Appendices for "Strategic Merger Approvals Under Incomplete Information"

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March 19, 2024

Appendix

Appendix 1 - Semiseparating equilibria

In order to check for a semiseparating equilibrium, we consider that the high-type firm randomizes between submitting and not submitting with probability σ_H and $(1-\sigma_H)$, respectively, while the low-type firm randomizes with probabilities σ_L and $(1-\sigma_L)$. The CA approves the merger with probability σ_{CA} and blocks it with probability $1-\sigma_{CA}$. Importantly, note that if the low-type firm is indifferent between submitting and not submitting, then the high-type firm must strictly prefer to submit, that is,

$$\sigma_{CA}\left[\left(\frac{1-c+x_L}{2}\right)^2 - R\right] + (1-\sigma_{CA})2\left(\frac{1-c}{3}\right)^2 = 2\left(\frac{1-c}{3}\right)^2$$

entails that

$$\sigma_{CA}\left[\left(\frac{1-c+x_H}{2}\right)^2 - R\right] + (1-\sigma_{CA})2\left(\frac{1-c}{3}\right)^2 > 2\left(\frac{1-c}{3}\right)^2$$

which implies that $\sigma_H = 1$.

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First step. For the CA to mix, his beliefs μ must satisfy

$$\mu\frac{1-c+x_H}{2}+(1-\mu)\frac{1-c+x_L}{2}=k\frac{1-c}{3}$$

where the left side represents the expected output when the CA approves the merger and the right side indicates its certain output when the CA blocks the merger. Rearranging, yields

$$\frac{\mu x_H+(1-\mu)x_L}{1-c}=\frac{1}{3}\equiv\overline{\theta}$$

which we can also express as

$$E[\theta]\equiv\mu\frac{x_H}{1-c}+(1-\mu)\frac{x_L}{1-c}=\mu\theta_H+(1-\mu)\theta_L=\overline{\theta}.$$

or, after solving for μ, we obtain that the CA's beliefs must satisfy $\mu=\frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L}\equiv\widehat{\mu}$; otherwise, it would not be mixing between approving the merger (if $\mu>\widehat{\mu}$) and blocking it (if $\mu<\widehat{\mu}$).
Second step. Given the CA's beliefs, we find from Bayes' rule that

$$\mu=\frac{\overline{\theta}-\theta_L}{\theta_H-\theta_L}=\frac{p}{p+(1-p)\sigma_L}$$

where the numerator captures the probability that the high-type firm submits a merger request (since $\sigma_H=1$), while the denominator reflects the probability that the CA receives a merger request from any firm type. Solving for probability σ_L, yields

$$\sigma_L^*=\frac{p}{1-p}\frac{\theta_H-\overline{\theta}}{\overline{\theta}-\theta_L}\tag{1}$$

which is unambiguously positive, and less than 1 if $\frac{p}{1-p}<\frac{\overline{\theta}-\theta_L}{\theta_H-\overline{\theta}}$.
Third step. Given our above results about μ and σ_L, we can now find σ_{CA}. The low-type firm mixes if and only if

$$\sigma_{CA}\left[\left(\frac{1-c+x_L}{2}\right)^2-R\right]+(1-\sigma_{CA})2\left(\frac{1-c}{3}\right)^2=2\left(\frac{1-c}{3}\right)^2$$

where the left (right) side denotes the expected (certain) profit from submitting (not submitting) a merger request, which is approved with probability σ_{CA}. However, after rearranging, we find that

$$\sigma_{CA}\left[\left(\frac{1-c+x_L}{2}\right)^2-R-k\left(\frac{1-c}{3}\right)^2\right]=0.$$

If $\theta_L>\widehat{\theta}$, the term in brackets is positive, entailing that the CA's probability, σ_{CA}, becomes $\sigma_{CA}=0$. In other words, the CA blocks all merger requests, implying that no firm type would have incentives to spend R into the submission process, that is, $\sigma_H=\sigma_L=0$, as in the PE where no firm type submits a merger request (see Proposition 3). Therefore, a semiseparating PBE cannot be sustained when $\theta_L>\widehat{\theta}$.

If, instead, $\theta_L < \widehat{\theta}$ holds, the term in brackets is negative, implying that probability σ_{CA} would have to be negative too, which cannot occur, entailing that a semiseparating PBE cannot be supported in this case either.
Finally, if $\theta_L = \widehat{\theta}$, the term in brackets is exactly zero, implying that probability σ_{CA} is undefined, $\sigma_{CA} \in [0,1]$. In this context, a semiseparating PBE can be sustained, where the firm randomizes with probability $\sigma_H^* = 1$ and $\sigma_L^* = \frac{p}{1-p}\frac{\theta_H - \overline{\theta}}{\overline{\theta} - \theta_L}$ and the CA responds approving mergers with any probability $\sigma_{CA} \in [0,1]$, if and only if $\theta_H > \theta_L = \widehat{\theta}$ holds.

Appendix 2 - Allowing for more merging firms

No merger. In a case of no mergers, every firm i solves,

$$\max_{q_i \geq 0} \quad (1 - q_i - Q_{-i})q_i - cq_i$$

where Q_{-i} denotes the aggregate output of firm i's rivals. Differentiating with respect to q_i, and solving for q_i, we find firm i's best response function

$$q(Q_i) = \frac{1-c}{2} - \frac{1}{2}Q_{-i}$$

In a symmetric equilibrium, $q_i = q_j = q$ for every two firms $i \neq j$, which entails $Q_{-i} = (n-1)q$. Therefore, the equilibrium output in this setting is

$$q_i^{NM} = \frac{1-c}{n+1}$$

and equilibrium profits become $\pi_i^{NM} = \left(\frac{1-c}{n+1}\right)^2 = \left(q_i^{NM}\right)^2$.
Merger. Now consider a merger of k firms is approved. The merging entity solves

$$\max_{q^M \geq 0} \quad (1 - q^M - Q_{-i})q^M - (c-x)q^M$$

where q^M denotes the merging entity's output, and Q_{-i} represents the aggregate output of all outsiders combined. Differentiating with respect to q^M, and solving for q^M, we find the best response function

$$q^M(Q_{-i}) = \frac{1-c+x}{2} - \frac{1}{2}Q_{-i}.$$

Similarly, every outsider firm i solves

$$\max_{q_i^M \geq 0} \quad (1 - q_i^M - q^M - Q_{-i}^M)q_i^M - cq_i^M$$

where Q_{-i}^M denotes the aggregate production level of all other $(n-k)-1$ firms that are outsiders in the merger. Differentiating with respect to q_i^M, and solving for q_i^M, we obtain the best response function q

$${}_i^M(q^M, Q_{-i}^M) = \frac{1-c}{2} - \frac{1}{2}\left(q^M + Q_{-i}^M\right).$$

In a symmetric equilibrium, all outsiders produce the same output, $q_i^M = q_j^M = q_O^M$ for all $n-k$ firms, implying that $Q_{-i}^M = [(n-k)-1]q_O^M$. Inserting this result in the above best response functions, and rearranging, yields equilibrium output levels

$$q^M = \frac{1-c+(n-k+1)x}{n-k+2} \quad \text{and} \quad q_O^M = \frac{1-c-x}{n-k+2}$$

Finally, equilibrium profits for the merging entity is

$$\pi^M = (1-q^M-(n-k)q_O^M)q^M-(c-x)q^M = \left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2 = \left(q^M\right)^2$$

whereas every outsider earns

$$\pi_O^M = (1-q^M-(n-k)q_O^M)q_O^M - cq_O^M = \left(\frac{1-c-x}{n-k+2}\right)^2 = \left(q_O^M\right)^2.$$

Cutoff $\overline{\theta}(k,n)$. In this setting, an increase in consumer surplus is equivalent to an increase in output. In particular, $q^M \geq kq_i^{NM}$ holds if and only if

$$\frac{1-c+(n-k+1)x}{n-k+2} \geq k\frac{1-c}{n+1}.$$

Rearranging, and solving for yields $x \geq \frac{(1-c)(k-1)}{n+1}$ or, alternatively,

$$\theta \equiv \frac{x}{1-c} \geq \frac{k-1}{1+n} \equiv \overline{\theta}(k,n)$$

Cutoff $\widehat{\theta}(k,n)$. A merger between k out of n firms is profitable if the post-merger profits exceed the pre-merger profits, that is, $\pi_I^M - R \geq k\pi_i^{NM}$, which holds if

$$\left(\frac{1-c+(n-k+1)x}{n-k+2}\right)^2 - R \geq k\left(\frac{1-c}{n+1}\right)^2$$

After simplifying, we obtain

$$\frac{1-c+(n-k+1)x}{n-k+2} \geq \sqrt{k\left(\frac{1-c}{n+1}\right)^2 + R}$$

and upon further rearranging, we find

$$\theta \equiv \frac{x}{1-c} \geq \frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2+R} - \frac{1}{n-k+1} \equiv \widehat{\theta}(k,n)$$

which increases in R. Comparing cutoffs $\widehat{\theta}(k,n)$ and $\overline{\theta}(k,n)$, we obtain that $\widehat{\theta}(k,n) > \overline{\theta}(k,n)$ holds if and only if

$$\frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2+R} - \frac{1}{n-k+1} > \frac{k-1}{1+n}$$

or, rearranging, and solving for R,

$$R > \left(\frac{1-c}{n+1}\right)^2 \left[\left(\frac{(n+1)+(n-k+1)(k-1)}{(n-k+2)}\right)^2 - k\right] \equiv \widehat{R}.$$

Therefore, $\widehat{\theta}(k,n) > \overline{\theta}(k,n)$ holds if and only if $R > \widehat{R}$.

Appendix 3 - Uninformed outsiders

No merger. In a case of no mergers, every firm i solves the same maximization problem as in Appendix 2, yielding the same equilibrium output, $q_i^{NM} = \frac{1-c}{n+1}$, and profits, $\pi_i^{NM} = \left(\frac{1-c}{n+1}\right)^2 = \left(q_i^{NM}\right)^2$.

Merger. When k out of n firms merge, the merging entity solves

$$\max_{q^M \geq 0} \quad (1 - q^M - Q_{-i})q^M - (c-x)q^M$$

where q^M denotes the insiders' output, and Q_{-i} represents the aggregate output of all outsiders combined. Differentiating with respect to q^M, and solving for q^M, we find the best response function

$$q_H^M(Q_{-i}) = \frac{1-c+x_H}{2} - \frac{1}{2}Q_{-i}$$

when the merging entity's type is high and, similarly, $q_L^M(Q_{-i}) = \frac{1-c+x_L}{2} - \frac{1}{2}Q_{-i}$ when its type is low.

Every outsider (uninformed about the realization of x), chooses its output q_i to solve the following expected profit-maximization problem

$$\max_{q_i \geq 0} \; p\left[(1-q_H-q_i-Q_{-i})q_i - cq_i\right] + (1-p)\left[(1-q_L-q_i-Q_{-i})q_i - cq_i\right]$$

where q_H (q_L) denotes the merging entity's output when its type is high (low, respectively); and Q_{-i} represents the aggregate output of all other outsiders (except for firm i). Differentiating with respect to q_i, and solving for q_i^M, we obtain the best response function

$$q_i^M(q_H^M, q_L^M, Q_{-i}) = \frac{1-c}{2} - \frac{pq_H^M + (1-p)q_L^M + Q_{-i}}{2}$$

where $pq_H^M + (1-p)q_L^M$ denotes the merging entity's expected output. In a symmetric equilibrium, all outsiders produce the same output, $q_i^M = q_j^M$ for all $n-k$ firms, implying that $Q_{-i} = [(n-k)-1]q_i^M$. Inserting this result in the above best response function, and rearranging, yields

$$q_i^M(q_H^M, q_L^M, Q_{-i}) = \frac{1-c}{n-k+1} - \frac{pq_H^M + (1-p)q_L^M}{n-k+1}.$$

Simultaneously solving for q_H^M, q_L^M, and q_i^M in the above three best response functions, yields Bayesian Nash equilibrium outputs

$$\begin{aligned} q_H^M &= \frac{2(1-c) + (n-k+2)x_H + (n-k)E[x]}{2(n-k+2)}, \\ q_L^M &= \frac{2(1-c) + (n-k+2)x_L + (n-k)E[x]}{2(n-k+2)}, \text{ and} \\ q_i^M &= \frac{1-c+E[x]}{n-k+2}, \end{aligned}$$

where $E[x] \equiv px_H + (1-p)x_L$ represents the expected cost-reduction effect.

The outsiders' equilibrium profit does not affect our results regarding regions 1-3, but the merging entity's does, becoming

$$\begin{aligned}\pi^{M,L}(\beta) &= (1-q_L^M-[(n-k)-1]q_i^M)q_L^M-(c-x_L)q_L^M \\ &= \left(\frac{2(1-c)+(n-k+2)x_L+(n-k)E[x]}{2\,(n-k+2)}\right)^2\end{aligned}$$

which collapses to that in the previous section when the merging entity's type is low with certainty, $p=0$, $\pi^{M,L}=\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2$. Therefore, the profit gain for the low-type entity is

$$\pi^{M,L}(\beta)-k\pi_i^{NM}=\left(\frac{2(1-c)+(n-k+2)x_L+(n-k)E[x]}{2\,(n-k+2)}\right)^2-k\left(\frac{1-c}{n+1}\right)^2$$

which in the case that $\beta=0$ simplifies to $\pi^{M,L}(0)-k\pi_i^{NM}=\left(\frac{1-c+(n-k+1)x_L}{n-k+2}\right)^2-k\left(\frac{1-c}{n+1}\right)^2$; as in section 6.1.

Setting $\pi^{M,L}(\beta)-k\pi_i^{NM}\geq R$ and solving for θ_L, we obtain $\theta_L\geq\widehat{\theta}(p)$. When the merging entity's type is low with certainty, $p=0$, cutoff $\widehat{\theta}(p)$ simplifies to that in section 6.1, that is, $\widehat{\theta}(0)=\widehat{\theta}(k,n)$, where

$$\widehat{\theta}(k,n)\equiv\frac{n-k+2}{(1-c)(n-k+1)}\sqrt{k\left(\frac{1-c}{n+1}\right)^2+R}-\frac{1}{n-k+1}.$$

Appendix 4 - Allowing for continuous responses by the CA

Updated beliefs. In this pooling strategy profile, the CA cannot update its beliefs according to Bayes' rule. Therefore, upon observing R, where $R\geq f$, its beliefs are $\mu\left(\theta_H|R\right)=p$ and $\mu\left(\theta_L|R\right)=1-p$, whereas upon receiving any off-the-equilibrium message $R'\neq R$, where $R'\geq f$, its off-the-equilibrium beliefs are $\mu\left(\theta_H|R'\right)=0$.

Receiver's response. Given the above beliefs, upon observing R, in equilibrium, the CA responds exerting a challenging effort α, upon observing R, that solves

$$\max_{\in\alpha[0,1]}\ \alpha\left(2\frac{1-c}{3}\right)+(1-\alpha)\left(p\frac{1-c+x_H}{2}+(1-p)\frac{1-c+x_L}{2}\right)-\frac{1}{2}\lambda\alpha^2.$$

which, differentiating with respect to α, yields

$$2\frac{1-c}{3}-\left(p\frac{1-c+x_H}{2}+(1-p)\frac{1-c+x_L}{2}\right)-\lambda\alpha=0.$$

and, solving for α, we obtain the CA's optimal response after observing the pooling submission cost R,

$$\alpha^*=\frac{1-c-3E[\theta]}{6\lambda}$$

where $E[\theta]\equiv p\theta_H+(1-p)\theta_L$. In addition, α^* satisfies $\alpha^*>0$ if $1-c>3E[\theta]$ or, after rearranging, $\frac{1-c}{3}>E[\theta]$, which is incompatible with the initial condition $E[\theta]>\frac{1}{3}\equiv\overline{\theta}$ since $\frac{1}{3}>\frac{1-c}{3}$. Therefore, it is never optimal for the CA to challenge a merger in a pooling equilibrium.

In contrast, upon observing the off-the-equilibrium message R', the CA responds blocking the merger since $\mu\left(\theta_H|R'\right)=0$ and $\theta_L<\overline{\theta}$ by assumption.
Sender's messages. Anticipating these responses, the θ_H-type entity invests R, as prescribed in this pooling strategy profile, if

$$\alpha\left(2\pi_i^{NM}\right)+(1-\alpha)\left(\pi_I^{M,H}-R\right)\geq 2\pi_i^{NM},$$

where the right side assumes that the high-type deviates to zero investment (no merger request) because any deviation to $R'\neq R$ guarantees a merger decline and $R'=0$ minimizes the firm's submission cost. Inserting the CA's optimal response, $\alpha^*=0$ identified above, into this inequality, we obtain

$$\pi_I^{M,H}-R\geq 2\pi_i^{NM}.$$

or, after solving for R, we find $R\leq\pi_I^{M,H}-2\pi_i^{NM}$. Solving for θ_H, we know from Proposition 2 that this inequality yields $\theta_H>\widehat{\theta}(R)$.
Similarly, the θ_L-type entity chooses R, instead of deviating to any other $R'\neq R$, which guarantees a merger decline, if and only if

$$\alpha\left(2\pi_i^{NM}\right)+(1-\alpha)\left(\pi_I^{M,L}-R\right)\geq 2\pi_i^{NM},$$

(The right side of this inequality follows a similar argument as for the high-type firm.). Inserting $\alpha^*=0$ into this inequality, yields

$$\pi_I^{M,L}-R\geq 2\pi_i^{NM}$$

which simplifies to $R\leq\pi_I^{M,L}-2\pi_i^{NM}$. Solving for θ_L, we know from Proposition 2 that this inequality yields $\theta_L>\widehat{\theta}(R)$. Combining the inequalities we found from the high- and low-type firms, we obtain that a PE can be sustained if $R\leq\pi_I^{M,L}-2\pi_i^{NM}$. Since $\theta_H>\theta_L$ by definition, a sufficient condition for inequalities $\theta_H>\widehat{\theta}(R)$ and $\theta_L>\widehat{\theta}(R)$ to hold is $\theta_L>\widehat{\theta}(R)$, which is equivalent to $\pi_I^{M,L}-2\pi_i^{NM}\geq R$.
Therefore, equilibrium behavior coincides with that in the pooling PBEs where the CA faces a binary strategy space (approve or block merger requests), surviving both the Intuitive and Divinity Criteria.