12. Find all of the Nash equilibria for the three-player game shown below.

Player 3: *A*

Player 2

		x	*y*	*z*
Player 1	*a*	2,0,4	1,1,1	1,2,3
	b	3,2,3	0,1,0	2,1,0
	c	1,0,2	0,0,3	3,1,1

Player 3: *B*

Player 2

		x	*y*	*z*
Player 1	*a*	2,0,3	4,1,2	1,1,2
	b	1,3,2	2,2,2	0,4,3
	c	0,0,0	3,0,3	2,1,0

ANSWER: The Nash equilibria are (*b*,*x*,*A*), (*c*,*z*,*A*), (*a*,*y*,*B*).

13. On Friday night, Elton and his partner Rodney are deciding where to go for dinner. The choices are Indian, Korean, and Mexican. Elton most likes Indian food and most dislikes Mexican food, whereas Mexican is Rodney's favorite and Indian is his least favorite. Each cares about the food but also about dining together. As long as the food is Indian or Korean, Elton prefers to go to the restaurant he thinks Rodney will choose. However, he abhors Mexican food and would choose to dine alone at either the Indian or the Korean restaurant rather than joining Rodney at the Mexican place. As long as the food is Mexican or Korean, Rodney will decide to go where he thinks Elton will choose. However, Rodney is allergic to some of the Indian spices and prefers dining alone to eating Indian food. Both of them are at their separate workplaces and must simultaneously decide on a restaurant. Find all Nash equilibria.

ANSWER: Given that Indian (denoted I) and Korean (K) strictly dominate Mexican (M) for Elton, a Nash equilibrium cannot involve Elton choosing Mexican. Given that Mexican and Korean strictly dominate Indian for Rodney, a Nash equilibrium cannot involve Rodney choosing Indian. This then leaves 4 strategy pairs as candidates for Nash equilibrium (with Elton's strategy listed first): (I,M), (I,K), (K,M), (K,K). The Nash equilibria are (I,M) and (K,K). (I,K) is not a Nash equilibrium as, given Rodney chooses K, Elton would prefer to choose K and dine with Rodney than choose I and dine alone. (K,M) is not a Nash equilibrium because given Elton is choosing K, Rodney would prefer to choose K and dine with Elton. It is straightforward to argue that both (K,K) and (I,M) are Nash equilibria.

14. Let us return to Juan and María from CYU 4.2 but modify their preferences. It is still the case that they are competitive and are deciding whether to show up at their mom's house at 8:00 A.M., 9:00 A.M., 10:00 A.M., or 11:00 A.M. But now they don't mind waking up early. Assume that the payoff is 1 if he or she shows up before the other sibling, it is 0 if he or she shows up after the other sibling, and it is −1 if they show up at the same time. The time of the morning does not matter. Find all Nash equilibria.

ANSWER: The strategic form of the game is as below.

Maria

		8	*9*	*10*	*11*
Juan	*8*	−1,−1	1,0	1,0	1,0
	9	0,1	−1,−1	1,0	1,0
	10	0,1	0,1	−1,−1	1,0
	11	0,1	0,1	0,1	−1,−1

If both arrive at 8, each would strictly prefer to arrive at any other time. Hence (8,8) is not a Nash Equilibrium. If neither of them arrive at 8, each of them strictly prefers to arrive at 8. Hence any profile in which neither of them arrive at 8 is not a Nash Equilibrium.

If one of them arrives at 8, the other is indifferent between 9, 10, and 11 and prefers each of those strategies to arriving at 8. Hence, any strategy pair in which one player chooses 8 and the other player chooses a strategy different from 8 is a Nash Equilibrium. All of the Nash equilibria have been underlined.

15. Two companies are deciding at what point to enter a market. The market lasts for four periods and companies simultaneously decide whether to enter in period 1, 2, 3, or 4, or not enter at all. Thus, the strategy set of a company is {1,2,3,4,do not enter}. The market is growing over time, which is reflected in growing profit from being in the market. Assume that the profit received by a monopolist in period t (where a monopoly means that only one company has entered) is $10 \times t - 15$, whereas each duopolist (so both have entered) would earn $4 \times t - 15$. A company earns zero profit for any period that it is not in the market. For example, if company 1 entered in period 2 and company 2 entered in period 3, then company 1 earns zero profit in period 1; 5 ($= 10 \times 2 - 15$) in period 2; -3 ($= 4 \times 3 - 15$) in period 3; and 1 ($= 4 \times 4 - 15$) in period 4, for a total payoff of 3. Company 2 earns zero profit in periods 1 and 2, -3 in period 3, and 1 in period 4, for a total payoff of -2.

a. Derive the payoff matrix.

ANSWER:

Company 2

Company 1	1	2	3	4	Do not enter
1	−20,−20	−14,−9	−2,−2	16,1	40,0
2	−9,−14	−9,−9	3,−2	21,1	45,0
3	−2,−2	−2,3	−2,−2	16,1	40,0
4	1,16	1,21	1,16	1,1	25,0
Do not enter	0,40	0,45	0,40	0,25	0,0

b. Derive a company's best reply for each strategy of the other company.

ANSWER:

If the other company is to enter at $t = 1$ then it is optimal to enter at $t = 4$
If the other company is to enter at $t = 2$ then it is optimal to enter at $t = 4$
If the other company is to enter at $t = 3$ then it is optimal to enter at $t = 2$
If the other company is to enter at $t = 4$ then it is optimal to enter at $t = 2$
If the other company is not to enter at all then it is optimal to enter at $t = 2$.

c. Find the strategies that survive the IDSDS.

ANSWER: Round 1: For each company, strategy 1 is strictly dominated by strategy 2, and strategy "do not enter" is strictly dominated by strategy 4. After eliminating those strategies,

Company 2

Company 1	2	3	4
2	−9,−9	3,−2	21,1
3	−2,3	−2,−2	16,1
4	1,21	1,16	1,1

Round 2: No strategies are strictly dominated.
Strategies 2, 3, and 4 survive the iterative deletion of strictly dominated strategies.

d. Find the Nash equilibria.

ANSWER:

Company 1's strategy	Company 2's strategy	Company 1's payoff	Company 2's payoff	Company 1's strategy optimal?	Company 2's strategy optimal?
1	1	−20	−20	No	No
1	2	−14	−9	No	No
1	3	−2	−2	No	No
1	4	16	1	No	Yes
1	DNE	40	0	No	No
2	2	−9	−9	No	No
2	3	3	−2	Yes	No
2	4	21	1	Yes	Yes
2	DNE	45	0	Yes	No
3	3	−2	−2	No	No
3	4	16	1	No	No
3	DNE	40	0	Yes	No
4	4	1	1	No	No
4	DNE	25	0	No	No
DNE	DNE	0	0	No	No

Recognizing that the game is symmetric, the Nash equilibria are (2,4) and (4,2).

16. Consider an odd type of student who prefers to study alone except when the group is large. We have four of these folks: Melissa, Josh, Samina, and Wei. Melissa and Josh are deciding between studying in the common room in their dorm (which we will denote D) and the library (denoted L). Samina and Wei are choosing between the library and the local cafe (denoted C). If someone is the only person at a location, then his or her payoff is 6. If he or she is one of two people at a location, then the payoff is 2. If he or she is one of three people, then the payoff is 1. If all four end up together, then the payoff is 8.

a. Is it a Nash equilibrium for Melissa and Josh to study in the common room and for Samina and Wei to study in the cafe?

ANSWER: No, because each has a payoff of 2 when a payoff of 6 can be had by Melissa, Josh, Samina, or Wei by choosing the library and studying alone.

b. Is it a Nash equilibrium for Josh to study in the common room, Samina to study in the cafe, and Melissa and Wei to study in the library?

ANSWER: Yes. Melissa and Wei each have a payoff of 2 from studying in the library because there is one other person there. If Melissa (Wei) was to go to the common room (café) then he or she would still receive a payoff of 2 as there would be another person in the common room (café). Josh is receiving a payoff of 6 from the common room and his payoff would be 1 from the library, while Samina's payoff is 6 from the café and would be 1 from the library.

prefer to stay home and receive a zero payoff. What about an equilibrium in which some citizens protest? To answer that question, let's derive an important property. If m' citizens protest then it is optimal for citizen i to be one of those protesting citizens when

$$v_i - \frac{c}{m'} \geq 0. \quad \textbf{[SOL5.10.1]}$$

Now consider citizen j, where $j < i$. Since $v_j > v_i$ (that is, citizen j values protesting more than does citizen i), then it follows from (SOL5.10.1) that

$$v_j - \frac{c}{m'} > 0.$$

In other words, if citizen i finds it best to protest than so does citizen j. Hence, at a Nash equilibrium, if citizen i protests, then so must citizens $1, 2, \ldots, i-1$. This makes sense because all citizens face the same cost, but citizens $1, 2, \ldots, i-1$ attach greater benefit to protesting than does citizen i With this property, we can proceed. Consider a strategy profile in which m' citizens protest where $2 \leq m' \leq n-1$. By the argument just made, it must be citizens $1, 2, \ldots, m'$. For this to be an equilibrium, all of those m' citizens must earn a nonnegative payoff from protesting (so that protesting is better than not protesting), and the other $n - m'$ citizens must earn a nonpositive payoff from protesting (so that they prefer not to protest). This involves n conditions. However, note that if citizen m' prefers to protest, then so do citizens 1, $2, \ldots, m'-1$, since all of them earn a payoff from protesting that is at least as high as that of citizen m'. To ensure that protesting is optimal for citizens $1, 2, \ldots, m'$, we just need to make sure it is optimal for citizen m', which is the case when

$$v_{m'} - \frac{c}{m'} \geq 0. \quad \textbf{[SOL5.10.2]}$$

Now consider the citizens who are not protesting. Since the benefit to protesting is greatest among them for citizen $m'+1$, if that citizen finds it optimal to stay home, then so do citizens $m'+2, m'+3, \ldots, n$. Thus, we just need to verify that citizen $m'+1$ finds it optimal not to protest, which is the case when

$$0 \geq v_{m'+1} - \frac{c}{m'+1}. \quad \textbf{[SOL5.10.3]}$$

Note that the cost from protesting is $\frac{c}{m'+1}$, since, if she also protested, there would be $m'+1$ people at the protest. Putting conditions (SOL5.10.2) and (SOL5.10.3) together, we then require that

$$v_{m'} - \frac{c}{m'} \geq 0 \geq v_{m'+1} - \frac{c}{m'+1}. \quad \textbf{[SOL5.10.4]}$$

$v_{m'} - \frac{c}{m'}$ is the payoff to protesting for citizen m' and $v_{m'+1} - \frac{c}{m+1}$ is the payoff to protesting for citizen $m'+1$. A value for m' that satisfies **(SOL5.10.4)** means that if m' citizens are expected to protest, then citizens $1, 2, \ldots, m'$ will protest (with citizen m' being the most reluctant protestor in that group) and citizens $m'+1$, $m'+2, \ldots, n$ will not protest (with citizen $m'+1$ the one in that group most inclined to protest). m' is the equilibrium size of a protest.

11. n pre-med students are planning to take the MCAT. Each student must decide whether to take a preparatory course prior to taking the test. Let x_i denote the choice of student i, where $x_i = 0$ indicates that she will not take the course and $x_i = 1$ indicates that she will take the course. A student cares about her ranking in terms of her MCAT score and whether or not she took the prep course. Let s_i denote student i's MCAT score and r_i denote the ranking of student i among the n students who took the test. Specifically, r_i equals 1 plus the number of students who scored strictly higher than student i. To clarify this specification, here are three examples: If $s_i \geq s_j$ for all $j \neq i$, then $r_i = 1$. (In other words, if nobody's score is higher than that of student i, then her rank is 1.) If $s_i < s_j$ for all $j \neq i$, then $r_i = n$. (In other words, if student i has the lowest score, then her rank is n.) Finally, if $s_1 > s_2 > s_3 = s_4 > s_5$, then $r_1 = 1, r_2 = 2, r_3 = r_4 = 3, r_5 = 5$. Now, assume that student i's payoff equals $b(n - r_i) - x_i c$, where $b > c > 0$. Note that taking the prep course entails a cost to a student equal to c. Note also that a student adds to her payoff by an amount b if her rank increases by 1. Student i's score is assumed to be determined from the formula $s_i = a_i + x_i z$, where $a_i > 0$ and $z > 0$. a_i is related to the innate ability of the student and is what she would score if she did not take the prep course. If she takes the prep course, she adds to her score by an amount z. Assume that

$$a_1 > a_2 > \cdots > a_{n-1} = a_n.$$

This means that student 1 is, in a sense, smarter than student 2, student 2 is smarter than student 3, . . . , student $n - 2$ is smarter than student $n - 1$, and students $n - 1$ and n are equally smart. The final assumption is

$$a_{i+1} + z > a_i \text{ for all } i = 1, 2, \ldots, n - 1$$

In this game, n students simultaneously deciding whether or not to take the MCAT preparatory course. Derive a Nash equilibrium.

ANSWER: This game has a unique Nash equilibrium, which is for all to take the course: $x_i = 1$ for all i . Suppose all students other than i take the course: $x_j = 1$ for all $j \neq i$. Suppose $i = 1$. Since $a_2 + z > a_1$, then student 1's ranking without having taken the course is no higher than second (it could be lower if $a_3 + z > a_1$.) Thus, her payoff from not taking the course is no higher than $b(n - 2)$. If she takes the course, she will be ranked first, since $a_1 + z > a_j + z$ for all $j \neq i$. Hence, her payoff is $b(n - 1) - c$. Next, note that $b(n - 1) - c > b(n - 2)$, which is equivalent to $b > c$, which is true by assumption. It has then been shown that the payoff to student 1 from taking the course exceeds her payoff from not taking the course, given everyone else takes the course. It is then optimal for student 1 to take the course. The gain in score and ranking from taking the prep course exceeds the cost to student 1.

Next, consider student i, where $2 \leq i \leq n - 2$. The analysis is similar to that for student 1. If student i takes the course—given all other students take the course—her payoff is $b(n - i) - c$. Her payoff from not taking the course is no higher than $b(n - i - 1)$. Since $b(n - i) - c > b(n - i - 1)$, she prefers to take the course. Finally, consider student i, where $i = n - 1$ or $i = n$. Her payoff from not taking the course is $b(n - n) = 0$, as she is ranked last. Her payoff from taking the course is $b(n - (n - 1)) - c = b - c > 0$, so she prefers to take the course. This completes the proof that all students taking the course is a Nash equilibrium. One can show that this is the unique Nash equilibrium.

12. For the operating systems game, let us now assume the intrinsic superiority of Mac is not as great and that network effects are stronger for Windows. These modifications are reflected in different payoffs. Now, the payoff from adopting Windows is $50 \times w$ and from adopting Mac is $15 + 5 \times m$; n consumers are simultaneously deciding between Windows and Mac.

a. Find all Nash equilibria.

ANSWER:

All choosing Windows is a Nash equilibrium when $50n \geq 20$ or $n \geq 1$.

All choosing Mac is a Nash equilibrium when $15 + 5n \geq 50$ or $n \geq 7$.

b. With these new payoffs, let us now suppose that a third option exists, which is to not buy either operating system; it has a payoff of 1,000. Consumers simultaneously decide among Windows, Mac, and no operating system. Find all Nash equilibria.

ANSWER: All choosing Windows is a Nash equilibrium when $50n \geq 1000$ or $n \geq 20$. All choosing Mac is a Nash equilibrium when $15 + 5n \geq 50$ or $n \geq 7$ and $15 + 5n \geq 1000$ or $n \geq 197$. All choosing neither is always a Nash equilibrium because, given all others are not buying, the payoff from buying Windows is 50, from buying Mac is 20, and from not buying is 1000. However, there is no Nash equilibrium in which some buy and some do not.

ANSWER: In this case, the friends maximize $v_p + 5v_c - 5\text{exp}$ where v_p is the cost of producing the meal, v_c is the value to producing the meal and exp is the expense of making the meal. Refer to the table below to find the meal that maximizes $v_p + 5v_c - \text{exp}$

Expense	Value to producing	Value to consuming	$v_p + 5v_c - \text{exp}$
50	100	20	150
80	175	30	245
150	200	40	250
250	230	50	230
400	300	60	200

The common expense that maximizes a player's payoff is 150.

17. China and the United States are racing for an innovation that is worth \$100 million. Each has created a research and development (R&D) joint venture and invited companies to participate. Five U.S. companies are each deciding whether to join the U.S. joint venture, and five Chinese companies are considering whether to join the Chinese joint venture. In both cases, entry into the joint venture costs a firm \$20 million. If n companies are in the American consortium and m are in the Chinese consortium, then the probability that the United States wins is $.5 + .1 \times (n - m)$, and the probability that China wins is $.5 - .1 \times (n - m)$. Each company in the winning venture gets an equal share of the \$100 million prize, while firms in the losing venture get nothing. Thus, if n firms join the U.S. venture, then each has an expected payoff (in millions of dollars) of $(.5 + .1 \times (n - m)) \times (100/n) - 20$, and if m firms join the Chinese venture, then each has an expected payoff of $(.5 - .1 \times (n - m)) \times (100/m) - 20$. The payoff to not joining is zero.

a. Is it a Nash equilibrium for all five American companies to join the U.S. venture and all five Chinese companies to join the Chinese venture?

ANSWER: The payoff to each is $.5(100/5) - 20 = -10$ which is lower than not joining. It is not a Nash Equilibrium.

b. Is it a Nash equilibrium for three American companies to join the U.S. venture and one company to join the Chinese venture?

ANSWER: The payoff to each of the U.S. companies that join is

$$(.5 + .1(3 - 1))(100/3) - 20 = 3.33$$

so joining is optimal. If another U.S. company joins then its payoff is

$$(.5 + .1(4 - 1))(100/4) - 20 = 0$$

which is not higher than that from not joining. Hence, all U.S. companies have optimal strategies. Now consider the lone company to join the Chinese venture. Its payoff is

$$(.5 - .1(3 - 1))(100) - 20 = 10$$

so it is optimal for it to join. The payoff to one of the other Chinese companies from joining is

$$(.5 - .1(3 - 2))(100/2) - 20 = 0$$

so joining is not an improvement. This strategy profile is a Nash Equilibrium.

c. Find all Nash equilibria.

ANSWER: Use the property in part (b) that the payoff to joining a venture is non-increasing in how many other companies are joining it. When there are $(n' + 1)$ firms in the U.S. venture, the payoff to joining is

$$(.5 + .1(n' - m))(100/n') - 20,$$

while if there are n' firms in the U.S. venture, the payoff to joining is

$$(.5 + .1(n' + 1 - m))(100/(n' + 1)) - 20.$$

The former is greater than the latter if and only if

$$\begin{aligned}
(.5 + .1(n' - m))(100/n') - 20 &\geq (.5 + .1(n' + 1 - m))(100/(n' + 1)) - 20 \\
(.5 + .1(n' - m))(n' + 1) &\geq (.5 + .1(n' + 1 - m))n' \\
.5(n' + 1) + .1(n' - m)(n' + 1) &\geq .5n' + .1(n' + 1 - m)n' \\
.5 + .1(n' - m)n' + .1(n' - m) &\geq .1(n' - m)n' + .1n' \\
.5 &\geq .1m \\
5 &\geq m
\end{aligned}$$

which is true by assumption. Thus, the payoff to joining a venture is lower when more companies are in the venture. By symmetry, the same can be shown for the Chinese venture. Find the number of companies in each venture such that each has a zero payoff:

$$\begin{aligned}
(.5 + .1(n - m))(100/n) - 20 &= 0 \\
50 + 10(n - m) &= 20n \\
50 - 10n - 10m &= 0 \\
n + m &= 5
\end{aligned}$$

Thus, if $n + m = 5$ then the firms that join earn a zero payoff in which case each has an optimal strategy given that the payoff from not joining is zero. If an additional firm was to join, it would receive a non-positive payoff because payoffs are the same or lower when more companies join the venture. By the same logic, if a total of four companies have chosen to join $(n + m = 4)$ then the addition of another company—either U.S. or Chinese—would result in that firm having a zero payoff which is the same as if it did not join. It is then a Nash equilibrium for a total of four companies to join. In sum, a strategy profile is a Nash equilibrium if and only if it has a total of four or five companies joining the two ventures.

18. Consider the following game played by three people. Each person writes down a number from {1,2, . . . ,10}. Let x_i denote the number selected by player i. Whichever player has the number closest to $(1/3)(x_1 + x_2 + x_3)$, which is the average of the three numbers, pays an amount equal to the number he or she wrote down; all other players pay 5. For example, if $x_1 = 2$, $x_2 = 4$, $x_3 = 7$, then player 2 pays 4 (because his number is closest to the average of 13/3), and players 1 and 3 each pay 5; or if $x_1 = 3$, $x_2 = 3$, $x_3 = 6$, then players 1 and 2 each pay 3 (which is closer to the average of 4, so this is player 3's number) and player 3 pays 5. Find all symmetric Nash equilibria.

ANSWER: A symmetric strategy profile is a Nash equilibrium if and only if all bidders submit a common bid less than or equal to 5. Consider symmetric bids less than or equal to 5 so that $x_1, x_2, x_3 = x \leq 5$. The average is x, and each player pays x. Note that any deviation leads to a payment of 5 (since a player's bid is now farther from the average than others). As there is no incentive to deviate, any symmetric bid less than or equal to 5 is a Nash equilibrium.

Next consider a symmetric bid $x > 5$. Each player pays $x > 5$. However, a player can instead choose $x - 1$ and guarantee herself a payment of 5. Hence, any symmetric bid above 5 is not a Nash equilibrium.

5. At a company, 20 employees are making contributions for a retirement gift. Each person is choosing how many dollars to contribute from the interval [0,10]. The payoff to person i is $b_i \times x_i - x_i$, where $b_i > 0$ is the "warm glow" he receives from each dollar he contributes, and he incurs a personal cost of 1.

a. Assume $b_i < 1$ for all i. Find all Nash equilibria. How much is collected?

ANSWER: $x_i = 0$ for all i. 0 is collected.

b. Assume $b_i > 1$ for all i. Find all Nash equilibria. How much is collected?

ANSWER: $x_i = 10$ for all i. 200 is collected.

c. Assume $b_i = 1$ for all i. Find all Nash equilibria. How much is collected?

ANSWER: Any value from [0,10] for all i. Anywhere from 0 to 200 is collected.

Now suppose the manager of these 20 employees has announced that she will contribute $d > 0$ dollars for each dollar that an employee contributes. The warm glow effect to employee i from contributing a dollar is now $b_i \times (1 + d)$ because each dollar contributed actually results in a total contribution of $1 + d$. Assume $b_i = 0.1$ for $i = 1, \dots, 5$; $b_i = 0.2$ for $i = 6, \dots, 10$; $b_i = 0.25$ for $i = 11, \dots, 15$; and $b_i = 0.5$ for $i = 16, \dots, 20$.

d. What value must the manager choose for d in order to get her employees to contribute \$100?

ANSWER: An employee is willing to contribute only if $b_i(1 + d) \geq 1$ which requires $d \geq (1 - b_i)/b_i$. Hence, $d \geq 3$ is required in order to get employees 11, . . . , 20 to contribute \$10 each which will then deliver a total contribution from them of \$100.

e. What value must the manager choose for d in order to raise \$750 in total from both her employees and her own matching contribution?

ANSWER: If the manager sets $d = 4$ then $0.2(1 + 4) = 1$ which will induce employees 6, . . . , 20 to contribute. With 15 employees each contributing \$10 and a matching contribution of four times that—or \$600—then the total amount raised is \$750.

6. Three married couples in the state of Maryland—Bob and Carol, Ted and Alice, and Ross and Mike (remember, same-sex marriage is legal in the state of Maryland)—are thinking about renting a boat to go sailing on the Chesapeake Bay. The cost of a boat rental is \$600. Each of the three couples puts some amount of money in an envelope. Thus, each player in this game is a couple. If the total amount collected is at least \$600, then the boat is rented. If the amount collected is more than \$600, then the money left over after renting the boat is spent on wine. If the total amount collected is less than \$600, then they do not rent the boat, and the money is spent on a dinner. Assume the benefit to a couple from the boat trip is 400, the benefit from each dollar spent on wine is 50 cents, the benefit from each dollar spent on dinner is 40 cents, and the personal cost of the contribution to a couple equals the amount of contribution. For example, if the boat is rented, \$50 of wine is purchased (so a total of 650 was contributed), and Bob and Carol contributed \$100 then their payoff is $400 + 0.5 \times 50 - 100 = 325$. If \$400 was collected (so the boat was not rented) and spent on dinner, and Ross and Mike contributed \$200 then their payoff is $400 \times 0.4 - 200 = -40$. Let x denote the contribution of Bob and Carol, y the contribution of Ted and Alice, and z the contribution of Ross and Mike.

a. Is it a Nash equilibrium if $x = 0$, $y = 0$, and $z = 0$?

ANSWER: Yes. The payoff to a couple is zero. By instead spending w, a couple's payoff is $.4 \times w - w = -.6w$ which is negative when $0 < w < 600$; and is $300 + .5 \times (600 - w) - w = 600 - 1.5 \times w$ which is again negative when $w \geq 600$. Therefore, none of the couples can improve their payoff.