Advanced Microeconomic Theory

Chapter 10: Contract Theory

Outline

- Moral Hazard
- Moral Hazard with a Continuum of Effort Levels—The First-Order Approach
- Moral Hazard with Multiple Signals
- Adverse Selection—The “Lemons” Problem
- Adverse Selection—The Principal–Agent Problem
- Application of Adverse Selection—Regulation

Moral Hazard

Moral Hazard

- **Moral hazard**: settings in which an agent does not observe the actions of the other individual(s).
  - Also referred to as "hidden action"
- Example:
  - A manager in a firm cannot observe the effort of employees in the firm even if the manager is perfectly informed about the worker's ability or productivity.
  - The worker might have incentives to *slack* from exerting a costly effort, thus giving rise to moral hazard problems.

Moral Hazard

- The manager can offer contracts that provide incentives to the worker to work hard
  - Paying a higher salary (bonus) if the worker's output is high but a low salary otherwise.
- Providing incentives to work hard is costly for the manager
- The manager only induces a high effort if the firm's expected profits are higher than those of inducing a low effort

Moral Hazard

- Consider a principal with benefit function
$$B(\pi - w)$$
where π is the profit that arises from the agent's effort and w is the salary that the principal pays to the agent.
- The benefit function satisfies $B' \geq 0$ and $B'' \leq 0$.
- The agent's (quasi-linear) utility function is
$$U(w, e) = u(w) - g(e)$$
where $u(w)$ is utility from the agent's salary, for $u' > 0$ and $u'' \leq 0$, and $g(e)$ is the agent's disutility from effort (e), for $g' > 0$ and $g'' \geq 0$.

Moral Hazard

- The agent's effort level e affects the probability that a certain profit occurs.
- For a given effort e, the conditional probability that a profit $\pi = \pi_i$ is

$$f(\pi_i|e) = \text{Prob}\{\pi = \pi_i|e\} \geq 0$$

  where $i = \{1,2, \ldots, N\}$ is the profits that can emerge for a given effort e.
- Hence a high profit could arise even if the worker slacks
  - That is, a given profit level $\pi = \pi_i$ can arise from every effort level

Symmetric Information

- The principal **can observe** the agent's effort level e.
- The principal's maximization problem is

$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot B(\pi_i - w(\pi_i))$$

$$\text{s.t.} \quad \sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$$

- The principal seeks to maximize expected profits, subject to the agent participating in the contract.
  - The constraint guarantees the agent's voluntary participation in the contract.
  - Hence it is referred to as the participation constraint (PC) or the "individual rationality" condition.
- The constraint must be binding (holding with equality).

Symmetric Information

- The Lagrangian that solves the maximization problem is

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e) \cdot B\big(\pi_i - w(\pi_i)\big)$$
$$+ \lambda \left[\sum_{i=1}^{N} f(\pi_i|e)\, u\big(w(\pi_i)\big) - g(e) - \bar{u}\right]$$

- Take FOC with respect to w to obtain

$$f(\pi_i|e) \cdot B'\big(\pi_i - w(\pi_i)\big) \cdot (-1)$$
$$+\lambda f(\pi_i|e) u'\big(w(\pi_i)\big) = 0$$

where B' and u' are the derivative of $B(\cdot)$ and $u(\cdot)$ with respect to w, respectively.

Symmetric Information

- Rearranging

$$\lambda u'\big(w(\pi_i)\big) = B'\big(\pi_i - w(\pi_i)\big)$$

- Solving for λ

$$\lambda = \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} \qquad \textbf{(1)}$$

which is positive since $B'(\cdot) > 0$ and $u'(\cdot) > 0$.

- $\lambda > 0$ entails that the agent's participation constraint must bind (i.e., hold with equality)

$$\sum_{i=1}^{N} f(\pi_i|e)u\big(w(\pi_i)\big) - g(e) = \bar{u}$$

Symmetric Information

- Example 1:
  - Consider a risk-neutral principal hiring a risk-averse agent with utility function $u(w) = \sqrt{w}$, disutility of effort $g(e) = e$, and reservation utility $\bar{u} = 9$.
  - There are two effort levels $e_H = 5$ and $e_L = 0$.
  - When $e_H = 5$, the principal's sales are \$0 with probability 0.1, \$100 with probability 0.3, and \$400 with probability 0.6.
  - When $e_L = 0$, the principal's sales are \$0 with probability 0.6, \$100 with probability 0.3, and \$400 with probability 0.1.
  - In the case of $e_H = 5$, the expected profit is \$270, while in the case of $e_L = 0$, the expected profit is \$70.

Symmetric Information

- Example 1: (con't)
  - When effort is observable, the principal can induce an effort $e_H = 5$ by paying a wage w_e^* that solves

$$u(w_e^*) = \bar{u} + g(e)$$
$$\sqrt{w_e^*} = 9 + 5$$
$$w_e^* = 14^2 = 196$$

  - Similarly, the principal can induce a low effort $e_L = 0$ by offering a wage

$$\sqrt{w_e^*} = 9 + 0$$
$$w_e^* = 9^2 = 81$$

Symmetric Information

- Example 1: (con't)
  - Given these salaries, the profits that the principal obtains are

$$\$270 - \$196 = \$74 \text{ from } e_H = 5$$

$$\$70 - \$81 = -\$11 \text{ from } e_L = 0$$

  - Thus the principal prefers to induce e_H when effort is observable.

Risk Attitudes

- Continuing with the moral hazard setting under symmetric information, let us now consider the role of **risk aversion**.
- Three cases:
  1. The principal is risk-neutral but the agent is risk-averse
  2. The principal is risk-averse but the agent is risk-neutral
  3. Both the principal and the agent are risk-averse

Risk Attitudes: Case 1

- The principal is risk-neutral but the agent is risk-averse.
- The principal's benefit function is

$$B\left(\pi_i - w(\pi_i)\right) = \pi_i - w(\pi_i)$$

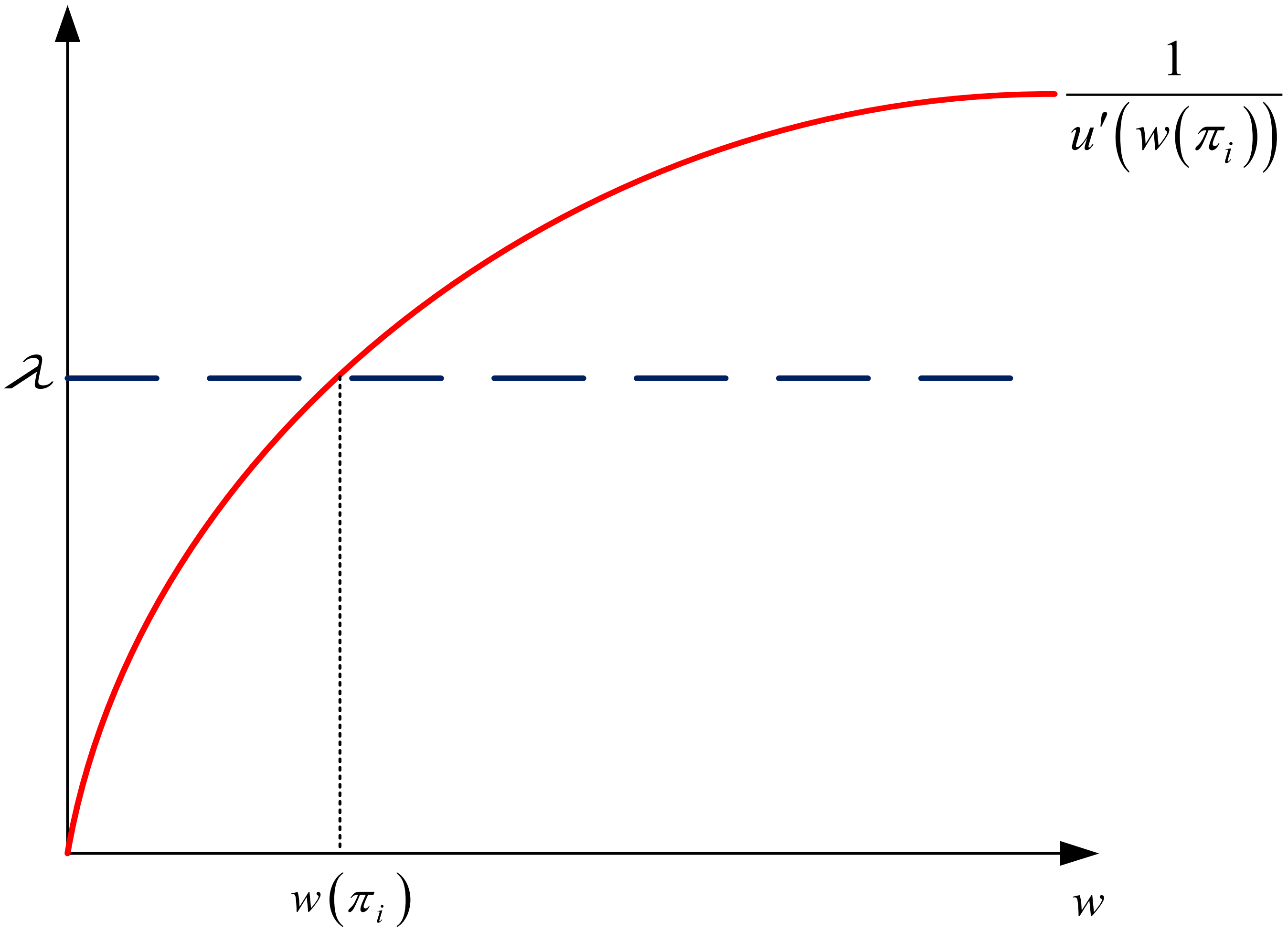
- Hence,

$$B'\left(\pi_i - w(\pi_i)\right) = 1$$

- In this context, FOC in expression **(1)** becomes

$$\lambda = \frac{1}{u'(w(\pi_i))} \text{ for all } \pi_i \qquad \textbf{(2)}$$

Risk Attitudes: Case 1



Risk Attitudes: Case 1

- FOC in **(2)** entails that the principal pays a **fixed wage** level for all profit realizations.
- For any $\pi_i \neq \pi_j$,

$$\lambda = \frac{1}{u'(w(\pi_i))} = \frac{1}{u'\left(w(\pi_j)\right)}$$

$$u'(w(\pi_i)) = u'\left(w(\pi_j)\right)$$

$$w(\pi_i) = w(\pi_j) \text{ given } u' > 0$$

- This is a **standard risk-sharing result**
  - The risk-neutral principal offers a contract to the risk-averse agent that guarantees the latter a fixed salary of w_e^* regardless of the specific profit realization that emerges.
  - The risk-neutral principal bears all the risk.

Risk Attitudes: Case 1

- Since the agent's PC binds, we can express it
$$u(w_e^*) - g(e) = \bar{u}$$
- Rearranging the PC expression
$$u(w_e^*) = \bar{u} + g(e)$$
- Applying the inverse
$$w_e^* = u^{-1}(\bar{u} + g(e))$$
- This expression helps to identify the salary that the principal needs to offer in order to induce a specific effort level e from the agent.

Risk Attitudes: Case 1

- For two effort levels e_L and e_H, the disutility of effort function satisfies

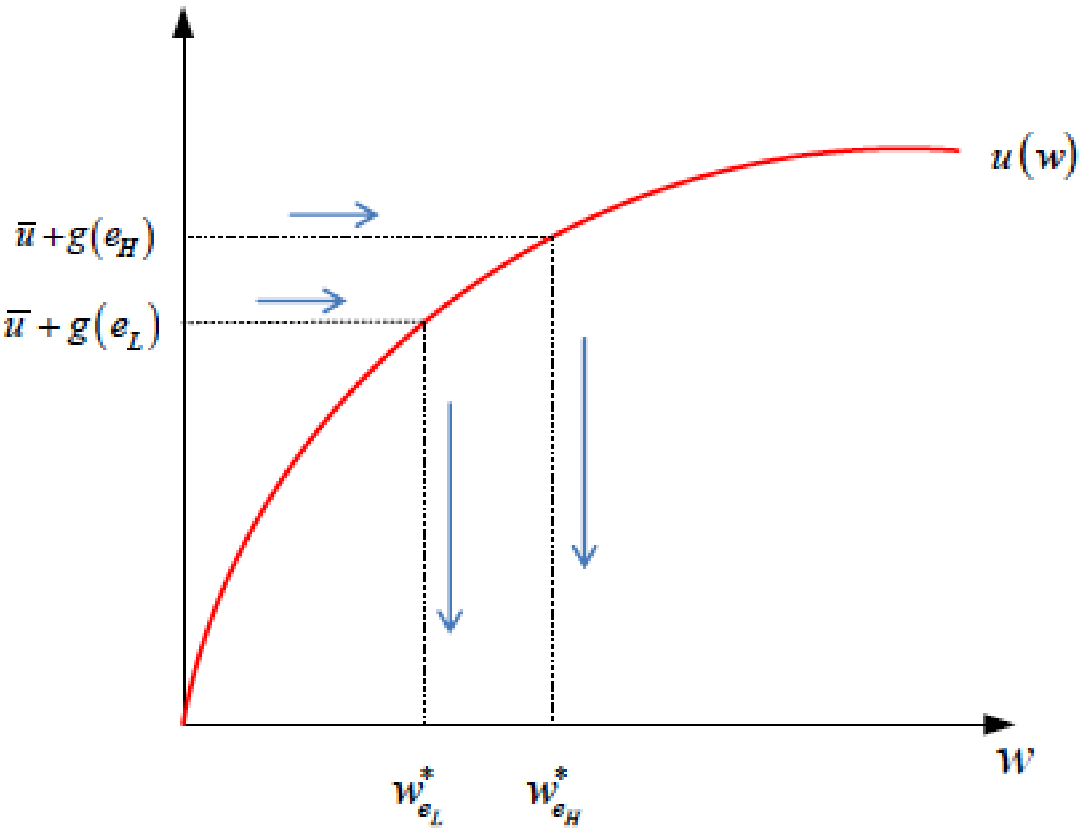
$$g(e_L) < g(e_H)$$

- This entails

$$w_{e_L}^{*} = u^{-1}\big(\bar{u} + g(e_L)\big) < u^{-1}\big(\bar{u} + g(e_H)\big) = w_{e_H}^{*}$$

- In order to induce e_H, we need to evaluate the utility function at a height of $\bar{u} + g(e_H)$.
- Inducing a higher effort implies offering a higher salary.

Risk Attitudes: Case 1



Risk Attitudes: Case 1

- We can plug in salary $w_e^* = u^{-1}\big(\bar{u} + g(e)\big)$ into the principal's objective function in order to find the effort level that maximizes the principal's expected profits

$$\max_e \sum_{i=1}^{N} f(\pi_i|e)\big(\pi_i - w(\pi_i)\big)$$

$$= \max_e \sum_{i=1}^{N} f(\pi_i|e) \cdot \pi_i - \underbrace{u^{-1}\big(\bar{u} + g(e)\big)}_{w_e^*}$$

where w_e^* does not depend on π_i.

- This helps to reduce the number of choice variables to only the effort level e.

Risk Attitudes: Case 1

- Taking FOC with respect to e yields

$$\sum_{i=1}^{N} f'(\pi_i|e) \cdot \pi_i - \frac{\partial u^{-1}\left(\bar{u} + g(e)\right)}{\partial e} g'(e) = 0$$

  where $\frac{\partial u^{-1}(\bar{u}+g(e))}{\partial e}$ can be expressed as $(u^{-1})'\left(\bar{u} + g(e)\right)$.

- By the implicit function theorem,

$$(u^{-1})'\left(\bar{u} + g(e)\right) = (u')^{-1}\left(\bar{u} + g(e)\right)$$

- Hence the above FOC can be rewritten as

$$\sum_{i=1}^{N} f'(\pi_i|e) \cdot \pi_i = \frac{g'(e)}{u'\left(\bar{u} + g(e)\right)}$$

Risk Attitudes: Case 1

- Intuition:
  - Effort e is increased until the point at which its marginal expected profit (left-hand side) coincides with its certain costs (in the right-hand side), which stems from a larger marginal disutility of effort for the agent (numerator) that needs to be compensated with a more generous salary (denominator).
- See textbook for the second-order condition that guarantees concavity.

Risk Attitudes: Case 2

- The principal is risk-averse but the agent is risk-neutral.
- The principal's benefit function is $B\left(\pi_i - w(\pi_i)\right)$, with $B' > 0$ and $B'' < 0$.
- The agent's utility function is
$$u(w_i) - g(e) = w_i - g(e)$$
- In this context, FOC in expression **(1)** becomes
$$\lambda = B'\left(\pi_i - w(\pi_i)\right)$$
where $u'\left(w(\pi_i)\right) = 1$.

Risk Attitudes: Case 2

- FOC entails that it is now the principal who obtains a **fixed payoff** for all profit realizations.
- For any $\pi_i \neq \pi_j$,

$$\lambda = B'(\pi_i - w(\pi_i)) = B'\left(\pi_j - w(\pi_j)\right)$$

$$\pi_i - w(\pi_i) = \pi_j - w(\pi_j) = K \text{ given } B' > 0$$

- That is, the risk-averse principal receives the same payoff regardless of the profit realization π, whereas the risk-neutral agent now bears all the risk.

Risk Attitudes: Case 2

- The agent's salary is
$$w(\pi_i) = \pi_i - K$$
where K is found by making the agent indifferent between accepting and rejecting the franchise contract
- Fee K solves
$$\sum_{i=1}^{N} f(\pi_i|e)[\pi_i - K] - g(e) = \bar{u}$$
$$K = \sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)$$

Risk Attitudes: Case 2

- The principal's expected profit is

$$\sum_{i=1}^{N} f(\pi_i|e)B\big(\pi_i - w(\pi_i)\big)$$
$$= \sum_{i=1}^{N} f(\pi_i|e)B\big(\pi_i - (\pi_i - K)\big)$$
$$= \sum_{i=1}^{N} f(\pi_i|e)B(K) = B(K)$$

- The principal's problem can then be written as

$$\max_{e} \; B(K) = B\left(\sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)\right)$$

Risk Attitudes: Case 2

- Taking FOC with respect to e yields

$$B'\left(\sum_{i=1}^{N} f(\pi_i|e)\pi_i - \bar{u} - g(e)\right)\left(\sum_{i=1}^{N} f'(\pi_i|e)\pi_i - g'(e)\right) = 0$$

which simplifies to

$$\sum_{i=1}^{N} f'(\pi_i|e)\pi_i = g'(e)$$

- Intuition:
  - Effort e is increased until the point where marginal expected profit from having the agent exert more effort (left-hand side) coincides with his marginal disutility (right-hand side).

Risk Attitudes: Case 2

- The second-order condition is

$$\sum_{i=1}^{N} f''(\pi_i|e)\pi_i - g''(e) \leq 0$$

where $g''(e) \geq 0$.

Risk Attitudes: Case 3

- Both the principal and the agent are risk-averse.
- Recall the FOCs with respect to w

$$B'\big(\pi_i - w(\pi_i)\big)\cdot(-1) + \lambda u'\big(w(\pi_i)\big) = 0 \qquad \textbf{(3)}$$

$$\lambda = \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} \qquad \textbf{(4)}$$

- To better understand how the profit-maximizing salary is affected by the profit realization π_i, differentiate **(3)** with respect to π_i

$$-B''\big(\pi_i - w(\pi_i)\big) + B''\big(\pi_i - w(\pi_i)\big)w'(\pi_i) + \lambda u''\big(w(\pi_i)\big)w'(\pi_i) = 0$$

Risk Attitudes: Case 3

- Plugging λ from **(4)** yields

$$-B''\big(\pi_i - w(\pi_i)\big) + B''\big(\pi_i - w(\pi_i)\big)w'(\pi_i) + \frac{B'\big(\pi_i - w(\pi_i)\big)}{u'\big(w(\pi_i)\big)}u''\big(w(\pi_i)\big)w'(\pi_i) = 0$$

- Factoring out $w'(\pi_i)$ yields

$$B''\big(\pi_i - w(\pi_i)\big) = \left[B''\big(\pi_i - w(\pi_i)\big) + B'\big(\pi_i - w(\pi_i)\big)\frac{u''\big(w(\pi_i)\big)}{u'\big(w(\pi_i)\big)}\right]w'(\pi_i)$$

Risk Attitudes: Case 3

- Solving for $w'(\pi_i)$ yields

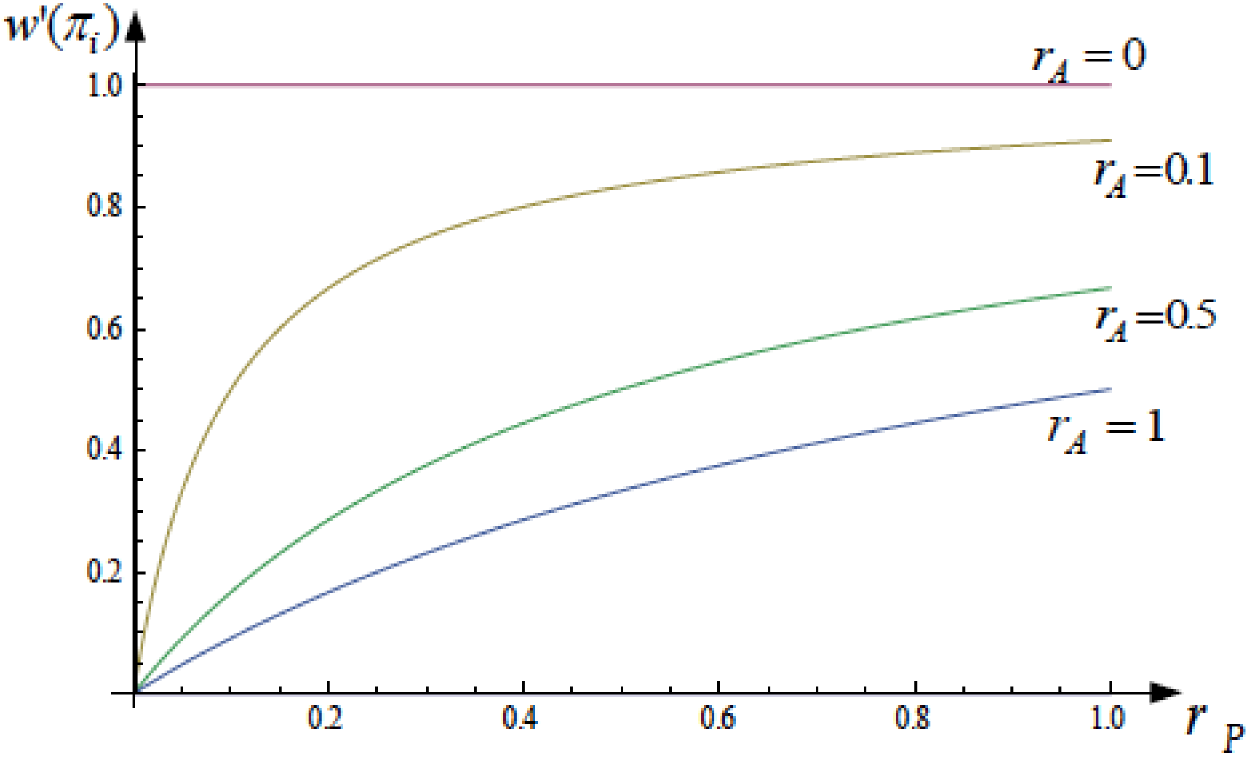
$$w'(\pi_i) = \frac{B''\big(\pi_i - w(\pi_i)\big)}{B''(\cdot) + \frac{u''\big(w(\pi_i)\big)}{u'\big(w(\pi_i)\big)} B'(\cdot)}$$

- Dividing numerator and denominator by $B'(\cdot)$ yields

$$w'(\pi_i) = \frac{B''(\cdot)/B'(\cdot)}{B''(\cdot)/B'(\cdot) + \frac{u''(\cdot)}{u'(\cdot)}} = \frac{r_P}{r_P + r_A} \qquad \textbf{(5)}$$

where r_P and r_A denote the the **Arrow–Pratt coefficient of absolute risk aversion** of the principal and the agent, respectively.

Risk Attitudes: Case 3



Risk Attitudes: Case 3

- Let us next evaluate the ratio in expression **(5)** at different values of r_P and r_A.
- **Risk-neutral principal**: $r_P = 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 0$.
  - This result holds regardless of the agent's coefficient of risk aversion $r_A > 0$.
- This setting coincides with that in Case 1, where the agent receives a fixed wage to insure him against the profit realization π_i, whereas the risk-neutral principal bears all the risk.

Risk Attitudes: Case 3

- **Risk-neutral agent**: $r_A = 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 1$.
  - This holds regardless of principal's coefficient of risk aversion $r_P > 0$.
  - This setting coincides with that in Case 2, where the risk-neutral agent bears all the risk while the principal receives a fixed payment K that insures the principal against different profit realization π_i.

Risk Attitudes: Case 3

- **Agent is more risk-averse than principal**: $r_A > r_P > 0$
  - The expression in **(5)** becomes $w'(\pi_i) < 1/2$.
  - It is optimal for the agent's salary $w'(\pi_i)$ to exhibit small variations in the profit realization π_i.
  - The more risk-averse agent bears less payoff volatility.

Risk Attitudes: Case 3

- **Principal is more risk-averse than agent**: $r_P > r_A > 0$
  - The expression in **(5)** becomes $w'(\pi_i) > 1/2$.
  - The less risk-averse agent bears more payoff volatility.

Risk Attitudes: Case 3

- **Same degree of risk aversion**: $r_A = r_P = r > 0$
  - The expression in **(5)** becomes $w'(\pi_i) = 1/2$.
  - Both the agent and the principal bear the same risk in the contract.

Asymmetric Information

- The principal **cannot observe** the agent's effort level e.
- The principal needs to offer to the agent enough incentives to exert the profit-maximizing effort level.
- How can the principal achieve this objective?
  - Make the salary an increasing function of the realized profit.
  - This is optimal even if the agent is risk-averse.

Asymmetric Information

- The principal's problem is

$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot B(\pi_i - w(\pi_i))$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\,[u(w(\pi_i)) - g(e)] \geq \bar{u}$

$$e^* \in \arg\max_{e} \sum_{i=1}^{N} f(\pi_i|e)\,[u(w(\pi_i)) - g(e)]$$

- The principal seeks to maximize its expected profits subject to:
  1. The voluntary participation of the agent (PC condition);
  2. The effort that he anticipates the agent will optimally choose in order to maximize his expected utility after receiving the contract from the principal (incentive compatibility, IC, condition).

Asymmetric Information

- Assume there are only two different effort levels available to the agent (e_L and e_H, where $e_H > e_L$).
- The agent can choose to work a positive number of hours or completely slack from the job ($e_H = e > 0$ and $e_L = 0$).
- Consider that the principal seeks to induce the high effort level e_H and that the principal is **risk-neutral** while the agent is **risk-averse**.

Asymmetric Information

- The principal's problem reduces to

$$\max_{\{e, w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e_H) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e_H)\,[u(w(\pi_i)) - g(e_H)] \geq \bar{u}$ (PC)

$\sum_{i=1}^{N} f(\pi_i|e_H)\,\big[u\big(w(\pi_i)\big) - g(e_H)\big] \geq$
$\sum_{i=1}^{N} f(\pi_i|e_L)\,[u(w(\pi_i)) - g(e_L)]$ (IC)

where the IC condition induces the agent to choose effort level e_H as such effort yields a higher expected utility than e_L for the agent.

Asymmetric Information

- The Lagrangian becomes

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e_H) \cdot [\pi_i - w(\pi_i)]$$
$$+ \lambda \left[\sum_{i=1}^{N} f(\pi_i|e_H) \left[u\big(w(\pi_i)\big) - g(e_H) \right] - \bar{u} \right]$$
$$+ \mu \left\{ \sum_{i=1}^{N} f(\pi_i|e_H) \left[u\big(w(\pi_i)\big) - g(e_H) \right] \right.$$
$$\left. - \left[\sum_{i=1}^{N} f(\pi_i|e_L) \left[u\big(w(\pi_i)\big) - g(e_L) \right] \right] \right\}$$

Asymmetric Information

- Taking FOC with respect to w yields

$$-f(\pi_i|e_H) + \lambda f(\pi_i|e_H)u'\big(w(\pi_i)\big)$$
$$+\mu\big[f(\pi_i|e_H)u'\big(w(\pi_i)\big) - f(\pi_i|e_L)u'\big(w(\pi_i)\big)\big] = 0$$

- Rearranging

$$\lambda + \underbrace{\mu\left[1 - \frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}\right]}_{\text{New}} = \frac{1}{u'\big(w(\pi_i)\big)} \qquad \textbf{(6)}$$

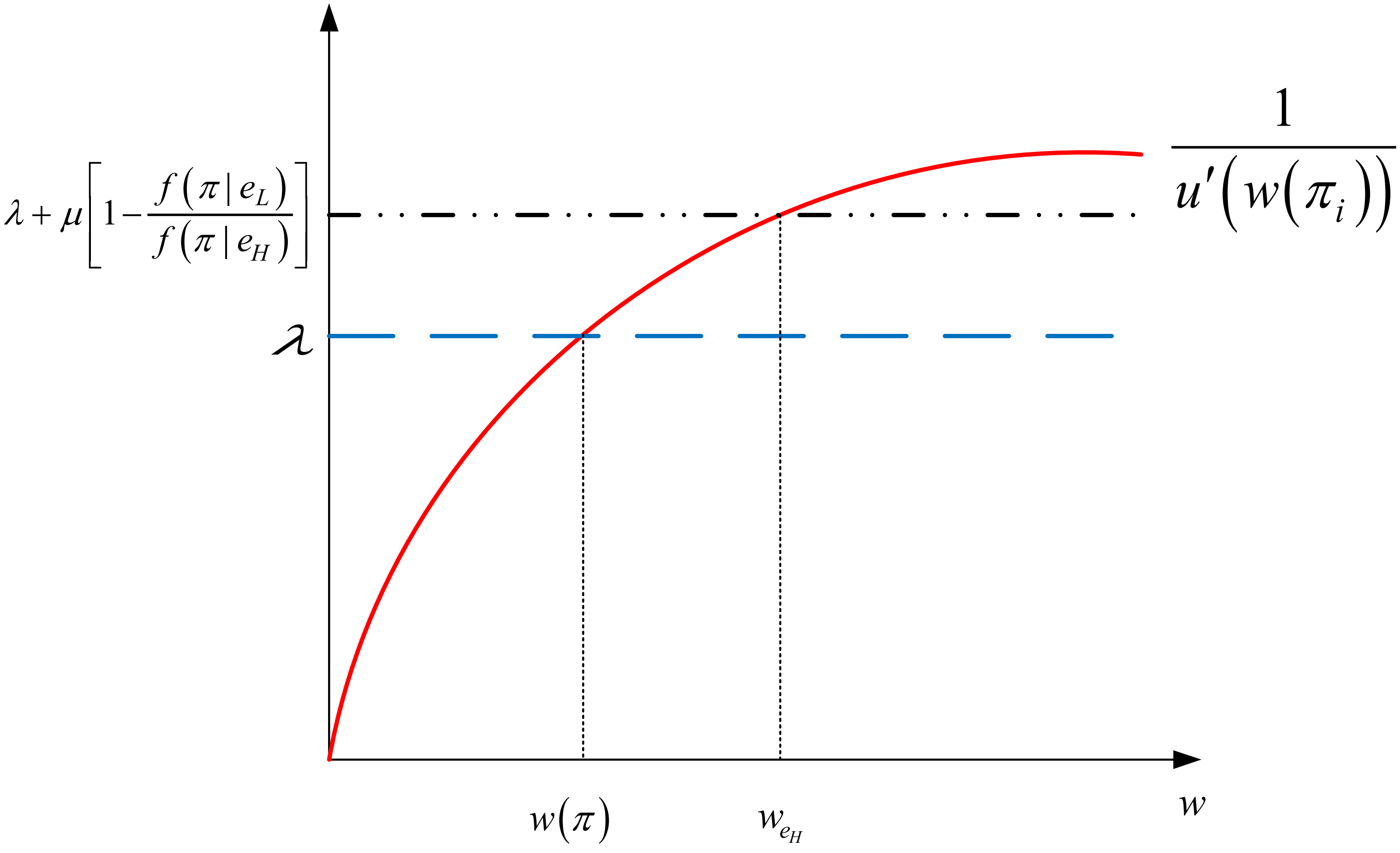
- Compare **(6)** with expression **(2)** in case 1, where the principal was risk-neutral but the agent was risk-averse.

Asymmetric Information

- Because $\lambda > 0$, $\mu > 0$, and $\frac{f(\pi_i|e_L)}{f(\pi_i|e_H)} < 1$, then

$$\underbrace{\lambda + \mu\left[1 - \frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}\right]}_{\text{asymmetric info.}} > \underbrace{\lambda}_{\text{symmetric info.}}$$

Asymmetric Information



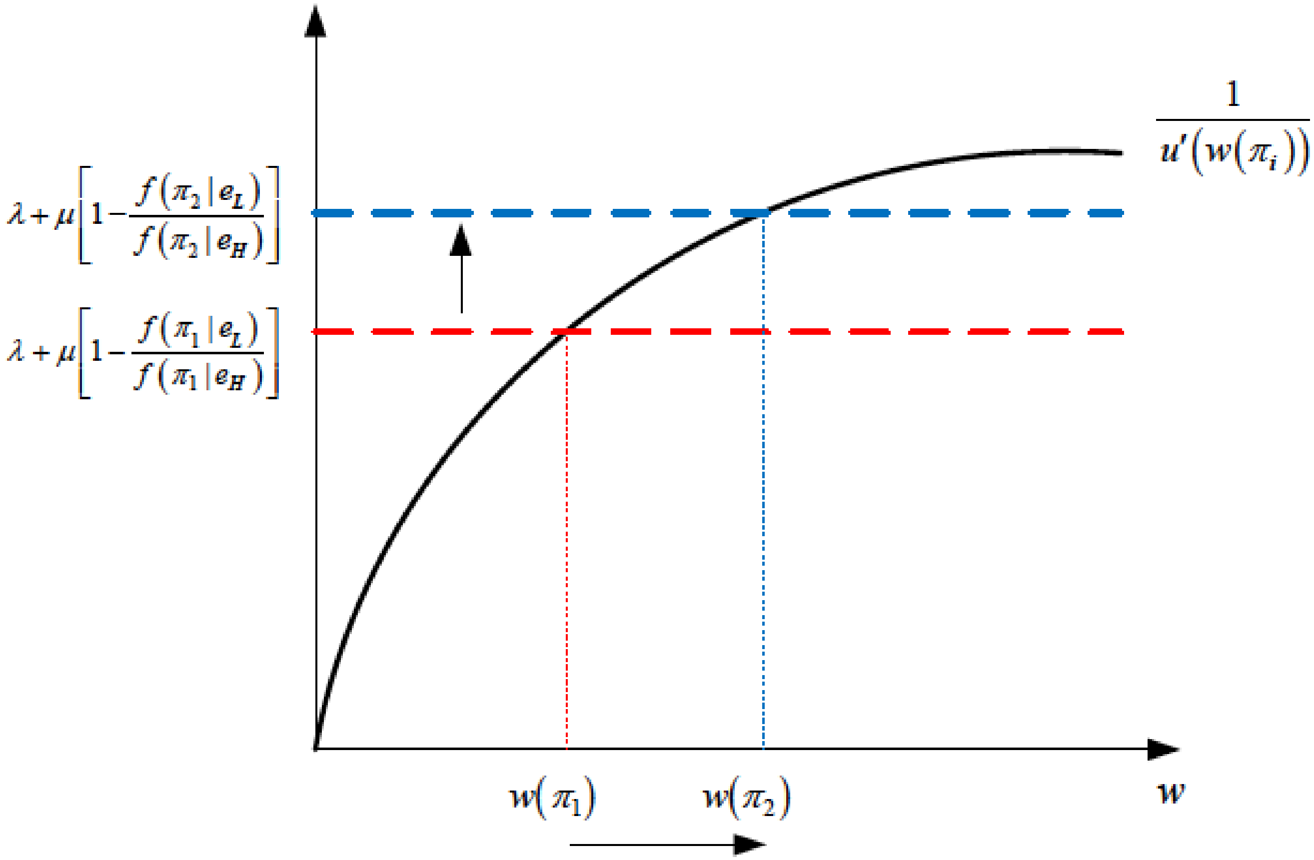
Asymmetric Information

- When deciding which effort to implement, the principal compares the effects of inducing a high effort level e_H.
  - Effort e_H yields a **positive effect** on profits since it increases the likelihood of higher profits.
  - This positive effect emerges under both symmetric and asymmetric information.
  - Effort e_H also entails a **negative effect** on profits since the salary that induces such effort is higher under asymmetric than under symmetric information $w_{e_H} > w(\pi)$.
  - Hence the principal is less willing to induce e_H when the agent's effort is unobservable than when it is observable.

Comparative Statics

- How does the salary above change as a function of the profit realization?
  - For that to happen, the left-hand side of **(6)** needs to increase in π.
  - This occurs if the likelihood ratio $\frac{f(\pi_i|e_L)}{f(\pi_i|e_H)}$ decreases in π.
  - Intuitively, as profits increase, the likelihood of obtaining a profit level of π from effort e_H increases faster than the probability of obtaining such a profit level from e_L.
  - This probability is commonly known as the **monotone likelihood ratio property**, MLRP.

Comparative Statics



Comparative Statics

- Example 2:
  - Consider Example 1, but assuming that the principal cannot observe the agent's effort.
  - In this incomplete information setting, the principal must offer a salary that increases in profit if he seeks to induce $e_H = 5$.
  - The principal's maximization problem becomes

$$\max_{\{w(\pi_i)\}_{i=1}^{3}} \quad 270 - [0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)]$$

$$\text{s.t.} \quad 0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 5 \geq 9 \quad \text{(PC)}$$

$$0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 5 \geq$$
$$0.6\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.1\sqrt{w(\pi_3)} \quad \text{(IC)}$$

Comparative Statics

- Example 2: (con't)
  - Since the principal's revenue is a constant (\$270), he can alternatively minimize his expected costs

$$\min_{\{w(\pi_i)\}_{i=1}^{3}} \; 0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)$$

$$\text{s.t. } 0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 14 \geq 0 \quad \text{(PC)}$$

$$-0.5\sqrt{w(\pi_1)} + 0.5\sqrt{w(\pi_3)} - 5 \geq 0 \quad \text{(IC)}$$

where the IC constraint has been simplified.

Comparative Statics

- Example 2: (con't)
  - The associated Lagrangian is

$$\mathcal{L} = \quad 0.1w(\pi_1) + 0.3w(\pi_2) + 0.6w(\pi_3)$$
$$- \lambda\left[0.1\sqrt{w(\pi_1)} + 0.3\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} - 14\right]$$
$$- \mu\left[-0.5\sqrt{w(\pi_1)} + 0.5\sqrt{w(\pi_3)} - 5\right]$$

  - Taking FOC with respect to $w(\pi_1)$, $w(\pi_2)$, and $w(\pi_3)$ yields

$$\frac{\partial \mathcal{L}}{\partial w(\pi_1)} = 0.1 - \frac{0.1\lambda}{2\sqrt{w(\pi_1)}} + \frac{0.5\mu}{2\sqrt{w(\pi_1)}} = 0 \qquad \textbf{(7)}$$

$$\frac{\partial \mathcal{L}}{\partial w(\pi_2)} = 0.3 - \frac{0.3\lambda}{2\sqrt{w(\pi_2)}} = 0 \qquad \textbf{(8)}$$

$$\frac{\partial \mathcal{L}}{\partial w(\pi_3)} = 0.6 - \frac{0.6\lambda}{2\sqrt{w(\pi_3)}} - \frac{0.5\mu}{2\sqrt{w(\pi_3)}} = 0 \qquad \textbf{(9)}$$

Comparative Statics

- Example 2: (con't)
  - Rearranging **(7)** and **(8)**

$$\lambda = 2\sqrt{w(\pi_2)}$$

$$\mu = 0.4\sqrt{w(\pi_2)} - 0.4\sqrt{w(\pi_1)}$$

  - Plugging these values into **(9)** and rearranging

$$0.1\sqrt{w(\pi_1)} - 0.7\sqrt{w(\pi_2)} + 0.6\sqrt{w(\pi_3)} = 0 \quad \textbf{(10)}$$

  - Combining equation **(10)** with the (PC) and (IC) equations, we have three equations and three unknowns $w(\pi_1)$, $w(\pi_2)$, and $w(\pi_3)$.

Comparative Statics

- Example 2: (con't)
  - The (IC) equation yields
    $$\sqrt{w(\pi_3)} = 10 + \sqrt{w(\pi_1)}$$
  - Substituting this into the (PC) equation
    $$3\sqrt{w(\pi_2)} = 80 - 7\sqrt{w(\pi_1)}$$
  - Last, substituting the values of $w(\pi_2)$ and $w(\pi_3)$ in equation **(10)**
    $$w(\pi_1) = \$29.47, w(\pi_2) = \$196, w(\pi_3) = \$238.04$$
  - The principal's expected profit is then
    $$270 - [0.1 \cdot 29.47 + 0.3 \cdot 196 + 0.6 \cdot 238.04] = \$65.43$$
    which is lower than its profit when effort is observable (\$74).

Moral Hazard with a Continuum of Effort Levels—The First-Order Approach

Continuum of Effort Levels

- So far we assumed that a worker could only have a discrete number of effort levels.
- Let us now consider a continuum of effort levels.
- The principal seeks to maximize its expected profits by anticipating the effort level that the agent selects in the second stage of the game:

$$\max_{\{e, w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$ (PC)

$e^* \in \arg\max_{e} \sum_{i=1}^{N} f(\pi_i|e) \left[u\big(w(\pi_i)\big) - g(e)\right]$ (IC)

Continuum of Effort Levels

- Difference/similarities between discrete and continuum effort levels
  - The objective function of the principal and the PC condition for the agent coincide.
  - The agent's IC condition, however, differs as it now allows him to choose among a continuum of effort levels.
  - Intuitively, the IC condition represents the agent's UMP where, for a given salary $w(\pi_i)$, the agent selects an effort level e that maximizes his expected utility.

Continuum of Effort Levels

- Differentiating the agent's expected utility with respect to e yields

$$\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e) = 0$$

- The agent's FOC above can be used as the IC condition in the principal's problem.
- This approach is known as the **first-order approach**.

Continuum of Effort Levels

- The principal's problem, using a "first-order approach," is then

$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N} f(\pi_i|e)\, u(w(\pi_i)) - g(e) \geq \bar{u}$ (PC)

$\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e) = 0$ (IC)

Continuum of Effort Levels

- The the Lagrangian becomes

$$\mathcal{L} = \sum_{i=1}^{N} f(\pi_i|e) \cdot [\pi_i - w(\pi_i)]$$
$$+ \lambda \left[\sum_{i=1}^{N} f(\pi_i|e)\, u\big(w(\pi_i)\big) - g(e) - \bar{u} \right]$$
$$+ \mu \left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e) \right]$$

- Taking FOC with respect to w yields

$$\frac{\partial \mathcal{L}}{\partial w} = -f(\pi_i|e) + \lambda f(\pi_i|e) u'\big(w(\pi_i)\big)$$
$$+\mu f'(\pi_i|e) u'\big(w(\pi_i)\big) = 0$$

Continuum of Effort Levels

- Dividing both sides by $f(\pi_i|e)$

$$-1+\lambda u'\big(w(\pi_i)\big)+\mu\frac{f'(\pi_i|e)}{f(\pi_i|e)}u'\big(w(\pi_i)\big)=0$$

- Factoring out $u'\big(w(\pi_i)\big)$ on the left-hand side and rearranging

$$\lambda+\mu\frac{f'(\pi_i|e)}{f(\pi_i|e)}=\frac{1}{u'\big(w(\pi_i)\big)}$$

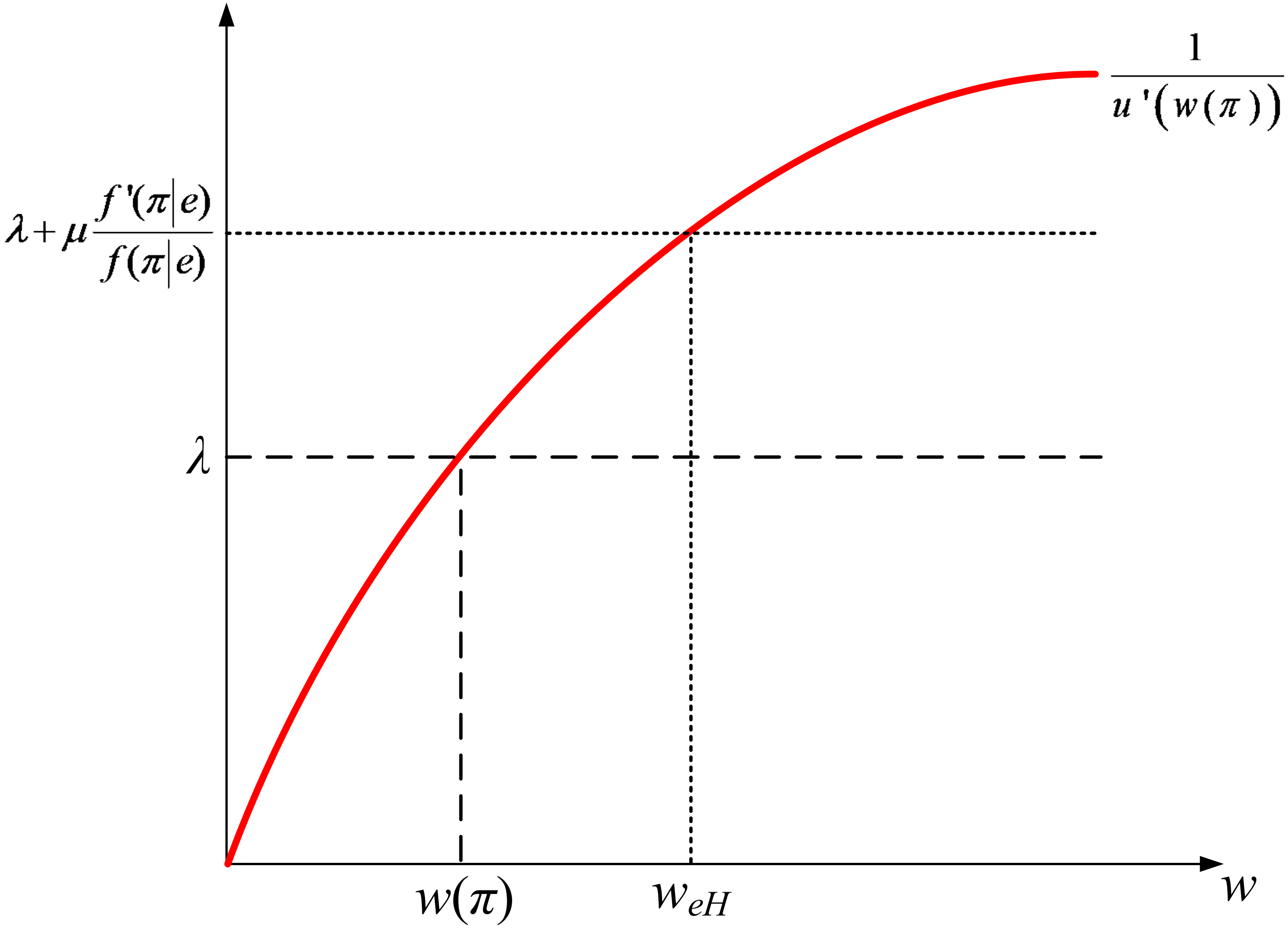
- This result is similar to that in previous sections.
- Because $\lambda>0$ and $\mu>0$ (since PC and IC bind), the left-hand side satisfies

$$\lambda+\mu\frac{f'(\pi_i|e)}{f(\pi_i|e)}>\lambda$$

Continuum of Effort Levels

- Since u' is decreasing in w (by concavity), its inverse, $1/u'$, is increasing in w.
- Hence the principal offers a larger salary under asymmetric information than symmetric information.
- $\frac{f'(\pi_i|e)}{f(\pi_i|e)}$ is the likelihood ratio, which measures how a marginally higher effort entails a larger probability of obtaining a given profit level π_i relative to an initial effort level.

Continuum of Effort Levels



Continuum of Effort Levels

- Taking FOC with respect to e yields

$$\frac{\partial \mathcal{L}}{\partial e} = \sum_{i=1}^{N} f'(\pi_i|e) \cdot [\pi_i - w(\pi_i)] + \mu \left[\sum_{i=1}^{N} f''(\pi_i|e)\, u\big(w(\pi_i)\big) - g''(e)\right]$$

$$+ \lambda \left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e)\right] = 0$$

- Rearranging

$$\sum_{i=1}^{N} f'(\pi_i|e)\, \pi_i = \sum_{i=1}^{N} f'(\pi_i|e)\, w(\pi_i)$$
$$-\mu \left[\sum_{i=1}^{N} f''(\pi_i|e)\, u\big(w(\pi_i)\big) - g''(e)\right]$$
$$-\lambda \left[\sum_{i=1}^{N} f'(\pi_i|e)\, u\big(w(\pi_i)\big) - g'(e)\right] \quad \textbf{(11)}$$

- Intuitively, effort is increased until the point where its expected profits (left-hand side) coincide with its associated costs (right-hand side).

Continuum of Effort Levels

- The cost of inducing a higher effort originates from two sources:
  1. A higher effort increases the probability of obtaining a higher profit, and thus the salary that the principal pays the agent once the profit is realized (first term on the right-hand side).
  2. The principal must provide more incentives (higher salary) in order for the agent to exert the effort level that the principal intended (second term on the right-hand side).

Continuum of Effort Levels

- Example 3:
  - Moral hazard with continuous effort but only two possible outcomes.
  - Consider a setting in which the conditional probability satisfies
$$f(\pi_i|e) = ef_H(\pi_i) + (1-e)f_L(\pi_i),\ e \in [0,1]$$
  - When effort is relatively high $e \to 1$, the probability of obtaining a profit level π_i is $f_H(\pi_i)$, where $f_H(\pi_i) > f_L(\pi_i)$.
  - When $e \to 0$, the probability of obtaining a profit level π_i is $f_L(\pi_i)$.

Continuum of Effort Levels

- Example 3: (con't)
  - The agent's expected utility is

$$EU(e) = \sum_{i=1}^{N} [ef_H(\pi_i) + (1-e)f_L(\pi_i)]u(w(\pi_i)) - g(e)$$

  - Since

$$ef_H(\pi_i) + (1-e)f_L(\pi_i) = e[f_H(\pi_i) - f_L(\pi_i)] + f_L(\pi_i)$$

  - Then

$$EU(e) = \sum_{i=1}^{N} e[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) + \sum_{i=1}^{N} f_L(\pi_i)u(w(\pi_i)) - g(e)$$

  - Differencing $EU(e)$ twice with respect to effort e, yields $-g''(e)$, which is negative by definition.
  - So we can use the first-order approach.

Continuum of Effort Levels

- Example 3: (con't)
  - The agent's FOC with respect to e is
$$\sum_{i=1}^{N}[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) = g'(e)$$
  - Plugging this FOC into the principal's problem
$$\max_{\{e,w(\pi_i)\}_{i=1}^{N}} \sum_{i=1}^{N}[ef_H(\pi_i) + (1-e)f_L(\pi_i)] \cdot [\pi_i - w(\pi_i)]$$

s.t. $\sum_{i=1}^{N}[ef_H(\pi_i) + (1-e)f_L(\pi_i)]\, u\big(w(\pi_i)\big) - g(e) \geq \bar{u}$ (PC)

$\sum_{i=1}^{N}[f_H(\pi_i) - f_L(\pi_i)]u(w(\pi_i)) = g'(e)$ (IC)

Continuum of Effort Levels

- Example 3: (con't)
  - The Lagrangian of this program is

$$\mathcal{L} = \sum_{i=1}^{N} [e f_H(\pi_i) + (1-e) f_L(\pi_i)] \cdot [\pi_i - w(\pi_i)]$$
$$+ \lambda \left[\sum_{i=1}^{N} [e f_H(\pi_i) + (1-e) f_L(\pi_i)]\, u\big(w(\pi_i)\big) - g(e) - \bar{u} \right]$$
$$+ \mu \left[\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)] u\big(w(\pi_i)\big) - g'(e) \right]$$

  - Taking FOC with respect to w and rearranging

$$\lambda + \mu \frac{f_H(\pi_i) - f_L(\pi_i)}{e f_H(\pi_i) + (1-e) f_L(\pi_i)} = \frac{1}{u'\big(w(\pi_i)\big)}$$

Continuum of Effort Levels

- Example 3: (con't)
  - Taking FOC with respect to e and rearranging

$$\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,\pi_i$$
$$= \sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,w(\pi_i) + \mu g''(e)$$
$$- \lambda \left[\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,u\big(w(\pi_i)\big) - g'(e) \right]$$

Continuum of Effort Levels

- Example 3: (con't)
  - From the binding (IC), we can further simplify and obtain

$$\sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,\pi_i$$
$$= \sum_{i=1}^{N} [f_H(\pi_i) - f_L(\pi_i)]\,w(\pi_i) + \mu g''(e)$$

  - The expected profit to the principal (left-hand side) is exactly balanced by the expected cost of inducing effort e from the agent (right-hand side).

Continuum of Effort Levels

- Example 4:
  - Moral hazard using the first-order approach
  - Assume the expected utility function of the agent is

$$u(w,e) = E(w) - \frac{1}{2}\rho Var(w) - c(e)$$

  where:

  - ρ is the Arrow–Pratt coefficient of absolute risk aversion for utility function $u(w) = -\mathrm{e}^{-\rho w}$,
  - $e \in [0,1]$ is the agent's effort, and
  - $c(e) = 0.5e^2$ is the cost of effort.
  - The outcome of the project, x, is stochastic and given by

$$x = f(e,\varepsilon) = e + \varepsilon, \text{ where } \varepsilon \sim N(0,\sigma^2)$$

Continuum of Effort Levels

- Example 4: (con't)
  - The agent's reservation utility is $\bar{u} = \frac{1}{2}$.
  - The principal offers a linear contract to the agent
$$w(x) = a + bx$$
  - where $a > 0$ is a fixed payment, and $b \in [0,1]$ is the share of profits that the agent receives (bonus).
  - The principal's expected profits are
$$E(\pi) = E(x - w) = E(x) - E(w)$$
$$= E(x) - [a + bE(x)] = (1 - b)e - a$$

Continuum of Effort Levels

- Example 4: (con't)
  - Since $E(x) = e$, the expected utility of the agent when he exerts effort level e is

$$E[u(w,e)] = E(w) - \frac{1}{2}\rho Var(w) - c(e)$$

$$= a + be - \frac{1}{2}\rho b^2\sigma^2 - \frac{1}{2}e^2$$

where $E(w) = a + be, Var(w) = b^2\sigma^2$, and $c(e) = \frac{1}{2}e^2$.

Continuum of Effort Levels

- Example 4: (con't)
  - Taking FOC with respect to e, we can find the effort that the agent chooses

$$\frac{\partial E[u(w,e)]}{\partial e} = b - e = 0$$

$$e = b$$

  - The principal's problem is to choose the fixed payment, a, and the bonus, b, to solve

$$\max_{e,a,b} \; (1-b)e - a$$

s.t. $$a + be - \frac{1}{2}\rho b^2 \sigma^2 - \frac{1}{2} e^2 \geq \frac{1}{2} \qquad \text{(PC)}$$

$$e = b \qquad \text{(IC)}$$

Continuum of Effort Levels

- Example 4: (con't)
  - Plugging $e = b$ into the program and simplifying

$$\max_{a,b} (1-b)b - a$$

$$\text{s.t.} \quad a + \frac{1}{2}b^2(1-\rho\sigma^2) \geq \frac{1}{2} \qquad \text{(PC)}$$

  - The Lagrangian is

$$\mathcal{L} = (1-b)b - a + \lambda\left[a + \frac{1}{2}b^2(1-\rho\sigma^2) - \frac{1}{2}\right]$$

Continuum of Effort Levels

- Example 4: (con't)
  - The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial a} = -1 + \lambda = 0 \rightarrow \lambda = 1 \qquad \textbf{(12)}$$

$$\frac{\partial \mathcal{L}}{\partial b} = 1 - 2b + \lambda b(1 - \rho\sigma^2) = 0 \qquad \textbf{(13)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = a + \frac{1}{2}b^2(1 - \rho\sigma^2) - \frac{1}{2} = 0 \qquad \textbf{(14)}$$

  - Plugging **(12)** into **(13)** yields

$$1 - 2b + b(1 - \rho\sigma^2) = 0$$

$$b = \frac{1}{1 + \rho\sigma^2}$$

Continuum of Effort Levels

- Example 4: (con't)
  - Plugging $b = \frac{1}{1+\rho\sigma^2}$ into the binding (PC) constraint yields

$$a + \frac{1-\rho\sigma^2}{2(1+\rho\sigma^2)^2} = \frac{1}{2}$$

  - Solving for the fixed payment a

$$a = \frac{1}{2}\left[1 - \frac{1-\rho\sigma^2}{(1+\rho\sigma^2)^2}\right]$$

Continuum of Effort Levels

- Example 4: (con't)
  - If $\sigma^2 = 0$, effort e is deterministic (a perfect predictor of profits)

$$x = f(e) = e$$

  - Then,

$$b = \frac{1}{1 + \rho \cdot 0} = 1$$

$$a = \frac{1}{2}\left[1 - \frac{1 - \rho \cdot 0}{(1 + \rho \cdot 0)^2}\right] = 0$$

  - Intuitively, the principal does not offer a fixed payment, and the agent is benefited from high-powered incentives.

Continuum of Effort Levels

- Example 4: (con’t)
  - If $\sigma^2 = 1$, effort e is imprecise predictor of outcomes.
  - Then,

$$b = \frac{1}{1+\rho}$$

$$a = \frac{\rho(\rho+3)}{2(1+\rho)^2}$$

Continuum of Effort Levels

- Example 4: (con't)
  - When the agent becomes more risk-averse (ρ increases), the agent is offered a higher fixed payment but a lower bonus, since

$$\frac{\partial b}{\partial \rho} = -\frac{1}{(1+\rho)^2} < 0$$

$$\frac{\partial a}{\partial \rho} = \frac{3-\rho}{2(1+\rho)^3} > 0$$

Continuum of Effort Levels

- Example 4: (con't)
  - In general, for $0 \leq \sigma^2 \leq 1$, we show that a increases but b decreases in σ^2 since

$$\frac{\partial b}{\partial \sigma^2} = -\frac{\rho}{(1+\rho\sigma^2)^2} < 0$$

$$\frac{\partial a}{\partial \sigma^2} = -\frac{-\rho(1+\rho\sigma^2)^2 - 2\rho(1+\rho\sigma^2)(1-\rho\sigma^2)}{2(1+\rho\sigma^2)^4}$$

$$= \frac{\rho(3-\rho\sigma^2)}{2(1+\rho\sigma^2)^3} > 0$$

Continuum of Effort Levels

- Example 4: (con't)
- When σ^2 is low (i.e., all effort levels yield a similar outcome x), the fixed payment a is low while the bonus b is high, which we call **high-powered incentives**.
- When σ^2 is high (i.e., an effort level is possible to yield many different outcomes x), the fixed payment a is high while the bonus b is low, which we call **low-powered incentives**.

Moral Hazard with Multiple Signals

Multiple Signals

- Consider a setting in which the principal, still not observing effort e, observes:
  - the profits π of the firm;
  - a signal s, based on a middle management report about the agent's performance.
- Signal s provides no intrinsic economic value but it provides information about effort e.
- Hence the probability density function has two observables, π and s.
- Then, similar to equation **(6)**, we have

$$\frac{1}{u'(w)} = \gamma + \mu\left[1 - \frac{f(\pi, s|e_L)}{f(\pi, s|e_H)}\right]$$

Multiple Signals

- Hence variations in s affect wages only if
$$f(\pi, s|e) \neq f(\pi|e)$$
- That is, if π is not a sufficient statistic of e.
- Intuitively, the pair (π, s) contains more information about the agent's exerted effort e than π alone.
- Signal s is uninformative (provides no more information than π alone), if
$$f(\pi, s|e) = f(\pi|e)$$
- We can examine under which conditions w increases in signal s.

Multiple Signals

- For two signals s_1 and s_2, where $s_2 > s_1$, if salary increases in the signal, $w(\pi, s_2) > w(\pi, s_1)$, then $u'(w)$ decreases and its inverse, $1/u'(w)$, increases.
- Therefore,

$$\gamma + \mu\left[1 - \frac{f(\pi, s_2|e_L)}{f(\pi, s_2|e_H)}\right] > \gamma + \mu\left[1 - \frac{f(\pi, s_1|e_L)}{f(\pi, s_1|e_H)}\right]$$

- Simplifying this inequality to express it in terms of the likelihood ratio, $\frac{f(\pi,s|e_L)}{f(\pi,s|e_H)}$, we obtain

$$\frac{f(\pi, s_2|e_L)}{f(\pi, s_2|e_H)} < \frac{f(\pi, s_1|e_L)}{f(\pi, s_1|e_H)}$$

- In words, this condition says that, for the salary to increase in the intermediate signal s that the principal receives, we need such a signal to have a *decreasing* likelihood ratio.

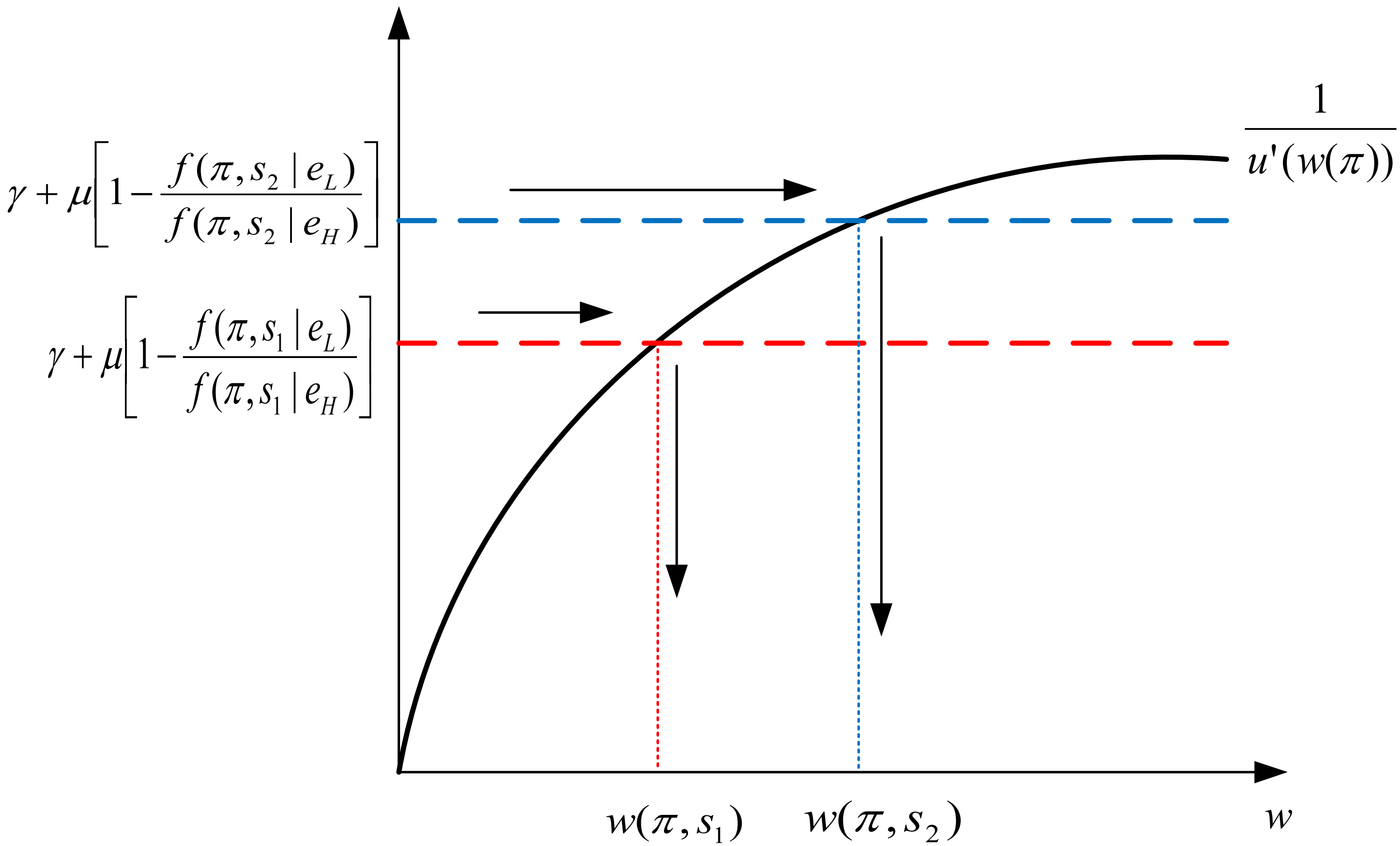
Multiple Signals

- Alternatively, we can rearrange expression $\frac{f(\pi,s_2|e_L)}{f(\pi,s_2|e_H)} < \frac{f(\pi,s_1|e_L)}{f(\pi,s_1|e_H)}$ as follows

$$\frac{f(\pi,s_1|e_H)}{f(\pi,s_1|e_L)} < \frac{f(\pi,s_2|e_H)}{f(\pi,s_2|e_L)}$$

- Intuitively, signal s_2 is more likely to originate from the high than the low effort, relative to signal s_1.

Multiple Signals



Adverse Selection
The “Lemons” Problem

Adverse Selection

- **Adverse selection**: settings in which an agent does not observe the payoff of the other individual.
  - Also referred to as "hidden information"
- Example:
  - A manager in a firm might not observe the worker's ability
  - The manager could err in its selection of candidates for a job if he does not observe their ability, thus giving rise to adverse selection
- Under symmetric information markets often work well.
- Under asymmetric information, however, markets do not necessarily work well.

Adverse Selection

- Akerloff's (1970) model:
  - Consider a market of used cars, whose quality is denoted by q, where $q \in U[0, Q]$ and $Q \in (1,2)$.
  - A car of quality q is valued as such by the buyer, and as q/Q by the seller.
  - Since $\frac{q}{Q} < q$, the buyer assigns a higher value to the car than the seller.
  - This allows both parties to exchange the car at a price p between q/Q and q and make a profit (for the seller) and a surplus (for the buyer).

Adverse Selection

- Akerloff's (1970) model:
  - If a car of quality q is exchanged at price p the buyer obtains a utility
  $$u(p,q) = q - p$$
  while the seller makes a profit of
  $$\pi(p,q,Q) = p - \frac{q}{Q}$$
  - Assume that there are a sufficient number of buyers so that all gains from trade are appropriated by the seller.

Symmetric Information

- When the buyer can **perfectly** observe the car quality q, he buys at a price p if and only if
$$q - p \geq 0$$
- That is, his utility from such a trade is positive.
- A seller with a car of quality q anticipates such an acceptance rule by the buyer and sets a price p that solves
$$\max_{p \geq 0} \ p - \frac{q}{Q}$$
$$\text{s.t. } p \leq q$$

where $p \leq q$ is the buyer's participation constraint (PC).

Symmetric Information

- Since condition (PC) must bind, $p = q$, the seller's objective function can be represented as unconstrained problem:

$$\max_{p \geq 0} \; p - \frac{p}{Q}$$

- Taking the FOC with respect to p yields

$$1 - \frac{1}{Q} > 0 \;\text{ or }\; \frac{Q-1}{Q} > 0$$

- Since $Q > 1$ by definition, a corner solution exists whereby the seller raises the price p as much as possible

$$p^{SI} = q$$

Asymmetric Information

- When the buyer is **unable** to observe the car's true quality q, he forms an expectation $E(q)$.
- The buyer accepts a trade if the car's asking price p satisfies

$$p = E(q)$$

- The seller anticipates such an acceptance rule by the buyer and sets a price p that solves

$$\max_{p \geq 0} \; p - \frac{q}{Q}$$

$$\text{s.t. } \; p \leq E(q)$$

where $p \leq E(q)$ is the buyer's PC constraint.

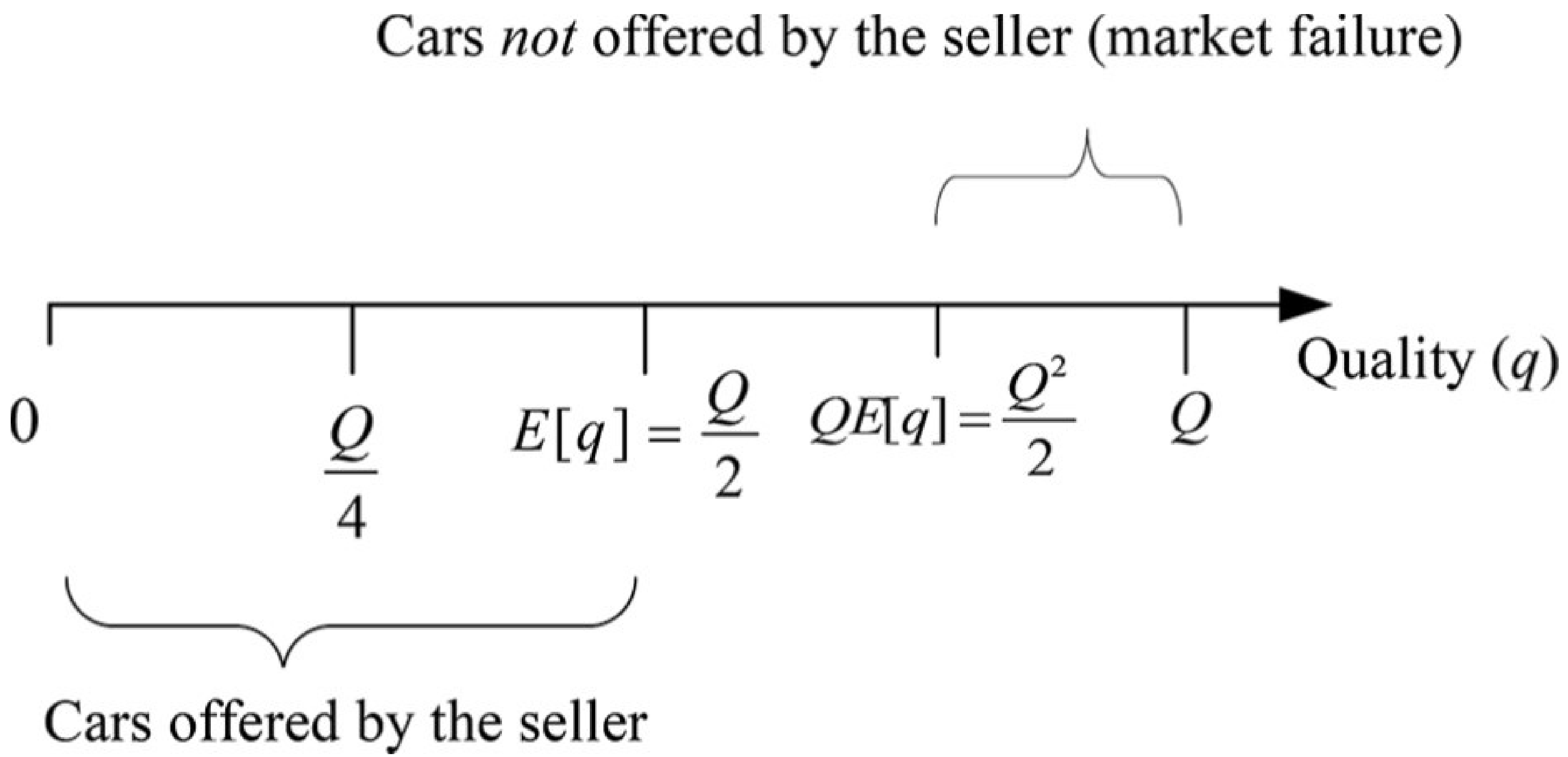
Asymmetric Information

- Since condition (PC) must bind, $p = E(q)$, the price that the seller sets

$$p - \frac{q}{Q} = E(q) - \frac{q}{Q} \geq 0$$

$$q \leq Q \cdot E(q)$$

Asymmetric Information



Asymmetric Information

- When q is uniformly distributed, that is, $q \sim U[0, Q]$, its expected value becomes

$$E(q) = \frac{Q - 0}{2} = \frac{Q}{2}$$

- Then, $Q \cdot E(q) = Q^2/2$.
- Hence all cars with relatively **low quality**, $q \leq Q^2/2$, are offered by the seller at a price

$$p = E(q) = \frac{Q}{2}$$

yielding profit of $\frac{Q}{2} - \frac{q}{2}$ for the seller and a zero (expected) utility for the buyer since $p = E(q)$.

Asymmetric Information

- Cars with relatively **high quality**, $q \geq Q^2/2$, are not offered by the seller since the highest price he can charge to the uninformed buyer, $p = E(q)$, does not compensate the seller's costs.
- This is problematic.
- The buyer's inability to observe q leads to the non-existence of the market for good cars ("peaches"), whereas only bad cars ("lemons") exist in the market.

Asymmetric Information

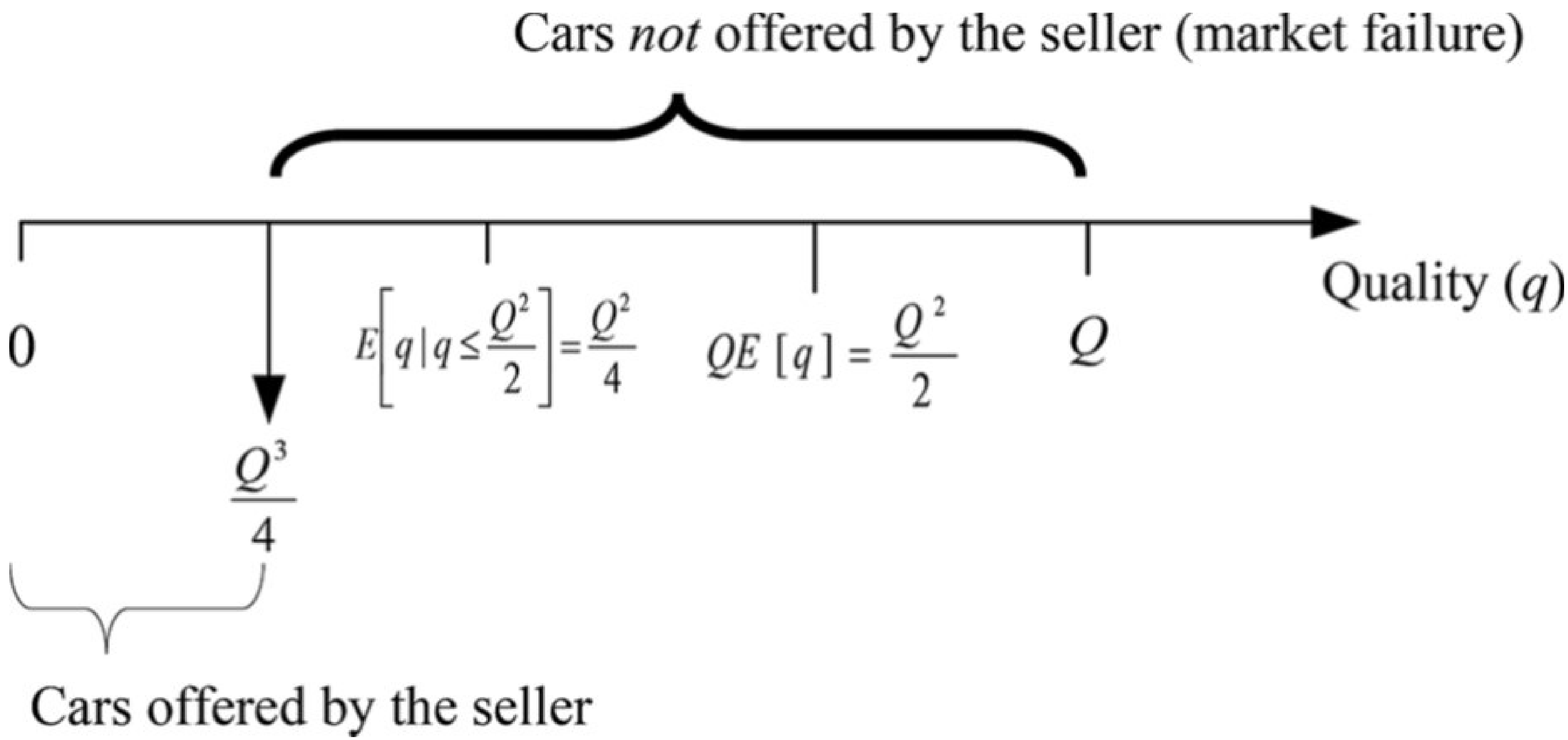
- A fully rational buyer would anticipate such a pricing decision by the seller
  - That the seller finds it worthy to only offer low quality cars, $p \leq Q^2/2$.
- In that case, the buyer anticipates that only cars of quality $q \in (0, Q^2/2)$ are offered.
- Then, if $q \sim U[0, Q]$, buyers can compute the expected quality of those offered cars

$$E\left[q \middle| q \leq \frac{Q^2}{2}\right] = \frac{\frac{Q^2}{2} - 0}{2} = \frac{Q^2}{4}$$

Asymmetric Information

- Hence the buyer would only buy cars whose price satisfies $p = Q^2/4$.
- The seller would then set the price at $p = Q^2/4$, yielding a profit of
$$p - \frac{q}{Q} = \frac{Q^2}{4} - \frac{q}{Q}$$
which is positive only if quality q satisfies
$$q \leq \frac{Q^3}{4}$$

Asymmetric Information



Asymmetric Information

- A rational buyer would now update its expected car quality to those satisfying $Q^3/4$
- This yields an expected quality of only

$$E\left[q \middle| q \leq \frac{Q^3}{4}\right] = \frac{\frac{Q^3}{4} - 0}{2} = \frac{Q^3}{8}$$

- The seller offers cars that yield a positive profit, that is, those with quality q satisfying

$$p - \frac{q}{Q} = \frac{Q^3}{8} - \frac{q}{Q} \geq 0 \quad \text{or} \quad q \leq \frac{Q^4}{8}$$

which lies closer to zero than cutoff $\frac{Q^3}{4}$.

Asymmetric Information

- Intuition:
  - The seller would shift the set of offered cars even more to the left of the quality line toward worse cars (closer to zero).
  - Repeating the same argument enough times, we find that the market "unravels."
  - It only offers cars of the worst possible quality, $q = 0$.
  - The buyer is only willing to pay a price of $p = 0$, leaving all other types of cars unsold.

Asymmetric Information

- Example 5:
  - Consider a market of used cars with maximum available quality $Q = 1.9$, and that $q \sim U[0, Q]$.
  - Recall that $Q \in (1,2)$, i.e., the availability of several cars of relatively good quality.
  - The buyer's expected value is $\frac{1.9}{2} = 0.95$.
  - The cutoff $Q \cdot E(q)$ of cars offered by the seller is $1.9 \cdot 0.95 = 1.805$.
  - Unoffered cars $(1.805, 1.9)$.
  - Under complete information, these cars would have been bought by the buyer who values them at q, and sold by the seller who values them at only $\frac{q}{1.9} = 0.52q$.

Asymmetric Information

- Example 5: (con't)
  - A rational buyer will anticipate that cars in the interval $(1.805, 1.9)$ are unoffered by the seller.
  - Thus buyer updates expected value of offered cars to
  $$E[q|q \leq 1.805] = \frac{1.805 - 0}{2} = 0.9$$
  - This leads the seller to only offer those cars with quality
  $$q \leq \frac{Q^3}{4} = 1.71$$
  - The set of offered cars is thus restricted from $(0, 1.805)$ to $(0, 1.71)$.
  - A similar argument applies to further iterations in the buyer's expected car quality.
  - The presence of asymmetric information between buyer and seller prevents mutually beneficial trades from occurring.

Asymmetric Information

- Application to Labor Markets
  - Consider a competitive labor market with many firms seeking to hire a worker for a specific position.
  - The worker (seller of labor services) privately observes his own productivity θ, but firms (the buyer of labor) cannot observe it.
  - Firms offer a wage according to the worker's expected productivity

$$E(\theta) = 1/2, \; \theta \sim U[0,1]$$

  - For this salary, only workers with a productivity $\theta \leq 1/2$ would be interested in accepting the position, while those with $\theta > 1/2$ will be left unemployed.

Asymmetric Information

- Application to Labor Markets
  - A fully rational manager will only offer a salary of

$$w = E\left(\theta | \theta \leq \frac{1}{2}\right) = \frac{1}{4}$$

  - Then only those workers with productivity $\theta \leq \frac{1}{4}$ accept the job.
  - Extending the argument infinite times, workers with lowest productivity level $\theta = 0$ are employed, while the labor market for all other worker types $\theta > 0$ unravels.

Solutions to Adverse Selection

- The market failure described above can be overcome by a number of tools.
  - Sellers can offer warranties for their cars in order to signal their quality.
  - **Screening**: The principal (buyer) offers a menu of contracts to the agent (seller) that induce each type of agent to voluntarily select only one contract, whereby the contracts induce self-selection.

Adverse Selection
The Principal–Agent Problem

The Principal–Agent Problem

- Consider a setting where a firm (the principal) seeks to hire a worker (an agent).
- The firm cannot observe the worker's cost of effort
  - This affects the amount of effort that the worker exerts and thus the firm's profits.
- The firm's manager would like to know the worker's cost of effort in order to design his salary.
- The firm's profit function is

$$\pi(e, w) = x(e) - w$$

  where $x(e)$ is the benefit that the firm obtains when the worker supplies e units of effort, $x'(e) \geq 0$, $x''(e) \leq 0$.

The Principal–Agent Problem

- The worker's utility function is
$$v(w, e|\theta) = u(w) - c(e, \theta)$$
where $u(w)$ is the value from the salary w, $u'(w) > 0$, $u''(w) \leq 0$; $c(e, \theta)$ is the worker's cost of exerting e units of effort when his type is θ.
- Assume the worker can only be of two types, θ_L and θ_H, where $\theta_L < \theta_H$, with probabilities p and $1 - p$.
- A high-type worker faces a higher total and marginal cost of effort
$$c(e, \theta_L) < c(e, \theta_H)$$
$$c'(e, \theta_L) < c'(e, \theta_H)$$
for every e.

Symmetric Information

- When the principal (firm) **knows** that the agent is type $i = \{L, H\}$, it solves

$$\max_{w_i, e_i} \; x(e_i) - w_i$$

$$\text{s.t.} \quad u(w_i) - c(e_i, \theta_i) \geq 0 \qquad \text{(PC)}$$

- (PC) constraint guarantees that the worker willingly accepts the contract.
- Since the firm can reduce w_i until (PC) holds with equality, (PC) must bind

$$u(w_i) = c(e_i, \theta_i)$$

$$w_i = u^{-1}[c(e_i, \theta_i)]$$

Symmetric Information

- The principal's unconstrained maximization problem can then be written as

$$\max_{e_i} \; x(e_i) - u^{-1}[c(e_i, \theta_i)]$$

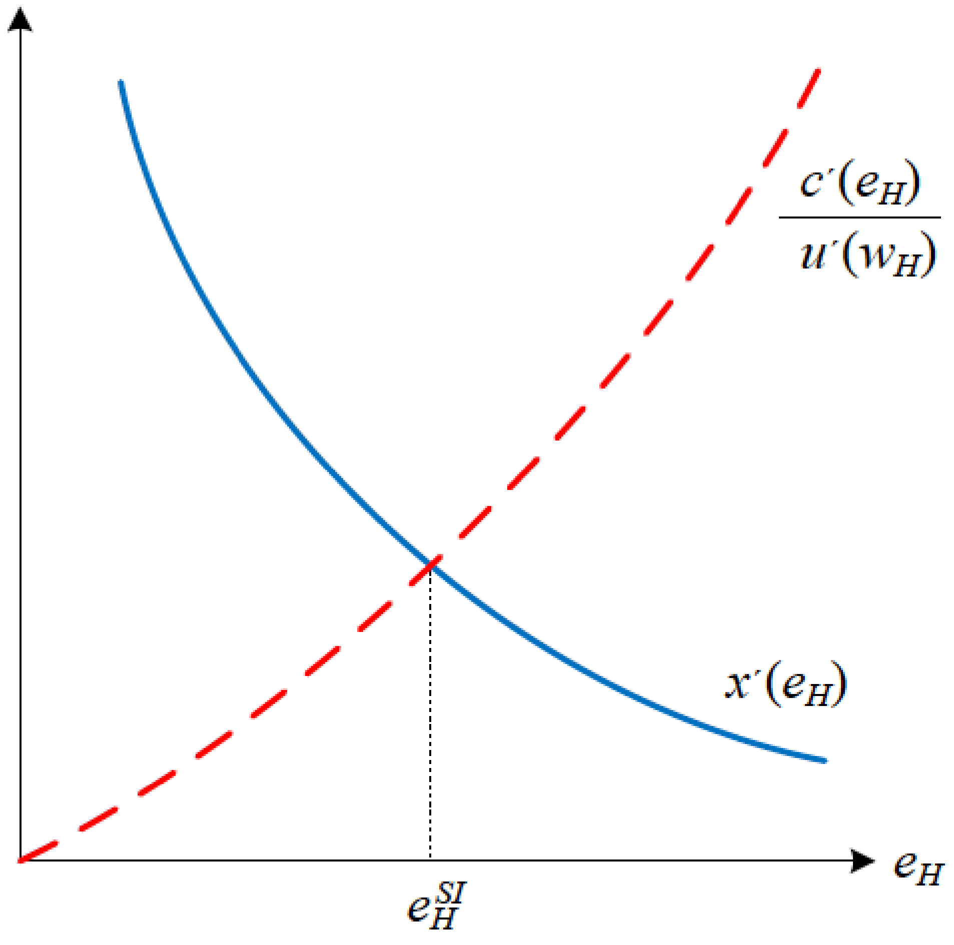
- Taking FOC with respect to e_i yields

$$x'(e_i) = \frac{1}{u'\{u^{-1}[c(e_i, \theta_i)]\}} c'(e_i, \theta_i)$$

$$x'(e_i) = \frac{c'(e_i, \theta_i)}{u'\{w_i\}}$$

- Hence effort is increased until the point at which the marginal rate of substitution of effort and wage for the firm (left-hand side) coincides with that of the worker (right-hand side).

Symmetric Information



Symmetric Information

- Example 6:
  - Consider a principal and an agent of type $\theta_L = 1$, $\theta_H = 2$.
  - The probability of facing a low type is $p = 1/2$.
  - Productivity of effort is $x(e) = \log(e)$, and $u(w) = w$.
  - The cost of effort is $c(e, \theta) = \theta_i e^2$, with the marginal cost of effort of $2\theta_i e$, which is positive and increasing in e.
  - The principal's profit function is

$$\pi(e, w) = \log(e) - w$$

  - The agent's utility is

$$v(w, e|\theta_i) = w - \theta_i e^2$$

Symmetric Information

- Taking FOC

$$x'(e_i) = \frac{c'(e_i, \theta_i)}{u'\{w_i\}} \Rightarrow \frac{1}{e_i} = \frac{2\theta_i e_i}{1}$$

- Solving for e_i

$$e_i^2 = \frac{1}{2\theta_i} \to e_i^{SI} = \left(\frac{1}{2\theta_i}\right)^{1/2}$$

- Use the (PC) constraint, $u(w_i) = c(e_i, \theta_i)$, to find optimal salary

$$w_i = \theta_i \left(e_i^{SI}\right)^2 = \theta_i \left(\left(\frac{1}{2\theta_i}\right)^{1/2}\right)^2 = \frac{1}{2}$$

Symmetric Information

- Plugging in $\theta_L = 1$ and $\theta_H = 2$, we find optimal contracts

$$(w_H^{SI}, e_H^{SI}) = \left(\frac{1}{2}, \frac{1}{2}\right) = (0.5, 0.5)$$

$$(w_L^{SI}, e_L^{SI}) = \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right) = (0.5, 0.707)$$

- The firm will pay both types of workers the same wage under symmetric information, but expect a higher effort level from the low-cost worker, $e_L^{SI} > e_H^{SI}$.

Asymmetric Information

- When the firm **cannot** observe the worker's type, it seeks to maximize the expected profits by designing a pair of contracts, (w_H, e_H) and (w_L, e_L), that satisfy four constraints:
  1. voluntary participation of the high-type worker;
  2. voluntary participation of the low-type worker;
  3. the high-type worker prefers the contract (w_H, e_H) rather than that for the low-type, (w_L, e_L);
  4. the low-type worker prefers the contract (w_L, e_L) rather than that for the high-type, (w_H, e_H).
- Since every type of worker has an incentive to select the contract meant for him, these contracts induce "**self-selection**."

Asymmetric Information

- The firm solves the following profit maximization problem

$$\max_{w_L, e_L, w_H, e_H} \quad p[x(e_L) - w_L] + (1-p)[x(e_H) - w_H]$$

$$\text{s.t.} \quad u(w_H) - c(e_H, \theta_H) \geq 0 \qquad (\text{PC}_\text{H})$$

$$u(w_L) - c(e_L, \theta_L) \geq 0 \qquad (\text{PC}_\text{L})$$

$$u(w_H) - c(e_H, \theta_H) \geq u(w_L) - c(e_L, \theta_H) \qquad (\text{IC}_\text{H})$$

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L) \qquad (\text{IC}_\text{L})$$

Asymmetric Information

- Note that (PC_L) is implied by (IC_L) and (PC_H)

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L)$$
$$> u(w_H) - c(e_H, \theta_H) \geq 0$$

  - The first (weak) inequality stems from (IC_L).
  - The second (strict) inequality stems from the assumption $c(e_H, \theta_L) < c(e_H, \theta_H)$.
  - The third (weak) inequality stems from (PC_H).
- Hence we obtain (PC_L)

$$u(w_L) - c(e_L, \theta_L) > 0$$

Asymmetric Information

- The Lagrangian is

$$
\begin{aligned}
\mathcal{L} \quad &= p[x(e_L) - w_L] + (1-p)[x(e_H) - w_H] \\
&+ \lambda_1[u(w_H) - c(e_H, \theta_H)] \\
&+ \lambda_2[u(w_H) - c(e_H, \theta_H) - u(w_L) + c(e_L, \theta_H)] \\
&+ \lambda_3[u(w_L) - c(e_L, \theta_L) - u(w_H) + c(e_H, \theta_L)]
\end{aligned}
$$

Asymmetric Information

- Taking FOCs

$$\frac{\partial \mathcal{L}}{\partial w_L} = -p - \lambda_2 u'(w_L) + \lambda_3 u'(w_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_H} = -(1-p) + \lambda_1 u'(w_H) + \lambda_2 u'(w_H) - \lambda_3 u'(w_H) = 0$$

$$\frac{\partial \mathcal{L}}{\partial e_L} = px'(e_L) + \lambda_2 c'(e_L, \theta_H) - \lambda_3 c'(e_L, \theta_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial e_H} = (1-p)x'(e_H) - \lambda_1 c'(e_H, \theta_H) - \lambda_2 c'(e_H, \theta_H) + \lambda_3 c'(e_H, \theta_L) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = u(w_H) - c(e_H, \theta_H) \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = u(w_H) - c(e_H, \theta_H) - u(w_L) + c(e_L, \theta_H) \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = u(w_L) - c(e_L, \theta_L) - u(w_H) + c(e_H, \theta_L) \geq 0$$

Asymmetric Information

- For simplicity, consider that the cost of effort takes the following form
$$c(e, \theta_i) = \theta_i c(e) \quad \text{for all } \; i = \{H, L\}$$
where $c(e)$ is increasing and convex in effort, $c'(e) \geq 0$ and $c''(e) \geq 0$.
- Rearranging the first two FOCs yields
$$-\lambda_2 + \lambda_3 = \frac{p}{u'(w_L)}$$
$$\lambda_1 + \lambda_2 - \lambda_3 = \frac{1-p}{u'(w_H)}$$
- Then adding them together
$$\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$$

Asymmetric Information

- Hence $\lambda_1 > 0$, implying that the constraint associated with Lagrange multiplier λ_1, (PC_H), binds:

$$u(w_H) - c(e_H, \theta_H) = 0$$

Asymmetric Information

- The third FOC can be written as

$$px'(e_L) = \lambda_3 \theta_L c'(e_L) - \lambda_2 \theta_H c'(e_L)$$

- Rearranging

$$\frac{px'(e_L)}{c'(e_L)} = \lambda_3 \theta_L - \lambda_2 \theta_H$$

- The fourth FOC can be written as

$$(1-p)x'(e_H) = \lambda_1 \theta_H c'(e_H) - \lambda_3 \theta_L c'(e_H) + \lambda_2 \theta_H c'(e_H)$$

- Rearranging

$$\frac{(1-p)x'(e_H)}{c'(e_H)} = \lambda_1 \theta_H - (\lambda_3 \theta_L - \lambda_2 \theta_H)$$

Asymmetric Information

- Combining the two (rearranged) FOCs yields

$$\frac{(1-p)x'(e_H)}{c'(e_H)} = \lambda_1\theta_H - \frac{px'(e_L)}{c'(e_L)}$$

- Solving for $\lambda_1\theta_H$ and using $\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$ from our results above, we obtain

$$\left[\frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}\right]\theta_H = \frac{px'(e_L)}{c'(e_L)} + \frac{(1-p)x'(e_H)}{c'(e_H)}$$

Asymmetric Information

- Moreover, $\lambda_3 > \lambda_2$, since otherwise the first FOC, $(\lambda_3 - \lambda_2)u'(w_L) = p$, could not hold.
- Therefore, $\lambda_3 > 0$, which means (IC_L) binds:
$$u(w_L) - \theta_L c(e_L) = u(w_H) - \theta_L c(e_H)$$
- Rearranging the right-hand side
$$u(w_L) - \theta_L c(e_L)$$
$$= u(w_H) - \theta_H c(e_H) + (\theta_H - \theta_L)c(e_H)$$
- Since (PC_H), binds, $u(w_H) - c(e_H, \theta_H) = 0$, hence
$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H)$$

Asymmetric Information

- Intuition:
  - The most efficient agent, θ_L, obtains in equilibrium a positive utility level, $(\theta_H - \theta_L)c(e_H)$, that increases in his difference with respect to the least efficient worker, $\theta_H - \theta_L$.

Asymmetric Information

- The incentive compatibility condition of the least efficient worker, (IC_H) , does not bind, implying that its associated Lagrange multiplier $\lambda_2 = 0$.
- Using this result in the first and third FOCs yields

$$\lambda_3 = \frac{p}{u'(w_L)} \quad \text{and} \quad \frac{px'(e_L)}{c'(e_L)} = \lambda_3 \theta_L$$

- Solving for λ_3 and combining the two FOCs

$$\frac{p}{u'(w_L)} = \frac{px'(e_L)}{\theta_L c'(e_L)}$$

- Solving for $x'(e_L)$

$$x'(e_L) = \frac{\theta_L c'(e_L)}{u'(w_L)}$$

Asymmetric Information

- Intuition:
  - For the most efficient worker, the equilibrium outcome under asymmetric information **coincides** with the socially optimal result under symmetric information.

Asymmetric Information

- Using $\lambda_1 = \frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}$, $\lambda_2 = 0$, $\lambda_3 = \frac{p}{u'(w_L)}$ in the fourth FOC, we obtain

$$(1-p)x'(e_H) - \left[\frac{p}{u'(w_L)} + \frac{1-p}{u'(w_H)}\right]\theta_H c'(e_H)$$
$$+ \left[\frac{p}{u'(w_L)}\right]\theta_L c'(e_H) = 0$$

- Rearranging

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$$

- The effort level that solves this equation is the **optimal** effort under asymmetric information, e_H^{AI}.

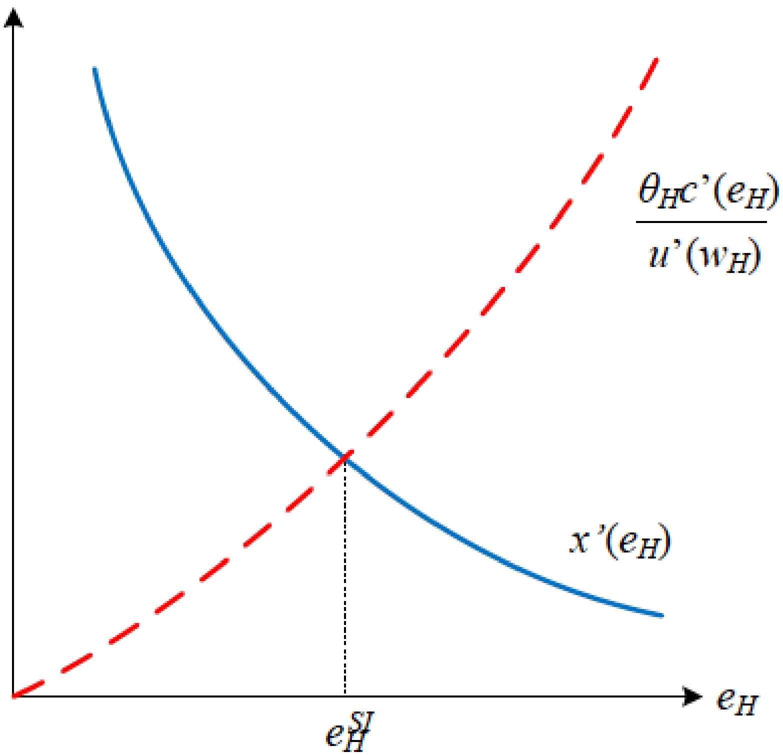
Asymmetric Information

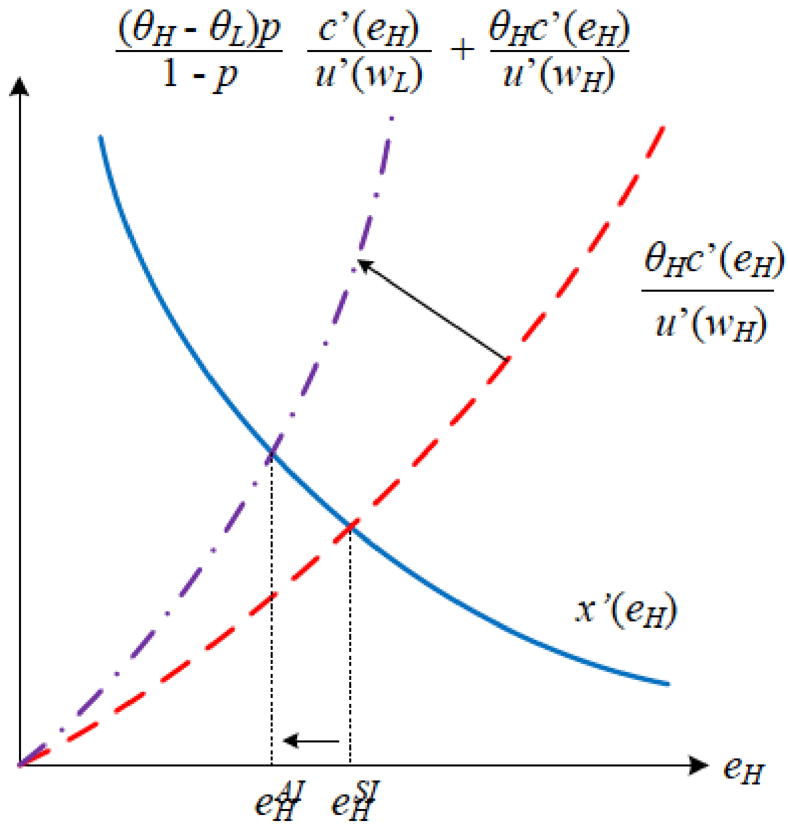
- Compare e_H^{AI} against the effort arising under symmetric information e_H^{SI}, $\frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$.
- Given $\theta_H - \theta_L > 0$, $p > 0$, $c'(e_H) > 0$ and $u'(w_L) > 0$,

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} > \frac{\theta_H c'(e_H)}{u'(w_H)}$$

- Hence the effort level under asymmetric information is **lower** than that under symmetric information, $e_H^{AI} < e_H^{SI}$.

Asymmetric Information





Asymmetric Information

- In summary, the pair of contracts (w_H, e_H) and (w_L, e_L) must satisfy the following equations

$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H)$$

$$u(w_H) - c(e_H, \theta_H) = 0$$

$$\frac{\theta_L c'(e_L)}{u'(w_L)} = x'(e_L)$$

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H)$$

Monotonicity in Effort

- Consider that effort levels satisfy $e_L \geq e_H$.
  - That is, the worker with the lowest cost of effort exerts a larger effort level than the worker with a high cost of effort.
- Combining (IC_L) and (IC_H) to obtain

$$u(w_L) - c(e_L, \theta_L) \geq u(w_H) - c(e_H, \theta_L) >$$
$$u(w_H) - c(e_H, \theta_H) \geq u(w_L) - c(e_L, \theta_H)$$

  - The first inequality stems from (IC_L).
  - The second inequality is due to $c(e_L, \theta_L) < c(e_H, \theta_H)$.
  - The third inequality is due to (IC_H).
- Hence, the above inequality can be rearranged as

$$c(e_H, \theta_L) - c(e_L, \theta_L) \geq u(w_H) - u(w_L)$$
$$> c(e_H, \theta_H) - c(e_L, \theta_H)$$

Monotonicity in Effort

- Multiplying this expression by -1, and using the first and last terms
$$c(e_L,\theta_L) - c(e_H,\theta_L) < c(e_L,\theta_H) - c(e_H,\theta_H)$$
- This condition indicates that the marginal cost of increasing effort from e_H to e_L is higher for the high-type than for the low-type worker.
- Evaluating this condition in the cost of effort function $c(e,\theta) = \theta c(e)$
$$\theta_L[c(e_L) - c(e_H)] < \theta_H[c(e_L) - c(e_H)]$$
- Since $\theta_L < \theta_H$, we must have $c(e_L) > c(e_H)$.
- Hence effort is larger for the worker with the low cost of effort, $e_L > e_H$

Monotonicity in Effort

- Example 7:
  - Let us use Example 6 to calculate the optimal contracts under asymmetric information.
  - Taking FOCs from above

$$u(w_L) - \theta_L c(e_L) = (\theta_H - \theta_L)c(e_H) \Rightarrow w_L - e_L^2 = e_H^2$$

$$u(w_H) - \theta_H c(e_H) = 0 \Rightarrow w_H = 2e_H^2$$

$$x'(e_L) = \frac{\theta_L c'(e_L)}{u'(w_L)} \Rightarrow \frac{1}{e_L} = \frac{2e_L}{1} \rightarrow 2e_L^2 = 1 \rightarrow e_L = \frac{1}{\sqrt{2}}$$

$$\frac{(\theta_H - \theta_L)p}{1-p}\frac{c'(e_H)}{u'(w_L)} + \frac{\theta_H c'(e_H)}{u'(w_H)} = x'(e_H) \Rightarrow \frac{2e_H}{1} + \frac{4e_H}{1}$$

$$= \frac{1}{e_H} \rightarrow e_H = \frac{1}{\sqrt{6}}$$

Monotonicity in Effort

- Example 7: (con't)
  - From the last two FOCs, we obtain the equilibrium effort levels $e_L = \frac{1}{\sqrt{2}}$ and $e_H = \frac{1}{\sqrt{6}}$.
  - From the first equation

$$w_L - \frac{1}{2} = \frac{1}{6} \rightarrow w_L = \frac{2}{3}$$

  - From the second equation

$$w_H = 2 \cdot \frac{1}{6} \rightarrow w_H = \frac{1}{3}$$

  - Therefore, the optimal pair of contracts is

$$\left(w_H^{AI}, e_H^{AI}\right) = \left(\frac{1}{3}, \frac{1}{\sqrt{6}}\right) = (0.333, 0.408)$$

$$\left(w_L^{AI}, e_L^{AI}\right) = \left(\frac{2}{3}, \frac{1}{\sqrt{2}}\right) = (0.667, 0.707)$$

Monotonicity in Effort

- Example 7: (con't)
  - The introduction of asymmetric information entails:
    - *No changes* in effort for the low-cost worker relative to symmetric information
$$e_L^{AI} = e_L^{SI} = 0.707$$
    - *Lower* effort for the high-cost worker than under symmetric information
$$e_H^{AI} = 0.408 < 0.5 = e_H^{SI}$$
    - *Higher* salaries for the low-cost worker than under symmetric information
$$w_L^{AI} = 0.667 > 0.5 = w_L^{SI}$$
    - *Lower* salaries for the high-cost worker
$$w_H^{AI} = 0.333 < 0.5 = w_H^{SI}$$

Monotonicity in Effort

- Example 7: (con't)
  - The net utility that each type of worker obtains under asymmetric information is
$$u_H^{AI} = w_H - 2e_H^2 = 0$$
$$u_L^{AI} = w_L - e_L^2 = 0.167$$
  - Hence the worker with a low cost of effort captures an information rent
$$u_L^{AI} - u_L^{SI} = 0.167 - 0 = 0.167$$
  - The worker with a high cost of effort does not
$$u_H^{AI} = u_H^{SI} = 0$$
  - Intuitively, the firm must compensate the low-cost worker above symmetric information terms in order for him to reveal his type.

Application of Adverse Selection—Regulation

Regulation

- Regulatory agencies often cannot observe some characteristics of the regulated firm or of individual.
- Examples:
  - A firm’s production costs
  - A firm’s costs from pollution abatement
  - A consumer’s willingness to pay for certain products
- In these scenarios the privately informed party (e.g., firm) has incentives to overstate its costs.
- Hence the regulator cannot directly ask firms about their production costs since responses would be unreliable.
- Adverse selection models offer an alternative contracting tool to extract information from privately informed firms (or consumers).

Regulation

- Consider that a government regulating a monopoly with cost function
$$c(q) = C + cq$$
where C is fixed costs and $c > 0$ is marginal costs.
- The consumer pays F for the bulk of q units consumed, and the monopolist may receive a lump-sum subsidy from the government of S.
- Assume that the shadow cost of raising public funds is $g \in (0,1)$, thus implying that the total cost of providing a subsidy S to the monopolist is $(1 + g)S$.
- Analyze settings where government has symmetric and asymmetric information about the monopolist's costs.

Regulation- Symmetric Information

- Consider that the government can **perfectly** observe the monopolist's marginal cost of production c.
- The government solves the following problem subject to PCs of both the monopolist and the consumer:

$$\max_{F,S,q} \; [u(q) - F] + [F + S - C - cq] - (1 + g)S$$

$$\text{s.t.} \quad F + S - C - cq \geq 0 \qquad (\text{PC}_{\text{Monop}})$$

$$u(q) - F \geq 0 \qquad (\text{PC}_{\text{Consum}})$$

where $u(q) - F$ is the consumer's utility after paying F for q units; and $F + S - C - cq$ is the monopolist's profits.

- The Lagrangian is

$$\mathcal{L} = [u(q) - F] + [F + S - C - cq] - (1 + g)S$$
$$+ \lambda_1[F + S - C - cq] + \lambda_2[u(q) - F]$$

Regulation- Symmetric Information

- Taking FOCs yields

$$\frac{\partial \mathcal{L}}{\partial F} = \lambda_1 - \lambda_2 = 0 \rightarrow \lambda_1 = \lambda_2$$

$$\frac{\partial \mathcal{L}}{\partial S} = 1 - (1 + g) + \lambda_1 = 0 \rightarrow \lambda_1 = g$$

$$\frac{\partial \mathcal{L}}{\partial q} = u'(q) - c - \lambda_1 c + \lambda_2 u'(q) = 0$$

$$\lambda_1[F + S - C - cq] = 0$$

$$\lambda_2[u(q) - F] = 0$$

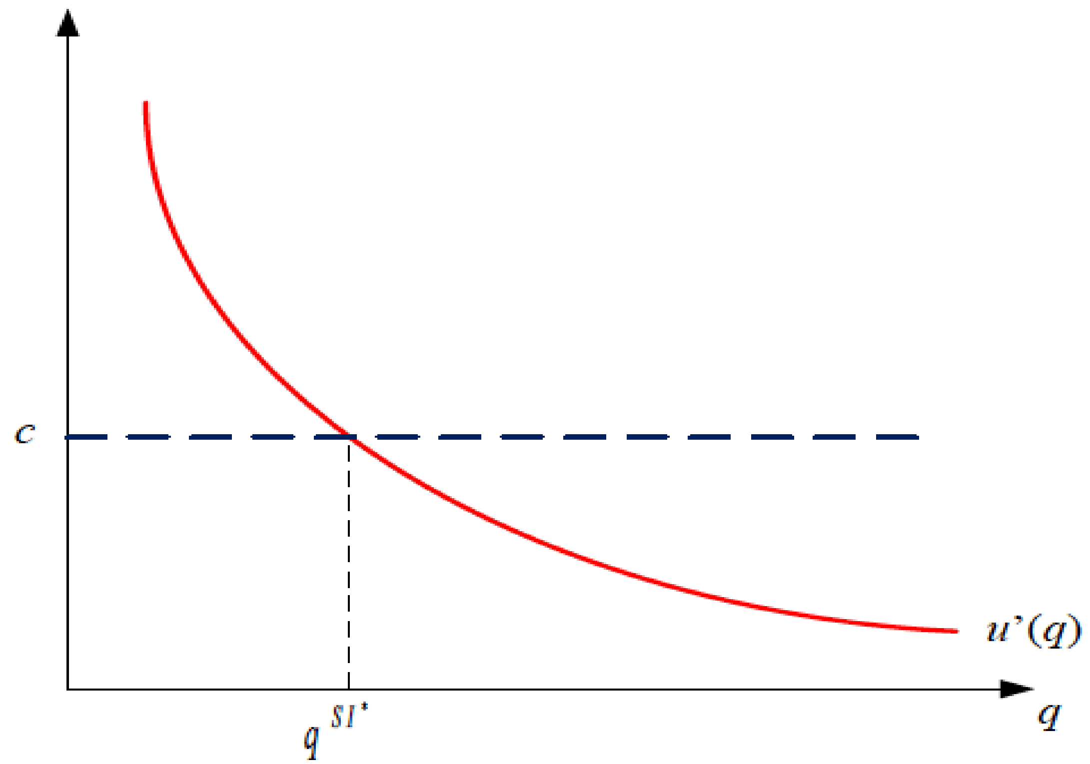
- Combining the first and second FOC

$$\lambda_1 = \lambda_2 = g$$

Regulation- Symmetric Information

- Plugging this result into the third FOC yields
$$u'(q) - c - gc + gu'(q) = 0$$
- Rearranging
$$(1+g)u'(q) = (1+g)c \Leftrightarrow u'(q) = c$$
- That is, q is increased until the point where marginal utility from further units coincides with its marginal cost.
- Hence, under symmetric information, the monopolist's production is **efficient**.

Regulation- Symmetric Information



Regulation- Asymmetric Information

- Consider now that the government **cannot** observe the monopolist's marginal cost of production c.
- Marginal cost can be low or high $c = \{c_L, c_H\}$, where $c_L < c_H$, with associated probabilities p and $1 - p$, respectively.
- The government offers two menus (F_L, S_L, q_L) and (F_H, S_H, q_H) to maximize the expected social welfare subject to PCs of both the monopolist and the consumer.

Regulation- Asymmetric Information

- The government's maximization problem is

$$\max_{(F_L,S_L,q_L),(F_H,S_H,q_H)} p[u(q_L) - F_L + F_L + S_L - C - c_L q_L - (1+g)S_L]$$
$$+ (1-p)[u(q_H) - F_H + F_H + S_H - C - c_H q_H - (1+g)S_H]$$

s.t.

$$F_L + S_L - C - c_L q_L \geq 0 \quad (\text{PC}_{\text{Monop,L}})$$
$$F_H + S_H - C - c_H q_H \geq 0 \quad (\text{PC}_{\text{Monop,H}})$$
$$F_L + S_L - C - c_L q_L \geq F_H + S_H - C - c_L q_H \quad (\text{IC}_{\text{Monop,L}})$$
$$F_H + S_H - C - c_H q_H \geq F_L + S_L - C - c_H q_L \quad (\text{IC}_{\text{Monop,H}})$$
$$u(q_L) - F_L \geq 0 \quad (\text{PC}_{\text{Consum,L}})$$
$$u(q_H) - F_H \geq 0 \quad (\text{PC}_{\text{Consum,H}})$$

Regulation- Asymmetric Information

- Timeframe:
  - The government offers contracts
  - The monopolist chooses one of contracts, and then the K-type monopolist offers q_K units to the consumer at a lump-sum price of F_K where $K = \{L, H\}$.
  - The consumer can accept or reject the offer.
- *Practice:* Solve the problem on your own.
  - Output of the low type coincides with that under symmetric information, whereas, that of the high type is smaller.
  - However, the subsidy that the high-cost firm receives is lower than under symmetric information, while that of the low-cost firm is the same.

Regulation- Asymmetric Information

- Example 8:
  - Consider consumers with utility function $u(q) = \sqrt{q}$, a monopoly with cost function $c(q) = \frac{1}{4} + cq$, where marginal costs can be high $c_H = \frac{1}{8}$ or low $c_L = \frac{1}{16}$, with probability $p = \frac{1}{2}$.
  - The shadow cost of raising public funds is $g = \frac{1}{24}$.
  - Symmetric information entails an output level that solves
  $$\frac{1}{2\sqrt{q}} = c_K$$
  which yields $q_H^{SI} = 16$ and $q_L^{SI} = 64$.
  - Asymmetric information entails output levels
  $$q_L^{AI} = q_L^{SI} = 64$$
  $$q_H^{AI} \cong 15.38 < q_H^{SI} = 16$$