# Application of adverse selection: *Menu pricing by a monopolist*

**EconS 503 – Advanced Microeconomics II**
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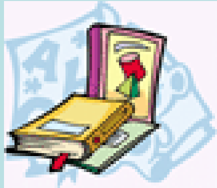
# Menu vs. group pricing

- Group (and personalized) pricing
  - Seller can infer consumers' willingness to pay from observable and verifiable characteristic (e.g., age)
- Menu pricing
  - Willingness to pay = private information
  - Seller must bring consumer to reveal this information.
  - How?
    - Identify product dimension valued differently by consumers
    - Design several versions of the product along that dimension
    - Price versions to induce consumers' self-selection
      → **Menu pricing** (a.k.a. versioning, 2nd-degree price discrimination, nonlinear pricing)
      → *Screening problem*: uninformed party brings informed parties to reveal their private information

# Case. Menu pricing in the information economy

- Versioning based on quality
  - 'Nagware': software distributed freely but displaying ads or screen encouraging users to buy full version
    → annoyance = discriminating device



- Versioning based on time
  - Books: first in hardcover, later in paperback
  - Movies: first in theaters, next on DVD, finally on TV.
    → price decreases as delay increases

- Versioning based on quantity
  - Software site licenses
  - Newspaper subscription
    → quantity discounts

# Case. **Geographical pricing by LCCs**

- Low Cost Carriers have abandoned many of the price discrimination tactics of the airline industry
  - ‘Point-to-point’ tickets, ‘no-frills’ flights
- But, geographical price discrimination on their website (Bachis and Piga, 2006)
  - Example: London-Madrid flight
    - 1st leg for British traveller, fare offered in £
    - Return leg for Spanish traveller, fare offered in €
  - If booking occurs at same time and no price discrimination, then ratio of prices = exchange rate
  - Yet, difference of at least 7£ for 450 000 observations
  - Despite possibility of arbitrage.

# Monopoly menu pricing

- Quality-dependent prices
  - Consumer's indirect utility when buying one unit of quality $s \geq 0$ at price $p \geq 0$ :

$$\vartheta = \begin{cases} U(\theta, s) - p, & \text{if consumer buys one unit} \\ 0 & \text{, if consumer does not buy} \end{cases}$$

  - $U$ increases in $s$ and in $\theta$ (taste parameter)

# Monopoly menu pricing (cont'd)

- Suppose: 2 types of consumers
  - 'Low type', $1-\lambda$ in proportion, with taste parameter $\theta_1$
  - 'High type', $\lambda$ in proportion, with taste parameter $\theta_2 > \theta_1$
    - care more about quality than low types:
      $$U(\theta_2, s) > U(\theta_1, s)$$
    - High types value more any *increase* in quality than low types (single-crossing property): for any $s_2 > s_1$,
      $$U(\theta_2, s_2) - U(\theta_2, s_1) > U(\theta_1, s_2) - U(\theta_1, s_1)$$
- Monopolist can produce $s_1$ and $s_2$ at constant marginal costs $c_1$ and $c_2$.

# Monopoly menu pricing (cont'd)

- Monopolist can produce two **exogenously** given qualities:
  - $s_1$ and $s_2$, at constant marginal costs $c_1$ and $c_2$ (with $c_1 < U(\theta_1, s_i)$) .

- The question is whether the monopolist will choose to:
  a. Price-discriminate by offering the two qualities priced appropriately, or
  b. Prefer to offer a single quality

- In option (b), assume that the monopolist always prefers to offer the high quality $s_{2.}$ A sufficient condition for this is:

$$U(\theta_1, s_2) - U(\theta_1, s_1) > c_2 - c_1$$

That is, the value low-type consumers attribute to an increase in quality is larger than the cost difference between the two qualities.

# Monopoly menu pricing (cont'd)

- Then, the monopolist has two options:

1. Charges the high price equal to $U(\theta_2, s_2)$ and sells to high-type consumers only, or
2. Lowers the price to $U(\theta_1, s_2)$ and sells to all consumers.

# Monopoly menu pricing (cont'd)

- If firm sells to only high-type consumer

$$p = U(\theta_2, s_2) \rightarrow [U(\theta_2, s_2) - c_2]\lambda$$

- If firm sells to everyone

$$p = U(\theta_1, s_2) \rightarrow [U(\theta_1, s_2) - c_2][\lambda + (1 - \lambda)]$$
$$\rightarrow [U(\theta_1, s_2) - c_2]$$

- Therefore, firm sells to high-type consumer only if

$$[U(\theta_2, s_2) - c_2]\lambda > [U(\theta_1, s_2) - c_2]$$

$$\Rightarrow \lambda > \frac{U(\theta_1, s_2) - c_2}{U(\theta_2, s_2) - c_2} \equiv \lambda_0$$

- Profit from selling only the high quality

$$\Pi_s = \begin{cases} [U(\theta_2, s_2) - c_2]\lambda, & if\ \lambda \geq \lambda_0 \\ U(\theta_1, s_2) - c_2, & if\ \lambda < \lambda_0 \end{cases}$$

# Monopoly menu pricing (cont'd)

- Under menu pricing, the monopolist must find the profit-maximizing price pair $(p_1, p_2)$ that induces type-$i$ consumers to select quality $s_i$.
- There are two concerns:

  i. Participation – each consumer must do at least well as well consuming the good as not consuming

  ii. Self-selection – or incentive compatibility, each type of consumer must prefer their consumption to the consumption of the other type of consumer

# Monopoly menu pricing (cont'd)

- Under menu pricing

$$\max_{p_1, p_2} \quad \Pi_m = \lambda(p_2 - c_2) + (1-\lambda)(p_1 - c_1)$$

$$U(\theta_1, s_1) \geq p_1 \qquad (PC_1)$$

$$U(\theta_2, s_2) \geq p_2 \qquad (PC_2)$$

$$U(\theta_1, s_1) - p_1 \geq U(\theta_1, s_2) - p_2$$

$$\Rightarrow p_1 \leq p_2 - [U(\theta_1, s_2) - U(\theta_1, s_1)] \qquad (IC_1)$$

$$U(\theta_2, s_2) - p_2 \geq U(\theta_2, s_1) - p_1$$

$$\Rightarrow p_2 \leq p_1 + [U(\theta_2, s_2) - U(\theta_2, s_1)] \qquad (IC_2)$$

# Monopoly menu pricing (cont'd)

- Of course, the monopolist wants to choose $p_1$ and $p_2$ to be as large as possible.
- It follows, in general, that one of the first two inequalities, and one of the second two inequalities will be binding.
- Intuitively, we can guess that what matters is participation of low-type, and self-selection of the high-type.
- We, thus, expect that $PC_1$ and $IC_2$ to bind.

# Monopoly menu pricing (cont'd)

To show that $PC_1$ and $IC_2$ are binding:

- Suppose that by contradiction, $PC_2$ is binding, i.e. $p_2 = U(\theta_2, s_2)$, then $IC_2$ implies that:

$$p_2 \leq p_1 + p_2 - U(\theta_2, s_1)$$

$$\Rightarrow U(\theta_2, s_1) \leq p_1$$

# Monopoly menu pricing (cont'd)

To show that $PC_1$ and $IC_2$ are binding:

- Using the assumption that high types care more about quality, we can write

$$U(\theta_1, s_1) < U(\theta_2, s_1) \leq p_1$$

- which contradicts $PC_1$.
- It follows that $PC_2$is not binding, and that $IC_2$ is binding. That is:

$$p_1^* = U(\theta_1, s_1)$$
$$\Rightarrow p_2^* = U(\theta_1, s_1) + [U(\theta_2, s_2) - U(\theta_2, s_1)]$$
$$= p_1^* + [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

# Monopoly menu pricing (cont'd)

- Now, consider $PC_1$ and $IC_1$.
- If $IC_1$ were binding, we would have

$$p_1 = p_2 - [U(\theta_1, s_2) - U(\theta_1, s_1)]$$

- Using the binding $IC_2$, the latter equality can be rewritten as:

$$p_2^* = p_1^* + [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

$$\Rightarrow p_1 + [U(\theta_1, s_2) - U(\theta_1, s_1)] = p_1 + [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

$$\Rightarrow p_1 = p_1 + [U(\theta_2, s_2) - U(\theta_2, s_1)] - [U(\theta_1, s_2) - U(\theta_1, s_1)]$$

$$\Rightarrow [U(\theta_1, s_2) - U(\theta_1, s_1)] = [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

# Monopoly menu pricing (cont'd)

$$\Rightarrow [U(\theta_1, s_2) - U(\theta_1, s_1)] = [U(\theta_2, s_2) - U(\theta_2, s_1)]$$

- As menu pricing supposes, $s_2 > s_1$, this contradicts out initial assumption (single-crossing property).
- It follows that $IC_1$is not binding, and that $PC_1$ is binding so:

$$p_1^* = U(\theta_1, s_1), \text{ and}$$
$$p_2^* = U(\theta_2, s_2) - [U(\theta_2, s_1) - U(\theta_1, s_1)]$$

- Because $U(\theta_2, s_1) > U(\theta_1, s_1)$, we observe that
  - $p_2^* < U(\theta_2, s_2)$: the monopolist is not able to fully extract full surplus from high-type consumers.

# Monopoly menu pricing (cont'd)

When is menu pricing optimal?

- We need now to compare profits when:
  - the monopolist only sells the high quality, and
  - when the monopolist price discriminates by selling both qualities.

- In the latter case, profits are given by:

$$\Pi_m = \lambda[U(\theta_2, s_2) - [U(\theta_2, s_1) - U(\theta_1, s_1)] - c_2] + (1-\lambda)(U(\theta_1, s_1) - c_1)$$

- Consider the first case where the proportion of high-type consumers is large enough, so that the monopolist sells to them only when it produces a single quality $(\lambda \geq \lambda_0)$.

# Monopoly menu pricing (cont'd)

- Then menu pricing modifies profits as follows

$$\Delta\Pi = \Pi_m - \Pi_s$$

- If $\lambda \geq \lambda_0$

$$\begin{aligned}\Delta\Pi &= \Pi_m - [U(\theta_2, s_2) - c_2]\lambda \\ &= (1-\lambda)[U(\theta_1, s_1) - c_1] - \lambda[U(\theta_2, s_1) - U(\theta_1, s_1)]\end{aligned}$$

- Menu pricing involves two opposite effects.
  1. It increases profits through ***market expansion***: low-type consumers now buy the low quality, which yields a margin of $U(\theta_1, s_1) - c_1$ per consumer.
  2. It decreases profits because of ***cannibalization:*** high-type consumers still buy the high quality but now price reduced by $U(\theta_2, s_1) - U(\theta_1, s_1)$.

- The net effects is positive provided that high-type consumers are not too numerous:

$$\Delta\Pi > 0 \Leftrightarrow \lambda < \frac{U(\theta_1, s_1) - c_1}{U(\theta_2, s_1) - c_1} \equiv \bar{\lambda}$$

# Monopoly menu pricing (cont'd)

- The latter condition is compatible with our starting point iff $\bar{\lambda} > \lambda_0$, which is equivalent to

$$\Rightarrow \frac{U(\theta_1, s_1) - c_1}{U(\theta_2, s_1) - c_1} > \frac{U(\theta_1, s_2) - c_2}{U(\theta_2, s_2) - c_2}$$

$$\Rightarrow \frac{U(\theta_2, s_2) - c_2}{U(\theta_2, s_1) - c_1} > \frac{U(\theta_1, s_2) - c_2}{U(\theta_1, s_1) - c_1}$$

(We interpret this ratio ranking later.)

# Monopoly menu pricing (cont'd)

- Second case: If $\lambda < \lambda_0$
- Here, the monopolist sells the high quality at a low price to everyone if he decides to sell only one quality.
  - The change in profits induced by menu pricing is then given by:

$$\Delta\Pi = \Pi_m - U(\theta_1, s_2) - c_2$$
$$= \lambda\big[\big(U(\theta_2, s_2) - U(\theta_2, s_1)\big) - \big(U(\theta_1, s_2) - U(\theta_1, s_1)\big)\big]$$
$$+(1-\lambda)[(U(\theta_1, s_1) - c_1) - (U(\theta_1, s_2) - c_2)]$$

- Two opposite effects:
  1. Profit from low-type consumers decreases (because they buy low quality instead of high-quality, which is detrimental for the monopolist by assumption)
  2. Profit from high-type consumers increases (they continue to buy the high quality but now pay a higher price by assumption.

# Monopoly menu pricing (cont'd)

- Overall, profits increase if high-type agents are numerous enough:

$$\Delta\Pi > 0 \Leftrightarrow \lambda > \frac{U(\theta_1, s_2) - U(\theta_1, s_1) - (c_2 - c_1)}{U(\theta_2, s_2) - U(\theta_2, s_1) - (c_2 - c_1)} \equiv \underline{\lambda}$$

- For this condition to be compatible with our starting assumption, we need that $\underline{\lambda} < \lambda_0$,


- or

$$\frac{U(\theta_2, s_2) - c_2}{U(\theta_2, s_1) - c_1} > \frac{U(\theta_1, s_2) - c_2}{U(\theta_1, s_1) - c_1}$$

which is the exact same condition as in the previous case.

# Monopoly menu pricing (cont'd)

- Condition

$$\frac{U(\theta_2, s_2) - c_2}{U(\theta_2, s_1) - c_1} > \frac{U(\theta_1, s_2) - c_2}{U(\theta_1, s_1) - c_1}$$

- This condition says that going from low to high quality increases surplus *proportionally more* for high-type consumers than for low-type consumers.

# Monopoly menu pricing (cont'd)

- In summary, menu pricing is optimal if:

a) The proportion of high-type consumers, $\lambda$, is neither too small nor too large.

b) Going from low to high quality increases surplus *proportionally more* for high-type consumers than for low-type consumers.

# Monopoly menu pricing: summary

- **Lesson**: Consider a monopolist who offers 2 pairs of price and quality to 2 types of consumers.
- Prices are chosen so as to fully appropriate low-type's consumer surplus.
- High-type consumers obtain a positive surplus ('information rent') as they can always choose the low-quality instead.

# Monopoly menu pricing (cont'd)

- All previous analysis assumes qualities $s_1$ and $s_2$ are given.
  - Therefore, the monopolist only chooses prices $p_1$ and $p_2$.

- Now, we will allow the monopolist to choose these two quality levels too (four choice variables).

# Monopoly menu pricing (cont'd)

## Distortion of quality

- In the previous analysis, we assumed that qualities were given and that the only task left to the monopolist was to choose prices.
- Suppose, now, that the monopolist can also choose which quality to offer.
- If menu pricing is the optimal conduct, then the monopolist will choose to offer two different qualities.
  - But which qualities exactly?
- To answer this, we modify the previous model slightly as follows:

# Monopoly menu pricing (cont'd)

## Distortion of quality

- Let $c(s)$ denote the monopolist's cost per unit of output of producing quality $s$.
- Assume that $c'(s) > 0$ and $c''(s) > 0$: it is more expensive and increasingly more expensive to produce higher quality.

$$\max_{s_1, s_2} \Pi_m = \lambda[U(\theta_2, s_2) - [U(\theta_2, s_1) - U(\theta_1, s_1)] - c(s_2)] + (1-\lambda)(U(\theta_1, s_1) - c(s_1))$$

- $FOC_{s_1}$, $\frac{\partial \Pi_m}{\partial s_1} = 0$, which yields

$$\Rightarrow c'(s_1) = \frac{\partial U(\theta_1, s_1)}{\partial s_1} - \frac{\lambda}{1-\lambda}\left(\frac{\partial U(\theta_2, s_1)}{\partial s_1} - \frac{\partial U(\theta_1, s_1)}{\partial s_1}\right)$$

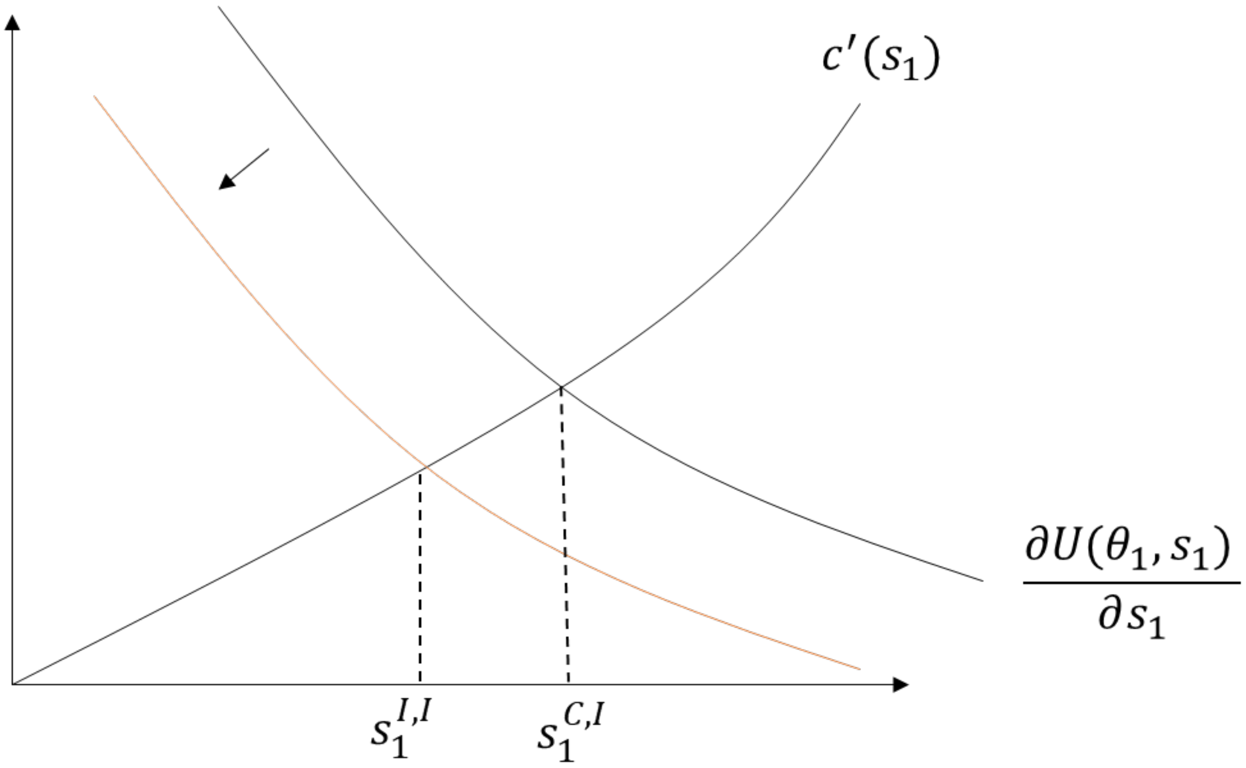
- $FOC_{s_2}$, $\frac{\partial \Pi_m}{\partial s_2} = 0$, which yields

$$\Rightarrow c'(s_2) = \frac{\partial U(\theta_2, s_2)}{\partial s_2}$$

No distortion relative to complete information

# Monopoly menu pricing (cont'd)

- **Distortion of quality** (cont'd)



# Monopoly menu pricing (cont'd)

## Welfare Effects

- We now seek to measure welfare effects in the base model, were qualities $s_1$ and $s_2$ are given.
  - Recall that their costs are $c_1$ and $c_2$, respectively.
- Social welfare is the sum of consumer surplus and monopolist's profit:
  - Case: No menu pricing

If $\lambda \geq \lambda_0$: $\pi_s = \lambda(U(\theta_2, s_2) - c_2)$, and

$$CS_S = 0 \text{ because } p^* = U(\theta_2, s_2)$$

$$\Rightarrow W_s \equiv \pi_s + CS_S = \lambda(U(\theta_2, s_2) - c_2)$$

If $\lambda < \lambda_0$: $\pi_s = (U(\theta_1, s_2) - c_2)$ &

$$CS_S = \lambda \left[U(\theta_2, s_2) - \underbrace{U(\theta_1, s_2)}_{\boldsymbol{p^*}}\right] + (1-\lambda)\left[U(\theta_1, s_2) - \underbrace{U(\theta_1, s_2)}_{\boldsymbol{p^*}}\right]$$

$$= \lambda[U(\theta_2, s_2) - U(\theta_1, s_2)]$$

$$\Rightarrow W_s \equiv \pi_s + CS_S = U(\theta_1, s_2) - c_2 + \lambda[U(\theta_2, s_2) - U(\theta_1, s_2)]$$

# Monopoly menu pricing (cont'd)

## Welfare Effects

- Case: With menu pricing

$$\pi_m = (1-\lambda)[p_1 - c_1] + \lambda[p_2 - c_2]$$
$$= (1-\lambda)[U(\theta_1, s_1) - c_1] + \lambda[U(\theta_2, s_2) - U(\theta_2, s_1) + U(\theta_1, s_1) - c_2]$$

$$CS_m = (1-\lambda)[U(\theta_1, s_1) - p_1] + \lambda[U(\theta_2, s_2) - p_2]$$

$$= (1-\lambda)\Big[\underbrace{U(\theta_1, s_1) - \underbrace{U(\theta_1, s_1)}_{p_1^*}}_{=0}\Big] + \lambda\Big[U(\theta_2, s_2) - \underbrace{U(\theta_2, s_2) + U(\theta_2, s_1) - U(\theta_1, s_1)}_{p_2^*}\Big]$$

Which simplifies to

$$CS_m = \lambda[U(\theta_2, s_1) - U(\theta_1, s_1)]$$

# Monopoly menu pricing (cont'd)

## Welfare Effects

- Case: With menu pricing

Therefore, total welfare is:

$$\Rightarrow W_m \equiv \pi_m + CS_m$$
$$= (1-\lambda)[U(\theta_1, s_1) - c_1] + \lambda[U(\theta_2, s_2) - U(\theta_2, s_1) + U(\theta_1, s_1) - c_2]$$
$$+ \lambda[U(\theta_2, s_1) - U(\theta_1, s_1)]$$

$$= (1-\lambda)[U(\theta_1, s_1) - c_1] + \lambda[U(\theta_2, s_2) - c_2]$$

# Monopoly menu pricing (cont'd)

## Welfare Effects

- In summary, the monopolist's profit:

$$W_s = \begin{cases} \lambda(U(\theta_2, s_2) - c_2) \; if \; \lambda \geq \lambda_0 \\ (U(\theta_1, s_2) - c_2) + \lambda[U(\theta_2, s_2) - U(\theta_1, s_2)] \; if \; \lambda < \lambda_0 \end{cases}$$

$$W_m = (1 - \lambda) \; (U(\theta_1, s_1) - c_1) + \lambda \; (U(\theta_2, s_2) - c_2)$$

- The change in welfare induced by menu pricing is, then:

$$\Delta W = W_m - W_s = \begin{cases} (1 - \lambda)(U(\theta_1, s_1) - c_1) > 0 \quad if \; \lambda \geq \lambda_0 \\ -(1 - \lambda)[U(\theta_1, s_2) - U(\theta_1, s_1) - (c_2 - c_1)] < 0 \quad if \; \lambda < \lambda_0 \end{cases}$$

# Monopoly menu pricing (cont'd)

## Welfare Effects

- We observe that welfare increases when $\lambda \geq \lambda_0$.
  - In that case, menu pricing *expands* the market:
    - low-type consumers are sold the low quality,
    - they are left out of the market when only the high quality is sold.
- In contrast, welfare decreases when $\lambda < \lambda_0$.
  - Here, the monopolist chooses to cover the whole market when it sells only the high quality;
  - Under menu pricing, low-type consumers are sold the low quality instead of the high one, although what they are willing to pay for high quality is larger than the extra cost of producing higher quality;
    - gains of trade are thus left unexploited, and welfare is lower.

# Monopoly menu pricing (cont'd)

## First application: Linear utility

- $U(\theta, s) = \theta s \;\rightarrow U = \theta s - p$
- The condition for menu pricing to be profitable simplifies to:

$$\frac{\theta_2 s_2 - c_2}{\theta_2 s_1 - c_1} > \frac{\theta_1 s_2 - c_2}{\theta_1 s_1 - c_1}$$

$$\Rightarrow \frac{c_2}{s_2} > \frac{c_1}{s_1}$$

Average cost of quality of high quality

Average cost of quality of low quality

# Monopoly menu pricing (cont'd)

## First application: Linear utility

- In information goods, the marginal cost of production is often unaffected by product quality, meaning that $c_1 = c_2 = c$.
- The above inequality further simplifies to

$$\frac{c_2}{s_2} > \frac{c_1}{s_2} \Rightarrow s_1 > s_2$$

But this is not true!

- This would imply that sellers of information goods wouldn't practice menu pricing.
- But they do! How to reconcile this model and the real-world observation?
  - Allowing for consumers' utility to not be linear in the product's quality, as we do next.

# Monopoly menu pricing (cont'd)

## First application: Linear utility

- Consumers' utility is not linear in product quality
- Assume, for example, $U(\theta, s) = \theta s + k$, where $k \geq 0$.
- Menu pricing is, then, profitable if

$$\frac{k + \theta_2 s_2 - c_2}{k + \theta_2 s_1 - c_1} > \frac{k + \theta_1 s_2 - c_2}{k + \theta_1 s_1 - c_1}$$

- Which, after solving for $k$, yields

$$k > \frac{c_1 s_2 - c_2 s_1}{s_2 - s_1}$$

- If $c_2 = c_1 = c$, this inequality simplifies to:

$$k > \frac{c s_2 - c s_1}{s_2 - s_1} = c$$

# Monopoly menu pricing (cont'd)

## Information goods

- If, in addition, $c_2 = c_1 = 0$, as in the case of most information goods, the above inequality further simplifies to $k > c = 0$.
- Menu pricing needs that the proportion of high-type consumers is intermediate ("goldie locks" condition):

$$\underline{\lambda} < \lambda_0 < \bar{\lambda}$$

Let's find the value of each cutoff in this setting (next slide).

# Monopoly menu pricing (cont'd)

## Information goods

$$\underline{\lambda} < \lambda_0 < \bar{\lambda}$$

where

$$\bar{\lambda} = \frac{k + \theta_1 s_1 - c_1}{k + \theta_2 s_1 - c_1} = \frac{k + \theta_1 s_1}{k + \theta_2 s_1}$$

$$\lambda = \frac{k + \theta_1 s_2 - c_2}{k + \theta_2 s_2 - c_2} = \frac{k + \theta_1 s_2}{k + \theta_2 s_2}$$

$$\underline{\lambda} = \frac{k + \theta_1 s_2 - c_2}{k + \theta_1 s_1 - c_1} = \frac{k + \theta_1 s_2}{k + \theta_1 s_1}$$

Therefore, menu pricing is profitable if:

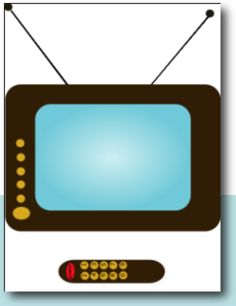
$$\Rightarrow \frac{\theta_1}{\theta_2} < \lambda < \frac{k + \theta_1 s_1}{k + \theta_2 s_1}$$

# Monopoly menu pricing (cont'd)

## Damaged goods

- Interestingly, condition $k > \frac{c_1 s_2 - c_2 s_1}{s_2 - s_1}$ can be satisfied even when $c_1 > c_2$.
- This means that the monopolist incurs an extra cost to produce the low-quality version of the same product.
  - Damaged goods strategy:
  - Firms intentionally damaging a portion of their goods to price discriminate.
  - Examples in software markets: full-featured vs. low-quality version.

# Monopoly menu pricing: further results



## Case. **Damaged goods**

- IBM LaserPrinter E → identical to original printer, but software limited printing to 5 rather than 10 pages/minute
- Sony MiniDisc 60’ → curbed 74’ disc
- Sharp DVD players → DVE611 and DV740U are almost identical, but DV740U does not allow user to play output encoded in PAL format on NTSC televisions (a critical button is hidden on the remote)

# Monopoly menu pricing: further results (cont'd)

- Previous quality model
  - Suppose linear utility: $U(\theta, s) = \theta s$
  - Cost of producing one unit of given quality: $c(s_i)$
- Transposition to time-dependent prices
  - Let $s = e^{-rt}$, where $t$ = date when the good is produced and delivered, and $r$ = interest rate
  - The monopolist, here, chooses two delivery/release dates.

$$\max_{t_1, t_2}(1-\lambda)\left[\theta_1 e^{-rt_1} - c(e^{-rt_1})\right] + \lambda\left[\theta_2 e^{-rt_2} - (\theta_2 - \theta_1)e^{-rt_1} - c(e^{-rt_2})\right]$$

# Monopoly menu pricing: further results (cont'd)

- Transposition to quantity-dependent prices
  - Unit price depends on quantity purchased (but not on the identity of the consumer).
  - This type of menu pricing is also known as *non-linear pricing (e.g., two-part tariffs* which are widely used by utilities*)*.
  - Consumer of type $i$ can buy a certain quantity $q_i$ at price $p_i$
  - Unit price may depend on quantity purchased (nonlinear pricing). Let $q_i = c(s_i)$ $\rightarrow s_i = c^{-1}(q_i) = V(q_i)$

$$\max_{q_1, q_2} (1-\lambda)\left[\theta_1 V(q_1) - q_1\right] + \lambda\left[\theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) - q_2\right]$$