EconS 503 - Microeconomic Theory II
Homework #1 - Answer key

2.5 **Strict dominance, IDSDS, and IDWDS**^{B}**.** Consider the following two-player game, adapted from Tadelis (2013).

Player 2

Player 1		L	C	R
	U	3, 3	5, 1	6, 2
	M	4, 1	8, 4	3, 6
	D	4, 0	9, 6	6, 8

(a) Find the strict dominant equilibrium of this game.

- This game does not have a strict dominant equilibrium because none of the players has a strictly dominant strategy. For a strict dominant equilibrium to exist, we need all players to use a strictly dominant strategy (or strategies). In this context, we need that, every player i, strategy s_i satisfies

$$u_i(s_i, s_j) \geq u_i(s_i', s_j)$$

for every $s_i' \neq s_i$ and for all $s_j \in S_j$.

(b) Which strategy profile/s that survive IDSDS?

- Let us start with player 1, who does not have strictly dominated strategy. To see this, note that:
  - $u_1(U, s_2) < u_1(M, s_2)$ when player 2 selects $s_2 = L$ (in the left-hand column) and when he selects $s_2 = C$ (in the center column), but
  - $u_1(U, s_2) > u_1(M, s_2)$ when player 2 selects $s_2 = R$ in the right-hand column.
- A similar argument applies when comparing player 1's payoffs from choosing M and D:
  - $u_1(M, L) = u_1(D, L)$ when player 2 chooses L,
  - $u_1(M, C) < u_1(D, C)$ when player 2 chooses C, and
  - $u_1(M, R) < u_1(D, R)$ when player 2 chooses R.
- We can now move to player 2, where C is strictly dominated by R since $u_2(C, s_1) < u_2(R, s_1)$ for every strategy s_1 chosen by player 1. We can then reproduce the remaining matrix after the first two rounds of IDSDS, i.e., after deleting nothing for player 1 and strategy C for player 2.

Player 2

Player 1		L	R
	U	3, 3	6, 2
	M	4, 1	3, 6
	D	4, 0	6, 8

- We cannot find any more strictly dominated strategies relying on pure strategies. (As a practice, check that allowing for player 1 to randomize would not help us to further reduce the set of strategy profiles surviving IDSDS.) Then, the set of strategies surviving IDSDS is the six strategy profiles in the reduced matrix:

$$\{(U, L), (U, R), (M, L), (M, R), (D, L), (D, R)\}$$

(c) Which strategy profile/s survive IDWDS?

- From part (b), we know that player 2 finds that C is strictly dominated by R since $u_2(C, s_1) < u_2(R, s_1)$ for every strategy s_1 chosen by player 1, leaving us with the following reduced matrix.

Player 1 \ Player 2	L	R
U	3, 3	6, 2
M	4, 1	3, 6
D	4, 0	6, 8

- Turning to player 1, he finds that D weakly dominates M because:
  - $u_1(M, L) = u_1(D, L)$ when player 2 chooses L,
  - $u_1(M, R) < u_1(D, R)$ when player 2 chooses R.

  We can then delete M from the previous payoff matrix, leaving us with the following reduced matrix.

Player 1 \ Player 2	L	R
U	3, 3	6, 2
D	4, 0	6, 8

- We can keep analyzing player 1, finding that D weakly dominates U (D yields a higher payoff than U when player 2 chooses L, but they both yield the same payoff when player 2 chooses R). Deleting row U from the previous matrix, we obtain the following reduced matrix.

Player 1 \ Player 2	L	R
D	4, 0	6, 8

  Finally, turning to player 2, we can say that R strictly dominates L, which leaves us with the following reduced matrix, entailing that (D, R) is the unique strategy profile that survives IDWDS.

Player 1 \ Player 2	R
D	6, 8

  Therefore, the set of strategies surviving IDWDS is (D, R).

2.8 **Unique prediction in IDWDS and IDSDS^C.** Consider a game with N players, where player i's strategy space is denoted as S_i. In addition, assume that in strategy profile $s = (s_1, ..., s_N)$ strategy s_i is strictly dominant for every player i, that is,

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \text{ for every } s_i' \neq s_i \text{ and for all } s_{-i} \in S_{-i}.$$

(a) Show that s_i must be the only strategy profile surviving IDSDS.

- For every $s_i' \neq s_i$, strategy s_i' must be strictly dominated by s_i. Therefore, the application of IDSDS deletes strategy s_i' at every round of deletion, that is, s_i' is strictly dominated by s_i at every round of deletion. As a result, only strategy s_i survives IDSDS for player i.

(b) Show that s_i must be the only strategy profile surviving IDWDS.

- A similar argument as in part (b) applies. Every strategy $s_i' \neq s_i$ is strictly dominated by s_i, and thus weakly dominated, too. Therefore, the application of IDWDS must delete s_i' at every round of deletion.

(c) Show that, when applying IDSDS, the order of deletion does not produce different equilibrium outcomes.

- From (b), only strategy s_i survives for every player i. Since s_i strictly dominates all other strategies $s_i' \neq s_i$, no other strategy can survive the application of IDSDS or IDWDS at any stage, implying that the order of deletion does not affect our equilibrium results.

2.13 **Finding Dominant Strategies in games with $N \geq 2$ players^B.** Consider a public project between $N \geq 2$ individuals, each of them simultaneously and independently contributing effort x_i (such as a group project in class), incurring cost $c_i x_i^2$ where $c_i > 0$. Individual i's payoff function is

$$u_i(x_i, x_{-i}) = x_i + \alpha \sum_{j \neq i} x_j - c_i x_i^2$$

where $x_{-i} = (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_N)$ represents the effort profile from all i's rivals, and parameter $\alpha > 0$.

(a) Provide a verbal interpretation behind parameter α.

- The aggregate effort from individual i's rivals, $\sum_{j \neq i} x_j$, produces a positive externality on individual i, since she can benefit from a share, α, of the aggregate effort that they contributed. Parameter α, then, can be understood as the intensity of this positive externality. If $\alpha = 0$, the aggregate effort from other players does not affect i's payoff, reducing her utility function to $u_i(x_i) = x_i - c_i x_i^2$; but when $\alpha > 0$, their aggregate effort increases her utility.

(b) Do players have a strictly dominant strategy? If so, find the SDE.

- For player i to have a strictly dominant strategy, x_i, she must prefer to use x_i regardless of the particular vector of effort levels, x_{-i}, selected by her rivals. We next show that such a strictly dominant strategy exists. First, we set up player i's utility function with respect to x_i to find the effort level that maximizes her utility, that is,

$$\max_{x_i \geq 0} \; x_i + \alpha \sum_{j \neq i} x_j - c_i x_i^2$$

Differentiating with respect to x_i, yields

$$\frac{\partial u_i}{\partial x_i} = 1 - 2c_i x_i \leq 0$$

In an interior solution, this result holds with equality, $1 - 2c_i x_i = 0$. Solving for x_i, yields

$$x_i^* = \frac{1}{2c_i}.$$

- This expression is decreasing in player i's cost of additional units of effort, c_i, but, more importantly, is *unaffected* by her rivals' effort levels (i.e., it is not a function of x_{-i}). Intuitively, player i finds that choosing effort $x_i^* = \frac{1}{2c_i}$ yields her a strictly higher payoff than any of her other strategies, $x_i \neq x_i^*$, and this holds regardless of how her rivals behave (for all x_{-i}), that is,

$$u_i(x_i^*, x_{-i}) > u_i(x_i, x_{-i}) \;\text{ for all } x_i \neq x_i^* \text{ and all } x_{-i},$$

which is exactly the definition a strictly dominant strategy.

(c) Is the SDE found in part (b) socially optimal? Interpret.

- In the social optimum, the vector of effort levels, $x = (x_1, x_2, ..., x_N)$, maximizes the sum of individuals' utilities (social welfare, W), that is,

$$\max_{x_1, x_2, ..., x_N \geq 0} \; W = \left(x_1 + \alpha \sum_{j \neq 1} x_j - c_1 x_1^2 \right) + ... + \left(x_N + \alpha \sum_{j \neq N} x_j - c_N x_N^2 \right)$$

Differentiating with respect to each x_i, yields

$$\frac{\partial W}{\partial x_i} = 1 + \alpha(N-1) - 2c_i x_i \leq 0$$

In an interior solution, this result holds with equality, $1+\alpha(N-1)-2c_i x_i = 0$. Solving for x_i, yields

$$x_i^{SO} = \frac{1 + \alpha(N-1)}{2c_i}$$

for every player i, where the superscript SO denotes "socially optimal."

- Comparing x_i^* and x_i^{SO}, we find that

$$\begin{aligned} x_i^{SO} - x_i^* &= \frac{1 + \alpha(N-1)}{2c_i} - \frac{1}{2c_i} \\ &= \frac{\alpha(N-1)}{2c_i} \end{aligned}$$

which is positive since $\alpha > 0$ and $N \geq 2$ by assumption. Therefore, when every individual independently and simultaneously chooses her effort in part (b), she ignores the positive externality that her effort generates on other players, leading to an underprovision of effort in equilibrium relative to the social optimum. In contrast, in the social optimum, individuals internalize this positive externality, leading to higher effort levels for every player.

(d) Are your equilibrium results in part (b) affected if the number of players, N, increases? What if parameter α becomes negative?

- The strictly dominant strategy we found in part (b), where player i exerts effort $x_i^* = \frac{1}{2c_i}$, is only affected by her marginal cost of effort, c_i; but unaffected by the number of players, N, or by the intensity of the externality (whether α is positive or negative).

(e) Are your equilibrium results in part (c) affected if the number of players, N, increases? What if parameter α is asymmetric across players (i.e, player i's parameter is α_i, where $\alpha_i \neq \alpha_j$ for all $j \neq i$)? What if parameter α is symmetric across players but becomes negative?

- The difference between the socially optimal effort, x_i^{SO}, and the SDE, x_i^*, is $x_i^{SO} - x_i^* = \frac{\alpha(N-1)}{2c_i}$, which measures the degree of effort underprovision in equilibrium. This underprovision is:
  - Increasing in the number of players, N. Intuitively, every player i ignores the positive externality that her effort produces on a larger group of rivals.
  - Negative when parameter α is negative. Intuitively, $\alpha < 0$ indicates that the aggregate effort from player i's rivals imposes a negative (not positive) externality on player i. Alternatively, every player i's effort, x_i, generates a negative externality on her rivals, which she does not internalize when choosing x_i, leading to an overprovision of effort relative to the social optimum.

3.2 **More general Pareto coordination game**^{A}**.** Consider the following Pareto coordination game where every firm simultaneously chooses between technology A or B, and payoffs satisfy $a_i > b_i > 0$ for every firm $i = \{1, 2\}$, implying that both firms regard technology A as superior. However, the premium that firm i assigns to technology A, as captured by $a_i - b_i$, can be different between firms 1 and 2.

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	a_1, a_2	$0, 0$
	Tech. B	$0, 0$	b_1, b_2

(a) Find the best responses of every firm.

- *Firm 1.*
  - When firm 2 chooses A (in the left column), firm 1's best response is $BR_1(A) = A$ since payoffs satisfy $a_1 > 0$.

  - When firm 2 chooses B (in the right column), firm 1's best response is $BR_1(B) = B$ because $b_1 > 0$.
- *Firm 2.* Since players' payoffs are not symmetric (for that, we would need $a_1 = a_2$ and $b_1 = b_2$), so we cannot invoke symmetry to analyze firm 2's best responses.
  - When firm 1 chooses A (in the top row), firm 2's best response is $BR_2(A) = A$ since payoffs satisfy $a_2 > 0$.
  - When firm 1 chooses B (in the bottom row), firm 2's best response is $BR_2(B) = B$ because $b_2 > 0$.
- As a reference, the following payoff matrix underlines the best response payoffs for each firm. Intuitively, every firm seeks to choose the same strategy as its rival, ultimately selecting the same technology.

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	$\underline{a_1}, \underline{a_2}$	$0, 0$
	Tech. B	$0, 0$	$\underline{b_1}, \underline{b_2}$

(b) Find the NEs of this game.

- From part (a), we see that there are two NEs in this game:

$$\text{NE} = \{(A, A), (B, B)\}.$$

  However, since payoffs satisfy $a_i > b_i > 0$ for every player $i = \{1, 2\}$, the NE (A, A) Pareto dominates the other NE, (B, B).

(c) Are your results affected if firm 1 assigns a larger premium to technology A than firm 2 does, $a_1 - b_1 > a_2 - b_2$? Interpret.

- Strategy profiles (A, A) and (B, B) are the NEs of the game with (A, A) Pareto dominating (B, B). This result is unaffected by how symmetric or asymmetric payoffs are between firm 1 and 2. In other words, (A, A) and (B, B) remain the NE of the game when $a_1 - b_1 > a_2 - b_2$ holds and when $a_1 - b_1 = a_2 - b_2$ is satisfied. It also remains the unique NE when the players' payoffs are perfectly symmetric, that is, $a_1 = a_2$ and $b_1 = b_2$.

3.11 **A unique strategy profile surviving IDSDS must also be a NE^B.** Consider that strategy profile $s^* = (s_1^*, s_2^*, \cdots, s_N^*)$ is the unique strategy profile surviving IDSDS in a game with $N \geq 2$ players. Show that s^* must also be the unique NE the game.

- If strategy s_i^* survives IDSDS for player i, it means that such strategy yields a strictly higher payoff than some other strategy $s_i \neq s_i^*$ against the surviving strategies of his opponents, s_{-i}, that is

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i})$$

  In particular, the above inequality holds when the surviving strategy of player i's opponents is s_{-i}^*, that is

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*)$$

Hence, s_i^* is a best response to s_{-i}^*. Since this argument applies to every player i, strategy profile s^* is the unique Nash equilibrium of the game.

3.15 **Strict Nash equilibrium**^{B}**.** Consider the following definition: A strategy profile $s^* \equiv (s_1^*, ..., s_N^*)$ is a *strict Nash equilibrium* (SNE) if it satisfies

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*) \text{ for every player } i, \text{ and every } s_i \in S_i.$$

You probably noticed that this definition is almost identical to the definition of Nash equilibrium (NE) strategy profiles, except for using a strict, rather than weak, inequality. In this exercise we connect both solution concepts, but first examine the relationship between a strict Nash equilibrium and IDSDS.

(a) Show that a game can have more than one SNE. An example suffices.

- Consider the following game matrix

Player 2

Player 1		L	R
	U	$\underline{3}$,$\underline{3}$	0, 0
	D	0, 0	$\underline{2}$,$\underline{2}$

Underlining best responses, we see that (U, L) and (D, R) are SNE. In (U, L), for instance, player 1 earns a strictly higher payoff choosing U than D; and so does player 2, who earns a strictly higher payoff choosing L than R. A similar argument applies to (D, R).

(b) Provide an example of a game with more than one NE but just one SNEs.

- Consider the following game matrix, which modifies the matrix in part (a) by assuming that player 2 earns a payoff of 2 in strategy profile (D, L), in the bottom left corner of the matrix.

Player 2

Player 1		L	R
	U	$\underline{3}$,$\underline{3}$	0, 0
	D	0,$\underline{2}$	$\underline{2}$,$\underline{2}$

Underlining the best responses, we see that (U, L) and (D, R) are NEs. However, only (U, L) is a SNE. In strategy profile (D, R), player 2 earns the same payoff with L and R, see his underlined payoffs in the bottom row, implying that (D, R) is a weak (non-strict) NE.

(c) Provide an example with a game where SNEs are a subset of all strategy profiles surviving IDSDS.

- Considering the matrix in part (b) again, we see that, if we apply IDSDS, we see that player 1 has no strictly dominated strategy, nor does player 2, implying that all four original strategy profiles survive IDSDS. Therefore, if s^* is an SNE, it must also survive IDSDS, but the converse is not necessarily true.

(d) Show that if strategy profile s^* is the only strategy profile surviving IDSDS, then s^* must be a SNE.

- By contradiction, assume that strategy profile s^* is the only stratey profile surviving IDSDS, but s^* is not an SNE. Then, there must be at least one player i and at least one of his strategies $s_i' \neq s_i^*$ for which

$$u_i(s_i', s_{-i}^*) \geq u_i(s_i^*, s_{-i}^*).$$

  This inequality would imply that s_i' was not dominated by s_i^*, nor could s_i^* eliminate any profile which could have eliminated s_i' in previous steps of applying IDSDS, which contradicts strategy s_i^* being the unique strategy that survives IDSDS.

(e) Show that if strategy profile s^* is a SNE, it must also be a NE.

- By the definition of NE, if s^* is a NE it satisfies

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \text{ for every player } i, \text{ and every } s_i \in S_i.$$

  If this definition holds with strict inequality (>), strategy profile s^* is a SNE. Therefore, a SNE must also be a NE, but the converse is not necessarily true.

1. **Exercises from Tadelis:**

   (a) Chapter 4: Exercise 4.6.

   (b) Chapter 5: Exercise 5.18.

   - See scanned answer keys at the end of this handout.

less than your valuation is weakly dominated by actually bidding your valuation. ■

(c) Use your analysis above to make sense of eBay's recommendation. Would you follow it?

Answer: The recommendation is indeed supported by an analysis of rational behavior.[1]

5. In the following normal-form game, which strategy profiles survive iterated elimination of strictly dominated strategies?

Player 1 \ Player 2	L	C	R
U	$6,8$	$2,6$	$8,2$
M	$8,2$	$4,4$	$9,5$
D	$8,10$	$4,6$	$6,7$

Answer: First, U is dominated by M for player 1. In the remaining game, C is dominated by R for player 2. No more strategies are strictly dominated, and hence $(M, L), (M, R), (D, L)$ and (D, R) all survive IESDS. (Note: after the last stage above, D is *weakly* dominated by M for player 1, after which L is dominated by R for player 1, so that (M, R) would be the only strategy profile that would survive iterated elimination of weakly dominated strategies. ■

6. **Roommates**: Two roommates need to each choose to clean their apartment, and each can choose an amount of time $t_i \geq 0$ to clean. If their choices are t_i and t_j, then player i's payoff is given by $(10 - t_j)t_i - t_i^2$. (This payoff function implies that the more one roommate cleans, the less valuable is cleaning for the other roommate.)

[1] Those familiar with eBay know about sniping, which is bidding in the last minute. It still is a weakly dominated strategy to bid your valuation at that time, and waiting for the last minute may be a "best response" if you believe other people may respond to an early bid. More on this is discussed in chapter 13.

(a) What is the best response correspondence of each player i?

Answer: Player i maximizes $(10-t_j)t_i - t_i^2$ given a belief about t_j, and the first-order optimality condition is $10 - t_j - 2t_i = 0$ implying that the best response is $t_i = \frac{10-t_j}{2}$. ■

(b) Which choices survive one round of IESDS?

Answer: The most player i would choose is $t_i = 5$, which is a BR to $t_j = 0$. Hence, any $t_i > 5$ is dominated by $t_i = 5$.[2] Hence, $t_i \in [0, 5]$ are the choices that survive one round of IESDS.

(c) Which choices survive IESDS?

Answer: The analysis follows the same ideas that were used for the Cournot duopoly in section 4.2.2. In the second round of elimination, because $t_2 \leq 5$, the best response $t_i = \frac{10-t_j}{2}$ implies that firm 1 will choose $t_1 \geq 2.5$, and a symmetric argument applies to firm 2. Hence, the second round of elimination implies that the surviving strategy sets are $t_i \in [2.5, 5]$ for $i \in \{1, 2\}$. If this process were to converge to an interval, and not to a single point, then by the symmetry between both players, the resulting interval for each firm would be $[t_{\text{min}}, t_{\text{max}}]$ that simultaneously satisfy two equations with two unknowns: $t_{\text{min}} = \frac{10-t_{\text{max}}}{2}$ and $t_{\text{max}} = \frac{10-t_{\text{min}}}{2}$. However, the only solution to these two equations is $t_{\text{min}} = t_{\text{max}} = \frac{10}{3}$. Hence, the unique pair of choices that survive IESDS for this game are $t_1 = t_2 = \frac{10}{3}$. ■

7. **Campaigning:** Two candidates, 1 and 2, are running for office. They each have one of three choices in running their campaign: focus on the positive aspects of one's own platform, call this a positive campaign (or P), focus on the positive aspects of one's own platform while attacking one's opponent's

[2] This can be shown directly: The payoff from choosing $t_i = 5$ when the opponent is choosing t_j is $v(5, t_j) = (10 - t_j)5 - 25 = 25 - 5t_j$. The payoff from choosing $t_i = 5 + k$ where $k > 0$ when the opponent is choosing t_j is $v(5+k, t_j) = (10-t_j)(5+k) - (5+k)^2 = 25 - 5t - k^2 - t_j k_j$, and because $k > 0$ it follows that $v(5+k, t_j) < v(5, t_j)$.

of supporters of candidate D is is exactly 1 more than that of candidate R, that is, $d' = r' + 1$. (A symmetric argument will apply to the case of $r' = d' + 1$.) In this case any one of the R supporters who does not plan to vote knows that his vote can turn a loss into a tie, and hence he would prefer to vote and change the election giving him a payoff of 1 instead of 0. Hence, this too cannot be a Nash equilibrium. This covers all the possible scenarios and shows that everyone voting is the unique Nash equilibrium. ■

(c) Assume now that the costs of voting are equal to 3. How does your answer to (a) and (b) change?

Answer: The two player game is now

		Player 2 Y	Player 2 N
Player 1	Y	$-1,-1$	$1,0$
	N	$0,1$	$2,2$

and the dominated strategy is voting, implying that the unique Nash equilibrium is for the players not to vote, (N, N). A similar argument to part (b) above shows that all players not voting is the unique Nash equilibrium. ■

18. **Political Campaigning:** Two candidates are competing in a political race. Each candidate i can spend $s_i \geq 0$ on adds that reach out to voters, which in turn increases the probability that candidate i wins the race. Given a pair of spending choices (s_1, s_2), the probability that candidate i wins is given by $\frac{s_i}{s_1+s_2}$. If neither spends any resources then each wins with probability $\frac{1}{2}$. Each candidate values winning at a payoff of $v > 0$, and the cost of spending s_i is just s_i.

(a) Given two spend levels (s_1, s_2), write the expected payoff of a candidate i.

Answer: Player i's payoff function is

$$v_i(s_1, s_2) = \frac{s_i v}{s_1 + s_2} - s_i \ .$$

■

(b) What is the function that represents each player's best response function?

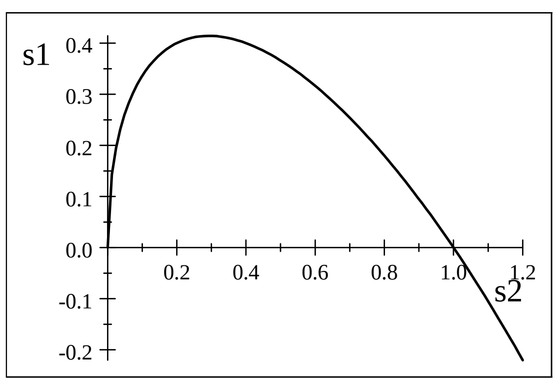
Answer: Player 1 maximizes his payoff $v_1(s_1, s_2)$ shown in (a) above and the first order optimality condition is,

$$\frac{v(s_1 + s_2) - s_1 v}{(s_1 + s_2)^2} - 1 = 0$$

and if we use $s_1(s_2)$ to denote player 1's best response function then it explicitly solves the following equality that is derived from the first-order condition,

$$[s_1(s_2)]^2 + 2s_1(s_2)s_2 + (s_2)^2 - vs_2 = 0 \ .$$

Because this is a quadratic equation we cannot write an explicit best response function (or correspondence). However, if we can graph $s_1(s_2)$ as shown in the following figure (the values correspond for the case of $v = 1$).



Similarly we can derive the symmetric function for player 2. ■

(c) Find the unique Nash equilibrium.

Answer: The best response functions are symmetric mirror images and have a symmetric solution where $s_1 = s_2$ in the unique Nash equilibrium. We can therefore use any one of the two best response functions and replace both variables with a single variable s,

$$s^2 + 2s^2 + s^2 - vs = 0 \ ,$$

or,

$$s = \frac{v}{4}$$

so that the unique Nash equilibrium has $s_1^* = s_2^* = \frac{v}{4}$.■

(d) What happens to the Nash equilibrium spending levels if v increases?

Answer: It is easy to see from part (c) that higher values of v cause the players to spend more in equilibrium. As the stakes of the prize rise, it is more valuable to fight over it. ■

(e) What happens to the Nash equilibrium levels if player 1 still values winning at v, but player 2 values winning at kv where $k > 1$?

Answer: Now the two best response functions are not symmetric. The best response function of player 1 remains as above, but that of player 2 will now have kv instead of v,

$$(s_1)^2 + 2s_1 s_2 + (s_2)^2 - vs_2 = 0 \ . \qquad \text{((BR1))}$$

and

$$(s_2)^2 + 2s_1 s_2 + (s_1)^2 - kvs_1 = 0 \ . \qquad \text{((BR2))}$$

Subtracting (BR2) from (BR1) we obtain,

$$ks_1 = s_2,$$

which implies that the solution will no longer be symmetric and, moreover, $s_2 > s_1$, which is intuitive because now player 2 cares more about the prize. Using $ks_1 = s_2$ we substitute for s_2 in (BR1) to obtain,

$$(s_1)^2 + 2k(s_1)^2 + k^2(s_1)^2 - kvs_1 = 0$$

which results in,

$$s_1 = \frac{kv}{1 + 2k + k^2} < \frac{v}{1 + 2k + k^2} < \frac{v}{4}$$

where both inequalities follow from the fact that $k > 1$. From $ks_1 = s_2$ above we have

$$s_2 = \frac{k^2 v}{1 + 2k + k^2} > \frac{k^2 v}{k^2 + 2k^2 + k^2} = \frac{v}{4}$$

where the inequality follows from $k > 1$. ■