Greener or Cheaper Goods: Economies of Scope in R&D Investments

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KFUPM - February 2026

Greener or Cheaper Goods?

- Firms investments in cost-reducing R&D are large and increasing:
    - $625 billion in the US and $310 billion in the EU; OECD.
- Environmental R&D (ER&D) has also increased,
    - Over $100 billion in 2024; IEA.
- Also applies to Saudi Arabia:
    - Petrochemicals, refining, and desalination engage in large cost-reducing investments.

Greener or Cheaper Goods?

- Each investment has separately received attention.
- But firms' simultaneous choice of R&D and ER&D has been largely overlooked.
    - Most chemical companies recognize investing in both.
    - Surveys by Potters and Grassano (2019).

- We allow for both investments, helping us:
  - Better understand firms' decisions.
  - Avoid potential regulatory mistakes (undertaxation).

- Why not just analyze R&D and ER&D separately?
  - We could...
  - if their marginal benefits and costs were additively separable.
    - A larger investment in one didn't affect firms' incentives to invest in the other.
- But are they separable?

Motivation

- Benefits are likely not separable.
    - If a firm invests in R&D, it lowers its production costs, increasing pollution,
    - This triggers a more stringent emission fee,
    - ultimately increasing firms' incentives to invest in abatement.

Motivation

- Costs may not be separable either:
  - Waterless dying technologies in the textile industry, Heida (2014).
  - Innovations originally developed to reduce emissions can also be used to reduce costs.
  - We refer to them as "economies of scope" in investments:
    - Investing in multiple forms of R&D is less costly than separately investing in each of them.

- We also allow for "diseconomies of scope" in investments:
  - Innovations developed to reduce emissions end up increasing costs.
  - Examples abound in firms' green investments in the EU.

Motivation - Regulatory implications

- Ignoring firms' simultaneous investment decisions gives rise to an *undertaxation* problem.
- First, consider no economies of scope.
  - We show that R&D and ER&D are strategic complements...
    - leading to more investments in both.
    - but especially more investments in cost-reducing R&D.
  - Anticipating more production and pollution, the regulator sets a more stringent fee.
  - Ignoring the multiplicity of investments leads to lower-than-optimal fees.
- Second, allowing for economies of scope.

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  - The reg. can set less stringent fees...
  - Ameliorating undertaxation.

Literature-I

- We contribute to three branches:
- Firms' abatement decisions.
  - Poyago-Theotoky (2007), Montero (2011), Lambertini et al. (2017), and Strandholm et al. (2018, 2023), among others.
  - We allow for both investments, and how emission fees are affected.
  - Petrakis and Poyago-Theotoky (2002) study subsidies in a model with both types of investments, but assume an exogenous emission fee and no economies of scope.

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- Cost-reducing R&D.
  - Seminal article by d'Aspremont and Jacquemin (1988), followed by Kamien et al. (1992), and Matsumura et al. (2013).
  - We show that their results can underestimate firms' investment in R&D.

- Investing in abatement because of CSR reasons.
  - Baron (2001, 2008), Farzin (2003), and Calveras and Ganuza (2006), among others.
  - Alternative channel for green investment
    - Without having to rely on green consumers.
    - Even in the absence of environmental regulation.

Outline of the presentation

- Model
- Equilibrium behavior.
    - Unregulated oligopoly (benchmark).
    - Regulated oligopoly

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- Comparing the equilibrium fee against that in standard models.
- Sequential investments:
    - First, invest in abatement; then, in R&D.
    - Is regulation more effective under sim. or seq. investments?

- **Time structure:**
  - Stage 1. Every firm i chooses its investment in cost-reducing R&D, k_i, and ER&D, z_i.
  - Stage 2. The regulator responds with emission fee $t \geq 0$.
  - Stage 3. Firms compete à la Cournot.

- $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.

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- $n \geq 2$ firms facing inverse demand function $p(Q) = 1 - Q$, where $Q \geq 0$ denotes aggregate output.
- Marginal cost, $c \in [0, 1]$, decreases to $c - k_i$.
- Emissions from firm i are $e = q_i - z_i$.

Model

- Total investment cost is

$$C(k_i, z_i) = \frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i,$$

  where γ and α denote the efficiency in R&D and ER&D, respectively.

- Marginal costs: $C_{k_i} = \gamma k_i - \lambda z_i$ and $C_{z_i} = \alpha z_i - \lambda k_i$.

- Special cases:

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- Special cases:
    - If $k_i = 0$, total cost simplifies to $\frac{1}{2}\alpha z_i^2$ [Poyago-Theotoky (2007)].
    - If $z_i = 0$, total cost collapses to $\frac{1}{2}\gamma k_i^2$ [Traditional R&D models].

- Economies of scope:
  - If $\lambda = 0$, total costs in R&D and ER&D are independent.
  - If $\lambda > 0$ ($\lambda < 0$) economies (diseconomies) of scope arise.
- Assumption I (γ and α are high enough, $\gamma, \alpha \geq \frac{2n^2}{(n+1)^2}$)
- Assumption II ($\lambda < \overline{\lambda} \equiv \frac{\sqrt{\alpha(\gamma(n+1)^2 - 2n)}}{(n+1)}$ is not excessive).

Unregulated Oligopoly

- **Last stage:**
  - Every firm i takes $(k_1, ..., k_n)$ and $(z_1, ..., z_n)$ as given, and solves
  $$\max_{q_i \geq 0} \ (1 - q_i + Q_{-i})q_i - (c - k_i)q_i$$
  where $Q_{-i} = \sum_{j \neq i} q_j$.
  - Cournot model with n cost-asymmetric firms:
    - The investment profile $(k_1, \ldots, k_n)$ can entail a different net production cost $c - k_i$ for each firm i.

- **Lemma 1 (summary)**
  - Equilibrium output is $q_i^{NR} = \frac{1-c+nk_i-K_{-i}}{n+1}$, which increases in k_i, but decreases in c, n, and $K_{-i} = \sum_{j \neq i} k_j$;

- Therefore every firm's output increases in its cost advantage, either because:
  - its own R&D investment k_i is higher, or
  - its rivals' investment K_{-i} is lower.

- **First stage:**
  - In the first stage, each firm i anticipates profit $\pi_i^{NR} = (q_i^{NR})^2$ and solves

  $$\max_{k_i, z_i \geq 0} \ \frac{(1 - c + nk_i - K_{-i})^2}{(n+1)^2} - \left(\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i \right).$$

  Differentiating with respect to k_i, yields best response function

  $$k_i(K_{-i}) = \frac{2n(1-c) + \lambda(n+1)^2 z_i}{\gamma + n[(\gamma - 2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma - 2)n + 2\gamma]} K_{-i}.$$

- Because of Assumption I, $k_i(K_{-i})$ originates in the positive quadrant and decreases in K_{-i} (strategic substitutes).

- **First stage:**
- Special cases:
  - When $\lambda = 0$, this best response function simplifies to

    $$k_i(K_{-i}) = \frac{2n(1-c)}{\gamma + n[(\gamma-2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma-2)n + 2\gamma]} K_{-i},$$

    meaning that abatement decisions, z_i, do not affect R&D investment.
  - When $\lambda > 0$, however, $k_i(K_{-i})$ shifts upward, without changing its slope:
    - Firms have stronger incentives to invest in R&D, but its strategic substitutability is unaffected.

- What about abatement?

- **First stage:**
- Special cases:
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- What about abatement?
  - Differentiating with respect to z_i, yields $z_i = \frac{\lambda}{\alpha}k_i$.

- **Lemma 2 (summary)**
- When $\lambda \leq 0$, corner solution where $z_i^{NR} = 0$ and $k_i^{NR} > 0$.
  - As in the models where firms can only invest in abatement.
  - No abatement without regulation.

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- When $\lambda > 0$, we find an interior solution:
    - $z_i^{NR} = \frac{2(1-c)\lambda n}{(n+1)^2(\alpha\gamma - \lambda^2) - 2\alpha n}$ and $k_i^{NR} = \frac{2\alpha(1-c)n}{(n+1)^2(\alpha\gamma - \lambda^2) - 2\alpha n}$,
      both positive.

Benchmark - No regulation

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  - Despite the absence of emission fees, investments lower each other's marginal costs.

Introducing Environmental Regulation

- **Third stage:**
  - Observing the investment profile $(k_1, \ldots, k_n, z_1, \ldots, z_n)$ and emission fee t, every firm i solves

    $$\max_{q_i \geq 0} (1 - q_i - Q_{-i})q_i - (c - k_i)q_i - t(q_i - z_i)$$

    which yields output $q_i^R = \frac{1-c-t+nk_i-K_{-i}}{n+1}$, with associated profit $\pi_i^R = (q_i^R)^2 + tz_i$.

- **Second stage:**
  - The regulator chooses t to maximize

    $$\max_{t} \quad SW = CS(Q) + PS(Q) + T(Q) - ED(Q)$$

    where $CS(Q) = \frac{1}{2}Q^2$ denotes consumer surplus,
  - $PS(Q)$ represents aggregate profits net of taxes,
  - $T(Q) = t(Q - Z)$ denotes total tax collection, with Z denoting aggregate abatement, and
  - $ED(Q) = d(Q - Z)^2$ measures aggregate environmental damages, where $d > 1$ denotes pollution severity.
- Aggregate output is evaluated at $Q^R = nq_i^R$.
- The optimal emission fee is the following.

Introducing environmental regulation

- **Second stage:**
  - **Lemma 3.** The emission fee t is
    $$t(K, Z) = \frac{(2dn-1)[K + n(1-c)] - dn(n+1)Z}{(2d+1)n^2}$$
    which is increasing in environmental damage, d, and R&D, K, but decreasing in ER&D, Z, and marginal cost, c.
  - In addition, the fee is positive if and only if $d > d(n) \equiv \frac{(1-c)n+K}{2n[n(1-c-Z)+K-Z]}$, where cutoff $d(n)$ is increasing in c and Z, but decreasing in K.

- Increase in z_i lowers the stringency of $t(K, Z)$; positive externality.

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- Increase in z_i lowers the stringency of $t(K, Z)$; positive externality.
- Increase in k_i increases this stringency; negative externality (novel in this literature).
- In addition, $t(K, Z)$ is separable in K and Z, $\frac{\partial^2 t(K,Z)}{\partial Z \partial K} = 0$, i.e., no cross effects.

- **First stage:**
  - Anticipating fee $t(K, Z)$, every firm i solves

$$\max_{k_i, z_i > 0} \quad \left(\frac{1 - c - t(K, Z) + nk_i - K_{-i}}{n+1} \right)^2 + t(K, Z)z_i$$
$$- \left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda(k_i z_i) \right]$$

- Differentiating with respect to k_i and z_i yields $k_i(z_i, Z_{-i})$ and $z_i(k_i, K_{-i})$.
  - Each form of investment is increasing in the other type, which holds even when $\lambda = 0$
  - k_i and z_i are, then, strategic complements.

- k_i **and** z_i **are strategic complements.**
- *Intuition:*
- A larger abatement induces a less stringent emission fee,
    - allowing firms to invest more in R&D.

- Similarly, a larger investment in R&D triggers a more stringent emission fee,

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Introducing environmental regulation

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- This complementarity provides firms with more incentives to invest in both forms of R&D
  - than in models that consider a single type of investment, as we next show.

Introducing environmental regulation

- Equilibrium k^R and z^R.

(a) Equilibrium R&D k^R at different levels of λ. (b) Equilibrium R&D k^R at different levels of α.
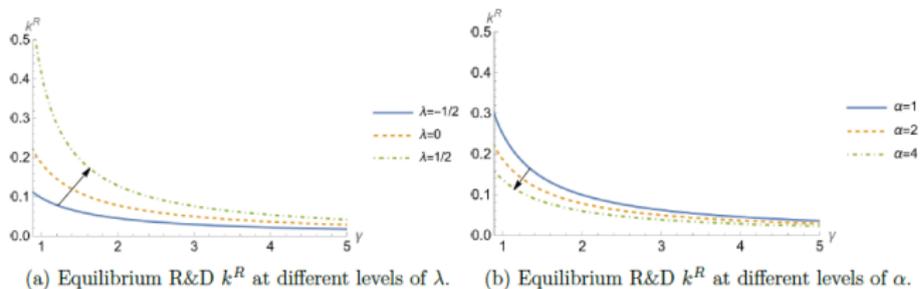
Figure 1: Comparative statics for equilibrium R&D k^R.

(a) Equilibrium ER&D z^R at different levels of λ. (b) Equilibrium ER&D z^R at different levels of γ.
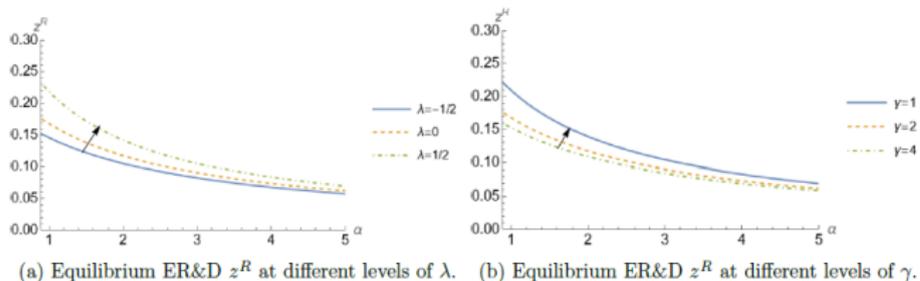
Figure 2: Comparative statics for equilibrium ER&D z^R.

Introducing environmental regulation
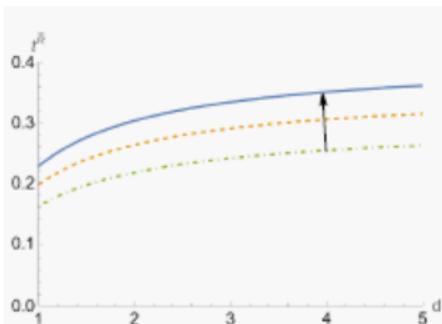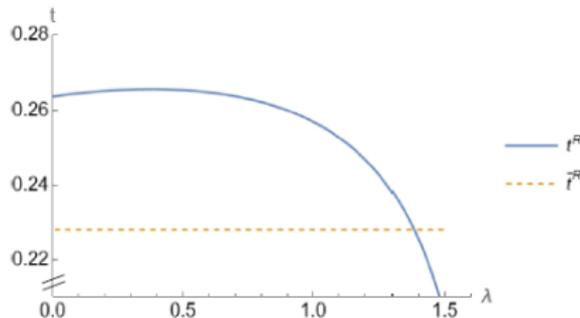

(a) Equilibrium fee t^R at different levels of α.
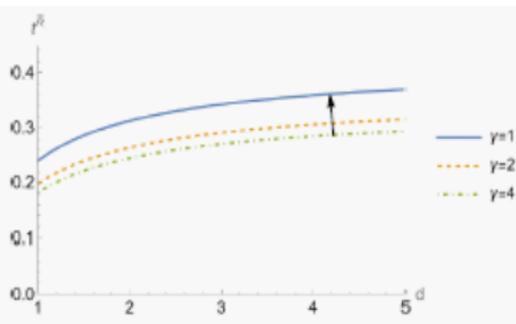
(b) Equilibrium fee t^R at different levels of γ.

(c) Equilibrium fee t^R at different levels of n.

- **Effect of regulation:**

where $\overline{t}^R = \lim\limits_{\gamma \to +\infty} t^R$, and coincides with that in PT (2007).

- Undertaxation for most values of λ.
- Ameliorated when the market is more competitive, pollution is not severe, and investments are more expensive.

Introducing environmental regulation

- **Effect of regulation:**
  - Increases abatement, $z^R > z^{NR}$.
  - Decreases R&D, $k^R < k^{NR}$
- This happens regardless of eco. of scope, λ.
- It is emphasized when investments are more costly (higher γ or α).

Comparison with models
assuming separable investments

## Introducing environmental regulation

- **Investment ratios:**
  - Let $\overline{z}^R \equiv \lim_{\gamma \to +\infty} z^R$, as in Poyago-Theotoky (2007); and
    $\overline{k}^R \equiv \lim_{\alpha \to +\infty} k^R$, as in standard R&D models.
  - Then, define investment ratios

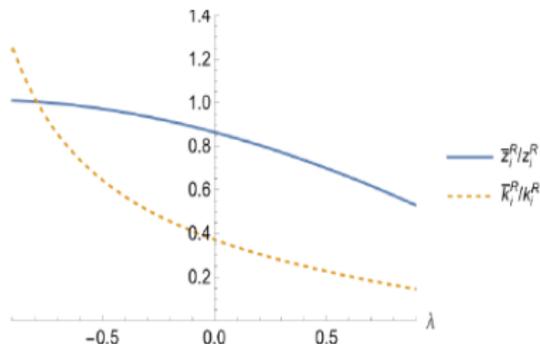 $$\frac{\overline{z}^R}{z^R} \quad \text{and} \quad \frac{\overline{k}^R}{k^R}.$$

- *Interpretation:*
  - If close to 1, we are "not losing much" by assuming separable investments
    - We could consider $\overline{z}^R$ and $\overline{k}^R$ to design policy, instead of z^R and k^R.
  - If lower than 1, we are *underestimating* investments.
  - If higher than 1, we are *overestimating* investments.
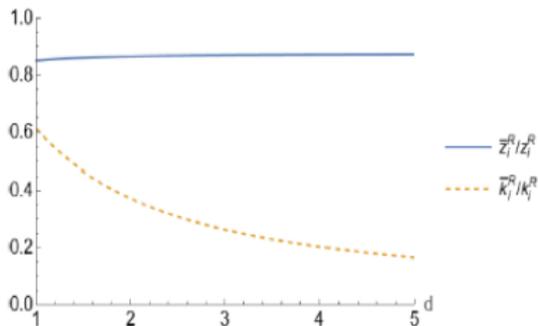
(a) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of d. (b) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of λ.

Extension: Sequential investment decisions

Sequential investments - No regulation

- **Third stage.** Output decisions coincide with Lemma 1.
- **Second stage.** R&D decisions, taking abatement $(z_1, ..., z_n)$ as given:
  - $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n + (n+1)^2 \lambda z_i}{\gamma(n+1)^2 - 2n}$, which is positive and increasing in z_i and λ.
- Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$...
  - R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2 - 2n}$, coinciding with Lemma 2.
- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,

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- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,
  - Every firm benefits from lower marginal costs in its traditional R&D,

Sequential investments - No regulation

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  - $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n + (n+1)^2 \lambda z_i}{\gamma(n+1)^2 - 2n}$, which is positive and increasing in z_i and λ.
- Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$...
  - R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2 - 2n}$, coinciding with Lemma 2.
- However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$,
  - Every firm benefits from lower marginal costs in its traditional R&D,
  - responding by increasing this investment.

Sequential investments - No regulation

- **First stage.** Anticipating $k_i^{NR,Seq}(z_i)$, every firm solves

$$
\max_{z_i \geq 0} \; \frac{(1 - c + k_i^{NR,Seq}(z_i))^2}{(n+1)^2}
$$
$$
- \left(\frac{1}{2} \gamma \left(k_i^{NR,Seq}(z_i) \right)^2 + \frac{1}{2} \alpha z_i^2 - \lambda \left(k_i^{NR,Seq}(z_i) \right) z_i \right).
$$

yielding the first-order condition

$$
\underbrace{\frac{\partial k_i^{NR,Seq}(z_i)}{\partial z_i}}_{(+,\ \text{Strategic effect})} \underbrace{\left[\frac{2(1 - c + k_i^{NR,Seq}(z_i))}{(n+1)^2} - (\gamma k_i^{NR,Seq}(z_i) - \lambda z_i) \right]}_{(-)}
$$

$$
+ \alpha z_i - \lambda k_i^{NR,Seq}(z_i) = 0,
$$

- Relative to sim., seq. decisions give rise to a new strategic effect.

- **First stage.**
- *Simultaneous setting*:
    - An increase in abatement lowers R&D costs if $\lambda > 0$;
    - helping firms increase their R&D investments which, in turn, reduces abatement costs;
    - Positive "*feedback*" effect.

- *Sequential investments*: break this feedback.

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- *Sequential investments*: break this feedback.
    - An increase in abatement still lowers R&D costs, inducing more R&D investments, but...

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  - an increase in R&D does not affect abatement costs because abatement decisions are now taken as given in the second stage.

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  - An increase in abatement still lowers R&D costs, inducing more R&D investments, but...
  - an increase in R&D does not affect abatement costs because abatement decisions are now taken as given in the second stage.
  - Hence, $z_i^{NR,Seq} < z_i^{NR}$.

- **First stage.**
- **Proposition 2.** In the sequential-investment game without regulation, equilibrium investments satisfy:
  - If $\lambda = 0$, equilibrium abatement is $z_i^{NR,Seq} = 0$, yielding an equilibrium R&D of $k_i^{NR,Seq} = \frac{2(1-c)n}{\gamma(n+1)^2 - 2n}$.
  - If $\lambda > 0$, equilibrium abatement is $z_i^{NR,Seq} = \frac{2\lambda(1-c)\left[n^2(\gamma-2)+\gamma(2n+1)\right]}{D}$, yielding an equilibrium R&D of $k_i^{NR,Seq} = \frac{2(1-c)[nD+\lambda^2(n+1)^2(n^2(\gamma-2)+\gamma(2n+1))]}{D[(n+1)^2\gamma-2n]}$.
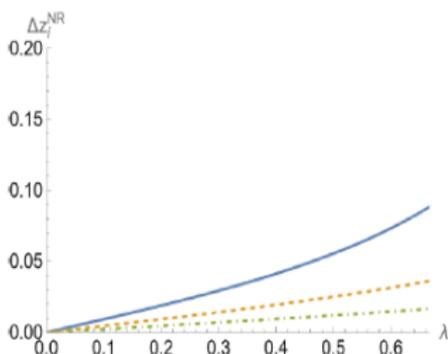- Role of sequential investments?

- **Measuring the effect of sequential investments.**
- Abatement differential

$$\Delta z_i^{NR} \equiv z_i^{NR} - z_i^{NR,Seq},$$

- where z_i^{NR} from Lemma 2 and $z_i^{NR,Seq}$ in Proposition 2.
- When $\Delta z_i^{NR} > 0$, firms invest more in abatement when their R&D decisions are sim than seq;
  - the opposite applies when $\Delta z_i^{NR} < 0$.

Sequential investments - No regulation

- When $\lambda = 0$, abatement coincides, $\Delta z_i^{NR} = 0$.
- When $\lambda > 0$, abatement is larger under sim, $\Delta z_i^{NR} > 0$ (feedback effect).

(a) Differential Δz_i^{NR} at different values of α.
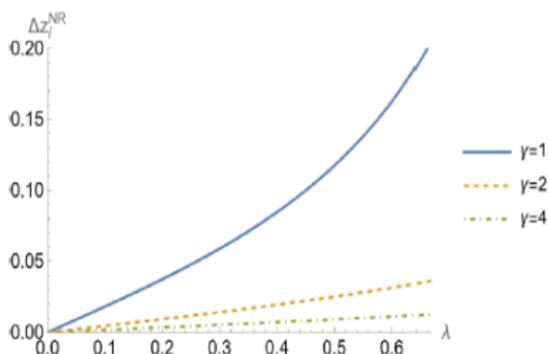
(b) Differential Δz_i^{NR} at different values of γ.

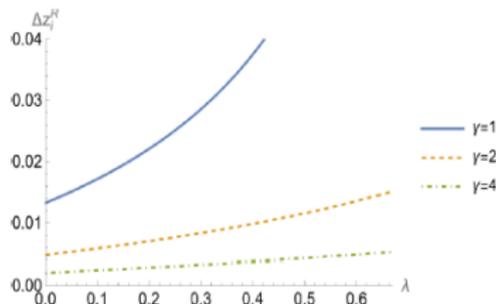Figure 8: Differential Δz_i^{NR} over a range of λ.

- Similar approach with regulation, but:
- The strategic effect can now be positive or negative.
- *Interpretation*:
    - As under no reg., no feedback effect: less incentives to invest in z_i relative to sim.
    - Reg. gives rise to a second effect: investing in z_i to lower emissions, yielding less stringent fee.
    - Under simultaneous investments, firms face strategic uncertainty about the $(k_1, ..., k_n)$ profile.
    - Under sequential investments, they can anticipate this profile, investing more in z_i to lower production costs.
- If feedback effect dominates, firms would invest less under seq. than sim.

Sequential investments - Regulation

- Abatement differential $\Delta z_i^R \equiv z_i^R - z_i^{R,Seq}$.
  - Feedback effect dominates, $z_i^R > z_i^{R,Seq}$.
  - Even if $\lambda = 0$, unlike without regulation.
  - Increasing in λ, but decreasing in α and γ.
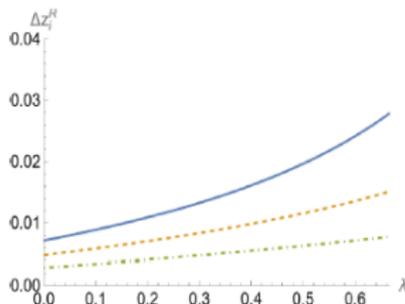
(a) Differential Δz_i^R at different values of α.   (b) Differential Δz_i^R at different values of γ.

Figure 9: Differential Δz_i^R over a range of λ.

- **Simultaneous vs. Sequential Fees**
  - Because $z_i^R > z_i^{R,Seq}$, fees respond in opposite direction, $t^{R,Seq} < t^R$.
  - Then, seq. investments emphasize undertaxation problems, $t^{R,Seq} - \bar{t}^R > t^R - \bar{t}^R$.
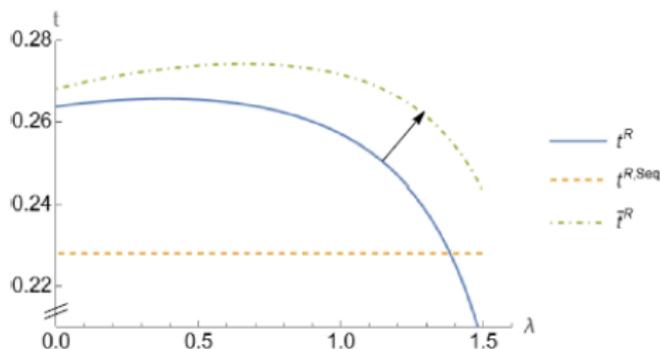

Figure 10: Equilibrium fees $t^{R,Seq}$, t^R, and $\bar{t}^R$ over a range of λ.

- **Is regulation more effective with sim. or seq. investments?**
- Consider $\Delta z_i^{NR} - \Delta z_i^R$, or after rearranging,

$$(z_i^{R,Seq} - z_i^{NR,Seq}) - (z_i^R - z_i^{NR}).$$

- If positive, regulation would provide firms with stronger incentives to increase z_i under seq. than sim. investments.
- If negative, the opposite ranking applies.

Sequential investments - Regulation

- **Is regulation more effective with sim. or seq. investments?**
- Positive differential, meaning regulation is more effective with seq. investments.
- Strategic uncertainty attenuates the feedback effect.

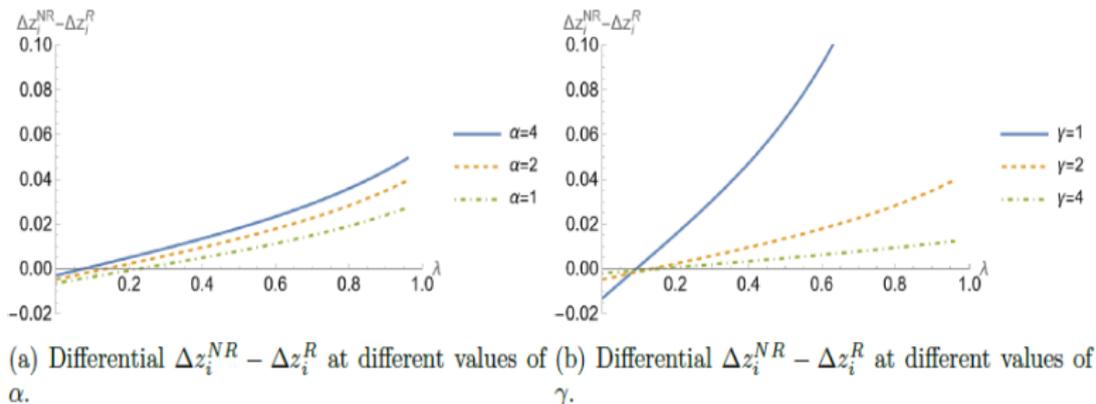(a) Differential $\Delta z_i^{NR} - \Delta z_i^R$ at different values of α.

(b) Differential $\Delta z_i^{NR} - \Delta z_i^R$ at different values of γ.

Figure 11: Differentials $\Delta z_i^{NR} - \Delta z_i^R$ over a range of λ.

Discussion

Three externalities

1. In the absence of reg. and economies of scope.

    - An increase in k_i makes firm i more competitive.
    - #1: Negative externality on its rivals.

Still without reg., but allow for economies of scope.

2. An increase in abatement z_i helps firm i to lower its R&D costs.

- The firm can, then, invest more in k_i.
- #2: Emphasizing the above negative externality.

Three externalities

3. Now introduce regulation.

   - An increase in z_i lowers the stringency of t.
   - #3: Positive externality on its rivals.

- The literature only considers #3,

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  - underestimate k_i,
  - anticipate less pollution, and
  - undertax.

Undertaxation.

- It arises even in the absence of economies of scope, $\lambda = 0$.
    - In that setting, #2 does not exist,
    - But #1 still gives rise to more k_i, output, and pollution than in traditional models.
    - The regulator, then, responds with a fee that is too lax.

- When economies of scope are present, #2 arises.
    - Leading firms to invest more in both, but specially in abatement.
    - This attenuates the magnitude of undertaxation.

- *Practitioners:* When is undertaxation the largest?
  - When pollution is severe, cost of R&D or ER&D is low, or when industry is less competitive.
  - Otherwise, ignoring the multiplicity of investments generates small inefficiencies.

Sequential investments.

- Under no reg., a positive feedback effect arises under sim. investments.
  - It helps firms lower their abatement costs.
- In a sequential setting, this feedback effect is broken.
  - R&D investments do not lower abatement costs.
  - Investing more in abatement under sim. than seq. ("abatement differential").

- Introducing regulation:
  - Attenuates the abatement differential.
  - But this differential is still positive.
  - This implies that emission fee should be more stringent than under simultaneous investments.
  - Emphasizing the undertaxation problem.

Further research

- Firms coordinating their investment decisions.
  - Their k_i's, their z_i's, or both.
  - Helping at internalizing the above three externalities.

- Asymmetric economies of scope across firms.
  - λ_i and λ_j, but it would make results more untractable.

- Allowing for investments to generate spillover effects across firms:
  - R&D investments, $c - k_i - \beta K_{-i}$, where $\beta \geq 0$.
  - ER&D investments, $e_i = q_i - z_i - \beta_E Z_{-i}$, where $\beta_E \geq 0$.

Thank you!