriation to be a strictly dominant strategy; making Low appropriation a strictly dominated strategy.

II. Finding Nash equilibria

- We find the best responses of each player, and then identify which strategy profile implies every player choosing a best response to her opponent’s strategy.

I. Finding strictly dominated strategies for firm 1

- When firm 2 chooses a **High appropriation**:
  - Firm 1 chooses a High appropriation, because the payoff of choosing High appropriation (\$$d$) is higher than when it chooses Low appropriation (\$$c$) since $d > c$ by assumption.

- When firm 2 chooses a **Low appropriation**:
  - Firm 1 chooses a High appropriation, because the payoff of choosing High appropriation (\$$b$) is higher than when it chooses Low appropriation (\$$a$) since $b > a$ by assumption.

5.2 Modeling repeated interaction

- Firm 1 finds it optimal to choose High appropriation *regardless* of the strategy that firm 2 selects; so ***High appropriation is strictly dominant***.
- Since players are symmetric, a similar argument applies to firm 2, which finds High appropriation to be strictly dominant as well.
- Since rational players would never choose strictly dominated strategies (low appropriation for each firm), we can delete them from Matrix 5.1, leaving us with a one-cell matrix containing only the High appropriation row and column.

		Firm 2
		High approp.
Firm 1	High approp.	*d, d*

- *Therefore, the strategy profile (High, High) survives strict dominance yielding a payoff pair (d, d).*

II. Finding Nash equilibria (NE)

- ***Steps:***

1. We need to find the best responses for each firm.
2. We underline the best response payoff for each firm.
3. The Nash equilibrium (NE) occurs at where the best response payoffs of all players are underlined.

➢ Starting with the best responses of firm 1

- When firm 2 chooses High appropriation:
  - ✓ Firm 1 is better off responding with High than Low appropriation since $d > c$.
- When firm 2 chooses Low appropriation:
  - ✓ Firm 1 is better off responding with High than Low appropriation since $b > a$.

		Firm 2	
		High approp.	Low approp.
Firm 1	High approp.	***d***, *d*	***b***, *c*
	Low approp.	*c*, *b*	*a*, *a*

➢ Moving to find the best responses of firm 2

- When firm 1 chooses High appropriation:
  - ✓ Firm 2 is better off responding with High than Low appropriation since $d > c$.
- When firm 1 chooses Low appropriation:
  - ✓ Firm 2 is better off responding with High than Low appropriation since $b > a$.

		Firm 2	
		High approp.	Low approp.
Firm 1	High approp.	*d*, ***<u>d</u>***	*b*, *c*
	Low approp.	*c*, ***<u>b</u>***	*a*, *a*

- In short, **<u>there is only one cell</u>** where all payoffs are underlined, (High, High), indicating that in this cell every firm plays a best response to its opponent's strategies.

		Firm 2: High approp.	Firm 2: Low approp.
Firm 1	High approp.	$\underline{\mathbf{d}}$, $\underline{\mathbf{d}}$	$\underline{\mathbf{b}}$, c
	Low approp.	c, $\underline{\mathbf{b}}$	a, a

Matrix 5.2. CPR game – underlining best responses

- *Therefore, the strategy profile (High, High) is a Nash equilibrium (NE) of the CPR game.*

➢ Static inefficiency

- Matrix 5.2 helps us illustrate the static inefficiency that arises in the NE of the CPR game.

- In particular, firms choose a High appropriation, each earning $\$d$ at the top left-hand corner of the matrix.

➢ Low appropriation

- However, firms could coordinate on Low appropriation to earn $\$a$ at the bottom right-hand corner of the matrix.

➢ Interpretation

- **Low appropriation:**
  - ✓ The socially optimal appropriation level (at least when the welfare function only considers producer surplus, $W = PS$).

- **High appropriation:**
  - ✓ The equilibrium that emerges in the game, which difference can be understood as the static inefficiency that arises when firms simultaneously and independently choose their appropriation decisions, with an individual profit loss of $a – d$ for each firm.

- Therefore, the cooperative outcome (Low, Low) **cannot be** sustained in the equilibrium of the unrepeated game.

- We need to explore under which conditions it can be sustained when the game is repeated, that is, when firms interact many times, playing the game of Matrix 5.1 in each round.

- For simplicity, our presentation abstracts from the dynamic inefficiencies that emerge in the CPR game by assuming that the resource fully regenerates across periods. A richer analysis should, however, consider that the resource **does not fully regenerate**:

  - Implying that every firm's appropriation decisions in period *t* are affected by the net stock available at the beginning of this period:

    - ✓ It depends on the stream of appropriation decisions by both players in all previous periods, and involves dynamic optimization techniques.

Finite repetitions

- We consider that the game is repeated ***T* periods**, where T is a finite number (*which can be 2 or 500, but not infinite number of times*).
  - **The timeline**

1. Period t:
   - Every firm chooses its appropriation decision in period $t = \{1,2, \ldots, T\}$, yielding an outcome (i.e., a cell in Matrix 5.1) for period t, which is perfectly observed by both firms.
2. Period $t + 1$:
   - Observing the outcome of period t, every firm chooses its appropriation decision in period $t + 1$, which yields an outcome for period $t + 1$.

3. Period T:

- After observing the outcome of period $T - 1$, ever firm chooses its appropriation and the game ends.

➢ **This is a sequential-move game:**

- Since every firm, when considering its appropriation in period $t + 1$, perfectly observes the past history of appropriation decisions by both firms from period 1 until period t.

- Given this history, every firm responds with its appropriation in period $t + 1$.

➢ *How to solve the game?*

- We deploy the Subgame Perfect Equilibrium (SPE) solution concept by applying backward induction (as in Chapters 3 & 4)

- We analyze equilibrium appropriation decisions in the last period, T, for any previous history of appropriation decisions:

➢ Period *T*

- Starting from the last round of play at $t = T$, **every firm's strictly dominant strategy is High appropriation**, thus providing us with (High, High) as the NE of the last-period game.

➢ Period $T - 1$

- In the previous-to-last period, every firm anticipates that (High, High) will ensue in period T, where both firms will choose High appropriation regardless of the outcome in period $T - 1$.

- Consequently, **every firm finds that High appropriation is a strictly dominant strategy.** Therefore, strategy profile (High, High) is, again, the NE of the game in period $T - 1$.

➢ Period $T - 2$

- A similar argument applies if we move one step up to period $T - 2$, where both firms anticipate that (High, High) will be the equilibrium outcome in the subsequent periods, $T - 1$ and T.

- Thus, every firm chooses **High appropriation in the current period $T - 2$,** which yields (High, High) as the NE outcome in this period, too.

➢ ***In summary,***

- We find that (High, High) is the NE <u>in every period *t*</u>, from the beginning of the game at $t = 1$ to the last stage $t = T$.

- The SPE of the game has every firm choosing High appropriation at every round, regardless of the outcomes in previous rounds.

➢ ***Intuitively,***

- The existence of a <u>terminal period makes every firm anticipate</u> that <u>**both firms will select High appropriation**</u> in that period.

- Since the last stage outcome is unaffected by previous moves, firms in prior periods do not benefit from <u>**Low appropriation**</u>.

❖ *This behavior leads to the depletion of some natural resources.*

Infinite repetitions

- How can we operationalize an infinitely repeated game in Matrix 5.1?

- We assume that:
  - At any given moment, firms play the game one more round with some probability p.
  - This probability is always possible, so players could interact for infinite rounds. ***Why*?**
- **If *p* = 0.9**
  - The probability that players interact for 10 rounds is $0.9^{10} \cong 0.35$.
  - The probability that they continue playing for 100 rounds is extremely small, $0.9^{100} \cong 0.000027$.

- We know that from previous section that when the game is played once or a finite number of times the only equilibrium prediction is (High, High) in every single round of play.
- How can we sustain cooperation if the game is played an infinite number of times?
  - By the use of a Grim-Trigger Strategy (GTS):

In the first period $t = 1$:

- Every firm cooperates to choose **Low appropriation**.

In all subsequent periods $t > 1$:

- Every firm continues to cooperate as long as it observes that all firms cooperated in all past periods.
- If instead, one firm defects at any previous period, all respond playing **High appropriation** thereafter (deviating from the GTS.)

- We need to show:
  - Every firm finds the GTS optimal at every time period *t*.
  - GTS can be sustained as a SPE of the infinitely repeated game.
- GTS **must be** optimal after any previous history of play:

(a) No cheating history

(b) Some cheating history

➢ Case (a): No cheating history

- The GTS dictates that every player cooperates in the next period, earning a payoff of a, yielding a stream of discounted payoffs:

$$a + \delta(a) + \delta^2(a) + \cdots = a(1 + \delta + \delta^2 + \cdots)$$

- Since the infinite geometric progression, $1 + \delta + \delta^2 + \cdots$, can be simplified to $\frac{1}{1-\delta}$, the above expression becomes

$$\frac{a}{1-\delta}$$

- where $\delta \in (0,1)$ represents the player's discount factor, indicating how much it cares about future payoffs.

- If the player cheats by choosing High appropriation, and its opponent chooses a Low appropriation:

  - The player's payoff increases from a to b, where $b > a$ by assumption.

  - The player's defection is detected by the other firm, which responds with **<u>High appropriation</u> thereafter** *(the punishment)*, yielding a payoff of d thereafter, where $d < a$.

- The player's stream of discounted payoffs from cheating becomes:

$$\underbrace{b}_{\text{Firm cheats}} + \underbrace{\delta(d) + \delta^2(d) + \cdots}_{\text{Punishment thereafter}}$$

- which simplifies to

$$b + d(\delta + \delta^2 + \delta^3 + \cdots) = b + d\delta(1 + \delta + \delta^2 + \cdots)$$

$$= b + \frac{d\delta}{1-\delta}$$

- Therefore, after a history of cooperation, every firm keeps cooperating, obtaining $\frac{a}{1-\delta}$, rather than defecting if

$$\frac{a}{1-\delta} \geq b + \frac{d\delta}{1-\delta}$$

- Multiplying both sides by the denominator, 1 – δ, yields

$$a \geq b(1-\delta) + d\delta$$

- Solving for δ, we obtain

$$\delta \geq \frac{b-a}{b-d}$$

- In words, this condition states that firms cooperate every single round of the game (choosing Low appropriation) as long as they assign a sufficiently high weight on future payoffs.

- The discount factor δ is increasing in b because

$$\frac{\partial\delta}{\partial b} = \frac{a-d}{(b-d)^2} > 0$$

so the higher the deviation payoff, the more difficult to cooperate.

- The discount factor δ is decreasing in a since

$$\frac{\partial\delta}{\partial a} = -\frac{1}{b-d} < 0$$

so the higher the cooperation payoff, the more likely to cooperate.

- The discount factor δ is increasing in d as

$$\frac{\partial\delta}{\partial d} = \frac{b-a}{(b-d)^2} > 0$$

so the higher the stage game payoff, the less likely to cooperate.

Case (b): Some cheating history

- If some (or all) firms cheated in a previous period $t - 1$:
  - The GTS prescribes that every firm should choose High appropriation thereafter, yielding a stream of discounted payoffs

$$d + \delta(d) + \delta^2(d) + \cdots = d(1 + \delta + \delta^2 + ..) = \frac{d}{1-\delta}$$

- If, instead, a firm unilaterally deviates from this punishment scheme (playing Low while its opponent chooses High), its stream of discounted payoffs becomes

$$\underbrace{c}_{\text{Firm deviates}} + \underbrace{\delta(d) + \delta^2(d) + \cdots}_{\text{Punishment thereafter}}$$

  - Intuitively, when the firm chooses Low while its opponent selects High, this firm's payoff decreases from d to c during one period (since $c < d$ by assumption.)

- The choice of High by one firm triggers an infinite punishment by all firms, as prescribed by the GTS, yielding a payoff of *d* thereafter:

$$c + d(\delta + \delta^2 + \delta^3 + \cdots) = c + d\delta(1 + \delta + \delta^2 + \cdots)$$
$$= c + d\frac{\delta}{1-\delta}$$

- Upon observing a deviation to High:
  - Every firm prefers to stick to the GTS rather than deviating if:

$$\frac{d}{1-\delta} \geq c + d\frac{\delta}{1-\delta}$$

- which simplifies to

$$d\frac{1-\delta}{1-\delta} \geq c$$

- Since $c < d$ by definition, if your opponent chooses High appropriation thereafter, you <u>do not have any incentives</u> to unilaterally deviate towards a Low appropriation level, not even for one period!

➢ *In summary,*

- case (a), no cheating history, was the only condition we require for cooperation to be sustained as an equilibrium of this infinitely repeated game (GTS to be SPE of the game).

$$\delta \geq \frac{b-a}{b-d}$$

- **The numerator**, $b - a$, measures **the instantaneous gain that a firm obtains** by deviating from cooperation to defection (that is, from Low to High appropriation).

- **The denominator**, $b - d$, measures **the loss that the firm suffers** thereafter as a consequence of its deviation.

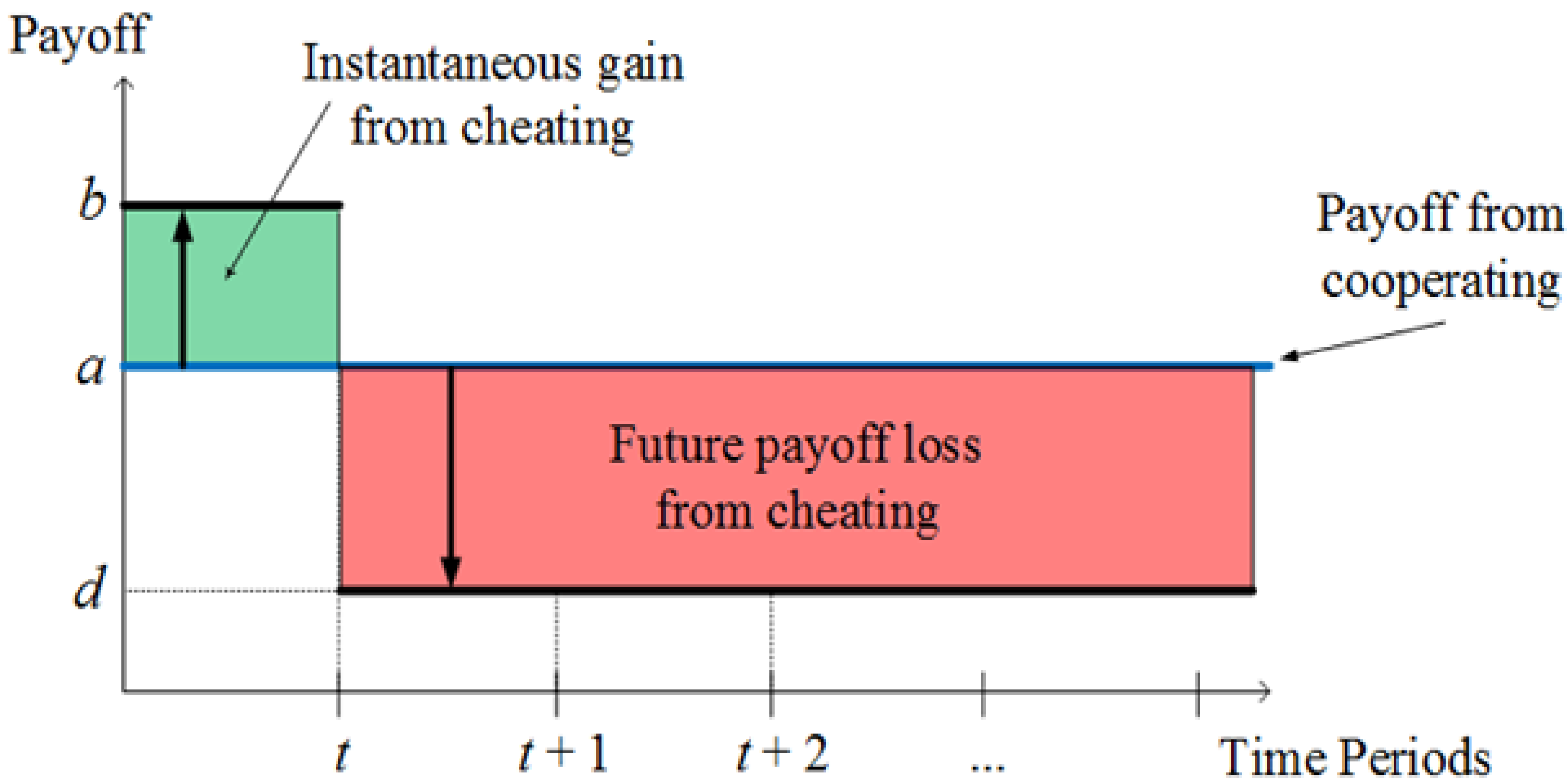


Figure 5.1. Incentives to cheat in repeated games.

Figure 5.1:

- Illustrates the trade-off that every player faces when, upon observing that all firms chose Low appropriation in previous rounds, it must choose whether to continue cooperating (with a Low appropriation) or to cheat (with a High appropriation).

If the firm cooperates:

- Its payoff remains at *a* in all subsequent periods.

If the firm cheats:

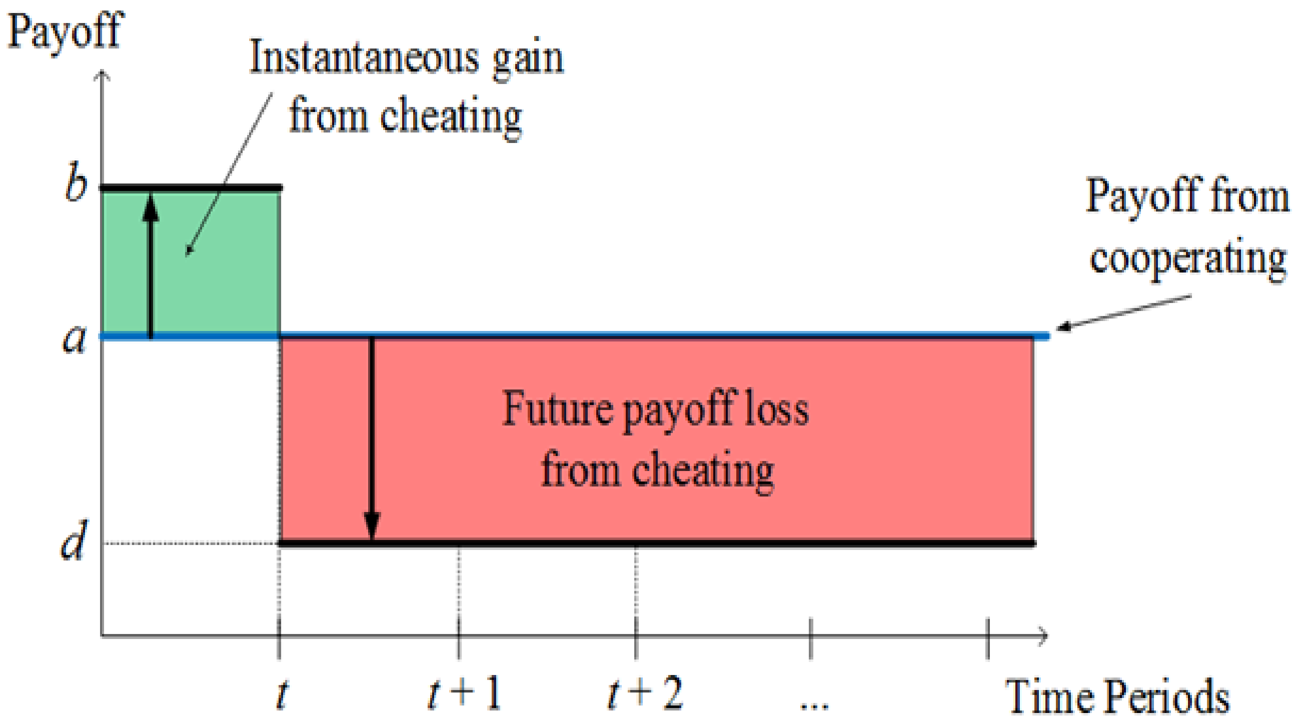
- Its payoff increases from *a* to *b* today; but its defection is thereafter punished by the other firms, decreasing its payoff from *b* to *d* in all subsequent periods.

- Graphically:
  - The instantaneous gain from cheating today is represented by the left-hand square in green, whereas the future loss from cheating is illustrated with the right-hand rectangle in red.
- Figure 5.1:
  - Helps us predict in which CPRs cooperation can more easily occur.

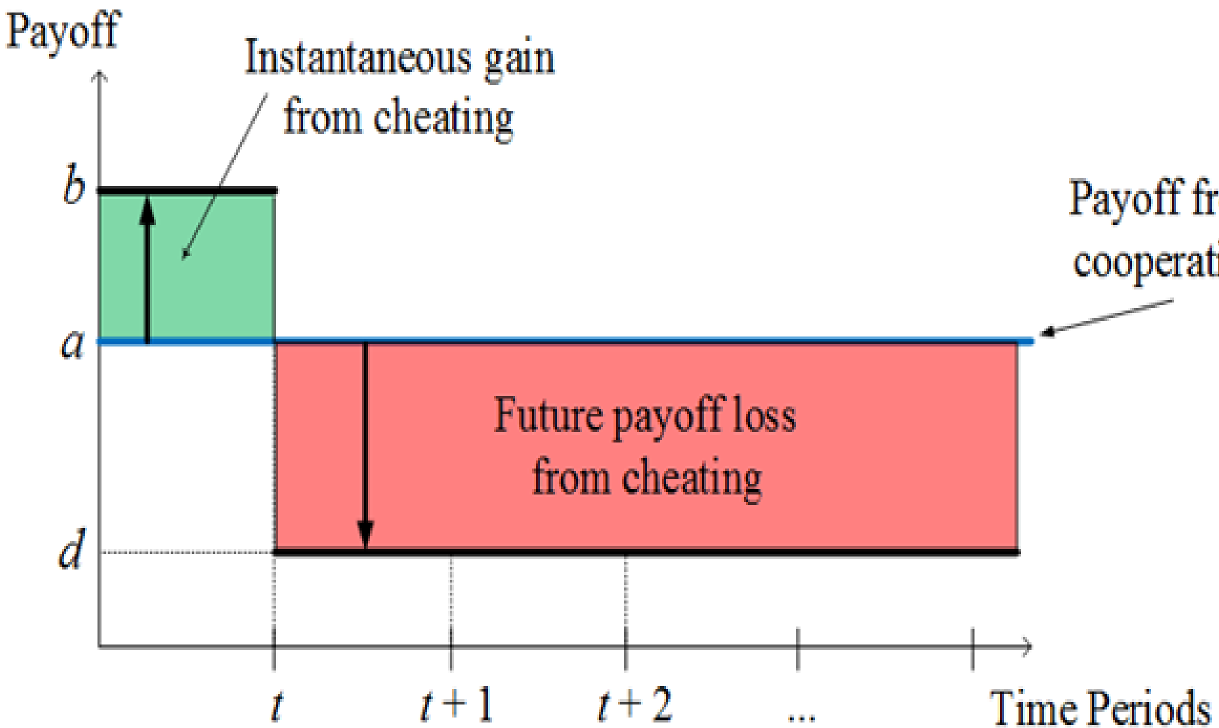
5.4 Infinite repetitions

➢ If the **instantaneous gain from choosing** a High appropriation decreases, the incentives to cheat also decrease.

- This occur when the fishermen face relatively convex appropriation costs, limiting the unilateral increase in appropriation that an individual fisherman can choose when deviating from the cooperative agreement.



- A similar argument applies when **cheating is detected immediately** rather than requiring several periods for other fishermen to detect.
  - The right-hand rectangle is narrow, rather than wide, thus shrinking the area that represents the firm's instantaneous gain from cheating.



➢ Extensions

1. Temporal reversion

- When firms choose a High appropriation for *T* rounds (e.g., 2 periods) but return to cooperation once the punishment has been inflicted, firms have more incentives to cheat since the future payoff loss from cheating (right-hand rectangle) is narrower, ultimately making defection more attractive.

2. Probabilistic detection

- When firms monitor each other's catches at the port, it is raw estimation of the actual appropriation. Thus, cheating on a High appropriation is not detected with certainty, ultimately making defection more attractive.

3. Exploitation by *N* firms

- When more firms exploit the CPR, it is harder to coordinate the catches of all fishermen, so that every firm finds a High appropriation more lucrative.

Experimental studies of repeated interaction in the commons

➢ Theoretical results

- The previous theoretical results have been tested in controlled experimental labs across several countries.
- The standard experimental session asks students to sit at computer terminals, separate from each other to prevent them from seeing other individuals' responses.
- The experimenter then reads the experiment instructions aloud or presents them on the computer screen for each participant to read.

5.5.1 Experimental design

- In a CPR game
  - The standard design provides the subject with a set of tokens.
- These tokens are distributed into two accounts.

I. Public account (A)
  - To avoid biases in favor/against public projects.
  - Provide a benefit to all subjects who participate in the experiment, whether or not they deposited tokens here.

II. Private account (B)
  - To maintain the experiment as neutral as possible.
  - Only benefit the individual who deposited them here.

➢ The experimenter

- The experimenter designs the return of each account so that it becomes a strictly dominant strategy to contribute to the private account (account B) alone.

  ✓ Implying that depositing a positive amount of tokens in the public account is strictly dominated.

- After reading the instructions, the experimenter lets subjects ask questions. As in most experiments nowadays, subjects go through a trial run to gain some practice before the experiment starts.

Instructions

- Specify whether the CPR game will be played only once, twice, T times, or whether there is a positive probability p that the subject will play the game in the next period.
- Describe clearly if the payoff from each period is affected by the players' behaviors in the previous rounds.
- If the subject will be paired with a different individual or with the same individual in the subsequent rounds.
- Whether the other individuals received the same information as the subject did, and etc.

5.5.1 Experimental results

➢ Start

- Each individual selects how many tokens to deposit in account A and B, and is informed about her payoff in that round (as in most experiments, how other subjects deposited their tokens).

- If the game is repeated, the subject is then asked to submit her token deposits to accounts A and B, which may be different from those in period 1.

➢ End

- The subject collects the tokens earned in each round, and exchange them for money at the exit of the experimental lab; often around US$20-$50.

I. Finitely-repeated games

- The experiments found that **in the last round** of interaction, players behave as if they are in an unrepeated (one-shot) game.
- However, **in the first round**, players cooperate, i.e., deposit positive amounts in the public account.
- This behavior **contradicts the theoretical prediction** discussed in section 5.3, where firms choose a High appropriation level, thus not cooperating with each other in any round.

II. Infinitely-repeated games

- Individuals participating in the experiment were informed that they will play one more round of the game with some probability, e.g., p = 80%, since they cannot play the game forever.

➢ Results

- Players' cooperation increases in the probability they interact in future rounds (e.g., p increases from 80% to 90%).

- Consistent with our findings of cooperation being easier to sustain when players care more about the future.

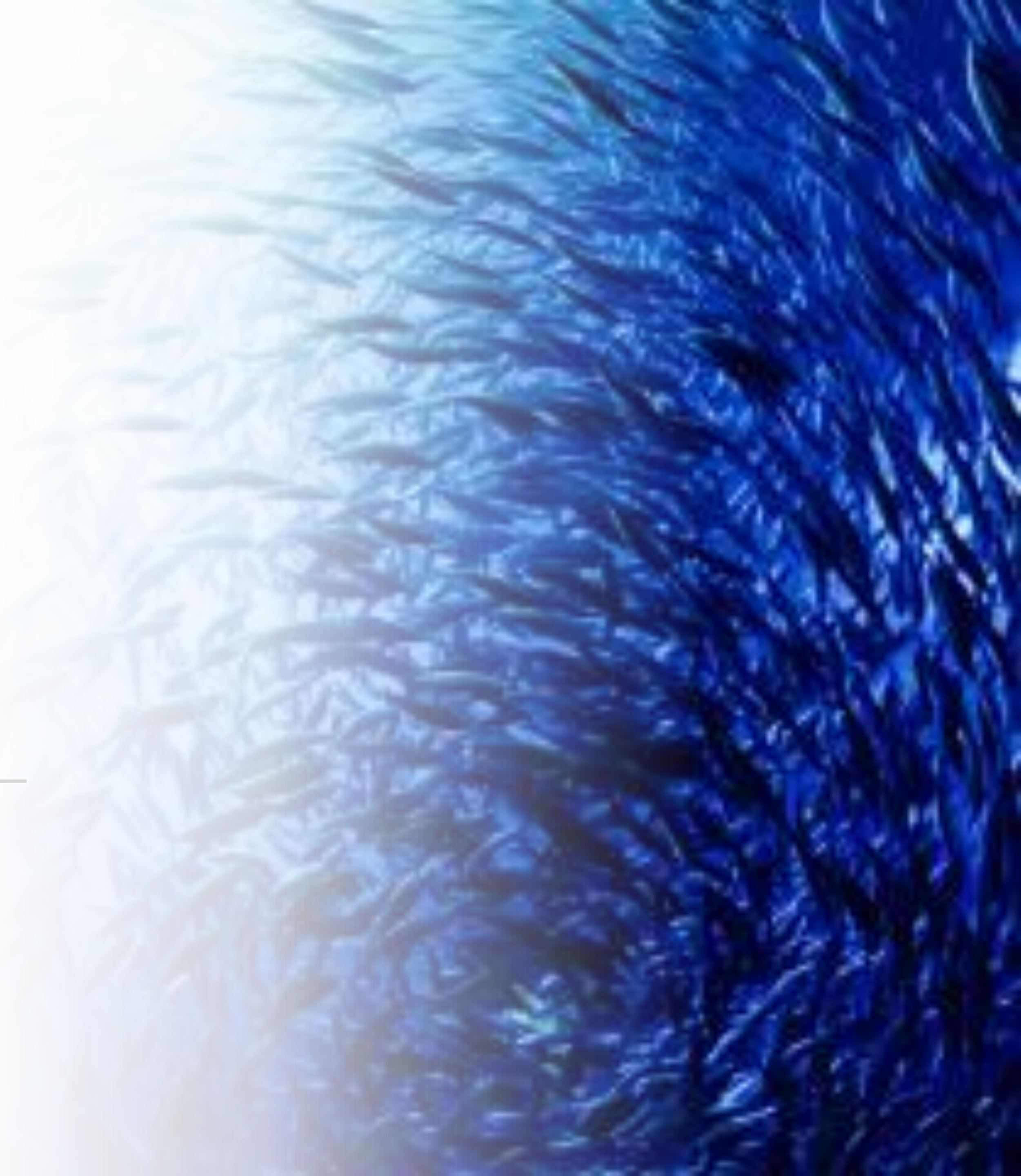
5.5.2 Experimental results

- However, when players **interact during many rounds**, they start **defecting more frequently**, anticipating that they may not interact in the future.
  - This occurs because since the probability they encounter each other again declines rapidly.
  - Therefore, they try to reap the gains from a unilateral defection in one of the last rounds of play.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 6: Commons under incomplete information



Outline

- Symmetrically uninformed firms
  - Everyone is in the dark
- Asymmetrically uninformed firms
  - Only some firms are in the dark

Symmetrically uninformed firms – Everyone is in the dark

6.2 Symmetrically uninformed firms

- *Assumption*

- Assume that N firms have free access to the resource.

- Every firm i must simultaneously and independently choose its appropriation level x_i.

- All firms face a common uncertainty: They cannot observe the available stock, S, but knows that it is high S_H with probability p or low S_L with probability $1 - p$, where $p \in [0,1]$.

- The unobservability of the stock can be rationalized as (i) poor technology, or (ii) higher variations of the stock due to weather conditions.

6.2 Symmetrically uninformed firms

➢ Every firm faces the following cost function;

$$C(x_i, X_{-i}) = \frac{x_i(x_i + X_{-i})}{S}$$

➢ where

- $S > 0$ denotes the stock of the resource, which reduces firm i's extraction cost when the resource is more abundant.
- x_i represents firm i's appropriation level.
- $X_{-i} = \sum_{i \neq j} x_j$ are aggregate appropriations by firms other than i.
- Every firm i takes market price p as given (normalize to $p = \$1$).

6.2 Symmetrically uninformed firms

- Every firm *i* chooses appropriation x_i to maximize its expected profit:

$$\max_{x_i \geq 0} \; p \underbrace{\left(x_i - \frac{x_i(x_i + X_{-i})}{S_H} \right)}_{\text{Profit if stock is high}} + (1-p) \underbrace{\left(x_i - \frac{x_i(x_i + X_{-i})}{S_L} \right)}_{\text{Profit if stock is low}}$$

  - Intuitively, firm *i* chooses its appropriation level x_i without being able to condition its choice on the stock of the resource.

- Differentiating with respect to x_i, we obtain

$$p - \frac{p(2x_i + X_{-i})}{S_H} + 1 - p - \frac{(1-p)(2x_i + X_{-i})}{S_L} = 0$$

where subscripts *H* and *L* indicate high and low stock, respectively.

- Rearranging the first-order condition, we have

$$[pS_L + (1-p)S_H](2x_i + X_{-i}) = S_L S_H$$

- Solving for x_i, the best response function becomes

$$x_i(X_{-i}) = \frac{S_L S_H}{2[pS_L + (1-p)S_H]} - \frac{1}{2}X_{-i}$$

which decreases in aggregate appropriation of all its rivals, X_{-i}.

- When $p = 1$, all firms know that the stock is high, and the above best response function collapses to $x_i(X_{-i}) = \frac{S_H}{2} - \frac{1}{2}X_{-i}$.
- When $p = 0$, all firms observes a low stock, and the above best response function simplifies to $x_i(X_{-i}) = \frac{S_L}{2} - \frac{1}{2}X_{-i}$.

- In a symmetric equilibrium, $x_i^* = x_j^* = x^*$ entails $X_{-i}^* = (N-1)x^*$.
- Substituting into the above best response function yields

$$x^* = \frac{S_L S_H}{2[pS_L + (1-p)S_H]} - \frac{1}{2}(N-1)x^*$$

- Solving for x^*, we find the equilibrium appropriation level

$$x^* = \frac{S_L S_H}{(N+1)[pS_L + (1-p)S_H]}$$

  - When $p = 1$, equilibrium appropriation x^* reduces to $x_H = \frac{S_H}{N+1}$.
  - When $p = 0$, equilibrium appropriation x^* becomes $x_L = \frac{S_L}{N+1}$.

- When the resource is low, total appropriation becomes

$$Nx^* = \frac{NS_L S_H}{(N+1)[pS_L + (1-p)S_H]} \leq S_L$$

which we simplify to

$$p(N+1)S_L + [(N+1)(1-p) - N]S_H \geq 0$$

- Since $S_H > S_L$, the above inequality becomes

$$[p(N+1) + (N+1)(1-p) - N]S_L \geq 0$$

that further reduces to $S_L > 0$ which holds by definition.

- When the resource is high, total appropriation becomes

$$Nx^* = \frac{NS_L S_H}{(N+1)[pS_L + (1-p)S_H]} \leq S_H$$

which we simplify to

$$[p(N+1) - N]S_L + (N+1)(1-p)S_H \geq 0$$

- Since $S_H > S_L$, the above inequality becomes

$$[p(N+1) - N + (N+1)(1-p)]S_L \geq 0$$

that further reduces to $S_L > 0$ which holds by definition.

- Comparing equilibrium appropriation under incomplete information, x^*, against its complete information counterpart, x_H, we have

$$x_H - x^* = \frac{S_H}{N+1} - \frac{S_L S_H}{(N+1)[pS_L + (1-p)S_H]}$$

$$= \frac{S_H\left[1 - \frac{S_L}{pS_L + (1-p)S_H}\right]}{N+1}$$

- which is positive if

$$1 - \frac{S_L}{pS_L + (1-p)S_H} > 0$$

- or rearranging, $pS_L + (1-p)S_H > S_L$ that simplifies to $S_H > S_L$.

6.2.1 Comparing equilibrium appropriation in different information contexts – I

- Comparing equilibrium appropriation under incomplete information, x^*, against its complete information counterpart, x_L, we have

$$x_L - x^* = \frac{S_L}{N+1} - \frac{S_L S_H}{(N+1)[pS_L + (1-p)S_H]}$$

$$= \frac{S_L\left[1 - \frac{S_H}{pS_L + (1-p)S_H}\right]}{N+1}$$

- which is negative if

$$1 - \frac{S_H}{pS_L + (1-p)S_H} < 0$$

- or rearranging, $pS_L + (1-p)S_H < S_H$ that simplifies to $S_H > S_L$.

6.2.1 Comparing equilibrium appropriation in different information contexts – I

- From $x_H > x^* > x_L$, every firm exploits the resources more (less) intensively when informed of the stock being high (low) than when facing some uncertainty about the stock's value.

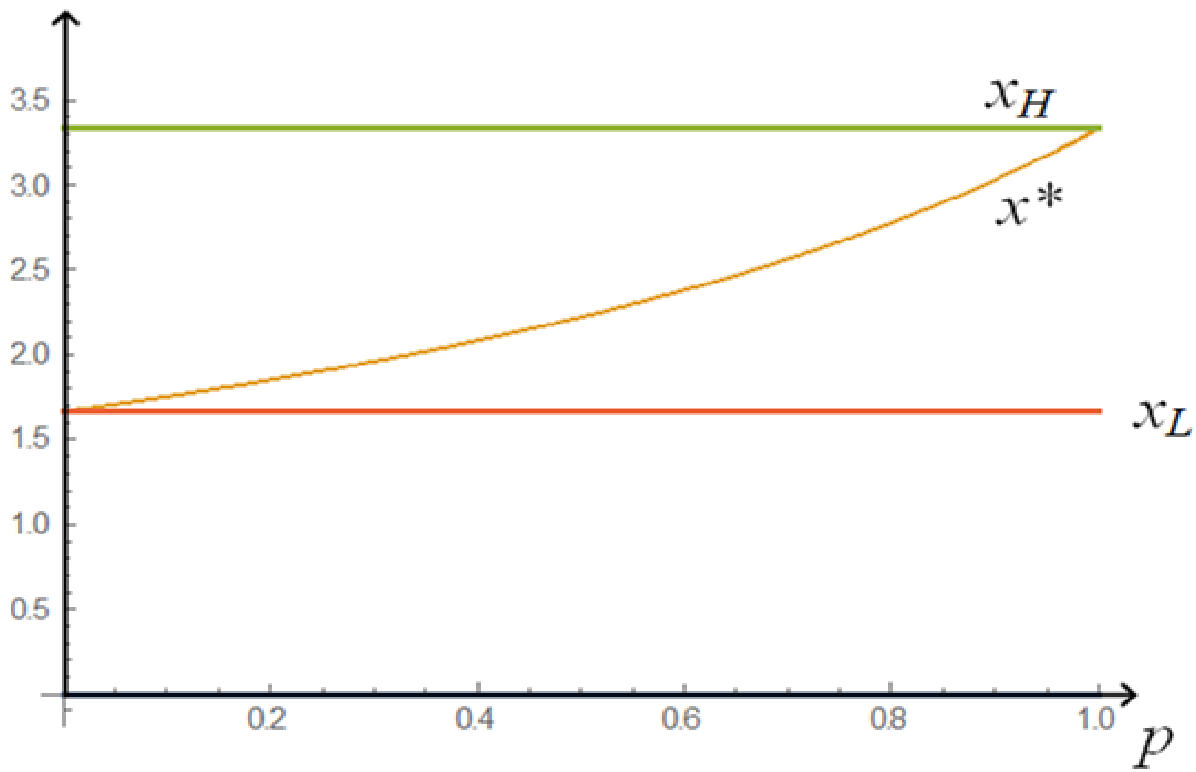


Figure 6.1: Equilibrium appropriation under complete and incomplete information

➢ Figure 6.1

- Equilibrium appropriation under incomplete information x^* lies between complete information when stock is high and low, $x_H > x^* > x_L$, and x^* increases in the probability that the stock is high.
- When $p = 0$, $x^* = x_L$ but when $p = 1$, $x^* = x_H$.
- For illustration purposes, consider $S_H = 10$, $S_L = 5$, and $N = 2$.

$$x^* = \frac{5 \times 10}{(2+1)[5p + 10(1-p)]} = \frac{10}{3(2-p)}$$

$$x_H = \frac{10}{2+1} = 3\frac{1}{3} \approx 3.33$$

$$x_L = \frac{5}{2+1} = 1\frac{2}{3} \approx 1.67$$

6.2.2 Comparative Statics

➢ Differentiating x^* with respect to p, S_L, and S_H

$$\frac{\partial x^*}{\partial p} = \frac{(S_H - S_L)S_L S_H}{(N+1)[pS_L + (1-p)S_H]^2} > 0$$

$$\frac{\partial x^*}{\partial S_H} = \frac{pS_L^2}{(N+1)[pS_L + (1-p)S_H]^2} > 0$$

$$\frac{\partial x^*}{\partial S_L} = \frac{(1-p)S_H^2}{(N+1)[pS_L + (1-p)S_H]^2} > 0$$

- Equilibrium appropriation increases as the stock becomes more abundant, either S_H or S_L, and in probability p that the stock is high.
- Yet, x^* decreases in the number of firms competing for the commons.

Asymmetrically uninformed firms – Only some firms are in the dark

- **Assume two firms**:
  - The privately informed firm can observe the value of the stock. So, this firm conditions its appropriation strategy on the stock.
  - The uninformed firm cannot observe the value of the stock. So, it does not condition its appropriation strategy on the stock.
- **Use BNE to solve the game in three steps**.
  - Derive the best response functions of the privately informed firm when it observes a high stock and when it observes a low stock.
  - Find the best response function of the uninformed firm.
  - Use the three best response functions we found above to solve a system of three equations with three unknowns, x_H, x_L, and x_U.

➢ Privately informed firm – High stock:

- The privately informed firm (firm i) observes a high stock and chooses x_H to solve

$$\max_{x_i \geq 0} \; x_i - \frac{x_i(x_i + x_U)}{S_H}$$

where subscript U denotes "uninformed".

- Differentiating with respect to x_i, we obtain

$$1 - \frac{2x_H + x_U}{S_H} = 0$$

- Rearranging, its best response function observing a high stock is

$$x_H(x_U) = \frac{S_H}{2} - \frac{1}{2}x_U \qquad (BRF_H)$$

➢ Privately informed firm – Low stock:

- The privately informed firm (firm i) observes a low stock and chooses x_H to solve

$$\max_{x_i \geq 0} \; x_i - \frac{x_i(x_i + x_U)}{S_L}$$

- Differentiating with respect to x_i, we have

$$1 - \frac{2x_L + x_U}{S_L} = 0$$

- Rearranging, its best response function observing a low stock is

$$x_L(x_U) = \frac{S_L}{2} - \frac{1}{2}x_U \qquad (BRF_L)$$

➢ Uninformed Firm:

- The uninformed firm (firm j) chooses x_j to solve

$$\max_{x_j \geq 0} \; p \underbrace{\left(x_j - \frac{x_j(x_j + x_H)}{S_H} \right)}_{\text{Profit if the stock is high}} + (1-p) \underbrace{\left(x_j - \frac{x_j(x_j + x_L)}{S_L} \right)}_{\text{Profit if the stock is low}}$$

- In Section 6.2, no firm could condition its appropriation decisions on the level of the stock (i.e., all firms were operating in the dark).

- However, the uninformed firm in this context anticipates that the informed firm will choose a different appropriation level when the stock is high than when the stock is low.

Uninformed Firm:

- Differentiating with respect to x_j, we have

$$1-\frac{p\left(2x_j+x_H\right)}{S_H}-\frac{(1-p)\left(2x_j+x_L\right)}{S_L}=0$$

- Solving for x_j, the uninformed firm *j*'s best response function is

$$x_U(x_H,x_L)=\frac{S_H[S_L-(1-p)x_L]-px_HS_L}{2[pS_L+(1-p)S_H]} \qquad (BRF_U)$$

- This can alternatively be expressed as

$$x_U(x_H,x_L)=\underbrace{\frac{S_HS_L}{2[pS_L+(1-p)S_H]}}_{\text{Vertical intercept}}-\underbrace{\frac{pS_L}{2[pS_L+(1-p)S_H]}}_{\text{Slope for } x_H}x_H-\underbrace{\frac{(1-p)S_H}{2[pS_L+(1-p)S_H]}}_{\text{Slope for } x_L}x_L$$

➢ Combining All Best Responses:

- Substituting $x_H(x_U)$ and $x_L(x_U)$ into the best response function of the uninformed firm, $x_U(x_H, x_L)$, yields

$$x_U = \frac{S_H[2S_L - (1-p)(S_L - x_U)] - p(S_H - x_U)S_L}{4[pS_L + (1-p)S_H]}$$

which simplifies to

$$x_U = \frac{S_H S_L}{4[pS_L + (1-p)S_H]} + \frac{1}{4}x_U$$

- Solving for x_U, equilibrium appropriation of the uninformed firm is

$$x_U^* = \frac{S_H S_L}{3[pS_L + (1-p)S_H]}$$

➢ Combining All Best Responses:

- Substituting x_U^* into $x_H(x_U)$, equilibrium appropriation of the informed firm when it observes a high stock is

$$x_H^* = \frac{S_H}{2} - \frac{S_H S_L}{6[pS_L + (1-p)S_H]}$$

$$= \frac{S_H}{2}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

➢ Combining All Best Responses:

- Substituting x_U^* into $x_L(x_U)$, equilibrium appropriation of the informed firm when it observes a low stock is

$$x_L^* = \frac{S_L}{2} - \frac{S_H S_L}{6[pS_L + (1-p)S_H]}$$

$$= \frac{S_L}{2}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)$$

➢ Combining All Best Responses:

- Considering the BNE triplet at the same parameter values as in Figure 6.1, $S_H = 10$ and $S_L = 5$, equilibrium appropriations are

$$x_U^* = \frac{10 \times 5}{3[5p + 10(1-p)]} = \frac{10}{3(2-p)}$$

$$x_H^* = \frac{10}{2}\left(1 - \frac{5}{3[5p + 10(1-p)]}\right) = \frac{5(5-3p)}{3(2-p)}$$

$$x_L^* = \frac{5}{2}\left(1 - \frac{10}{3[5p + 10(1-p)]}\right) = \frac{5(4-3p)}{6(2-p)}$$

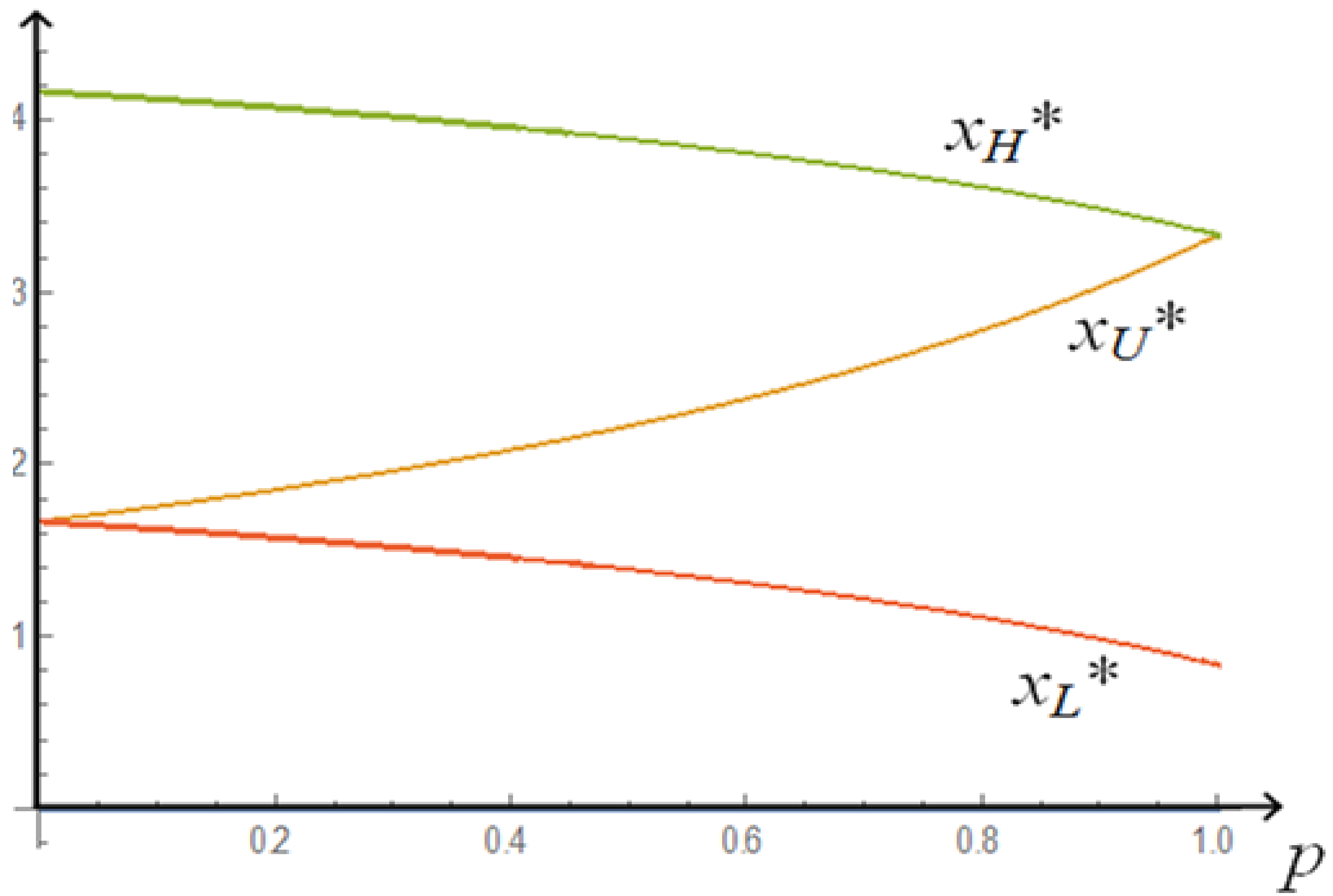


Figure 6.2: Equilibrium appropriation

- When the resource is low, total appropriation becomes

$$x_L^* + x_U^* = \frac{S_L}{2}\left(1 + \frac{S_H}{3[pS_L + (1-p)S_H]}\right) \leq S_L$$

- that can be supported by the resource S_L because it reduces to

$$1 + \frac{S_H}{3[pS_L + (1-p)S_H]} \leq 2$$

which further simplifies to

$$3pS_L + (2-3p)S_H \geq 0$$

- Since $S_H > S_L$, the above inequality becomes

$$(3p + 2 - 3p)S_L \geq 0$$

that further reduces to $S_L > 0$ which holds by definition.

- When the resource is high, total appropriation becomes

$$x_H^* + x_U^* = \frac{S_H}{2}\left(1 + \frac{S_L}{3[pS_L + (1-p)S_H]}\right) \leq S_H$$

- that can be supported by the resource S_H because it reduces to

$$1 + \frac{S_L}{3[pS_L + (1-p)S_H]} \leq 2$$

which further simplifies to

$$(3p-1)S_L + 3(1-p)S_H \geq 0$$

- Since $S_H > S_L$, the above inequality becomes

$$(3p - 1 + 3 - 3p)S_L \geq 0$$

that further reduces to $S_L > 0$ which holds by definition.

- Figure 6.2 depicts that the uninformed firm appropriates
  - same as the privately informed low-stock firm when $p = 0$
  - more intensively as p increases
  - same as the privately informed high-stock firm when $p = 1$
- When the uninformed firm assigns a higher probability p to facing a high-stock CPR, the privately informed firm anticipates that its uninformed rival will appropriate more intensively, so that it decreases its exploitation both when facing a high and a low stock.

➢ Uninformed Firm

- Comparing equilibrium appropriation between the uninformed and the informed firm under complete information:

$$x_H - x_U^* = \frac{S_H}{3} - \frac{S_H S_L}{3[pS_L + (1-p)S_H]} = \frac{(1-p)S_H(S_H - S_L)}{3[pS_L + (1-p)S_H]} > 0$$

$$x_L - x_U^* = \frac{S_L}{3} - \frac{S_H S_L}{3[pS_L + (1-p)S_H]} = -\frac{pS_L(S_H - S_L)}{3[pS_L + (1-p)S_H]} < 0$$

- Therefore, $x_H > x_U^* > x_L$, entailing that the uninformed firm exploits the resource more (less) intensively than when it knew that the stock is low (high) with certainty.

6.3.1 Comparing equilibrium appropriation in different information contexts – II

➢ Informed Firm – High Stock

- Comparing equilibrium appropriation of the informed firm under complete and incomplete information observing a high stock:

$$x_H - x_H^* = \frac{S_H}{3} - \frac{S_H}{2}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right) = -\frac{(1-p)S_H(S_H - S_L)}{6[pS_L + (1-p)S_H]} < 0$$

- The informed firm observing a high stock exploits the resource more intensively when it knows that its rival is uninformed about the stock value than when its rival is informed about it, $x_H^* > x_H$.

- Since appropriation decisions are *strategic substitutes* because they enter negatively in all three best response functions, when the uninformed firm exploits less, the informed firm does more.

➢ Informed Firm – Low Stock

- Comparing equilibrium appropriation of the informed firm under complete and incomplete information observing a low stock:

$$x_L - x_L^* = \frac{S_L}{3} - \frac{S_L}{2}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right) = \frac{pS_L(S_H - S_L)}{6[pS_L + (1-p)S_H]} > 0$$

- The informed firm observing a low stock exploits the resource less intensively when it knows that its rival is uninformed about the stock value than when its rival is informed about it, $x_L^* < x_L$.
- Since the uninformed firm appropriates more under uncertainty, the informed firm, in response, exploits less of the resource.

➢ More or less intense overall exploitation?

- When the stock is high:

$$-\underbrace{(x_H - x_H^*)}_{-} - \underbrace{(x_H - x_U^*)}_{+}$$

$$= \frac{(1-p)S_H(S_H - S_L)}{6[pS_L + (1-p)S_H]} - \frac{(1-p)S_H(S_H - S_L)}{3[pS_L + (1-p)S_H]}$$

$$= -\frac{(1-p)S_H(S_H - S_L)}{6[pS_L + (1-p)S_H]} < 0$$

- The decrease in the uninformed firm's appropriation, $x_H - x_U^*$, offsets the increase in the informed firm's appropriation, $x_H - x_H^*$, ultimately decreasing overall exploitation of the resource.

➢ More or less intense overall exploitation?

- When the stock is low:

$$-\underbrace{(x_L - x_L^*)}_{+} - \underbrace{(x_L - x_U^*)}_{-}$$

$$= -\frac{pS_L(S_H - S_L)}{6[pS_L + (1-p)S_H]} + \frac{pS_L(S_H - S_L)}{3[pS_L + (1-p)S_H]}$$

$$= \frac{pS_L(S_H - S_L)}{6[pS_L + (1-p)S_H]} > 0$$

- The increase in the uninformed firm's appropriation, $x_L - x_U^*$, offsets the decrease in the informed firm's appropriation, $x_L - x_L^*$, ultimately increasing overall exploitation of the resource.

6.3.1 Comparing equilibrium appropriation in different information contexts – II

- In summary,
  - When only one firm is informed about the stock of the resource, and the stock is actually abundant (scarce), overall appropriation falls below (lies above) than that under complete information.
  - Paradoxically, conservation is more intense when the CPR is abundant, but exploitation becomes particularly severe when the resource is scarce.

6.3.2 Efficiency Properties

- Under complete information:
  - Firms overexploit the CPR as they interact for only one period.
- Under incomplete information:
  - When the resource is abundant, overall exploitation is lower under incomplete information than under complete information. Hence, incomplete information ameliorates static inefficiencies.
  - When the resource is scarce, overall exploitation is higher under incomplete information than under complete information. Thus, incomplete information makes static inefficiencies more severe.

6.3.2 Efficiency Properties

- When the stock is low, static inefficiency is

$$SI_L = (x_L^* + x_U^*) - 2x_L^{SO}$$

$$= \frac{S_L}{2}\left(1 + \frac{S_H}{3[pS_L + (1-p)S_H]}\right) - 2\left(\frac{S_L}{4}\right)$$

$$= \frac{S_L S_H}{6[pS_L + (1-p)S_H]}$$

6.3.2 Efficiency Properties

- When the stock is high, static inefficiency is

$$SI_H = (x_H^* + x_U^*) - 2x_H^{SO}$$

$$= \frac{S_H}{2}\left(1 + \frac{S_L}{3[pS_L + (1-p)S_H]}\right) - 2\left(\frac{S_H}{4}\right)$$

$$= \frac{S_L S_H}{6[pS_L + (1-p)S_H]}$$

6.3.2 Efficiency Properties

- Interestingly, static inefficiencies coincide across both settings.

  - This happens because static inefficiencies are ameliorated when the stock is abundant but aggravated when it is scarce, so that $SI = SI_H = SI_L$, which are increasing in p, S_H, and S_L owing to

$$\frac{\partial SI}{\partial p} = \frac{(S_H - S_L)S_L S_H}{6[pS_L + (1-p)S_H]^2} > 0$$

$$\frac{\partial SI}{\partial S_H} = \frac{pS_L^2}{6[pS_L + (1-p)S_H]^2} > 0$$

$$\frac{\partial SI}{\partial S_L} = \frac{(1-p)S_H^2}{6[pS_L + (1-p)S_H]^2} > 0$$

- Let us consider the case of asymmetrically uninformed firms.
- The profits of the uninformed firm when the stock is low are

$$\pi_U^L = x_U^* - \frac{x_U^*(x_U^* + x_L^*)}{S_L}$$

$$= \frac{x_U^*}{S_L}\left[S_L - \frac{S_L}{2}\left(1 + \frac{S_H}{3[pS_L + (1-p)S_H]}\right)\right]$$

$$= \frac{x_U^*}{2}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)$$

6.3.2 Efficiency Properties

- The profits of the informed firm when the stock is low are

$$\pi_L^* = x_L^* - \frac{x_L^*(x_L^* + x_U^*)}{S_L}$$

$$= \frac{x_L^*}{S_L}\left[S_L - \frac{S_L}{2}\left(1 + \frac{S_H}{3[pS_L + (1-p)S_H]}\right)\right]$$

$$= \frac{x_L^*}{2}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)$$

6.3.2 Efficiency Properties

- Since $x_U^* > x_L^*$, it is obvious that $\pi_U^L > \pi_L^*$, so that the uninformed firm can take advantage of information asymmetry to produce more output and make more profits than the informed firm.
- Summing up the profits, social welfare in this context becomes

$$W_L^* = \frac{x_L^* + x_U^*}{2}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)$$

$$= \frac{S_L}{4}\left(1 + \frac{S_H}{3[pS_L + (1-p)S_H]}\right)\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)$$

$$= \frac{S_L}{4}\left(1 - \frac{S_H^2}{9[pS_L + (1-p)S_H]^2}\right)$$

6.3.2 Efficiency Properties

- The profits of the uninformed firm when the stock is high are

$$\pi_U^H = x_U^* - \frac{x_U^*(x_U^* + x_H^*)}{S_H}$$

$$= \frac{x_U^*}{S_H}\left[S_H - \frac{S_H}{2}\left(1 + \frac{S_L}{3[pS_L + (1-p)S_H]}\right)\right]$$

$$= \frac{x_U^*}{2}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

6.3.2 Efficiency Properties

- The profits of the informed firm when the stock is high are

$$\pi_H^* = x_H^* - \frac{x_H^*(x_H^* + x_U^*)}{S_H}$$

$$= \frac{x_H^*}{S_H}\left[S_H - \frac{S_H}{2}\left(1 + \frac{S_L}{3[pS_L + (1-p)S_H]}\right)\right]$$

$$= \frac{x_H^*}{2}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

6.3.2 Efficiency Properties

- Since $x_H^* > x_U^*$, it is obvious that $\pi_H^* > \pi_U^H$, so that the informed firm can take advantage of information asymmetry to produce more output and make more profits than the uninformed firm.
- Summing up the profits, social welfare in this context becomes

$$W_H^* = \frac{x_H^* + x_U^*}{2}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

$$= \frac{S_H}{4}\left(1 + \frac{S_L}{3[pS_L + (1-p)S_H]}\right)\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

$$= \frac{S_H}{4}\left(1 - \frac{S_L^2}{9[pS_L + (1-p)S_H]^2}\right)$$

6.3.2 Efficiency Properties

- Social welfare is higher under high stock than low stock because

$$W_H^* = \frac{S_H}{4}\left(1 - \frac{S_L^2}{9[pS_L + (1-p)S_H]^2}\right) > \frac{S_L}{4}\left(1 - \frac{S_H^2}{9[pS_L + (1-p)S_H]^2}\right) = W_L^*$$

reduces to

$$(S_H - S_L)\left(1 + \frac{S_H S_L}{9[pS_L + (1-p)S_H]^2}\right) > 0$$

which unambiguously holds, and such difference widens in p where

$$\frac{\partial(W_H^* - W_L^*)}{\partial p} = \frac{2S_H S_L (S_H - S_L)^2}{9[pS_L + (1-p)S_H]^3} > 0$$

so the expected welfare gain becomes more substantial when the stock is more likely to be high.

6.3.2 Efficiency Properties

- Let us consider the case of symmetrically uninformed firms.
  - Plugging x^* into the profit function, yields

$$\pi_i^k = x^* - \frac{x^*(Nx^*)}{S_k} = \frac{x^*(S_k - Nx^*)}{S_k}$$

  - When the stock is low, profits of every firm i become

$$\pi_i^L = \frac{S_L S_H}{(N+1)[pS_L + (1-p)S_H]}\left(1 - \frac{NS_H}{(N+1)[pS_L + (1-p)S_H]}\right)$$

  - When the stock is high, profits of every firm i become

$$\pi_i^H = \frac{S_L S_H}{(N+1)[pS_L + (1-p)S_H]}\left(1 - \frac{NS_L}{(N+1)[pS_L + (1-p)S_H]}\right)$$

6.3.2 Efficiency Properties

- When 2 firms operate in a low stock, every uninformed firm earns

$$\pi_i^L = \frac{S_L S_H}{3[pS_L + (1-p)S_H]}\left(1 - \frac{2S_H}{3[pS_L + (1-p)S_H]}\right)$$

that fall below the profits of the informed firm under low stock since

$$\pi_L^* - \pi_i^L = \frac{S_L}{4}\left(1 - \frac{S_H}{3[pS_L + (1-p)S_H]}\right)^2 - \pi_i^L$$

reduces to

$$\frac{p^2 S_L (S_H - S_L)^2}{4[pS_L + (1-p)S_H]^2} > 0$$

which holds, so information benefits the informed low stock firm.

6.3.2 Efficiency Properties

- When the stock is low, profits are ranked as $\pi_U^L > \pi_L^* > \pi_i^L$.

  - When every firm is uninformed, it extracts most aggressively, with profits below that when the firm is informed of the stock.
  - While information opaqueness benefits the uninformed firm when it knows that the other firm is informed of the stock abundance, it hurts the uninformed firm when the other firm is also uninformed.

6.3.2 Efficiency Properties

- When 2 firms operate in a high stock, every uninformed firm earns

$$\pi_i^H = \frac{S_L S_H}{3[pS_L + (1-p)S_H]}\left(1 - \frac{2S_L}{3[pS_L + (1-p)S_H]}\right)$$

that fall below the profits of the informed firm under high stock since

$$\pi_H^* - \pi_i^H = \frac{S_H}{4}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)^2 - \pi_i^H$$

reduces to

$$\frac{(1-p)^2 S_H (S_H - S_L)^2}{4[pS_L + (1-p)S_H]^2} > 0$$

which holds, so information benefits the informed high stock firm.

6.3.2 Efficiency Properties

- However, they exceed the profits of the uninformed firm because

$$\pi_i^H - \pi_U^H = \pi_i^H - \frac{S_H S_L}{6[pS_L + (1-p)S_H]}\left(1 - \frac{S_L}{3[pS_L + (1-p)S_H]}\right)$$

reduces to

$$\frac{(1-p)S_L S_H (S_H - S_L)}{6[pS_L + (1-p)S_H]^2} > 0$$

which holds, so information hurts the uninformed high stock firm.

6.3.2 Efficiency Properties

➢ When the stock is high, profits are ranked as $\pi_H^* > \pi_i^H > \pi_U^H$.

- When every firm is uninformed, it extracts more aggressively than the firm which is the only firm uninformed of the stock, but less than the firm which is the only firm informed of the stock.
- This happens because when every firm faces the opaque stock, it does not take advantage of information asymmetry to appropriate more intensively if informed or conservatively if uninformed.

6.3.2 Efficiency Properties

- Summing up the profits, social welfare of the uninformed firms operating in a low stock environment becomes

$$W_U^L = \frac{2S_L S_H}{3[pS_L + (1-p)S_H]}\left(1 - \frac{2S_H}{3[pS_L + (1-p)S_H]}\right)$$

that falls below the social welfare when one firm is informed since

$$W_U^L \leq \frac{S_L}{4}\left(1 - \frac{S_H^2}{9[pS_L + (1-p)S_H]^2}\right) = W_L^*$$

reduces to

$$3\left(\frac{pS_L + (1-p)S_H}{S_H}\right)^2 - 8\left(\frac{pS_L + (1-p)S_H}{S_H}\right) + 5 \geq 0$$

6.3.2 Efficiency Properties

- Applying the quadratic formula, we find

$$\frac{pS_L + (1-p)S_H}{S_H} \leq 1 \qquad \text{or} \qquad \frac{pS_L + (1-p)S_H}{S_H} \geq \frac{5}{3}$$

- Since $pS_L + (1-p)S_H \leq S_H$, we reject $\frac{pS_L+(1-p)S_H}{S_H} \geq \frac{5}{3}$.
- Hence, $W_L^* \geq W_U^L$ holds, and information asymmetry improves social welfare. Therefore, the regulator may release the stock level to either firm rather than concealing it to both firms.
- Intuitively, when both informed and uninformed firms reduce extraction once information is concealed, every firm generates lower profits that ultimately lowers social welfare.

6.3.2 Efficiency Properties

- Summing up the profits, social welfare of the uninformed firms operating in a high stock environment becomes

$$W_U^H = \frac{2S_L S_H}{3[pS_L + (1-p)S_H]}\left(1 - \frac{2S_L}{3[pS_L + (1-p)S_H]}\right)$$

that exceeds the social welfare when one firm is informed since

$$W_U^H \geq \frac{S_H}{4}\left(1 - \frac{S_L^2}{9[pS_L + (1-p)S_H]^2}\right) = W_H^*$$

reduces to

$$3\left(\frac{pS_L + (1-p)S_H}{S_L}\right)^2 - 8\left(\frac{pS_L + (1-p)S_H}{S_L}\right) + 5 \leq 0$$

6.3.2 Efficiency Properties

➢ Applying the quadratic formula, we find

$$1 \leq \frac{pS_L + (1-p)S_H}{S_L} \leq \frac{5}{3}$$

- Since $S_L \leq pS_L + (1-p)S_H$, the above condition holds if $\frac{S_L}{S_H} \geq \frac{3(1-p)}{5-3p} \equiv \hat{p}$, which cutoff decreases in p since $\frac{\partial \hat{p}}{\partial p} = -\frac{6}{(5-3p)^2} < 0$.
- Hence, $W_U^H \geq W_H^*$ holds if the resource disparity is not too extreme, for which the regulator better conceals the resource.
- However, when the resource is more likely to be scarce (p decreases), then cutoff $\hat{p}$ increases making condition $S_L/S_H \geq \hat{p}$ more difficult to hold, so when $W_H^* > W_U^H$, the regulator should inform either firm of stock abundance when it turns out to be high.

Common Pool Resources:

Strategic Behavior, Inefficiencies, and Incomplete Information

Chapter 7: Signaling in the commons

Outline

- Modeling signals in the commons
  - Prior and posterior beliefs
- Separating equilibrium
  - Separating effort
- Pooling Equilibrium
  - Pooling effort
- Welfare implications

Modeling signals in the commons

7.2 Modeling signals in the commons

- Consider an incomplete information setting where the incumbent firm has access to better information about the stock than the potential entrant.
- This can be rationalized on this firm's longer experience exploiting the CPR, or asymmetric technologies between the incumbent and the entrant.
- The time structure of the sequential-move game is depicted in Figure 7.1.

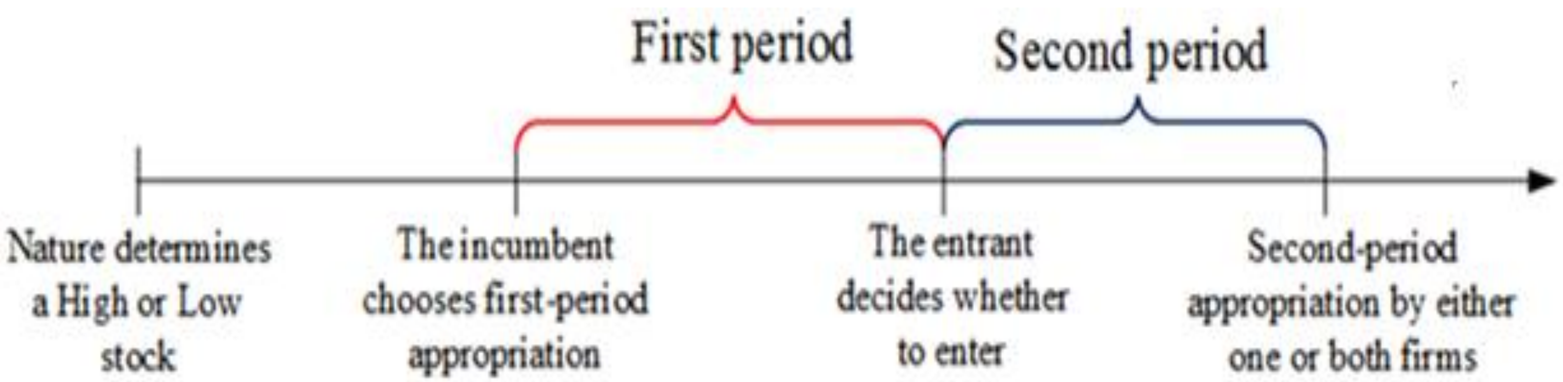


Figure 7.1. Time structure of the signaling game

7.2 Modeling signals in the commons

The time structure is as follows.

1. Stock realization: Nature determines the realization of the stock, which is either high, S_H with probability p, or low, S_L with probability $1-p$, where $S_H > S_L$ and $p \in (0,1)$. The incumbent privately observes this realization while the entrant is only informed of the probability distribution of this stock.

2. First-period appropriation: In the first period, the incumbent chooses an appropriation level x.

7.2 Modeling signals in the commons

3. Belief updating: Observing the incumbent's appropriation x, the entrant forms beliefs about the initial stock S. Let $\mu(S|x)$ denotes the entrant's posterior belief about S being high after observing x.

4. Entry Decision: Given the above beliefs, the entrant decides whether to enter the CPR or not.
   - If entry does not occur, the incumbent remains the only firm exploiting the resource in the second period.
   - If entry ensues, both the incumbent and the entrant compete for the CPR.

7.2 Modeling signals in the commons

- Let firms face a given market price $P = \$1$ and the same cost functions as in Chapter 3, where $C(x_i, X_{-i}) = \frac{x_i(x_i+X_{-i})}{S}$ in the first period and $C(q_i, Q_{-i}) = \frac{q_i(q_i+Q_{-i})}{S-(1-r)X}$ in the second period, letting us measure how the incumbent's behavior changes when this firm deals with a potential entrant that does not know the exact amount of the available stock.

- For simplicity, we assume that if the entrant was perfectly informed about the stock, it would (not) enter when the stock is high (low).

- This allows for information about the stock to play an entry-deterring role, which the incumbent may exploit to prevent entry.

7.2 Modeling signals in the commons

- Players then interact in a sequential-move game of incomplete information, where the incumbent uses first-period appropriation to convey or conceal information about the available stock form the potential entrant, thus inducing the latter to enter into the CPR or not.

- Using the Perfect Bayesian Equilibrium (PBE) solution concept:
  - Every firm finds its appropriation profit-maximizing, given the point of the game at which the firm is called upon to move, and given the information that the firm observes at that point; and
  - The potential entrant updates its beliefs about the stock using Bayes' rule, whenever possible.

7.2.1 Prior and posterior beliefs

- The potential entrant, uninformed of the stock level, uses the incumbent's actions (first-period appropriation x) to increase or decrease the probability of dealing with a high-stock CPR.

- For instance, if the incumbent chooses a stock-dependent first-period appropriation (e.g., $x_L = 2$ tons when the stock is low but $x_H = 10$ tons when the stock is high), then the entrant can perfectly infer the stock's level by just observing the incumbent's first-period appropriation.

7.2.1 Prior and posterior beliefs

- **Separating Equilibria**, also known as *informative equilibria*, highlights the idea that each type of incumbent separates from each other by choosing a different first-period appropriation.

- Bayes' rule helps us update the potential entrant's posterior belief of facing a high stock S_H from prior probability p to $\mu(S_H|x)$:

$$\mu(S_H|x) = \frac{p \times \alpha_H}{p \times \alpha_H + [(1-p) \times \alpha_L]}$$

- where α_k denotes the probability that the incumbent facing a k-type stock chooses a first-period appropriation of exactly x units, and $k = \{H, L\}$.

7.2.1 Prior and posterior beliefs

- When only the high-stock incumbent chooses such a first-period appropriation, these probabilities become $\alpha_H = 1$ and $\alpha_L = 0$.

- In this context, the potential entrant's posterior belief becomes

$$\mu(S_H|x) = \frac{p \times 1}{p \times 1 + [(1-p) \times 0]} = \frac{p}{p} = 1$$

  - If only the high-stock incumbent chooses this level of x, then the entrant can assign full probability, $\mu(S_H|x) = 1$, on facing this type of incumbent.

  - The incumbent, by choosing a different first-period appropriation depending on the stock, signals information about this stock to the potential entrant.

7.2.1 Prior and posterior beliefs

- **Pooling Equilibria**, also known as *uninformative equilibria*, emphasizes the feature that both types of incumbent pool by choosing the same first-period appropriation.

- For instance, the incumbent selects a stock-independent first-period appropriation (e.g., $x = 5$ both when the stock is high and low), then the entrant cannot infer the available stock after observing the incumbent's first-period appropriation.

- In this context, Bayes' rule does not help the entrant change its posterior belief about the stock being high, p.

7.2.1 Prior and posterior beliefs

- When both types of incumbent choose x, $\alpha_H = \alpha_L = 1$, yielding

$$\mu(S_H|x) = \frac{p \times 1}{(p \times 1) + [(1-p) \times 1]} = \frac{p}{1} = p$$

  - In words, upon observing the incumbent's first-period appropriation x, the entrant cannot refine its beliefs about the incumbent's type since x does not reveal any information about the available stock.

  - Alternatively, this is to say, by choosing the same first-period appropriation, the incumbent conceals the state of the stock from the potential entrant.

7.2.1 Prior and posterior beliefs

➢ If a first-period appropriation level x is not chosen by either type of incumbent, then $\alpha_H = \alpha_L = 0$, implying that **beliefs are unrestricted off-the-equilibrium path**, where

$$\mu(S_H|x) = \frac{p \times 0}{(p \times 0) + [(1-p) \times 0]} = \frac{0}{0}$$

- Since Bayes' rule does not apply to off-the-equilibrium appropriation levels, for generality, we allow for any probability $\mu(S_H|x) \in [0,1]$.

- However, for presentation purposes, we assume here that $\mu(S_H|x) = 1$, making the CPR more attractive for the entrant, and thereby reducing the incumbent's incentives to deviate toward x.

Separating Equilibrium

7.3 Separating Equilibrium

- The incumbent chooses a first-period appropriation x_H when the stock is high, S_H, but x_L when it is low, S_L, where $x_H > x_L$.

- The entrant's beliefs are $\mu(S_H|x_H) = 1$ after observing x_H, $\mu(S_H|x_L) = 0$ after observing x_L, and $\mu(S_H|x) = 1$ after observing any off-the-equilibrium appropriation $x \neq x_H, x_L$.

- Observing x_L, the entrant stays out of the CPR, but enters otherwise. Anticipating this response of the entrant, the incumbent chooses x in the first stage of the game.

7.3 Separating Equilibrium

➢ High-stock incumbent

- In Chapter 4, the high stock incumbent chooses the same first-period appropriation as in a complete information setting with subsequent entry, denoted as $x^* = \frac{S_H[9-\delta(1-r)]}{18}$. For clarity, we relabel x^* as x_H.

- Choosing first-period appropriation x_H, its overall profits are

$$\Pi_H^{AE} = \left[x_H - \frac{x_H^2}{S_H}\right] + \delta\left[\frac{S_H - (1-r)x_H}{9}\right]$$

where the superscript *AE* denotes "allow entry".

- By choosing x_H, the high-stock incumbent maximizes its discounted stream of profits (good news!), but does not deter entry (bad news!).

7.3 Separating Equilibrium

- If, instead, the high-stock incumbent chooses the first-period appropriation of the low-stock incumbent, x_L, it induces the entrant to believe that the stock is low, yielding overall profits of

$$\Pi_H^{ED} = \left[x_L - \frac{x_L^2}{S_H}\right] + \delta\left[\frac{S_H - (1-r)x_L}{4}\right]$$

where the superscript *ED* denotes "entry deterrence".

- In words, the incumbent chooses a suboptimal first-period appropriation but deters entry in the second period of the game.

7.3 Separating Equilibrium

- The high-stock incumbent chooses x_H rather than mimicking the low-stock incumbent's appropriation x_L if overall profits from allowing entry exceed those of deterring entry, $\Pi_H^{AE} \geq \Pi_H^{ED}$.

$$\left[x_H - \frac{x_H^2}{S_H}\right] - \left[x_L - \frac{x_L^2}{S_H}\right] \geq \delta\left[\frac{S_H - (1-r)x_L}{4} - \frac{S_H - (1-r)x_H}{9}\right]$$

which is often known as an "incentive compatibility" condition of the high-stock incumbent, or IC_H.

- Intuitively, the first-period profit gain from choosing x_H yields a larger profit than choosing x_L, which exceeds the second-period loss from attracting entry that occurs when the incumbent chooses x_H but not x_L.

- That is, x_L must be unprofitable for the high-stock incumbent to mimic.

7.3 Separating Equilibrium

➢ Low-stock incumbent

- If the low-stock incumbent chooses x_L, as prescribed by this separating equilibrium, it deters entry, yielding overall profits of

$$\Pi_L^{ED} = \left[x_L - \frac{x_L^2}{S_L}\right] + \delta\left[\frac{S_L - (1-r)x_L}{4}\right]$$

- If, instead, this firm deviates toward any other appropriation level, it attracts entry. Conditional on entry, however, x_H does not yield the highest profits, but choosing x to solve

$$\max_{x \geq 0}\left[x - \frac{x^2}{S_L}\right] + \delta\left[\frac{S_L - (1-r)x}{9}\right]$$

7.3 Separating Equilibrium

- Solving, $x_{L,E} = \frac{S_L[9-\delta(1-r)]}{18}$, where the subscript L,E denotes entry allowance for the low-stock incumbent.

- Attracting entry, the low-stock incumbent's overall profits are

$$\Pi_L^{AE} = \left[x_{L,E} - \frac{x_{L,E}^2}{S_L}\right] + \delta\left[\frac{S_L - (1-r)x_{L,E}}{9}\right]$$

7.3 Separating Equilibrium

- The low stock incumbent chooses x_L to deter entry rather than $x_{L,E}$ to attract entry if and only if $\Pi_L^{ED} \geq \Pi_L^{AE}$, that is,

$$\left[x_{L,E} - \frac{x_{L,E}^2}{S_L}\right] - \left[x_L - \frac{x_L^2}{S_L}\right] \leq \delta\left[\frac{S_L - (1-r)x_L}{4} - \frac{S_L - (1-r)x_{L,E}}{9}\right]$$

which is known as a "incentive compatibility" condition of the low-stock incumbent, or IC_L.

- Intuitively, the first-period profit gain of choosing $x_{L,E}$ rather than x_L does not compensate the second-period profit loss that this incumbent experiences when attracting entry.

7.3 Separating Equilibrium

- Substituting $x_H = \frac{S_H[9-\delta(1-r)]}{18}$ into IC_H, we find

$$\left[\frac{S_H[9-\delta(1-r)]}{18} - \frac{S_H[9-\delta(1-r)]^2}{324}\right] - \left[x_L - \frac{x_L^2}{S_H}\right]$$

$$\geq \delta\left[\frac{S_H-(1-r)x_L}{4} - \frac{S_H[9(1+r)+\delta(1-r)^2]}{162}\right]$$

- which we rearrange to yield

$$324x_L^2 - 81[4-\delta(1-r)]S_H x_L + \{[9-\delta(1-r)]^2 - 45\delta\}S_H^2 \geq 0$$

7.3 Separating Equilibrium

- Similarly, substituting $x_{L,E} = \frac{S_L[9-\delta(1-r)]}{18}$ into IC_L, we find

$$\left[\frac{S_L[9-\delta(1-r)]}{18} - \frac{S_L[9-\delta(1-r)]^2}{324}\right] - \left[x_L - \frac{x_L^2}{S_L}\right]$$

$$\leq \delta\left[\frac{S_L-(1-r)x_L}{4} - \frac{S_L[9(1+r)+\delta(1-r)^2]}{162}\right]$$

- which we rearrange to yield

$$324x_L^2 - 81[4-\delta(1-r)]S_L x_L + \{[9-\delta(1-r)]^2 - 45\delta\}S_L^2 \leq 0$$

7.3 Separating Equilibrium

- Solving for x_L in IC_H

$$x_L \leq \underline{x}_H \equiv \frac{S_H\left[9\big(4-\delta(1-r)\big)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

- or

$$x_L \geq \overline{x}_H \equiv \frac{S_H\left[9\big(4-\delta(1-r)\big)+\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

7.3 Separating Equilibrium

- Solving for x_L in IC_L

$$x_L \geq \underline{x}_L \equiv \frac{S_L\left[9\big(4-\delta(1-r)\big)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

- and

$$x_L \leq \overline{x}_L \equiv \frac{S_L\left[9\big(4-\delta(1-r)\big)+\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

7.3 Separating Equilibrium

- Since $S_L < S_H$, $\bar{x}_L < \bar{x}_H$ implies that $x_L \geq \bar{x}_H$ is slack.

- In addition, $\underline{x}_H \leq x_{L,NE}$ when

$$\frac{S_H\left[9\big(4-\delta(1-r)\big)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72} \leq \frac{S_L[4-\delta(1-r)]}{8}$$

  - which we rearrange to

$$\frac{S_L}{S_H} \geq \bar{S} \equiv \frac{9[4-\delta(1-r)]-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)]}$$

- This means when $\frac{S_L}{S_H} \geq \bar{S}$, appropriation level $\underline{x}_H$ is binding but when $\frac{S_L}{S_H} < \bar{S}$, the low stock incumbent chooses $x_{L,NE}$ rendering $\underline{x}_H$ slack.

7.3 Separating Equilibrium

- Furthermore, $\underline{x}_H < \overline{x}_L$ when

$$\frac{S_H\left[9\big(4-\delta(1-r)\big)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72} < \frac{S_L\left[9\big(4-\delta(1-r)\big)+\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

  - which we rearrange to

$$\frac{S_L}{S_H} > \hat{S} \equiv \frac{9[4-\delta(1-r)]-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)]+\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}$$

7.3 Separating Equilibrium

- It is straightforward to show that cutoffs satisfy $\overline{S} > \hat{S}$ because

$$\frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)]}$$

$$> \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)] + \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}$$

reduces to $\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]} > 0$ which holds.

- Thus, when $\underline{x}_H \leq x_{L,NE}$ (due to the stronger condition $\frac{s_L}{s_H} \geq \overline{S}$), the condition $\underline{x}_H < \overline{x}_L$ is slack (satisfying the weaker condition $\frac{s_L}{s_H} > \hat{S}$).

7.3 Separating Equilibrium

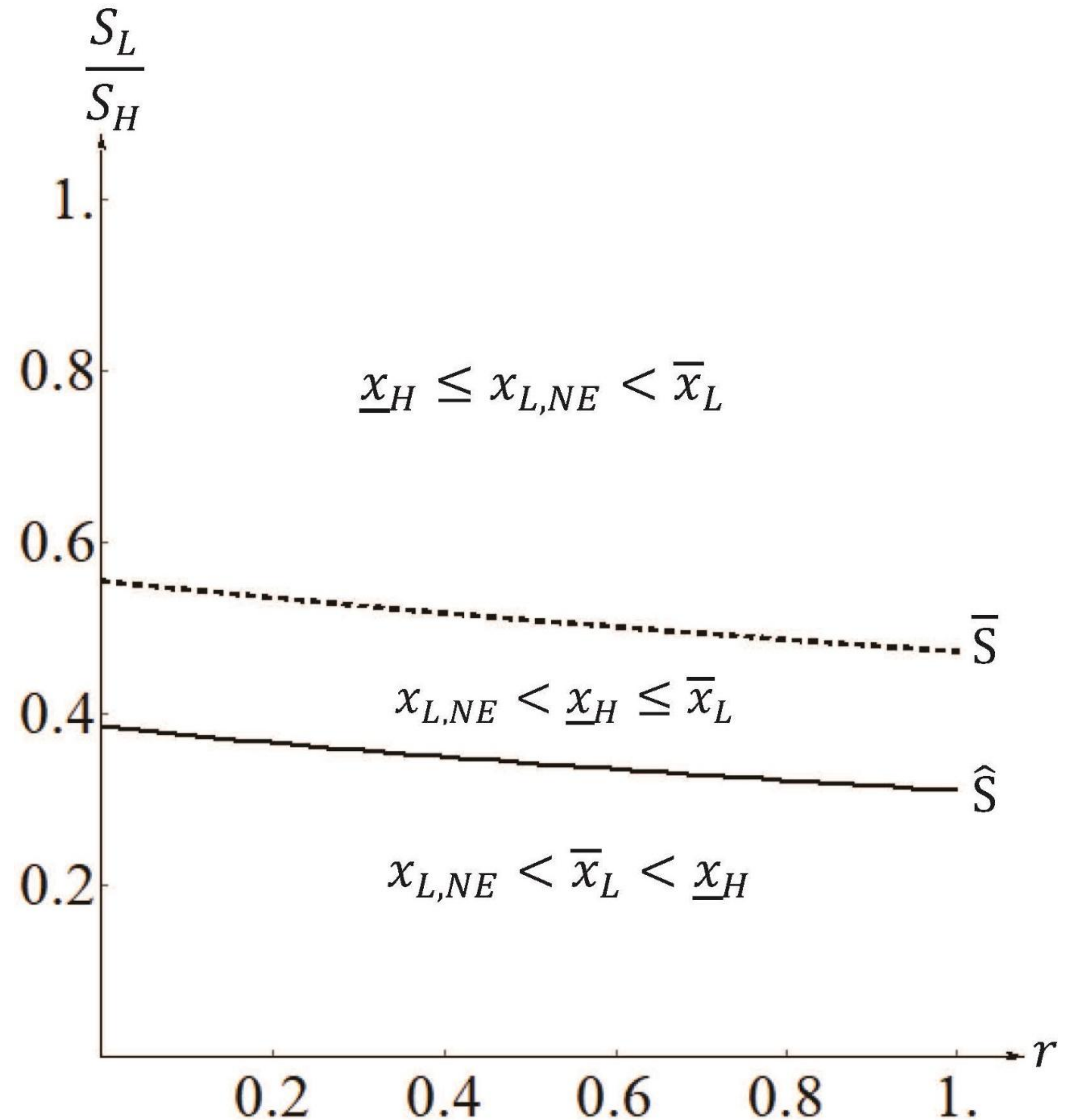
- Finally, we check that $x_{L,NE} < \overline{x}_L$ holds because

$$\frac{S_L[4-\delta(1-r)]}{8} < \frac{S_L\left[9\left(4-\delta(1-r)\right)+\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

reduces to $\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]} > 0$ which holds.

- Therefore, we can dissect the stock ratio into three ranges of values.
  - When $S_L/S_H \geq \overline{S}$, ranking satisfies $\underline{x}_H \leq x_{L,NE} < \overline{x}_L$, and the low stock incumbent chooses $\underline{x}_H$ to be identified as a low-stock firm.
  - When $\hat{S} \leq S_L/S_H < \overline{S}$, ranking satisfies $x_{L,NE} < \underline{x}_H \leq \overline{x}_L$, and the low stock incumbent chooses $x_{L,NE}$ that renders $\underline{x}_H$ slack.
  - When $S_L/S_H < \hat{S}$, ranking satisfies $x_{L,NE} < \overline{x}_L < \underline{x}_H$, and the low stock incumbent chooses $x_{L,NE}$ that renders $\overline{x}_L$ slack.

7.3 Separating Equilibrium



7.3 Separating Equilibrium

- Last but not least, we show that $x_{L,NE} > \underline{x}_L$ because

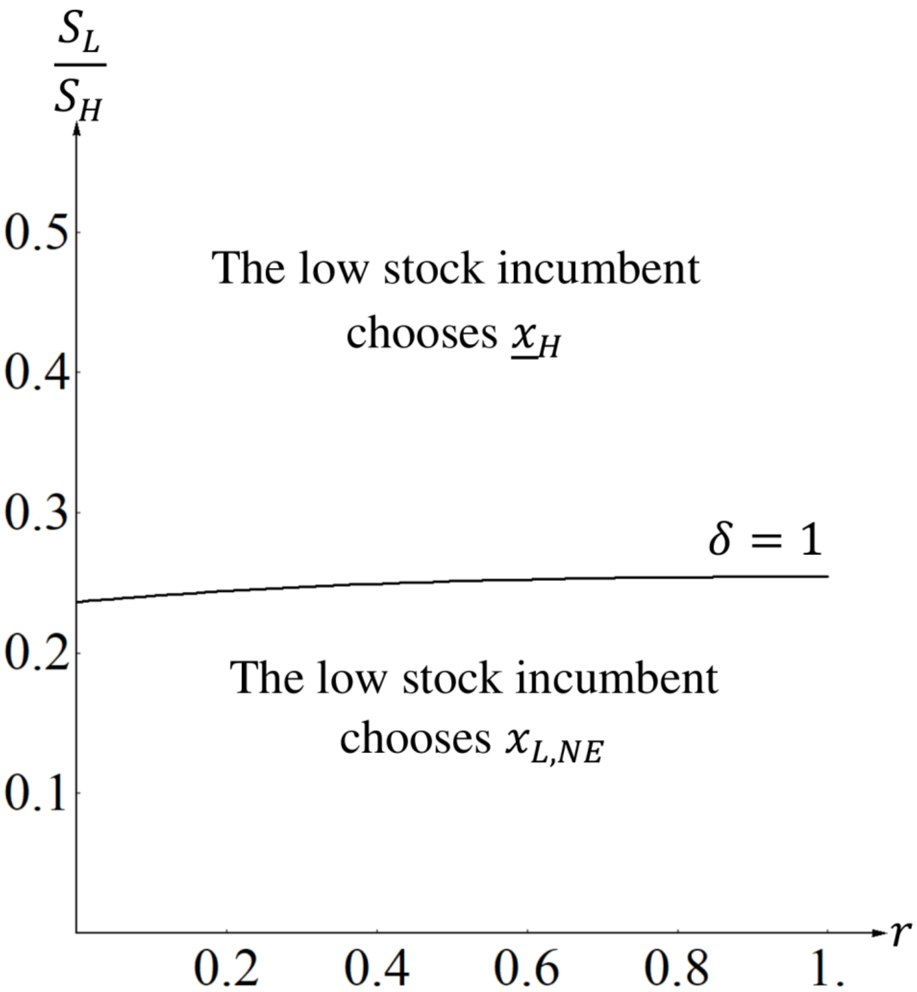
$$\frac{S_L[4-\delta(1-r)]}{8} > \frac{S_L\left[9\big(4-\delta(1-r)\big)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

reduces to $\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]} > 0$ which holds.

- This means that the incumbent is either bound by $\underline{x}_H$ when $S_L/S_H \geq \overline{S}$ or unconstrained to choose $x_{L,NE}$ otherwise, but never finds $\underline{x}_L$ optimal.

7.3 Separating Equilibrium

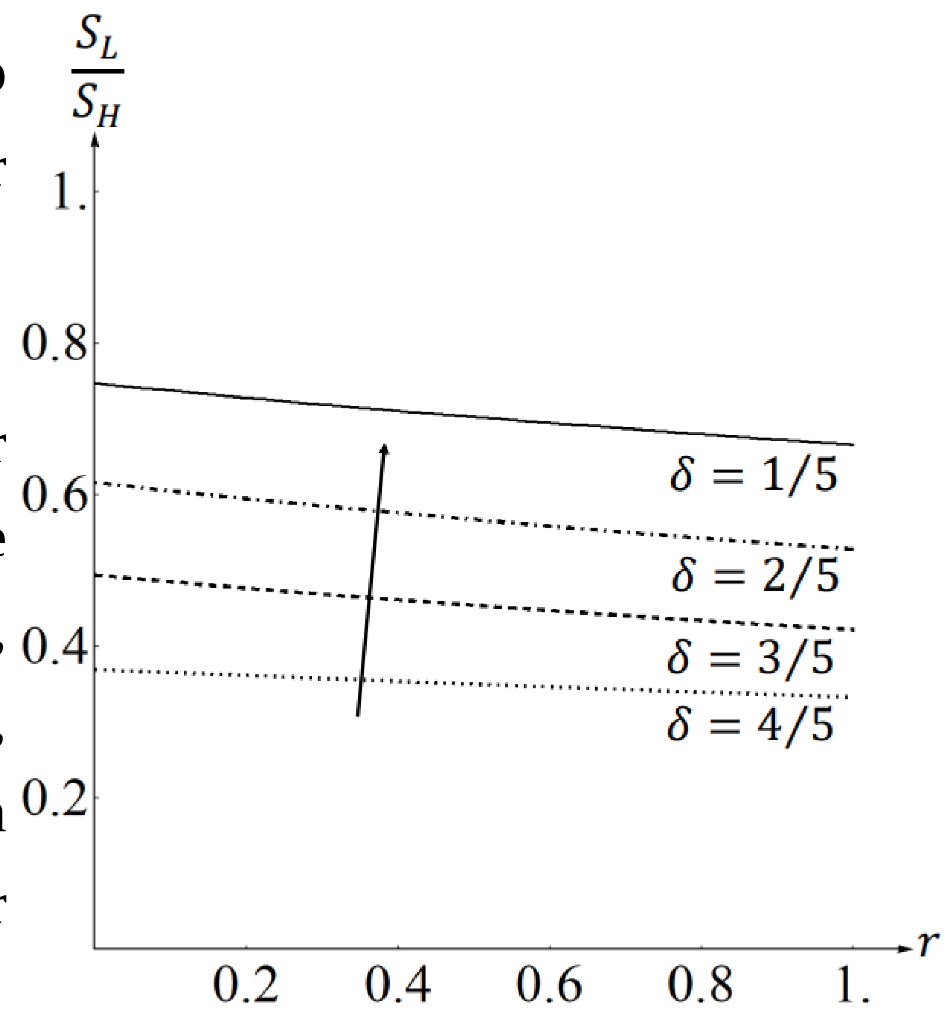
- The following figure plots cutoff $\overline{S}$ as a function of r assuming $\delta = 1$.
- Above this cutoff, separating level $\underline{x}_H$ is binding but below, it is slack.
- When the stock regenerates faster (higher r), the low-stock firm is less likely to be bound by the separating appropriation level $\underline{x}_H$.



7.3 Separating Equilibrium

- When δ decreases, the firm assigns lower weights to future payoffs, so it is subject to the binding $\underline{x}_H$ under narrower conditions (higher $\overline{S}$).

- However, cutoff $\overline{S}$ falls with r for sufficiently low δ because when the resource regenerates more rapidly, competition becomes more intense, so that it has to separate itself from the high stock incumbent under wider parameter conditions.



7.3 Separating Equilibrium

- Differentiating $\underline{x}_H$ with respect to S_H, we have

$$\frac{\partial \underline{x}_H}{\partial S_H} = \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{72} > 0$$

  - so the more abundant the high stock becomes, the less demanding is the maximum level of appropriation that still deters entry.

- Differentiating $\underline{x}_H$ with respect to δ, we find

$$\frac{\partial \underline{x}_H}{\partial \delta} = -\frac{S_H}{72}\left[9(1-r) + \frac{\sqrt{5}[36(1+r)+13\delta(1-r)^2]}{\sqrt{\delta[72(1+r)+13\delta(1-r)^2]}}\right] < 0$$

  - so the more weights that firms assign to future payoffs, the lower is the maximum level of appropriation that still deters entry.

7.3 Separating Equilibrium

➢ Differentiating $\underline{x}_H$ with respect to r, we obtain

$$\frac{\partial \underline{x}_H}{\partial r} = \frac{\sqrt{\delta} S_H}{72}\left[9\sqrt{\delta} - \frac{\sqrt{5}[36 - 13\delta(1-r)]}{\sqrt{72(1+r) + 13\delta(1-r)^2}}\right]$$

- which is positive if and only if

$$13(1-r)^2\delta^2 + 9(73+8r)\delta - 405 > 0$$

- that entails if δ is sufficiently high, where

$$\delta > \underline{\delta} \equiv \frac{9\left[\sqrt{5589 + 648r - 196r^2} - (73+8r)\right]}{26(1-r)^2}$$

- firms focus more on future payoffs so when the stock regenerates faster, the maximum entry-deterring appropriation $\underline{x}_H$ increases. When $r > \sqrt{2} - 1 \approx 0.41$, $\underline{\delta} < 0$ so $\underline{x}_H$ increases in r for all values of $\delta \in [0,1]$.

7.3 Separating Equilibrium

➢ Assume parameter values $S_H = 10$, $S_L = 5$, and $\delta = 1$, we obtain an upper and lower bound for x_L simultaneously:

- Solving for x_L in IC_H

$$x_L \leq \underline{x}_H \equiv \frac{5\left[9(3+r) - \sqrt{5[72(1+r)+13(1-r)^2]}\right]}{36}$$

- Solving for x_L in IC_L

$$x_L \geq \underline{x}_L \equiv \frac{5\left[9(3+r) - \sqrt{5[72(1+r)+13(1-r)^2]}\right]}{72}$$

7.3 Separating Equilibrium

- Therefore, x_L must lie in the interval $x_L \in [\underline{x}_L, \underline{x}_H]$, that is,

$$x_L \in \left[\frac{5\left[9(3+r)-\sqrt{5(85+46r+13r^2)}\right]}{72}, \frac{5\left[9(3+r)-\sqrt{5(85+46r+13r^2)}\right]}{36}\right]$$

- Since all appropriation levels in this range convey low stock to the potential entrant, the incumbent chooses the maximum, $x_L = \underline{x}_H$, to deviate the least from complete-information level to deter entry, i.e.,

$$x_L = \frac{5\left[9(3+r)-\sqrt{5(85+46r+13r^2)}\right]}{36}$$

- which is the "least-costly separating equilibrium," also known as the "Riley outcome," that survives Cho and Kreps' Intuitive Criterion (1987).

7.3.1 Separating Effort

- The "separating effort" that the incumbent exerts to convey the low stock to the potential entrant and thus deter entry is

$$x_{L,NE} - x_L$$

$$= \frac{S_L[4 - \delta(1 - r)]}{8}$$

$$- \frac{S_H\left[9\big(4 - \delta(1 - r)\big) - \sqrt{5\delta[72(1 + r) + 13\delta(1 - r)^2]}\right]}{72}$$

- where $x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8}$ is the first-period appropriation level that the low-stock incumbent selects under complete information.

7.3.1 Separating Effort

- Since $S_H = 10$, $S_L = 5$, and $\delta = 1$, this separating effort becomes

$$\frac{5\sqrt{5 \times 1[72(1+r) + 13 \times 1(1-r)^2]}}{36} - \frac{(10-5)(3+r)}{8}$$

$$= \frac{5\left[2\sqrt{5(85+46r+13r^2)} - 9(3+r)\right]}{72}$$

- We show that this separating effort is positive for all values of r since $x_{L,NE} - x_L > 0$ entails $2\sqrt{5(85+46r+13r^2)} > 9(3+r)$, which reduces to $971 + 434r + 179r^2 > 0$, suggesting that the incumbent's appropriation falls below the socially optimal level.

7.3.1 Separating Effort

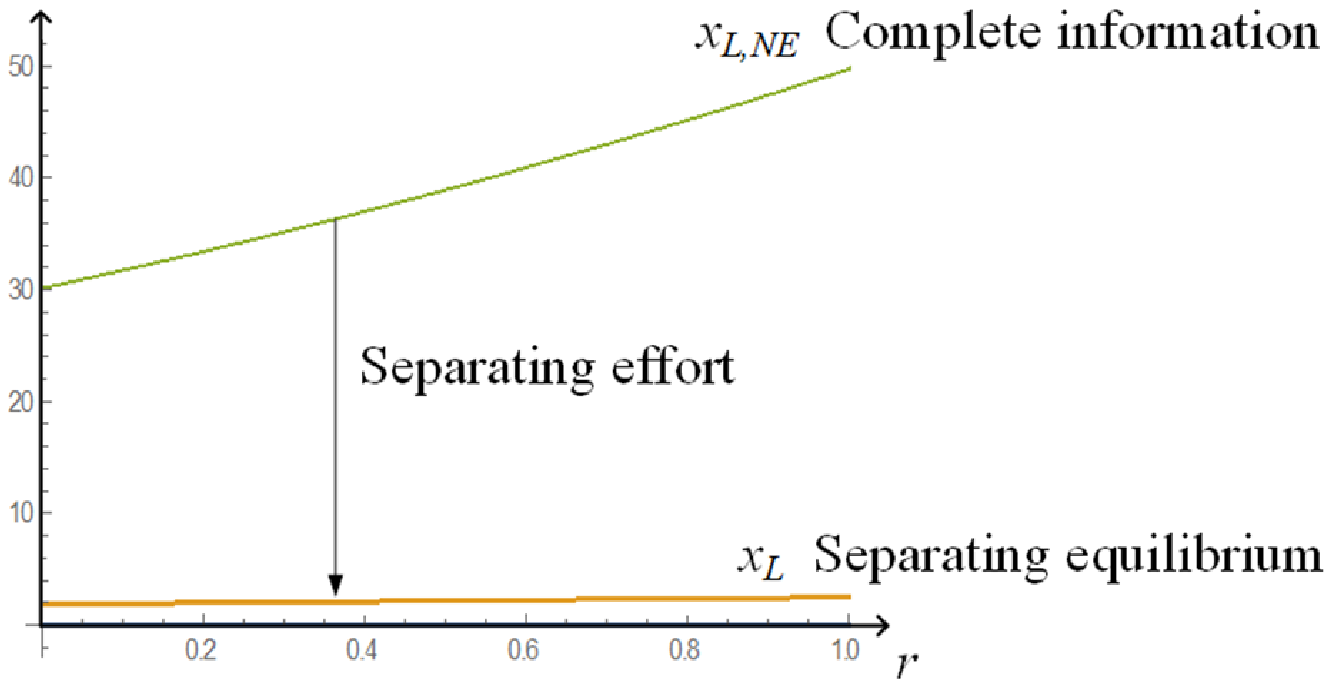


Figure 7.2. Separating Effort

- The above figure depicts that the low-stock incumbent chooses a lower appropriation level x_L than that under complete information, $x_{L,NE}$, thus exerting a positive separating effort, $x_{L,NE} - x_L$, for all $r \in [0,1]$.

7.3.1 Separating Effort

- This separating effort is increasing in the regeneration rate r since

$$\frac{\partial(x_{L,NE}-x_L)}{\partial r} = \frac{5}{72}\left[\frac{2\sqrt{5}(13r+23)}{\sqrt{13r^2+46r+85}} - 9\right] > 0$$

  - for all values of $r \in [0,1]$, as the bracket is unambiguously positive that is rearranged to $3695 + 8234r + 2327r^2 > 0$ which holds.

- When a larger share of the resource regenerates across periods, the CPR becomes more attractive, and the high-stock incumbent has stronger incentives to mimic the low-stock incumbent.

- As a response, the low-stock incumbent must underexploit the CPR more intensively so that the high-stock incumbent no longer has incentives to mimic its choice of appropriation level x_L.

7.3.1 Separating Effort

- Figure 7.2 shows that as the regeneration rate r increases, the separating effort that the low-stock incumbent exerts expands.

- This separating effort can be interpreted as the “strategic conservation effort” that the low-stock incumbent exerts to distinguish itself from the high-stock incumbent, which has incentives to mimic the low-stock incumbent to deter entry.

- Thus, CPRs with high regeneration rates should exhibit relatively substantial conservation efforts while those with low regeneration rates should have a low conservation effort.

7.3 Separating Equilibrium

- When $S_L/S_H \geq \overline{S}$, aggregate appropriation becomes

$$(1-r)x_L + \frac{S_L - (1-r)x_L}{2} = \frac{S_L + (1-r)x_L}{2}$$

- Substituting $x_L = \frac{S_H\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$, we have

$$\frac{1}{2}\left[S_L + (1-r)\frac{S_H\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}\right] \leq S_L$$

- which we rearrange to yield

$$72S_L - (1-r)S_H\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right] \geq 0$$

- so that resource S_L can be supported if

$$\frac{S_L}{S_H} \geq \underline{S} \equiv \frac{(1-r)\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

7.3 Separating Equilibrium

- We further show that $\overline{S} > \underline{S}$ because

$$\frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)]}$$

$$> \frac{(1-r)\left[9\big(4-\delta(1-r)\big) - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72}$$

- simplifies to

$$9[4-\delta(1-r)] > \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}$$

- that further reduces to

$$1296 - 144(7\delta + 2r) + 16\delta^2(1-r)^2 > 0$$

- which unambiguously holds, so when the condition $x_L < x_{L,NE}$ holds, resource S_L can be supported in equilibrium that renders cutoff $\underline{S}$ slack.

7.3.2 Efficiency Properties

➢ ***When the initial stock is high***

- The incumbent chooses the same appropriation as that under complete information, $x_H = x^*$, which is socially excessive.

- This yields the same dynamic inefficiency that its first-period appropriation imposes on the entrant's second-period profits.

- Therefore, under a high stock, the same inefficiencies emerge under complete and incomplete information.

7.3.2 Efficiency Properties

➢ ***When the initial stock is low***

- The incumbent underexploits the resource relative to the first-period appropriation under complete information, that is, $x_L < x_{L,NE}$.
- Under complete information, no inefficiencies arise since the incumbent is the only firm operating in both periods, thus internalizing the intertemporal effect of its appropriation of the low-stock CPR.
- In the separating equilibrium, the underexploitation gives rise to a new form of inefficiency: a socially inefficient appropriation of the CPR in the incomplete information environment which firms interact.
- This has been reported in fishing grounds such as those of the Silver hake in the North Atlantic and blackfin tuna in the Caribbean.

7.3.2 Efficiency Properties

- If the regulatory authorities are perfectly informed about the available stock, our results suggest that they should strategically distribute this information among potential entrants, for example, publicizing the low stock in different media outlets.

- This would transform the structure of firms' interaction from an incomplete information game, where entry is deterred via underexploiting the resource, to a complete information game, where entry does not occur and appropriation is socially optimal.

Pooling Equilibrium

7.4 Pooling Equilibrium

- In this equilibrium, both types of incumbent choose the same first-period appropriation, $x_{L,NE}$, that corresponds to the level of the low-stock incumbent under complete information.

- Since the observation of the incumbent's appropriation decision does not help the entrant restrict its belief about the available stock, the potential entrant's posterior belief, $\mu(S_H|x)$, coincides with its prior p, which is the probability that the stock is high.

- In other words, the pooling appropriation level $x_{L,NE}$ conceals information about the stock from the entrant.

7.4 Pooling Equilibrium

- ***Low-stock incumbent***
  - By selecting the same appropriation level $x_{L,NE}$ as under complete information, this type of incumbent deters entry and maximizes its overall profits conditional on no entry in the next period.
  - If, instead, it deviates to any other appropriation level, $x \neq x_{L,NE}$, entry ensues, yielding an unambiguously lower overall profits.
  - In short, this type of incumbent has no incentives to deviate.

7.4 Pooling Equilibrium

➢ ***High-stock incumbent***

- Selecting $x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8}$, this incumbent deters entry, earning

$$\Pi_H^{ED} = \left[x_{L,NE} - \frac{x_{L,NE}^2}{S_H}\right] + \delta\left[\frac{S_H - (1-r)x_{L,NE}}{4}\right]$$

$$= \frac{S_L[4-\delta(1-r)][8S_H - 4S_L + \delta(1-r)S_L]}{64S_H} + \delta\left[\frac{8S_H - 4(1-r)S_L + \delta(1-r)^2 S_L}{32}\right]$$

$$= \frac{16\delta S_H^2 + (2S_H - S_L)[4-\delta(1-r)]^2 S_L}{64S_H}$$

- Suppose $S_H = 10$, $S_L = 5$, and $\delta = 1$, entry-deterring profits are

$$\frac{5(3+r)(13-r)}{128} + \frac{5(13+2r+r^2)}{32}$$

7.4 Pooling Equilibrium

- If, instead, it deviates to $x \neq x_{L,NE}$, entry occurs, solving

$$\max_{x \geq 0} \left[x - \frac{x^2}{S_H}\right] + \delta \left[\frac{S_H - (1-r)x}{9}\right]$$

- Evaluating at $x_{H,E} = \frac{S_H[9-\delta(1-r)]}{18}$, overall profits become

$$\Pi_H^{AE} = \left[x_{H,E} - \frac{x_{H,E}^2}{S_H}\right] + \delta \left[\frac{S_H - (1-r)x_{H,E}}{9}\right]$$

$$= \frac{S_H[9-\delta(1-r)][9+\delta(1-r)]}{324} + \delta S_H \left[\frac{9(1+r) + \delta(1-r)^2}{162}\right]$$

$$= \frac{S_H\{9[9+2\delta(1+r)] + \delta^2(1-r)^2\}}{324}$$

- Suppose $S_H = 10$, $S_L = 5$, and $\delta = 1$, entry-accommodating profits are

$$\frac{5(8+r)(10-r)}{162} + \frac{5(10+7r+r^2)}{81}$$

7.4 Pooling Equilibrium

- Therefore, the high-stock incumbent chooses $x_{L,NE}$ to deter entry rather than $x_{H,E}$ that attracts entry if and only if $\Pi_H^{ED} \geq \Pi_H^{AE}$, yielding

$$\left[x_{L,NE} - \frac{x_{L,NE}^2}{S_H}\right] - \left[x_{H,E} - \frac{x_{H,E}^2}{S_H}\right] \geq \delta \left[\frac{S_H - (1-r)x_{H,E}}{9} - \frac{S_H - (1-r)x_{L,NE}}{4}\right]$$

- Substituting $x_{H,E} = \frac{S_H[9-\delta(1-r)]}{18}$ and $x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8}$, we have

$$81[4-\delta(1-r)]^2 \left(\frac{S_L}{S_H}\right)^2 - 162[4-\delta(1-r)]^2 \frac{S_L}{S_H} + 16[81 - 9\delta(7-2r) + \delta^2(1-r)^2] \leq 0$$

- Invoking the quadratic formula, the above inequality solves

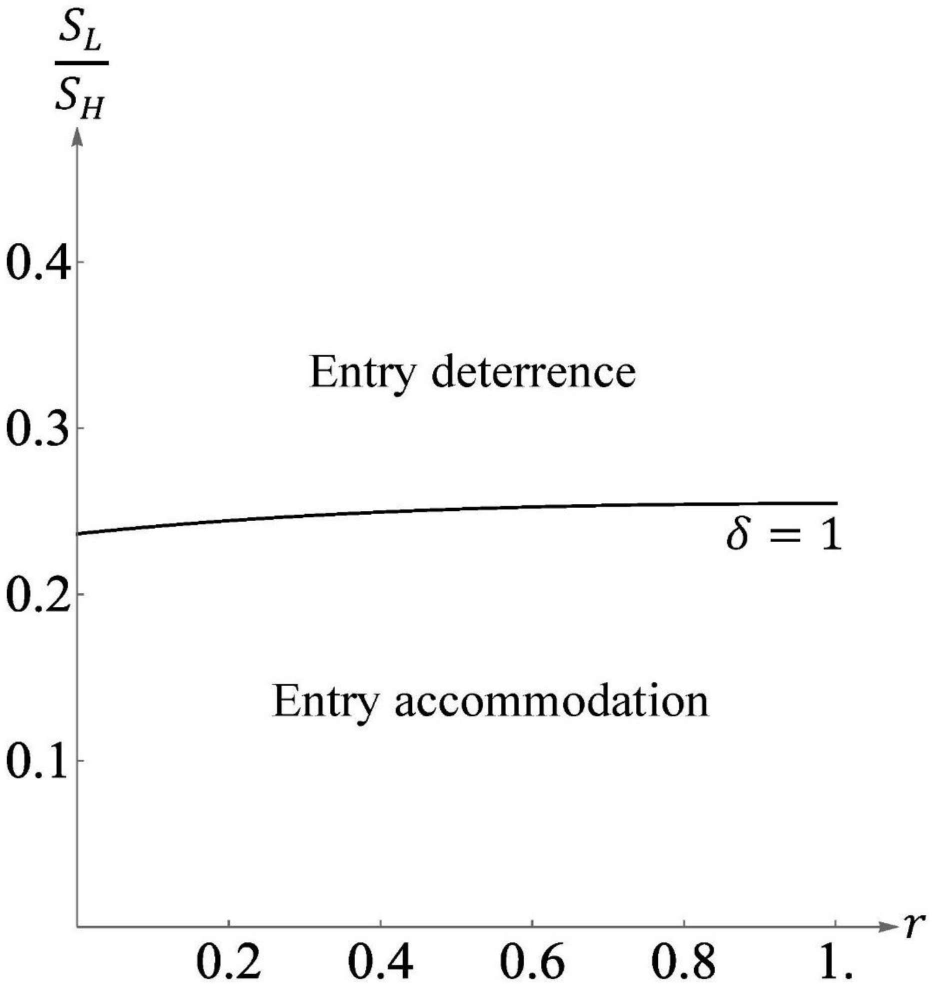
$$\frac{S_L}{S_H} \geq \underline{\Sigma} \equiv \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r) + 13\delta(1-r)^2]}}{9[4-\delta(1-r)]}$$

7.4 Pooling Equilibrium

- The following figure plots cutoff $\underline{\Sigma}$ as a function of r assuming $\delta = 1$.

- Above this cutoff, entry is deterred but below, entry is accommodated.

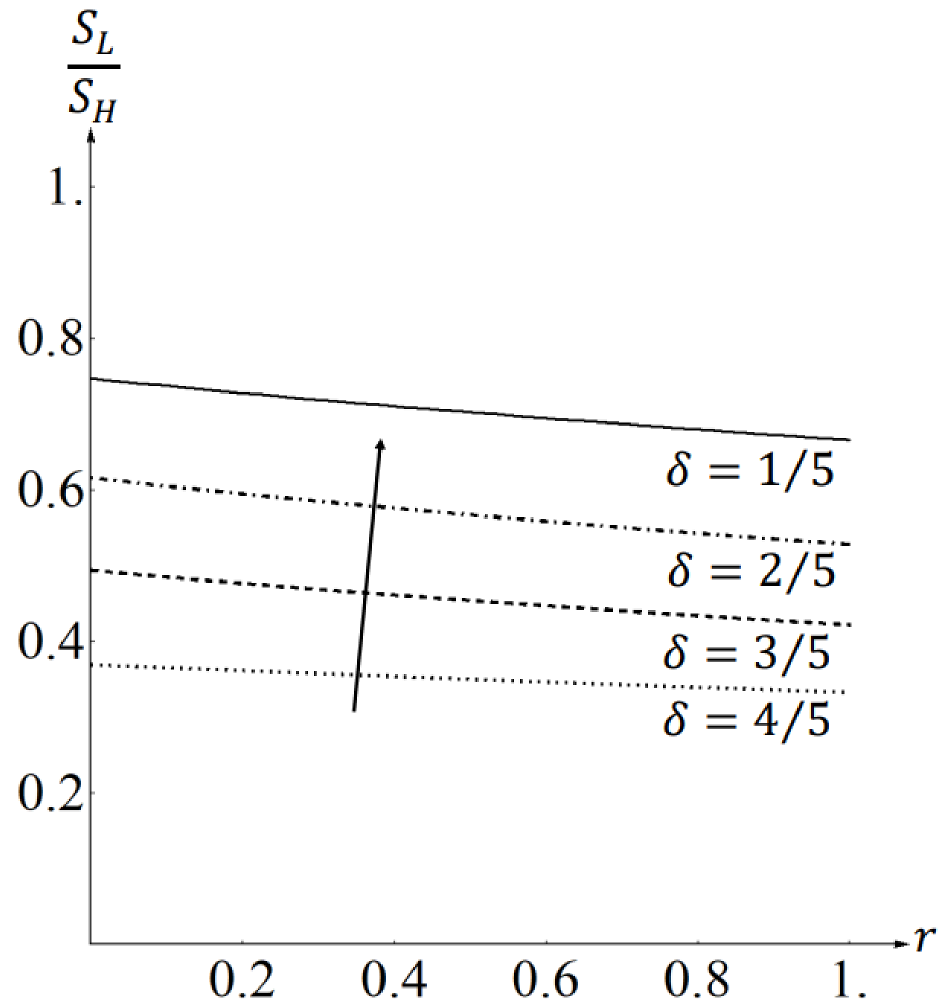
- When the stock regenerates faster, (higher r), the incumbent is more likely to accommodate entry.



7.4 Pooling Equilibrium

- When δ decreases, the incumbent assigns lower weights to future payoffs, so entry is accommodated under wider conditions (higher $\underline{\Sigma}$).

- However, cutoff $\underline{\Sigma}$ decreases in r for sufficiently low δ since when the resource regenerates at a faster rate, competition becomes more intense, so that the incumbent is less inclined to accommodate entry.



7.4 Pooling Equilibrium

- Evaluating at $S_H = 10$, $S_L = 5$, and $\delta = 1$, the above incentive compatibility condition for the high-stock incumbent becomes

$$\frac{3(3+r)^2+64}{128} \geq \frac{(8+r)^2+36}{162}$$

which yields $971 + 434r + 179r^2 \geq 0$ that holds for all $r \in [0,1]$.

- The high stock incumbent has incentives to operate like a low stock counterpart and reduce first-period appropriation from $x_{H,E}$ to $x_{L,NE}$.

7.4 Pooling Equilibrium

- In this context, aggregate appropriation becomes

$$(1-r)x_{L,NE} + \frac{S_H - (1-r)x_{L,NE}}{2} = \frac{S_H + (1-r)x_{L,NE}}{2}$$

- Substituting $x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8}$ into the above expression, we find

$$\frac{1}{2}\left[S_H + (1-r)\frac{S_L[4-\delta(1-r)]}{8}\right] \leq S_H$$

- which we rearrange to yield

$$8S_H - S_L(1-r)[4-\delta(1-r)] \geq 0$$

- Since $S_H > S_L$, we further obtain

$$S_L[4(1+r) + \delta(1-r)^2] \geq 0$$

- that holds, so resource S_H can be supported for any values of $\delta, r \in [0,1]$.

7.4 Pooling Equilibrium

- This type of behavior has been observed in several coastal communities in Loreto, Mexico where fishermen reduced their appropriation when new firms show an interest in exploiting the fishing ground.

- In this type of small-scale fishery, it is reasonable to assume that local fishermen have more accurate information about the available stock than fishermen operating in different locations.

- Underexploitation has also been observed in other fishing grounds, such as those of the yellowfin sole in the Pacific Northwest, the blackfin tuna and diamond black squid in the Caribbean region, and the Argentine anchovy in the Southern Atlantic (Haughton, 2002; FAO, 2005).

7.4.1 Pooling Effort

- The decrease in exploitation that the high-stock incumbent exerts to mimic the low stock incumbent is known as the pooling effort, which is strategic conservation seeking to deter potential entrants, as follows:

$$x_{H,E} - x_{L,NE} = \frac{S_H[9-\delta(1-r)]}{18} - \frac{S_L[4-\delta(1-r)]}{8}$$

$$= \frac{4S_H[9-\delta(1-r)] - 9S_L[4-\delta(1-r)]}{72}$$

- For illustration purposes, when $S_H = 10$, $S_L = 5$, and $\delta = 1$, the appropriation level of the high-stock incumbent under complete information is $x_{H,E} = \frac{5(8+r)}{9}$, that under the pooling equilibrium is $x_{L,NE} = \frac{5(3+r)}{8}$, and the pooling effort becomes $x_{H,E} - x_{L,NE} = \frac{5(37-r)}{72}$.

7.4.1 Pooling Effort

- This pooling effort is positive if and only if

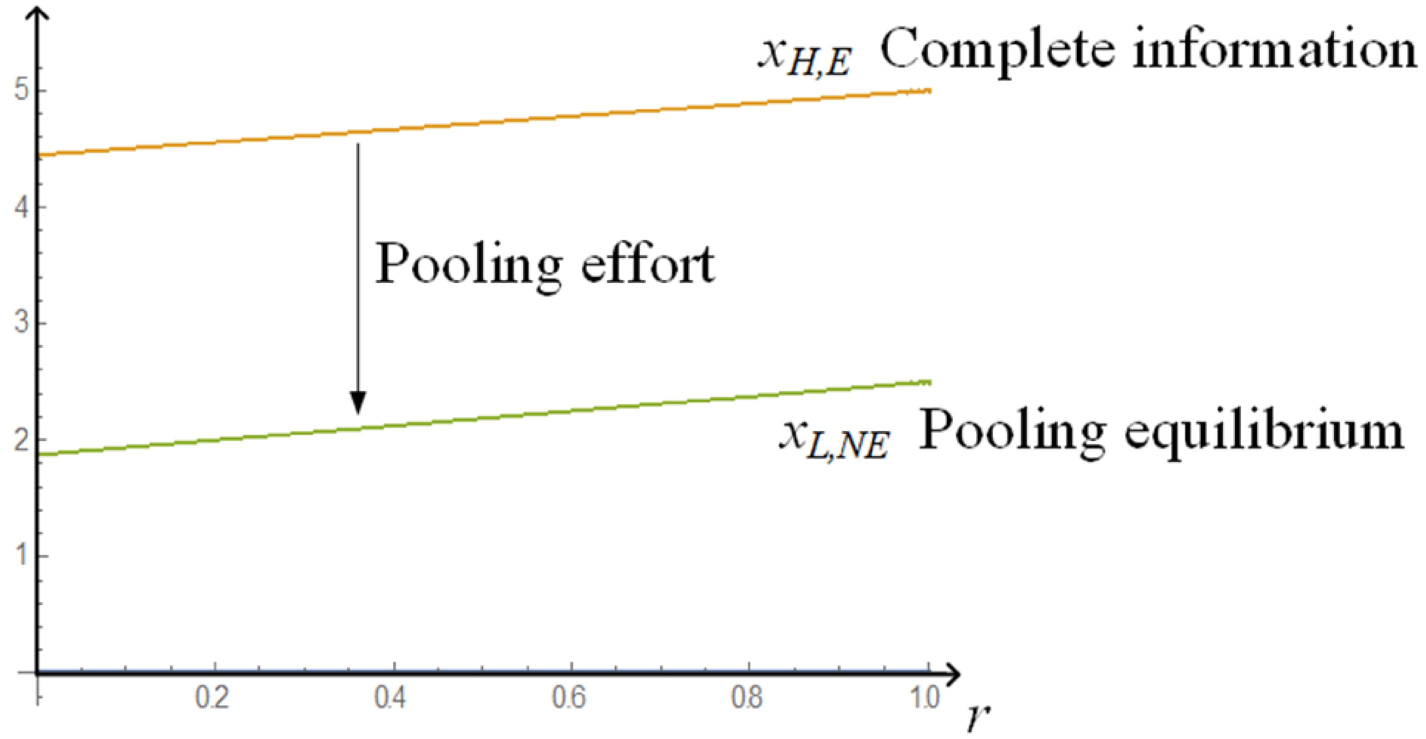
$$4S_H[9-\delta(1-r)]-9S_L[4-\delta(1-r)]>0$$

  - which we rearrange to yield

$$36(S_H-S_L)-\delta(1-r)(4S_H-9S_L)>0$$

  - A sufficient condition is $4S_H<9S_L$, indicating that when the high stock is not too much abundant than the low stock (or $\frac{S_L}{S_H}>\frac{4}{9}$), then the high-stock incumbent has incentives to appropriate the same level as the low-stock firm to deter entry.

7.4.1 Pooling Effort



- The above figure (Figure 7.3) depicts this pooling effort, which is
  - increasing in S_H, as the high-stock incumbent needs to decrease its first-period appropriation more significantly to mimic the low-stock incumbent.
  - decreasing in S_L, since the incumbent lowers its appropriation to a smaller extent.

7.4.1 Pooling Effort

- The pooling effort increases in the discount factor δ when

$$\frac{\partial\left(x_{H,E} - x_{L,NE}\right)}{\partial\delta} = -\frac{(4S_H - 9S_L)(1-r)}{72}$$

is positive, which holds when $4S_H < 9S_L$, or $\frac{S_L}{S_H} > \frac{4}{9}$.

- For example, when $S_H = 10$ and $S_L = 5$, the above condition becomes $\frac{5}{10} > \frac{4}{9}$ that holds, indicating that when the high-stock incumbent cares more about its future profits during the second period, it has stronger incentives to deter entry by undergoing a costly underexploitation of the resource today.

7.4.1 Pooling Effort

- The pooling effort decreases in the regeneration rate r when

$$\frac{\partial\left(x_{H,E} - x_{L,NE}\right)}{\partial r} = \frac{\delta(4S_H - 9S_L)}{72}$$

is negative, which holds when $4S_H < 9S_L$, or $\frac{S_L}{S_H} > \frac{4}{9}$.

- Intuitively, the high-stock incumbent is less willing to mimic the low-stock incumbent when the CPR regenerates faster.

- When the commons become more attractive, the high-stock incumbent has fewer incentives to undergo a costly pooling effort and is willing to share the CPR with the potential entrant.

7.4.1 Pooling Effort

- The pooling effort increases in the high stock S_H since

$$\frac{\partial\left(x_{H,E} - x_{L,NE}\right)}{\partial S_H} = \frac{9 - \delta(1 - r)}{18} > 0$$

that happens as the high stock incumbent needs to exert more effort in restraining its appropriation when the stock becomes more abundant.

- The pooling effort decreases in the low stock S_L since

$$\frac{\partial\left(x_{H,E} - x_{L,NE}\right)}{\partial S_L} = -\frac{4 - \delta(1 - r)}{8} < 0$$

that happens as the high stock incumbent does not need to cut its appropriation so severely in order to deter entry.

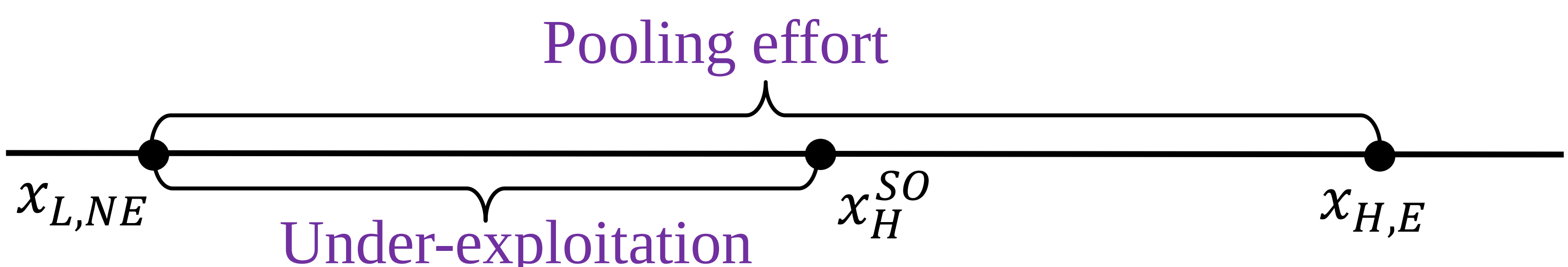
7.4.2 Efficiency Properties

➢ ***What about efficiency properties of pooling equilibrium?***

- Under complete information, the high-stock incumbent overexploits the resource in the first (dynamic inefficiency) and in the second period (static inefficiency once entry occurs).
- Under incomplete information, however, entry is deterred, thus eliminating static inefficiency during the second-period. Indeed, the incumbent exploits below the socially optimal level because

$$x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8} < \frac{S_H[4-\delta(1-r)]}{8} = x_H^{SO}$$

reduces to $S_H > S_L$ which holds by definition.



7.4.2 Efficiency Properties

- ***No precise efficiency result!***
  - If eliminating the second-period static inefficiency offsets the first-period dynamic inefficiency, overall welfare increases relative to the complete information setting.
    - ✓ Regulatory agencies perfectly informed about the available stock should not disseminate information to potential entrants.
  - Otherwise, welfare would be lower under incomplete than complete information.
    - ✓ In this context, distributing information about the available stock becomes welfare-enhancing.

7.4.2 Efficiency Properties

➢ ***Across information settings***

- Dynamic inefficiency under complete information is

$$DI = \frac{5\delta(1-r)S_H}{72}$$

- Over-exploitation under complete information is less severe than under-exploitation under incomplete information when

$$\frac{5\delta(1-r)S_H}{72} < \frac{(S_H - S_L)[4 - \delta(1-r)]}{8}$$

which reduces to

$$\delta(1-r) < \frac{36(S_H - S_L)}{14S_H - 9S_L}$$

7.4.2 Efficiency Properties

- ***Consider $S_H = 10$ and $S_L = 5$***
  - The above inequality reduces to $\delta(1-r) < \frac{36}{19}$ which holds for all values of $\delta, r \in [0,1]$, so dynamic inefficiency is more substantial under incomplete than complete information.
  - In contrast, static inefficiency is positive under complete but vanished under incomplete information, since entry is allowed in the former but deterred in the latter, yielding

$$SI = \frac{S_H - (1-r)\overbrace{\frac{S_H[9-\delta(1-r)]}{18}}^{x_{H,E}}}{12} = \frac{S_H[9(1+r)+\delta(1-r)^2]}{216}$$

7.4.2 Efficiency Properties

- ***In summary, the regulator should***
  - publicize information of the stock, giving rise to a complete information context, where entry ensues in equilibrium if it emphasizes more on dynamic efficiency.
  - conceal information of the stock, leading to an incomplete information context, where entry does not ensue in equilibrium if it is more focused on static efficiency.

Welfare implications

7.5.1 What if the regulator is uninformed?

7.5.2. What if the regulator is informed?

7.5.1 What if the regulator is uninformed?

- Our previous implications hinge upon a regulator as perfectly informed about the available stock as the incumbent.
  - This can occur in CPRs where the regulator closely monitors the appropriation levels for decades and has access to similar technology as the incumbent.
  - However, the regulator may have just started to monitor the incumbent's appropriation level or may not have access to the same type of technology as the incumbent, leaving the regulator as poorly informed as the potential entrant about the available stock.

7.5.1 What if the regulator is uninformed?

- ***Separating Equilibrium***
  - The first-period appropriation is lower than that under complete information, recalling that the low-stock incumbent underexploits the CPR to reveal its type to the potential entrant, $x_L < x_{L,NE}$.
  - Since the increase in second-period appropriation is relatively large, $q_L(x_L) > q_L(x_{L,NE})$, there is an overall increase in exploitation of the CPR, that is, $(1-r)x_L + q_L(x_L) > (1-r)x_{L,NE} + q_L(x_{L,NE})$.
  - Overall welfare is greater than that under complete information when the increase in consumer surplus (a positive effect) dominates the reduction in the incumbent's overall profits (a negative effect).

7.5.1 What if the regulator is uninformed?

- ***Pooling Equilibrium***
  - Appropriations in the first and second periods fall below those under complete information, $x_{H,NE} < x_{H,E}$ and $q_H\left(x_{H,NE}\right) < 2q_H\left(x_{H,E}\right)$.
  - While consumer surplus decreases due to lower overall appropriation, the incumbent's profits increase as it finds profitable to deter entry.
  - Overall welfare is larger under incomplete than complete information when the second effect (PS↑) dominates the first (CS↓).

7.5.1 What if the regulator is uninformed?

- ***Welfare Comparisons***

1. When both equilibria generate a ***lower welfare than that under complete information***, firms' strategic appropriation under incomplete information yields an unambiguous welfare loss regardless of the value of the stock. In this context, the regulator should incur the cost of researching the available stock and publicize such information in media outlets.

2. When both equilibria generate a ***larger welfare than that under complete information***, firms' appropriation under incomplete information yields an unambiguous welfare gain regardless of the value of the stock. In this case, the regulator does not have incentives to investigate the available stock and distribute that information among potential entrants.

7.5.1 What if the regulator is uninformed?

➢ ***Welfare Comparisons***

3. When only the separating (pooling) equilibrium yields a welfare gain, where the stock is low (high), the regulator can carry out research on the available stock, disseminating its finding only when the stock is scarce (abundant, respectively).

   - One caveat is that the absence of information dissemination from the regulator reveals the stock's value to the potential entrant if this firm knows that the regulator has conducted research on the stock.
   - To prevent such possibility, the regulator's research should not be publicly known.

7.5.2 What if the regulator is informed?

➢ ***Suppose the regulator only considers producer surplus.***

- In a separating equilibrium, consider the low stock incumbent.
- Under entry deterrence, first and second period appropriations are

$$\underline{x}_H \equiv \frac{S_H\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72} \text{ and } q^{NE} = \frac{S_L-(1-r)\underline{x}_H}{4}.$$

- Social welfare is comprised of the incumbent's profits alone, where

$$W_L^{ED} = \left[\underline{x}_H - \frac{\underline{x}_H^2}{S_L}\right] + \frac{\delta\left[S_L - (1-r)\underline{x}_H\right]}{4}$$

$$= \frac{\delta S_L^2 + [4-\delta(1-r)]S_L\underline{x}_H - 4\underline{x}_H^2}{4S_L}$$

7.5.2 What if the regulator is informed?

➢ ***Suppose the low stock incumbent accommodates entry.***

- First period appropriation of the incumbent is $x_{L,E} = \frac{S_L[9-\delta(1-r)]}{18}$ while second period appropriation is $q^E = \frac{S_L-(1-r)x_{L,E}}{3}$ for every firm, so that social welfare is the sum of the firms' profits, where

$$W_L^{AE} = \left[x_{L,E} - \frac{x_{L,E}^2}{S_L}\right] + 2\delta\left[\frac{S_L - (1-r)x_{L,E}}{9}\right]$$

$$= \frac{S_L[9-\delta(1-r)][9+\delta(1-r)]}{324} + 2\delta S_L\left[\frac{9(1+r)+\delta(1-r)^2}{162}\right]$$

$$= \frac{S_L\{9[9+4\delta(1+r)]+3\delta^2(1-r)^2\}}{324}$$

7.5.2 What if the regulator is informed?

➢ ***Entry deterrence is socially optimal if*** $W_L^{ED} \geq W_L^{AE}$***, solving***

$$\frac{\delta S_L^2 + [4-\delta(1-r)]S_L \underline{x}_H - 4\underline{x}_H^2}{4S_L} \geq \frac{S_L\{9[9+4\delta(1+r)]+3\delta^2(1-r)^2\}}{324}$$

- Define $A \equiv 9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}$, then

$$A^2\left(\frac{S_H}{S_L}\right)^2 - 18A[4-\delta(1-r)]\frac{S_H}{S_L} + 16[81-9\delta(5-4r)+3\delta^2(1-r)^2] \leq 0$$

- Invoking the quadratic formula, the above inequality solves

$$\frac{S_L}{S_H} \geq \tilde{S} \equiv \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)] + \sqrt{3\delta[24(1+r)+11\delta(1-r)^2]}}$$

7.5.2 What if the regulator is informed?

➢ ***We show that cutoff $\tilde{S}$ is slack, owing to $\tilde{S} < \bar{S}$ where***

$$\tilde{S} = \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)] + \sqrt{3\delta[24(1+r)+11\delta(1-r)^2]}}$$

$$< \frac{9[4-\delta(1-r)] - \sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}}{9[4-\delta(1-r)]} = \bar{S}$$

is simplified to $\sqrt{3\delta[24(1+r)+11\delta(1-r)^2]} > 0$ that holds.

- Therefore, when the low stock incumbent is bounded by $\underline{x}_H$, it is socially optimal to practice entry deterrence since $S_L/S_H \geq \bar{S}$ implies $S_L/S_H > \tilde{S}$.

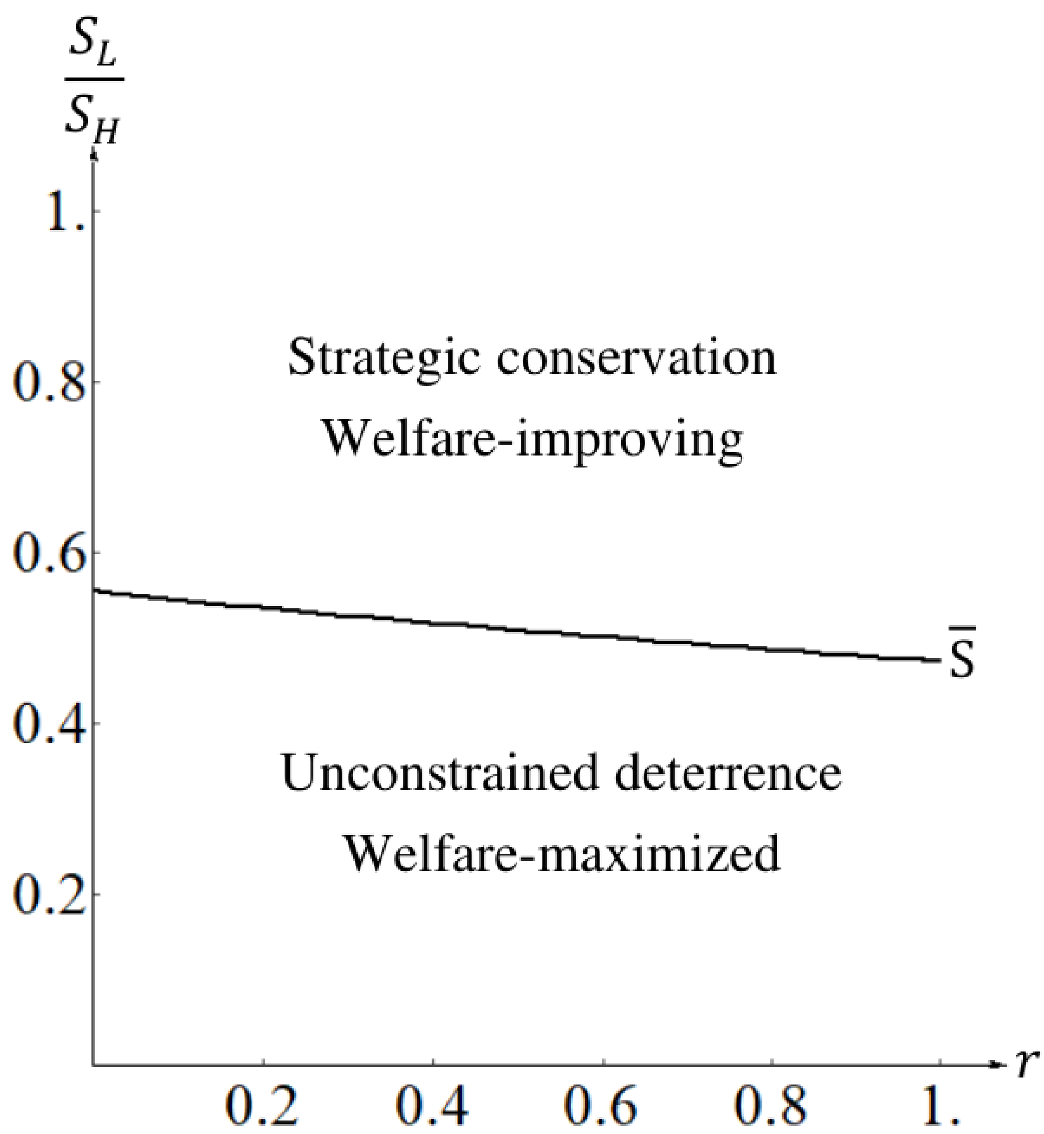
7.5.2 What if the regulator is informed?

- In other words, when the low-stock incumbent is unconstrained to choose $x_{L,NE}$, the high-stock counterpart does not find it profitable to mimic. So, the entrant believes with certainty that the stock is low and does not enter.

- As the low-stock firm wards off competitors, it faces no dynamic and static inefficiencies, such that welfare is maximized when $S_L/S_H < \overline{S}$.

- In sum, depending on stock abundance, we have two ranges of values (for illustration purposes, we take $\delta = 1/2$ in the following figure).
  - When $S_L/S_H \geq \overline{S}$, a separating equilibrium exists at $\underline{x}_H$.
  - When $S_L/S_H < \overline{S}$, a separating equilibrium exists at $x_{L,NE}$.

7.5.2 What if the regulator is informed?



7.5.2 What if the regulator is informed?

➢ ***Suppose the regulator only considers producer surplus.***

- In a pooling equilibrium, social welfare under entry deterrence coincides with profits of the incumbent, that is, $W_H^{ED} = \Pi_H^{ED}$.
- Under accommodation, however, social welfare is the incumbent's intertemporal profits and the entrant's second-period profits, where

$$W_H^{AE} = \left[x_{H,E} - \frac{x_{H,E}^2}{S_H}\right] + 2\delta\left[\frac{S_H - (1-r)x_{H,E}}{9}\right]$$

$$= \frac{S_H[9-\delta(1-r)][9+\delta(1-r)]}{324} + 2\delta S_H\left[\frac{9(1+r)+\delta(1-r)^2}{162}\right]$$

$$= \frac{S_H\{9[9+4\delta(1+r)]+3\delta^2(1-r)^2\}}{324}$$

7.5.2 What if the regulator is informed?

- Entry deterrence is socially optimal if $W_H^{ED} \geq W_H^{AE}$, solving

$$\frac{16\delta S_H^2 + (2S_H - S_L)[4 - \delta(1-r)]^2 S_L}{64 S_H} \geq \frac{S_H\{9[9 + 4\delta(1+r)] + 3\delta^2(1-r)^2\}}{324}$$

which reduces to

$$81[4 - \delta(1-r)]^2 \left(\frac{S_L}{S_H}\right)^2 - 162[4 - \delta(1-r)]^2 \frac{S_L}{S_H} + 16[81 - 9\delta(5 - 4r) + 3\delta^2(1-r)^2] \leq 0$$

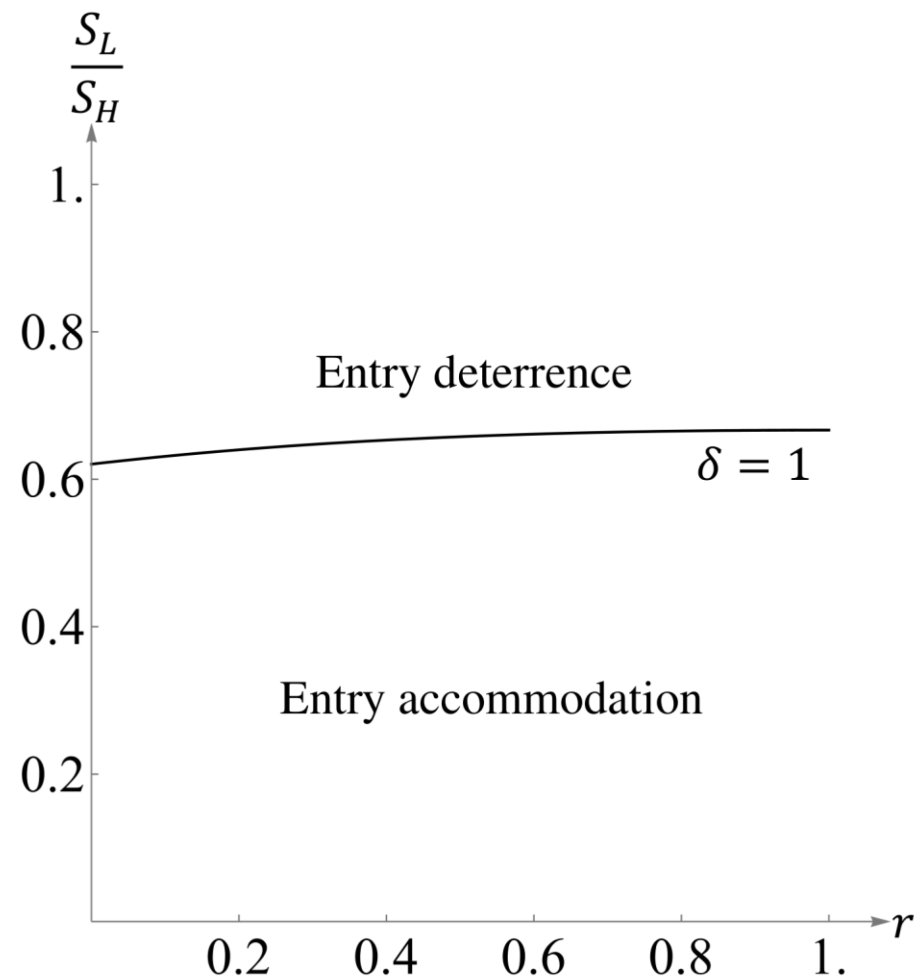
- Invoking the quadratic formula, the above inequality solves

$$\frac{S_L}{S_H} \geq \overline{\Sigma} \equiv \frac{9[4 - \delta(1-r)] - \sqrt{3\delta[24(1+r) + 11\delta(1-r)^2]}}{9[4 - \delta(1-r)]}$$

7.5.2 What if the regulator is informed?

- The following figure plots cutoff $\overline{\Sigma}$ as a function of r assuming $\delta = 1$; above which, it is socially optimal for the regulator to deter entry, but below, entry is accommodated.

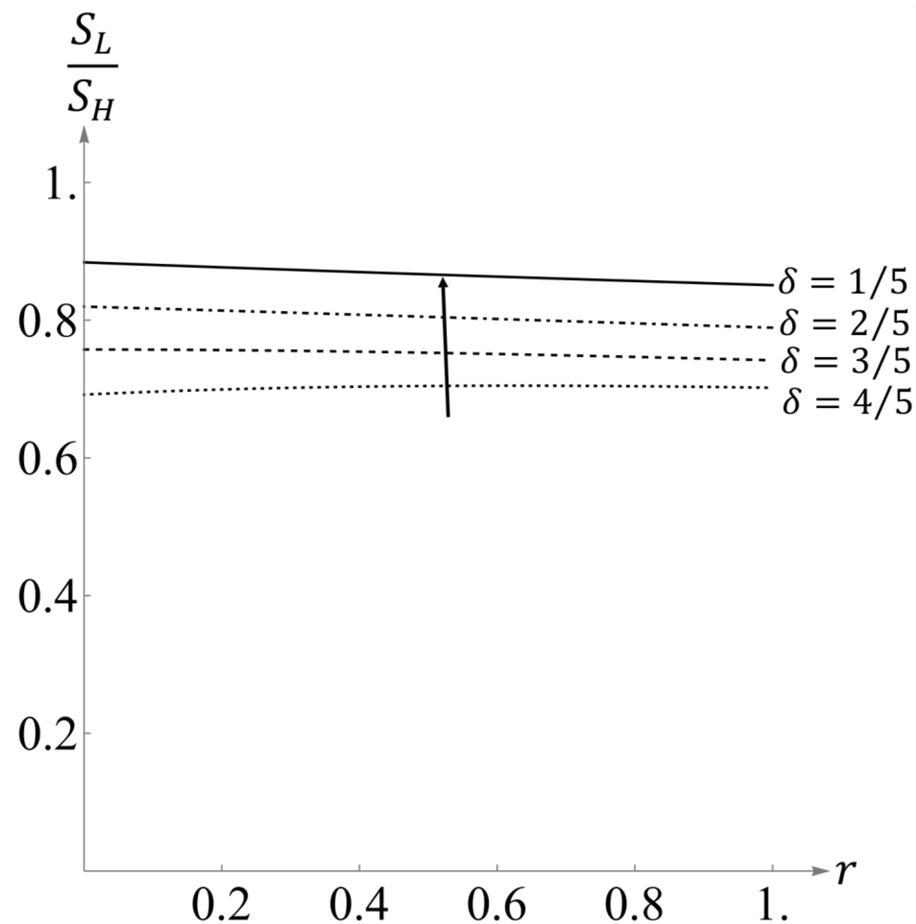
- When the stock regenerates faster, welfare is more likely to be higher when entry is accommodated than when entry is deterred.



7.5.2 What if the regulator is informed?

- When δ decreases, the regulator assigns lower weights to future payoffs, so entry is accommodated under wider conditions (higher $\bar{\Sigma}$).

- However, cutoff $\bar{\Sigma}$ decreases in r at an increasing rate for sufficiently low δ since when the resource regenerates more quickly, potential entrants find it more attractive, so entry deterrence is more likely to be welfare superior to accommodation.



7.5.2 What if the regulator is informed?

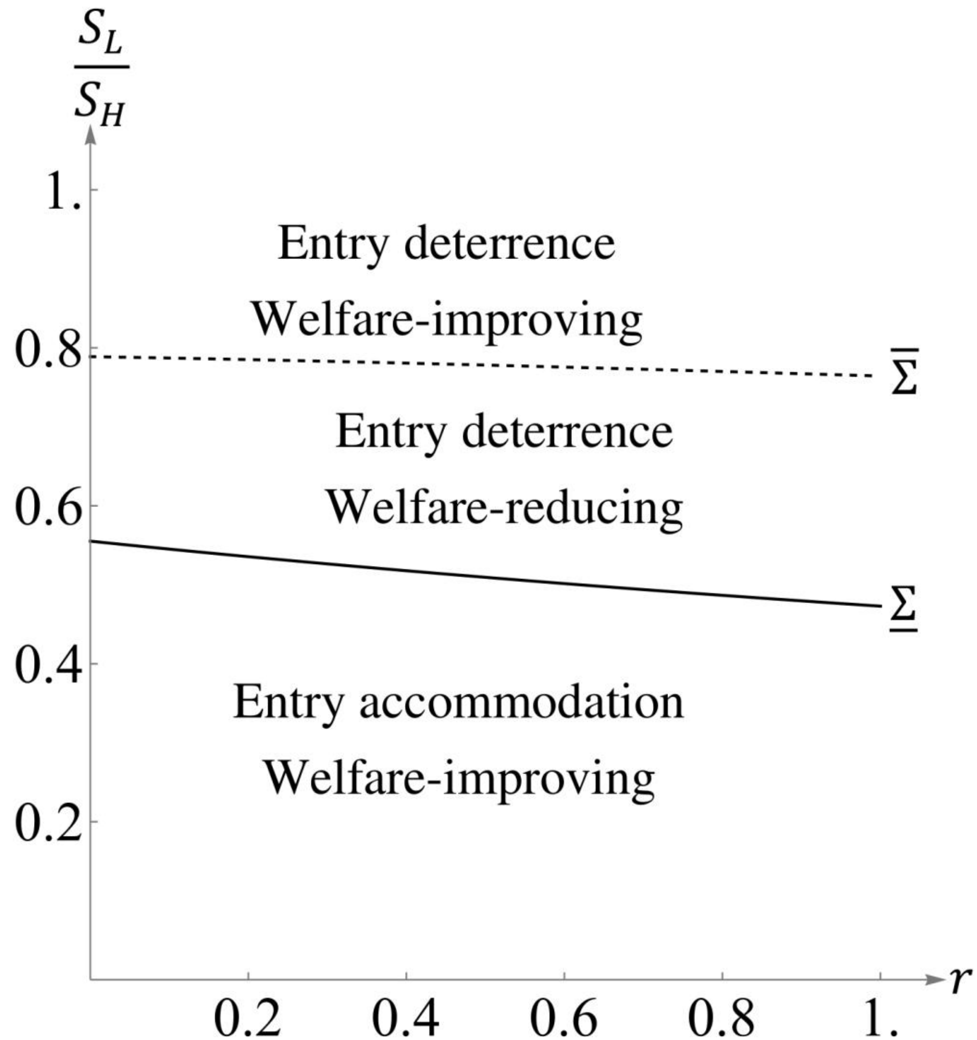
- Define $\underline{C} \equiv 81 - 9\delta(7-2r) + \delta^2(1-r)^2$ and $\overline{C} \equiv 81 - 9\delta(5-4r) + 3\delta^2(1-r)^2$, it is straightforward to show that $\overline{C} > \underline{C}$ since $\overline{C} - \underline{C} = 18\delta(1+r) + 2\delta^2(1-r)^2 > 0$ for all values of $\delta, r \in [0,1]$.

- As a result, when the welfare function has a higher constant term, it intersects the horizontal axis at a higher cutoff than the profit function.

- To see this point, consider the quadratic equation $Ax^2 - Bx + C \leq 0$ that yields the cutoff $x = \frac{B-\sqrt{B^2-4AC}}{2A}$ which is increasing in C because $\frac{\partial x}{\partial C} = (B^2 - 4AC)^{-\frac{1}{2}} > 0$, entailing that cutoffs $\overline{\Sigma}$ and $\underline{\Sigma}$ satisfy $\overline{\Sigma} > \underline{\Sigma}$.

7.5.2 What if the regulator is informed?

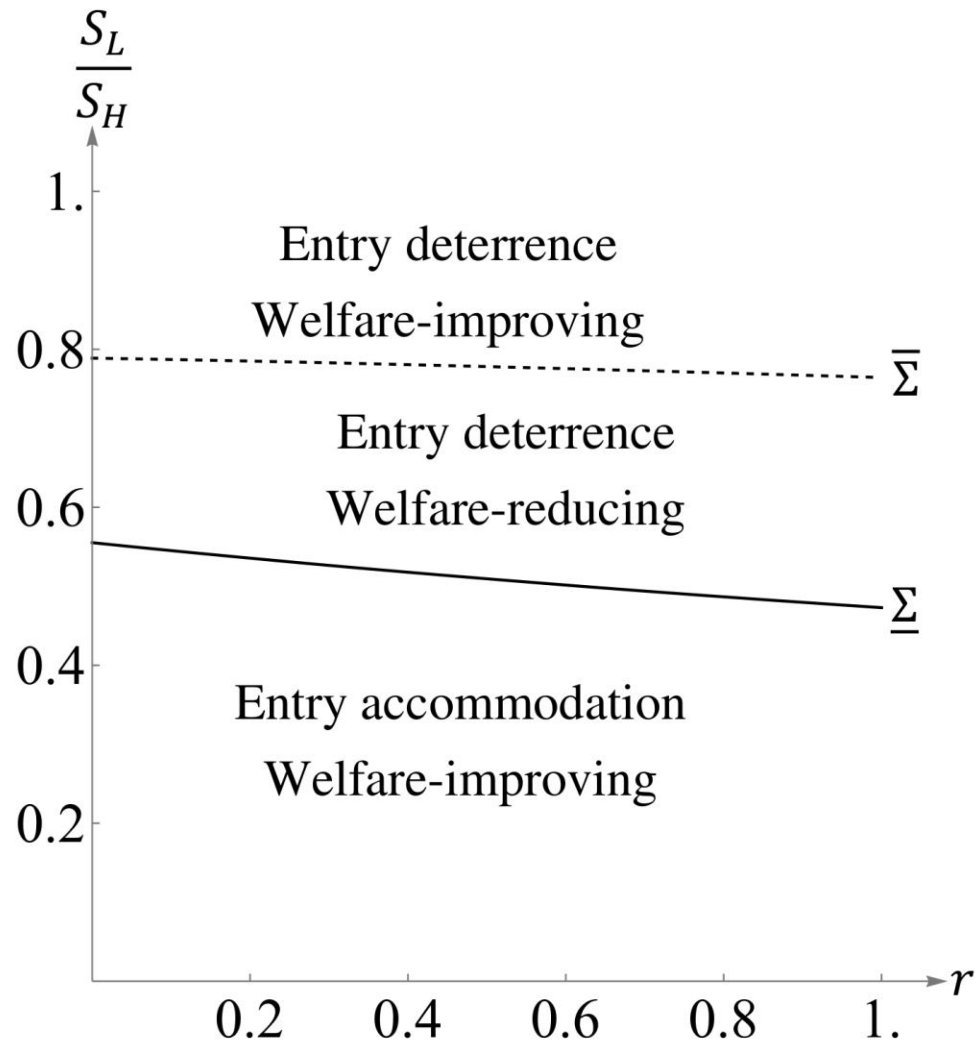
- Depending on stock abundance, we have three ranges of values (as an illustration, take $\delta = 1/2$).



- When $\frac{S_L}{S_H} \geq \overline{\Sigma}$, it is profitable and welfare-enhancing to practice entry deterrence to protect the commons.
  - In this context, the regulator should avoid disseminating information on abundance of the stock that will induce the entrant.

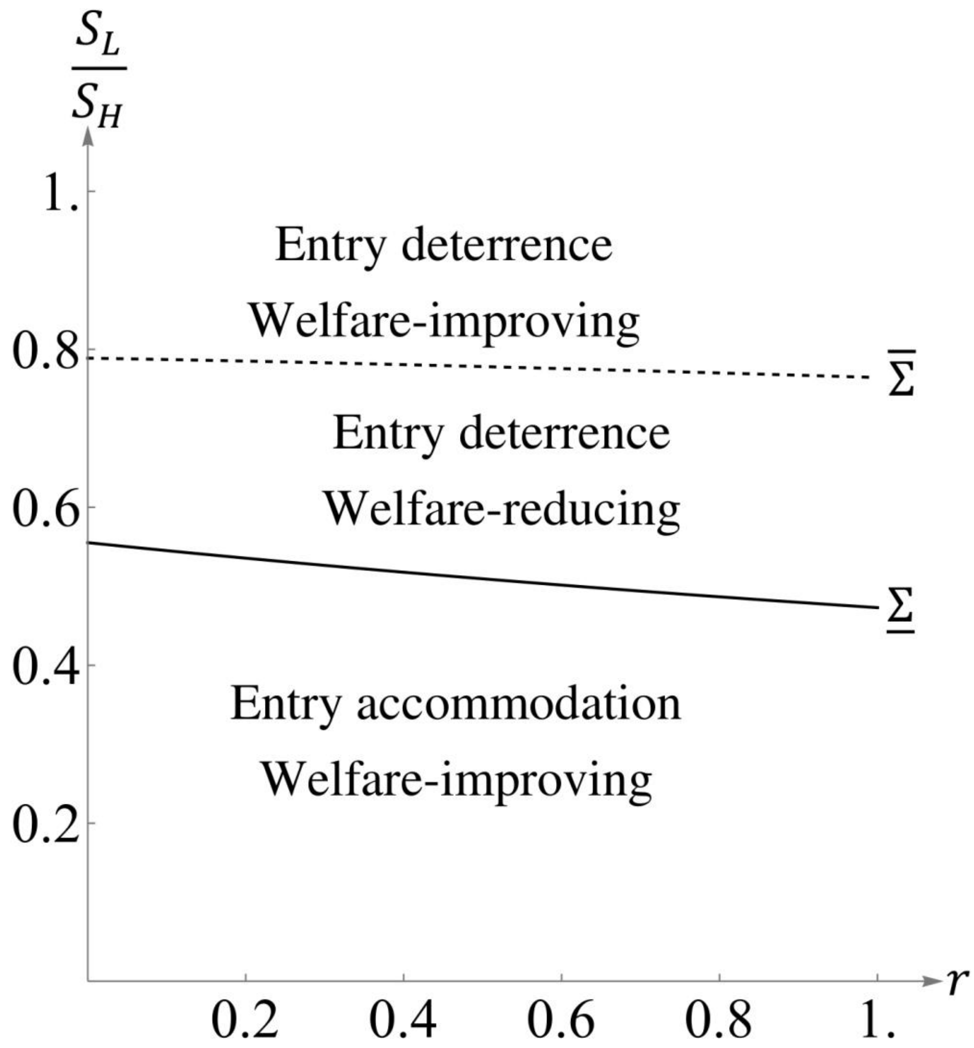
7.5.2 What if the regulator is informed?

- When $\underline{\Sigma} \leq \frac{S_L}{S_H} < \overline{\Sigma}$, it is still profitable to practice entry deterrence, so the high stock incumbent disguises itself as a low-stock counterpart,
  - and entry does not ensue that results in under-exploitation of the resource.



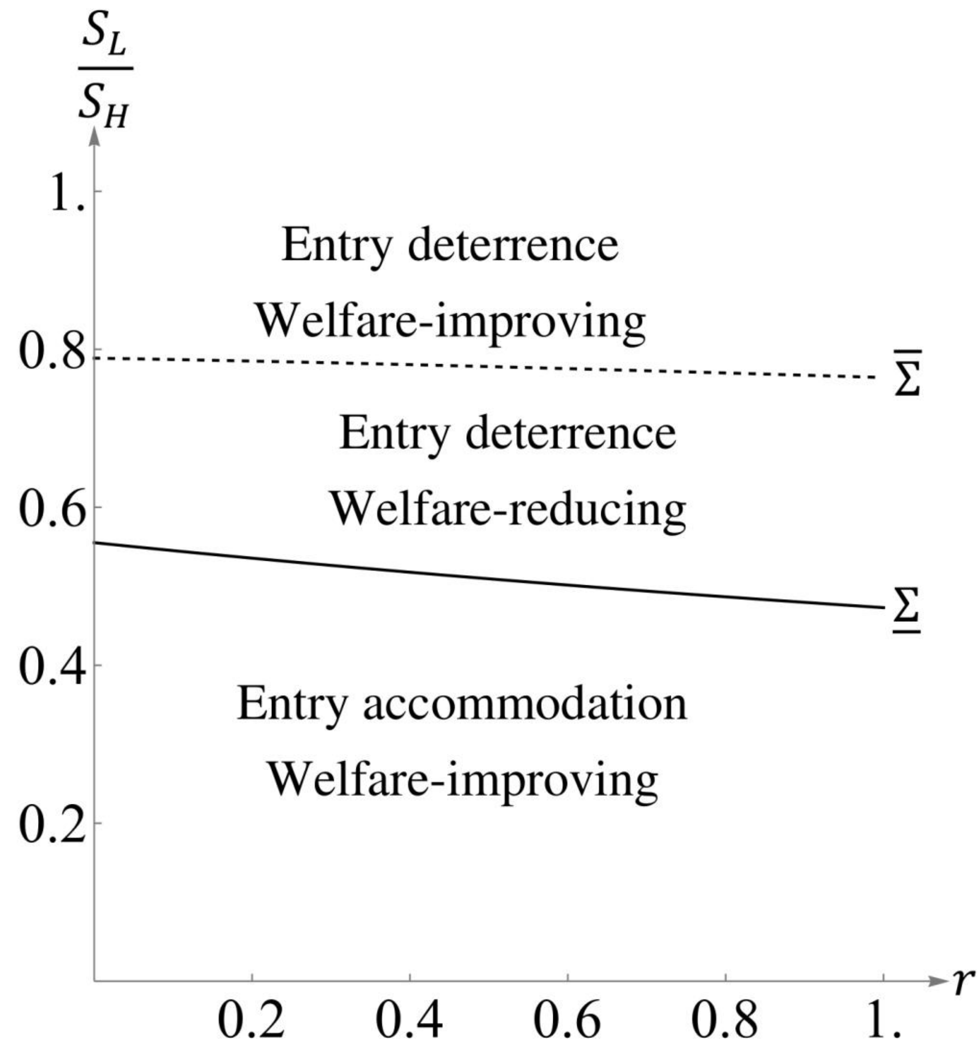
7.5.2 What if the regulator is informed?

- When $\underline{\Sigma} \leq \frac{S_L}{S_H} < \overline{\Sigma}$, it is still profitable to practice entry deterrence.
  - However, this comes at the expense of social welfare. In this setting, if the regulator is informed of abundance of the stock, it should publicize such information to create an environment conducive to entry.



7.5.2 What if the regulator is informed?

- Finally, when $\frac{S_L}{S_H} < \underline{\Sigma}$, the high stock incumbent does not have incentives to deter entry,
  - so market accommodation increases joint profits that ultimately promote social welfare.
- Therefore, the regulator can be *laissez-faire* and impose no restriction on entry into the commons.



7.5.2 What if the regulator is informed?

- ***Suppose the regulator considers consumer and producer surplus.***
  - Consider the inverse demand function $p(Q) = a - Q$. Social welfare is the area under the demand curve minus the total cost, where

$$W(Q) = \int_0^Q p(\hat{Q})d\hat{Q} - \frac{Q^2}{S_k} = \left[a\hat{Q} - \frac{\hat{Q}^2}{2}\right]_0^Q - \frac{Q^2}{S_k} = \frac{[2aS_k - (2 + S_k)Q]Q}{2S_k}$$

  - Assume a discount rate of δ, intertemporal welfare becomes

$$W = \frac{[(2a - x)S_k - 2x]x}{2S_k} + \delta \frac{\{(2a - Q)[S_k - (1 - r)x] - 2Q\}Q}{2[S_k - (1 - r)x]}$$

where $S_k = \{S_L, S_H\}$ and Q is the aggregate output in the second stage.

7.5.2 What if the regulator is informed?

➢ ***Consider the low stock incumbent in a separating equilibrium.***

- When it accommodates entry, the first-period extraction is $x_{L,E} = \frac{S_L[9-\delta(1-r)]}{18}$ while the second-period appropriation becomes $q^E\left(x_{L,E}\right) = \frac{S_L-(1-r)x_{L,E}}{3}$ for every firm, yielding $Q_L^E = \frac{2\left[S_L-(1-r)x_{L,E}\right]}{3}$.

- When entry is deterred, it faces no competition, so the first-period extraction is $x_L = \begin{cases} \frac{S_H\left[9\left(4-\delta(1-r)\right)-\sqrt{5\delta[72(1+r)+13\delta(1-r)^2]}\right]}{72} & if \quad \frac{S_L}{S_H} \geq \overline{S} \\ \frac{S_L[4-\delta(1-r)]}{8} & if \quad \frac{S_L}{S_H} < \overline{S} \end{cases}$ while the second-period appropriation is $q^{NE}(x_L) = \frac{S_L-(1-r)x_L}{2} \equiv Q_L^{NE}$.

7.5.2 What if the regulator is informed?

➢ ***It is welfare superior to deter entry if*** $\boldsymbol{W_L^{ED} \geq W_L^{AE}}$***, where***

- $$W_L^{ED} = \frac{[(2a - x_L)S_L - 2x_L]x_L}{2S_L} + \delta \frac{\{(2a - Q_L^{NE})[S_L - (1-r)x_L] - 2Q_L^{NE}\}Q_L^{NE}}{2[S_L - (1-r)x_L]}$$

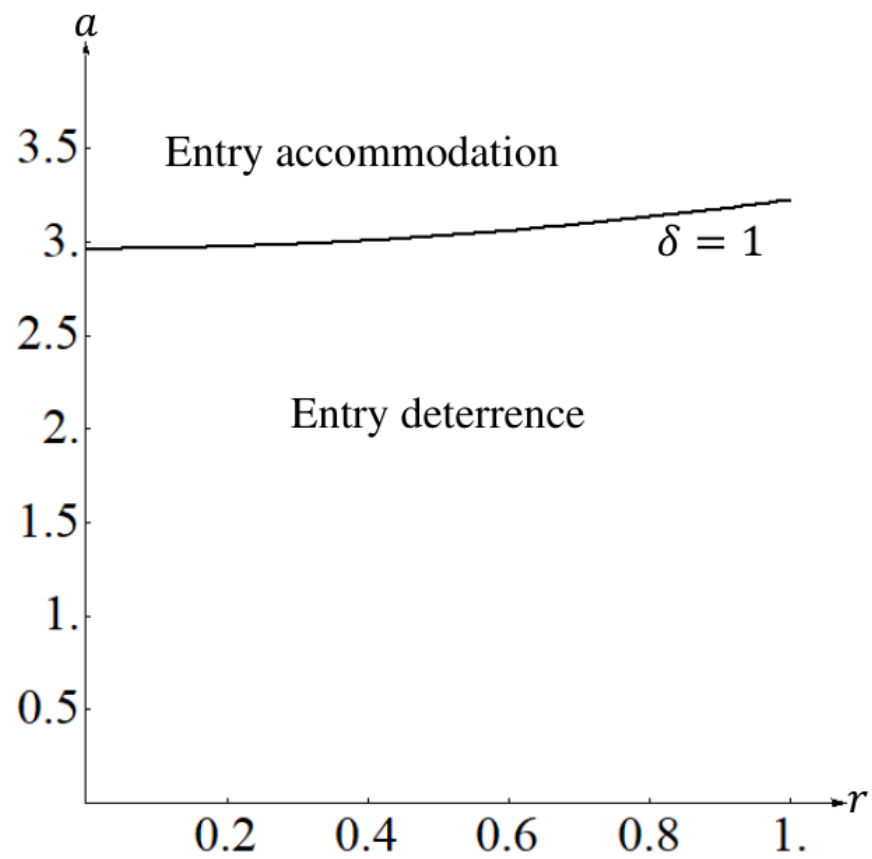
- $$W_L^{AE} = \frac{[(2a - x_{L,E})S_L - 2x_{L,E}]x_{L,E}}{2S_L} + \delta \frac{\{(2a - Q_L^{E})[S_L - (1-r)x_{L,E}] - 2Q_L^{E}\}Q_L^{E}}{2[S_L - (1-r)x_{L,E}]}$$

- Solving the above inequality, which is highly nonlinear, is intractable, so we assume $S_H = 10$, $S_L = 5$, and $\delta = 1$ to find the space of (a, r) pairs where the regulator prefers entry deterrence to accommodation.

7.5.2 What if the regulator is informed?

- The following figure plots the cutoff in a as a function of r assuming $\delta = 1$; above which, the market size is large enough such that the regulator finds it socially optimal to accommodate entry, but below, entry should be deterred.

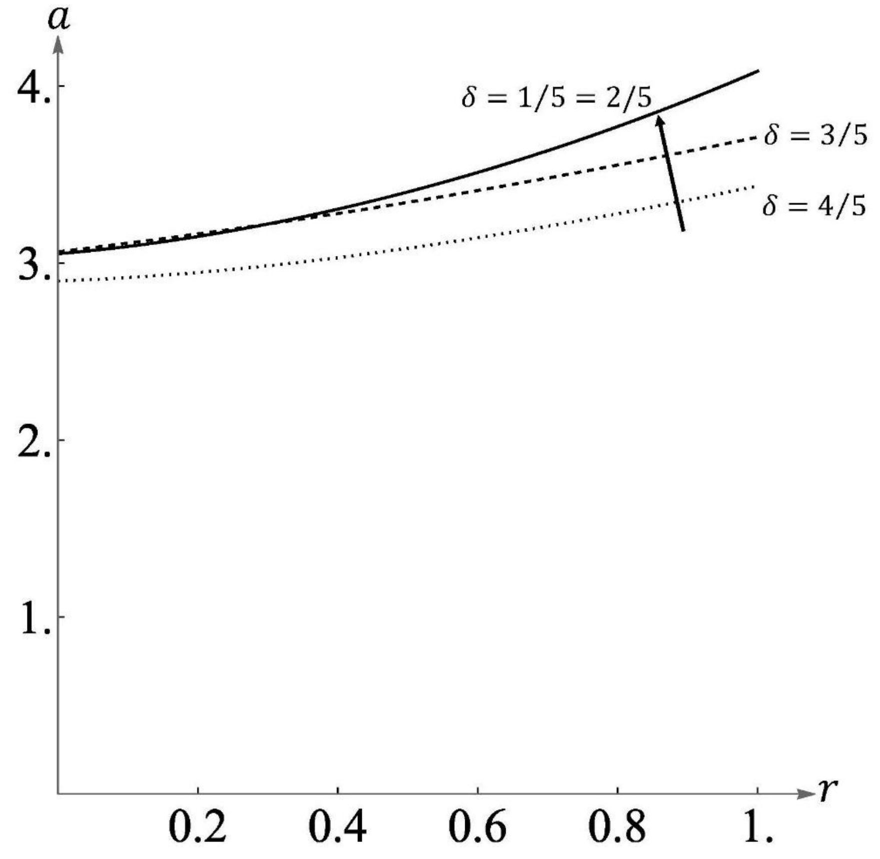
- When the stock regenerates quicker, it becomes more attractive, so that entry deterrence is more likely to be welfare superior in protecting the commons.



7.5.2 What if the regulator is informed?

- When δ decreases, the regulator pays less attention on future payoffs, so entry is accommodated under more restrictive conditions (higher cutoff).

- When δ is sufficiently low, cutoff $\underline{x}_H$ is slack. As the separating effort is nil, the low stock incumbent appropriates at the socially optimal level across both periods, where cutoffs coincide when $\delta = 1/5$ and when $\delta = 2/5$.



7.5.2 What if the regulator is informed?

➢ ***Consider the high stock incumbent in a pooling equilibrium.***

- When it accommodates entry, the first-period extraction is $x_{H,E} = \frac{S_H[9-\delta(1-r)]}{18}$ while the second-period appropriation is $q^E(x_{H,E}) = \frac{S_H-(1-r)x_{H,E}}{3}$ for every firm, yielding $Q_H^E = \frac{2[S_H-(1-r)x_{H,E}]}{3}$.

- When entry is deterred, the first-period extraction is $x_{L,NE} = \frac{S_L[4-\delta(1-r)]}{8}$ and the second-period extraction is $q^{NE}(x_{L,NE}) = \frac{S_H-(1-r)x_{L,NE}}{2} \equiv Q_H^{NE}$ since this firm monopolizes the market.

7.5.2 What if the regulator is informed?

➢ ***It is welfare superior to deter entry if*** $\boldsymbol{W_H^{ED} \geq W_H^{AE}}$***, where***

- $W_H^{ED} = \frac{[(2a - x_{L,NE})S_H - 2x_{L,NE}]x_{L,NE}}{2S_H} + \delta\frac{\{(2a - Q_H^{NE})[S_H - (1-r)x_{L,NE}] - 2Q_H^{NE}\}Q_H^{NE}}{2[S_H - (1-r)x_{L,NE}]}$

- $W_H^{AE} = \frac{[(2a - x_{H,E})S_H - 2x_{H,E}]x_{H,E}}{2S_H} + \delta\frac{\{(2a - Q_H^{E})[S_H - (1-r)x_{H,E}] - 2Q_H^{E}\}Q_H^{E}}{2[S_H - (1-r)x_{H,E}]}$

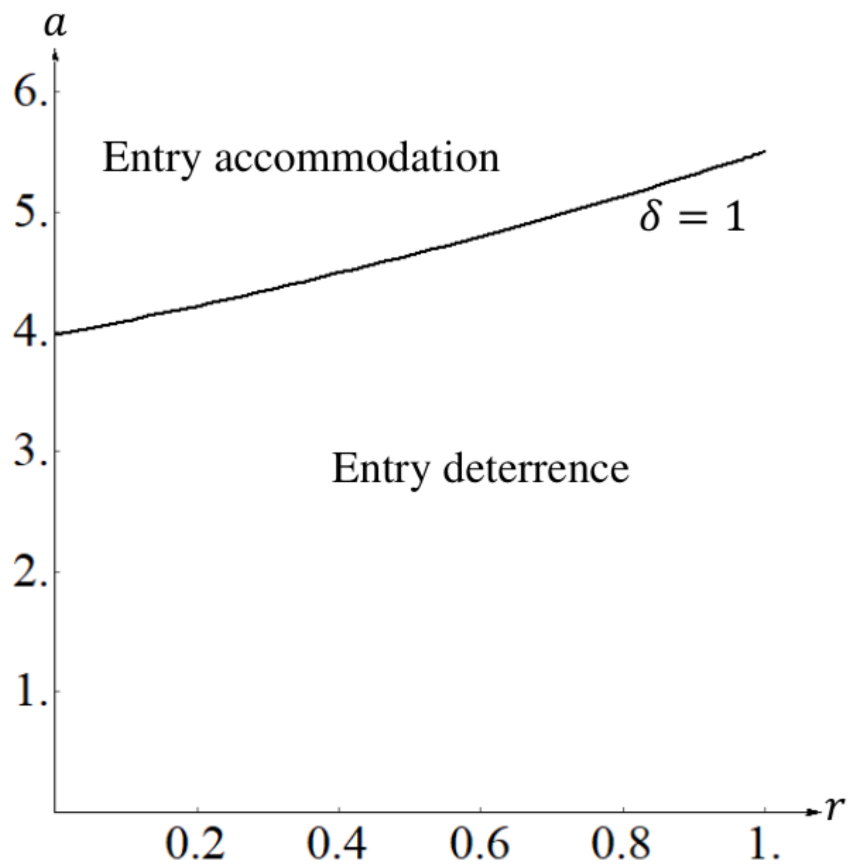
- Solving the above inequality, which is highly nonlinear, is intractable, so we assume $S_H = 10$, $S_L = 5$, and $\delta = 1$ to find the space of (a, r) pairs where the regulator prefers entry deterrence to accommodation.

7.5.2 What if the regulator is informed?

- The following figure plots the cutoff in a as a function of r assuming $\delta = 1$.

- Above this cutoff, when the market size a is sufficiently large, accommodating entry is welfare superior to deterrence.

- When the regeneration rate r increases, however, it is better off deterring entry to avoid over-exploitation of the stock.



7.5.2 What if the regulator is informed?

- When δ decreases, the regulator is less emphasized on future payoffs, so there is a clockwise rotation of the cutoff for more accommodation when the regeneration rate is relatively high.

- In contrast, when the resource is slow to replenish (low r), it is better off not to accommodate entry when the effect of over-exploitation on firms' profits is more substantial than the effect of market power on consumer surplus.