Time Inconsistent Preferences

Advanced Microeconomics II- Washington State University

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Time Inconsistent Preferences – Tadelis 8.3.4

- Previously, we assumed that players maximize their discounted sum of payoffs.
  - This is typically done using *exponential discounting,* where every future period is multiplied by δ^t.
- We can show that using exponential discounting, an individual's consumption choices are consistent across time periods.

Time Inconsistent Preferences

- Consider a player with $u(x) = \ln(x)$ who needs to allocate a fixed budget K across three periods.
- Assume that prices all equal 1.
- His maximization problem is

$$\max_{x_2, x_3} \ln\Bigg(\underbrace{K - x_2 - x_3}_{x_1}\Bigg) + \delta ln(x_2) + \delta^2 ln(x_3)$$

FOCs:

$$\frac{\partial v}{\partial x_2} = -\frac{1}{K - x_2 - x_3} + \frac{\delta}{x_2} = 0$$

$$\frac{\partial v}{\partial x_3} = -\frac{1}{K - x_2 - x_3} + \frac{\delta^2}{x_3} = 0$$

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- Solving these two equations, we find:

$$x_2 = \frac{\delta(K-x_3)}{1+\delta}, \text{ from the first FOC, and}$$

$$(1+\delta^2)x_3 = \delta^2(K-x_2) \text{ from the second FOC.}$$

- Plugging in x_2 into x_3:

$$\Rightarrow (1+\delta^2)x_3 = \delta^2\left(K - \underbrace{\frac{\delta(K-x_3)}{1+\delta}}_{x_2}\right)$$

- Rearranging:

$$\left(1 - \frac{\delta^3}{1+\delta} + \delta^2\right)x_3 = \left(\delta^2 - \frac{\delta^3}{1+\delta}\right)K$$

$$\Rightarrow \left(\frac{\delta^2(1+\delta)-\delta^3+1+\delta}{\delta^2(1+\delta)-\delta^3}\right)x_3 = K$$

- Solving for x_3^*, we get:

$$x_3^* = \left(\frac{\delta^2}{1+\delta+\delta^2}\right)K$$

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- Plugging x_3^* into $x_2 = \frac{\delta(K - x_3)}{1+\delta}$, we get:

$$x_2^* = \frac{\delta}{1+\delta}\left(K - \frac{\delta^2}{1+\delta+\delta^2}K\right) = \left(\frac{\delta}{1+\delta+\delta^2}\right)K$$

- Finally, plugging in x_2^* and x_3^* into $x_1 = K - x_2 - x_3$, we obtain:

$$x_1^* = K - \left(\frac{\delta^2+\delta}{1+\delta+\delta^2}\right)K = \frac{1}{1+\delta+\delta^2}K$$

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- In summary, we find:

$$x_1^* = \frac{1}{1+\delta+\delta^2}K$$

$$x_2^* = \frac{\delta}{1+\delta+\delta^2}K$$

$$x_3^* = \frac{\delta^2}{1+\delta+\delta^2}K$$

- These three choices are made at the beginning of the game, assuming the player has perfect commitment.
- But, what if the player made his choices sequentially?
  - Would he reach the same equilibrium?
  - Let’s try backward induction.

Time Inconsistent Preferences

- In period 2, the player solves

$$\max_{x_2} \ln(x_2) + \delta ln\left(\underbrace{K_2 - x_2}_{x_3}\right)$$

where he is left with a budget of K_2

$$K_2 = K - x_1^* = K - K\frac{1}{1+\delta+\delta^2} = K\frac{\delta+\delta^2}{1+\delta+\delta^2}$$

Solving, we find:

$$x_2^* = \frac{K_2}{1+\delta} = K\frac{\delta}{1+\delta+\delta^2}$$

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- Plugging $K_2 = K\frac{\delta+\delta^2}{1+\delta+\delta^2}$ into $x_2^* = \frac{K_2}{1+\delta}$, yields:

$$x_2^* = K\frac{\delta}{1+\delta+\delta^2}$$

which is identical to the player's original choice of x_2 under commitment.

Time Inconsistent Preferences

- Thus, regardless of his previous actions, the player will choose the same consumption amounts sequentially as he will simultaneously.
  - Technically, we say that this player has *time consistent preferences*.
  - This is a useful property of exponential discounting.

Time Inconsistent Preferences

- What about other types of discounting, namely hyperbolic discounting?
  - In this case, a player uses the discount rate δ as seen in exponential discounting, but he uses an additional discount factor $\beta \in (0,1)$ to discount all future consumption.
  - Our previous example would look as follows under hyperbolic discounting:

$$\max_{x_2, x_3} \ \ln\left(\underbrace{K - x_2 - x_3}_{x_1}\right) + \beta\delta \ln(x_2) + \beta\delta^2 \ln(x_3)$$

- Intuitively, if the player looks toward future payoffs:
  - The discount factor he uses between periods $t = 1$ and $t = 2$ is stronger than…
  - the one he uses between periods $t = 2$ and $t = 3$.

Time Inconsistent Preferences

- Hyperbolic discounting will cause problems with self-control:
  - A player will plan to do one thing but later choose to revise his plan.
  - Commitment problems.
  - Largely studied in Behavioral Economics.

Time Inconsistent Preferences

- Let $\delta = 1$ for simplicity. We know that a rational person using exponential discounting will equalize his consumption across the three periods, i.e., $x_i^* = \frac{K}{3}$, for every period $i = 1,2,3$.
- Now, let's look at the case where a player uses hyperbolic discounting, and for simplicity, we'll assume that $\beta = \frac{1}{2}$.
- Our simultaneous problem (with perfect commitment) becomes

$$\max_{x_2, x_3} \ \ln\left(\underbrace{K - x_2 - x_3}_{x_1}\right) + \frac{1}{2}\ln(x_2) + \frac{1}{2}\ln(x_3)$$

Time Inconsistent Preferences

- First-order conditions:

$$\frac{\partial v}{\partial x_3} = -\frac{1}{K - x_2 - x_3} + \frac{1}{2x_3} = 0$$

$$\Rightarrow \; 2x_3 = K - x_2 - x_3 \Rightarrow 3x_3 = K - x_2 \Rightarrow x_3 = \tfrac{1}{3}(K - x_2)$$

$$\frac{\partial v}{\partial x_2} = -\frac{1}{K - x_2 - x_3} + \frac{1}{2x_2} = 0$$

$\Rightarrow 2x_2 = K - x_2 - x_3$, plugging in $x_3 = \frac{1}{3}(K - x_2)$ from above:

$$\Rightarrow 3x_2 = K - \tfrac{1}{3}(K - x_2) \Rightarrow x_2^* = \tfrac{1}{4}K$$

Time Inconsistent Preferences

- Plugging in $x_2^* = \frac{1}{4}K$ into $x_3 = \frac{1}{3}(K - x_2)$, we get:

$$x_3^* = \frac{1}{3}\left(K - \frac{1}{4}K\right)$$

$$\Rightarrow x_3^* = \frac{1}{4}K$$

- Recall $x_1 = K - x_2 - x_3$, so:

$$x_1^* = K - \frac{1}{4}K - \frac{1}{4}K = \frac{1}{2}K$$

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- In summary:

$$x_1^* = \frac{1}{2}K$$

$$x_2^* = \frac{1}{4}K$$

$$x_3^* = \frac{1}{4}K$$

Time Inconsistent Preferences

- We now check if the player's sequential solution is the same (when the player has no commitment).
- We use backward induction. Our period 2 problem becomes:

$$\max_{x_2} \ \ln(x_2) + \frac{1}{2}\ln\left(\underbrace{K_2 - x_1 - x_2}_{x_3}\right)$$

Time Inconsistent Preferences

- First-order conditions:

$$\frac{\partial v}{\partial x_2} = \frac{1}{x_2} - \frac{1}{2(K - x_1 - x_2)} = 0$$

$$\Rightarrow x_2 = 2(K - x_1 - x_2)$$

$$\Rightarrow x_2(x_1) = \frac{2}{3}(K - x_1)$$

Plugging $x_3 = K - x_1 - x_2$ into $x_2(x_1) = \frac{2}{3}(K - x_1)$, we obtain that

$$\Rightarrow x_3(x_1) = \frac{1}{3}(K - x_1)$$

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In summary, the best-response functions are:

$$x_2(x_1) = \frac{2}{3}(K - x_1)$$

$$x_3(x_1) = \frac{1}{3}(K - x_1)$$

Time Inconsistent Preferences

- We can substitute these best-response functions back into our period 1 problem to obtain:

$$\max_{x_1} \ \ln(x_1) + \frac{1}{2}\ln\left(\frac{2}{3}(K - x_1)\right) + \frac{1}{2}\ln\left(\frac{1}{3}(K - x_1)\right)$$

- First-order condition:

$$\frac{\partial v}{\partial x_1} = \frac{1}{x_1} - \frac{1}{2}\left[\frac{2}{3} \cdot \frac{3}{2(K - x_1)}\right] - \frac{1}{2}\left[\frac{1}{3} \cdot \frac{3}{(K - x_1)}\right] = 0$$

$$\Rightarrow \frac{1}{x_1} - \frac{1}{2(K - x_1)} - \frac{1}{2(K - x_1)} = 0$$

$$\Rightarrow \frac{1}{x_1} = \frac{1}{K - x_1} \Rightarrow \ x_1^* = \frac{1}{2}K$$

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- Plugging in $x_1^* = \frac{1}{2}K$ into the best response functions above, we get:

$$x_2^* = \frac{2}{3}\left(K - \frac{1}{2}K\right) = \frac{1}{3}K$$

$$x_3^* = \frac{1}{3}\left(K - \frac{1}{2}K\right) = \frac{1}{6}K$$

Time Inconsistent Preferences

- In summary,

$$x_1^* = \frac{1}{2}K$$

$$x_2^* = \frac{1}{3}K$$

$$x_3^* = \frac{1}{6}K$$

- *Note*: This differs from Tadelis' solution. He has an error in his FOCs. (This is why we check them!)

Time Inconsistent Preferences

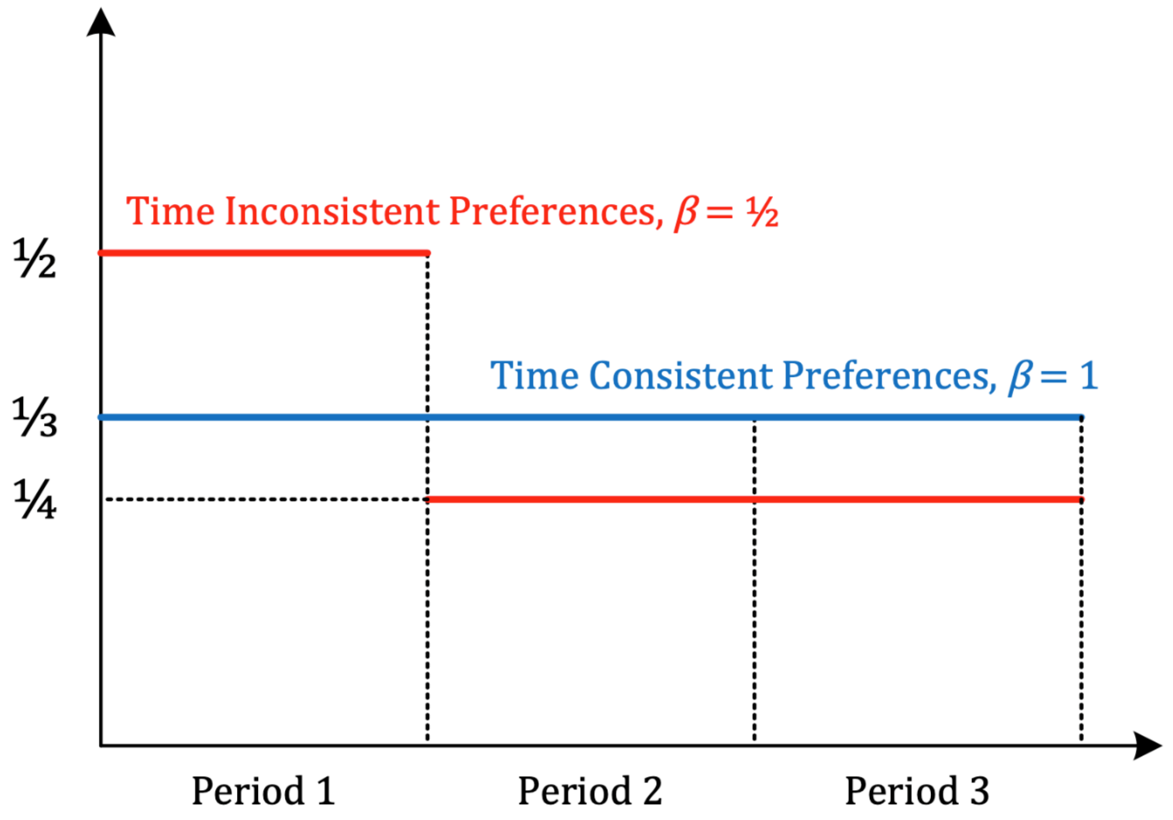
- This results in a very different solution than our simultaneous benchmark (perfect commitment), where we found

$$x_1^* = \frac{1}{2}K;\ x_2^* = \frac{1}{4}K;\ \text{and}\ x_3^* = \frac{1}{4}K.$$

- In this situation, the player consumes:
  - the same amount as he would normally in period 1, but…
  - in period 2, he become impatient once again, and “overconsumes” relative to what he planned to consume in period 1.
- We refer to this as the player having *time inconsistent preferences*.

Time Inconsistent Preferences

- Comparing time consistent with time inconsistent preferences in the simultaneous game ($\delta = 1$):



Time Inconsistent Preferences

- Comparing time consistent with time inconsistent preferences in the sequential game ($\delta = 1$):