Chapter 13: Cheap talk games

Game Theory:

An Introduction with Step-by-Step Examples

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Introduction

- So far we signals that were costly for the sender
  - Years of education.
- We will now consider settings where signals are costless
  - Such signals are called “cheap talk”, since “talk is cheap” or costless.
- Some examples are
  - A lobbyist (sender) informing a member of Congress (receiver) about the situation of an industry she works for,
  - An investment banker (sender) recommending a client (receiver) which stocks to purchase.
- In chapters 10-12, we assumed utility to be a function of the message. Mathematically, $u(m, a, \theta)$, where m is a costly signal or message.

Introduction

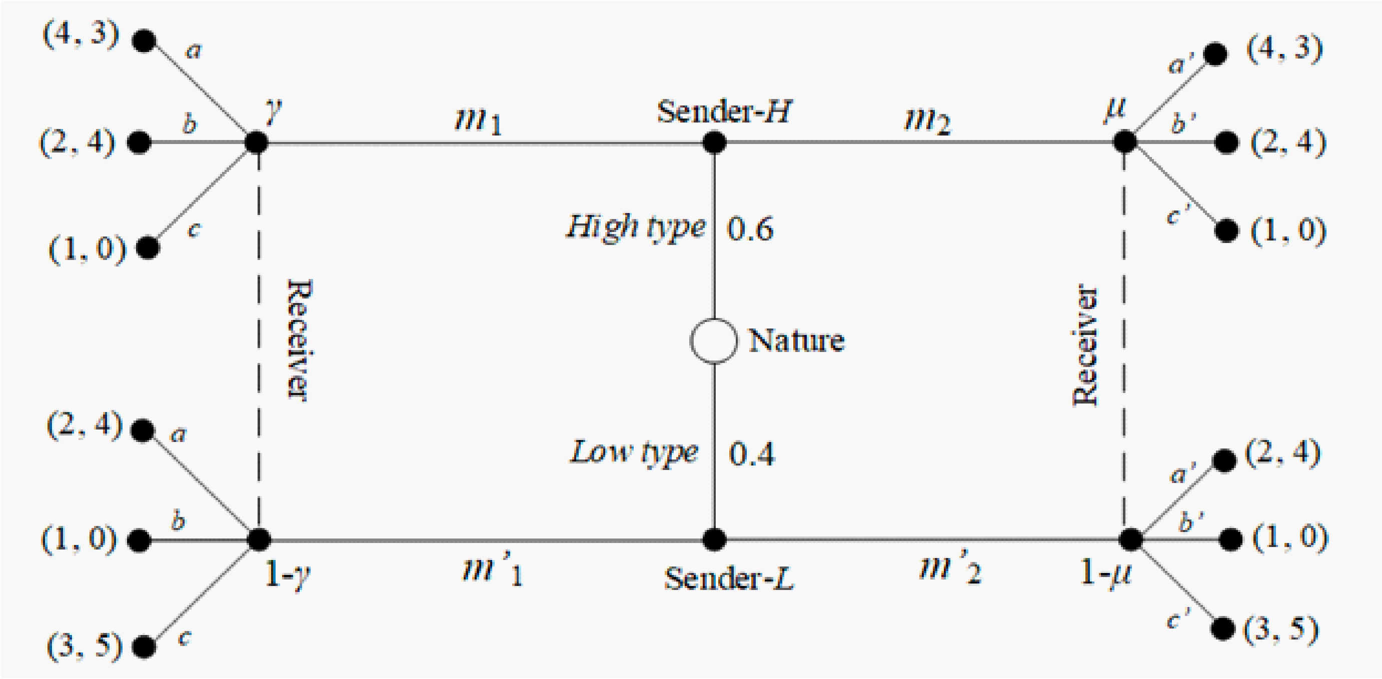
- In this chapter,

$$u(m, a, \theta) = u(m', a, \theta) \text{ for every } m \neq m'$$

- This property allows us to present utility more compactly as $u(a, \theta)$.

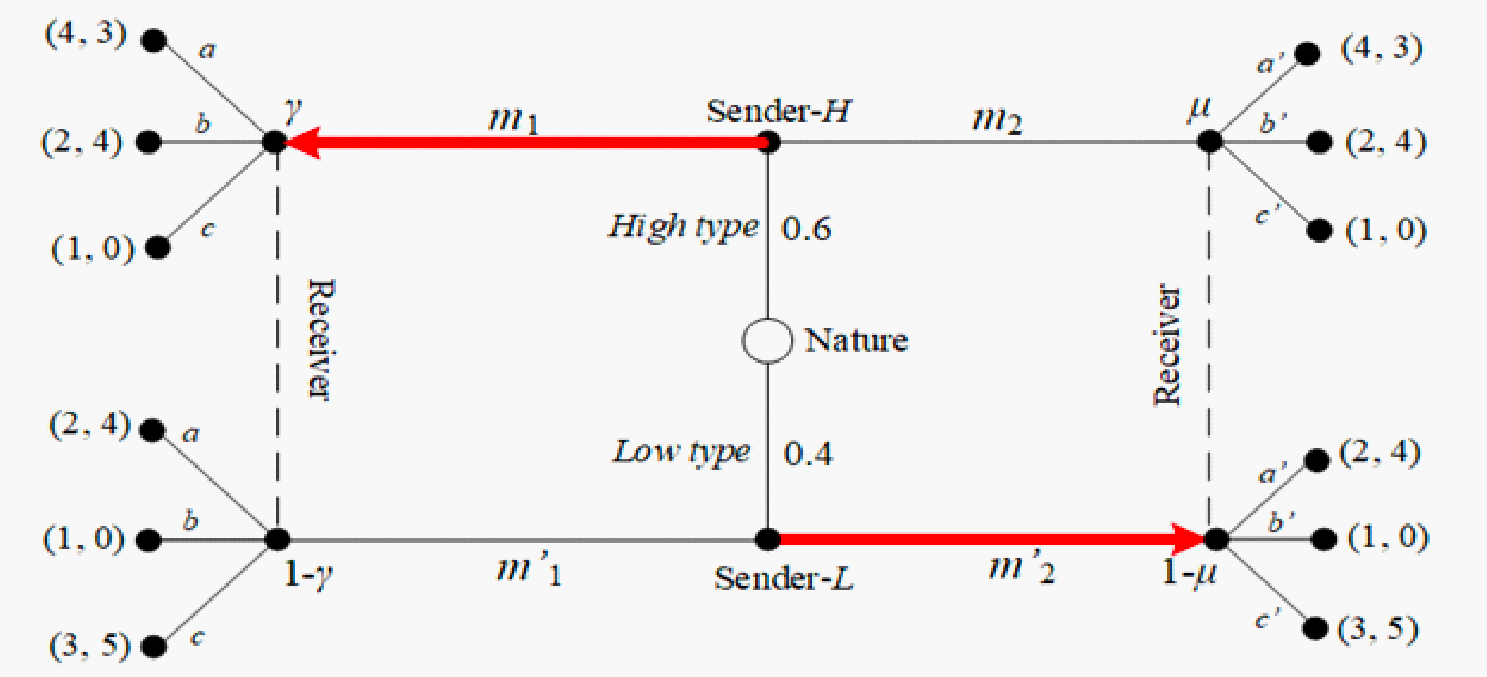
- Main goal: To identify if separating PBE is still possible.
- In previous chapters, to sustain separating PBEs, we need the cost of sending messages to differ across sender types.
- In this chapter, we show that separating PBEs can still emerge, but sender and receiver preferences have to be sufficiently similar.

Cheap talk with discrete messages and responses



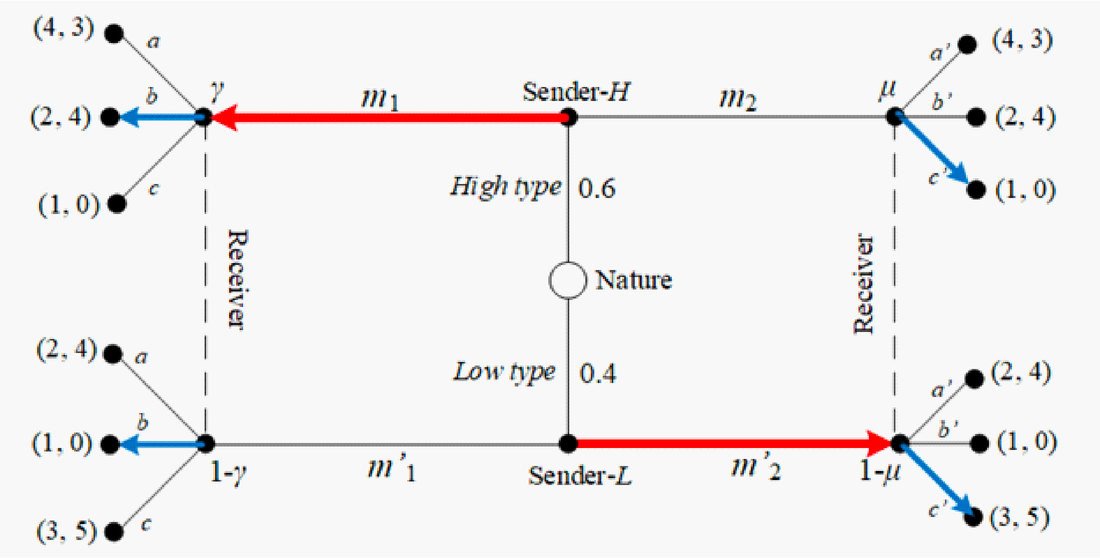
- Nature determines the sender's type (high or low).
- Sender observes this and sends message m_1 or m_2. Receiver only observes the signal.
- Receiver response with a, b or c after observing m_1(or a', b' or c' after observing m_2).
- Payoffs on the right and left coincide, because m_1 and m_2 do not affect utility (though they may affect the response of the receiver).
- Represents a cheap-talk setting because the sender's payoff is unaffected by her message.

Separating PBE



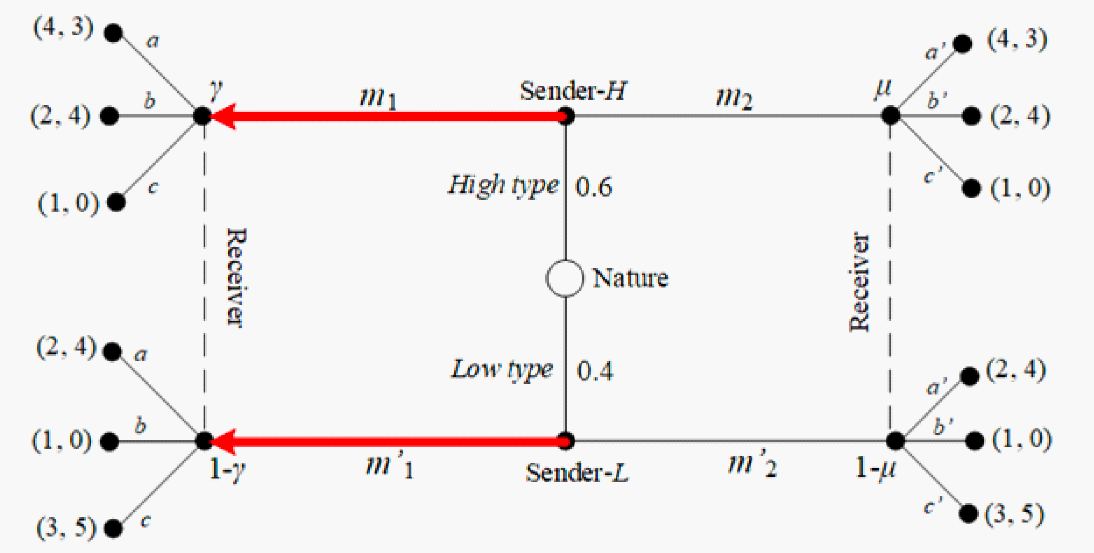
1. *Specifying a strategy profile.* Consider (m_1, m_2').
2. *Bayes' rule.* Upon observing $m_1 (m_2')$, the receiver believes that this message must originate from the high (low) type, entailing that $\mu(H|m_1) = 1$ at the top left side ($\mu(H|m_2) = 0$ at the top right side).
3. *Optimal response.* Receiver responds with:
   - b upon observing m_1, since it yields a payoff of 4 > 3 (payoff from a) > 0 (payoff from c)
   - c' upon observing m_2', since it yields a payoff of 5 > 4 (payoff from a') > 0 (payoff from b')

Separating PBE



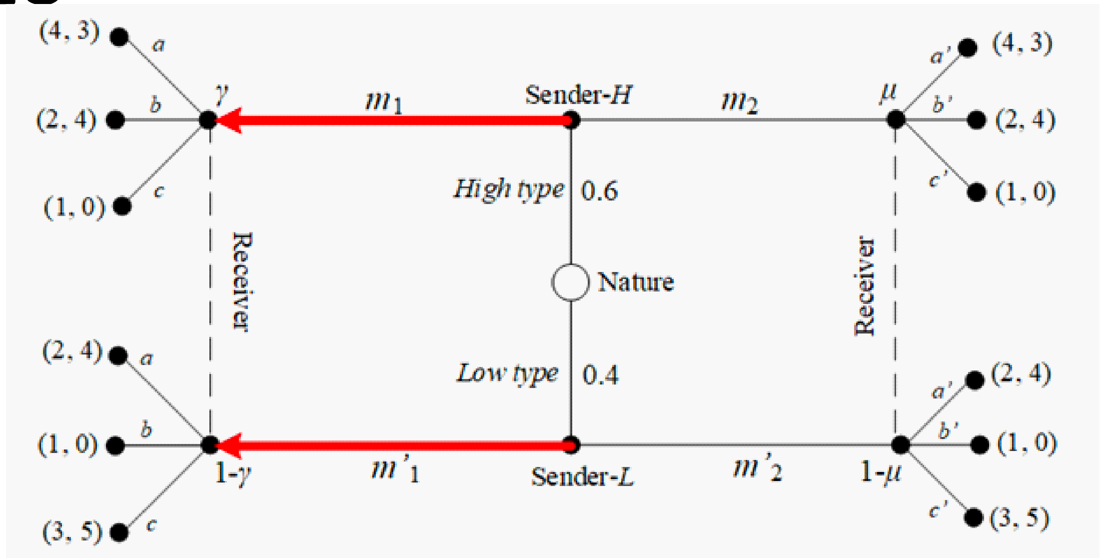
4. *Optimal messages.* Based on the receiver's optimal responses:
   - *High type.* If she sends a message of m_1, as prescribed in this strategy profile, she earns a payoff of 2 (since m_1 is subsequently responded with b), which exceeds her payoff from deviating towards m_2, 1, where the receiver responds with c'.
   - *Low type.* If she sends a message of m_2', as prescribed in this strategy profile, she obtains a payoff of 3 (as m_2' is responded with c'), which exceeds her payoff from deviating towards m_1, 1, where the receiver responds with b.
5. *Summary*. Here, (m_1, m_2') can be supported as a separating PBE.

Pooling PBEs



1. *Specifying a strategy profile.* Consider (m_1, m_1').
2. *Bayes' rule.* Upon observing m_1, the receiver's posterior beliefs coincide with her priors, entailing that $\mu(H|m_1) = 0.6$. Upon observing m_2, Bayes' rule does not help the receiver update her belief, which remains generic at $(\mu(H|m_2) \in [0,1]$.

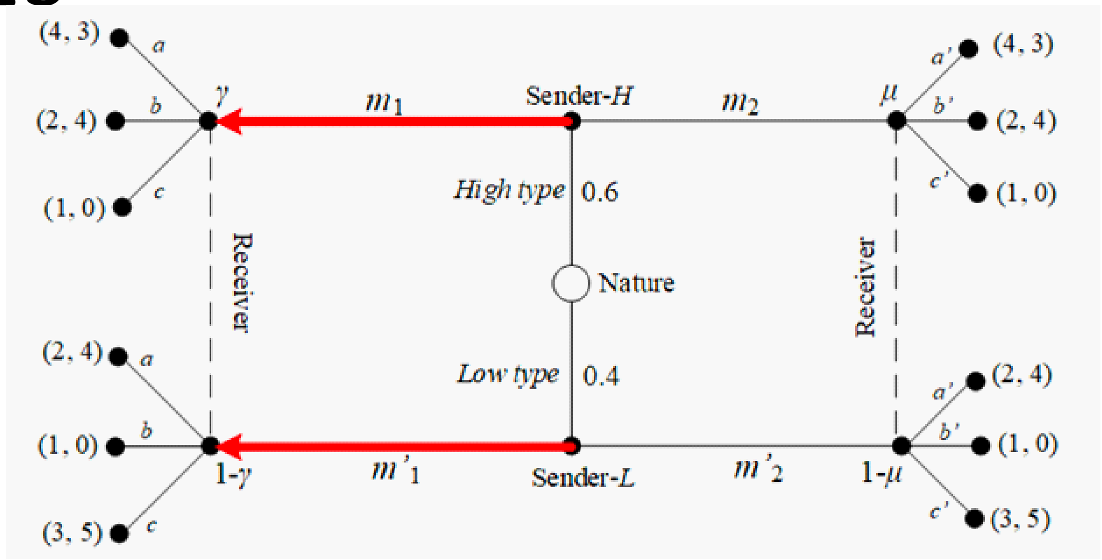
Pooling PBEs



3. *Optimal response.* Receiver responds with:
    - Upon observing m_1, responds with a (the expected utility for this is the highest of the three at 3.4).

$$EU_R(a) = (0.6 \times 3) + (0.4 \times 4) = 3.4$$
$$EU_R(b) = (0.6 \times 4) + (0.4 \times 0) = 2.4$$
$$EU_R(b) = (0.6 \times 0) + (0.4 \times 5) = 2.0$$

Pooling PBEs



3. *Optimal response.* Receiver responds with:
   - Upon observing m_2 , we need to find expected payoffs from each response:

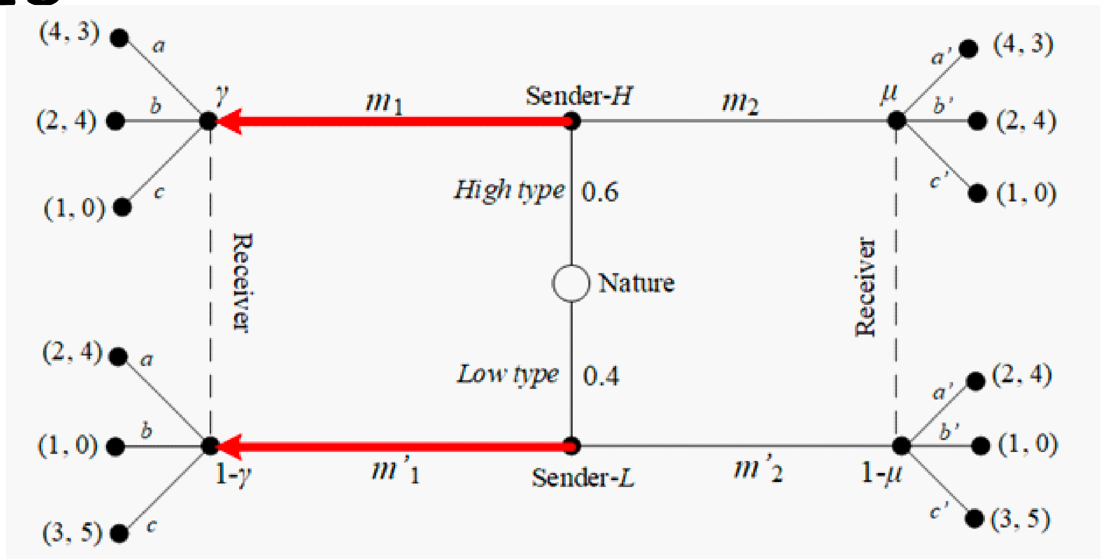
$$EU_R(a') = (\mu \times 3) + \big((1-\mu) \times 4\big) = 4 - \mu$$

$$EU_R(b') = (\mu \times 4) + \big((1-\mu) \times 0\big) = 4\mu$$

$$EU_R(b) = (\mu \times 0) + \big((1-\mu) \times 5\big) = 5(1-\mu)$$

   - Plot the three lines of $4-\mu$, 4μ, and $5(1-\mu)$; all as a function of μ.
     - This helps us see their crossing points.

Pooling PBEs



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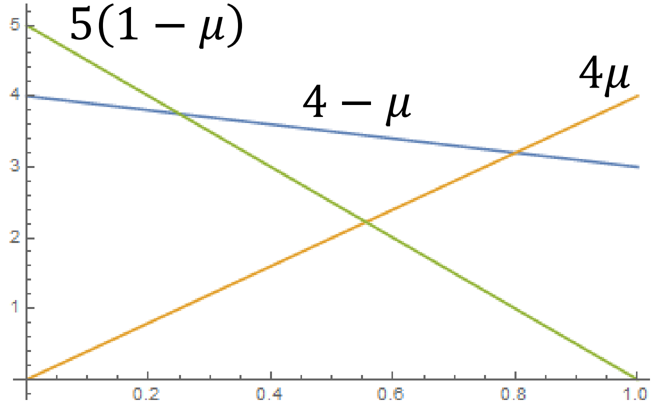
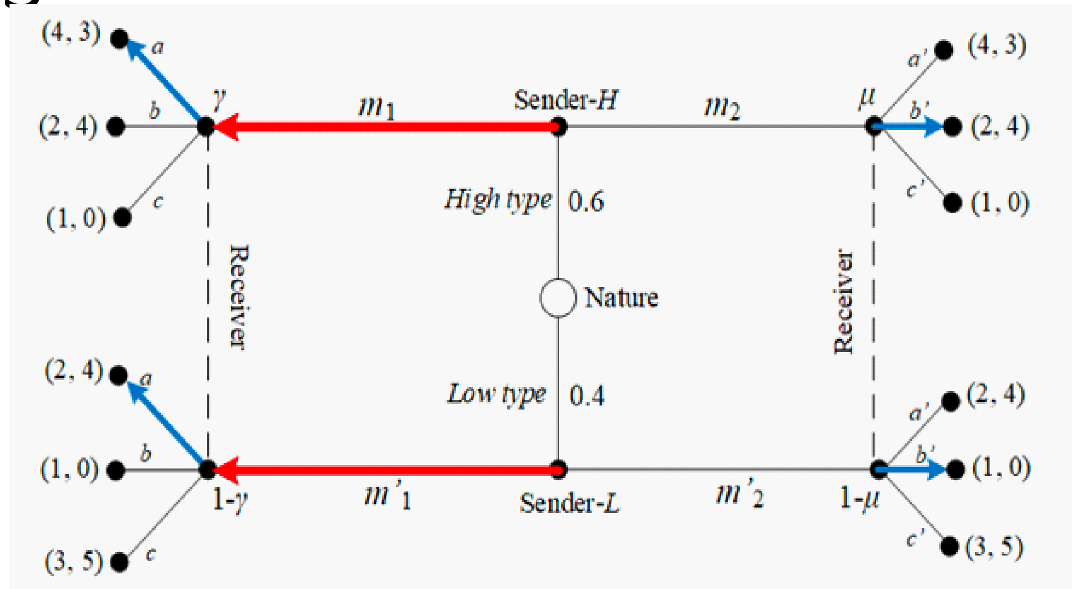
$$EU_R(b') = (\mu \times 4) + \big((1-\mu) \times 0\big) = 4\mu$$
$$EU_R(b) = (\mu \times 0) + \big((1-\mu) \times 5\big) = 5(1-\mu)$$

   - This implies that the best responses are
     - c' if $\mu > \frac{1}{4}$,
     - a' if $\frac{1}{4} < \mu \leq \frac{4}{5}$, and
     - b' otherwise.

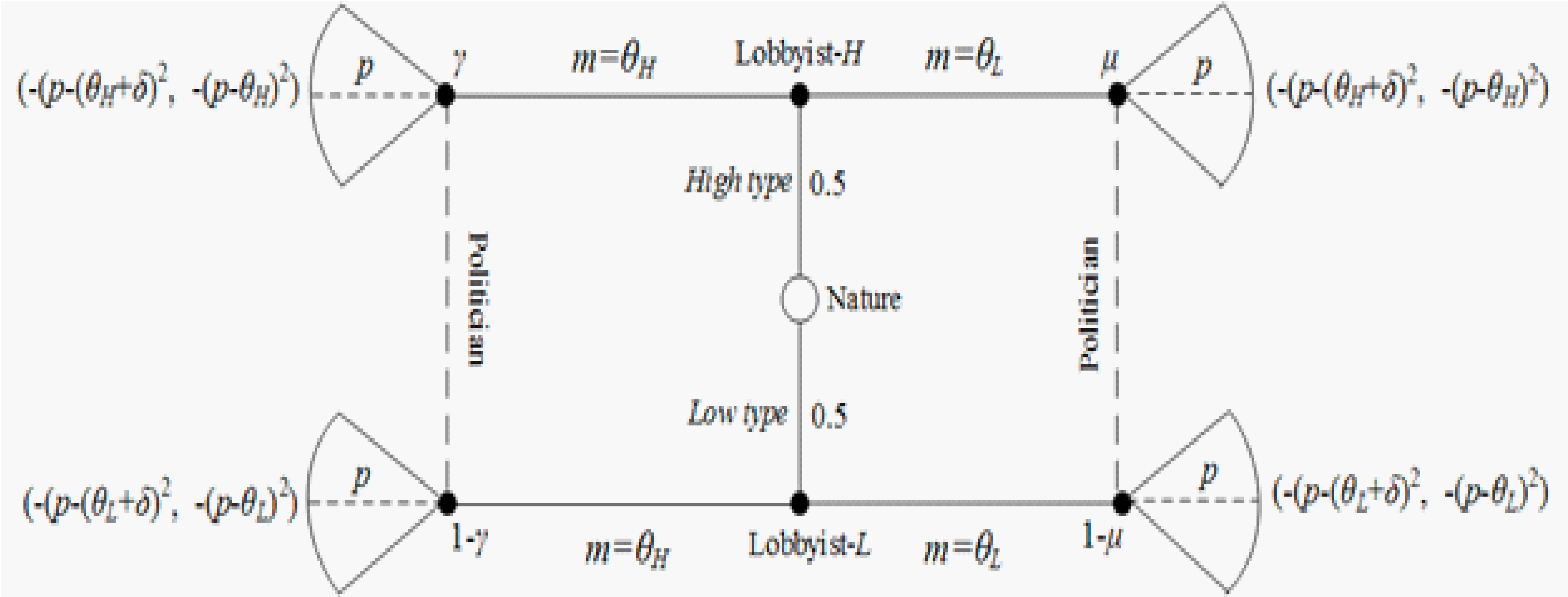


Pooling PBEs



4. *Optimal messages.* Based on the receiver's optimal responses:
   - *High type.* If she sends a message of m_1, as prescribed in this strategy profile, she earns a payoff of 4 (since m_1 is subsequently responded with a), which exceeds her payoff from deviating towards m_2, 2, where the receiver responds with b'.
   - *Low type.* If she sends a message of m_1' , as prescribed in this strategy profile, she obtains a payoff of 2 (as m_1' is responded with a), which exceeds her payoff from deviating towards m_2', 1, where the receiver responds with b'.
5. *Summary.* Here, (m_1, m_1') can be supported as a separating PBE with responses (a, b') and equilibrium beliefs $\mu(H|m_1) = 0.6$ and off-the-equilibrium beliefs $\mu(H|m_2) > \frac{4}{5}$.

Cheap talk with discrete messages but continuous responses



- We now allow for continuous responses by the receiver.
- After observing her own type privately (θ_H or θ_L), the lobbyist (sender), chooses a message to send to the politician which message to send (still binary).
- Upon observing this message, the politician responds with a policy $p > 0$.

Quadratic loss functions - Sender

- The payoffs of the government (politician) follow a quadratic loss function, as in Crawford and Sobel (1982)

$$U_G(p, \theta) = -(p - \theta)^2$$

- which becomes zero when the politician chooses policy $p = \theta$,
- but is negative otherwise (both when $p < \theta$ and when $p > \theta$, as depicted in the figure two slides from now).

Quadratic loss functions - Receiver

- Similarly, the lobbyist's utility is given by a quadratic loss function

$$U_L(p, \theta) = -\big(p - (\theta + \delta)\big)^2$$

- which becomes zero when the politician chooses $p = \theta + \delta$, that we can refer to as the lobbyist's ideal policy.
  - Otherwise, the lobbyist's payoff is negative (see figure in next slide).
- *Cases*:
  - When $\delta = 0$, the utility functions of both lobbyist and politician coincide, and we can say that their preferences are aligned.
  - Otherwise, the lobbyist's ideal policy is $p = \theta + \delta$, exceeding the politician's, $p = \theta$.
  - This explains why $\delta > 0$ is known as the lobbyist's "bias" relative to the politician.

Quadratic loss function (graphically)

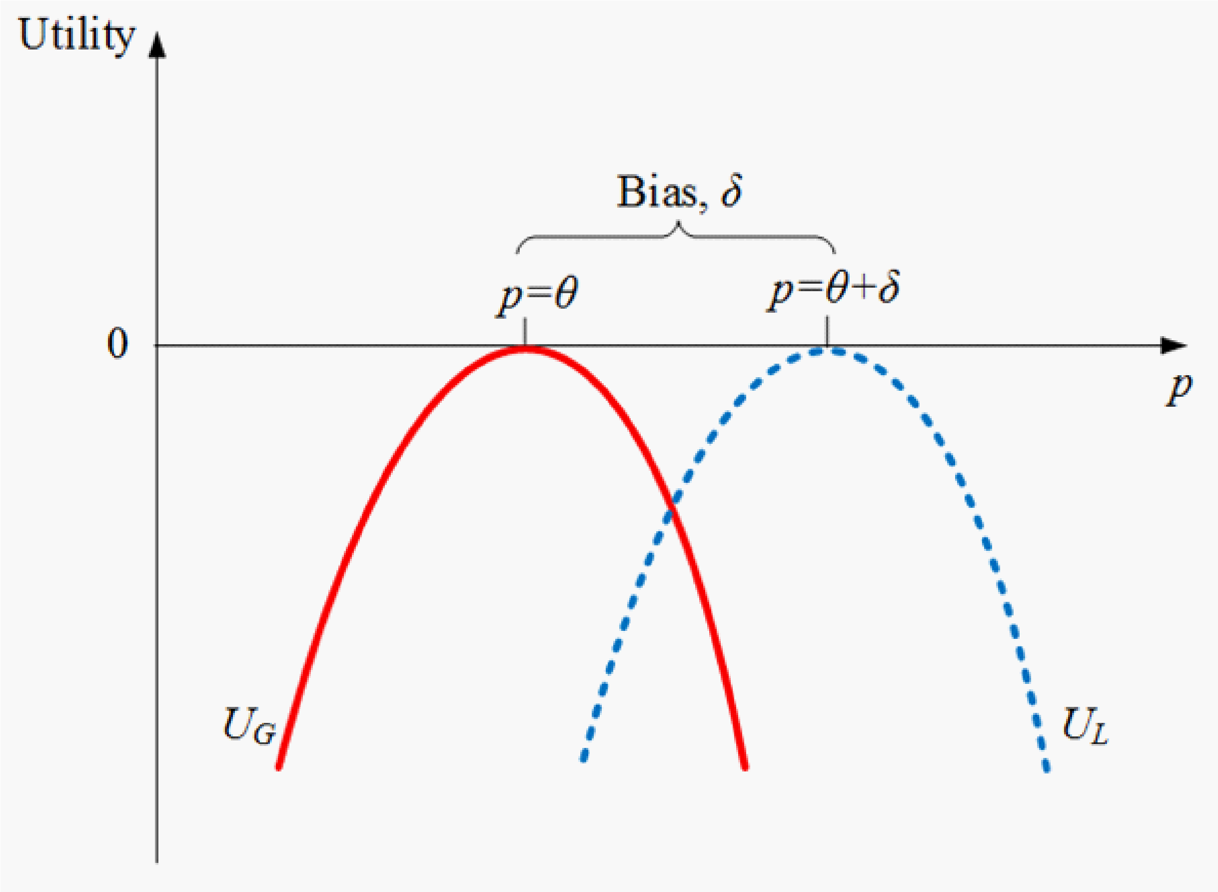
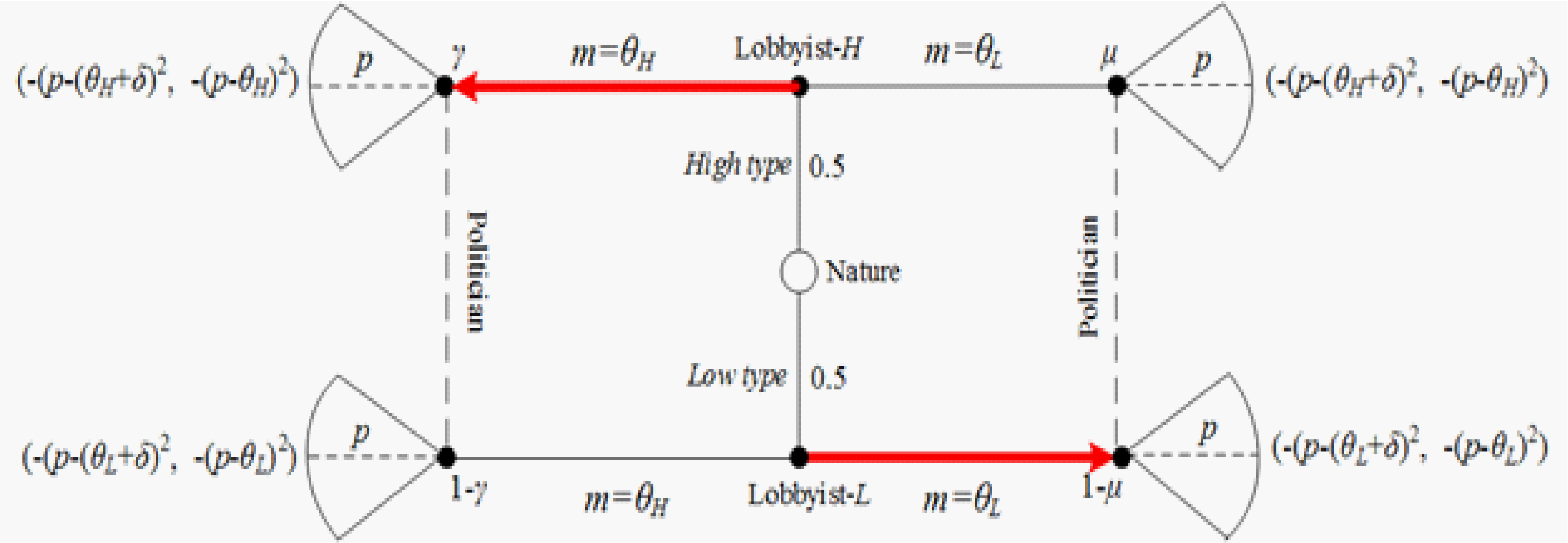


Figure 13.5 Quadratic loss function for each player

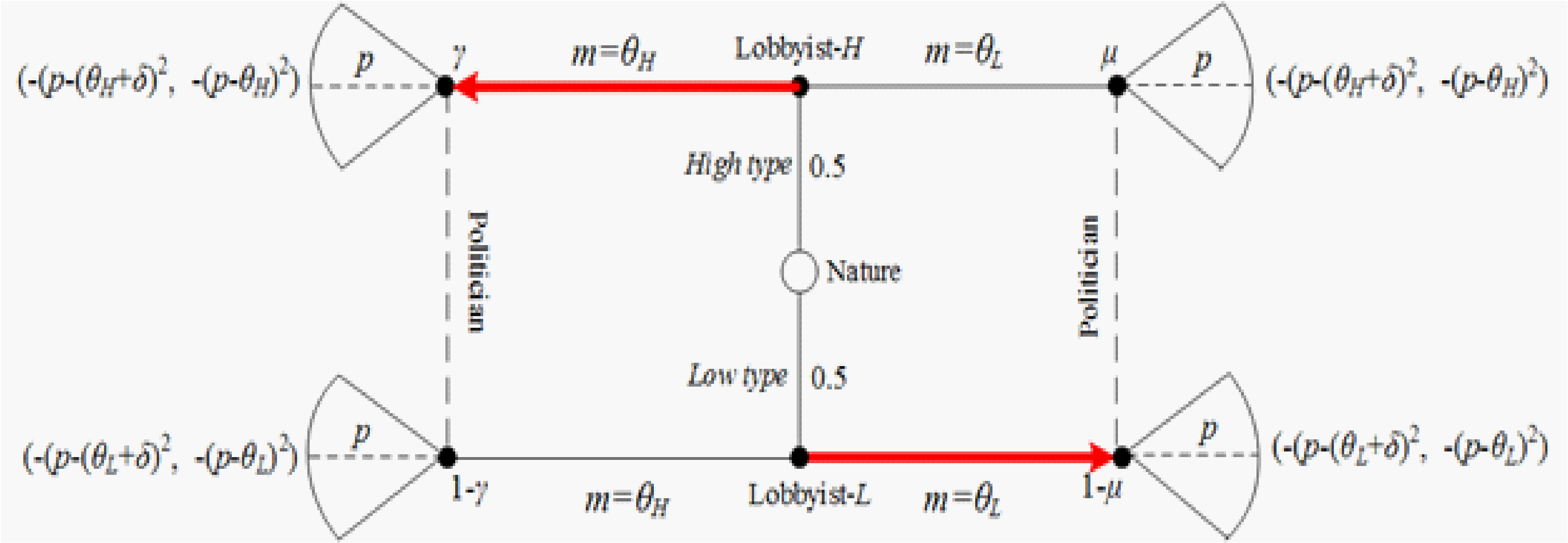
Separating PBE



1. *Specifying a strategy profile.* Consider (θ_H, θ_L).

2. *Bayes' rule.*
   - Upon observing θ_H, the politician (receiver) believes that this message must originate from the high type lobbyist (sender), entailing that $\mu(\theta_H|\theta_H) = 1$ and $\mu(\theta_L|\theta_H) = 0$.
   - Similarly, upon observing θ_L, the politician's beliefs are $\mu(\theta_H|\theta_L) = 0$ and $\mu(\theta_L|\theta_L) = 1$.

Separating PBE



3. Optimal response. Politician responds with responds with:

- $p = \theta_H$ upon observing θ_H, as this policy minimizes her quadratic loss, yielding a payoff of zero.
- $p = \theta_L$ upon observing θ_L, also minimizes her quadratic loss, yielding a payoff of zero.

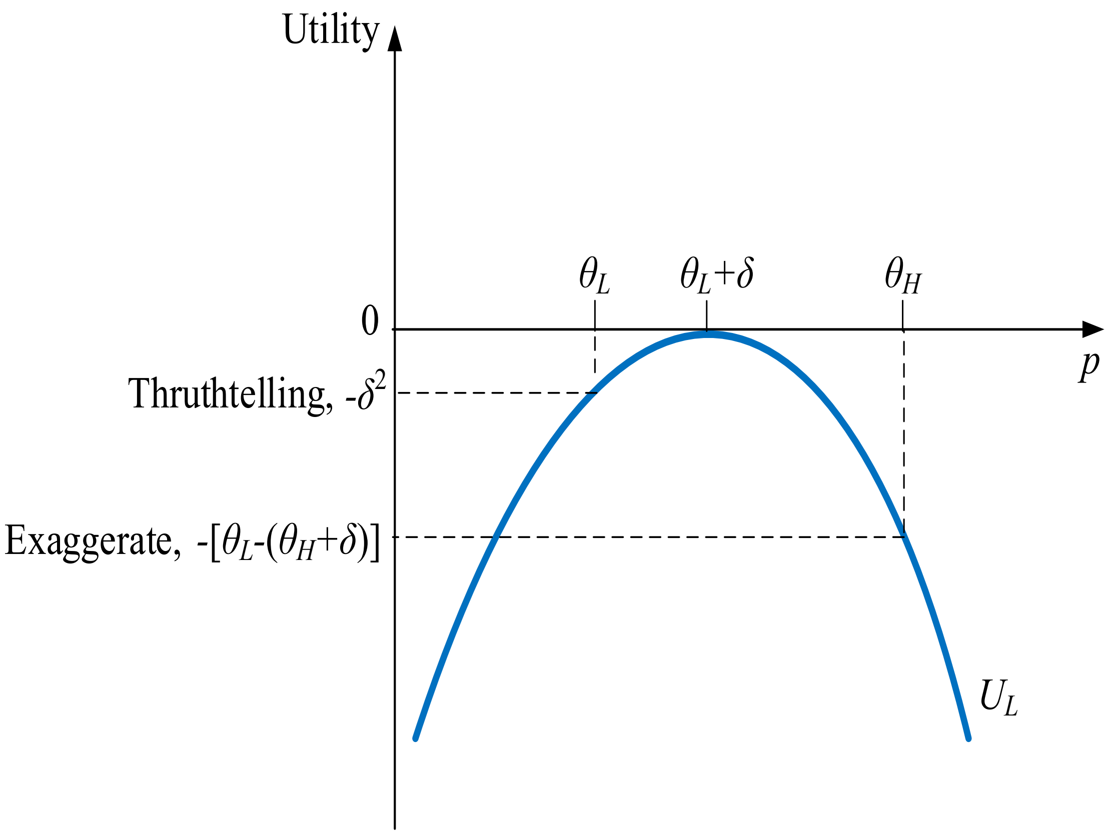
Separating PBE

4. *Optimal messages.* Based on the receiver's optimal responses:
   - *High type.* If she sends a message of θ_H, as prescribed in this strategy profile, she earns a payoff of $-\delta^2$, which exceeds her payoff from deviating towards θ_L,$-(\theta_L - \theta_H - \delta)^2$.
     - Role of bias.

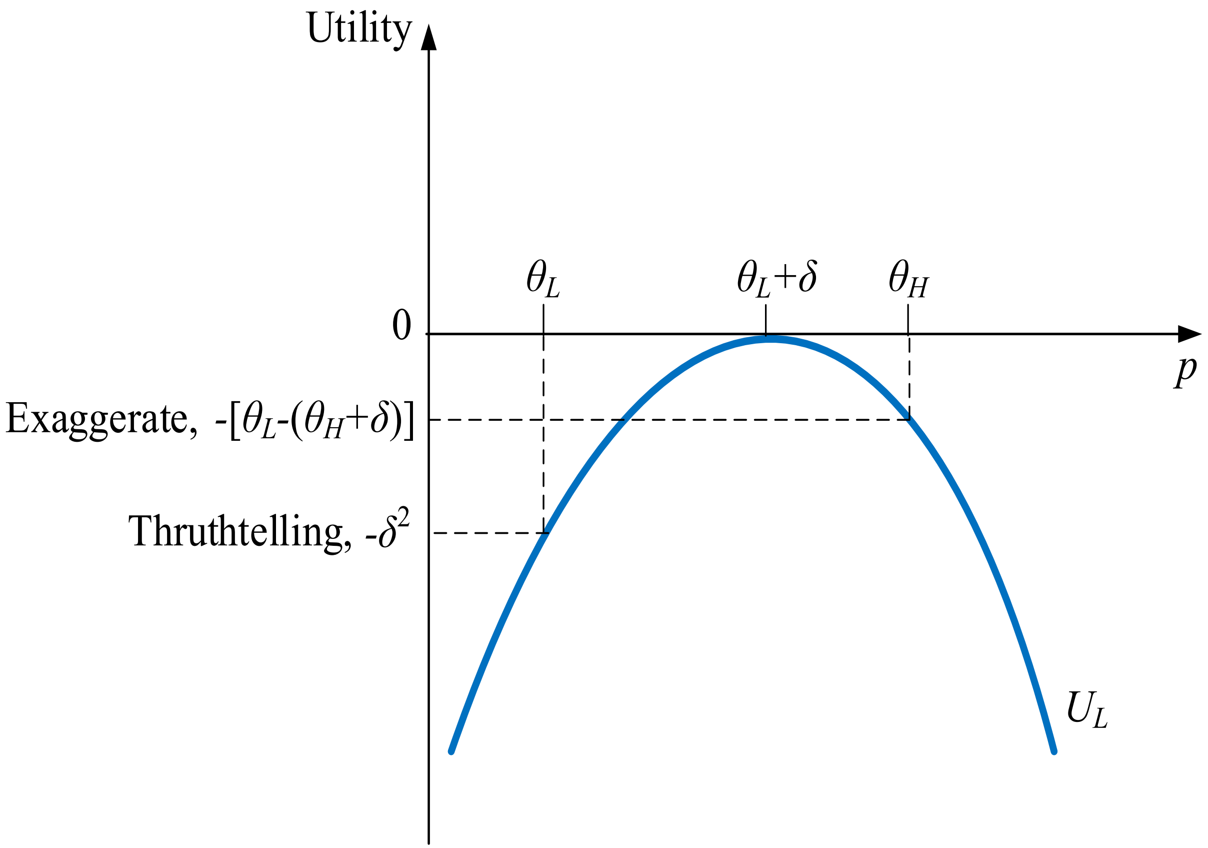
   - *Low type.* If she sends a message of θ_L, as prescribed in this strategy profile, she obtains a payoff of $-\delta^2$, which exceeds her payoff from deviating towards θ_H, $-\left(\theta_H - (\theta_L + \delta)\right)^2$ iff $\delta \leq \frac{\theta_H - \theta_L}{2}$.
     - See figures in next slide.

Separating PBE

Condition $\delta \leq \frac{\theta_H - \theta_L}{2}$ holds



Condition $\delta \leq \frac{\theta_H - \theta_L}{2}$ does not hold

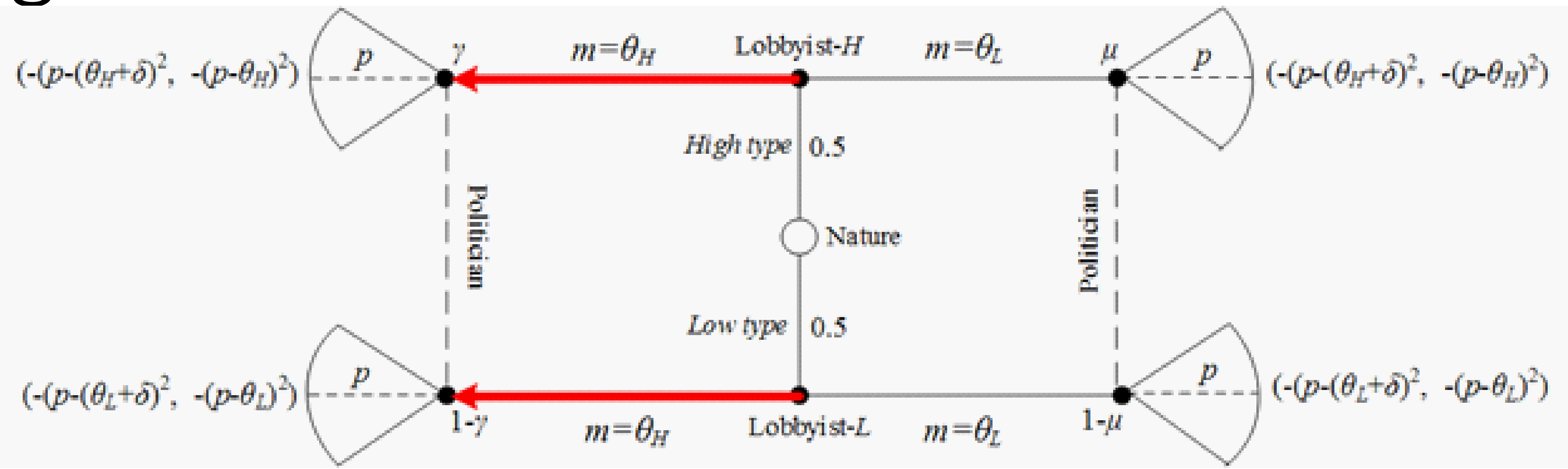


Separating PBE

5. *Summary.* Hence, (θ_H, θ_L) can be supported as a separating PBE if $\delta \leq \frac{\theta_H - \theta_L}{2}$, or if the lobbyist's bias to report the state of nature is sufficiently small.

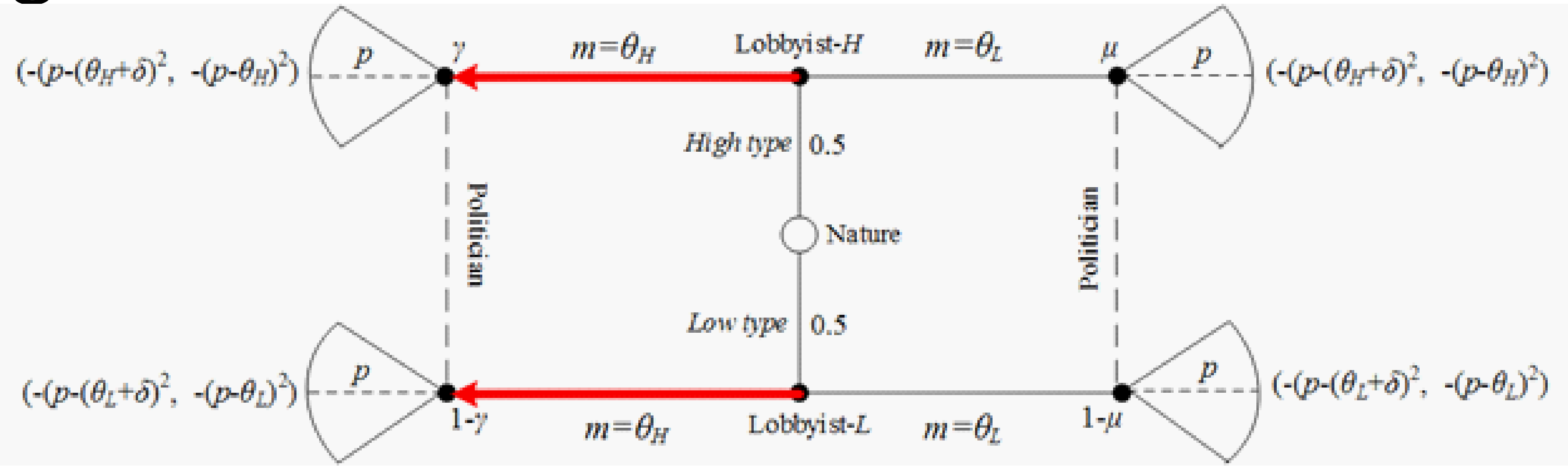
   - This happens when her preferences and the politician's are sufficiently *aligned*.

Pooling PBEs



1. *Specifying a strategy profile.* Consider (θ_H, θ_H).
2. *Bayes' rule.*
   - Upon observing θ_H, the politician's beliefs coincide with her priors, $\mu(\theta_H|\,\theta_H) = 0.5$.
   - Upon observing θ_L, Bayes' rule does not help the receiver update her belief, which remains generic at $(\mu(\theta_L|\theta_L) \in [0,1]$.

Pooling PBEs



3. *Optimal response.* Receiver responds with:

- $p = \frac{\theta_H+\theta_L}{2}$ upon observing θ_H, from $\max_{p\geq 0} \underbrace{-\frac{1}{2}(p-\theta_H)^2}_{if\ \theta=\theta_H} - \underbrace{\frac{1}{2}(p-\theta_L)^2}_{if\ \theta=\theta_L}$

- $p = \mu\theta_H + (1-\mu)\theta_L$ upon observing θ_L, from $\max_{p\geq 0} \underbrace{-\mu(p-\theta_H)^2}_{if\ \theta=\theta_H} - \underbrace{(1-\mu)(p-\theta_L)^2}_{if\ \theta=\theta_L}$

which can also be interpreted as the expected state of nature, given the politician's off-the-equilibrium beliefs μ and $(1-\mu)$ on the high and low state occurring, respectively.

Pooling PBEs

4. *Optimal messages.* From our results in Step 3, we now identify the lobbyist's optimal messages.
   - *High type.* The high type sends θ_H and this holds for all values of δ. Recall that this is due to the lobbyist's upward bias.

Pooling PBEs

- *Low type.*
  - Chooses between sending message θ_H, which induces that the politician responds with policy $p = \frac{\theta_H + \theta_L}{2}$ as shown in 3(a); or
  - Deviating towards the off-the-equilibrium message θ_L, which induces the politician to respond with policy $p = \mu\theta_H + (1-\mu)\theta_L$ as shown in 3(b). This holds if and only if

$$-\left[\frac{\theta_H + \theta_L}{2} - (\theta_L + \delta)\right]^2 \geq -[(\mu\theta_H + (1-\mu)\theta_L) - (\theta_L + \delta)]^2$$

After rearranging, it simplifies to $\mu \geq \frac{1}{2}$.

This means that, when the receiver believes the off-the-equilibrium message of θ_L is more likely to originate from the high-type sender, the low-type lobbyist has incentives to choose the pooling message θ_H.

Pooling PBEs

- This includes the case in which message θ_L is responded with policy $p = \theta_H$, i.e., $\mu = 1$.
- Importantly, this entails that the pooling PBE can be sustained for all values of δ.
- This is a typical result in cheap-talk games: while the separating PBE requires sender and receiver to exhibit similar preferences (low δ), the pooling PBE can be sustained regardless of players' preference divergence (for all δ).
- Informally, concealing information from the receiver can easily arise in equilibrium, but conveying it requires more demanding conditions.

Pooling PBEs

5. Summary. From step 4, we found that no sender type has incentives to deviate from the pooling strategy profile,(θ_H, θ_H), sustaining it as a PBE if $\mu \geq \frac{1}{2}$, for all value of δ.

In this PBE, the politician:

- upon observing message θ_H in equilibrium, holds beliefs $\mu(\theta_H, \theta_H) = \frac{1}{2}$, responding with $p = \frac{\theta_H + \theta_L}{2}$; and
- upon observing θ_L off-the-equilibrium path, her beliefs are unrestricted, $\mu(\theta_L, \theta_L) = \mu$, responding with policy $p = \mu\theta_H + (1 - \mu)\theta_L$.

Cheap talk with continuous messages and responses

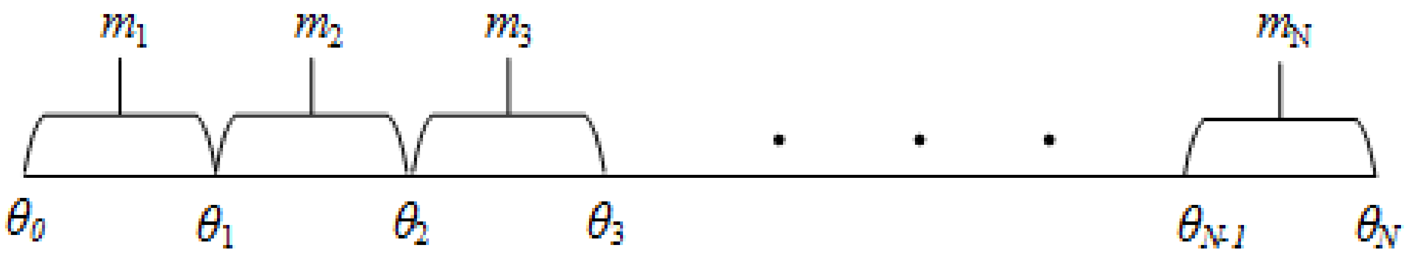
- We now extend our cheap talk model to allow for both
  - Continuous messages.
  - Continuous responses.
  - In addition, we allow for the state of nature to be continuous, i.e., $\theta \sim U[0,1]$.

Cheap talk with continuous messages and responses

- Our discussion, based on Crawford and Sobel (1982), helps us confirm that separating strategy profiles can emerge in equilibrium if the sender's and receiver's preferences are relatively aligned.

- This more general setting, however, allows us a new result:
  - That the "quality of information," understood as the number of different messages that the lobbyist sends, also depends on the players' preference alignment.

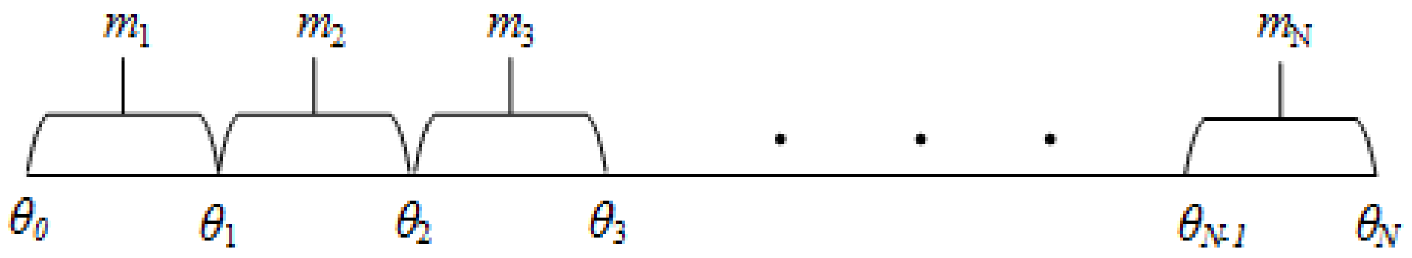
Separating PBE



1. *Specifying a strategy profile.* The lobbyist sends message m_k when $\theta \in [\theta_{k-1}, \theta_k)$, where k∈ $\{1,2, \dots, N\}$.

2. *Bayes' rule.*
   - Upon observing m_k, the politician (receiver) believes that:
     - $\mu(\text{Interval } k|m_k) = 1$ and
     - $\mu(\text{Interval } j|m_k) = 0$ for all $j \neq k$.

Separating PBE



3. *Optimal responses.* Politician responds with responds with:
   - $p = \frac{\theta_{k-1}+\theta_k}{2}$ upon observing m_k, which minimizes the quadratic loss in that interval.
   - Derived from:
     - $\max\limits_{p\geq 0} \; -E[(p-\theta)^2]$ such that $\theta \in [\theta_{k-1}, \theta_k] \;\; \Rightarrow \;\; \max\limits_{p\geq 0} \; -\left(p - \frac{\theta_{k-1}+\theta_k}{2}\right)^2$
     - Differentiating with respect to policy p, yields:
       $$-2\left(p - \frac{\theta_{k-1}+\theta_k}{2}\right)(-1) = 2\left(p - \frac{\theta_{k-1}+\theta_k}{2}\right) = 0 \Rightarrow p = \frac{\theta_{k-1}+\theta_k}{2} \text{ (optimal policy)}$$
     - In other words, after receiving message m_k,the politician (receiver) responds with a policy that coincides with the expected state of the nature in this interval, $p = \frac{\theta_{k-1}+\theta_k}{2}$.

Separating PBE

4. *Optimal messages.* To check optimal messages, without loss of generality, it suffices to show that the k-th type sender sends neither m_{k-1} or m_{k+1} (in the intervals immediately below and above interval k, respectively).

- *No incentives to overreport* – occurs if

$$\underbrace{-\left(\frac{\theta_{k-1}+\theta_k}{2} - (\theta_k + \delta)\right)^2}_{\substack{Utility\ from\ sending \\ message\ m_k}} \geq \underbrace{-\left(\frac{\theta_{k+1}+\theta_k}{2} - (\theta_k + \delta)\right)^2}_{\substack{Utility\ from\ sending \\ message\ m_{k+1}}}$$

Term $(\theta_k + \delta)$ lies to right-hand side of $\frac{\theta_{k-1}+\theta_k}{2}$, but to the left side of $\frac{\theta_{k+1}+\theta_k}{2}$, implying that

$$\frac{\theta_{k+1} + \theta_k}{2} > \theta_k + \delta > \frac{\theta_{k-1} + \theta_k}{2}.$$

This implies that the LHS of this inequality is negative while that of the RHS is positive, helping us rewrite it as follows

$$-\left[(-1)\left(\frac{\theta_{k-1} + \theta_k}{2} - (\theta_k + \delta)\right)\right]^2 \geq -\left(\frac{\theta_{k+1} + \theta_k}{2} - (\theta_k + \delta)\right)^2$$

Separating PBE

Multiplying both sides by -1 and taking square roots on both sides, we obtain that

$$\frac{\theta_{k-1} + \theta_k}{2} - (\theta_k + \delta) \geq (\theta_k + \delta) - \frac{\theta_{k+1} + \theta_k}{2}$$

Rearranging the above expression, we find that

$$\theta_{k-1} + \theta_k - 2(\theta_k + \delta) \geq 2(\theta_k + \delta) - \theta_{k+1} - \theta_k$$

And further simplifying, yields

$$\theta_{k+1} \geq 2\theta_k - \theta_{k-1} + 4\delta$$

Separating PBE

- *No incentives to underreport* – checking the $(k+1)-th$ sender, occurs if

$$-\left(\frac{\theta_{k+1}+\theta_k}{2}-(\theta_{k+1}+\delta)\right)^2 \geq -\left(\frac{\theta_k+\theta_{k-1}}{2}-(\theta_{k+1}+\delta)\right)^2$$

- Following the same approach in the case of "no incentives to overreport", we now have that $\frac{\theta_{k+1}+\theta_k}{2}<\frac{\theta_k+\theta_{k-1}}{2}$, implying that the LHS of the above inequality is negative, whereas the RHS is positive. Rearranging the above expression, we obtain:

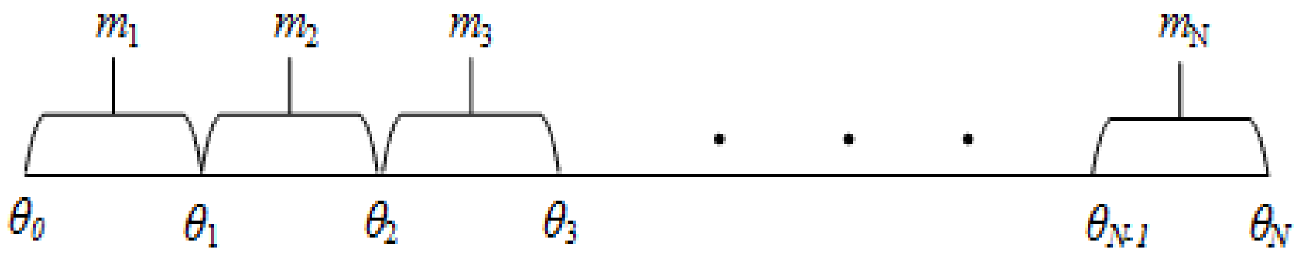
$$\theta_{k+1}+\delta-\frac{\theta_{k+1}+\theta_k}{2} \geq \frac{\theta_k+\theta_{k-1}}{2}-(\theta_{k+1}+\delta)$$

$$\Rightarrow \theta_{k+1} \geq \frac{1}{3}(2\theta_k-\theta_{k-1}-4\delta)$$

- Since, $\theta_{k+1}>\theta_k$ by construction, the k^{th} sender has no incentives to underreport.
  - Intuitively, the lobbyist finds it unprofitable to report a lower type to the politician, for all values of the bias parameter δ.
- Therefore, in general, the condition for the k^{th} sender to send the appropriate message is

$$\theta_{k+1} \geq 2\theta_k-\theta_{k-1}+4\delta$$

Separating PBE



5. *Summary.* The sender has no incentives to underreport, and does not have incentives to over-report iff $\theta_{k+1} \geq 2\theta_k - \theta_{k-1} + 4\delta$.

Separating PBE

- We have shown the conditions under which θ is truthfully reported by the sender.
- There are a few things we have not characterized.
  i. The number of partitions that can be sustained in equilibrium, N
  ii. How is this number of partitions affected by the preference divergence parameter, δ; and
  iii. The length of each of these partitions (intervals), as they are not necessarily equally long.

Equilibrium number of partitions

- $\theta \sim U[0,1]$, first interval starts at 0, last interval finishes at 1.
- Let $d = \theta_1 - \theta_0$.
- We can now arrange the 'no incentives to over-report' as

$$\theta_{k+1} - \theta_k \geq (\theta_k - \theta_{k-1}) + 4\delta.$$

- If this condition binds,

$$\theta_2 - \theta_1 = \underbrace{(\theta_1 - \theta_0)}_{d} + 4\delta$$

- We can keep doing this exercise and get

Equilibrium number of partitions

$$
\begin{aligned}
\theta_k - \theta_{k-1} &= (\theta_{k-1} - \theta_{k-2}) + 4\delta \\
&= (\theta_{k-2} - \theta_{k-3}) + (2 + 4\delta) \\
&= (\theta_{k-3} - \theta_{k-4}) + (3 + 4\delta) \\
&= \ \dots \\
&= \ (\theta_1 - \theta_0) + [(k-1) \ \times \ 4\delta] \\
&= \ d + 4(k-1)\delta
\end{aligned}
$$

- When $k = N$, $\theta_N - \theta_{N-1} = d + 4(N-1)\delta$
- Since $\theta_N - \theta_0 = 1$

Equilibrium number of partitions

- We can now express the length of the unit interval, $\theta_N - \theta_0 = 1$, as the sum of N partitions as follows:

$$\theta_N - \theta_0 = \underbrace{(\theta_N - \theta_{N-1})}_{d+4(N-1)\delta} + \underbrace{(\theta_{N-1} - \theta_{N-2})}_{d+4(N-2)\delta} + \cdots + \underbrace{(\theta_2 - \theta_1)}_{d+4\delta} + \underbrace{(\theta_1 - \theta_0)}_{d}$$

$$\underbrace{\theta_N - \theta_0}_{=1} = Nd + 4\delta \quad \underbrace{[(N-1) + (N-2) + \cdots + 1]}_{=\frac{N(N-1)}{2} \; since \; 1+2+\cdots+(N-1)=\frac{N(N-1)}{2}}$$

Therefore, the above expression simplifies to:

$$\theta_N - \theta_0 = Nd + 4\delta \frac{N(N-1)}{2} = Nd + 2\delta N(N-1)$$

Equilibrium number of partitions

- And since the LHS is $\theta_N - \theta_0 = 1$ (θ lies in the unit interval), we can write the above equation as

$$1 = Nd + 2\delta N(N-1),$$

- and solve for the bias parameter, δ, to obtain:

$$\underline{\delta}\,(d) = \frac{1 - Nd}{2N(N-1)} \Rightarrow \underline{\delta}\,(0) = \frac{1}{2N(N-1)}$$

- Cutoff $\underline{\delta}\,(d)$ decreases in d.
- As d increases, a given number of partitions N is harder to sustain as a PBE.
- Given $\underline{\delta}\,(0)$, more partitions (higher N) can only be supported as a PBE if the bias parameter δ becomes smaller.
    - That is, more informative PBEs can be sustained when the preferences of lobbyist and politician are more similar (lower δ).

Equilibrium number of partitions

- Solving for N,

$$N^2 - N - \frac{1}{2\delta} \leq 0$$

- Factorizing yields

$$\left(N - \frac{1+\sqrt{1+\frac{2}{\delta}}}{2}\right)\left(N - \frac{1-\sqrt{1+\frac{2}{\delta}}}{2}\right) \leq 0$$

- Furthermore, since N is a positive integer, we can rule out the negative root and restrict the result to

$$N \leq \bar{N}(\delta) \equiv \frac{1-\sqrt{1+\frac{2}{\delta}}}{2}$$

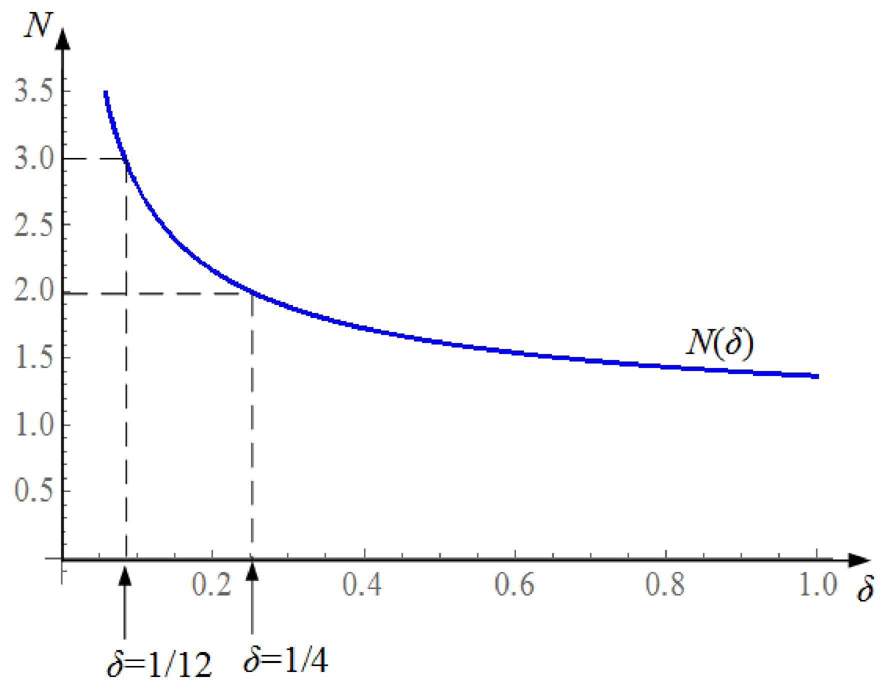
- Where $\lfloor \cdot \rfloor$ rounds to the next integer from below, e.g., $\lfloor 3.7 \rfloor = 3$.

Example 13.1. Equilibrium number of partitions

- The figure to the right depicts $\overline{N}(\delta)$.
- PBE yields a smaller number of partitions as the bias parameter δ increases. Mathematically,

$$\delta \leq \frac{1}{2N(N-1)}$$

- This also indicates that as we seek a larger number of partitions (higher N) in equilibrium, the preference divergence parameter must be lower (see figure).



Example 13.1. Equilibrium number of partitions.

- *Example 13.1*.
- If we seek to support $N = 2$ partitions, we need

$$\delta \leq \frac{1}{2 \times 2(2-1)} = \frac{1}{4}$$

- To sustain $N = 3$ partitions, we need

$$\delta \leq \frac{1}{2 \times 3(3-1)} = \frac{1}{12}$$

- The latter imposes a more restrictive condition on players' preference alignment.

Interval lengths in equilibrium

- We now try to find equilibrium length d^*.
- Consider again:

$$\theta_{k+1} - 2\theta_k + \theta_{k-1} = 4\delta.$$

- Since the unit interval starts at $\theta_0 = 0$, for $k = 1$ we can write

$$\theta_2 = 2\theta_1 - \theta_0 + 4\delta = 2\theta_1 + 4\delta$$

- Similarly,

$$\theta_3 = 2\theta_2 - \theta_1 + 4\delta = \underbrace{2(2\theta_1 + 4\delta)}_{\theta_2} - \theta_1 + 4\delta = 3\theta_1 + 12\delta$$

- More generally, for any value value of k, $\theta_k = k\theta_1 + 2k(k-1)\delta$.

Interval lengths in equilibrium

- Evaluating at $k = N$, we obtain $\theta_N = Nd + 2N(N-1)\delta$.
- We can express $\theta_N - \theta_0 = 1 = Nd + 2N(N-1)\delta$ since $\theta_0 = 0$ and $\theta_N - \theta_0 = 1$.
- Solving for d yields.

$$d^* = \frac{1}{N} - 2(N-1)\delta.$$

- This satisfies $d^* = \theta_1 - \theta_0$ since $\theta_0 = 0$.
- This length decreases in the number of partitions in that equilibrium, N, since $\frac{\partial d^*}{\partial N} = -2\delta - \frac{1}{N^2} \leq 0$ which means that the first interval shrinks to "make room" for subsequent partitions to its right side.

Interval lengths in equilibrium

- Example 13.2. First interval decreasing in N.
  - If $\delta = \frac{1}{20}$ and $N = 2$, we obtain $d^* = \frac{1}{2} - 2(2-1)\frac{1}{20} = \frac{2}{5}$.
  - When N increases to 3, $d^* = \frac{2}{15}$.
- We now use these results to find the length of the k-th interval, $\theta_k - \theta_{k-1}$.

$$\theta_k = \underbrace{k\left(\frac{1}{N} - 2(N-1)\delta\right)}_{\theta_1} + 2k(k-1)\delta = \frac{k}{N} - 2k(N-k)\delta$$

which implies that the length of the k^{th} interval is

$$\theta_k - \theta_{k-1} = \underbrace{\left(\frac{k}{N} - 2k(N-k)\delta\right)}_{\theta_k} - \underbrace{\left(\frac{k-1}{N} - 2(k-1)(N-k+1)\delta\right)}_{\theta_{k-1}}$$

$$= \frac{1}{N} - 2(N+1-2k)\delta$$

Interval lengths in equilibrium

- We can confirm that the length of the first interval coincides with the expression found above, d^*.

$$\theta_1 - \theta_0 = \frac{1}{N} - 2(N + 1 - 2)\delta$$
$$= \frac{1}{N} - 2(N - 1)\delta = d^*$$

- This coincides with the result of d^* found above.

Example 13.3 Length of each interval in equilibrium

- Following Example 13.2, when $\delta = \frac{1}{20}$ and $N = 2$, we find that $d^* = \frac{2}{5}$.
- The length of the second interval is

$$\theta_2 - \theta_1 = \frac{1}{2} - 2[2 + 1 - (2 \times 2)]\frac{1}{20} = \frac{3}{5}$$

- We get that $\frac{2}{5} + \frac{3}{5} = 1$, as required.
- When $N = 3$:
  - The first interval's length is $d^* = \frac{2}{5}$
  - That of the second interval is: $\theta_2 - \theta_1 = \frac{1}{3} - 2[3 + 1 - (2 \times 2)]\frac{1}{20} = \frac{1}{3}$;
  - That of the third interval is: $\theta_3 - \theta_2 = \frac{1}{3} - 2[3 + 1 - (2 \times 3)]\frac{1}{20} = \frac{8}{15}$
- So when $N = 3$, the interval lengths are $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{8}{15}$ respectively, which add up to 1, spanning the unit interval.

Extensions - I

- Allowing for conversations, Krishna and Morgan (2004).
  - How the above results would be affected if the politician could also send messages to the lobbyist.
  - Authors identify conditions under which this conversation between the players yields more partitions than otherwise.

Extensions - II

- Cheap talk vs. Delegation, Dessein (2002).
  - Delegating the decision of p to the lobbyist entails a trade-off:
    - On one hand, it shifts the policy decision to the informed player (the lobbyist), which should be positive;
    - But, on the other hand, it allows the lobbyist to choose a policy according to her bias, δ, which reduces the politician's utility.
  - When δ is relatively small, the first (positive) effect dominates the second (negative) effect, and the politician is better off delegating.
  - Otherwise, the bias is too large, and the politician is better off not delegating (operating as in the Crawford and Sobel's model we studied above).

Extensions - III

- Open vs. Closed rules.
  - Open rule: Politician can freely amend the bill.
  - Closed rule: Politician faces limited ability to amend the bill.
  - Despite "tying the hands" of the politician, the closed rule can yield more information transmission in equilibrium than the open rule (Gilligan and Krehbiel, 1987).

Extensions - IV

- Several lobbyists.
  - Is information transmission facilitated when the politician receives messages from more than one lobbyist (sender).
  - When lobbyists send their messages simultaneously, they convey the true state to the politician (Krishna and Morgan, 2001).
  - When lobbyists send their messages sequentially, if experts have similar (opposite) biases, the politician is better off ignoring the lobbyist with the smaller bias (considering the messages sent by both lobbyists, respectively).

Other Extensions

- Multiple receivers (politicians) (Farrell and Gibbons, 1989).
- Repeated cheap talk (Ottaviani and Sorensen, 2006).
- Allowing for the expert's private information to be multidimensional (Chakraborty and Harbaugh, 2007).
- The presence of a mediator (Ganguly and Ray, 2005).
- Noise in the lobbyist messages before the politician receives them (Blume et al., 2007).
- Equilibrium refinement criteria in cheap talk environments (Farrell, 1993; Chen et al., 2008; Groot Ruiz et al., 2015).