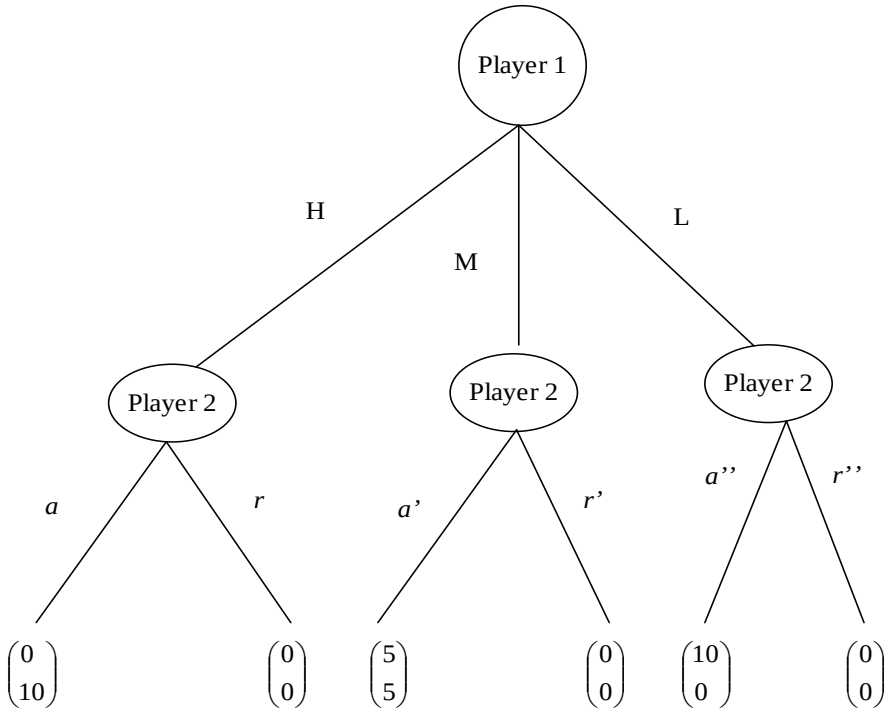
EconS 424
Strategy and Game Theory
Homework #1 – Answer Key

Exercise 1–*From extensive form to normal form representation*
Consider the following extensive form game



a) Which are the strategies for player 1?
Three strategies $S_1=\{H,M,L\}$

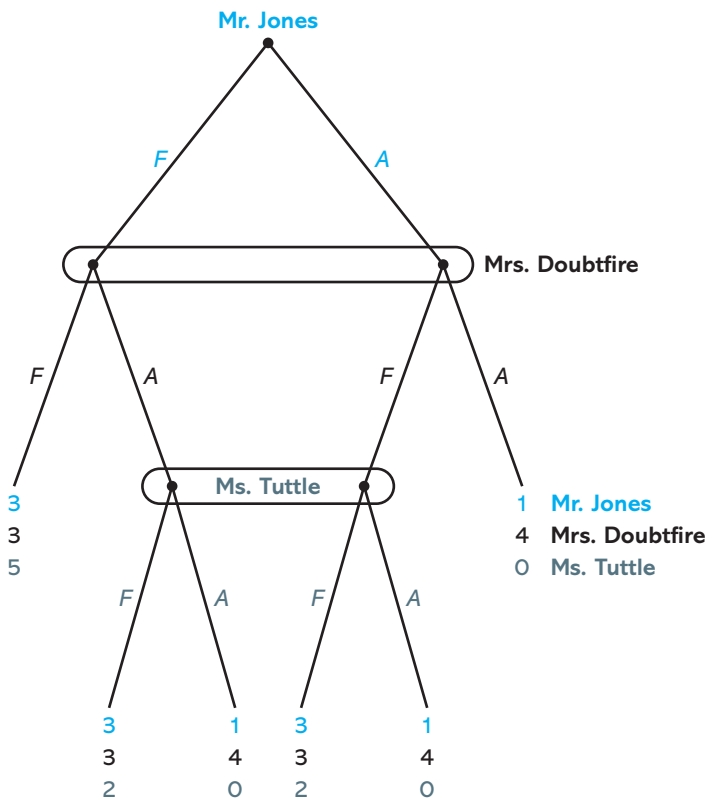
b) What are the strategies for player 2?
Now the second mover can condition his move to the first player's action, since he is able to observe it (unlike in the prisoners' dilemma game). Hence, $S_2=\{$*aaa, aar, arr, rrr, rra, raa, ara, rar*$\}$ where each of them represents a complete plan of action that specifies what to do in the event that player 1 chooses H, in the event that player 1 chooses M and in the case that he chooses L, respectively.

c) Take your results from *a)* and *b)* and construct a matrix representing its normal form game representation.
If you take the about three strategies for player 1, and the above eight strategies for player 2, we have the following normal form game.

		Player 2							
Player 1		*aa'a''*	*aa'r''*	*ar'r''*	*rr'r''*	*rr'a''*	*ra'a''*	*ar'a''*	*ra'r''*
	H	0,10	0,10	0,10	0,0	0,0	0,0	0,10	0,0
	M	5,5	5,5	0,0	0,0	0,0	5,5	0,0	5,5
	L	10,0	0,0	0,0	0,0	10,0	10,0	10,0	0,0

5. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the council's other two members are Mr. Jones and Mrs. Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either *for* or *against* the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally. If the secretary reports that both slips say *for*, then the tax increase is implemented and the game is over. If both vote *against,* then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one *for* and one *against*, then Ms. Tuttle has to vote. If she votes *for,* then the tax increase is implemented, and if she votes *against,* then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Jones's payoff is 1. As for Ms. Tuttle, she prefers to have a tax increase—believing that it will provide the funds to improve the city's schools—but would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her *for* vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted). Write down the extensive form of the game composed of Ms. Tuttle, Mr. Jones, and Mrs. Doubtfire.

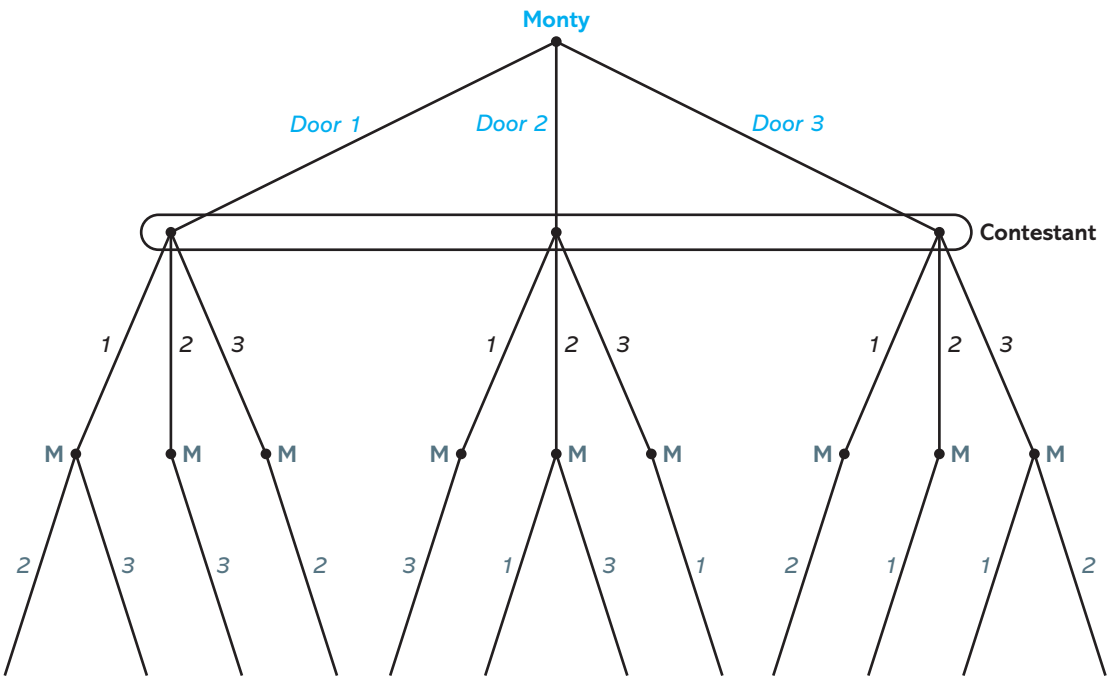
ANSWER:



6. Consider a contestant on the legendary game show *Let's Make a Deal*. There are three doors, and behind two doors is a booby prize (i.e., a prize of little value), while behind one door is a prize of considerable value, such as an automobile. The doors are labeled 1, 2, and 3. The strategic situation starts when, prior to the show, the host, Monty Hall, selects one of the three doors behind which to place the good prize. Then, during the show, a contestant selects one of the three doors. After its selection, Monty opens up one of the two doors not selected by the contestant. In opening up a door, a rule of the show is that Monty is prohibited from opening the door with the good prize. After Monty opens a door, the contestant is then given the opportunity to continue with the door originally selected or switch to the other unopened door. After the contestant's decision, the remaining two doors are opened.

a. Write down an extensive form game of *Let's Make a Deal* up to (but not including) the stage at which the contestant decides whether to maintain his original choice or switch to the other unopened door. Thus, you are to write down the extensive form for when (1) Monty Hall chooses the door behind which the good prize is placed; (2) the contestant chooses a door; and (3) Monty Hall chooses a door to open. You may exclude payoffs.

ANSWER:



b. For the stage at which the contestant decides whether or not to switch, write down the contestant's collection of information sets. In doing so, denote a node by a triple, such as 3/2/1, which describes the sequence of play leading up to that node. 3/2/1 would mean that Monty Hall put the good prize behind door 3, the contestant initially selected door 2, and Monty Hall opened door 1.

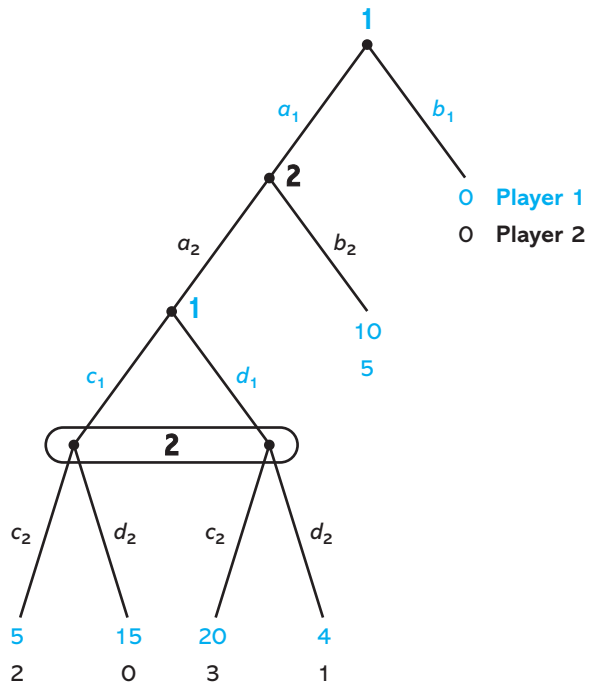
ANSWER: There are six information sets for the contestant at the point when he has to decide whether or not to switch doors: (1) nodes 1/1/2 and 3/1/2; (2) nodes 1/1/3 and 2/1/3; (3) nodes 1/2/3 and 2/2/3; (4) nodes 1/3/2 and 3/3/2; (5) nodes 2/2/1 and 3/2/1; and (6) nodes 2/3/1 and 3/3/1. For example, the first information set comprises nodes 1/1/2 and 3/1/2. At node 1/1/2, Monty put the prize behind door 1, the contestant chose door 1, and Monty opened door 2. At node 3/1/2, Monty put the prize behind door 3, the contestant chose door 1, and Monty opened door 2. The contestant cannot discriminate between those two nodes since they entail the same sequence of observed actions—the contestant chose door 1 and Monty opened door 2—and differ only in terms of where Monty put the prize. That the information set includes both nodes 1/1/2 and 3/1/2 means that the contestant doesn't know whether the good prize is behind door 1 or door 3.

7. For the Iraq War game in Figure 2.11, write down the strategy sets for the three players.

ANSWER: Iraq has three information sets: (1) the initial node; (2) the set in which it does not have WMD and the UN requested inspections; and (3) the set in which it does have WMD and the UN requested inspections. A strategy for Iraq is then a triple of actions. At two of those information sets it has two feasible actions and at the other one it has three actions. The total number of strategies for Iraq is then 12 strategies. The United States has four information sets: (1) the UN did not request inspections; (2) the UN requested inspections and Iraq rejected the

request; (3) the UN requested inspections, Iraq acquiesced to the request, and WMD were not found (that is, either Iraq doesn't have them or has them and hid them); and (4) the UN requested inspections, Iraq acquiesced to the request,, and WMD were found (that is, Iraq had them and did not hide them). The first three information sets each comprise two nodes, one corresponding to Iraq's having WMD and one to its not having WMD. The final information set is a singleton because of the implicit assumption that a UN inspection will reveal that Iraq has WMD when Iraq does not attempt to hide them. A strategy for the U.S. is then a 4-tuple of actions. Since at each of its four information sets the U.S. has two feasible actions—attacking Iraq or not attacking Iraq—the U.S. has 16 strategies. Finally, the UN has one information set. Like the U.S., it has two nodes with one corresponding to Iraq's having WMD and one not. A strategy for the UN is then a single action; its strategy set is composed of *request inspections* and *do not request inspections*.

8. Derive the corresponding strategic form for the extensive form game in the figure below.



ANSWER: Player 1 has two information sets, the initial node and the information set associated with a_1 and a_2 having been played. Let x/y denote a strategy for player 1 that assigns action x to the initial node and action y to the other information set. Player 1's strategy then contains four elements: a_1/c_1, a_1/d_1, b_1/c_1, and b_1/d_1. Player 2 also has two information sets, the singleton associated with 1 having used a_1 and the information set with two nodes—one when the path is $a_1 \rightarrow a_2 \rightarrow c_1$ (read as "a_1 is chosen then a_2 is chosen then c_1 is chosen") and one when the path is $a_1 \rightarrow a_2 \rightarrow d_1$. If strategy x/y assigns action x to the first information set and action y to the second one, then player 2 has four strategies: a_2/c_2, a_2/d_2, b_2/c_2, and b_2/d_2. The payoff matrix associated with these strategies is shown in the figure below.

		Player 2			
		a_2/c_2	a_2/d_2	b_2/c_2	b_2/d_2
Player 1	a_1/c_1	5,2	15,0	10,5	10,5
	a_1/d_1	20,3	4,1	10,5	10,5
	b_1/c_1	0,0	0,0	0,0	0,0
	b_1/d_1	0,0	0,0	0,0	0,0

(Continued)

Project creator: 1500

Contributor #2

Contributor #1	*250/250*	*250/750*	*750/250*	*750/750*
250/250	0,0,0	0,0,0	0,0,0	0,0,0
250/750	0,0,0	750,750,4500	0,0,0	750,750,4500
750/250	0,0,0	0,0,0	0,0,0	0,0,0
750/750	0,0,0	750,750,4500	0,0,0	750,750,4500

16. Consider drivers who commonly traverse a major highway. Each driver is deciding whether to buy E-ZPass. E-ZPass electronically charges a driver for going through a toll, which avoids having to stop and hand over money. E-ZPass costs $4 and allows a driver to go through the E-ZPass lane. Without E-ZPass, a driver goes through the Cash lane. With either lane, the toll is $6. The average time it takes for a car to get through the E-ZPass line is 10 seconds multiplied by the number of cars in the E-ZPass lane (which is assumed to equal the number of cars with E-ZPass). For the Cash lane, the average time it takes for a car to get through is 30 seconds multiplied by the number of cars in the Cash lane (which is assumed to equal the number of cars without E-ZPass). The value of a driver's time is 30 cents per minute. Assume there are 100 drivers, each of whom has a payoff equal to 20 minus the value of time spent in line minus expenditure (the latter is $4 without E-ZPass and $10 with E-ZPass). Drivers make simultaneous decisions about whether or not to buy E-ZPass.

a. The strategy set for a driver is (*E-ZPass, No E-ZPass*). Derive a driver's payoff function depending on his choice and the choices of the other 99 drivers.

ANSWER: Let m denote the number of other drivers that choose E-ZPass. A driver's payoff from buying E-ZPass is $20 - .3(\frac{m+1}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$

b. Now suppose a driver with E-ZPass can use either lane. Assume that it takes the same amount of time to go through the Cash lane whether a driver has E-ZPass or not. Drivers without E-ZPass can still go through the Cash-only lane. The strategy set for a driver is (*E-ZPass & E-ZPass lane, E-ZPass & Cash lane, No E-ZPass & Cash lane*). Derive a driver's payoff function, depending on her choice and the choices of the other 99 drivers.

ANSWER: Let m denote the number of other drivers that choose E-ZPass & E-ZPass lane and n denote the number of other drivers that choose E-ZPass & Cash lane. A driver's payoff from E-ZPass & E-ZPass lane is $20 - .3(\frac{m+1}{6}) - 10$, from E-ZPass & Cash lane is $20 - .3(\frac{100-m}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$

ANSWER: Examining player 1's strategies, first note that d is the unique optimal strategy for player 1 when player 2 is expected to use w. Thus, d cannot be weakly dominated since to be weakly dominated requires that there is another strategy that yields at least as high a payoff for all strategies of the other player and a strictly higher payoff for some strategies of the other player. Since b is the unique optimal strategy for player 1 when player 2 uses x, then b is not weakly dominated either. c is not weakly dominated since it yields a strictly higher payoff than a and b when player 2 uses w and a strictly higher payoff than d when player 2 uses x. a is weakly (and strictly) dominated by c (and also by d). We then find that the set of strategies for player 1 which survive the first round of the iterative deletion of weakly dominated strategies (IDSDS) is $\{b,c,d\}$.

Turning to player 2's strategies, x is the unique optimal strategy for player 2 when player 1 uses a, and z is the unique optimal strategy when player 1 uses d. w weakly dominates y since it yields an identical payoff when player 1 uses a, b, or c and a strictly higher payoff when player 1 uses d. w is not weakly dominated by x since it yields a strictly higher payoff when player 1 uses b, and it is not weakly dominated by z since it yields a strictly higher payoff when player 1 uses a. Therefore, the set of strategies for player 1 that survive the first round of the IDSDS is $\{w,x,z\}$.

After one round of IDSDS, the game is as shown in the figure below.

		Player 2		
		w	*x*	*z*
Player 1	*b*	1,3	5,2	2,0
	c	2,3	4,0	6,2
	d	3,4	2,1	7,5

d is not weakly dominated since it is the unique optimal strategy when player 2 uses w, and b is not weakly dominated since it is the unique optimal strategy when player 2 uses x. c is not weakly dominated by b since it yields a strictly higher payoff when player 2 uses w, and it is not weakly dominated by d since it yields a strictly higher payoff when player 2 uses x. None of player 1's strategies is eliminated in the second round of IDSDS.

Turning to player 2, w is the unique optimal strategy when player 1 uses b and z is the unique optimal strategy when player 1 uses d. x is strictly and thus weakly dominated by w.

After two rounds of IDSDS, the game is as shown in the figure below.

		Player 2	
		w	*z*
Player 1	*b*	1,3	2,0
	c	2,3	6,2
	d	3,4	7,5

For player 1, d strictly and therefore weakly dominates both b and c. Since none of player 1's strategies was eliminated in the second round, none of player 2's strategies can be eliminated in the third round.

After three rounds of IDSDS, the game is as shown in the figure below.

		Player 2	
		w	*z*
Player 1	*d*	3,4	7,5

There is nothing left for player 1 to do. Since z yields a strictly higher payoff than w when player 1 uses d, then z strictly and therefore weakly dominates w.

8. Consider the three-player game shown. Player 1 selects a row, either a_1, b_1 or c_1. Player 2 selects a column, either a_2 or b_2. Player 3 selects a matrix, either a_3 or b_3. The first number in a cell is player 1's payoff, the second number is player 2's payoff, and the last number is player 3's payoff. Derive the strategies that survive the IDSDS.

a_3

	a_2	b_2
a_1	3,1,0	2,3,1
b_1	0,3,1	1,1,0
c_1	1,0,2	1,2,1

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1
c_1	1,1,1	0,2,0

ANSWER: To begin, consider player 1. Neither a_1 nor b_1 is strictly dominated, as a_1 yields the highest payoff for player 1 when players 2 and 3 choose (a_2,a_3), while b_1 is best when players 2 and 3 choose (b_2,b_3). However, a_1 strictly dominates c_1. Thus, the surviving strategies for player 1 are a_1 and b_1. Turning to player 2, neither of her strategies is strictly dominated since a_2 is best when players 1 and 3 choose (b_1,a_3) and b_2 is best when players 1 and 3 choose (a_1, a_3). Finally, neither of player 3's strategies is strictly dominated as a_3 is best when players 1 and 2 choose (c_1,a_2) and b_3 is best when players 1 and 2 choose (a_1,a_2). After the first round, the reduced game is as shown in the figure below.

a_3

	a_2	b_2
a_1	3,1,0	2,3,1
b_1	0,3,1	1,1,0

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1

Since no strategies of players 2 and 3 were eliminated in the first round, no strategies of player 1 can be eliminated in the second round. Neither of player 2's strategies is strictly dominated, as a_2 is best when players 1 and 3 choose (b_1, a_3) and b_2 is best when players 1 and 3 choose (a_1, a_3). For player 3, b_3 strictly dominates a_3. After the second round, the reduced game is as shown in the figure below.

b_3

	a_2	b_2
a_1	3,1,1	1,3,2
b_1	2,0,2	2,2,1

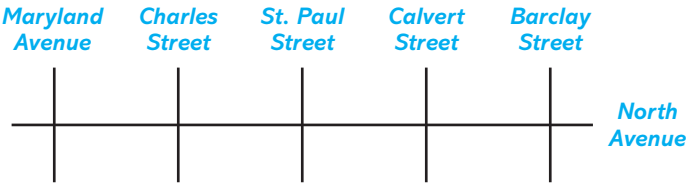
In round 3, neither of player 1's strategies is strictly dominated, but for player 2, b_2 strictly dominates a_2. After the third round, the reduced game is as shown in the figure below.

b_3

	b_2
a_1	1,3,2
b_1	2,2,1

In the fourth round, b_1 strictly dominates a_1 and this solves the game since each player has only one strategy remaining. Thus, by the iterative deletion of strictly dominated strategies, we conclude that the strategy profile that will be played is (b_1,b_2,b_3).

9. A gang controls the drug trade along North Avenue between Maryland Avenue and Barclay Street. The city grid is shown below.



b. In addition to that assumed in part (a), assume that player 2 knows player 1 knows player 2 is rational. Find strategies consistent with these beliefs.

ANSWER: Round 3: y strictly dominates x for player 2.

		Player 2	
		x	*y*
Player 1	*a*	5,1	4,2

Note that the information available to player 1 has not changed from part (a), therefore, Player 1 still plays a. However, now player 2 knows player 1 knows player 2 is rational.

Therefore, player 2 knows player 1 knows that player 2 will not play z from which it follows that player 2 knows player 1 plays a (because a strictly dominates b and c in the absence of z). Player 2 then plays y. Hence, player 1 plays a and player 2 plays y.

14. Len and Melanie are deciding what to do Saturday night. The options are to see Mozart's opera *Don Giovanni* or go to the local arena to watch Ultimate Fighter. Len prefers Ultimate Fighter, while Melanie prefers *Don Giovanni*. As a possible compromise, a friend suggests that they attend "*Rocky: The Ballet,*" which is a newly produced ballet about Rocky Balboa, the down-and-out boxer from the streets of Philadelphia who gets a shot at the title. Each would like to go to their most preferred performance, but each also cares about attending with the other person. Also, Len may feel guilty about spending a lot of money for a ticket to Ultimate Fighter when Melanie is not with him; *Rocky: The Ballet* is cheaper. *Don Giovanni* and Ultimate Fighter are both expensive tickets, but Melanie would not feel guilty about attending her first choice alone and spending a lot of money. Both Len and Melanie are flying back into town Saturday afternoon and each must independently decide which to attend. The strategic form of the game is shown below. Using the IDSDS, what will they do?

		Melanie		
		Don Giovanni	Ultimate Fighter	*Rocky: The Ballet*
Len	*Don Giovanni*	1,5	0,0	0,2
	Ultimate Fighter	3,3	6,1	3,2
	Rocky: The Ballet	4,3	2,0	5,4

ANSWER: Round 1: Ultimate Fighter (UF) and Rocky: The Ballet (R) strictly dominate Don Giovanni (DG) for Len. For Melanie, DG and R strictly dominate UF. After eliminating these strategies, the game is now:

		Melanie	
		Don Giovanni	*Rocky: The Ballet*
Len	*Ultimate Fighter*	3,3	3,2
	Rocky: The Ballet	4,3	5,4

Round 2: R strictly dominates UF for Len, while nothing is strictly dominated for Melanie. The game is now:

		Melanie	
		Don Giovanni	*Rocky: The Ballet*
Len	*Rocky: The Ballet*	4,3	5,4

Round 3: R strictly dominates DG for Melanie. Thus, they both go to see *Rocky: The Ballet*.