

EconS 424 - Strategy and Game Theory
Homework #4 - Due date: March 23rd, in class.

1. **Harrington, Chapter 13 (second edition):**

- Exercises 7 and 9.

2. **Collusion when n firms compete in quantities.** Consider $n \geq 2$ firms producing homogenous goods and choosing quantities (competing à la Cournot) in each period for an infinite number of periods. Demand in the industry is given by $p(Q) = 1 - Q$, where $Q = \sum_{i=1}^n q_i$ denotes aggregate output. Firms are symmetric in their marginal production cost, c , which satisfies $1 > c > 0$; and in their discount factor $\delta \in (0, 1)$. Consider the following grim-trigger strategy (GTS):

- In the first period, every firm produces $\frac{q^m}{n}$ units of output, where $q^m = \frac{1-c}{2}$ maximizes their joint profits.
- In all subsequent periods, every firm keeps producing $\frac{q^m}{n}$ units of output unless one or more firms deviate. After a deviation, each firm sets the quantity $q^c = \frac{1-c}{n+1}$, which is the Nash equilibrium of the one-shot Cournot game.

Answer the following questions:

- (a) Find the minimal discount factor that allows for collusion to be sustained in this industry.
- (b) Indicate how the number of firms in the industry, n , affects the possibility of reaching the collusive outcome.