Chapter 11: Equilibrium Refinements

Game Theory:

An Introduction with Step-by-Step Examples

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Introduction

- Chapter 10 highlights that signaling games and the PBE solution concept are an excellent tool to explain a wide array of economic situations
- However, we found that this class of games may yield:
  - A large number of PBEs
  - Some PBEs based on insensible off-the-equilibrium beliefs.
- In this chapter, we present two commonly used tools to “refine” set of PBEs
  - satisfying different consistency requirements in their off-the-equilibrium beliefs

Introduction

- Cho and Kreps’ Intuitive Criterion
- D1 Criterion
- Sequential equilibrium

Intuitive Criterion

- The Cho and Kreps' (1982) "Intuitive Criterion":
  - eliminates all PBEs that are sustained on insensible off-the-equilibrium beliefs,
  - such as the pooling PBE (NE^H, NE^L) in Figure 11.1.
- Recall that this PBE required an off-the-equilibrium belief $\mu \leq \frac{2}{5}$.
  - Informally, if the firm observes, surprisingly, a worker acquiring education, it believes the worker must likely be low productivity.
  - Crazy, right?

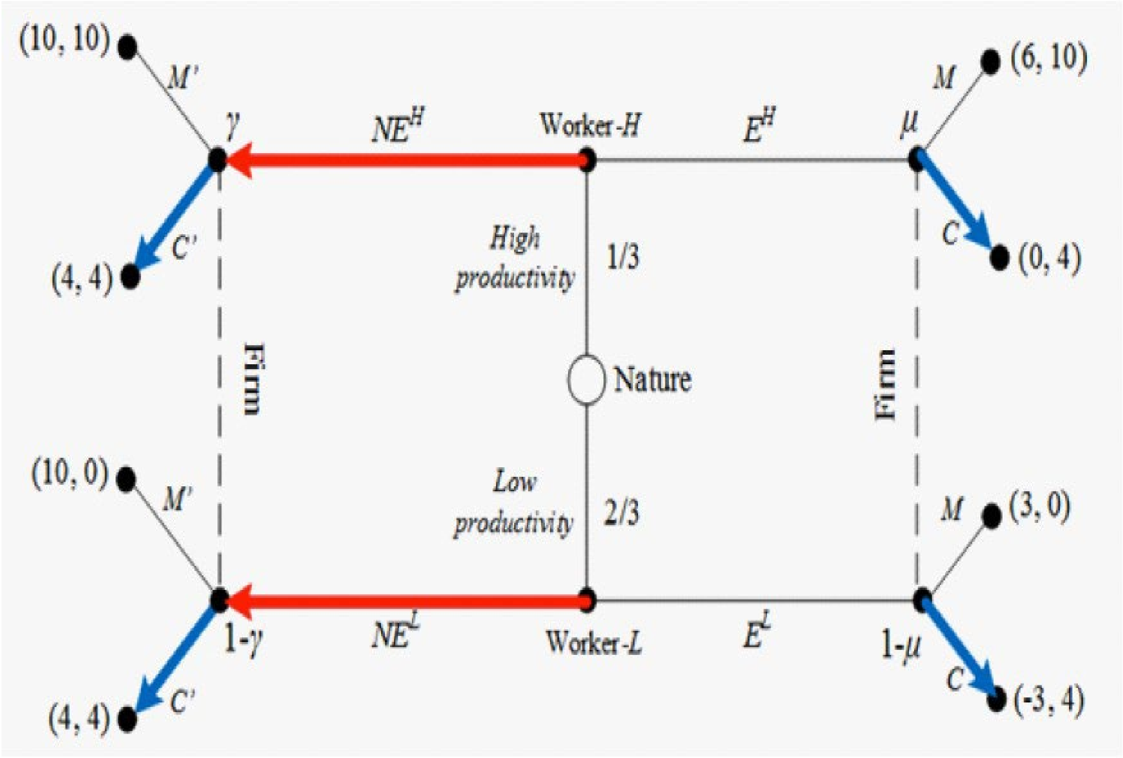


Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Intuitive Criterion

- Essentially, the Intuitive Criterion seeks to answer a relatively basic question:
  - "If the receiver observed an off-the-equilibrium message, such as education on the right side of Figure 11.1…
  - which sender types could benefit from sending such an off-the-equilibrium message?"

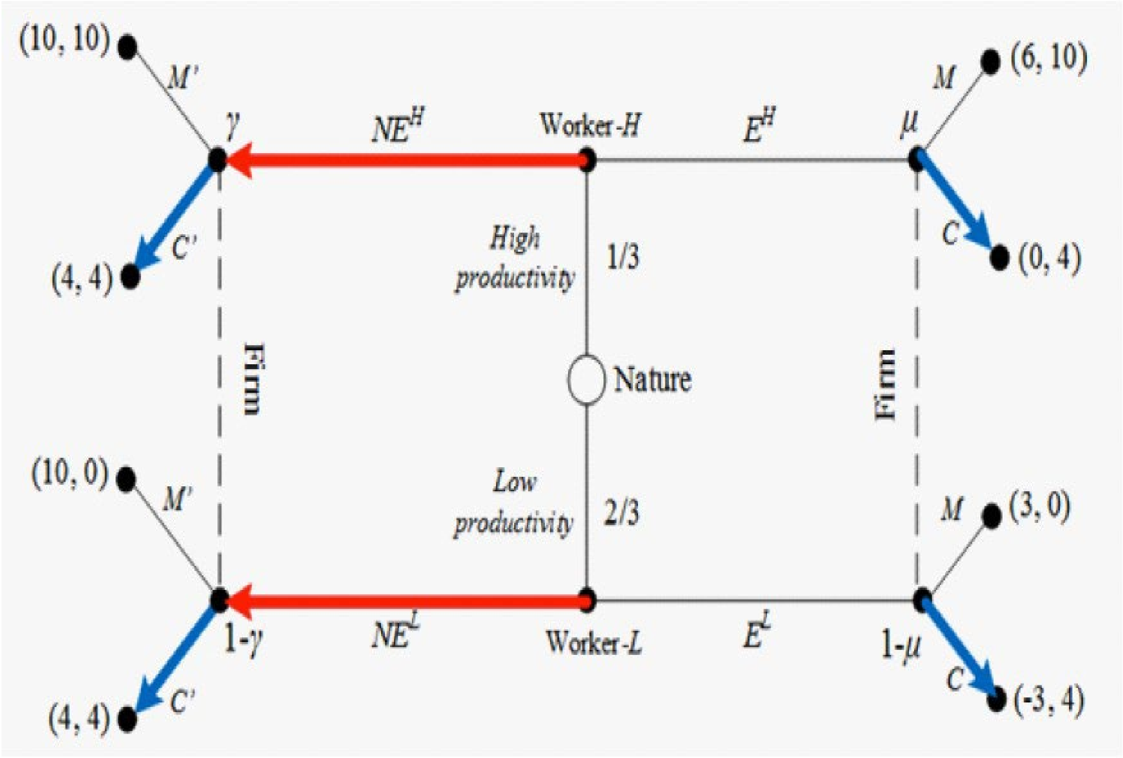


Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Intuitive Criterion

- If no sender types can benefit, then the initial PBE we considered *survives* the Intuitive Criterion.
- If some sender types can benefit, the receiver would then update her off-the-equilibrium beliefs and her response to this message.
- In turn, this updated response could induce some sender types to deviate from their equilibrium messages, implying that the initial PBE we considered *violates* the Intuitive Criterion.
- More details in Tool 11.1

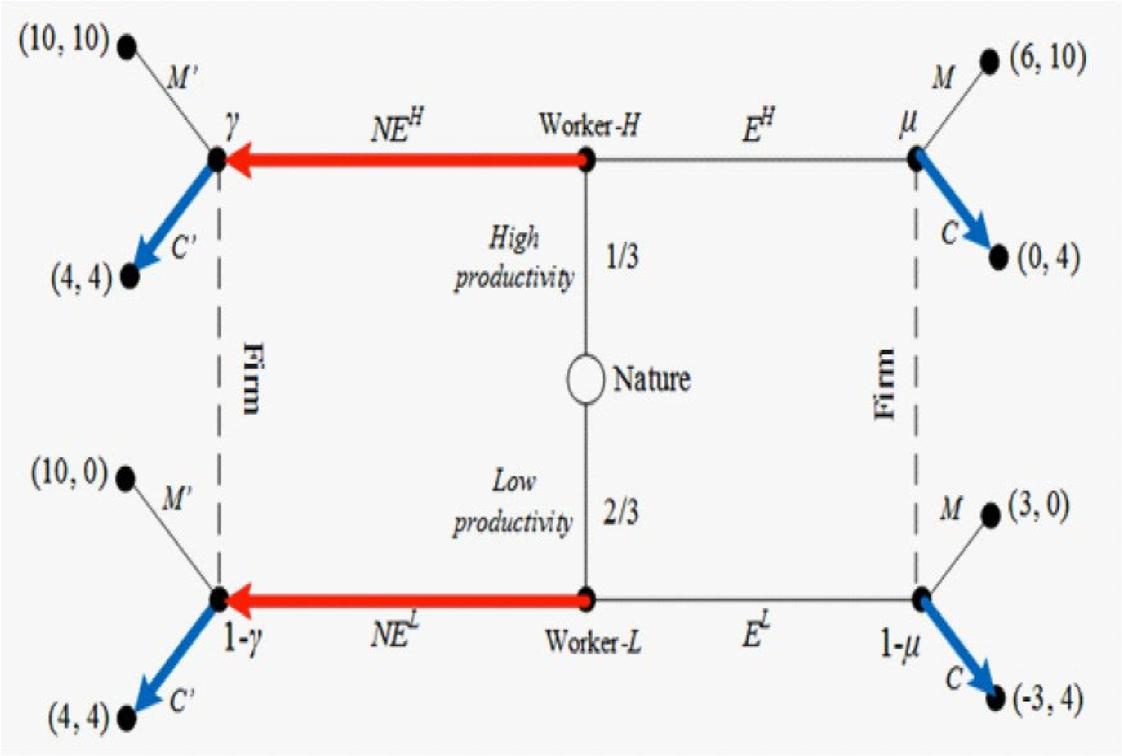
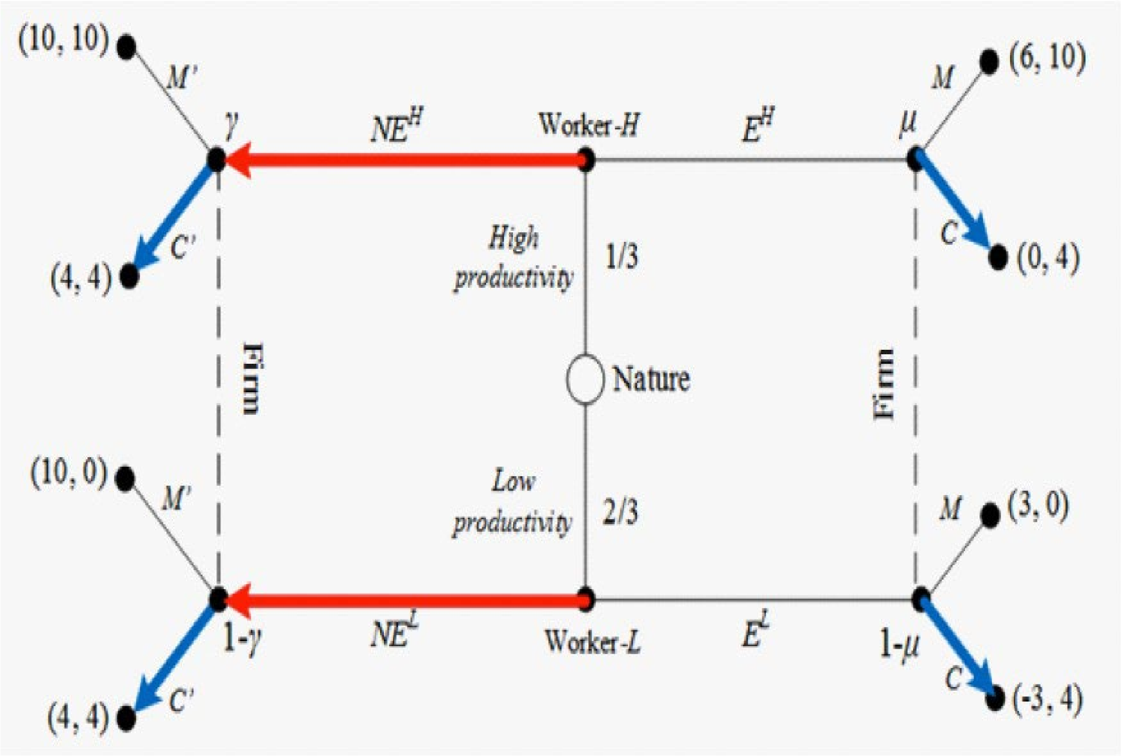


Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 1. Consider a specific PBE

- For instance, the pooling PBE (NE^H, NE^L).
- Recall that (NE^H, NE^L) could be supported as a PBE if the firm responds with (C, C') and its beliefs are:
  - $\gamma = \frac{1}{3}$ upon observing no education (in the equilibrium path), and
  - $\mu \leq \frac{2}{5}$ upon observing education (off-the-equilibrium path)



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Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 2. Identify an off-the-equilibrium message for the sender

- That is, find an action that no type of sender chooses in the PBE we are analyzing,
- e.g., education in the pooling PBE (NE^H, NE^L)

- Think about games with two types but *k* messages.
  - There would be *k*-1 off-the-equilibrium messages.

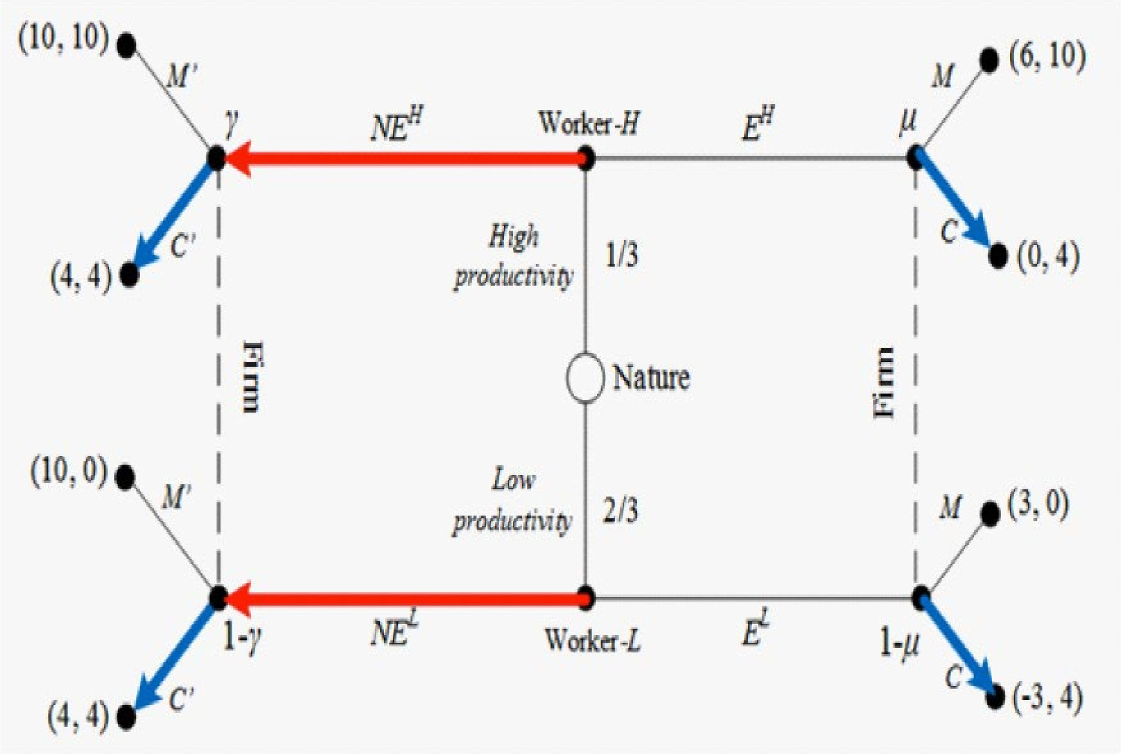

Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 3. Find which types of senders can obtain a higher utility level by deviating (i.e., when they send off-the-equilibrium messages) than by choosing their equilibrium message.

- The high-productivity worker *can benefit* when she deviates from NE^H, where she earns a payoff of 4, to E^H, where she can earn a payoff of 6 if the firm responds hiring her as a manager
- The low-productivity worker, however, *cannot benefit* when she deviates from NE^L, where she earns a payoff of 4, to E^L, where her highest payoff is 3 when the firm hires her as a manager.

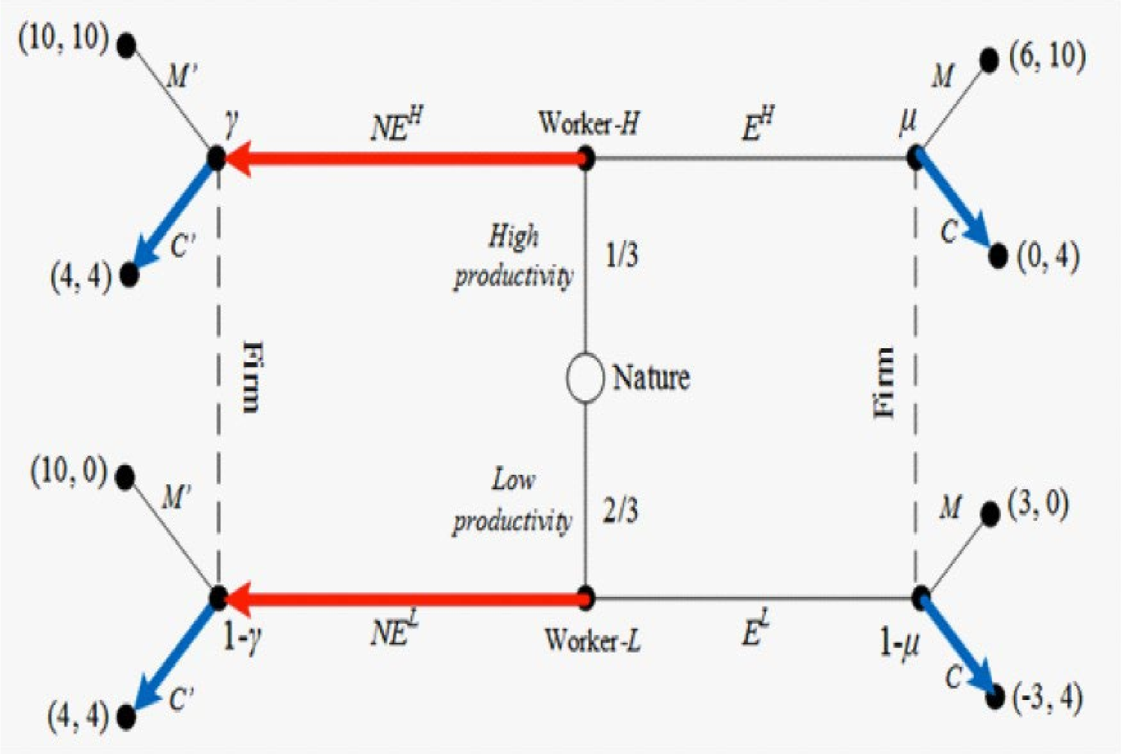

Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 4. Restrict the off-the-equilibrium beliefs of the responder using the results of Step 3.

- Intuitively, if education is observed,
  - the firm believes it can only originate from the high-productivity worker
  - since the low-productivity worker cannot benefit from acquiring education.
- Therefore, the firm's beliefs upon observing education are restricted from $\mu \leq \frac{2}{5}$ to $\mu = 1$.

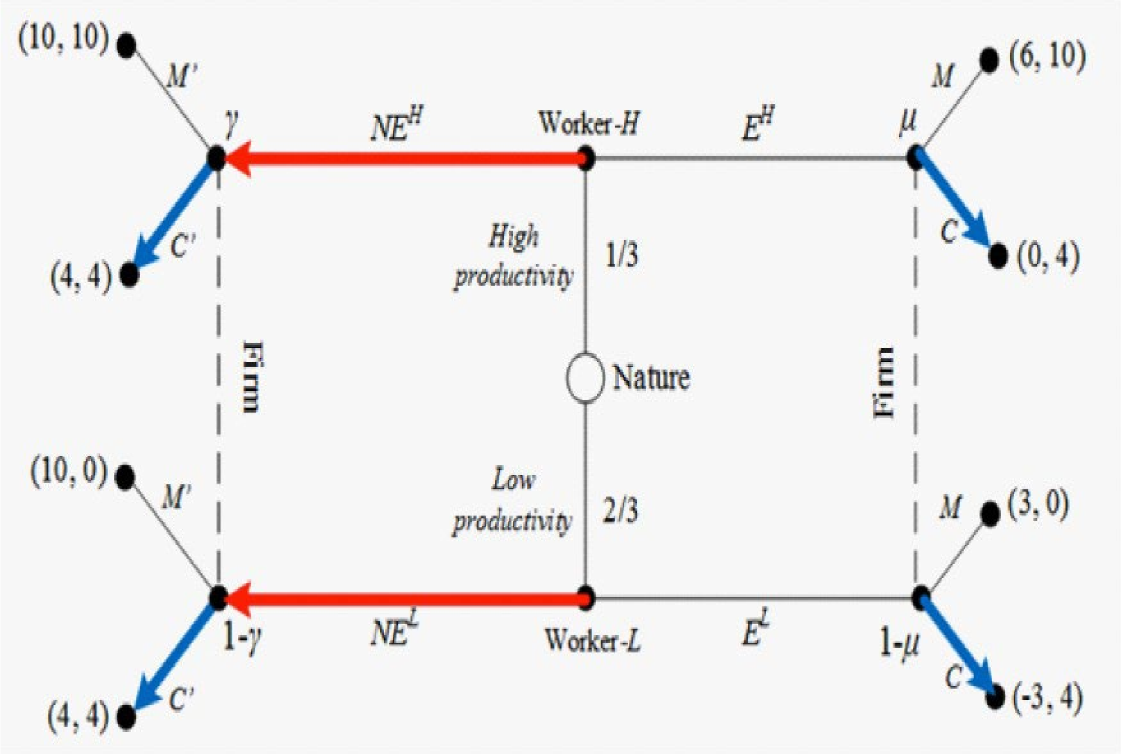

Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 5. Find optimal response given the restricted beliefs found in Step 4.

- In our ongoing example, upon observing education, the firm is convinced of dealing with a high-productivity worker, $\mu = 1$, and
- optimally responds hiring her as a manager, M (earning a profit of 10), which exceeds the profit of hiring the worker as a cashier (4).

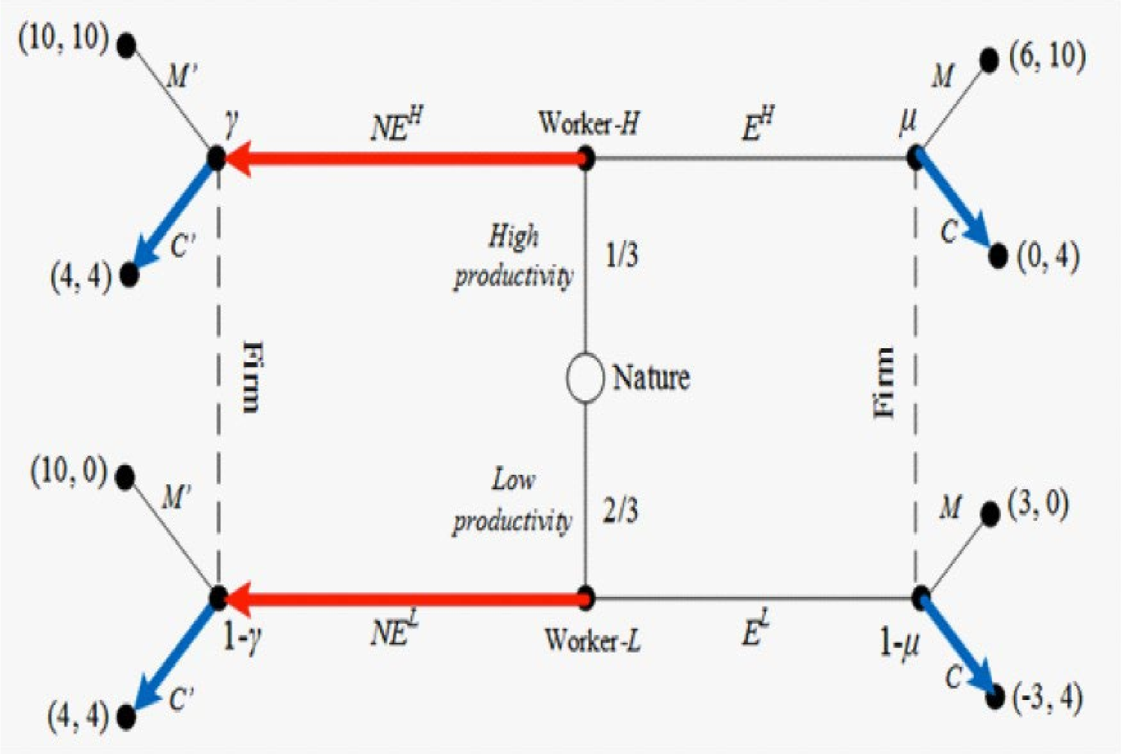

Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Tool 11.1 Applying the Cho and Kreps' (1982) Intuitive Criterion

Step 6. Given the optimal response found in Step 5, check if there is one of more sender types who can profitably deviate from her equilibrium message.

a. If there is, we say that the PBE we considered *violates* the Intuitive Criterion.
b. Otherwise, the PBE *survives* the Intuitive Criterion

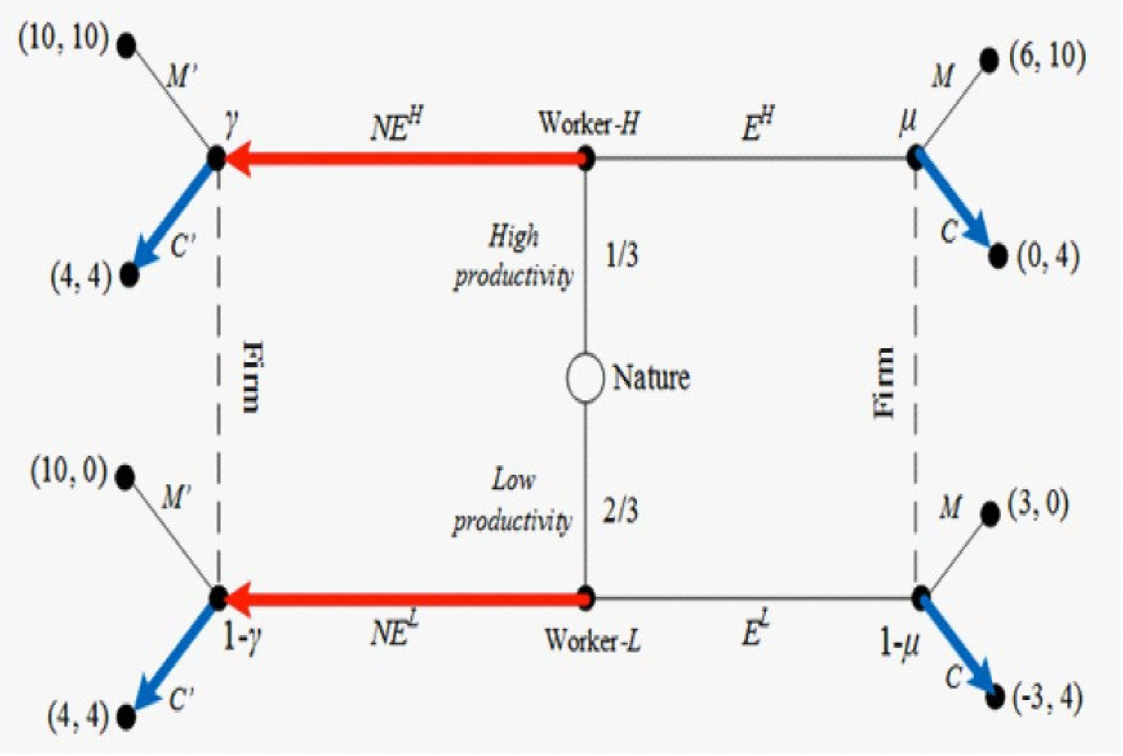

Figure 11.1. Pooling strategy profile (NE^H, NE^L) - Responses (C', C).

Pooling Equilibria May Violate the Intuitive Criterion

- In the pooling PBE (NE^H, NE^L), education is an off-the-equilibrium message.
- Moving on to Step 3, we note that:
  - only the high-productivity worker can profitably deviate towards education, as she could increase her payoff if the firm responds hiring her as a manager.
  - Formally, the highest payoff she earns from deviating to education is 6, which exceeds her equilibrium payoff of 4.
  - The low-productivity worker cannot profitably deviate, as the highest payoff she can earn from deviating is 3, which lies below her equilibrium payoff of 4.

Pooling Equilibria May Violate the Intuitive Criterion

- In other words, even if she fools the firm into believing that her productivity is high, and the firm hires her as a manager, her cost of acquiring education offsets her wage increase.
- In summary, we say that the low-productivity worker does not find education to be *equilibrium dominated,* since

$$u_L(NE^L, C') = 4 > 3 = \max_{a \geq 0} u_L(E^L, a)$$

where $a \in \{M, C\}$ denotes the firm's response.

- In contrast, the high-productivity worker does not find education to be equilibrium dominated.

Pooling Equilibria May Violate the Intuitive Criterion

- From Step 3, the firm can, in Step 4, restrict its off-the-equilibrium belief to $\mu = 1$
  - (if it observed education, it must stem from the high-productivity worker).
- As a consequence, in Step 5, we find that the firm's optimal response to education, given $\mu = 1$, is:
  - to hire the worker as a manager, M, as its profit, 10 exceeds that of hiring her as a cashier, 4.
- Finally, in Step 6, we obtain that:
  - one sender type (the high-productivity worker) has incentives to deviate from NE^H in the pooling PBE towards E^H,
  - as she anticipates that the firm will respond hiring her as a manager (Step 5).
- In conclusion, we can claim that:
  - this pooling PBE *violates* the Intuitive Criterion,
  - implying that we can eliminate the pooling PBE (NE^H, NE^L) as a solution to this game.

Separating Equilibria Survive the Intuitive Criterion

- In the separating PBE (E^H, NE^L) of Figure 10.5:
  - there is no off-the-equilibrium message in Step 2 of the Intuitive Criterion.
- In other words,
  - all possible messages (education and no education) are being used by at least one worker type.
  - Alternatively, no message is unused.

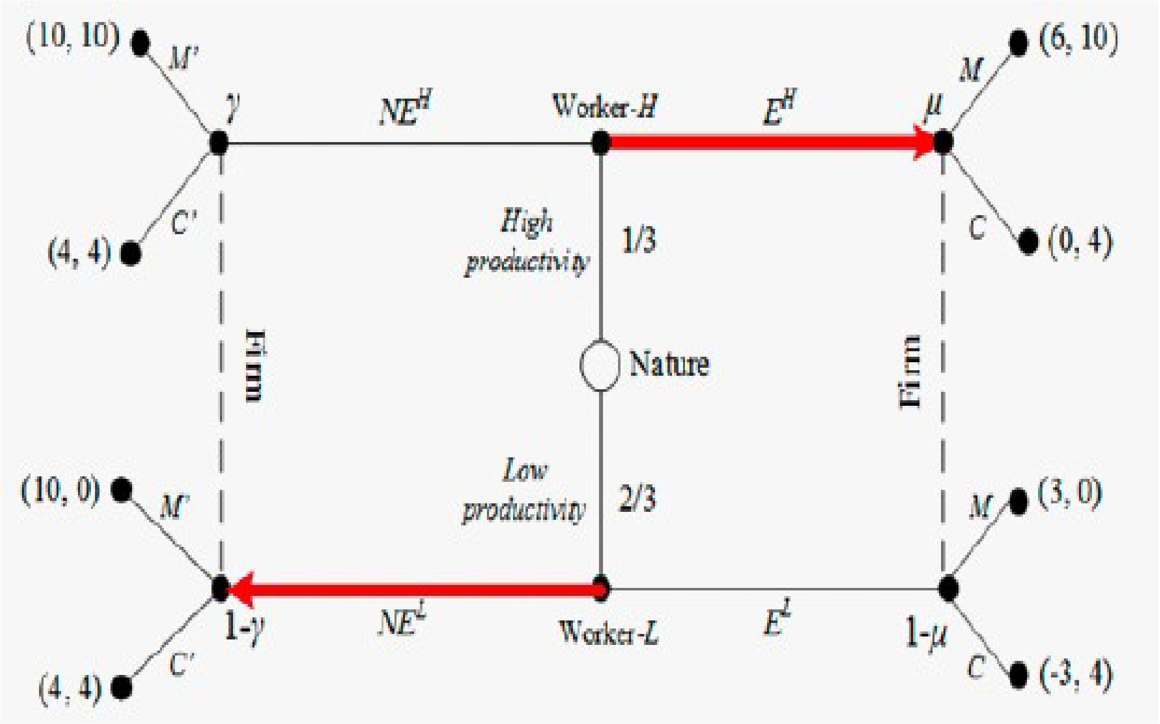

Figure 10.5. Separating strategy profile (E^H, NE^L).

Separating Equilibria Survive the Intuitive Criterion

- In this setting, the separating PBE survives the Intuitive Criterion.
- For a PBE to violate the Intuitive Criterion, we need to go through steps 1 to 6a,
  - and in this case Step 2 does not identify off-the-equilibrium messages,
  - entailing that we cannot continue with steps 3 and beyond.

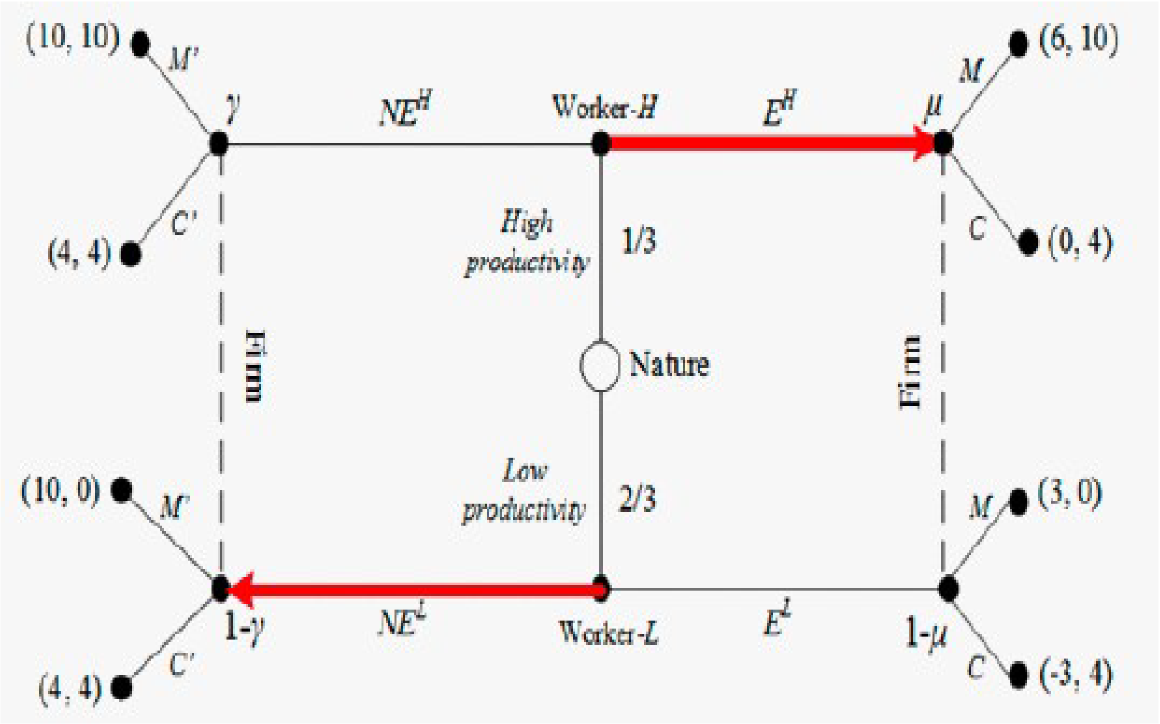

Figure 10.5. Separating strategy profile (E^H, NE^L).

Separating Equilibria Survive the Intuitive Criterion

- This argument applies to all separating PBEs:
  - With the same number of types as messages (as in the labor market signaling game).
  - With more sender types than messages.
- It doesn't necessarily apply to separating PBEs:
  - With fewer sender types than messages.
  - At least one of the messages is left unused, becoming an off-the-equilibrium message.
  - This is a "partially separating" PBE, which may survive or violate the Intuitive Criterion.

Separating Equilibria Survive the Intuitive Criterion

Separating PBEs	***Survives the Intuitive Criterion?***
# sender types = # messages	Yes
# sender types > # messages	Yes
# sender types < # messages	Maybe

D1 Criterion

- The D1 Criterion:
  - considers a similar approach as the Intuitive Criterion, as described in the six steps in Tool 11.1,
  - but differs in Step 3.

- In particular, for a given off-the-equilibrium message, the D1 Criterion seeks to answer the following question:

”If the receiver observes an off-the-equilibrium message,

such as education on the right side of Figure 11.1,

for which sender type are *most of the receiver’s actions beneficial*?”

D1 Criterion

- After that, we restrict the receiver's beliefs as in Step 4,
  - and proceed through Steps 5 and 6 as in Tool 11.1.
- That is, the Intuitive Criterion seeks to identify which sender types *could benefit* from deviating towards the off-the-equilibrium message,
  - which in some games could lead to a large number of sender types
- This approach doesn't restrict the receiver's beliefs in Step 4 enough to rule out PBEs sustained on insensible off-the-equilibrium beliefs.

D1 Criterion

- The D1 Criterion poses a similar question,
  - But, in a sense, it “goes deeper”
  - by finding not only which sender types have incentives to deviate…
  - but which sender type is “the most likely to deviate” as we describe next.

D1 Criterion

1. First, we measure the number of the receiver's responses (e.g., salary offers from the firm) that would weakly improve the sender's payoff, relative to her equilibrium payoff,
   - repeating this process for each sender type.
2. Second, we compare which sender type has the largest set of responses that would make her better off than in equilibrium.

- The application of these two steps slightly differs depending on whether the receiver has:
  - a discrete or continuous strategy space.
  - We separately consider them next.

D1 Criterion

- **Discrete responses:**
  - In a setting with discrete responses, such as when the firm hires the applicant as a CEO, manager, or cashier,

1. In the first step, we would just count!
   - How many of the firm responses can improve the high-productivity worker's equilibrium payoff,
   - How many responses can improve the low-productivity worker's, …
   - Similarly for each worker's type
2. In the second step:
   - we would compare which worker type has more payoff-improving responses,
   - identifying her as the one who is "the most likely to deviate."

D1 Criterion

- **Continuous Responses:**
  - In a context with continuous responses, such as when the firm responds with a salary $w > 0$ to the worker's observed education, these two steps are slightly different.

1. In particular, the first step would measure the set of firm responses that improve the worker's productivity (i.e., a wage interval).
2. The second step would compare the size of these wage intervals across worker types.
   - In this context, the worker with the longest "utility-improving wage interval" is the one who is the "most likely to deviate"

Applying the D1 Criterion – An Example

- We test whether the (NE^H, NE^L) pooling PBE survives the D1-criterion.

Step 1. Consider a specific PBE, such as the pooling PBE (NE^H, NE^L) we found in section 10.7.2.

Step 2. Identify an off-the-equilibrium message for the sender.

- In (NE^H, NE^L), the only off-the-equilibrium message is that of Education, on the right-side information set.

Applying the D1 Criterion – An Example

Step 3. Find which sender type is more *likely to benefit* from the deviation of Step 2.

(Recall: this is the main difference relative to the Intuitive Criterion).

To identify this type of sender, we go through the following steps:

a. When the high-productivity worker acquires education, she only has one response from the firm that can improve her equilibrium payoff, M, where her payoff increases from 4 to 6.
b. When the low-productivity worker acquires education, however, she has no responses from the firm that could improve her equilibrium payoff of 4.
c. Comparing the number of payoff-improving responses for the high- and low-productivity workers, we find that the former has more than the latter (1 vs. 0) implying that the high type is more likely to deviate.

Applying the D1 Criterion – An Example

Step 4. (From this point on, all steps coincide with those in the Intuitive Criterion).

- Restrict the off-the-equilibrium beliefs of the responder using the results of Step 3.
- Intuitively, if education is observed, the firm believes it must originate from the high-productivity worker, restricting its off-the-equilibrium beliefs from $\mu \leq \frac{2}{5}$ to $\mu = 1$.

Step 5. Find the optimal response given the restricted beliefs found in Step 4.

- Upon observing education, the firm is convinced of facing a high-productivity worker.
- If it responds hiring her as a manager, M, it earns a profit of 10, which exceeds the profit of hiring her as a cashier, 4.

Applying the D1 Criterion – An Example

Step 6. Given the optimal response found in Step 5, check if there is one or more sender types who can profitably deviate from her equilibrium message.

a. If there is, we say that the PBE we considered *violates* the D1-Criterion.
b. Otherwise, the PBE *survives* the D1-Criterion.

- In our ongoing example, after Step 5, we see that:
  - the high-productivity (low-productivity) deviates (does not deviate) as she anticipates the firm to respond to education hiring her as a manager, M.

- Therefore, we can conclude that pooling PBE (NE^H, NE^L) *violates* the D1-Criterion.

Comparing the Intuitive and D1 Criterion

- Therefore, we found the same refinement result as when applying the Intuitive Criterion;
  - but that is not generally the case.
- Both refinement criteria yield the same results in signaling games with only two sender types,
  - explaining why the Intuitive Criterion is used in this class of games,
  - as it is often easier to apply and provides the same refinement power.
- When the sender has three or more possible types, however,
  - the D1 Criterion gives us more restricted equilibrium predictions.

- Formally, this means that, if strategy profile $s = (s_1, s_2, \dots, s_N)$ can be sustained as a PBE,

$$s \text{ survives the D1 criterion} \Rightarrow s \text{ survives the Intuitive Criterion}$$

$$\not\Leftarrow$$

Comparing the Intuitive and D1 Criterion

- We return to this point in our analysis of signaling games with continuous action spaces in Chapter 12 (section 12.7).
- But intuitively, their difference arises from Step 3.
  - In the Intuitive Criterion, Step 3 only helps us to eliminate those sender types for which deviations are equilibrium dominated
    - i.e., they are better off behaving as prescribed by the PBE.
  - In the D1-Criterion, in contrast, Step 3 identifies which sender type is the "most likely to benefit from the deviation,"
    - implying that we focus our attention on a specific type of sender (or types) found in Step 3 of the Intuitive Criterion.

Other Refinement Criteria

- The literature offers other refinement criteria in signaling games which you may consider
  - when analyzing the PBEs where the Intuitive or D1 Criterion have no bite,
  - or when a large number of PBEs survive these two criteria.
- Examples include the:
  - “divinity” criterion and “universal divinity” criterion, both after Banks and Sobel (1988).

Other Refinement Criteria

- **Divinity Criterion.** It is a weakening of the D1 Criterion:
  - It requires that the receiver's posterior belief after observing an off-the-equilibrium message cannot increase the likelihood ratio of the sender being type θ relative to θ'.
  - Note that the D1 Criterion sets a stronger requirement, as it puts zero probability weight on this type of sender.

Other Refinement Criteria

- **University Divinity Criterion.** It requires that a sender type θ is eliminated, making updated beliefs independent on priors.

- These criteria are, however, beyond the scope of this book, and the advanced reader can refer to Banks and Sobel (1988) for more details.

- For experiments testing if subject behave as prescribed by the Intuitive and D1 Criteria, among other refinements, see:
  - Brandts and Holt (1992) and
  - Banks et al. (1994).

Sequential Equilibrium

- While refining PBEs, it yields similar results as the Intuitive or D1 Criteria and it is not as straightforward to apply as these two criteria, where we only need to check for sender types who could benefit from deviating to an off-the-equilibrium message.
- Consider a behavioral strategy profile $b = (b_i, b_{-i})$.
- Recall that player $i's$ behavioral strategy b_i prescribes which action she selects when she is called to move at information set h_i
- Allowing her to choose an action among those available at information set h_i , or instead, to randomize over two or more actions available to her at h_i.

Sequential Equilibrium

- Definition. **Sequential Equilibrium (SE).** Behavioral strategy profile $b = (b_i, b_{-i})$ and beliefs μ over all information sets are sustained as a sequential equilibrium if:
    1. *Sequential rationality.* Behavioral strategy b_i specifies optimal actions for every player i at each information set where she is called to move, given the strategies of the other players, b_{-i}, given the system of beliefs μ; and
    2. *Belief consistency.* There is a sequence $\{(b^k, \mu^k)\}_{k=1}^{\infty}$ such that, for all k:
        a. b^k is a totally mixed behavioral strategy, assigning a positive probability weight to all available actions at each information set h_i where player i gets to play
        b. μ^k is consistent with Bayes' rule given b^k
        c. Sequence $\{(b^k, \mu^k)\}_{k=1}^{\infty}$ converges to (b, μ)

Sequential Equilibrium

- While sequential rationality (point 1) is analogous to the requirement on the PBE definition, belief consistency (point 2) differs because it used totally mixed behavioral strategies.
- Graphically, point 2a implies that every information on the game tree is reached with positive probability, which did not necessarily happen in PBEs where some information sets may not be reached in equilibrium
- By reaching all information sets, Bayes' rule is no longer undefined, and helps us update beliefs given players' behavior
- Finally, point 2c states that the perturbations in the sequence become smaller as k increases, ultimately converging to (b, μ)

Sequential Equilibrium

- Intuitively, both PBE and SE require players to choose sequentially rational actions at every information set they are called to move.
- The SE solution concept, instead, required that off-the-equilibrium beliefs are consistent with Bayes' rule when players deviate from their equilibrium strategies with a positive probability
- Therefore, SE helps us identify which PBEs are sustained on consistent off-the-equilibrium beliefs.
- But exhibits no refining power on separating PBEs where all information sets are reached in equilibrium
- Every separating PBE is also a SE, but not all pooling PBEs are necessarily SEs as some may be sustained on insensible off-the-equilibrium beliefs.

$$(b, \mu) \text{ is a SE} \Rightarrow (b, \mu) \text{ is a PBE}$$

$$\not\Leftarrow$$

Finding Sequential Equilibria

- Consider a variation of the labor market signaling game where the probability of the worker's productivity being high is now $\frac{2}{3}$, as depicted in Figure 11.2.
- In this setting, three strategy profiles can be sustained as PBEs

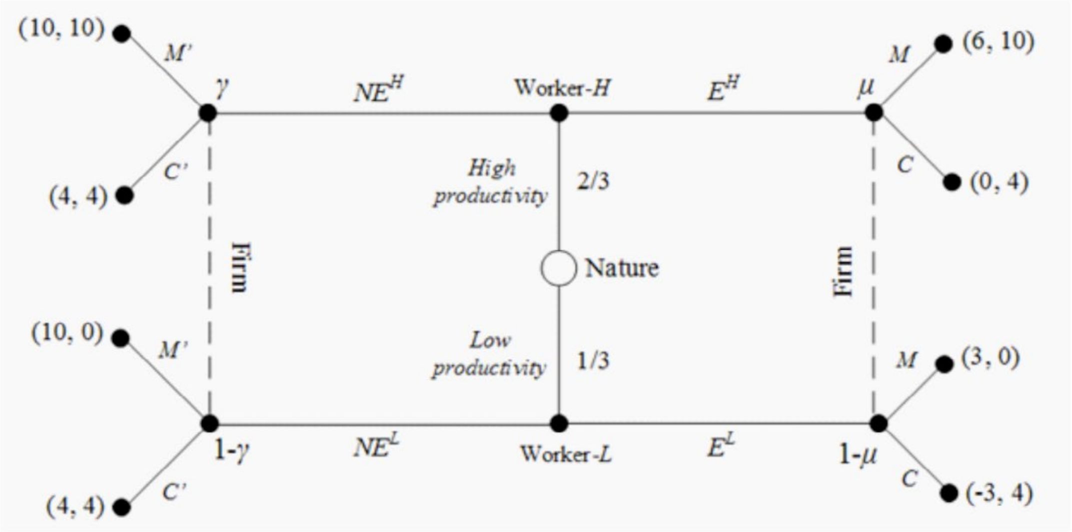

Figure 11.2. Modified labor market signaling game.

Finding Sequential Equilibria

1. The separating PBE $(E^H, NE^L; M, C')$, where only the high-productivity worker acquires education, sustained with equilibrium beliefs $\mu = 1(\gamma = 0)$ after observing education (no education, respectively).
2. The pooling PBE $(NE^H, NE^L; M, M')$, where no types of worker acquires education, the firm holds off-the-equilibrium beliefs $\mu > \frac{2}{5}$ and responds with M upon observing education.
3. The pooling PBE $(NE^H, NE^L; C, M')$, where no type of worker acquires education, the firm holds off-the-equilibrium beliefs $\mu \leq \frac{2}{5}$ and responds with C upon observing education.

Separating PBE that are also SEs

- We consider the separating strategy profile $(E^H, NE^L; M, C')$, as depicted in Figure 11.3, which is supported as a PBE with beliefs $\mu = 1$ and $\gamma = 0$.

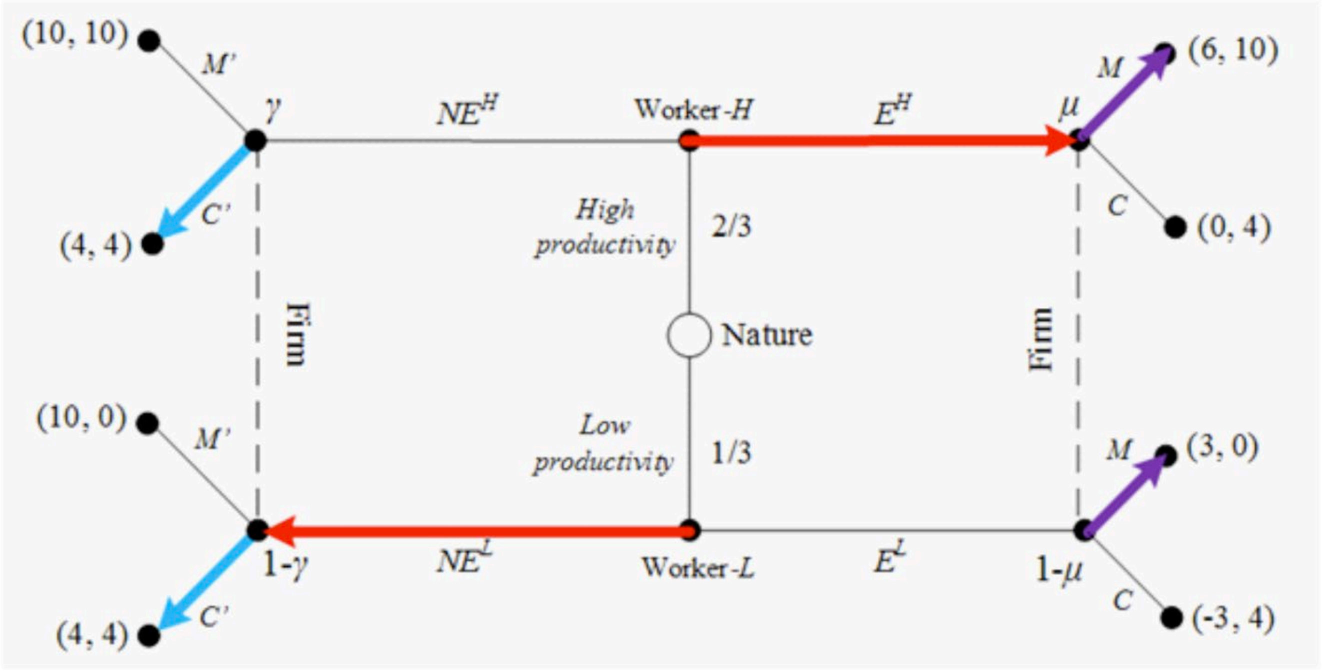

Figure 11.3. Separating strategy profile $(E^H, NE^L; M, C')$.

Separating PBE that are also SEs

- First, consider the following totally mixed behavioral strategy

$$b^k = \left(\underbrace{\left(1-\varepsilon^k, \varepsilon^k\right)}_{Worker-H}, \underbrace{\left(\varepsilon^k, 1-\varepsilon^k\right)}_{Worker-L}, \underbrace{\left(1-2\varepsilon^k, 2\varepsilon^k\right)}_{Firm\ after\ educ.}, \underbrace{\left(2\varepsilon^k, 1-2\varepsilon^k\right)}_{Firm\ after\ no\ educ.} \right)$$

where $\varepsilon^k > 0$ denotes a small perturbation, the first component of the high (low) types worker's behavioral strategy denotes the probability that she chooses $E^H(E^L)$, and the first component in the third (fourth) parenthesis represents the probability that the firm responds with $M(M')$ upon observing education (no education, respectively).

- This totally mixed behavioral strategy converges to $(1,0;1,0)$ when $k \to +\infty$, yielding outcome $(E^H, NE^L; M, C')$.

Separating PBE that are also SEs

- In addition, the belief system (μ^k, γ^k) is consistent with Bayes rule if

$$\mu^k = \frac{\frac{2}{3}(1-\varepsilon^k)}{\frac{2}{3}(1-\varepsilon^k)+\frac{1}{3}\varepsilon^k} = \frac{2(1-\varepsilon^k)}{2-\varepsilon^k} \text{ upon observing education, and}$$

$$\gamma^k = \frac{\frac{2}{3}\varepsilon^k}{\frac{2}{3}\varepsilon^k+\frac{1}{3}(1-\varepsilon^k)} = \frac{\frac{2}{3}\varepsilon^k}{1+\varepsilon^k} \text{ upon observing no education}$$

Separating PBE that are also SEs

- Figure 11.4 plots these beliefs as a function of k, showing that they converge to $\lim_{k \to \infty} \mu^k = 1$ and $\lim_{k \to \infty} \gamma^k = 0$.
- Therefore, updated beliefs coincide with those in the separating PBE, $\mu = 1$ and $\gamma = 0$, as required.

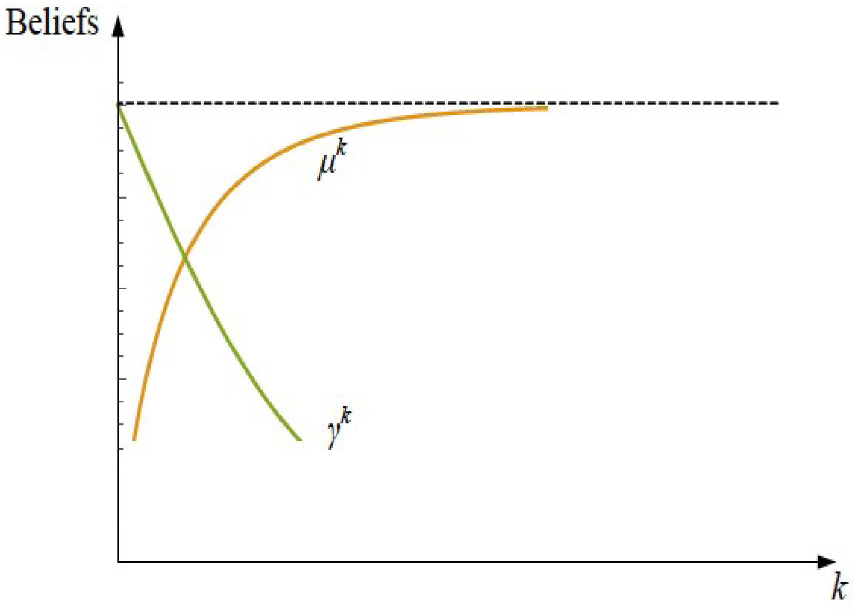

Figure 11.4. Updated beliefs μ^k and γ^k as a function of k.

Separating PBE that are also SEs

- Finally, given these beliefs, the firm's responses (M, C'), satisfy sequential rationality, as required by SE, since $\pi(M) = 10 > 4 = \pi(C)$ after observing education, and $\pi(C') = 4 > 0 = \pi(M')$ after no education.
- Therefore, the PBE $(E^H, NE^L; M, C')$ sustained with beliefs $\mu = 1$ and $\gamma = 0$ is also a SE of this game.

Pooling PBE that is not a SE

- We consider the pooling PBE$(E^H, NE^L; M, M')$ depicted in Figure 11.5, which is based on insensible off-the-equilibrium beliefs.
- Indeed, $\mu \leq \frac{2}{5}$ means that, upon observing the off-the-equilibrium message of education, the firm assigns a relatively low weight to such as message originating from the high-productivity worker.
- It even allows for the firm to be convinced that only the low-productivity worker acquires education $(\mu = 0)$!

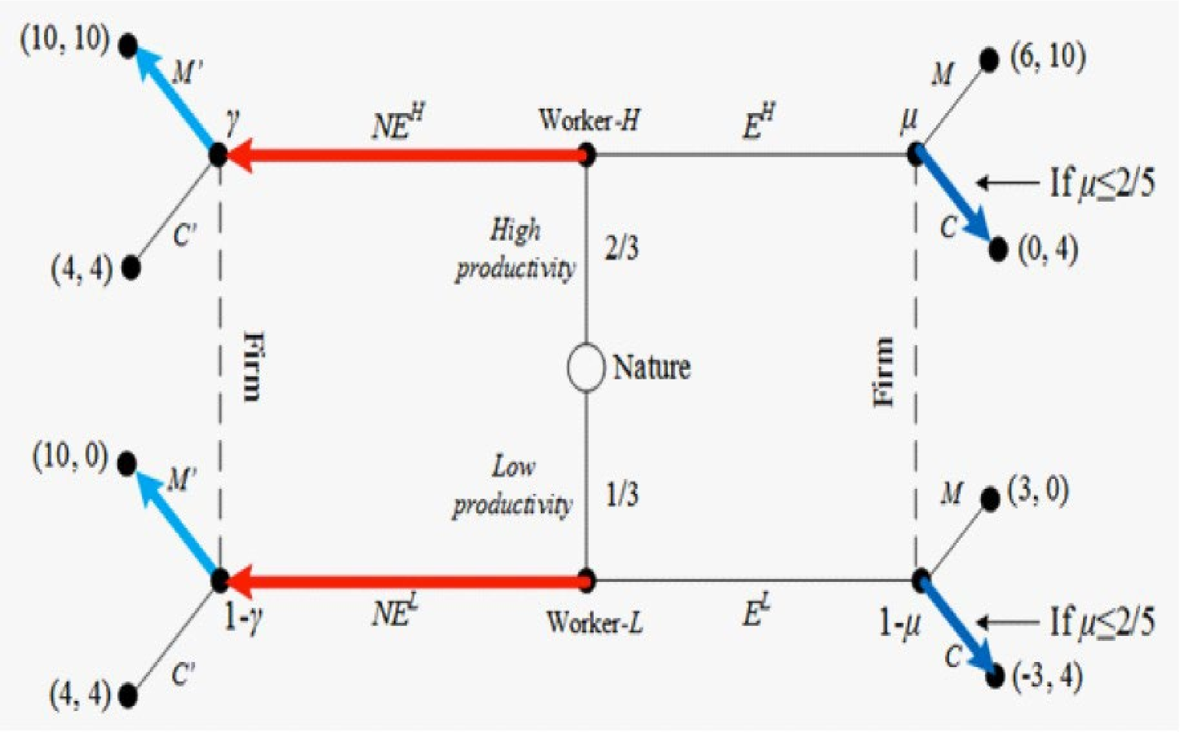

Figure 11.5. Pooling strategy profile $(NE^H, NE^L; M, M')$.

Pooling PBE that is not a SE

- We next demonstrate that this PBE is not a SE.
- To show this result, recall that when players choose a sequence of totally mixed behavioral strategies $\{b^k\}_{k=1}^{\infty}$, the firm's right-hand information set is reached with positive probability (i.e., E^H and E^L are played with positive probabilities).
- Because this information set is reached, we no longer face off-the-equilibrium information sets, and the firm's beliefs can be updated by Bayes' rule.

Pooling PBE that is not a SE

- In particular, if both types of workers choose education with the same small perturbation, ε^k, Bayes' rule entails that

$$\mu^k = \frac{\frac{2}{3}\varepsilon^k}{\frac{2}{3}\varepsilon^k + \frac{1}{3}\varepsilon^k} = \frac{2}{3}$$

implying that posteriors and priors coincide.

- Indeed, when perturbation ε^k is symmetric across players, the two nodes of the right-hand information set are reached with the same (small) probability, as in pooling strategy profiles.

Pooling PBE that is not a SE

- Given this updated belief, the firm responds with M after observing education since $E\pi(M) = \frac{20}{3} > 4 = E\pi(C)$.
- Therefore, responding with C cannot be part of a SE.
- We next provide a formal proof, describing the sequence of totally mixed behavioral strategies.
- Consider the following totally mixed behavioral strategy

$$b^k = \left(\underbrace{\left(\varepsilon^k, 1-\varepsilon^k\right)}_{Worker-H}, \underbrace{\left(\varepsilon^k, 1-\varepsilon^k\right)}_{Worker-L}, \underbrace{\left(2\varepsilon^k, 1-2\varepsilon^k\right)}_{Firm\ after\ educ.}, \underbrace{\left(1-2\varepsilon^k, 2\varepsilon^k\right)}_{Firm\ after\ no\ educ.} \right)$$

which converges to $(0,0;0,1)$ when $k \to +\infty$, yielding strategy profile $(NE^H, NE^L; C, M')$.

Pooling PBE that is not a SE

- In addition, the belief system (μ^k, γ^k) is consistent with Bayes rule if

$$\mu^k = \frac{\frac{2}{3}\varepsilon^k}{\frac{2}{3}\varepsilon^k + \frac{1}{3}\varepsilon^k} = \frac{2}{3} \text{ upon observing education, and}$$

$$\gamma^k = \frac{\frac{2}{3}(1-\varepsilon^k)}{\frac{2}{3}(1-\varepsilon^k) + \frac{1}{3}(1-\varepsilon^k)} = \frac{2}{3} \text{ upon observing no education}$$

which converge to $\lim_{k\to\infty} \mu^k = \gamma^k = \frac{2}{3}$.

- Finally, given these beliefs, the firm's responses, (C, M'), *violate* sequential rationality, since $E\pi(M) = \frac{20}{3} > 4 = E\pi(C)$ after observing education and, similarly $E\pi(M') = \frac{20}{3} > 4 = E\pi(C')$ after no education.
- In conclusion, the PBE $(NE^H, NE^L; C, M')$, supported with $\mu \leq \frac{2}{5}$, is not a SE.

Pooling PBE that is also a SE

- Consider now the pooling PBE $(NE^H, NE^L; M, M')$.

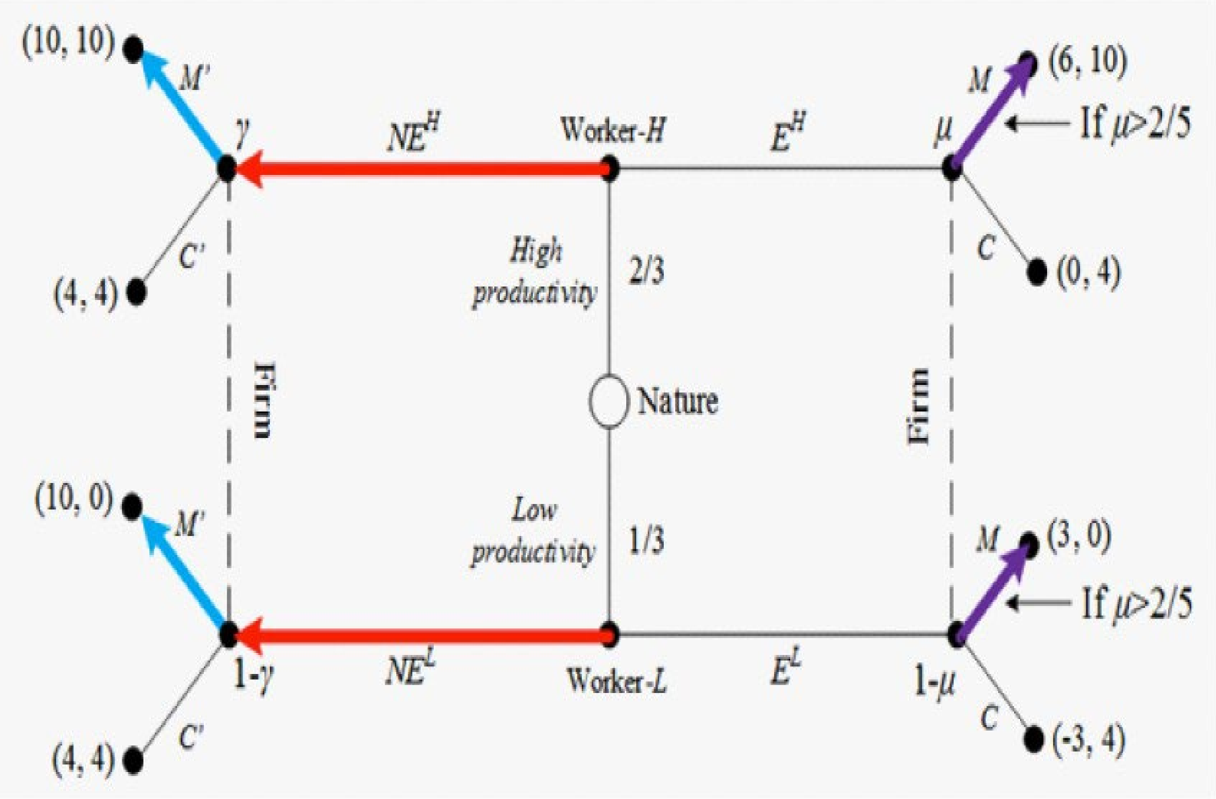

Figure 11.6 Pooling strategy profile $(NE^H, NE^L; M, M')$

Pooling PBE that is also a SE

- To see this point, consider the totally mixed behavioral strategy

$$b^k = \Big(\underbrace{(\varepsilon^k, 1-\varepsilon^k)}_{Worker-H}, \underbrace{(\varepsilon^k, 1-\varepsilon^k)}_{Worker-L}, \underbrace{(1-2\varepsilon^k, 2\varepsilon^k)}_{Firm\ after\ educ.}, \underbrace{(1-2\varepsilon^k, 2\varepsilon^k)}_{Firm\ after\ no\ educ.}\Big)$$

which converges to $(0,0;1,1)$ when $k \to +\infty$, yielding strategy profile $(NE^H, NE^L; M, M')$.

- In addition, the belief system (μ^k, γ^k) is consistent with Bayes rule, and produces the same results as b^k in section 11.4.1, that is, $\mu^k = \frac{2}{3}$ upon observing education, and $\gamma^k = \frac{2}{3}$ upon observing no education; converging to $\lim_{k\to\infty} \mu^k = \gamma^k = \frac{2}{3}$.

Pooling PBE that is also a SE

- Finally, given these beliefs, the firm's responses (M, M'), *satisfy* sequential rationality, since $E\pi(M) = \frac{20}{3} > 4 = E\pi(C)$ after observing education, and similarly, $E\pi(M') = \frac{20}{3} > 4 = E\pi(M')$ after no education.
- Consequently, the PBE $(NE^H, NE^L; M, M')$ supported with $\mu > \frac{2}{5}$, is also a SE.