Greener or Cheaper Goods: Economies of Scope in R&D Investments*

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Abstract

This paper examines firms' incentives to simultaneously invest in cost-reducing and in green R&D (abatement) under the presence of regulation. We show that, without regulation, firms only invest in cost-reducing R&D when economies of scope are absent, but invest in both types of R&D otherwise. With regulation, investments exhibit strategic complementarities, with and without economies of scope, leading to more investments in cost-reducing R&D, thus requiring more stringent emission fees. Assuming that firms invest in only one form of R&D, a traditional approach in the literature, gives rise to an undertaxation problem. This inefficiency is attenuated if R&D investments exhibit economies of scope, but emphasized if pollution is severe, and the market is concentrated; which is further increased when investment decisions are sequential.

Keywords: R&D, Emission fees, Abatement, Economies of scope.
JEL classification: D43, H23, Q55, Q58

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1 Introduction

Firm investments in cost-reducing research and development (R&D) have increased in the last decades, reaching $625 billion in the US and $310 billion in the EU in 2021.[1] Similarly, private investments in abatement (also known as "environmental R&D," ER&D) have continuously increased. For instance, BloombergNEF reported that total investment in low-carbon technologies reached $755 billion in 2021, representing a 25% rise over the previous year.[2] Potters and Grassano (2019) report that 60% of EU chemical companies participating in their survey recognize investing in both traditional and sustainable R&Ds. While each investment decision has separately received attention from the literature, firms' simultaneous choice of R&D and ER&D has been largely overlooked.

In this paper, we consider that firms simultaneously invest in both forms of R&D, investigate how investment levels and emission fees differ from those in standard models where firms invest in one type of R&D, and identify in which settings this difference becomes larger. Understanding firms' simultaneous investment: (i) helps to better characterize firms' decisions in real life; and (ii) avoids potential regulatory mistakes, especially in the form of undertaxation.

First, from a modeling standpoint, R&D and ER&D decisions could be independently analyzed if their marginal benefits and costs were additively separable, i.e., if a larger investment in one of them did not affect firms' incentives to invest in the other. We argue that the opposite is more likely to occur: even if investment costs are separable, its benefits are not. To see this point, note that a firm's investment in R&D improves its efficiency, triggering more stringent environmental regulation, ultimately affecting all firms' incentives to invest in abatement. Moreover, investment costs may not be separable either. For instance, waterless dying technology in the textile industry reduces both emissions and costs in an industry notorious for its pollution, as reported by Heida (2014). Hence, innovations originally developed for lowering emissions (abatement) can also be used to decrease production costs, thus lowering R&D costs;[3] or, instead, increase the firm's R&D costs.[4] A similar argument applies if R&D innovations can also be dedicated to abatement, lowering ER&D costs. We refer to these effects as economies (or diseconomies) of scope, since investing in multiple types of R&D is less (more) costly than separately investing in each of them.

Second, from a regulatory perspective, we find that policy designs that consider firms invest in only one form of R&D, when they actually simultaneously invest in both, gives rise to an "undertaxation" problem.[5] In the absence of economies of scope, we show that firms' investments

[1]According to the OECD Main Science and Technology Indicators database: https://www.oecd.org/sti/msti.htm.

[2]Likewise, PwC (Cox, 2023) reports that in 2022 aggregate climate technology investments were of $37.98 billion in the US, $21.11 billion in China, and $18.17 billion in Europe.

[3]Timilsina and Malla (2021) review several studies finding an ex-post positive relationship between clean investments and firms' cost reduction, especially in the energy-intensive manufacturing sector.

[4]Stucki (2019), for instance, shows that firms' investment in green technologies in Germany, Austria, and Switzerland leads to weakly higher production costs for most firms in their sample, with only 19 percent of them benefiting from lower production costs. For more empirical evidence in the EU energy sector, see Paramati et al. (2021), and for non-EU countries, see Aleluia Reis et al. (2023).

[5]This is not due to regulators being uninformed about firms' choice variables but, instead, because regulators may rely on more simplified policy designs where firms only invest in one form of R&D.

in R&D and abatement are strategic complements, leading to more investments in both forms, but especially in cost-reducing R&D. As a consequence, the regulator responds setting a more stringent emission fee than in models where firms only invest in abatement, such as Lambertini et al. (2017). Alternatively, policies that do not account for the multiplicity of R&D investments set a less stringent emission fee than what is socially optimal. The presence of economies of scope emphasizes the strategic complementarity between investments. Anticipating a larger investment in abatement, the regulator can now set less stringent fees, thus ameliorating the undertaxation problem.

Our model considers that, in the first stage, every firm chooses how much to invest in R&D and ER&D; in the second stage, the regulator responds setting the emission fee; and, in the third stage, firms compete à la Cournot; as in Poyago-Theotoky (2007). For comparison purposes, we also consider an alternative timing, where firms sequentially invest in abatement and R&D, to examine how sequential investments affect our equilibrium results.

For presentation purposes, we first analyze an unregulated oligopoly. When economies of scope are absent, we show that firms only invest in R&D, not having incentives to invest in abatement. When economies of scope are present, however, firms invest in both forms of R&D, since abatement lowers their marginal costs of R&D even without emission fees. This result highlights a motive for green investments, novel to our knowledge, where firms invest in abatement to facilitate their R&D projects, even without facing environmental policy, or demand being affected by public image or corporate social responsibility. The introduction of emission fees provides with more incentives to invest in abatement under all parameter conditions, thus giving rise to an abatement differential relative to no regulation. Economies of scope, however, attenuate this differential.

Our above results also help examine in which settings equilibrium investments are close to those in traditional models (small "measurement error") and when they become farther apart. When this error is small, one could conclude that considering a straightforward model where firms only invest in one form of R&D should suffice. We show that this is the case when pollution is not severe, the market is competitive, and diseconomies of scope are significant. Otherwise, using traditional models gives rise to large measurement errors, implying that the regulator anticipates less cost-reducing R&D and less pollution, thus setting a laxer emission fee and exacerbating undertaxation. This is the case in relatively concentrated industries with severe pollution and low investment costs.

Our results also hold when firms choose their investments sequentially. In particular, we show that, under no regulation, investments in abatement coincide with those in a simultaneous setting if, in addition, economies of scope are absent. If they are present, however, firms invest less in abatement when their decisions are sequential. This is due to the fact that, in a simultaneous context, economies of scope give rise to a positive "feedback" effect, namely, larger investments in abatement lower R&D costs and vice versa. A sequential context, however, "breaks" the feedback effect, since economies of scope cannot affect abatement costs because R&D investments happen in a subsequent stage. Therefore, firms invest more under a simultaneous timing, generating an abatement differential across settings. When firms face environmental regulation, this differential is attenuated, but remains positive in all contexts. The optimal emission fee should, then, be more stringent under a sequential than simultaneous setting, thus exacerbating the undertaxation problem discussed above. Our findings, hence, highlight the importance of policy makers considering that firms invest in both forms of R&D, particularly when investment decisions are sequential.

1.1 Related literature

Our paper contributes to three branches of the literature. First, we follow the literature analyzing firms' abatement decisions, such as Biglaiser and Horowitz (1995), Denicolò (1999), Poyago-Theotoky (2007), Ben Youssef and Dinar (2011), Montero (2011), Ouchida and Goto (2016), Lambertini et al. (2017), and Strandholm et al. (2018, 2023). For a related literature, see Ouchida and Goto (2021). We extend the setting in these articles by allowing firms to invest in both abatement and cost-reducing R&D, and examine how these investments interact with each other. Petrakis and Poyago-Theotoky (2002) also consider that firms invest in abatement and, subsequently, in cost-reducing R&D. While they examine R&D subsidies and consider that firms coordinate their R&D decisions, they assume that the emission fee is exogenous, thus not examining how each form of investment separately affects environmental policy, and do not allow for economies of scope.

Second, we consider a similar setting as the literature in industrial organization studying firms' investment decisions in cost-reducing R&D with the goal to strengthen their competitiveness in subsequent oligopoly competition. These studies include d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and Matsumura et al. (2013, who provide a review of this literature); but, as described above, we allow for firms to invest in R&D and ER&D, showing that equilibrium results in traditional models could underestimate firms' investment in R&D.

Finally, our results about firms having incentives to invest in abatement, when economies of scope are strong, goes in line with that in the corporate social responsibility literature, including Baron (2001, 2008), Farzin (2003), and Calveras and Ganuza (2016), among others. We then identify an alternative channel to rationalize green investments without having to rely on environmentally conscious consumers or incomplete information about the product environmental attributes, namely, firms may invest in ER&D to help reduce their own R&D costs at the same time. This incentive exists in the absence of environmental policy but is strengthened otherwise.

The paper is organized as follows. Section 2 presents the model, section 3 identifies equilibrium behavior under simultaneous investments, with and without environmental regulation. Section 4 extends our results to a setting with sequential investments in abatement and R&D, and section 5 concludes.

2 Model

Following Petrakis and Xepapadeas (1999) and Poyago-Theotoky (2007),[6] we consider the following three-stage oligopoly model with $n \geq 2$ firms:

Stage 1: Every firm i chooses how much to invest in cost-reducing R&D, k_i, and in ER&D, z_i.

Stage 2: Observing all firms' investment decisions, the regulator responds setting emission fee t.

Stage 3: Firms compete à la Cournot.

Firms sell a homogeneous good and face inverse demand $p(Q) = 1 - Q$, where Q denotes aggregate output. We consider a constant marginal cost of production $1 > c > 0$ which decreases to $c - k_i$ when firm i invests k_i in cost-reducing R&D. Firms emit pollution proportionally to output, which can be abated as a result of ER&D investment, and emissions become $e_i = q_i - z_i$.

Firm i's total investment cost is

$$C(k_i, z_i) = \frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i, \tag{1}$$

where γ and α represent the efficiency in R&D and ER&D, respectively.

Assumption I. *Investment costs satisfy* $\gamma,\ \alpha \geq \frac{2n^2}{(n+1)^2}$.

This assumption rules out extreme investments, and simplifies to $\gamma \geq \frac{1}{2}$ in the case of monopoly, which is standard in the literature. Otherwise, cutoff $\frac{2n^2}{(n+1)^2}$ increases in n, approaching a height of 2 when $n \to +\infty$.

When the firm does not invest in R&D, $k_i = 0$, its cost function simplifies to $\frac{1}{2}\alpha z_i^2$, being a function of z_i, as in traditional models of investment in abatement. Similarly, when the firm does not invest in ER&D, $z_i = 0$, the above cost function collapses to $\frac{1}{2}\gamma k_i^2$, being affected only by k_i, as in standard R&D models.

When $\lambda = 0$, the cost function considers that R&D and ER&D expenditures are independent. In contrast, when $\lambda > 0$, these investments give rise to economies of scope, which could occur if innovations in abatement can also be used by the firm in its traditional R&D, thus lowering its production costs, i.e., a higher z_i decreases the cost of k_i, and vice versa. Technically, both marginal costs, $C_{k_i} = \gamma k_i - \lambda z_i$ and $C_{z_i} = \alpha z_i - \lambda k_i$, decrease when economies of scope are present ($\lambda > 0$).

Assumption II. *Economies of scope are not excessive,* $|\lambda| < \overline{\lambda} \equiv \frac{1}{n+1}\sqrt{\alpha[\gamma(n+1)^2 - 2n]}$, *where* $\overline{\lambda} > 0$ *holds because of Assumption I.*

However, when $\lambda < 0$ holds, investments lead to diseconomies of scope, where innovations initially developed to improve abatement end up increasing the firm's R&D costs, ultimately leading to higher total and marginal costs.[7]

[6] Other articles considering similar time structures include Petrakis and Xepapadeas (2003), Buccella et al. (2021), and Agliardi and Lambertini (2024).

[7] For instance, when $n = 2$, cutoff $\overline{\lambda}$ becomes $\overline{\lambda} = \frac{1}{3}\sqrt{\alpha(9\gamma - 4)}$, which is positive since $\gamma \geq \frac{8}{9}$ by Assumption 1.

For presentation purposes, we first examine an unregulated oligopoly and then compare it against a regulated industry.

3 Equilibrium Analysis

3.1 Unregulated oligopoly

In the last stage, every firm i solves

$$\max_{q_i \geq 0} \; (1 - q_i + Q_{-i})q_i - (c - k_i)q_i \tag{2}$$

where $Q_{-i} = \sum_{j \neq i} q_j$. This setting gives rise to a Cournot model with n cost-asymmetric firms since the investment profile $(k_1, k_2, \ldots, k_n)$ can entail a different net production cost $c - k_i$ for each firm i. The next lemma identifies equilibrium output and profits in this context. For compactness, all proofs are relegated to Appendix 2.

Lemma 1. *Equilibrium output in the unregulated oligopoly is* $q_i^{NR} = \frac{1-c+nk_i-K_{-i}}{n+1}$*, where* $K_{-i} = \sum_{j \neq i} k_j$*, which increases in* k_i *but decreases in* c*,* n*, and* K_{-i}*; and equilibrium profit is* $\pi_i^{NR} = (q_i^{NR})^2$*, which increases in* k_i *but decreases in* c*,* n*, and* K_{-i}*.*

Therefore every firm's output increases in its cost advantage, either because its own R&D investment k_i is higher, or because its rivals' investment K_{-i} is lower.

In the first stage, each firm i anticipates profit π_i^{NR} and solves

$$\max_{k_i, z_i \geq 0} \frac{(1 - c + nk_i - K_{-i})^2}{(n+1)^2} - \left(\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i \right). \tag{3}$$

Differentiating (3) with respect to k_i, yields best response function

$$k_i(K_{-i}) = \frac{2n(1-c) + \lambda(n+1)^2 z_i}{\gamma + n[(\gamma - 2)n + 2\gamma]} - \frac{2n}{\gamma + n[(\gamma - 2)n + 2\gamma]} K_{-i}. \tag{4}$$

Because of Assumption I, this function originates in the positive quadrant and decreases in its rivals' R&D investment, K_{-i}, indicating that firms deem each others' investments as strategic substitutes. When economies of scope are absent ($\lambda = 0$), equation (4) simplifies to $k_i(K_{-i}) = \frac{2n(1-c)}{\gamma+n[(\gamma-2)n+2\gamma]} - \frac{2n}{\gamma+n[(\gamma-2)n+2\gamma]} K_{-i}$, implying that abatement decisions do not affect firms' incentives to invest in R&D. However, economies of scope (higher λ) and larger investments in abatement (higher z_i) make R&D more attractive for firm i, thus inducing an upward shift in its best response function without affecting the degree of strategic substitutability between R&D investments.

Similarly, differentiating (3) with respect to z_i, yields first-order condition $z_i = \frac{\lambda}{\alpha} k_i$, which is unaffected by its rivals' abatement, $Z_{-i} = \sum_{j \neq i} z_j$. When $\lambda \leq 0$, this first-order condition yields the well-known corner solution of firms not investing in abatement without regulation. When economies of scope are present ($\lambda > 0$), however, every firm i's abatement helps lower its R&D costs, leading to positive investments in both forms of R&D. The next lemma confirms this result.

Lemma 2. *Equilibrium investments of the unregulated oligopoly satisfy:*

a. If $\lambda \leq 0$, then $k_i^{NR} = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$ and $z_i^{NR} = 0$.

b. If $\lambda > 0$, then $k_i^{NR} = \frac{2\alpha(1-c)n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$ and $z_i^{NR} = \frac{2(1-c)\lambda n}{(n+1)^2(\alpha\gamma-\lambda^2)-2\alpha n}$, which are positive. Investments k_i^{NR} and z_i^{NR} are decreasing in the marginal cost c, the cost of R&D γ, the cost of ER&D α, and the number of firms n, but are increasing in the economies of scope λ.

Therefore, when the industry does not face regulation and investments do not affect each others' costs ($\lambda = 0$) or there are diseconomies of scope ($\lambda < 0$), every firm has no incentives to invest in abatement, only investing in cost-reducing R&D. In contrast, when investments give rise to economies of scope between R&D and ER&D ($\lambda > 0$), firms have incentives to invest in both. This occurs despite the absence of emission fees, because investments lower each other's marginal cost.

Our results also show that the presence of economies of scope induce less investment in both forms of R&D when either of them becomes more expensive (higher γ or α). However, every firm increases its investment in both types of R&D when economies of scope increase, which effectively reduces the cost of investment.[8]

3.2 Regulated Oligopoly

3.2.1 Third stage

Observing the investment profile $(k_1, \ldots, k_n, z_1, \ldots, z_n)$ and emission fee t, every firm i chooses its output q_i to solve

$$\max_{q_i \geq 0} \; (1 - q_i - Q_{-i})q_i - (c - k_i)q_i - t(q_i - z_i)$$

which yields output $q_i^R = \frac{1-c-t+nk_i-K_{-i}}{n+1}$, with associated profit $\pi_i^R = (q_i^R)^2 + tz_i$. Aggregate output in this stage is $Q^R = \sum_{i=1}^n q_i^R = \frac{n(1-c-t)+K}{n+1}$. When regulation is present, the emission fee directly reduces output in the third stage, but its effect on profits can be ameliorated by investing in ER&D z_i.

3.2.2 Second stage

The regulator chooses the emission fee to maximize social welfare as follows,

$$\max_t SW = CS(Q) + PS(Q) + T(Q) - ED(Q) \tag{5}$$

where $CS(Q) = \frac{1}{2}Q^2$ denotes consumer surplus, $PS(Q) = (1-Q)Q - \sum_{i=1}^n (c-k_i)q_i - t(Q-Z) - \sum_{i=1}^n \left(\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda(k_i z_i)\right)$ represents aggregate profits net of taxes, $T(Q) = t(Q - Z)$ denotes

[8] As an example, when $n = 2$ firms, the interior equilibrium becomes: $k_i^{NR}(2) = \frac{4\alpha(1-c)}{\alpha(9\gamma-4)-9\lambda^2}$ and $z_i^{NR}(2) = \frac{4\lambda(1-c)}{\alpha(9\gamma-4)-9\lambda^2}$. The investments are positive if $\lambda < \overline{\lambda} \equiv \frac{1}{3}\sqrt{\alpha(9\gamma-4)}$, which holds by Assumption II.

total tax collection, with Z denoting aggregate abatement, and $ED(Q) = d(Q - Z)^2$ measures aggregate environmental damages, where $d > 1$ denotes pollution severity. Aggregate output is evaluated at Q^R as defined in section 3.2.1. The following lemma describes the emission fee.

Lemma 3. *The emission fee t is*

$$t(K, Z) = \frac{(2dn - 1)[K + n(1 - c)] - dn(n + 1)Z}{(2d + 1)n^2}$$

which is increasing in environmental damage, d, and R&D, K, but decreasing in ER&D, Z, and marginal cost, c, and increasing in the number of firms n if and only if $K < \frac{n(1-c+dZ)}{2(dn-1)}$. *In addition, the fee is positive if and only if* $d > d(n) \equiv \frac{(1-c)n+K}{2n[n(1-c-Z)+K-Z]}$ *where cutoff d(n) is unambiguously increasing in c and Z, while decreasing in K, and decreasing in n if and only if* $Z < \overline{Z} \equiv \frac{[n(1-c)+K]^2}{n^2(1-c)+K(2n+1)}$.

When more firms enter the industry (higher n), the regulator anticipates more output in the subsequent stage and a positive emission fee becomes more likely if aggregate abatement is relatively low, $Z < \overline{Z}$. Otherwise, production becomes cleaner and the regulator is less likely to set a positive fee despite the industry being more competitive. However, when emissions are more damaging (higher d) or firms invest less in ER&D, Z, their output becomes more polluting, and the regulator responds setting a more stringent fee.

Our results highlight that the emission fee becomes more stringent in aggregate R&D investments, K, indicating that every firm i's investment, k_i, gives rise to a negative externality in its rivals' profits. The fee is, however, decreasing in aggregate ER&D, Z, entailing that every firm i's investment, z_i, generates a positive externality in its rivals' profits. While the latter was identified in previous studies, such as Poyago-Theotoky (2007), the former externality is novel to our setting, and originates from the coexistence of both forms of investment, arising even when economies of scope are absent.[9]

The emission fee in Lemma 3 satisfies $\frac{\partial t(K,Z)}{\partial K} = \frac{2dn-1}{n^2(1+2d)} > 0$ and $\frac{\partial t(K,Z)}{\partial Z} = \frac{-2d(n+1)}{n(1+2d)} < 0$, but $\frac{\partial^2 t(K,Z)}{\partial Z \partial K} = 0$, which implies that $t(K, Z)$ is separable in K and Z. Intuitively, the marginal decrease that aggregate abatement, Z, produces in $t(K, Z)$, is unaffected by firms' cost-reducing R&D investment. Alternatively, each investment separately affects the fee's stringency (abatement makes it less stringent, while cost-reducing R&D makes it more stringent), but they do not give rise to cross effects. Therefore, while the presence of both types of investment affects firms' decisions, the environmental agency can, essentially, consider each aggregate investment separately.

[9] Finally, note that when $K = 0$ and $n = 2$, the above emission fee simplifies to $t = \frac{(4d-1)(1-c)-6dZ}{2(2d+1)}$, which coincides with that in Poyago-Theotoky (2007) after adjusting the environmental damage function, where firms are only allowed to invest in abatement.

3.2.3 First stage

In the first stage, each firm i anticipates the output and emission fee $t(K, Z)$ in subsequent stages, and chooses its investment in k_i and z_i to solve

$$\max_{k_i, z_i > 0} \pi_i = \left(\frac{1 - c - t(K, Z) + nk_i - K_{-i}}{n+1} \right)^2 + t(K, Z)z_i - \left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda(k_i z_i) \right] \quad (6)$$

Differentiating with respect to k_i and z_i yields first-order conditions $k_i(z_i, Z_{-i})$ and $z_i(k_i, K_{-i})$ which, for compactness, are presented in the proof of Proposition 1. Each form of investment is increasing in the other type, which holds even in the absence of economies of scope, entailing that k_i and z_i are strategic complements. Intuitively, this occurs because a larger abatement induces a less stringent emission fee, allowing firms to invest more in R&D. Similarly, a larger investment in R&D triggers a more stringent emission fee, which induces firms to invest more in abatement. This complementarity provides firms with more incentives to invest in both forms of R&D than in models that consider a single type of investment, as we show in section 3.5. The equilibrium levels of R&D and ER&D are described in the following proposition.

Proposition 1. *Each firm i's equilibrium levels of R&D and ER&D are:*

$$k_i^R = \frac{1}{A}(1-c) \left[2\alpha + n \left(4d^2(\lambda+1)n^2 + 2d(\lambda + (n-1)(2\alpha+n+1) + \lambda(n-1)n) + 2\alpha(n-1) - \lambda n \right) + 1 \right],$$
$$z_i^R = \frac{1}{A}(1-c) \left[\gamma n[2d(n(2dn+n-1)+1) - n] + 2(2d+1)\lambda(n-1)n + 2\lambda \right].$$

When economies of scope are absent, $\lambda = 0$, equilibrium investments become

$$k_i^R = \frac{1}{B}(1-c)[2\alpha + 2n \left[d \left[(2d+1)n^2 - 1 \right] + \alpha(2d+1)(n-1) \right] + 1],$$
$$z_i^R = \frac{1}{B}(1-c)\gamma n[2d(n(2dn+n-1)+1) - n],$$

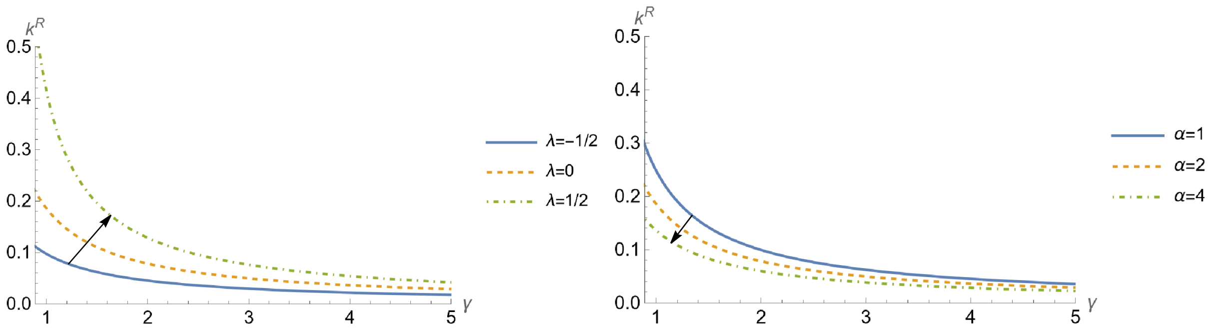
where terms A and B are presented, for compactness, in the appendix. If $\alpha \to +\infty$, equilibrium investments simplify to $k_i^R = \frac{2(1-c)[(2d+1)(n-1)n+1]}{(2d+1)n[n(n(\gamma+2\gamma d)-2)+2]-2} \equiv \overline{k}_i^R$ and $z_i^R = 0$; and if $\gamma \to +\infty$, these investments become $k_i^R = 0$ and $z_i^R = \frac{(1-c)[2d(n(2dn+n-1)+1)-n]}{n[\alpha(2d+1)^2 n + d(2d+1)n(n+2)+d]} \equiv \overline{z}_i^R$.

The last case in Proposition 1, where $\gamma \to +\infty$, induces firms to only invest in ER&D, and, when $n = 2$ firms, aligns with the equilibrium presented in Poyago-Theotoky (2007). Figure 1 depicts the equilibrium investment in R&D, k^R, as a function of its cost, γ, on the horizontal axis, and evaluates it at different economies of scope (figure 1a) and at different costs of abatement (figure 1b). For presentation purposes, figure 1 considers $c = 1/2$, $n = d = \alpha = 2$, and $\lambda = 0$.[10] Investment k^R is, as expected, unambiguously decreasing in its own cost, γ; shifts upwards when economies

[10] Parameter values are compatible with Assumption I, since $n = 2$ entails γ, $\alpha \geq \frac{8}{9}$; and with Assumption II, which in this context simplifies to $\lambda < |\overline{\lambda}| = \frac{1}{3}\sqrt{18\gamma - 8}$. Cutoff $\overline{\lambda}$ originates at a height of $\lambda = \frac{2\sqrt{2}}{3}$ when $\gamma = \frac{8}{9}$, and unambiguously increases in γ, implying that the lower bound of $\overline{\lambda}$ is $\left| \frac{2\sqrt{2}}{3} \right|$.

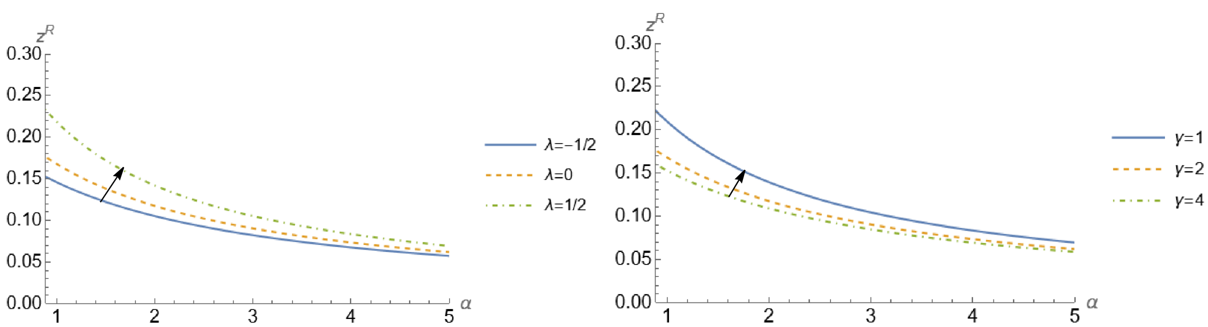
of scope increase (higher λ); but downwards when abatement becomes more expensive (higher α). Intuitively, the firm invests more in R&D when it can benefit from lower investment costs due to economies of scope originated in abatement, but also when such abatement becomes less expensive. This holds even when economies of scope are absent ($\lambda = 0$) since higher investments in abatement induce a less stringent fee, allowing more investment in R&D.

A similar argument applies to figure 2, depicting equilibrium investments in abatement, as a function of its own cost, α, and fixing $\gamma = 2$.[11] This investment is decreasing in its own cost, α; shift upwards in economies of scope (figure 2a) and when traditional R&D becomes less expensive (figure 2b). Aggregate abatement, $Z^R = nz_i^R$, is increasing in the number of firms, n, a typical Arrovian result, as in Poyago-Theotoky (2007).



(a) Equilibrium R&D k^R at different levels of λ. (b) Equilibrium R&D k^R at different levels of α.

Figure 1: Comparative statics for equilibrium R&D k^R.



(a) Equilibrium ER&D z^R at different levels of λ. (b) Equilibrium ER&D z^R at different levels of γ.

Figure 2: Comparative statics for equilibrium ER&D z^R.

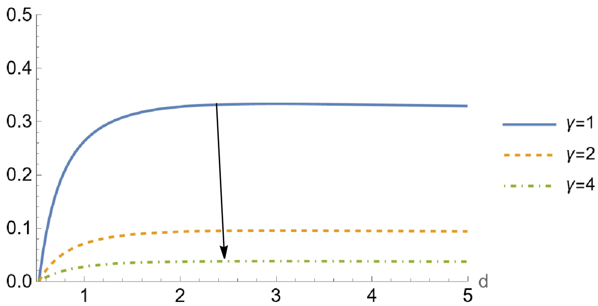
3.3 Comparing investments with and without regulation

We next evaluate the effect of regulation on equilibrium investment decisions. In particular, regulation affects traditional R&D investments by $\Delta k_i \equiv k_i^{NR} - k_i^R$ and abatement by $\Delta z_i \equiv z_i^{NR} - z_i^R$. While k_i^{NR} and z_i^{NR} are tractable, as found in Lemma 2, the expressions of k_i^R and z_i^R identified in

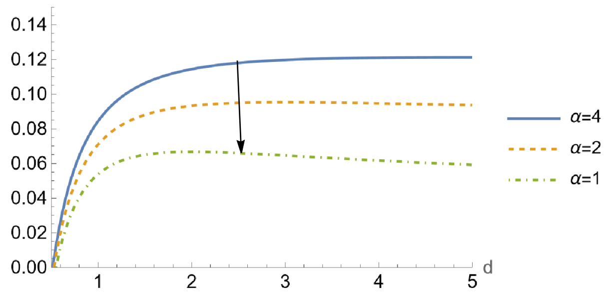
[11]Figure 1b considers $\gamma = 2$, which is consistent with Assumption I, which, in a context with $n = 2$ firms implies that γ, $\alpha \geq \frac{8}{9}$.

Proposition 1 are highly non-linear, thus not allowing for unambiguous comparative statics. Figure 3 depicts the investment differential Δk_i evaluated at the same parameter values as figure 1. Δk_i is positive and increasing in d, on the horizontal axis, thus indicating that the presence of regulation induces a more stringent fee, reducing investment k_i^R. Since k_i^{NR} is unaffected by parameter d, the differential Δk_i increases. In addition, this regulatory effect arises even in the absence of economies of scope, $\lambda = 0$, as in the baseline scenario.

Furthermore, when cost-reducing R&D becomes more expensive relative to investments in abatement (i.e., γ increases, α decreases, or both), curve Δk_i shifts downwards, as shown in figures 3a and 3b; implying that the presence of regulation has a smaller effect on firms' investment decisions. In this context, firms invest less in this R&D in the absence of regulation, which is relatively more expensive than abatement, and regulation does not significantly alter their investment decisions.[12]



(a) Difference in R&D $\Delta k_i \equiv k_i^{NR} - k_i^R$ at different levels of γ.



(b) Difference in R&D $\Delta k_i \equiv k_i^{NR} - k_i^R$ at different levels of α.

Figure 3: Comparative statics for the difference in R&D $\Delta k_i \equiv k_i^{NR} - k_i^R$.

The abatement differential Δz_i is, in contrast, unambiguously negative, implying that regulation induces more investment in abatement, $z_i^{NR} < z_i^R$. In the absence of economies of scope, $\lambda = 0$, z_i^{NR} is nil, implying that $\Delta z_i = z_i^R$, thus coinciding with figure 2. When economies of scope increase (higher λ), firms invest in abatement with and without regulation, entailing that firms increase their investments more significantly when regulation is absent than otherwise, increasing the differential Δz_i.

		k_i^{NR}	k_i^R	Δk_i	z_i^{NR}	z_i^R	Δz_i
Benchmark	$\lambda = 0$	0.1429	0.0782	0.00646	0	0.1176	-0.1176
Lower λ	$\lambda = -0.3$	0.1471	0.0572	0.0900	0	0.1086	-0.1086
	$\lambda = -0.2$	0.1477	0.0638	0.0809	0	0.1111	-0.1111
	$\lambda = -0.1$	0.1433	0.0708	0.0726	0	0.1141	-0.1141
Higher λ	$\lambda = 0.1$	0.1433	0.0864	0.0570	0.0072	0.1216	-0.1144
	$\lambda = 0.2$	0.1447	0.0952	0.0495	0.0145	0.1262	-0.1118
	$\lambda = 0.3$	0.1471	0.1050	0.0421	0.0221	0.1317	-0.1096

Table 1: Equilibrium investments where $c = 1/2$, and $d = \alpha = \gamma = n = 2$.

[12]A similar argument applies to more intense economies of scope, helping firms increase their investment more significantly when regulation is present than otherwise, ultimately shrinking Δk_i.

For illustration purposes, Table 1 evaluates equilibrium R&D with and without regulation, and their differential Δk_i; and abatement levels with and without regulation, along with their differential Δz_i. Intuitively, Δk_i and Δz_i measure the regulatory effect on investment decisions. If regulators overlooks the presence of economies of scope (assume that $\lambda = 0$ when this parameter is actually positive or negative), they could be overestimating the regulatory effect of their environmental policy (if Δk_i decreases in λ), or underestimating it otherwise. As shown in Table 1, Δk_i is decreasing in λ, implying that regulation has a smaller effect on R&D decisions when economies of scope are present than otherwise. In other words, policies that do not consider the presence of economies of scope would overestimate the effect of emission fees on R&D decisions, expecting a large investment reduction. In contrast, Δz_i is increasing in economies of scope (Δz_i is negative, but becomes closer to zero), entailing that regulation has a larger effect on abatement decisions when economies of scope are present than otherwise. In this case, a regulator who overlooks economies of scope would underestimate how much firms increase their abatement after facing environmental policy.

Tables A1-A4 in Appendix 1 are analogous to table 1 at higher levels of d, α, γ, and n, respectively. The R&D differential Δk_i expands when environmental damage d or the cost of ER&D α increases, thus enlarging the overestimation problem identified above. In contrast, Δk_i shrinks when the cost of R&D γ or the number of firms n increases, thus ameliorating overestimation issues. The ER&D differential Δz_i expands (in magnitude) when environmental damage d or the cost of R&D γ increases, emphasizing the underestimation of investments in abatement we discussed above. However, Δz_i shrinks when the cost of ER&D α or the number of firms n increases, reducing this underestimation.

3.4 Equilibrium fees

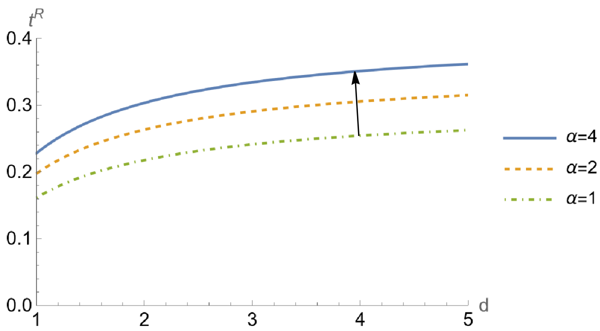
The next corollary evaluates the emission fee from Lemma 3, $t(K, Z)$, at the equilibrium R&D and ER&D found in Proposition 1, $t^R \equiv t(nk_i^R, nz_i^R)$.

Corollary 1. *The equilibrium emission fee is*

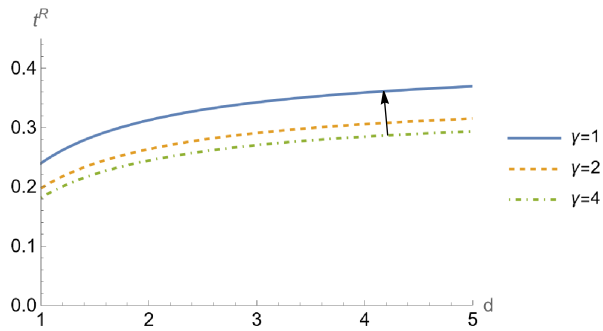
$$t^R = \frac{1}{C}(1-c)\left[2d^2n^3(2\alpha\gamma + \gamma - 2\lambda(\lambda+1)) - dn(\gamma(1 - 2\alpha(n-1)n + n) \right.$$
$$\left. +2\lambda((\lambda+1)(n-1)n - 1)) - \lambda - n^2\left(\alpha\gamma - \lambda^2\right)\right],$$

where term C is presented, for compactness, in the appendix. When $\gamma \to +\infty$, the equilibrium emission fee simplifies to $\bar{t}^R = \frac{(1-c)\left[d\left(2dn^2-n-1\right)+\alpha(2d+1)n(2dn-1)\right]}{n[\alpha(2d+1)^2n+d(2d+1)n(n+2)+d]}$, and when $\alpha \to +\infty$, the fee becomes $t^R = \frac{(1-c)\gamma(2d+1)n^2(2dn-1)}{(2d+1)n[n(n\gamma(2d+1)-2)+2]-2}$.

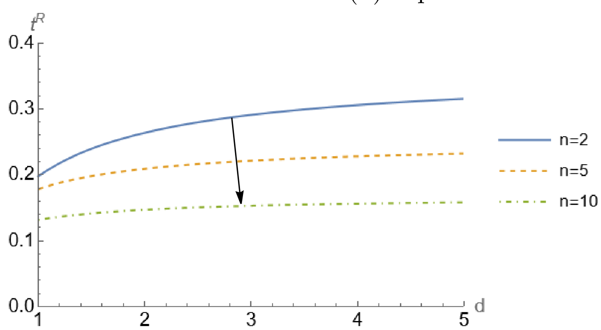
Figure 4 depicts the emission fee from Corollary 1 as a function of pollution severity, d, and considering the same parameter values as figure 3. The emission fee becomes more stringent when pollution is more severe (higher d), abatement is more expensive implying that firms invest less in clean technologies (higher α, figure 4a), or R&D is less expensive yielding more output and pollution



(a) Equilibrium fee t^R at different levels of α.



(b) Equilibrium fee t^R at different levels of γ.



(c) Equilibrium fee t^R at different levels of n.

Figure 4: Comparative statics for the equilibrium fee t^R.

(lower γ, figure 4b). When the market is more competitive, aggregate abatement increases, leading to less stringent emission fees (t^R shifts downwards in figure 4c).

The effect of economies of scope on the fee stringency is more involved. Figure 5 illustrates fee t^R, found in Corollary 1, as a function of λ, and evaluated at baseline parameters. When economies of scope are low, cost-reducing R&D is more responsive than abatement to a marginal increase in λ, thus leading to more pollution, and requiring a more stringent emission fee. When economies of scope are high, however, abatement becomes more responsive than traditional R&D, entailing less pollution and yielding a less stringent fee.

For comparison purposes, figure 5 also includes $\bar{t}^R$, which denotes the fee in traditional models assuming that firms only invest in abatement.[13] As shown in Corollary 1, fee $\bar{t}^R$ is not a function of λ, thus being unaffected by economies of scope. Fee t^R lies above $\bar{t}^R$ for most economies of scope, λ, implying that traditional models that prescribe $\bar{t}^R$ instead of t^R, would set a lax emission fee, giving rise to undertaxation and generating socially excessive pollution. When economies of scope are sufficiently strong, however, fee t^R lies below $\bar{t}^R$, indicating that traditional models prescribe a

[13] When $n = 2$, fee $\bar{t}^R$ coincides with that in Poyago-Theotoky (2007), equation (1) evaluated at $a = 1$, $\beta = 0$, and accounting for the difference in environmental damage, $\bar{t}^R = \frac{(1-c)(4d-1)-6dZ}{2(1+2d)}$. In addition, the parameter values in figures 4 and A1, where both α and γ are weakly larger than 2, are compatible with Assumption I: in the case of $n = 5$ firms this assumption entails $\gamma, \alpha \geq \frac{25}{18} = 1.389$; in the case of $n = 10$ it becomes $\gamma, \alpha \geq \frac{200}{121} = 1.653$; and does not exceed a height of 2 for any value of n. Similarly, this figure is compatible with Assumption II, since cutoff $\overline{\lambda}$ increases in the number of firms, and lies above $\lambda = 1$ for both $n = 5$ and $n = 10$, thus making condition $\lambda < |\overline{\lambda}|$ more likely to hold.

too stringent fee, leading to overtaxation.

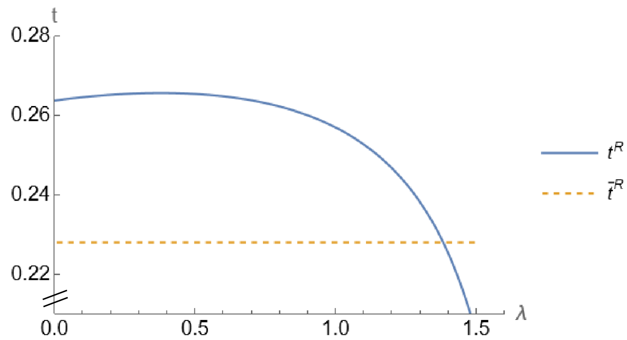


Figure 5: Equilibrium fees t^R and $\bar{t}^R$ over a range of λ.

As shown above, an increase in economies of scope raises both forms of investment, leading to opposite effects on the stringency of emission fee t^R. Specifically, cost-reducing R&D entails a more stringent fee, while abatement reduces this stringency. Since fee t^R is positive, the first effect dominates. When economies of scope are sufficiently strong, however, the second effect grows, yielding a lower t^R, which coincides with $\bar{t}^R$ at the crossing point.

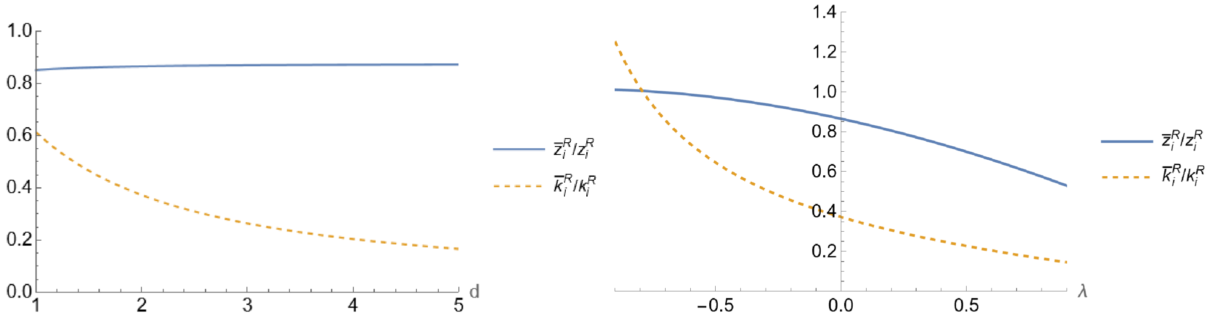
For completeness, figure A1 in Appendix 1 depicts the fee differential $\Delta t^R \equiv t^R - \bar{t}^R$, showing that it is positive under most parameter values, $t^R > \bar{t}^R$; but becomes less intense when the market is more competitive (higher n), pollution is not severe (lower d), and investments are more expensive (higher γ or α).

3.5 Investment Ratios

For illustration purposes, figures 6 and 7 depict the ratio of the investment in ER&D when R&D is absent ($\bar{z}_i^R$, as found at the end of Proposition 1), relative to the ER&D when both types of investments are present (z_i^R in Propostion 1), $\bar{z}_i^R/z_i^R$ in the solid line. We also depict a similar ratio for the investment in R&D, i.e., $\bar{k}_i^R/k_i^R$ in the dashed line. When either ratio is close to 1, our findings indicate that allowing for both forms of investment does not yield different results than models assuming firms separately invest in only one. In contrast, ratios closer to zero (above one) would indicate the opposite result: assuming that firms' investment decisions are separable would underestimate (overestimate, respectively) firms' ER&D decisions. In other words, firms have stronger incentives to invest in ER&D than what the literature found when the ratio is closer to 0, smaller incentives when the ratio is above one, but similar incentives otherwise.

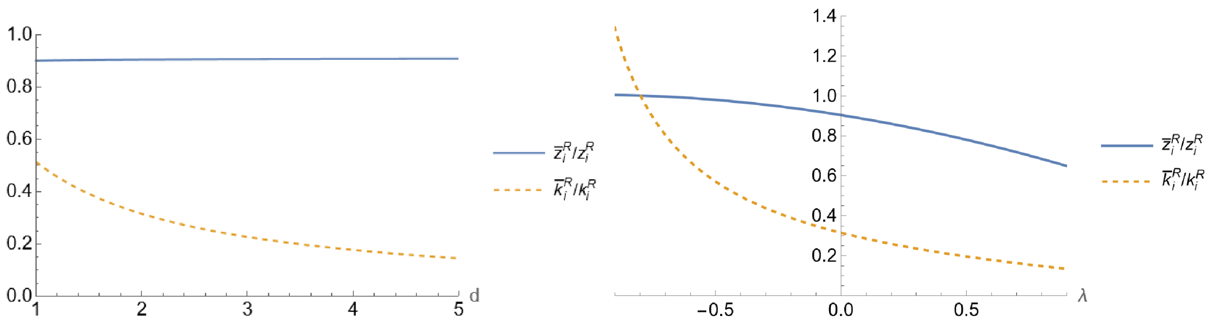
Overall, our findings indicate that the literature's underestimation of ER&D incentives is the largest (ratio $\bar{z}_i^R/z_i^R$ approaches zero) when economies of scope are significant (high λ). Otherwise, assuming separable investment decisions yields similar findings, entailing that standard models can accurately approximate investment incentives.

Regarding R&D, our results indicate that the literature underestimation is the largest (ratio $\bar{k}_i^R/k_i^R$ approaches zero) when pollution is severe (high d) and economies of scope exist ($\lambda > 0$);



(a) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of d. (b) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of λ.

Figure 6: Investment ratios when $n = 2$.



(a) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of d. (b) Ratios $\overline{z}_i^R/z_i^R$ and $\overline{k}_i^R/k_i^R$ as a function of λ.

Figure 7: Investment ratios when $n = 5$.

but become small otherwise. When pollution is sufficiently low and diseconomies of scope are strong, our findings indicate that ratio $\overline{k}_i^R/k_i^R$ can exceed 1, entailing that standard models would overestimate firms' investment in R&D.

As a summary, Table 2 evaluates equilibrium results with and without regulation at different economies of scope, λ. Stronger economies of scope lead firms to invest more in both forms of R&D with and without regulation. However, R&D investment is more responsive than abatement to a given increase in economies of scope, thus generating more output and pollution, and requiring a more stringent emission fee. Further, with higher economies of scope, aggregate output increases more than abatement, thus raising emissions. This increase in output has a smaller impact on consumer and producer surplus than on environmental damage, ultimately decreasing social welfare.

4 Extension: Sequential investment decisions

Previous sections consider that firms simultaneously choose their investments in R&D and abatement in the first stage of the game. Instead, we now examine sequential investment decisions: in the first stage, every firm chooses its abatement, z_i; in the second stage, every firm responds with its R&D investment, k_i; in the third stage, the EPA sets the emission fee t; and, in the last stage, firms compete. For compactness, we refer to this sequential timing as Seq, and focus on economies

		t	K	Z	Q	ED	SW
Regulation	Benchmark, $\lambda = 0$	0.2637	0.1565	0.2351	0.2097	0.0013	-0.1485
Lower λ	$\lambda = -0.3$	0.2598	0.1145	0.2171	0.1983	0.0007	-0.1271
	$\lambda = -0.2$	0.2613	0.1276	0.2222	0.2016	0.0008	-0.1332
	$\lambda = -0.1$	0.2626	0.1415	0.2282	0.2054	0.0010	-0.1403
Higher λ	$\lambda = 0.1$	0.2646	0.1727	0.2432	0.2145	0.0016	-0.1581
	$\lambda = 0.2$	0.2652	0.1905	0.2525	0.2200	0.0021	-0.1693
	$\lambda = 0.3$	0.2656	0.2101	0.2633	0.2263	0.0027	-0.1826
No regulation	Benchmark, $\lambda = 0$	0	0.2857	0	0.4286	0.3674	-0.2245
Higher λ	$\lambda = 0.1$	0	0.2866	0.0143	0.4289	0.3437	-0.2007
	$\lambda = 0.2$	0	0.2894	0.0289	0.4298	0.3214	-0.1781
	$\lambda = 0.3$	0	0.2942	0.0221	0.4314	0.3000	-0.1562

Table 2: Equilibrium values compared to a benchmark where $c = 1/2$ and $d = \alpha = \gamma = n = 2$.

of scope, $\lambda \geq 0$.

4.1 Unregulated oligopoly

In this setting, equilibrium behavior in the last stage (output levels) coincides with that in Lemma 1. Investment decisions, however, differ from those in Lemma 2. In particular, every firm now takes the abatement profile $(z_1, ..., z_n)$ as given, and chooses its R&D investment to solve

$$\max_{k_i \geq 0} \ \frac{(1 - c + nk_i - K_{-i})^2}{(n+1)^2} - \left(\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i\right), \tag{3'}$$

which is analogous to (3) but with only one choice variable, k_i.

Lemma 4. *In the sequential investment game without regulation, every firm i chooses R&D investment* $k_i^{NR,Seq}(z_i) = \frac{2(1-c)n+(n+1)^2\lambda z_i}{\gamma(n+1)^2-2n}$*, which is positive and increasing in* z_i *and* λ*.*

Therefore, when either abatement is nil, $z_i = 0$, or economies of scope are absent, $\lambda = 0$, R&D investment simplifies to $k_i^{NR,Seq}(0) = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$, thus coinciding with that in the simultaneous-investment version of the game (Lemma 2). However, when both abatement and economies of scope are positive, $z_i > 0$ and $\lambda > 0$, every firm benefits from lower marginal costs in its traditional R&D, and responds by increasing this investment. As suggested in previous sections, firms use abatement to help lower their traditional R&D costs, but this only applies when both λ and z_i are positive.

Anticipating $k_i^{NR,Seq}(z_i)$, every firm solves a problem similar to (3') but evaluated at $k_i^{NR,Seq}(z_i)$ and choosing its abatement level, z_i, as follows

$$\max_{z_i \geq 0} \ \frac{(1 - c + k_i^{NR,Seq}(z_i))^2}{(n+1)^2} - \left(\frac{1}{2}\gamma \left(k_i^{NR,Seq}(z_i)\right)^2 + \frac{1}{2}\alpha z_i^2 - \lambda \left(k_i^{NR,Seq}(z_i)\right) z_i\right). \tag{3''}$$

Differentiating with respect to z_i yields the first-order condition

$$\underbrace{\frac{\partial k_i^{NR,Seq}(z_i)}{\partial z_i}}_{(+)} \underbrace{\left[\frac{2(1-c+k_i^{NR,Seq}(z_i))}{(n+1)^2} - (\gamma k_i^{NR,Seq}(z_i) - \lambda z_i)\right]}_{(-)} + \alpha z_i - \lambda k_i^{NR,Seq}(z_i) = 0, \quad \text{(3"a)}$$

which, relative to Lemma 2, gives rise to a new strategic effect, as captured in the first term. While $\frac{\partial k_i^{NR,Seq}(z_i)}{\partial z_i} > 0$ from Lemma 4, the term in square brackets is negative because, in the second stage, differentiating problem (3') with respect to k_i, yields a first-order condition $\frac{2n(1-c+nk_i-K_{-i})}{(n+1)^2} - (\gamma k_i - \lambda z_i) = 0$ which, after invoking symmetry, $K_{-i} = (n-1)k_i$, simplifies to

$$\frac{2n(1-c+k_i)}{(n+1)^2} - (\gamma k_i - \lambda z_i) = 0. \quad \text{(3"b)}$$

Evaluating equation (3"b) at $k_i = k_i^{NR,Seq}(z_i)$, we can compare it against the term in brackets in (3"a), concluding that the latter must be negative; which entails that the strategic effect from sequentially choosing abatement is negative. Intuitively, under a simultaneous setting, an increase in abatement lowers R&D costs if economies of scope are present, helping firms increase their R&D investments which, in turn, reduces abatement costs; thus creating a positive "feedback" effect. Sequential investments, however, break this feedback: an increase in abatement still lowers R&D costs, inducing more R&D investments, but an increase in R&D does not affect abatement costs because abatement decisions are now taken as given in the second stage. As a consequence, firms have fewer incentives to invest in abatement in the sequential than simultaneous setting, $z_i^{NR,Seq} < z_i^{NR}$.

The next proposition identifies the equilibrium abatement level, and evaluates R&D in equilibrium as well.

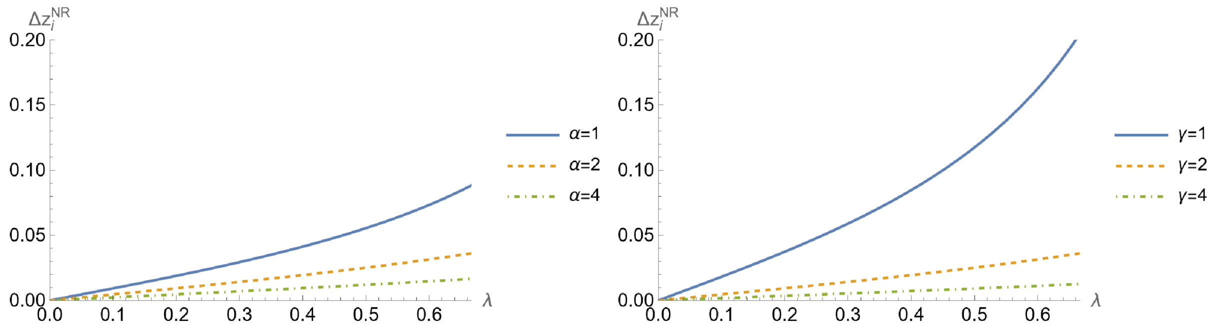
Proposition 2. *In the sequential-investment game without regulation, equilibrium investments satisfy:*

a. If $\lambda = 0$, equilibrium abatement is $z_i^{NR,Seq} = 0$, yielding an equilibrium R&D investment of $k_i^{NR,Seq} = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$.

b. If $\lambda > 0$, equilibrium abatement is $z_i^{NR,Seq} = \frac{2\lambda(1-c)\left[n^2(\gamma-2)+\gamma(2n+1)\right]}{D}$, yielding an equilibrium R&D investment of $k_i^{NR,Seq} = \frac{2(1-c)[nD+\lambda^2(n+1)^2(n^2(\gamma-2)+\gamma(2n+1))]}{D[(n+1)^2\gamma-2n]}$, where term D is presented, for compactness, in the appendix.

To understand how sequential decisions affect investment levels in equilibrium, figure 8 depicts the abatement differential $\Delta z_i^{NR} \equiv z_i^{NR} - z_i^{NR,Seq}$, where z_i^{NR} is presented in Lemma 2 and $z_i^{NR,Seq}$ in Proposition 2. When Δz_i^{NR} lies on the positive quadrant, firms invest more in abatement when their R&D decisions are simultaneous than sequential, while the opposite applies when Δz_i^{NR} lies in the negative quadrant. The figure considers the same parameter values as in the baseline scenario

($c = 1/2$ and $\alpha = \gamma = n = 2$, since these investments are not a function of d) and plots Δz_i^{NR} as a function of λ on the horizontal axis.



(a) Differential Δz_i^{NR} at different values of α.

(b) Differential Δz_i^{NR} at different values of γ.

Figure 8: Differential Δz_i^{NR} over a range of λ.

Figures 8a and 8b show that abatement coincides in both timings when economies of scope are absent ($\lambda = 0$, at the origin, where $z_i^R = z_i^{NR,Seq} = 0$). Otherwise, firms invest more when choosing their investments simultaneously, Δz_i^{NR} is positive. This differential is emphasized when economies of scope increase, since the feedback effect becomes more intense.

When abatement becomes more costly (higher α), firms invest less in abatement, both under a simultaneous and sequential context, but the inability to exploit economies of scope induces a more significant reduction in $z_i^{NR,Seq}$ than in z_i^{NR}, yielding a downward shift in Δz_i^{NR}, as shown in figure 8a. A similar argument applies when traditional R&D becomes more expensive (higher γ in figure 8b), leading firms to reduce their R&D in both settings, making abatement more costly due to economies of scope, yielding a similar result as when α increases.[14]

4.2 Regulated oligopoly

In the second stage, every firm takes abatement profile $(z_1, ..., z_n)$ as given and, anticipating the emission fee $t(K, Z)$ found in Lemma 3, chooses its R&D investment to solve

$$\max_{k_i \geq 0} \frac{(1 - c - t(K,Z) + nk_i - K_{-i})^2}{(n+1)^2} + t(K,Z)z_i - \left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda k_i z_i\right], \tag{6'}$$

which coincides with equation (6) in subsection 3.2.3, but with only one choice variable, k_i.

Lemma 5. *In the sequential investment game with regulation, every firm i chooses R&D investment*

$$\begin{aligned} k_i^{R,Seq}(z_i, Z_{-i}) = &\frac{1}{E}[2(2d+1)n^2\left(d(2z_i + Z_{-i}) + 1\right) - 2c((2d+1)(n-1)n + 1) \\ &- (2d+1)n\left(2d\left(z_i + Z_{-i}\right) + z_i + 2\right) + 2d\left(z_i + Z_{-i}\right) + (2d+1)^2\lambda n^3 z_i + 2], \end{aligned}$$

[14]Figure A2 in Appendix 1 examines the investment differential $\Delta k_i^{NR} \equiv k_i^{NR} - k_i^{NR,Seq}$ for traditional R&D, showing identical properties as the abatement differential in figures 8a and 8b.

where E is presented, for compactness, in the appendix. Investment $k_i^{R,Seq}(z_i, Z_{-i})$ is positive, and increasing in z_i, Z_{-i}, and λ, being more responsive to an increase in z_i than in Z_{-i}. In addition, the response in z_i is increasing in λ.

In the absence of economies of scope, $\lambda = 0$, individual abatement z_i reduces the stringency of the emission fee $t(K, Z)$, inducing firm i to increase its R&D investment $k_i^{R,Seq}(z_i, Z_{-i})$; as opposed to under no regulation. Economies of scope, then, provide an additional benefit, with individual abatement reducing R&D costs, thus leading to a further increase in R&D investment. Unlike no regulation, the aggregate abatement by firm i's rivals, Z_{-i}, now affects this firm's R&D decisions. In particular, a larger Z_{-i} lowers the stringency of the emission fee, benefiting all firms, which reduces every firm's production costs net of taxes, ultimately providing incentives to invest more in R&D than in the absence of regulation.

Anticipating $k_i^{R,Seq}(z_i, Z_{-i})$, every firm solves a problem similar to (6') but evaluated at $k_i^{R,Seq}(z_i, Z_{-i})$ and choosing its abatement level, z_i, as follows

$$\max_{z_i \geq 0} \ \frac{(1 - c + k_i^{R,Seq}(z_i, Z_{-i}))^2}{(n+1)^2} + t(nk_i^{R,Seq}(z_i, Z_{-i}), Z)z_i - \left(\frac{1}{2}\gamma\left(k_i^{R,Seq}(z_i, Z_{-i})\right)^2 + \frac{1}{2}\alpha z_i^2 - \lambda\left(k_i^{R,Seq}(z_i, Z_{-i})\right)z_i\right). \qquad (6")$$

Differentiating with respect to z_i yields the first-order condition

$$\frac{\partial t(K,Z)}{\partial z_i}\left(\frac{\partial \pi}{\partial t(K,Z)} + z_i\right) + t(K,Z) - \alpha z_i + \lambda k_i^{R,Seq}(z_i, Z_{-i})$$

$$+ \underbrace{\frac{\partial k_i^{R,Seq}(z_i, Z_{-i})}{\partial z_i}}_{(+)} \underbrace{\left[\frac{\partial \pi}{\partial k_i^{R,Seq}(z_i, Z_{-i})} + \frac{\partial t(K,Z)}{\partial k_i^{R,Seq}(z_i, Z_{-i})} z_i - \gamma k_i^{R,Seq}(z_i, Z_{-i}) + \lambda z_i\right]}_{(+) \text{ or } (-)} = 0 \qquad (6"a)$$

where, for compactness, $\pi \equiv \frac{(1-c-t(K,Z)+nk_i-K_{-i})^2}{(n+1)^2}$. The first line of equation (6"a) coincides with the first-order condition under simultaneous investment decisions (Proposition 1). Therefore, the second line of equation (6"a) identifies the strategic effect that arises when firms sequentially choose their investments. The term in square brackets can be positive or negative, depending on the magnitude of economies of scope, λ.

As discussed in subsection 4.1, investments in abatement without regulation benefit from a positive feedback loop in simultaneous settings, but do not under sequential contexts, providing firms with unambiguously stronger incentives to invest in abatement under simultaneous than sequential settings; a differential that expands in economies of scope. The presence of regulation gives rise to a second effect. In particular, regulation induces firms to increase their abatement to lower their net emissions, yielding less stringent fees. This generates a positive externality on its rivals' profits, under both simultaneous and sequential timings. Under a simultaneous context, however, firms face the strategic uncertainty of R&D investments $(k_1, k_2, \ldots, k_N)$ not being determined yet.

Under a sequential setting, in contrast, firms anticipate their R&D investment in the next stage, $k_i^{R,Seq}(z_i, Z_{-i})$. When economies of scope are absent, $\lambda = 0$, the strategic uncertainty induces firms to invest more in abatement under a simultaneous than sequential timing, giving rise to a positive abatement differential; as opposed to no regulation, where this differential was nil. When economies of scope increase (higher λ), the response of $k_i^{R,Seq}(z_i, Z_{-i})$ to a marginal increase in z_i is increasing in λ, as shown in Lemma 5, attenuating the positive abatement differential under no regulation. Overall, the positive feedback effect still dominates and yields $z_i^R > z_i^{R,Seq}$ under all parameter conditions.

The following proposition presents equilibrium abatement and the resulting R&D investments.

Proposition 3. *In the sequential investment game with regulation, every firm chooses an abatement level*

$$z_i^{R,Seq} = \frac{1}{G}(1-c)\left[16\gamma^2 d^4 n^6 + 8\gamma d^3 n^2 \left[n\left(n\left(\gamma + 3\gamma n^2 - (\gamma+2)n - 2\right) + 6\right) - 2\right] + F\right]$$

yielding an equilibrium R&D of

$$\begin{aligned} k_i^{R,Seq} = & \frac{1}{J}(1-c)\left[16\gamma d^4(\lambda+1)n^6 + 8d^3n^2\left[n\left(6(\lambda+1)\right.\right.\right. \\ & \left.\left.\left.+n\left(\gamma(\lambda-2\alpha-1) + 2\lambda^2 + 3\gamma(\lambda+1)n^2 + n(2\alpha\gamma - \lambda(\gamma+2\lambda+4) - 2) - 2\right)\right) - 2(\lambda+1)\right] + L\right] \end{aligned}$$

where terms F, G, J, and L are presented in the appendix.

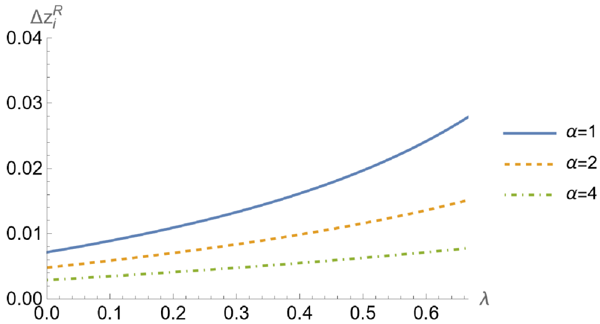
We next compare abatement levels under simultaneous and sequential investment decisions, and then we examine emission fees in each context.

4.2.1 Simultaneous vs. Sequential Abatement

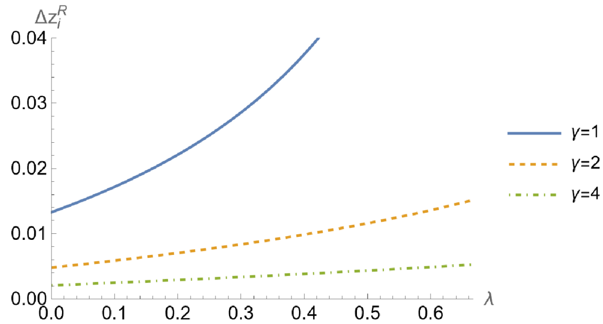
Figure 9 illustrates the abatement differential $\Delta z_i^R \equiv z_i^R - z_i^{R,Seq}$, as a function of λ and considers $d = 2$. Similar to the case of no regulation (figure 8), abatement is higher when firms choose investments simultaneously than sequentially, i.e., Δz_i^R lies in the positive quadrant in figure 9a, implying that the feedback effect described above dominates. A more costly abatement (higher α in figure 9a) or R&D (higher γ in figure 9b) emphasizes the strategic uncertainty effect, thus shifting Δz_i^R downwards.

4.2.2 Simultanous vs. Sequential Emission Fee

Figure 10 superimposes fee $t^{R,Seq}$ on figure 5, where $t^{R,Seq} = t(nk_i^{R,Seq}, nz_i^{R,Seq})$, and investments $k_i^{R,Seq}$ and $z_i^{R,Seq}$ are found in Proposition 3. As shown above, firms invest less in abatement under a sequential than simultaneous timing, $\Delta z_i^R > 0$ as depicted in figure 9, and the regulator responds setting a more stringent emission fee in a sequential setting, $t^{R,Seq} > t^R$, as shown in figure 10.



(a) Differential Δz_i^R at different values of α.



(b) Differential Δz_i^R at different values of γ.

Figure 9: Differential Δz_i^R over a range of λ.

Therefore, sequential decisions emphasize the undertaxation problem identified under simultaneous investments.

For robustness, figure A3 in Appendix 1 depicts the fee differential $\Delta t \equiv t^{Seq} - t^R$, as a function of λ, where fee t^R was identified in Corollary 1 (simultaneous investments). The fee differential Δt^R lies on the positive quadrant under all parameter values, thus confirming our above results where $t^{R,Seq} > t^R$. This fee differential is, however, increasing in the economies of scope (higher λ), but decreasing in investment costs (higher α or γ).

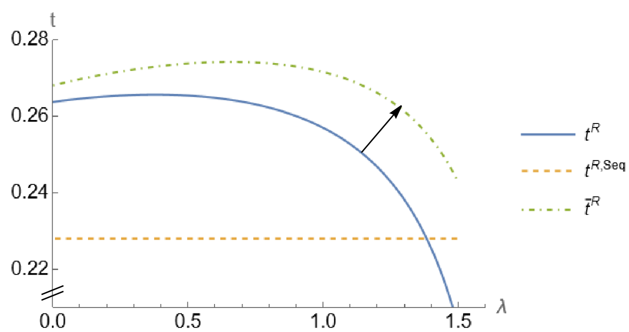


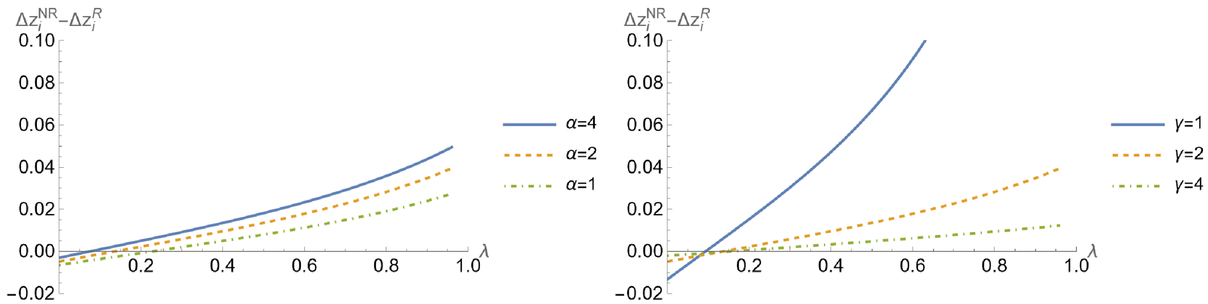
Figure 10: Equilibrium fees $t^{R,Seq}$, t^R, and $\bar{t}^R$ over a range of λ.

4.3 Is regulation more effective with simultaneous or sequential investments?

We next compare the abatement differentials, Δz_i^R and Δz_i^{NR}, to examine if sequential investments have a greater effect with than without regulation. Alternatively, $\Delta z_i^{NR} - \Delta z_i^R$ can be rearranged as

$$(z_i^{R,Seq} - z_i^{NR,Seq}) - (z_i^R - z_i^{NR}).$$

If this term is positive, regulation would have a stronger effect in boosting abatement when investments are sequential than otherwise. Figure 11 depicts the differential $\Delta z_i^{NR} - \Delta z_i^R$ as a function of λ. Except when economies of scope are very small, this differential lies on the positive quadrant, implying that firms are more responsive to regulation when choosing their investments sequentially than simultaneously, and confirming that the strategic uncertainty effect attenuates the feedback effect.



(a) Differential $\Delta z_i^{NR} - \Delta z_i^{R}$ at different values of α.

(b) Differential $\Delta z_i^{NR} - \Delta z_i^{R}$ at different values of γ.

Figure 11: Differentials $\Delta z_i^{NR} - \Delta z_i^{R}$ over a range of λ.

Table 3 summarizes our equilibrium results under sequential investment decisions, using the same parameter conditions as in table 2 to facilitate the comparison against simultaneous investments. In line with figure 9, aggregate abatement is larger when investments are simultaneous than otherwise, thus requiring a less stringent emission fee. Aggregate output is, then, lower when investments are sequential, yielding less environmental damage, and a higher welfare in equilibrium.[15]

		$t^{R,Seq}$	$K^{R,Seq}$	$Z^{R,Seq}$	Q	ED	SW
Regulation	Benchmark, $\lambda = 0$	0.2681	0.1525	0.2255	0.2055	0.0008	-0.1376
Higher λ	$\lambda = 0.1$	0.2697	0.1671	0.2314	0.2093	0.0010	-0.1445
	$\lambda = 0.2$	0.2710	0.1831	0.2384	0.2137	0.0012	-0.1526
	$\lambda = 0.3$	0.2722	0.2004	0.2466	0.2187	0.0016	-0.1622
No regulation	Benchmark, $\lambda = 0$	0	0.2857	0	0.4286	0.3673	-0.1420
Higher λ	$\lambda = 0.1$	0	0.2860	0.0051	0.4287	0.3588	-0.1247
	$\lambda = 0.2$	0	0.2870	0.0103	0.4290	0.3506	-0.1084
	$\lambda = 0.3$	0	0.2887	0.0157	0.4296	0.3426	-0.0928

Table 3: Equilibrium values under sequential investment, with benchmark considering $c = 1/2$ and $d = \alpha = \gamma = n = 2$.

5 Discussion

Three externalities. Our paper identifies three external effects. In the absence of regulation, when firm i invests in cost-reducing R&D, k_i, it becomes more competitive, thus lowering its rivals' profits (first negative externality). When economies (diseconomies) of scope are present,

[15] A similar argument applies under no regulation, where aggregate R&D and abatement are lower with sequential than simultaneous investments. The lack of regulation, however, switches the above ranking, with abatement falling more than traditional R&D. As a consequence, environmental damages are larger under a sequential setting.

investment in abatement decrease (increase) its R&D costs, thus emphasizing (ameliorating) the above externality. If regulation is present and abatement is not, firm i's investment in cost-reducing technologies, k_i, gives rise to a second external effect, namely, a more stringent emission fee which lowers its rivals' profits. Finally, when abatement is allowed, every firm's investment decreases its own emissions, but also lowers the stringency of the emission fee; thus entailing a third (positive) externality on its rivals' profits.

Relative to the existing literature, which only allows for investments in abatement, the introduction of R&D investment gives rise to two new external effects described above. Ignoring these externalities could induce undertaxation as we next discuss.

Undertaxation. Our results show that a policy design that overlooks firms' simultaneous investments in abatement and R&D leads to an underestimation of their cost-reducing R&D. This induces an emission fee that is less stringent than when considering both investments, yielding a socially excessive amount of pollution, and, hence, a policy inefficiency. This "undertaxation" arises even in the absence of economies of scope, is ameliorated when these economies are stronger, but emphasized when pollution becomes more severe, when the cost of investment in abatement or R&D is low, or when the industry is less competitive. Our findings, then, suggest that in these contexts regulators should examine whether firms invest in one or more forms of R&D before designing environmental policy. Otherwise, this fee inefficiency becomes smaller.

Sequential investments. Under no regulation and economies of scope, we show that a positive feedback effect arises under simultaneous investments, helping firms lower their abatement costs. Under a sequential setting, however, this feedback effect is "broken," since R&D investments do not reduce abatement costs. Therefore, firms invest more under simultaneous than sequential contexts, giving rise to an abatement differential across settings. The introduction of regulation, nonetheless, attenuates this differential, implying that firms still invest more in abatement under simultaneous than sequential settings; requiring a less stringent emission fee. As a consequence, overlooking that firms invest in both forms of R&D and that investment decisions are sequential would emphasize the undertaxation problem identified above.

Further research. Our model can be extended along several dimensions. First, one can consider firms coordinating their investment decisions in an R&D cartel, which allows them to jointly choose the vector of k_i's, that of z_i's, or both; which could help firms internalize some of the external effects listed above. Second, our paper considers that economies of scope are symmetric across investment types. While asymmetric economies of scope may better fit some industries, its equilibrium results are likely to be more intractable (two different λ's). Finally, one can allow for investment decisions to generate spillover effects across firms, potentially being different between R&D and ER&D.

6 Appendix 1

		k_i^{NR}	k_i^R	Δk_i	z_i^{NR}	z_i^R	Δz_i
Benchmark	$\lambda = 0$	0.1429	0.0742	0.0686	0	0.1296	-0.1296
Lower λ	$\lambda = -0.3$	0.1471	0.0514	0.0957	0	0.1204	-0.1204
	$\lambda = -0.2$	0.1447	0.0585	0.0862	0	0.1230	-0.1230
	$\lambda = -0.1$	0.1433	0.0661	0.0772	0	0.1260	-0.1260
Higher λ	$\lambda = 0.1$	0.1433	0.0830	0.0603	0.0072	0.1337	-0.1266
	$\lambda = 0.2$	0.1447	0.0926	0.0521	0.0145	0.1386	-0.1241
	$\lambda = 0.3$	0.1471	0.1032	0.0489	0.0221	0.1443	-0.1222

Table A1: Equilibrium investments where $c = 1/2$, $d = 4$, and $\alpha = \gamma = n = 2$.

		k_i^{NR}	k_i^R	Δk_i	z_i^{NR}	z_i^R	Δz_i
Benchmark	$\lambda = 0$	0.1429	0.0598	0.0831	0	0.0734	-0.0734
Lower λ	$\lambda = -0.3$	0.1450	0.0471	0.0979	0	0.0693	-0.0693
	$\lambda = -0.2$	0.1438	0.0511	0.0927	0	0.0705	-0.0705
	$\lambda = -0.1$	0.1431	0.0553	0.0877	0	0.0719	-0.0719
Higher λ	$\lambda = 0.1$	0.1431	0.0645	0.0786	0.0036	0.0752	-0.0716
	$\lambda = 0.2$	0.1438	0.0695	0.0742	0.0072	0.0772	-0.0700
	$\lambda = 0.3$	0.1450	0.0749	0.0700	0.0109	0.0794	-0.0685

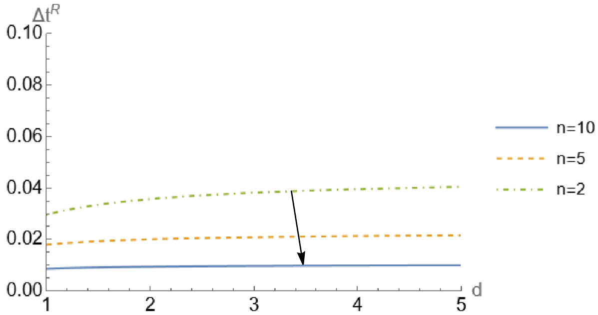
Table A2: Equilibrium investments where $c = 1/2$, $\alpha = 4$, and $d = \gamma = n = 2$.

		k_i^{NR}	k_i^R	Δk_i	z_i^{NR}	z_i^R	Δz_i
Benchmark	$\lambda = 0$	0.0625	0.0363	0.0262	0	0.1090	-0.1090
Lower λ	$\lambda = -0.3$	0.0633	0.0273	0.0359	0	0.1050	-0.1050
	$\lambda = -0.2$	0.0629	0.0302	0.0326	0	0.1061	-0.1061
	$\lambda = -0.1$	0.0626	0.0332	0.0294	0	0.1075	-0.1075
Higher λ	$\lambda = 0.1$	0.0626	0.0395	0.0231	0.0031	0.1108	-0.1076
	$\lambda = 0.2$	0.0629	0.0428	0.0200	0.0063	0.1127	-0.1064
	$\lambda = 0.3$	0.0633	0.0463	0.0170	0.0095	0.1149	-0.1054

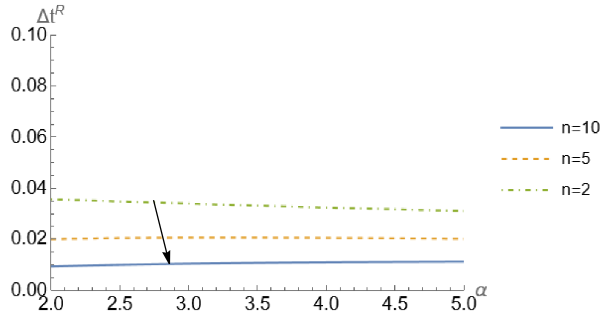
Table A3: Equilibrium investments where $c = 1/2$, $\gamma = 4$, and $d = \alpha = n = 2$.

		k_i^{NR}	k_i^R	Δk_i	z_i^{NR}	z_i^R	Δz_i
Benchmark	$\lambda = 0$	0.0806	0.0530	0.0276	0	0.0880	-0.0880
Lower λ	$\lambda = -0.3$	0.0828	0.0381	0.0447	0	0.0833	-0.0833
	$\lambda = -0.2$	0.0816	0.0428	0.0387	0	0.0846	-0.0846
	$\lambda = -0.1$	0.0809	0.0478	0.0331	0	0.0862	-0.0862
Higher λ	$\lambda = 0.1$	0.0809	0.0585	0.0224	0.0040	0.0901	-0.0861
	$\lambda = 0.2$	0.0816	0.0644	0.0172	0.0082	0.0925	-0.0843
	$\lambda = 0.3$	0.0828	0.0707	0.0121	0.0124	0.0952	-0.0828

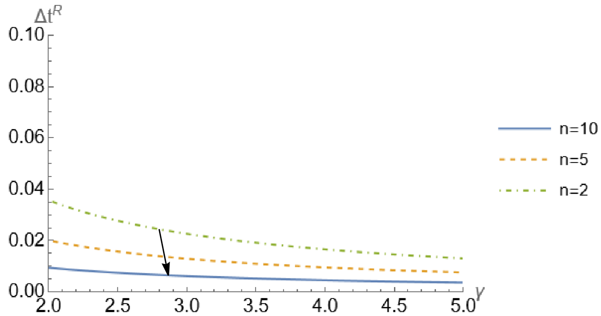
Table A4: Equilibrium investments where $c = 1/2$, $d = \alpha = \gamma = 2$, and $n = 5$.



(a) Δt^R at different levels of d.

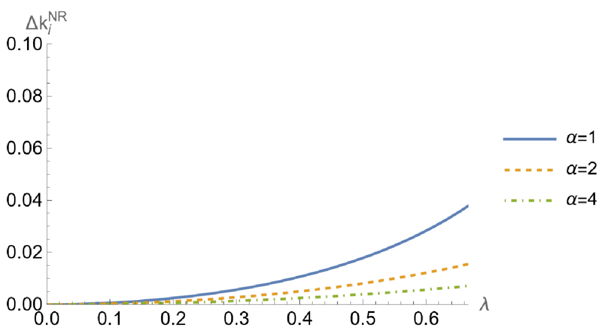


(b) Δt^R at different levels of α.

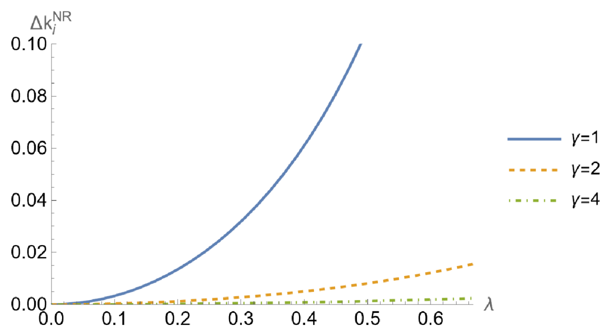


(c) Δt^R at different levels of γ.

Figure A1: Fee differential, $\Delta t^R \equiv t^R - \bar{t}^R$.

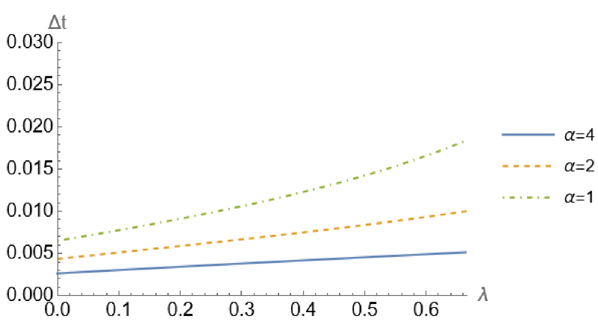


(a) Differential Δk_i^{NR} at different values of α.

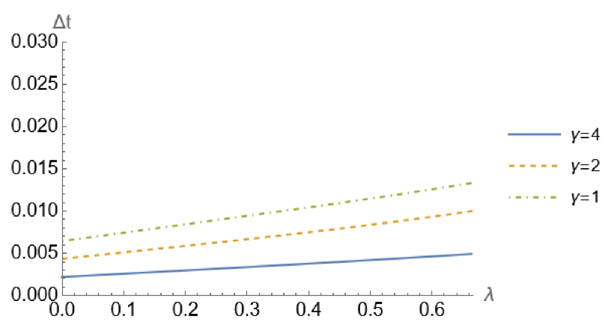


(b) Differential Δk_i^{NR} at different values of γ.

Figure A2: Differential $\Delta k_i^{NR} \equiv k_i^{NR} - k_i^{NR,Seq}$ over a range of λ.



(a) Differential Δt at different values of α.



(b) Differential Δt at different values of γ.

Figure A3: Differential $\Delta t \equiv t^{Seq} - t^R$ over a range of λ.

7 Appendix 2

7.1 Proof of Lemma 1

Differentiating equation 2 with respect to q_i yields $1 - c + k_i - 2q_i - Q_{-i} = 0$. In a symmetric equilibrium $q_i = q_j = q$ and the first-order condition for every firm i becomes $1-c+k_i = 2q+(n-1)q$, after rearranging. Summing the first-order conditions for all firms yields $n(1-c)+K = 2Q+(n-1)Q$ where $K = \sum_i k_i$. Solving for aggregate output Q yields $Q = \frac{n(1-c)+K}{n(1+n)}$.

To find individual equilibrium output, q_i^*, we return to firm i's first-order condition, $1-c+k_i = q_i + Q$. Inserting Q^* and solving for q_i, yields $q_i^* = \frac{1-c+nk_i-K_{-i}}{n+1}$, where $K_{-i} = \sum_{j \neq i} k_j$ denotes the aggregate R&D investment from firm i's rivals. This output unambiguously increases in k_i, and unambiguously decreases in c and K_{-i}, and decreases in n since $\frac{\partial q_i^*}{\partial n} = -\frac{1-c-k_i-K_{-i}}{(n+1)^2} < 0$.

Therefore, firm i's equilibrium profits are $\pi_i^* = (1 - q_i^* - Q_{-i}^*)q_i^* - (c - k_i)q_i^*$, where q_i^* was found above and $Q_{-i}^* = Q^* - q_i^* = \frac{n[1-(c+k_i)]-(1-c)+K+K_{-i}}{n+1}$. Hence, equilibrium profits are $\pi_i^* = \frac{(1-c+nk_i-K_{-i})[1-c+(n+1)k_i-K]}{(n+1)^2}$. Since $K = k_i + K_{-i}$, these profits can be more compactly expressed as $\pi_i^* = \frac{(1-c+nk_i-K_{-i})^2}{(n+1)^2} = (q_i^*)^2$.

7.2 Proof of Lemma 2

Every firm i's first-stage first-order conditions are

$$\frac{\partial \pi}{\partial k_i} = \frac{2n(1 - c + k_i n - K_{-i})}{(n + 1)^2} - \gamma k_i + \lambda z_i \leq 0$$
$$\frac{\partial \pi}{\partial z_i} = \lambda k_i - \alpha z_i \leq 0$$

If $\lambda \leq 0$, then it is clear that $-\alpha z_i + \lambda k_i < 0$ since $\alpha > 0$, implying that $z_i^{NR} = 0$ and, invoking symmetry (so that $K_{-i} = (n - 1)k_i$) and solving for equilibrium R&D for each firm yields $k_i^{NR} = \frac{2(1-c)n}{\gamma(n+1)^2-2n}$.

If $\lambda > 0$, then z_i and k_i can be interior. Invoking symmetry, every firm i invests:

$$k_i^{NR} = \frac{2\alpha(1 - c)n}{\alpha\gamma(n + 1)^2 - 2\alpha n - \lambda^2(n + 1)^2}$$
$$z_i^{NR} = \frac{2(1 - c)\lambda n}{\alpha\gamma(n + 1)^2 - 2\alpha n - \lambda^2(n + 1)^2}$$

The numerators on both expressions are positive given that $1 - c > 0$, $\alpha > 0$, and $\lambda > 0$ by assumption. Further, $k_i^{NR} > 0$ and $z_i^{NR} > 0$ if and only if $\lambda < \overline{\lambda}$ (which ensures that the denominator is positive).

The comparative statics of k_i^{NR} are

$$\begin{aligned}
\frac{\partial k_i^{NR}}{\partial c} &= -\frac{2\alpha n}{\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2} < 0 \\
\frac{\partial k_i^{NR}}{\partial \gamma} &= -\frac{2\alpha^2(1-c)n(n+1)^2}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0 \\
\frac{\partial k_i^{NR}}{\partial \alpha} &= -\frac{2(1-c)\lambda^2 n(n+1)^2}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0 \\
\frac{\partial k_i^{NR}}{\partial \lambda} &= \frac{4\alpha(1-c)\lambda n(n+1)^2}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} > 0 \\
\frac{\partial k_i^{NR}}{\partial n} &= -\frac{2\alpha(1-c)\left(n^2-1\right)\left(\alpha\gamma - \lambda^2\right)}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0
\end{aligned}$$

The final comparative static, $\frac{\partial k_i^{NR}}{\partial n}$, is negative since $\lambda < \overline{\lambda}$ implies that $\alpha\gamma > \lambda^2 + \frac{2\alpha n}{(n+1)^2}$. The comparative statics of z_i^{NR} are

$$\begin{aligned}
\frac{\partial z_i^{NR}}{\partial c} &= -\frac{2\lambda n}{\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2} < 0 \\
\frac{\partial z_i^{NR}}{\partial \gamma} &= -\frac{2\alpha(1-c)\lambda n(n+1)^2}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0 \\
\frac{\partial z_i^{NR}}{\partial \alpha} &= -\frac{2(1-c)\lambda n\left[\gamma(n+1)^2 - 2n\right]}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0 \\
\frac{\partial z_i^{NR}}{\partial \lambda} &= \frac{2(1-c)n\left[\alpha\gamma(n+1)^2 - 2\alpha n + \lambda^2(n+1)^2\right]}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} > 0 \\
\frac{\partial z_i^{NR}}{\partial n} &= -\frac{2(1-c)\lambda\left(n^2-1\right)\left(\alpha\gamma - \lambda^2\right)}{[\alpha\gamma(n+1)^2 - 2\alpha n - \lambda^2(n+1)^2]^2} < 0
\end{aligned}$$

7.3 Proof of Lemma 3

Social welfare in this stage is

$$SW = \underbrace{\frac{1}{2}Q^2}_{CS} + (1-Q)Q - \underbrace{\sum_{i=1}^{n}(c-k_i)q_i}_{\text{Cost term}} - t(Q-Z) - \sum_{i=1}^{n}\left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda(k_i z_i)\right] + t(Q-Z) - d(Q-Z)^2$$

The cost term simplifies as follows

$$\begin{aligned}\sum_{i=1}^{n}(c-k_i)q_i &= \sum_{i=1}^{n} cq_i - \sum_{i=1}^{n} k_i q_i \\ &= cQ - \sum_{i=1}^{n} k_i \left(\frac{1-c-t+nk_i-K_{-i}}{n+1}\right) \\ &= cQ + \sum_{i=1}^{n} k_i \frac{t}{n+1} - \sum_{i=1}^{n} k_i \left(\frac{1-c+nk_i-K_{-i}}{n+1}\right) \\ &= cQ + K\frac{t}{n+1} - \sum_{i=1}^{n} k_i \left(\frac{1-c+nk_i-K_{-i}}{n+1}\right)\end{aligned}$$

The regulator's problem becomes

$$\begin{aligned}\max_t SW = \frac{Q^2}{2} + (1-Q)Q - cQ - K\frac{t}{n+1} + \sum_{i=1}^{n} k_i \left(\frac{1-c+nk_i-K_{-i}}{n+1}\right) \\ - \sum_{i=1}^{n}\left[\frac{1}{2}\gamma k_i^2 + \frac{1}{2}\alpha z_i^2 - \lambda\left(k_i z_i\right)\right] - d(Q-Z)^2\end{aligned}$$

with first-order condition

$$\frac{K(2dn-1) - n[c(2dn-1) + d(n(t+Z-1)+Z) + nt + 1]}{(n+1)^2} = 0$$

Solving for t yields the equilibrium emission fee in Lemma 3 . The emission fee is positive if and only if the numerator is positive, which is satisfied by

$$d > d(n) \equiv \frac{(1-c)n + K}{2n[n(1-c-Z) + K - Z]}.$$

The comparative statics of cutoff $d(n)$ are $\frac{\partial d(n)}{\partial c} = \frac{(n+1)Z}{2[K-Z+n(1-c-Z)]^2} > 0$, $\frac{\partial d(n)}{\partial K} = -\frac{(n+1)Z}{2n[K-Z+n(1-c-Z)]^2} < 0$, $\frac{\partial d(n)}{\partial Z} = \frac{(n+1)[n(1-c)+K]}{2n[K-Z+n(1-c-Z)]^2} > 0$, and $\frac{\partial d(n)}{\partial n} = \frac{Z\left[n^2(1-c)+K(2n+1)\right]-[n(1-c)+K]^2}{2n^2[K-Z+n(1-c-Z)]^2} > 0$ for $Z > \frac{[n(1-c)+K]^2}{n^2(1-c)+K(2n+1)}$.

The comparative statics of the emission fee t are $\frac{\partial t}{\partial c} = -\frac{2dn-1}{2dn+n} < 0$, $\frac{\partial t}{\partial K} = \frac{2dn-1}{(2d+1)n^2} > 0$, $\frac{\partial t}{\partial Z} = -\frac{2d(n+1)}{n(2d+1)} < 0$, and $\frac{\partial t}{\partial d} = \frac{2(n+1)[K+n(1-c-Z)]}{(2d+1)^2 n^2} > 0$ for all $d > d(n)$. Finally, $\frac{\partial t}{\partial n} = \frac{n(1-c)+2dn(Z-K)+2K}{(2d+1)n^3}$, which is positive if and only if $K < \frac{n(1-c+2dZ)}{2dn-2}$.

7.4 Proof of Proposition 1

Differentiating with respect to k_i and z_i in problem (6), yields the following first-order conditions

$$\frac{\partial \pi}{\partial k_i} = \frac{2[(2d+1)(n-1)n+1]}{(2d+1)^2 n^4}\left[k_i + K_{-i} + n - cn - (1+2d)K_{-i}n + (1+2d)k_i(n-1)n + \right.$$
$$\left. dn(z_i + Z_{-i})\right] + \frac{z_i(2dn-1)}{(2d+1)n^2} - \gamma k_i + \lambda z_i = 0 \quad \text{(A1)}$$
$$\frac{\partial \pi}{\partial z_i} = \frac{2d((2d+1)k_i(n-1)n - cn - (2d+1)K_{-i}n + dn(z_i+Z_{-i}) + k_i + K_{-i} + n)}{(2d+1)^2 n^3}$$
$$+ \frac{(2dn-1)(n - cn + k_i + K_{-i}) - dn(n+1)(z_i + Z_{-i})}{(2d+1)n^2} - \frac{d(n+1)z_i}{2dn+n} + \lambda k_i - \alpha z_i = 0 \quad \text{(A2)}$$

Every firm j has a symmetric first-order condition. Invoking symmetry in k_i, so that $K_{-i} = (n-1)k_i$, in equation (A1) and solving for k_i, yields

$$k_i(z_i, Z_{-i}) = \frac{1}{E}[2(2d+1)n^2\left(dZ_{-i} + 2dz_i + 1\right) - 2c((2d+1)(n-1)n+1)$$
$$- (2d+1)n\left(2d\left(z_i + Z_{-i}\right) + z + 2\right) + 2d\left(z_i + Z_{-i}\right) + (2d+1)^2 \lambda n^3 z_i + 2], \quad \text{(A3)}$$

where term $E \equiv (2d+1)n[n(n\gamma(1+2d)-2)+2]-2$. It is sufficient for value $E > 0$ if $n\gamma(1+2d)-2 > 0$, which holds by Assumption I, therefore $k_i(z_i, Z_{-i}) > 0$. This investment level satisfies

$$\frac{\partial k_i(z_i, Z_{-i})}{\partial z_i} = \frac{(2d+1)^2\lambda n^3 + 4dn[d(2n-1)+n-1] + 2d - n}{E} > 0,$$
$$\frac{\partial k_i(z_i, Z_{-i})}{\partial Z_{-i}} = \frac{2d[(2d+1)(n-1)n+1]}{E} > 0, \text{ and}$$

Similarly, invoking symmetry in z_i, entailing that $Z_{-i} = (n-1)z_i$, in equation (A2) and solving for z_i, yields

$$z_i(k_i, K_{-i}) = \frac{1}{B}\left[4d^2 n(n(n(1-c) + K_{-i}) + k_i(n(\lambda n + 2) - 1) - K_{-i})\right.$$
$$+ 2d\left((1-c)n((n-1)n+1) + 2k_i n\left(\lambda n^2 + n - 1\right) + k_i + K_{-i}(n-1)^2\right)$$
$$\left. + n\left(k_i\left(\lambda n^2 - 1\right) - K_{-i} - (1-c)n\right)\right] \quad \text{(A4)}$$

where term $B \equiv n^2\left[\alpha(2d+1)^2 n + d(2d+1)n(n+2) + d\right] > 0$. The comparative statics on (A4) are

$$\frac{\partial z_i(k_i, K_{-i})}{\partial k_i} = \frac{(2d+1)^2\lambda n^3 + 4dn(d(2n-1)+n-1) + 2d - n}{n^2\left(\alpha(2d+1)^2 n + d(2d+1)n(n+2) + d\right)} > 0$$
$$\text{if and only if } \lambda > \frac{4dn[(1+d) - n(1+2d)] - 2d + n}{(2d+1)^2 n^3} \equiv \widehat{\lambda},$$
$$\frac{\partial z_i(k_i, K_{-i})}{\partial K_{-i}} = \frac{2d(n-1)(2dn+n-1) - n}{n^2\left(\alpha(2d+1)^2 n + d(2d+1)n(n+2) + d\right)} > 0,$$

where cutoff $\widehat{\lambda} < 0$ for all admissible parameters since $d > 1$ and $n \geq 2$ by definition. Therefore, condition $\lambda > \widehat{\lambda}$ unambiguously holds when economies of scope are absent, $\lambda = 0$, or when economies of scope are present, $\lambda > 0$. However, when diseconomies of scope are present, $\lambda < 0$, condition $\lambda > \widehat{\lambda}$ could be violated since cutoff $\widehat{\lambda} < 0$, but we next show that this condition holds under all admissible parameter values.

First, recall that Assumption II states that $|\lambda| < \overline{\lambda}$, where cutoff $\overline{\lambda} \equiv \frac{1}{n+1}\sqrt{\alpha\left[\gamma(n+1)^2 - 2n\right]}$ is unambiguously positive. Then, in a context with disesconomies of scope, $\lambda < 0$, Assumption II entails that $\lambda > -\overline{\lambda}$. Therefore, Assumption II, $\lambda > -\overline{\lambda}$, implies condition $\lambda > \widehat{\lambda}$ (i.e., condition $\lambda > \widehat{\lambda}$ holds for all admissible values, as restricted by Assumption II) if cutoffs $\overline{\lambda}$ and $\widehat{\lambda}$ satisfy $(-\overline{\lambda}) > \widehat{\lambda}$, which, solving for α, holds if and only if $\alpha > \widetilde{\alpha}$, where

$$\widetilde{\alpha} \equiv \frac{(n+1)^2\left[n - 2d + 4dn(1 + d - n(1+2d))\right]^2}{(1+2d)^4 n^6\left[\gamma(1+n)^2 - 2n\right]}.$$

Comparing cutoff $\widetilde{\alpha}$ against the condition for α in Assumption I, $\alpha \geq \frac{2n^2}{(n+1)^2}$, we find that $\frac{2n^2}{(n+1)^2} > \widetilde{\alpha}$ for all admissible parameter values. (The differential $\frac{2n^2}{(n+1)^2} - \widetilde{\alpha}$ is positive for all admissible parameters, decreases in d but remains positive, and increases in both n and γ.) Therefore, Assumption I entails that condition $\alpha > \widetilde{\alpha}$ must also hold. This, in turn, implies that condition $(-\overline{\lambda}) > \widehat{\lambda}$ must also be satisfied. As a consequence, condition $\lambda > \widehat{\lambda}$ also holds, even in the presence of diseconomies of scale.

Simultaneously solving for k_i and z_i in equations (A3) and (A4), yields equilibrium investments

$$k_i^R = \frac{1}{A}(1-c)\left[2\alpha + n\left(4d^2(\lambda+1)n^2 + 2d(\lambda + (n-1)(2\alpha + n + 1) + \lambda(n-1)n) + 2\alpha(n-1) - \lambda n\right) + 1\right],$$
$$z_i^R = \frac{1}{A}(1-c)\left[\gamma n[2d(n(2dn + n - 1) + 1) - n] + 2(2d+1)\lambda(n-1)n + 2\lambda\right].$$

where term $A \equiv n[2d^2n^2[\gamma(2\alpha + n + 2) - 2(\lambda+1)^2] + d(4\alpha - 2\lambda + n(\gamma - 4\alpha + 2\lambda + n(\gamma(4\alpha + n + 2) - 4\lambda(\lambda+1) - 2)) + 2) + \lambda + \alpha(n(\gamma n - 2) + 2) + \lambda n(1 - \lambda n)] - 2\alpha - 1$.

When economies of scope are absent, $\lambda = 0$, these investments simplify to

$$k_i^R = \frac{1}{B}(1-c)[2\alpha + 2n\left[d\left[(2d+1)n^2 - 1\right] + \alpha(2d+1)(n-1)\right] + 1],$$
$$z_i^R = \frac{1}{B}(1-c)\gamma n[2d(n(2dn + n - 1) + 1) - n],$$

where term $B \equiv n[\gamma n[\alpha(2d+1)^2 n + d(2d+1)n(n+2) + d] - 2d(2d+1)n^2 - 2\alpha(2d+1)(n-1) + 2d] - 2\alpha - 1$.

7.5 Proof of Corollary 1

The term $C \equiv n[2d^2n^2\left(\gamma(2\alpha + n + 2) - 2(\lambda+1)^2\right) + d(4\alpha - 2\lambda + n(\gamma - 4\alpha + 2\lambda + n(\gamma(4\alpha + n + 2) - 2) - 4\lambda(\lambda+1)) + 2) + \lambda + \alpha(n(\gamma n - 2) + 2) + \lambda n(1 - \lambda n)] - 2\alpha - 1$. When economies of scope are absent, $\lambda = 0$, the equilibrium emission fee simplifies to

$t^R = \frac{(1-c)\gamma n(\alpha(2d+1)n(2dn-1)+d(n(2dn-1)-1))}{n(\gamma n(\alpha(2d+1)^2 n+d(2d+1)n(n+2)+d)-2d(2d+1)n^2-2\alpha(2d+1)(n-1)+2d)-2\alpha-1}$

7.6 Proof of Lemma 4

Differentiating (3') with respect to k_i, yields

$$\frac{2n(1-c+nk_i-K_{-i})}{(n+1)^2} - \gamma k_i + \lambda z_i = 0$$

Invoking symmetry, $K_{-i} = (n-1)k_i$, which entails

$$\frac{2n(1-c+k_i)}{(n+1)^2} - \gamma k_i + \lambda z_i = 0$$

and, solving for k_i, we obtain $k_i^{NR,Seq}(z_i) = \frac{2n(1-c)+(n+1)^2\lambda z_i}{\gamma(n+1)^2-2n}$, which is positive by Assumption I. This investment level satisfies $\frac{\partial k_i^{NR,Seq}(z_i)}{\partial z_i} = \frac{\lambda(n+1)^2}{\gamma(n+1)^2-2n} > 0$ and $\frac{\partial k_i^{NR,Seq}(z_i)}{\partial \lambda} = \frac{z_i(n+1)^2}{\gamma(n+1)^2-2n} > 0$ because of Assumption I.

7.7 Proof of Proposition 2

Firm i's first-order condition is

$$\frac{\lambda\left[\lambda z_i(n+1)^2[\gamma+n(\gamma(n+2)-4)+2]-2(1-c)\left[2n^2-\gamma(n+1)^2\right]\right]}{\left(\gamma(n+1)^2-2n\right)^2} - \alpha z_i = 0$$

Solving for z_i, yields the expression of $z_i^{NR,Seq}$ in Proposition 2, where $D \equiv \alpha[2n-\gamma(n+1)^2]^2 - \lambda^2(n+1)^2[2+\gamma+n(\gamma(n+2)-4)]$. When $\lambda = 0$, the first-order condition simplifies to $-\alpha z_i = 0$ and $z_i^{NR,Seq} = 0$

7.8 Proof of Lemma 5

Differentiating (6') with respect to k_i, yields

$$\frac{2[(2d+1)(n-1)n+1]}{(2d+1)^2n^4}\left[k_i + K_{-i} + n - cn - (1+2d)K_{-i}n + (1+2d)k_i(n-1)n + \right.$$
$$\left. dn(z_i+Z_{-i})\right] + \frac{z_i(2dn-1)}{(2d+1)n^2} - \gamma k_i + \lambda z_i = 0,$$

which aligns with the first-order condition with respect to k_i from proposition 1. Invokying symmetry, $K_{-i} = (n-1)k_i$, which entails

$$\frac{2((2d+1)(n-1)n+1)(1-c+d(z_i+Z_{-i})+k_i)}{(2d+1)^2n^3} + \frac{z_i(2dn-1)}{(2d+1)n^2} - \gamma k_i + \lambda z_i = 0$$

and, solving for k_i, we obtain

$$k_i^{R,Seq}(z_i, Z_{-i}) = \frac{1}{E}[2(2d+1)n^2 (dZ_{-i} + 2dz_i + 1) - 2c((2d+1)(n-1)n+1) \\ - (2d+1)n(2d(z_i + Z_{-i}) + z + 2) + 2d(z_i + Z_{-i}) + (2d+1)^2\lambda n^3 z_i + 2],$$

where $E \equiv (2d+1)n[n(n\gamma(1+2d)-2)+2]-2$. This investment level satisfies

$$\frac{\partial k_i^{R,Seq}(z_i, Z_{-i})}{\partial \lambda} = \frac{(2d+1)^2 n^3 z_i}{E} > 0,$$

because of $E > 0$ (recall that we are assuming $\lambda \geq 0$ in this section, for simplicity).

Comparing $\frac{\partial k_i^{R,Seq}(z_i,Z_{-i})}{\partial z_i} > \frac{\partial k_i^{R,Seq}(z_i,Z_{-i})}{\partial Z_{-i}}$ simplifies to $-(2d+1)n\left(2dn(\lambda n+1)+\lambda n^2-1\right) < 0$ which holds for $\lambda \geq 0$.

In addition, the cross-partial satisfies

$$\frac{\partial^2 k_i^{R,Seq}(z_i, Z_{-i})}{\partial z_i \partial \lambda} = \frac{(1+2d)^2 n^3}{E} > 0$$

7.9 Proof of Proposition 3

The first-order condition is

$$\frac{1}{m^2}\left[2[n(d(n(\gamma+2\gamma d)+2)+(2d+1)\lambda n)-1\right]\left[\left((1-c)\gamma(2d+1)n^2\right)+\gamma d(2d+1)n^2 Z_{-i} \right. \\ \left. +z_i(n(d(2\gamma dn+\gamma n+2)+(2d+1)\lambda n)-1)\right]] \\ + \frac{n(2dn-1)\left[\frac{1}{m}\begin{pmatrix} 2-2c((2d+1)(n-1)n+1)+2(2d+1)n^2(dZ_{-i}+2dz_i+1) \\ -(2d+1)n(2d(Z_{-i}+z_i)+z+2)+2d(Z_{-i}+z_i)+(2d+1)^2\lambda n^3 z_i \end{pmatrix} - c + 1\right]}{(2d+1)n^2} \\ - \frac{dn(n+1)(Z_{-i}+z_i)}{(2d+1)n^2} + \frac{1}{m}\left[\lambda\left[-2c((2d+1)(n-1)n+1)+2(2d+1)n^2(dZ_{-i}+2dz_i+1)\right.\right. \\ \left.\left. -(2d+1)n(2d(Z_{-i}+z_i)+z_i+2)+2d(Z_{-i}+z_i)+(2d+1)^2\lambda n^3 z_i+2\right]\right] \\ - \frac{1}{m^2}\left[\gamma\left[(2d+1)^2\lambda n^3+4dn(d(2n-1)+n-1)+2d-n\right]\right. \\ \times\left[-2c((2d+1)(n-1)n+1)+2d((2d+1)(n-1)n+1)Z_{-i}+(2d+1)^2\lambda n^3 z_i+2(2d+1)n^2(2dz_i+1)\right. \\ \left.-(2d+1)n(2dz_i+z_i+2)+2dz_i+2\right]] + \frac{\lambda z_i\left((2d+1)^2\lambda n^3+4dn(d(2n-1)+n-1)+2d-n\right)}{m} \\ + \frac{z_I\left[1-\gamma d(2d+1)(n+1)n^2+(2d+1)\lambda n^2(2dn-1)+2d(n(d(4n-2)+n-3)+1)\right]}{m} - \alpha z_i = 0$$

where $m \equiv [(2d+1)n(n(n(\gamma+2\gamma d)-2)+2)-2]^2$. The sequential equilibrium ER&D is

$$z_i^{R,Seq} = \frac{1}{G}(1-c)\left[16\gamma^2 d^4 n^6 + 8\gamma d^3 n^2\left[n\left(n\left(\gamma+3\gamma n^2-(\gamma+2)n-2\right)+6\right)-2\right]+F\right]$$

where term

$$G \equiv 4\alpha + 8\gamma d^4 n^6 \left[\gamma(2\alpha + n + 2) - 2(\lambda + 1)^2\right] + 4d^3 n^2 [4(\lambda + 1) + n (n (8\alpha\gamma - 2\lambda(\gamma + 4\lambda) + 6\gamma$$
$$-4\lambda + 3\gamma^2 n^3 + 2\gamma n^2((4\alpha + 3)\gamma - 2\lambda(2\lambda + 3) - 4) + n \left(2\gamma(\lambda - 4\alpha - 1) + \gamma^2 + 4(\lambda + 1)(2\lambda + 1)\right) + 4)$$
$$-12(\lambda + 1))] + 2d^2 n \left[n \left(2(4\alpha + \gamma + 12\lambda + 14) + n \left(8(\lambda - 3)(\lambda + 1) - 8\alpha(\gamma + 2) - 10\gamma + 3\gamma^2 n^4\right.\right.\right.$$
$$+ 2\gamma n^3[(6\alpha + 3)\gamma - 6\lambda(\lambda + 1) - 5] + 2n^2 \left(4 - 2\gamma(6\alpha - 2\lambda + 1) + \gamma^2 + 12\lambda(\lambda + 1)\right)$$
$$\left.\left.\left. +4n \left(\alpha(6\gamma + 2) - (\gamma + 4)\lambda + 3\gamma - 7\lambda^2\right)\right)\right) - 8(\lambda + 1)\right]$$
$$+ d \left[4(\lambda + 1) + \gamma^2 n^7 + 2\gamma n^6 \left(4\alpha\gamma + \gamma - 2 \left(2\lambda^2 + \lambda + 1\right)\right) + n^5 \left(\gamma(\gamma - 24\alpha - 2) + 10\gamma\lambda + 24\lambda^2 + 12\lambda + 4\right)\right.$$
$$+ 2n^4(4\alpha(3\gamma + 2) - \gamma(\lambda - 3) - 2\lambda(8\lambda + 5) - 2) - 2n^3 \left(8\alpha(\gamma + 2) + 7\gamma - 8\lambda^2 + 2\lambda + 6\right)$$
$$\left. +4n^2(8\alpha + \gamma + 7\lambda + 7) - 2n(8\alpha + \gamma + 6\lambda + 10)\right]$$
$$+ n \left[n \left(\gamma + 4\lambda + \alpha \left(\gamma^2 n^4 - 4\gamma((n - 1)n + 1)n + 4(n - 2)n + 12\right) + n \left(\lambda^2(4 - n(n(\gamma n - 4) + 6)) - 2\gamma\right.\right.\right.$$
$$\left.\left.\left. +2\lambda n(\gamma n - 2)\right) + 4\right) - 4(2\alpha + \lambda + 1)\right] + 2$$

$$F \equiv 4d^2 n \left(\gamma \left(\gamma(3(n - 1)n + 2)n^3 - 2 \left(n \left(2n^2 + n - 6\right) + 6\right) n + 4\right) + 2\lambda n(n((\gamma - 2)n + 4) - 2)\right)$$
$$+ 2\gamma d \left(\gamma((n - 3)n + 1)n^4 - 2(n - 1) \left(n^3 - 3n + 3\right) n - 2\right) + 8d\lambda n(n(n((\gamma - 2)n + 4) - 4) + 2)$$
$$+ 2\lambda \left(\gamma n^4 - 2((n - 1)n + 1)^2\right) + \gamma n \left(-\gamma n^4 + 2n^3 - 2n + 2\right).$$

Plugging this equilibrium into $k_i^{R,Seq}(z_i, Z_{-i})$ from Lemma 5 gives

$$k_i^{R,Seq} = \frac{1}{J}(1 - c) \left[16\gamma d^4(\lambda + 1)n^6 + 8d^3 n^2 [n (6(\lambda + 1)\right.$$
$$\left. +n \left(\gamma(-2\alpha + \lambda - 1) + 2\lambda^2 + 3\gamma(\lambda + 1)n^2 + n(2\alpha\gamma - \lambda(\gamma + 2\lambda + 4) - 2) - 2\right)\right) - 2(\lambda + 1)] + L\right]$$

where term

$$J \equiv 4\alpha + 8\gamma d^4 n^6 \left[\gamma(2\alpha + n + 2) - 2(\lambda + 1)^2\right] + 4d^3 n^2 [4(\lambda + 1) + n (n (8\alpha\gamma - 2\lambda(\gamma + 4\lambda) + 6\gamma$$
$$-4\lambda + 3\gamma^2 n^3 + 2\gamma n^2((4\alpha + 3)\gamma - 2\lambda(2\lambda + 3) - 4) + n \left(2\gamma(-4\alpha + \lambda - 1) + \gamma^2 + 4(\lambda + 1)(2\lambda + 1)\right) + 4)$$
$$-12(\lambda + 1))] + 2d^2 n \left[n \left(2(4\alpha + \gamma + 12\lambda + 14) + n \left(8(\lambda - 3)(\lambda + 1) - 8\alpha(\gamma + 2) - 10\gamma + 3\gamma^2 n^4\right.\right.\right.$$
$$+ 2\gamma n^3((6\alpha + 3)\gamma - 6\lambda(\lambda + 1) - 5) + 2n^2 \left(\gamma^2 - 2\gamma(6\alpha - 2\lambda + 1) + 12\lambda(\lambda + 1) + 4\right)$$
$$\left.\left.\left. +4n \left(\alpha(6\gamma + 2) - (\gamma + 4)\lambda + 3\gamma - 7\lambda^2\right)\right)\right) - 8(\lambda + 1)\right] + d \left[4(\lambda + 1) + \gamma^2 n^7\right.$$
$$+ 2\gamma n^6 \left(4\alpha\gamma + \gamma - 2 \left(2\lambda^2 + \lambda + 1\right)\right) + n^5 \left(\gamma(-24\alpha + \gamma - 2) + 10\gamma\lambda + 24\lambda^2 + 12\lambda + 4\right)$$
$$+ 2n^4(4\alpha(3\gamma + 2) - \gamma(\lambda - 3) - 2\lambda(8\lambda + 5) - 2) - 2n^3 \left(8\alpha(\gamma + 2) + 7\gamma - 8\lambda^2 + 2\lambda + 6\right)$$
$$\left. +4n^2(8\alpha + \gamma + 7\lambda + 7) - 2n(8\alpha + \gamma + 6\lambda + 10)\right] + n \left[n \left(\gamma + 4\lambda + \alpha \left(\gamma^2 n^4 - 4\gamma((n - 1)n + 1)n\right.\right.\right.$$
$$\left.\left.\left. +4(n - 2)n + 12\right) + n \left(\lambda^2(4 - n(n(\gamma n - 4) + 6)) + 2\lambda n(\gamma n - 2) - 2\gamma\right) + 4\right) - 4(2\alpha + \lambda + 1)\right] + 2.$$

$$
\begin{aligned}
L \equiv & 4d^2 n \left[4(\lambda+1) + n\left(n\left(2\alpha(\gamma+4) + \gamma - 2(\lambda-6)(\lambda+1) + 3\gamma(\lambda+1)n^3 + n^2(6\alpha\gamma - \lambda(3\gamma+6\lambda+8) - 4)\right.\right.\right. \\
& \left.\left.\left. -2n(\alpha(3\gamma+2)+\gamma) + 2(\gamma+2)\lambda n + 8\lambda^2 n\right) - 2(2\alpha+6\lambda+7)\right)\right] + 2d\left[n\left(8\alpha + 6\lambda + \gamma(\lambda+1)n^5\right.\right. \\
& + n^4(6\alpha\gamma - \lambda(3\gamma+6\lambda+4) - 2) + n^3(-2\alpha(3\gamma+4) + \gamma(\lambda-1) + 2\lambda(5\lambda+4) + 2) \\
& \left.\left. +2n^2\left(2\alpha(\gamma+4) + \gamma - 2\lambda^2 + \lambda + 3\right) - 2n(8\alpha+6\lambda+7) + 10\right) - 2(\lambda+1)\right] \\
& + n\left[2(4\alpha+\lambda+2) + n\left(2\alpha\gamma n((n-1)n+1) - 4\alpha((n-2)n+3) - \lambda(n(n(\gamma n-4)+2)+2)\right.\right. \\
& \left.\left. +\gamma n - 2\lambda^2(n-1)^2 n - 4\right)\right] - 2 - 4\alpha
\end{aligned}
$$

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