Environmental policy helping antitrust decisions☆
Socially excessive and insufficient merger approvals

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Abstract

This paper considers firms’ incentives to merge under duopoly, where we allow for product differentiation, cost asymmetries, and pollution intensities (green and brown goods). We first analyze mergers in the absence of environmental regulation, showing that mergers induce an output shift towards the lowest cost firm. When emission fees are introduced, however, firms also consider their relative pollution intensities, potentially reverting the above output shift. We show that socially excessive mergers can arise when firms shift output to the more cost-efficient firm which may cause more pollution. In contrast, socially insufficient mergers can arise if output shifts reduce pollution.

Keywords: socially excessive/insufficient mergers, product differentiation, cost asymmetry, pollution intensity, emission fees, antitrust authorities, environmental regulation

JEL Classification: G34, H23, L41, Q50

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1. Introduction

Many industries nowadays include firms with different pollution intensities. The literature on environmental regulation, and mergers, however, mostly considers markets where firms are either all polluting or all green rather than a combination of both. In the energy sector, for instance, mergers are common between energy and utility service providers in which one firm is less polluting than its rivals (Creti and Sanin, 2017).

Allowing for firm-level heterogeneities can help us better understand firms’ behavior and predict the welfare effects from the mergers. If the antitrust authority (AA) incorrectly assume that firms are symmetric, mergers may be blocked on the basis that they can lead to an output reduction. When differences in pollution intensities are taken into account, however, mergers could induce output shifts that mitigate environmental damages, yielding an overall welfare gain in settings where mergers would have been blocked otherwise.

Our model considers an industry with a green and a polluting brown firm, where we allow for cost asymmetries and product differentiation. In the first stage, firms choose whether to file a merger request to the AA for approval. In the second stage, the AA approves or blocks the merger. In the third stage, the environmental regulator (such as the Environmental Protection Agency, EPA) sets emission fees, observing whether the merger was approved or not. In the fourth stage, firms respond choosing their output levels either cooperatively as part of the merger or competing à la Cournot. This timing indicates that merger plans and the AA’s response, approving or blocking the merger, are often taken into consideration in a long-run horizon, and staying in place for years or decades before any revisions. In contrast, emission fees and firms’ output decisions can be more easily revised after observing a change in the merger decisions in the early stages of the game.

1For example, Enbridge, a Canadian oil-pipeline operator, acquired the 50-megawatt Silver State North photovoltaic project from First Solar Inc., a Nevada-based solar power plant in 2012 to honor its neutral footprint commitment (Martin, 2012). Creti and Sanin (2017, Table 1) provide a list of mergers between renewable and non-renewable energy firms under the cap-and-trade program of the Regional Greenhouse Gas Initiative (RGGI). In addition, Hennessy and Roosen (1999) suggest that firms may have incentives to merge and aggregate pollution permits when emission is pooled within a certain attainment area under the “bubble” policy of the US Environmental Protection Agency (EPA).

2In addition, we consider the AA taking precedence over EPA in approving mergers when firms have incentives to merge in anticipation of more stringent environmental standards, such as in the energy sector (Creti and Sanin, 2017).
We first turn our attention to the case of no environmental regulation. In this setting, mergers induce an output shift from the less to the more cost-efficient firm irrespective of pollution differentials. Such mergers can, however, yield a welfare loss if firms produce relatively homogeneous goods, as in standard merger models without environmental regulation (Kao and Menezes, 2010). Nevertheless, welfare-enhancing mergers can still arise when output shifts to the less polluting firm, or output of both green and brown firms reduce to mitigate more severe environmental damages.3

We next consider environmental regulation in our analysis, examining under which conditions mergers entail a welfare loss, as in standard models of market power, and in which cases they yield a welfare gain. In this setting, emission fees create incentives to shift their output after the merger from the polluting to the green firm—an incentive that did not exist in the absence of environmental regulation. Most importantly, the increase in the green firm’s output exceeds the reduction in the polluting firm’s output, leading an overall decrease in pollution. Nevertheless, when firms produce relatively homogenous goods and pollution intensities are low, output reduction and price increase due to the mergers still decrease overall welfare, as in standard merger models (Salant et al., 1983).

We then identify the welfare effects of mergers, finding that they can be welfare-reducing if the polluting firm generates significantly higher environmental damages than its green rival. Therefore, it is possible that the AA, which ignores environmental damages, can approve welfare-reducing mergers, and we identify situations where those mergers should be blocked; specifically, when firms producing homogeneous goods are relatively symmetric in costs and pollution intensities.

Overall, we may observe the approval of welfare-reducing mergers or the blocking of welfare-enhancing mergers in equilibrium. When the AA ignores environmental damages, mergers can be approved (blocked) but they would have been blocked (approved) by the AA that considers environmental damages. However, the EPA has the ability, in the subsequent stage of the game,

3Monsanto, which manufactures Roundup™ weedkiller, is faced with multiple litigations for allegations of causing non-Hodgkin lymphoma (NHL) and multiple myeloma by its active ingredient glyphosate (Bayer AG, 2019, p.250). Bayer, which acquired Monsanto in 2018, manufactures Natria™ herbicide, an organic substitute for weed and broadleaf control that mitigates the environmental and health impacts associated with the use of chemical herbicides. Our model applies to a broad class of mergers between environmentally differentiated firms, such as the one above where an output shift from conventional to organic herbicides is predicted. Pelaez and Mizukawa (2017) identify a trend of agrochemical industry leaders, which face increasing regulatory costs, acquiring biopesticide companies, which produce more environmental-friendly pest control products.
to correct any inefficiencies arisen from the AA’s incorrect merger approval decisions. This gives rise to another benefit of environmental policy, often overlooked in the literature, as it can correct output inefficiencies that would not have existed if the AA considered environmental damages when evaluating merger requests. Our results then call for more coordinated efforts between the AA and EPA in evaluating mergers, especially in industries where firms have different pollution intensities.

Our findings contribute to the debate about the factors that competition policy should consider when evaluating mergers. Antitrust authorities often focus on consumer surplus and firms’ profits, without considering the potential output shifts that, after the merger, decrease (increase) the production of the relatively polluting (green) good. European competition law explicitly considers environmental reasons (see articles 6 and 174 of the Treaty), but they have rarely been used to approve or block mergers.\footnote{See, for instance, Martinez-Lopez (2000) for a discussion about the agreement between European appliances manufacturers to limit their production of energy-inefficient machines. While this agreement entailed an increase in prices, as the discontinued appliances were the most inexpensive, the European Commission considered that it would help address the environmental externalities that consumers did not take into account in their purchasing decisions.} In the case of the United States, the Horizontal Merger Guidelines (US Department of Justice and Federal Trade Commission, 2010) do not explicitly consider the broader impact of mergers on the environment, while the EPA does not actively monitor mergers.\footnote{If the emission fees are administratively costly, or if they require a close monitoring of firms’ production decisions, then the AA which considers environmental externalities may help the EPA’s task by approving (blocking) mergers that (do not) contribute to an overall welfare improvement.}

For generality, we extend our model to a setting with $n_G$ green firms and $n_B$ brown firms, allowing for mergers of a subset of green firms with a subset of brown firms (see Appendix 1). While the results become more intractable, our findings indicate that, in the absence of environmental policy, mergers can still be welfare improving under similar conditions as in an industry with one green and one brown firm but are profitable only when the merging firms hold a sufficiently high market share, as in Salant et al. (1983) and Gelves (2014). This result is emphasized when firms are subject to environmental regulation, increasing the minimal market share that merging firms must hold to make their merger profitable, which goes in line with Benchekroun et al. (2019) in the context of natural resource extraction.
1.1 Related Literature

A large body of the literature analyzes horizontal mergers in oligopolistic markets with environmental externalities. Specifically, Canton et al. (2012) consider the upstream ecoservice industry supplying pollution abatement goods to the downstream polluting market. In this context, horizontal mergers in the ecoservice industry are profitable if a sufficient number of firms merge, as in Salant et al. (1983), or if costs are relatively convex, as in Perry and Porter (1985). In addition, mergers are welfare-improving when emission fees are intermediate; that is, the fees are not too high (low) to yield insufficient output (abatement). Lambertini and Tampieri (2012, 2014) seek to understand mergers of Cournot triopolists under environmental regulation, and show that firm incentives and social objectives align if output reduction, which increases profits and decreases pollution, more than offsets consumer welfare losses. Recent developments include Benchekroun et al. (2019), Berchicci et al. (2012), Berchicci et al. (2017), Davidson and Mukherjee (2007), Erkal and Piccinin (2010), and Meglio and Park (2019). Our paper differs from the previous literature by considering endogenous emission fees and allowing for asymmetric firms. We show that mergers can be both profitable and welfare-improving under different forms of asymmetries, and how the EPA can strategically use emission fees to facilitate or hinder mergers among the firms.

A branch of literature in industrial organization analyzes differentiated oligopolies with asymmetric costs. Zanchettin (2006) extends Singh and Vives (1984)'s model of differentiated duopoly and Fauli-Oller (2002)'s model of asymmetric costs to consider pricing strategies and equilibrium profits in a setting of differentiated and asymmetric firms. Unlike our paper, however, these studies do not consider firms’ incentives to merge. Kao and Menezes (2010) show that welfare-enhancing mergers can arise in the context of output shifts from the less to the more cost-efficient firm, and the same result holds in our model without environmental damages. However, our model also shows that mergers can be welfare-improving if output shifts mitigate environmental damages.

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6Our analysis focuses on non-cumulative pollutants, such as carbon dioxide, sulphur dioxide, and suspended particulate; see Benchekroun and Ray Chaudhuri (2008) for an overview of studies on cumulative pollutants.

7A recent example in the pollution abatement industry is the acquisition of Auburn FilterSense LLC, a US provider of particulate emissions monitors and intelligent controls for industrial particulate/dust filtration systems, by Nederman, the Swedish environmental technology company (Filtration + Separation, 2018). Another example in the waste management industry is the acquisition of Quantex Environmental Inc., an Ontario based company, by Covanta Environmental Solutions, a New Jersey based environmental services provider (Waste 360, 2018).
Gelves (2014) considers two-firm mergers in an oligopolistic setting with \(N\) firms similar to that in Salant et al. (1983), but allowing for cost asymmetries and product differentiation, suggesting that welfare-enhancing mergers are more likely when products become more differentiated.

The closest articles to our paper are Fikru and Gautier (2016, 2017), who examine mergers in Cournot markets with product differentiation and emission fees but assuming symmetric costs and no pollution differentials between firms. We show that mergers can be welfare-improving when costs are asymmetric even in the absence of environmental externalities, since output shifts from the less efficient to the more cost-efficient firm help save production costs. When externalities are present, however, we propose another channel of welfare gains. Specifically, emission fees on the polluting firm induce this firm to reduce its output even when firms do not merge. Our research then complements Fikru and Gautier (2017, 2020), since we also find that regulation should be more stringent in markets with homogeneous goods and high pollution intensities. However, we demonstrate that stringent regulation is required when firms are relatively asymmetric in costs. In addition, we identify under which conditions welfare-enhancing (welfare-reducing) mergers are blocked (approved) by the AA, leading to an overall welfare loss.

Fikru and Gautier (2020) consider two countries (home and foreign), with \(n\) and \(m\) firms in each country, respectively, and \(k^h\) and \(k^f\) firms merging in each country, denoted as “insiders”, which leaves \(n - k^h\) and \(m - k^f\) unmerged firms as “outsiders”. In the first stage, each country’s government simultaneously and independently chooses the emission fee. In the second stage, every firm chooses its output and emissions, either to maximize joint profits if the firm is an insider or independently if the firm is an outsider. However, the number of merging firms in each country is exogenous, so every firm does not choose whether to merge or not at any point in the game, and the socially optimal emission fee is not defined (the authors study its effect on aggregate output and welfare).\(^8\) While we only consider firms in one country, we allow for firms’ merging decisions and the AA’s merger approvals to be endogenous, and identify socially optimal emission fees in different context, analyzing how the absence of environmental regulation gives rise to socially excessive or insufficient mergers. Our setting helps us examine the interaction between the EPA and the AA,

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\(^8\)As in our paper, their setting allows for firms to be asymmetric in their production costs or pollution intensities (although all firms located in a country are symmetric) and for product differentiation between firms located in different countries (all firms in a given country sell homogeneous products).
especially when their objective functions differ, and how emission fees can help correct the AA’s regulatory mistakes.

The remainder of this paper is organized as follows. Section 2 develops the model. Section 3 examines merger incentives and welfare effects without environmental regulation, whether the AA considers pollution (in section 3.2) or not (in section 3.2.1). Section 4 evaluates these effects when emission fees are present, whether the AA considers pollution (in section 4.3) or not (in section 4.5). Section 5 then compares welfare level across different regulatory regimes, and finally, section 6 concludes. The following table summarizes the different regulatory settings in our paper.

<table>
<thead>
<tr>
<th>AA considers pollution</th>
<th>EPA is absent</th>
<th>EPA is present</th>
</tr>
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<tbody>
<tr>
<td>AA ignores pollution</td>
<td>Section 3.2</td>
<td>Section 4.3</td>
</tr>
<tr>
<td>AA is absent</td>
<td>Section 3.2.1</td>
<td>Section 4.5</td>
</tr>
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</table>

Table 1: Different regulatory settings

2. Model

Consider a market with two firms, $i$ and $j$, where $i, j = \{B, G\}$ stand for the brown and green firms, respectively. Let $c_i$ denote firm $i$’s marginal cost of production, where $0 \leq c_i < 1$. Every firm $i$ faces an inverse demand function $p_i(q_i) = 1 - q_i - \beta q_j$, where $\beta \in [0, 1]$ represents the degree of product differentiation. When $\beta = 0$, goods are completely differentiated but when $\beta = 1$, goods are perfect substitutes; see Singh and Vives (1984). Environmental damages, $Env(Q_B, Q_G) = d (Q_B + \alpha Q_G)^2$, are increasing and convex in aggregate output of brown (green) firms $Q_B, Q_G$, where $d \geq 0$ stands for the pollution intensity. Parameter $\alpha \in [0, 1]$ denotes how clean the green firm is relative to its brown rival, being completely clean when $\alpha = 0$, as in Gelves (2014), or equally polluting when $\alpha = 1$.

We define $m_B = \frac{1-c_B}{1-c_G}$ to be the efficiency of the brown firm relative to the green firm. When the brown firm has a higher efficiency than the green firm, $c_B < c_G$, we obtain that $m_B > 1$, as

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empirically shown in a sample of Swedish exporting firms by Forslid et al. (2018). The opposite holds if the brown firm is less efficient than its green rival, where \( c_B > c_G \), yielding that \( m_B < 1 \), as found in Canadian firms by Najjar and Cherniwchan (2020).

The timing of the game is as follows:

1. Every firm \( i \) decides whether to merge with firm \( j \) or not. If one firm opposes, the merger does not ensue and the game proceeds to stage 3. If both firms agree to merge, the merger proposal is sent to the AA.

2. The AA, receiving the merger proposal, decides whether to approve or block the merger.

3. The EPA sets an emission fee \( t^k_i \) to every firm \( i \), which depends on its type \( i \in \{ B, G \} \) and the industry structure \( k \in \{ M, NM \} \), where \( M \) (NM) denotes merger (no-merger), respectively.

4. If firms merge, they jointly choose output pair \( (q^M_B, q^M_G) \). If the merger does not ensue, every firm \( i \) independently chooses output \( q^N_M \) and engages in Cournot competition.

We first consider a setting with no environmental regulation and solve for the firms’ equilibrium output and profits with and without the merger, examining merger incentives and the AA’s role in approving mergers. Next, we introduce the EPA which sets emission fees \( t^k_i \), analyzing its effects on firms’ output, profits, and social welfare. For presentation purposes, sections 3 and 4 consider a setting with one green and one brown firm, and appendix 1 extends our analysis to a market with \( n_G \geq 1 \) green and \( n_B \geq 1 \) brown firms. For simplicity, we assume \( m_B \geq \frac{\beta}{2} \) throughout the paper.\(^{10}\)

3. Equilibrium analysis without regulation

3.1 Third stage - output decisions

Operating by backwards induction, we begin our analysis with the third stage of the game.\(^{11}\) Lemma 1 identifies firm \( i \)’s output before and after the merger. For presentation purposes, all proofs are relegated to Appendix 2.

\(^{10}\)If, instead, \( m_B < \frac{\beta}{2} \) (\( m_B \geq \frac{\beta}{2} \)), the green (brown) firm dominates the market and its rival produces zero units when firms do not merge, rendering the merger problem uninteresting. Graphically, this condition sets an upper bound for \( m_B \) at 2, since \( \frac{2}{\beta} \geq 2 \) for all admissible values of \( \beta \).

\(^{11}\)Environmental regulation (stage 3) is absent in this section, implying that the game has only three stages.
Lemma 1. Firm i’s output is $q_i^{NM} = \frac{2(1-c_i) - \beta(1-c_i)}{4-\beta^2}$ if not merged. After the merger, firm i produces $q_i^M = 0$ if $m_i < \beta$, $q_i^M = \frac{(1-c_i) - \beta(1-c_i)}{2(1-\beta^2)}$ if $\beta \leq m_i < \frac{1}{\beta}$, and $q_i^M = \frac{1-c_i}{2}$ if $m_i \geq \frac{1}{\beta}$.

Lemma 2 evaluates every firm’s output change due to the merger, as captured by $\Delta q_i^{NR} = q_i^M - q_i^{NM}$.

Lemma 2. After the merger, the brown (green) firm: (1) increases output if $m_B \geq m_B \equiv \frac{2+\beta^2}{2\beta}$ ($m_B \leq \frac{1}{m_B}$); (2) reduces output if $\beta \leq m_B < \frac{1}{m_B} (\frac{1}{m_B} < m_B \leq \frac{1}{\beta})$; and (3) stops production if $m_B < \beta$ ($m_B \geq \frac{1}{\beta}$, respectively).

Figure 1a depicts that when the brown firm is more efficient than the green firm ($m_B \geq 1$), it increases (decreases) output after the merger when its relative efficiency parameter, $m_B$, lies above (below) cutoff $m_B$. Cutoff $m_B$ is monotonically decreasing in $\beta$, implying that when firms produce more homogeneous goods (higher $\beta$), it becomes less restrictive for the brown firm to increase output after the merger. When the brown firm becomes less efficient than the green firm ($m_B < 1$), however, it stops production when $m_B < \beta$.

Figure 1b depicts the green firm’s output change, still using the same vertical axis as in Figure 1a, $m_B$. Figure 1b shows that the green firm increases its output after the merger when it is relatively efficient ($m_B$ lying below cutoff $\frac{1}{m_B}$), but reduces output otherwise.\footnote{Our results go in line with the empirical findings in Berchicci et al. (2012, 2017), which show that when a relatively clean firm acquires a facility from a dirtier firm, the acquired facility’s environmental performance improves. Similarly, our results predict an output shift between the two firms reducing total emissions, especially when firms are subject to environmental regulation (section 4). For more examples of firms’ mergers and acquisitions incentives to improve their corporate social responsibility, see Meglio and Park (2019) and the references therein.}

When the green firm is relatively inefficient ($m_B \geq 1$), it stops production when $m_B \geq \frac{1}{m_B}$.\footnote{Some real-life examples include J.M. Smucker Company acquiring Millstone Coffee in 2008 and subsequently shutting it down in 2016 due to insufficient demand. In 2020, Cathay Pacific decided to shut down Dragon Air (acquired in 2006), becoming the sole aviation company based in Hong Kong, in a corporate restructuring effort to keep costs down (Lee, 2020).}
The following corollary compares the firms' output change from the merger, as follows.

**Corollary 1.** *The brown firm reduces output more than its green rival if and only if* $c_B > c_G$.

In the bottom half of Figure 1a, where $m_B < 1$, the brown firm reduces its output more than its green rival after the merger, as in this region the brown firm is relatively inefficient. In the top half of Figure 1a, however, the brown firm is more efficient than the green firm, reducing its output less significantly than the green firm after the merger when $m_B < m_B$, or increasing it otherwise. A similar analysis applies to the green firm in Figure 1b, which is relatively efficient (inefficient) at the bottom (top) half of the figure. In Corollary 2, we investigate the change in aggregate output due to the merger.

**Corollary 2.** *Merger reduces aggregate output under all parameter conditions.*

Therefore, our results indicate that aggregate output unambiguously decreases after the merger, entailing a loss in consumer surplus. The next section shows, however, that such a loss can be offset by the increase in profits stemming from the efficiency gains that arise from the shift in output across firms, and by the reduction in environmental damages.
3.2 Second stage - merger approval

In the second stage, the AA anticipates the output level that every firm $i$ produces in the third stage, and approves the merger if and only if it increases social welfare, as follows:

$$ SW(q_B, q_G) = CS + PS - Env(q_B, q_G) $$

where $CS = \frac{1}{2}(q_B^2 + 2\beta q_B q_G + q_G^2)$ represents consumer surplus,$^{14}$ $PS = \pi_B + \pi_G$ denotes producer surplus, and $Env(q_B, q_G) = d(q_B + \alpha q_G)^2$ captures environmental damages, as defined in section 2.$^{15}$

Further define $\Delta CS = CS^{NM} - CS^M$ and $\Delta PS = PS^M - PS^{NM}$ to be the loss of consumer surplus and gain in producer surplus due to the merger, respectively. Corollaries 3 and 4 examine how those surpluses vary when firms produce more homogeneous goods, and we analyze the overall welfare effects from the merger. For compactness, cutoffs $m_{BS}$ and $\overline{m}_{BS}$ are included in the proof.

**Corollary 3.** The loss in consumer surplus increases in $\beta$ if and only if $m_B$ satisfies $m_{BS} < m_B < \overline{m}_{BS}$, but decreases in $\beta$ otherwise.

Figure 2a depicts that the loss in consumer surplus becomes more severe in $\beta$ when the brown firm’s efficiency is intermediate, that is, $m_B$ lies in between cutoffs $m_{BS}$ and $\overline{m}_{BS}$. When firms are cost-symmetric ($m_B = 1$, as on the horizontal dashed line), such a loss unambiguously increases in $\beta$ since firms merge to reduce output. When firms become asymmetric in costs ($m_B \neq 1$), consumers still experience a larger loss from the merger as goods become more homogeneous. However, when firms are more cost-asymmetric, an increase in $\beta$ can lead to a smaller $\Delta CS$. To see this, consider, for instance, the case where $m_B = 2$ at the top of Figure 2a. Starting at $\beta = 0$, both firms reduce their output after the merger, as shown in Figures 1a and 1b. However, when $\beta$ increases enough, only the green firm decreases its output. While aggregate output is still lower than that before the merger, it is larger when goods are relatively homogeneous ($\beta \rightarrow 1$) than when they are highly

$^{14}$Our results do not suffer path dependence problem since our quasilinear-quadratic utility produces no wealth effects, see Johansson (1999, p.748) and Hsu (2005).

$^{15}$While parameter $d > 0$ measures the pollution intensity of both firms, it could alternatively capture the difference between the EPA and AA’s welfare function (e.g., their preference divergence). When $d = 0$, both agencies have the same welfare function. When $d > 0$, the EPA assigns a weight to environmental damages but the AA does not.
differentiated \((\beta \to 0)\). A similar argument applies at the bottom of Figure 2a, where it is now the green firm that decreases its output when \(\beta\) is low but increases it otherwise.

**Corollary 4.** The gain in producer surplus increases in \(\beta\) if and only if \(m_B\) satisfies \(m_B^{PS} < m_B < \bar{m}_B^{PS}\), but decreases in \(\beta\) otherwise, where \(m_B^{PS} \equiv \frac{2((4+3\beta^2)-(4-\beta^2)^{3/2})}{\beta(1+\beta^2)}\) and \(\bar{m}_B^{PS} \equiv \frac{2((4+3\beta^2)+(4-\beta^2)^{3/2})}{\beta(1+\beta^2)}\).

Figure 2b depicts that the gain in producer surplus increases in \(\beta\) when firms are relatively cost-symmetric, that is, \(m_B\) lies in between cutoffs \(m_B^{PS}\) and \(\bar{m}_B^{PS}\). When firms are cost-symmetric (\(m_B = 1\), as on the horizontal dashed line), such a gain clearly increases in \(\beta\) since firms merge to eliminate more intensive competition. When firms become more asymmetric in costs (\(m_B \neq 1\)), however, they still gain from the merger as they reduce aggregate output, but such a reduction becomes small when \(\beta\) increases enough to eliminate the inefficient rival, as shown in Corollary 2.

![Figure 2: Surplus variation under no regulation](image)

**Lemma 3** characterizes the merger approval decision of the AA, where for compactness, cutoffs \(m_B^{SW}\) and \(\bar{m}_B^{SW}\) are included in its proof.

**Lemma 3.** The AA blocks the merger when \(m_B^{SW} \leq m_B \leq \bar{m}_B^{SW}\), and approves it otherwise.
As a benchmark, Figure 3a depicts the case where neither type of firm pollutes \((d = 0)\), in which the merger is approved if goods are relatively homogeneous and one firm enjoys a large cost advantage relative to its rival; otherwise, the merger is blocked. Consider the case of symmetric firms with a dashed horizontal line at \(m_B = 1\). This line lies entirely in between cutoffs \(m_{SW}^B\) and \(\bar{m}_B^{SW}\), indicating that the merger is welfare-reducing for all values of \(\beta\). This setting considers, as a special case, that in which firms sell homogeneous goods \((\beta = 1)\), as in standard duopoly models where mergers are unambiguously welfare-reducing. Intuitively, mergers produce a loss in consumer surplus, \(\Delta CS > 0\), as shown in the analysis of aggregate output in Corollary 2, which holds under all parameter conditions, and an increase in producer surplus, \(\Delta PS > 0\), as firms maximize joint profits after merging. However, when firms are completely symmetric and sell homogeneous goods, \(m_B = \beta = 1\) as depicted in the bullet point of this figure, the former effect offsets the latter, yielding an overall welfare reduction.

This welfare decrease from the merger is emphasized when goods become more differentiated and \(m_B = 1\); graphically, we move leftward along the dashed horizontal line to lower values of \(\beta\). In this setting, every firm does not significantly affect its rival’s price before the merger and, as a consequence, firms do not need to substantially change their output levels after the merger. As a result, the increase in profits they enjoy from the merger is smaller than when goods are homogeneous, yielding a small gain in producer surplus, \(\Delta PS\). Overall, we obtain that the increase in producer surplus is so small that the above overall welfare decrease holds for larger parameter values.

When firms become more asymmetric, however, \(m_B \neq 1\), the merging firms now benefit from a new effect, as they can start shifting their output towards the most efficient firm. This new effect increases \(\Delta PS\), allowing for welfare increases to hold if firms are sufficiently asymmetric and goods are sufficiently homogeneous; as depicted in the top- and bottom-right corners of Figure 3a. When the dotted vertical line shifts leftward, our results suggest that the merger becomes welfare-reducing under larger parameter conditions (graphically, a longer segment of the vertical line between cutoffs \(m_{SW}^B\) and \(\bar{m}_B^{SW}\)). Intuitively, this occurs because, as goods become more differentiated (lower \(\beta\)), the

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16This result is similar to those in Davidson and Mukherjee (2007). Considering symmetric firms but allowing for free entry and exit, the authors show that firms have incentives to merge even if they benefit from a small cost synergy due to the merger and, as in our model, the merger becomes welfare-enhancing under large parameter conditions.
merger induces a less significant output shift from the least to the most efficient firm, thus yielding smaller welfare gains due to the merger.\textsuperscript{17}

Figures 3b to 3c analyze how merger approval decisions are affected if one or both firms are polluting, as opposed to Figure 3a where firms’ output does not generate pollution. First, Figure 3b depicts the case where only the brown firm is polluting \((d = \frac{1}{4} \text{ and } \alpha = 0)\). In this setting, the merger shutting down the relatively inefficient brown (green) firm is approved under larger (smaller) parameter conditions than when both firms generate no pollution \((d = 0, \text{ as in Figure 3a})\). In this setting, when the green firm becomes more efficient than its brown rival \((m_B < 1)\), the merger that induces an output shift towards the green firm reduces pollution, ultimately increasing welfare as in the region below cutoff \(m_B^{SW}\).\textsuperscript{18} Figure 3c depicts the case where both firms are polluting, although the green firm generates fewer emissions per unit of output \((d = \frac{1}{4} \text{ and } \alpha = \frac{1}{2})\). Compared to Figure 3b, cutoff \(m_B^{SW}\) \(m_B^{SW}\) shifts up (down), indicating that mergers which reduce output of both firms can enhance welfare when the reduction of environmental damages is sufficiently strong.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Social welfare under no regulation}
\end{figure}

In summary, if firms are polluting, \(d > 0\), but symmetric in their pollution intensities, \(\alpha = 1\), a new positive effect from the merger arises, since its aggregate output reduction improves social

\textsuperscript{17}Cutoff \(m_B^{SW}\) \(m_B^{SW}\) is downward- (upward-) sloping. Intuitively, this means that while aggregate output decreases after the merger, its reduction is less significant when \(\beta\) is relatively high, as shown in Corollary 2. As a result, an increase in \(\beta\) expands the set of parameters where the merger becomes welfare-improving.

\textsuperscript{18}Graphically, for every \((m_B, \beta)\)-pair that lies below cutoff \(m_B^{SW}\) in Figure 3a, it also lies below cutoff \(m_B^{SW}\) in Figure 3b.
welfare, thus expanding the region of parameter values for which the merger is welfare-improving. When firms are polluting but asymmetric in their pollution intensities, \( \alpha < 1 \), this new positive effect from the merger is attenuated, shrinking the region of parameter values for which the merger improves welfare (graphically, a decrease in parameter \( \alpha \) is illustrated when one moves from Figure 3c to 3b).

3.2.1 Antitrust overlooks environmental effects

If the AA does not consider environmental damages, its welfare function becomes

\[
\tilde{SW}(q_B, q_G) = CS + PS \tag{2}
\]

When environmental damages are absent \((d = 0)\), expressions (1) and (2) coincide, implying that the AA approves mergers under the same \((m_B, \beta)\)-pairs. However, when environmental damages are present \((d > 0)\), the AA behaves as in Figure 3b while the AA ignoring these damages behaves as in Figure 3a. Evaluating cutoffs \(m_{SW}^0\) and \(\bar{m}_{SW}^0\) in Figure 3a at \(d = 0\), we obtain cutoffs \(m_{B}^0\) and \(\bar{m}_{B}^0\), respectively. Figure 4a superimposes these two cutoffs on Figure 3b, showing that mergers that shift output to the brown firm and shut down the non-polluting green firm do not improve welfare above cutoff \(m_{SW}^0\) and below \(\bar{m}_{SW}^0\) (see yellow-shaded region at the top of Figure 4a). Mergers in this region should be blocked according to expression (1) since \(\Delta SW < 0\), but are approved according to the welfare criterion in expression (2) where \(\Delta \tilde{SW} \geq 0\), which we refer to as “socially excessive mergers” (SEM). In contrast, mergers that reduce output of the brown firm more than the green firm improves welfare above cutoff \(m_{B}^0\) and below \(m_{SW}^0\) (see purple-shaded region at the bottom of Figure 4a). Mergers in this region should be approved according to expression (1) since \(\Delta SW \geq 0\), but are blocked according to the welfare criterion in expression (2) where \(\Delta \tilde{SW} < 0\), which we refer to as “socially insufficient mergers” (SIM).

Figure 4b shows that when the green firm becomes polluting \((\alpha \text{ increases from 0 to } \frac{1}{2})\), SIM occurs in both shaded regions. In this context, SEM disappears because, when mergers are blocked according to expression (2), they are also blocked according to the welfare criterion (1).

Our above discussion did not consider the case in which the AA only considers consumer surplus, i.e., \(\tilde{SW}(q_B, q_G) = CS\), but its analysis is straightforward. By Corollary 2, mergers reduce aggregate output under all parameter conditions, and, thus, consumer surplus. Since all mergers in
this context would be blocked by the AA, welfare-enhancing mergers in Figures 3a to 3c would not be approved, only yielding SIM. In this context, SEM do not arise because all mergers are blocked.

![Figure 4](image.png)

**Figure 4**: Socially excessive and insufficient mergers

### 3.3 First stage - merger vote

Let us conclude this section by studying merger incentives in the first stage.

**Lemma 4.** Under no regulation, firms have incentives to merge under all parameter conditions.

Intuitively, when firms merge and coordinate output, the merged firm earns larger profits by internalizing output rivalry, that is, $\pi^M_B + \pi^M_G \geq \pi^{NM}_B + \pi^{NM}_G$, so that every firm has incentives to merge and produce positive units. This result holds even if the merger shuts down the relatively inefficient rival, entailing that the more efficient firm dominates the market.\(^{19}\)

### 4. Equilibrium analysis with environmental regulation

In this section, we introduce the regulator, which uses emission fees to affect firms’ behavior.

\(^{19}\)This follows the standard result in the literature, which suggests that the merger of duopolies into a monopoly is always profitable (Salant et al., 1983).
4.1 Fourth stage - output decisions

In the fourth stage, every firm $i$ makes output decisions in response to the emission fees set in the third stage, where the superscript $R$ denotes variables under environmental regulation.

**Lemma 5.** Under regulation, firm $i$’s output is $q_i^{NM,R} = \frac{2(1-c_i-t_i) - \beta (1-c_j-t_j)}{4-\beta^2}$ if not merged, $q_i^{M,R} = \frac{(1-c_i-t_i) - \beta (1-c_j-t_j)}{2(1-\beta^2)}$ if merged, and $q_i^{M,R} = \frac{1-c_i-t_i}{2}$ if it dominates the market.

Therefore, firm $i$ reduces (increases) output in response to emission fee $t_i$ ($t_j$, respectively).\(^{20}\)

In addition, output is more responsive to emission fees when firms merge since the merged firm considers how emission fees would affect the decision of both firms when they coordinate output.

4.2 Third stage - emission fees

In the third stage, the EPA seeks to identify the socially optimal output levels that solve

$$\max_{q_B, q_G \geq 0} SW(q_B, q_G) = CS + PS + Tax - Env(q_B, q_G)$$

where $Tax = t_B q_B + t_G q_G$ denotes the EPA’s tax revenue from the brown and green firms.

The following lemma identifies the first-best output levels that maximize social welfare, which apply to whether firms merge or not. We discuss our results and then find which emission fees induce firms to respond choosing a socially optimal output in the fourth stage.\(^{21}\)

\(^{20}\)Our model, then, implies that the regulator can set different emission fees on each firm type, given its relative pollution intensity as captured by parameter $\alpha$, which is a common practice when firms in the same industry exhibit asymmetric pollution intensities. In the freight industry, for instance, heavy-duty trucks and freight trains are subject to different air pollution regulation from the EPA. In particular, the EPA issued nitrogen oxides (NOx) and particulate matter emission standards on heavy-duty trucks, making them more stringent in 2007. In contrast, the major non-road freight modes of transportation (trains and marine vessels) were virtually unregulated until the late 1990s, and today remain much less regulated than freight trucks.

\(^{21}\)This approach yields the same results as maximizing social welfare, evaluated at the equilibrium output functions of Lemma 5, with respect to the emission fees. However, to facilitate our calculations, we first identify socially optimal output levels.
Lemma 6. The socially optimal output levels for the brown and the green firm are

\[
(q_{SO}^{B}, q_{SO}^{G}) = \begin{cases} 
(0, \frac{1-c_G}{1+2\alpha^2d}) & \text{if } m_B < m_B^R \\
\left( \frac{(1+2\alpha^2d)(1-c_B)-(\beta+2\alpha d)(1-c_G)}{(1+2\alpha^2d)(1+2d)-(\beta+2\alpha d)^2}, \frac{(1+2d)(1-c_G)-(\beta+2\alpha d)(1-c_B)}{(1+2\alpha^2d)(1+2d)-(\beta+2\alpha d)^2} \right) & \text{if } m_B^R \leq m_B < \overline{m}_B^R \\
(\frac{1-c_B}{1+2\alpha^2d}, 0) & \text{if } m_B \geq \overline{m}_B^R
\end{cases}
\]

where \( m_B^R = \frac{\beta+2\alpha d}{1+2\alpha^2d} \) and \( \overline{m}_B^R = \frac{1+2d}{\beta+2\alpha d} \).

Figure 5a considers the case where no firm pollutes \((d = 0)\). In this setting, it is socially optimal for the relatively inefficient brown (green) firm to shut down when \( m_B \) lies below (above) cutoff \( m_B^R \) (\( \overline{m}_B^R \)), thus coinciding with Figures 1a and 1b. Figure 5b illustrates that the brown firm generates pollution while its green rival is completely clean \((d = \frac{1}{4} \text{ and } \alpha = 0)\). In this context, it is socially optimal to stop the brown firm’s production under the same conditions as in Figure 5a, but it becomes more restrictive for the green firm, as cutoff \( \overline{m}_B^R \) shifts upwards. Finally, Figure 5c considers a setting where both brown and green firms are polluting \((d = \frac{1}{4}, \text{ although asymmetrically since } \alpha = \frac{1}{2})\), where cutoff \( m_B^R \) lies above (below) that in Figure 5a (5b), which implies that it is socially optimal to shut down the green firm under more (less) restrictive conditions than in Figure 5a (5b). However, cutoff \( m_B^R \) shifts up because when firms become less asymmetric in their pollution intensities (\( \alpha \) increases), it is socially optimal to shut down the brown firm under less restrictive parameter conditions.\(^{22}\)

\(^{22}\)Cutoff \( m_B^R \) (\( \overline{m}_B^R \)) is upward- (downward-) sloping, since when goods become more homogeneous, output shifts can achieve more significant efficiency gains. When environmental damages become more severe \((d \text{ increases})\), cutoff \( m_B^R \) (\( \overline{m}_B^R \)) shifts up if and only if \( 0 < \alpha < \frac{1}{\beta} \) \((\alpha < \beta)\), so it becomes more restrictive to shut down the relatively clean green firm. When the green firm becomes more polluting \((\text{higher } \alpha)\), however, cutoff \( m_B^R \) shifts up if and only if \( d < \frac{1-2\alpha\beta}{2\alpha^2} \), which occurs when environmental damages are not too severe, while cutoff \( \overline{m}_B^R \) shifts down for all parameter values.
Next, we consider how the socially optimal output changes in product differentiation and cost.

**Corollary 5.** When \( m_B^R \leq m_B < m_B^R \), socially optimal output \( q_{SO}^B \) (\( q_{SO}^G \)) increases in \( \beta \) if and only if \( m_B \geq m_B^\beta \equiv \frac{(1+2d)(1+2\alpha) + (\beta+2ad)}{2(\beta+2ad)(1+2\alpha)} \frac{m_B}{(1+2\alpha^2)} \) (\( m_B \leq m_B^\beta \equiv \frac{2(\beta+2ad)(1+2d)}{(1+2\alpha^2)} \)), and unambiguously decreases in \( c_B \) (\( c_G \)) but increases in \( c_G \) (\( c_B \)). Otherwise, \( q_{SO}^B \) (\( q_{SO}^G \)) is unaffected by \( \beta \) and only decreases in \( c_B \) (\( c_G \)).

Intuitively, when the brown (green) firm becomes more efficient, specifically, \( m_B \geq m_B^\beta \) (\( m_B \leq m_B^\beta \)), the EPA induces more units of output from this firm when goods are relatively homogeneous, but reduces output from this firm when it becomes less efficient.\(^{23}\)

We are also interested in how the socially optimal output affected by environmental damages.

**Corollary 6.** Socially optimal output, \( q_{SO}^B \) and \( q_{SO}^G \), decrease in \( d \) for all parameter conditions.

When goods become more polluting, the EPA induces fewer units of output from both firms. The next corollary evaluates how the socially optimal output varies in pollution intensities.

**Corollary 7.** When \( m_B^R \leq m_B < m_B^R \), socially optimal output \( q_{SO}^B \) (\( q_{SO}^G \)) increases (decreases) in \( \alpha \) if and only if \( m_B \geq m_B^\alpha \equiv \frac{(1+2d)(1-\alpha\beta) + (\beta+2ad)(\beta-\alpha)}{2(1-\alpha\beta)(\beta+2ad)} \) (\( m_B \leq m_B^\alpha \equiv \frac{2(\beta-\alpha)(1+2d)}{(1+2d)(1-\alpha\beta) + (\beta+2ad)(\beta-\alpha)} \)). Otherwise, \( q_{SO}^B \) (\( q_{SO}^G \)) is unaffected by (decreases in) \( \alpha \).

\(^{23}\) A direct comparison reveals that \( m_B^R < m_B^\beta < m_B^\alpha < m_B^R \), which makes the first result in Corollary 5 feasible.
When the green firm becomes more polluting ($\alpha$ increases), the EPA induces fewer units of both goods when firms are relatively cost-symmetric ($m_B^g < m_B < \bar{m}_B^g$). Further increases in $\alpha$, however, shift cutoff $\bar{m}_B^g$ ($\bar{m}_B^g$) down so that the EPA induces more (fewer) units of the brown (green) good over a wider range of $m_B$ when firms become less asymmetric in their pollution intensities.

Finally, we identify the fees that the EPA can use to induce socially optimal output, as follows.\(^{24}\)

**Proposition 1.** When $m_B^R \leq m < \bar{m}_B^R$, the EPA charges unmerged firms an emission fee of

$$t_B^{NM} = \frac{\left[ \beta + 2(2\alpha - \beta) d \right] (1 - c_G) - \left[ 1 - 2(1 - \alpha \beta - \alpha^2) d \right] (1 - c_B)}{(1 + 2\alpha^2 d)(1 + 2d) - (\beta + 2\alpha d)^2}$$

and merged firms an emission fee of

$$t_G^{NM} = \frac{\left[ \beta + 2\alpha(2 - \alpha \beta)d \right] (1 - c_B) - \left[ 1 + 2(1 + \alpha \beta - \alpha^2) d \right] (1 - c_G)}{(1 + 2\alpha^2 d)(1 + 2d) - (\beta + 2\alpha d)^2}$$

Otherwise, the EPA sets $t_G^M = \frac{(2\alpha^2d - 1)(1 - c_G)}{1 + 2\alpha^2d}$ when it is socially optimal that only the green firm is active ($m_B < \bar{m}_B^R$), and $t_B^M = \frac{(2d - 1)(1 - c_B)}{1 + 2d}$ when it is socially optimal that only the brown firm is active ($m_B \geq \bar{m}_B^R$).

The next corollary ranks the emission fees for each firm type and in each industry structure.

**Corollary 8.** When $\bar{m}_B^R \leq m_B < \bar{m}_B^R$, emission fees satisfy $t_G^{NM} > t_G^M$ and $t_B^{NM} > t_B^M$. Otherwise, when $m_B < \bar{m}_B^R$ ($m_B \geq \bar{m}_B^R$), the EPA shuts down the brown (green) firm and $t_G^{NM} = t_G^M$ ($t_B^{NM} = t_B^M$).

Intuitively, when firms merge and both remain active, the regulator offers a more generous subsidy to the green firm, or charges a less stringent emission fee to the brown firm, to offset the output-reduction effect of market monopolization; as in Collie (2003) and Huck and Konrad (2004), in order to align firms’ output in the subsequent stage to their respective socially optimal levels.\(^{25}\)

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\(^{24}\)The environmental regulator sets taxes on either or both firms, which we refer to as emission fees, to facilitate our presentation. These taxes, however, seek to correct two market failures: the environmental externality from pollution and the imperfect competition in this industry.

\(^{25}\)When $m_B$ is relatively low (high), Proposition 1 suggests that only the green (brown) firm remains active after the merger, and the EPA only needs to set a subsidy (an emission fee) to this firm if and only if $d < \frac{1}{2\alpha^2}$ ($d > \frac{1}{2}$).
**Corollary 9.** When $m_B^R \leq m_B < \overline{m}_B^R$, the brown (green) firm faces a positive emission fee after the merger, that is, $t_B^M > 0$ ($t_G^M > 0$), if (1) $m_B > \overline{m}_B^t \equiv \frac{4d(\beta-\alpha)}{2(1-\alpha^2)d-(1-\beta^2)}$ for $d > \frac{1-\beta^2}{2(1-\alpha^2)}$, or (2) $m_B < \overline{m}_B^t \equiv \frac{4d(\alpha-\beta)}{1-\beta^2-2(1-\alpha^2)d}$ for $d < \frac{1-\beta^2}{2(1-\alpha^2)}$ ($m_B > \overline{m}_B^t \equiv \frac{2(1-\alpha^2)d+1-\beta^2}{4\alpha d(1-\alpha\beta)}$ for all values of $d > 0$).

To understand the above results, note that when output is non-polluting ($d = 0$), the equilibrium fees in Proposition 1 become $t_B^M = -(1 - c_B)$ and $t_G^M = -(1 - c_G)$, implying that both firms receive a subsidy to increase their output to the socially optimal levels, whether both firms are active or the relatively inefficient firm is shut down. Further increases in $d$, however, shift cutoffs $\overline{m}_B^t$ and $\overline{m}_B^t$ down, so that both firms face a positive emission fee under a wider range of efficiency conditions.

### 4.3 Second stage - merger approval

In the second stage, we evaluate the AA’s merger approval decisions.

**Lemma 7.** Mergers are approved by the AA under all parameter conditions.

The AA, anticipating that the EPA in the third stage will respond setting emission fees that align firms’ output to the socially optimal levels, both when firms merge and when they do not, does not block any merger since mergers do not reduce social welfare under any parameter conditions.

### 4.4 First stage - merger vote

In the first stage, we analyze the firms’ incentives to merge.

**Lemma 8.** Firms have incentives to merge when the merger keeps both firms active.

When firms are relatively cost-symmetric and less polluting ($m_B^t \leq m_B < \overline{m}_B^t$), every firm receives a subsidy to increase output up to the socially optimal level, whether they choose to merge in the first stage or not. When environmental damages become more severe (high $d$), firms have incentives to merge and shift output to the less polluting firm in order to save emission fees.\(^{26}\)

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\(^{26}\)When firms are relatively cost-asymmetric, $m_B < \overline{m}_B^R$ ($m_B \geq \overline{m}_B^R$), the EPA induces the shut down of the inefficient brown (green) firm, allowing the green (brown) firm to dominate the market and produce socially optimal output level.
4.5 Antitrust overlooks environmental effects

Suppose the AA, because of its directives, focuses on consumer and producer surplus, overlooking environmental damages. In this context, we consider two cases: (1) AA anticipates emission fees that the EPA sets in the third stage; and (2) a naïve AA that does not.

In case (1), the AA expects that firms, because of emission fees, will produce socially optimal output whether they merge or not. In this setting, the AA adopts a “hands-free approach” and does not block the merger under any parameter conditions since approving or blocking it yields the same welfare. Interestingly, this happens because the AA anticipates that the EPA in the third stage will design emission fees to induce socially optimal output regardless of the market structure. This outcome is independent of the AA’s specific welfare function and whether it differs from the EPA’s or not. If their welfare functions differ, the AA can anticipate that the EPA will set emission fees inducing a welfare level which is suboptimal from its perspective, but this welfare level occurs regardless of whether the AA approves or blocks the merger. In case (2), however, the AA naïvely assumes that firms are not subject to emission fees in the third stage, so the AA approves (blocks) the merger following the same decision rule as the one considered in section 3.2.1.

In both cases, since the AA’s merger approval decisions only affect industry structure but not output levels, the regulator can subsequently charge different emission fees, following those in Proposition 1 to induce socially optimal output from each firm. In this setting, even if the AA overlooks environmental effects, or whether it ignores EPA’s activities in subsequent stages altogether, output levels are eventually corrected via environmental regulation. In this regard, the AA’s “mistakes” can be rectified by the EPA. However, if the administration of emission fees is costly, or if the EPA does not perfectly observe the firms’ output levels, then those fees may fall short of inducing first-best output levels. In these contexts, it becomes more necessary for the AA to also consider environmental externalities from the mergers.

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27Since aggregate output coincides with and without the merger, consumer surplus also coincides. In addition, producer surplus is in one case subject to emission fees while in the other case is not. Recalling that total tax collection is returned to the firms in the form of a lump-sum transfer, so emission fees are revenue-neutral, and we obtain that social welfare coincides in both settings.
5. Discussion

5.1 Welfare gains from the AA

In this section, we discuss the welfare gains of having an AA in different contexts. When the EPA is present, this welfare gain, as captured by the difference in welfare level when both the AA and EPA are active and when only the EPA is active, \( \Delta SW^{EPA} = SW_{AA,EPA} - SW_{EPA} \), is nil because the EPA induces a socially optimal output in both settings (whether the AA is present and absent).

When the EPA is absent, however, the welfare gain of having an AA may be positive or negative, as suggested in section 3.2. In particular, when the AA is absent, we assume that firms can merge whenever there is a profit gain from it, that is, mergers are allowed under all parameter values. However, when the AA is present, it may approve or block mergers under different settings. Thus, the AA yields no welfare gain when approving mergers, as they would have happened anyway in its absence. In other words, the AA only gives rise to a welfare gain when it blocks a merger, yielding \( SW_{NM,AA} - SW_{M} \). In this context, we must separately consider the following scenarios.

5.1.1 AA considers environmental damages

Graphically, the AA blocks mergers, and thus yields a welfare gain, in the regions between cutoffs \( m_B^{SW} \) and \( \bar{m}_B^{SW} \) in Figures 3a, 3b, and 3c. Otherwise, the AA produces no welfare gain, relative to a setting without this agency, in the regions below cutoff \( m_B^{SW} \) and above cutoff \( \bar{m}_B^{SW} \). In addition, welfare gain \( SW_{NM,AA}^{NM} - SW_{M} \) is particularly large, suggesting that its presence is critical, when firms sell differentiated products (low \( \beta \)), their costs are relatively symmetric (\( m_B \) close to 1), environmental damages are severe (high \( d \)), and the green firm is relatively clean (low \( \alpha \)).

5.1.2 AA ignores environmental damages

In this context, as analyzed in section 3.2.1, the AA considers the social welfare function \( \tilde{SW} \) rather than \( SW \), and may allow mergers that should have been blocked according to \( SW \) (referred to as SEM) and, similarly, block mergers that should have been allowed (denoted as SIM). For simplicity, let us start with the cases in which no regulatory mistakes occur (e.g., \( d = \alpha = 0 \), as in Figure 3a). First, when the AA approves a merger according to both \( SW \) and \( \tilde{SW} \), the presence of
the AA yields no welfare gains since, as discussed above, the merger would have ensued even in the absence of the AA, generating the same social welfare. Second, when the AA blocks a merger according to both $SW$ and $\tilde{SW}$, the presence of the AA yields the same welfare gain as when the AA considers environmental damages, $SW_{AA}^{NM} - SW^M$, capturing the difference in welfare when the merger is prevented (with the AA) and when it ensues (without the AA).

**SEM settings.** When SEMs can be supported, however, the merger is approved when the AA is present, which would have happened in the absence of the AA anyway, yielding no welfare gains, i.e., $\tilde{SW}_{AA}^M - SW^M = 0$. According to this welfare measure, the presence of an AA that ignores environmental damages yields no additional benefits. Nonetheless, from section 3.2, such a merger would have been blocked if the AA takes into account the environmental effects that arise after the merger, implying that this type of AA gives rise to a welfare loss, as captured by $SW_{AA}^{NM} - SW^M$. As discussed above, this welfare loss is particularly large in one of the regions where SEM can be sustained, namely, when goods are relatively homogeneous (high $\beta$), costs are asymmetric (high $m_B$), and environmental damages are strong (high $d$); see the top-right shaded area in Figure 4a.

**SIM settings.** When SIMs can be sustained, the AA blocks a merger that would occur in its absence, yielding a welfare loss $\tilde{SW}_{AA}^{NM} - SW^M$. However, if the AA considered the environmental effects from the merger, this welfare loss would be avoided since $SW_{AA}^{NM} - SW^M < 0$. This loss is particularly large when (i) costs are relatively symmetric ($m_B$ close to 1) and environmental damages are strong (high $d$), as in the bottom-right corner of Figures 4a and 4b; or (ii) firms produce relatively homogeneous goods (high $\beta$), costs are asymmetric (high $m_B$), and the green firm also pollutes (high $\alpha$), as in the top-right corner of Figure 4b. Therefore, in industries with these parameter conditions, we can expect that SIMs yield particularly large welfare losses, implying that the AA’s consideration of environmental damages is especially pronounced in this type of markets. Alternatively, the presence of an AA that ignores environmental damages is, in this setting, welfare-reducing, which implies that society would be better off if the AA was absent. This result suggests that either the AA considers the environmental effects from the merger or, instead, simply approves all merger requests from firms satisfying the above parameter conditions.
5.2 Welfare gains from the EPA

In the absence of the AA, the introduction of the EPA yields an unambiguous welfare gain since the latter induces firms to produce socially optimal output levels. When the AA is present, as discussed in section 4, the EPA keeps inducing socially optimal output, thus yielding the first-best outcome. This result holds when the AA considers environmental damages and the merger approval decision is correct, but the EPA provides firms with incentives to increase or decrease their output to approach the social optimum, ultimately producing a welfare gain. Similarly, this result holds even when the AA ignores the environmental effects from the merger because, while the merger approval decision may be incorrect, the EPA induces firms to produce the same socially optimal output, whether or not they should have merged according to the welfare definition $SW$.

Therefore, the welfare gain attributed to the EPA is the largest when it needs to use emission fees to rectify an incorrect merger approval decision by the AA (which occurs when SEM and SIM are sustained in equilibrium), second largest in settings where the AA is absent, and followed by contexts where the AA is present and made correct merger approval decisions. In other words, the EPA’s role is the most important when the AA overlooks the environmental effects from the merger, but less necessary when the AA considers these effects when evaluating merger proposals.

6. Conclusion

Our results indicate that, while mergers of symmetric firms under no environmental regulation can lead to a welfare loss, as in Tirole (1988) and Lambertini (2013), mergers can improve welfare when firms are relatively asymmetric in costs and pollution intensities. In these settings, mergers should be promoted. Furthermore, if the EPA can charge emission fees, then first-best outcomes are achieved whether firms are allowed to merge or not. Our paper then sheds light on the Horizontal Merger Guidelines (US Department of Justice and Federal Trade Commission, 2010), suggesting that the AA can approve mergers of differentiated firms if they can save costs and reduce pollution. These criteria may offset the anticompetitive effects of mergers, going beyond the conventional wisdom of consumer surplus and efficiency gains in merger evaluation, as in the merger between Boeing Company and McDonnell Douglas Corporation. Examples where a broader perspective in merger evaluations may be useful include mergers between food processing companies Sysco and

Our results further describe that, when the AA fails to consider the environmental effects associated with the mergers, socially excessive mergers can arise when output shifts to the relatively efficient firm after the merger aggravate environmental damages which over-compensate any efficiency gains. These occur when firms are relatively homogeneous but asymmetric in costs and environmental damages. In contrast, socially insufficient mergers that were originally blocked may be reconsidered in light of their reduction of environmental externalities. These happen when firms are relatively symmetric in costs but different in pollution intensities.

Our model can be extended in different directions. First, one could assume that EPA does not set emission fees to maximize social welfare but, instead, sets Pigouvian taxes that coincide with the marginal environmental damage from each firm’s emissions. Second, we could consider that the EPA cannot set firm-specific emission fees, choosing, instead, firm-uniform emission fees. This second-best environmental policy would yield lower welfare levels than those in the first-best scenario that we consider in this paper. Finally, our model can also be extended to allow for the AA, the EPA, or both agencies, not to observe firms’ production costs or pollution intensities, and how the expected welfare will be affected, when emission fees are used to induce socially optimal output levels (Sawaki, 2015).

7. Appendices

7.1 Appendix 1 - Allowing for $n_G$ green and $n_B$ brown firms

In this section, we extend our analysis to a setting with $n_G$ green and $n_B$ brown firms, where $k_G \leq n_G$ green and $k_B \leq n_B$ brown firms consider merging into one firm. Every firm $i$ faces an inverse demand function $p_i(q_i) = 1 - q_i - \sum_{k \neq i} q_k - \beta \sum_{j \neq i} q_j$, where $q_k$ ($q_j$) represents the output of other firms producing homogeneous (differentiated) goods. The timing of the game is as follows:

1. Every firm $i$ decides whether to merge with firm $j$ or not, where at least one firm must be of

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28In this setting, the AA’s role at the beginning of the game may become more relevant, since the AA no longer anticipates that the EPA will induce socially optimal outcomes in the third stage regardless of the merger approval decision.
the other type, \(i \in \{B, G\}\) and \(i \neq j\). If one firm opposes, the merger does not ensue and the game proceeds to stage 3. If all firms agree to merge, the merger proposal is sent to the AA.

(2) The AA, receiving the merger proposal, decides whether to approve or block the merger.

(3) The EPA sets an emission fee \(t^k_i\) to every firm \(i\), which depends on its type \(i \in \{B, G\}\) and the industry structure \(k \in \{M, NM\}\), where \(M (NM)\) denotes merger (no-merger), respectively.

(4) All firms that merge jointly choose output \(q^M_i\) and other firms that do not merge independently choose \(q^{NM}_i\). If the merger does not ensue, every firm \(i\) independently chooses output \(q^{NM}_i\).\(^{30}\)

As in the main body of this paper, we first examine the setting without environmental regulation (where stage 3 is absent), identifying equilibrium output, profits, and social welfare with and without the merger. Then, we evaluate how these results are affected when emission fees are introduced by the EPA in stage 3. For simplicity, we assume \(m_i \geq \frac{\beta(n_j-k_j+1)}{n_j-k_j+2}\) in this section.\(^{31}\)

7.1.1 Equilibrium analysis without regulation

7.1.1.1 Third stage - Output decisions

Operating by backward induction, we begin our analysis with the third stage of the game. For compactness, we define \(r \equiv \beta + 2\alpha_i\alpha_j d, s_i \equiv 1 + 2\alpha_i^2 d, v_i \equiv (n_i - k_i + 2)(n_j - k_j + 1) - \beta^2(n_i - k_i + 1)(n_j - k_j), w_i \equiv \beta^2(n_i - k_i) + n_j - k_j + 2, x \equiv (n_i - k_i + 2)(n_j - k_j + 2) - \beta^2(n_i - k_i)(n_j - k_j), y \equiv (n_i + 1)(n_j + 1) - \beta^2n_in_j, \) and \(z \equiv n_i - k_i + n_j - k_j + 2, \) where \(r, s_i, v_i, w_i, x, y, z > 0, \) as well as \(\alpha_B = 1 \) and \(\alpha_G \equiv \alpha \in [0, 1] \) denoting the pollution intensity of brown and green firm, respectively.

Lemma A1 identifies firm \(i\)'s output before and after the merger.

**Lemma A1.** Before the merger, every firm \(i\) produces \(q^{NM}_i = \frac{(n_j+1)(1-c_j)-\beta n_j(1-c_j)}{y}\) which is positive if and only if \(m_i \geq \frac{\beta n_j}{n_j+1}\). After the merger, every merged firm \(i\) produces: (I) \(q^M_i = 0\) if

\(^{29}\)See Canton et al. (2012) for mergers of homogeneous firms with environmental externalities.

\(^{30}\)Superscripts \(M, UM, \) and \(NM\) denote the merged firm, unmerged firm, and the case of no merger, respectively.

\(^{31}\)If, instead, \(m_i < \frac{\beta(n_j-k_j+1)}{n_j-k_j+2}\), then every firm \(i\) (the rival firm \(j\)) does not produce any output whether it is merged or not, rendering the merger problem uninteresting. In the case of one brown and one green firm, this condition becomes \(m_B < \frac{\beta}{2} (m_B \geq \frac{2}{\beta})\) for the brown (green) firm to remain dormant, as in section 2.
\[ m_i < \frac{\beta z}{w_i}, \quad (2) \quad q_i^M = \frac{w_i(1-c_i) - \beta z(1-c_i)}{(1-\beta^2)x} \text{ if } \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}, \quad \text{and (3) } \quad q_i^M = \frac{(n_j-k_j+1)(1-c_i) - \beta(n_j-k_j)(1-c_j)}{\nu_i} \text{ if } \frac{w_j}{\beta z} \leq m_i < \frac{n_t-k_i+2}{\beta(n_t-k_i+1)}, \text{ while the unmerged firm i produces } q_i^{UM} = \frac{(n_j-k_j+2)(1-c_i) - \beta(n_j-k_j)(1-c_j)}{\nu_i}. \]

When firms are relatively cost-symmetric, where \( \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \), both types of firms produce positive units of output before and after the merger. Otherwise, type-i firms shut down the relatively inefficient type-j firms after the merger and compete with all outsiders when \( \frac{w_j}{\beta z} \leq m_i < \frac{n_t-k_i+2}{\beta(n_t-k_i+1)} \).

Lemma A2 evaluates firm \( i \)'s output change due to the merger, as captured by \( \Delta q_i^{NR} \equiv q_i^M - q_i^{NM} \).

**Lemma A2.** After the merger, firm \( i \): (1) increases output if \( m_i \geq m_i \equiv \frac{\beta[yz-(1-\beta^2)n_jx]}{w_jy-(1-\beta^2)(n_j+1)x} \); (2) reduces output if \( \frac{\beta z}{w_i} \leq m_i < m_i \); and (3) stops production if \( m_i < \frac{\beta z}{w_i} \).

When \( m_i \geq m_i \), firm \( i \) is more efficient than firm \( j \) and increases output after the merger. Otherwise, firm \( i \) reduces output if its relative efficiency falls within \( \frac{\beta z}{w_i} \leq m_i < m_i \), and stops production if its cost disadvantage widens. Comparative statics of cutoff \( m_i \) with respect to \( n_i \) and \( n_j \) yield intractable expressions, so we next provide a numerical simulation of these cutoffs.

Figure A1a depicts the case of an equal number (10) of brown and green firms, within which, 9 firms of each type merge together. Above cutoff \( m_i \), firm \( i \)'s relative efficiency allows this firm to increase output after the merger, but decrease output otherwise. When it is less efficient than firm \( j \), firm \( i \) is shut down by firm \( j \) after the merger in the region between cutoffs \( \frac{\beta z}{w_i} \) and \( \frac{2\beta z}{\beta} \). When firm \( i \)'s cost disadvantage to firm \( j \) widens, it remains dormant before and after the merger below cutoff \( \frac{2\beta}{\beta} \). When firms produce more homogeneous goods (higher \( \beta \)), however, they engage in more intensive competition with the unmerged firms, shrinking (expanding) the range of \( m_i \) under which firm \( i \) can increase (must reduce) output after the merger, which occurs if \( m_i \) lies above (below) cutoff \( m_i \).

Figure A1b (A1c), in comparison, considers the case of 11 (10) type-i and 10 (11) type-j firms, still with the same number of firms merging together. Given one more homogeneous (heterogeneous) outsider, competition among firms now becomes more (less) intense, so cutoff \( m_i \) shifts up (down), and it becomes more (less) restrictive for firm \( i \) to increase output after the merger.

In addition, when \( n_i = n_j = k_i = k_j = 1 \), Figure A1d agrees with the case of one-to-one merger in Figure 1a. Still with the merger between one brown and one green firm \( (k_i = k_j = 1) \), when the market is comprised of 4 firms of each type \( (n_i = n_j = 4) \), competition becomes more intense, and cutoff \( m_i \) shifts up, indicating that it becomes more restrictive for every merged firm \( i \) to increase
output after the merger. Finally, when 2 out of 4 for each type of firms to merge \((n_i = n_j = 4\) and \(k_i = k_j = 2\)), cutoff \(m_i\) shifts down, indicating that as every merged firm \(i\) gains more market power, it becomes less restrictive for the merged firm to increase output after the merger.

![Graphs showing output changes under different conditions](image)

(a) \(n_i = n_j = 10\) and \(k_i = k_j = 9\)  
(b) \(n_i = 11, n_j = 10,\) and \(k_i = k_j = 9\)  
(c) \(n_i = 10, n_j = 11,\) and \(k_i = k_j = 9\)  
(d) \(n_i = n_j = 1\) and \(k_i = k_j = 1\)  
(e) \(n_i = n_j = 4\) and \(k_i = k_j = 1\)  
(f) \(n_i = n_j = 4\) and \(k_i = k_j = 2\)

Figure 6: Output change under no regulation

Next, Corollary A1 compares firm \(i\)'s output change to firm \(j\)'s as a result of the merger.

**Corollary A1.** Firm \(i\) increases (reduces) output more than firm \(j\)'s, that is, \(\Delta q_i^{NR} \geq \Delta q_j^{NR} \geq 0\) \((\Delta q_i^{NR} \leq \Delta q_j^{NR} < 0)\), if and only if \(m_i \geq \overline{m}_i\) \((m_i \leq \overline{m}_i)\), where \(\overline{m}_i \equiv \frac{(w_i + \beta z) y - (1 - \beta^2) (n_i + n_j + 1) x}{(w_i + \beta z) y - (1 - \beta^2) (\beta n_i + n_j + 1) x}\).

In this context, firm \(i\) increases output more than firm \(j\) after the merger when this firm is relatively more efficient than its rivals, that is, \(m_i \geq \overline{m}_i\). As in section 3, this result embodies three cases that: (1) firm \(i\) and firm \(j\) decrease their output after the merger, but firm \(i\)'s output reduction
is smaller than its rival; (2) firm $i$ increases its output while firm $j$ decreases its own; and (3) firm $i$ increases its output while firm $j$ shuts down its operation. For example, when $n_i = k_i = k_i = k_j = 1$, this cutoff becomes $\bar{m}_i = 1$, thus reconciling with our results in Corollary 1.

Corollary A2 studies the merged firms’ output change, $\Delta Q^{NR} \equiv q_i^M + q_j^M - k_i q_i^{NM} - k_j q_j^{NM}$.

**Corollary A2.** Mergers lead the merged firms to reduce output under all parameter conditions.

Intuitively, when a few firms merge, outsiders take advantage of the merger to increase output, which, in turn, reduce the merged firms’ output. When more firms merge, however, the merged firm obtains a larger market power to reduce output, command a higher price, and earn greater profits.

### 7.1.1.2 Second stage - Merger approval

In the second stage of the game, the AA anticipates the output that every firm $i$ produces in the third stage, and approves the merger if and only if it improves welfare, that is, $SW(Q^M_B, Q^M_G) \geq SW(Q^{NM}_B, Q^{NM}_G)$. The following Lemma characterizes the AA’s merger approval decisions.$^{32}$

**Lemma A3.** The AA blocks the merger when $\underline{m}^{SW}_B \leq m_B \leq \bar{m}^{SW}_B$, and approves it otherwise.

As an illustration, we consider the case of $n_B = n_G = 10$ and $k_B = k_G = 9$, as depicted in Figures A2a and A2b. When no firm generates environmental damages, $d = \alpha = 0$, mergers reduce welfare for all parameter values and are blocked by the AA. This occurs because the outsiders’ output increase does not offset the insiders’ output decrease, yielding a reduction in aggregate output after the merger, with an associated loss of consumer surplus that exceeds the gain in producer surplus.

When emissions only originate from the brown firms, $d = \frac{1}{4}$ and $\alpha = 0$, as depicted in Figure A2a, mergers above cutoff $\bar{m}^{SW}_B$ lead to a reduction in the aggregate output of the brown firms, thus decreasing their environmental damages. In the shaded region of Figure A2a, overall welfare improves due to the merger, implying that the AA approves it. When both firms are polluting, $d = \frac{1}{4}$ and $\alpha = \frac{1}{2}$, as depicted in Figure A2b, mergers above cutoff $\bar{m}^{SW}_B$ help reduce aggregate output of both firm types, but more significantly from the brown firms, which mitigates environmental damages. Relative to Figure A2a, $\bar{m}^{SW}_B$ shifts down as $\alpha$ increases, which implies that the shaded region of welfare-improving mergers expands.$^{33}$

$^{32}$For compactness, cutoffs $\underline{m}^{SW}_B$ and $\bar{m}^{SW}_B$ are provided in the proof of Lemma A3 in Appendix 2.

$^{33}$Out result suggests that cutoff $\bar{m}^{SW}_B$ does not fall within $\frac{2\beta}{3}$ and $\frac{3\beta}{2}$, so it is absent from Figures A2a and A2b.
Since welfare-improving mergers above cutoff $m_B^{SW}$ and below $\frac{3\beta}{2}$ are blocked by an AA that only considers consumer and producer surplus (ignoring environmental damages), SIM occurs in the shaded regions of Figures A2a and A2b. In contrast, SEM does not occur since no mergers are approved in the absence of environmental damages. Similarly, when the AA considers consumer surplus alone, all mergers proposals will be blocked, yielding SIM in the same shaded regions.

![Figure 7: Social welfare under no regulation](image)

7.1.1.3 First stage - Merger vote

In the first stage of the game, let us study firms’ incentives to merge. For compactness, we define $\Delta \pi_i^{NR} = \pi_i^M + \pi_j^M - k_i \pi_i^{NM} - k_j \pi_j^{NM}$ to be the merged firm’s profit gain from the merger.\(^{34}\)

**Lemma A4.** Under no regulation, firm $i$ has incentives to merge if and only if $m_i^{NR} \leq m_i \leq \overline{m}_i^{NR}$.

Figure A3a depicts the case with 10 firms of each type ($n_i = n_j = 10$), within which, 9 firms have incentives to merge ($k_i = k_j = 9$) in the region above cutoff $m_i^{NR}$ and below cutoff $\overline{m}_i^{NR}$, where mergers are profitable if firms are relatively cost-symmetric in producing more differentiated goods (low $\beta$), since the outsiders are less able to substitute for the output of the insiders. Figure A3b

\(^{34}\)For compactness, cutoffs $m_i^{NR}$ and $\overline{m}_i^{NR}$ are provided in the proof of Lemma A4 in Appendix 2.
considers the case with one more outsider producing homogeneous goods \((n_i = 11 \text{ and } n_j = 10)\). Facing more intense competition, firm \(i\) still has incentives to merge if it is relatively efficient in producing more differentiated goods, as seen by the downward shifting of cutoff \(m_{NR}^i\) that shrinks the region \((\beta, m)\)-pairs denoting profitable mergers. Figure A3c illustrates the case with one more outsider producing differentiated goods \((n_i = 10 \text{ and } n_j = 11)\), where cutoff \(m_{NR}^i\) rotates counterclockwise, so that a less efficient firm \(i\) has incentives to merge if goods are sufficiently differentiated.

![Diagram](image)

(a) \(n_i = n_j = 10 \text{ and } k_i = k_j = 9\)  
(b) \(n_i = 11, n_j = 10, \text{ and } k_i = k_j = 9\)  
(c) \(n_i = 10, n_j = 11, \text{ and } k_i = k_j = 9\)

Figure 8: Merger incentives under no regulation

Figure A4a identifies the subset of profitable mergers assuming \(\beta = \frac{1}{3}, m_B = \frac{2}{3}, \text{ and } n_B = n_G = 50\). The shaded region above (to the right of) the cutoff finds for each proportion of brown (green) firms on the horizontal (vertical) axis, \(k_B/n_B\) \((k_G/n_G)\), the proportion of green (brown) firms, \(k_G/n_G\) \((k_B/n_B)\), required for mergers to be profitable for every participating green (brown) firm. The unshaded region in the bottom-left corner, in contrast, indicates that neither the brown nor the green firm has incentives to merge, since there are too few merging firms to generate sufficient market power to offset the output substitution effect of the outsiders (Salant et al., 1983; Gelves, 2014). Figure A4b shows that when the number of firms increases \((n_B = n_G = 250 \text{ in dashed and } n_B = n_G = 1,000 \text{ in dotted lines})\), we need a larger proportion of firms merging to compete with an increased number of outsiders in a more competitive market. Similar results hold if we assume that goods are more homogeneous (higher \(\beta\)) or firms are more cost-asymmetric (higher \(m_i\)).
7.1.2 Equilibrium analysis with environmental regulation

7.1.2.1 Fourth stage - Output decisions

In this stage, every firm \( i \) produces output in response to emission fees set in the third stage.

**Lemma A5.** Under regulation, firm \( i \)'s output is \( q_{iNM}^N(t_i, t_j) = \frac{(n_j+1)(1-c_i-t_i)-\beta n_j(1-c_j-t_j)}{n_i} \) before the merger. After the merger, firm \( i \)'s output is \( q_{iM}^M(t_i, t_j) = \frac{w_i(1-c_i-t_i)-\beta z(1-c_j-t_j)}{(1-\beta^2)x} \) if merged and \( q_{iUM}^M(t_i, t_j) = \frac{(n_j-k_j+2)(1-c_i-t_i)-\beta(n_j-k_j)(1-c_j-t_j)}{x} \) if not merged. If firm \( i \) is the only active firm in the market, its output is \( q_{iM}^M(t_i) = \frac{1-c_i-t_i}{n_i-k_i+2} \).

Evaluating these output levels at \( n_i = n_j = k_i = k_j = 1 \), firm \( i \)'s output is \( q_{iNM}^N(t_i, t_j) = \frac{2(1-c_i-t_i)-\beta(1-c_j-t_j)}{4-\beta^2} \) before and \( q_{iM}^M(t_i, t_j) = \frac{1-c_i-t_i-\beta(1-c_j-t_j)}{2(1-\beta^2)} \) after the merger, which coincide with those in Lemma 5. In this setting, output \( q_{iUM}^M \) does not apply to the case of a full merger.
7.1.2.2 Third stage - Emission fees

In the third stage, the EPA chooses aggregate output $Q_i^{SO}$ for every firm $i$ that solves

$$SW(Q_B, Q_G) = CS + PS + Tax - Env(Q_B, Q_G)$$

where $Tax = t_B Q_B + t_G Q_G$ represents the total tax revenue collected from all brown and green firms. The following lemma finds this aggregate socially optimal output solving the EPA’s problem.

**Lemma A6.** The socially optimal aggregate output that applies to firm $i$ is

$$Q_i^{SO} = \begin{cases} 
0 & \text{if } m_i < \overline{m}_i^R \\
\frac{s_j(1-c_j) - r(1-c_j)}{s_i s_j - r^2} & \text{if } \overline{m}_i^R \leq m_i < \overline{m}_i^R \\
1-c_j & \text{if } m_i \geq \overline{m}_i^R
\end{cases}$$

where $\overline{m}_i^R = \frac{r}{s_j}$ and $\overline{m}_i^R = \frac{z_i}{r}$.

This socially optimal output is independent of the number of firms, and coincides with those in Lemma 6. In this context, the EPA sets emission fees to induce firms to produce the above output.

**Proposition A1.** When $\overline{m}_i^R \leq m_i < \overline{m}_i^R$, the EPA charges a fee to every firm $i$ before the merger of

$$t_i^{NM} = \frac{\left[n_i(s_is_j - r^2) - ((n_i + 1)s_j - \beta n_i r)\right](1-c_i) + \left[(n_i + 1)r - \beta n_i s_i\right](1-c_j)}{n_i(s_is_j - r^2)}$$

and after the merger of

$$t_i^{M} = \frac{\left[(s_is_j - r^2)(v_iv_j - \beta^2(x-z)^2) - (1-\beta^2)(s_j v_i - \beta r(x-z))x\right](1-c_j) + \left[(r v_i - \beta s_i(x-z))x\right](1-c_j)}{(v_iv_j - \beta^2(x-z)^2)(s_is_j - r^2)}$$

When $m_i \geq \overline{m}_i^R$, it is socially optimal that only firm $i$ is active, setting $t_i^{M} = \frac{[s_i(1-n_i k_i)+s_j-2](1-c_j)}{s_i(n_i k_i+1)}$.

Evaluating firm $i$’s optimal emission fees at $n_i = n_j = k_i = k_j = 1$, we obtain that $t_i^{NM} = \frac{(2\beta s_i)(1-c_j) - [s_j(2-s_j) + r(\beta - r)](1-c_j)}{s_i s_j - r^2}$ before the merger and $t_i^{M} = \frac{2(\beta s_i)(1-c_j) - [s_j(2-s_j) + r(\beta - r)](1-c_i)}{s_i s_j - r^2}$ after the merger when both firms are active, and $t_i^{M} = \frac{(2d^2 - 1)(1-c_i)}{s_i}$ when it is socially optimal that only firm $i$ is active, all of which coincide with those identified in Proposition 1 (see section 4).

Figure 5 plots those emission fees assuming $\alpha = \frac{1}{2}$, $d = \frac{1}{4}$, $n_B = n_G = 10$, and $k_B = k_G = 1$, above (below) the blue (black) cutoff which the relatively efficient brown (green) firm is subsidized.
to produce more homogeneous output; whereas, emission fees are charged to all firms in the shaded region between the two cutoffs when firms are not merged (Figure A5a) and merged (Figure A5b).

Figure 10: Optimal emission fees

7.1.2.3 Second stage - Merger approval

The following lemma characterizes the AA’s merger approval decisions in the second stage.

**Lemma A7.** Under regulation, mergers are approved by the AA under all parameter conditions.

Since the EPA can charge firm-specific fees as a function of industry structure (that is, how many firms are merged), socially optimal output can always be induced in the fourth stage of the game whether firms are merged or not. As a result, the AA can just approve all merger proposals.

7.1.2.4 First stage - Merger vote

The following lemma examines firm $i$’s incentives to merge with firm $j$ in the first stage.\footnote{For compactness, cutoffs $\overline{m}_i^\pi$ and $\underline{m}_i^\pi$ are provided in the proof of Lemma A8 in Appendix 2.}

**Lemma A8.** Firm $i$ has incentives to merge when $m_i \leq \overline{m}_i^\pi$ or $m_i \geq \underline{m}_i^\pi$. 

\footnote{For compactness, cutoffs $\overline{m}_i^\pi$ and $\underline{m}_i^\pi$ are provided in the proof of Lemma A8 in Appendix 2.}
Evaluating cutoffs at \( n_i = n_j = k_i = k_j = 1 \), we obtain \( m_R^i = \bar{m}_i^\pi \) and \( m_R^j = m_j^\pi \), implying that the conditions on \( m_i \) in Lemma A8 are satisfied for all admissible values of \( m_i \), thus confirming our result in Lemma 8, where both types of firms have incentives to merge under all parameter values.

As an illustration, Figure 6 considers \( n_B = n_G = 10 \) and \( k_B = k_G = 1 \), where the brown firm has incentives to merge in the following two shaded regions, (1) above cutoff \( \bar{m}_B^R \) and below cutoff \( m_B^R \), and (2) above cutoff \( m_B^R \) and below cutoff \( m_B^\pi \), where this or the rival firm is relatively efficient in producing more homogeneous goods, respectively. As a benchmark, Figure A6a considers the case of \( d = \alpha = 0 \), where neither firm pollutes. Figure A6b illustrates the case of \( d = \frac{1}{4} \) and \( \alpha = 0 \), where mergers can be supported under a wider range of parameter values when the brown firm is induced by emission fees to merge with the green firm. Figure A6c depicts the case of \( d = \frac{1}{4} \) and \( \alpha = \frac{1}{2} \), where mergers can be supported under less (more) restrictive conditions compared to those in Figure A6a (A6b) when the green firm also pollutes but to a lesser extent than the brown firm.

![Figure 11: Merger incentives under regulation](image)

Figure A7a also depicts the firms’ incentives to merge using a similar approach as in Figure 4, namely, plotting the market share of the merging green firms (vertical axis) and of the merging brown firms (horizontal axis). As in Figure 4, we assume that \( \beta = \frac{1}{3} \), \( m_B = \frac{2}{3} \), and \( n_B = n_G = 50 \), where the shaded region at the top-right corner identifies the subset of profitable mergers. Relative to Figure A4a (which assumes no environmental regulation), the region in which both firms have incentives to merge expands. This occurs because the green firm is subsidized to produce relatively clean products, and acquiring its brown rivals helps soften competition. Meanwhile, the brown
firm is taxed to reduce relatively polluting output, so merging with the green firm induces output shifts that help save emission fees.

![Diagram showing market concentration under regulation](image)

Figure 12: Market concentration under regulation

Figure A7b shows that when there are more firms in the market ($n_B = n_G = 250$ in dashed and $n_B = n_G = 1,000$ in dotted lines), the merger is only profitable if more firms from both types join. This result is analogous to that in Figure A4b without environmental regulation, as an increase in $n_B$ and $n_G$, keeping the number of insiders $k_B$ and $k_G$ constant, increases the number of outsiders, each of them individually increasing output level after the merger (output substitution effect), thereby making mergers more difficult to sustain in a more competitive market.

Benchekroun et al. (2019) evaluate merger profitability in industries exploiting a natural resource, such as oil, showing that, if every firm owns a small portion of the stock, mergers become profitable even if a small number of firms join. While we do not study natural resource exploitation, we demonstrate that environmental regulation makes mergers profitable only when a larger number of firms join, as in Benchekroun et al. (2019), where environmental policy can deter the merger.
7.2 Appendix 2 - Technical proofs

7.2.1 Proof of Lemma 1

**No merger.** Every firm $i$ chooses $q_i$ to solve the profit maximization problem, as follows,

$$\max_{q_i \geq 0} \pi(q_i) = (1 - q_i - \beta q_j) q_i - c_i q_i$$

Taking the first order condition with respect to $q_i$, and assuming interior solutions,

$$1 - 2q_i - \beta q_j - c_i = 0$$

such that the best response function of firm $i$, in response to firm $j$’s output, becomes

$$q_i(q_j) = \frac{1 - \beta q_j - c_i}{2}$$

Since in equilibrium firms’ output are mutual best response to each other, we obtain that

$$q_i^{NM} = \frac{2(1 - c_i) - \beta(1 - c_j)}{4 - \beta^2}$$

Defining $m_i \equiv \frac{1 - c_i}{1 - c_j}$, firm $i$ produces positive units before the merger, $q_i^{NM} > 0$, if and only if

$$m_i \geq \frac{\beta}{2}$$

Substituting $q_i^{NM}$ into the profit function, every unmerged firm $i$ earns equilibrium profits of

$$\pi_i^{NM} = \left(\frac{2(1 - c_i) - \beta(1 - c_j)}{4 - \beta^2}\right)^2$$

**Merger.** The merged firm solves the joint profit maximization problem, as follows,

$$\max_{q_i, q_j \geq 0} \pi(q_i, q_j) = (1 - q_i - \beta q_j - c_i) q_i + (1 - q_j - \beta q_i - c_j) q_j$$

Taking the first order condition with respect to $q_i$, and assuming interior solutions,

$$1 - 2q_i - 2\beta q_j - c_i = 0$$

Solving $q_i$ and $q_j$ simultaneously, we obtain

$$q_i^M = \frac{(1 - c_i) - \beta (1 - c_j)}{2(1 - \beta^2)}$$
In this context, firm $i$ produces positive units after the merger, $q_i^M > 0$, if and only if

$$m_i \geq \beta$$

Plugging $q_i^M$ and $q_j^M$ into the profit function, every merged firm $i$ earns equilibrium profits of

$$\pi_i^M = \frac{1 - c_i}{2} \cdot \frac{(1 - c_i) - \beta (1 - c_j)}{2 (1 - \beta^2)}$$

**Monopoly.** Suppose firm $i$ dominates the market after the merger, then it chooses $q_i$ to solve

$$\max \pi(q_i) = (1 - q_i) q_i - c_i q_i$$

Taking the first order condition with respect to $q_i$, and assuming interior solutions,

$$1 - 2q_i - c_i = 0$$

Solving for the output of firm $i$, we obtain

$$q_i^M = \frac{1 - c_i}{2}$$

with associated profits of

$$\pi_i^M = \frac{(1 - c_i)^2}{4}$$

### 7.2.2 Proof of Lemma 2

Firm $i$ increases output after the merger if and only if $\Delta q_i^{NR} = q_i^M - q_i^{NM} = \frac{3\beta^2(1-c_i) - \beta (2+\beta^2)(1-c_j)}{2(1-\beta^2)(4-\beta^2)} \geq 0$, which is rearranged to yield $m_i \geq \frac{2+\beta^2}{3\beta^2}$.

### 7.2.3 Proof of Corollary 1

$$\Delta q_i^{NR} < \Delta q_j^{NR}$$ if and only if $\frac{3\beta^2(1-c_i) - \beta (2+\beta^2)(1-c_j)}{2(1-\beta^2)(4-\beta^2)} < \frac{3\beta^2(1-c_i) - \beta (2+\beta^2)(1-c_j)}{2(1-\beta^2)(4-\beta^2)}$, yielding $m_i < 1$.

### 7.2.4 Proof of Corollary 2

When $\beta \leq m_i < \frac{1}{\beta}$, $q_i^M + q_j^M = \frac{2-c_i-c_j}{2(1+\beta)} < \frac{2-c_i-c_j}{2+\beta} = q_i^{NM} + q_j^{NM}$ for all values of $\beta$ and $0 \leq c_i < 1$.

When $\frac{1}{\beta} \leq m_i < \frac{2}{\beta}$, $q_i^M = \frac{1-c_i}{2} < \frac{2-c_i-c_j}{2(1+\beta)}$ reduces to $m_i < \frac{2}{\beta}$ that holds by assumption.
7.2.5 Proof of Corollary 3

Consumer surplus before the merger is

\[
CS^{NM} = \frac{(4 - 3\beta^2)(1 - c_i)^2 + 2\beta^3(1 - c_i)(1 - c_j) + (4 - 3\beta^2)(1 - c_j)^2}{2(4 - \beta^2)^2}
\]

When \(\beta \leq m_i < \frac{1}{\beta}\), consumer surplus after the merger is

\[
CS^M = \frac{(1 - c_i)^2 - 2\beta(1 - c_i)(1 - c_j) + (1 - c_j)^2}{8(1 - \beta^2)}
\]

Differentiating the loss of consumer surplus, \(\Delta CS = CS^{NM} - CS^M\), with respect to \(\beta\), we obtain

\[
\frac{\partial \Delta CS}{\partial \beta} > 0 \quad \text{when} \quad m_i^{CS} < m_i < \overline{m}_i^{CS}, \quad \text{where} \quad m_i^{CS} = \frac{V - \sqrt{B^2 - 4\beta^2}}{2A} > \beta \quad \text{and} \quad \overline{m}_i^{CS} = \frac{B + \sqrt{B^2 - 4\beta^2}}{2A} < \frac{1}{\beta}, \quad \text{with} \quad A \equiv \beta \left(80 - 68\beta^2 + 4\beta^4 + 11\beta^6\right) > 0 \quad \text{and} \quad B \equiv 64 + 64\beta^2 - 128\beta^4 + 51\beta^6 + 3\beta^8 > 0. \quad \text{When} \quad \frac{1}{\beta} \leq m_i < \frac{2}{\beta}, \quad CS^M = \left(\frac{1-c_j^2}{8}\right) \quad \text{that does not depend on} \ \beta, \quad \text{so that} \quad \frac{\partial \Delta CS}{\beta} = \frac{\partial CS^{NM}}{\beta} - \frac{\partial CS^M}{\beta} = \frac{\partial CS^{NM}}{\beta} < 0.
\]

7.2.6 Proof of Corollary 4

Producer surplus before the merger is

\[
PS^{NM} = \frac{(4 + \beta^2)(1 - c_i)^2 - 8\beta(1 - c_i)(1 - c_j) + (4 + \beta^2)(1 - c_j)^2}{(4 - \beta^2)^2}
\]

When \(\beta \leq m_i < \frac{1}{\beta}\), producer surplus after the merger is

\[
PS^M = \frac{(1 - c_i)^2 - 2\beta(1 - c_i)(1 - c_j) + (1 - c_j)^2}{4(1 - \beta^2)}
\]

We find that \(\Delta PS = PS^M - PS^{NM}\) increases in \(\beta\) for all \(\beta \in [0, 1]\). When \(\frac{1}{\beta} \leq m_i < \frac{2}{\beta}\), however, \(PS^M = \left(\frac{1-c_i^2}{4}\right) \quad \text{that does not depend on} \ \beta, \quad \text{so} \quad \frac{\partial \Delta PS}{\beta} = \frac{\partial PS^M}{\beta} = \frac{\partial PS^{NM}}{\beta} = \frac{\partial PS^{NM}}{\beta} > 0 \quad \text{if and only if} \quad \frac{2(4+3\beta^2)-(4-\beta^2)^3}{\beta(12+\beta^2)} \equiv m_i^{PS} < m_i < \overline{m}_i^{PS} \equiv \frac{2(4+3\beta^2)+(4-\beta^2)^3}{\beta(12+\beta^2)}, \quad \text{where} \quad \frac{1}{\beta} < m_i^{PS} < \beta \quad \text{and} \quad \frac{1}{\beta} < \overline{m}_i^{PS} < \frac{2}{\beta}.
\]

7.2.7 Proof of Lemma 3

Let \(r \equiv \beta + 2\alpha_i\alpha_j d\) and \(s_i \equiv 1 + 2\alpha_i^2 d\), where \(\alpha_B = 1\) and \(\alpha_G = \alpha\). Social welfare becomes

\[
SW(q_i, q_j) = \frac{q_i^2 + 2\beta q_i q_j + q_j^2}{2CS} + \left[p_i(q_i) - c_i\right]q_i + \left[p_j(q_j) - c_j\right]q_j - d\left(\alpha_i q_i + \alpha_j q_j\right)^2 - \frac{\partial \Delta PS}{\beta}
\]

\[
= \left(1 - \frac{s_i}{2} q_i - c_i\right)q_i + \left(1 - \frac{s_j}{2} q_j - c_j\right)q_j - r q_i q_j
\]

(A1)
Substituting $q_i^{NM} = \frac{2(1-c_i) - \beta(1-c_i)}{4 \beta^2}$, social welfare without the merger becomes

$$SW^{NM} = \frac{E_i (1 - c_i)^2 - 2F (1 - c_i) (1 - c_j) + E_j (1 - c_j)^2}{2 (4 - \beta^2)^2}$$

where $E_i = 4(4 - \beta^2 + \beta r - s_i) - \beta^2 s_j$ and $F = r(4 + \beta^2) - 2\beta (s_i + s_j - 4 + \beta^2)$.

When both firms are active, plug $q_i^M = \frac{(1-c_i) - \beta(1-c_i)}{2(1-\beta^2)}$ into the social welfare function, yields

$$SW^M = \frac{C_i (1 - c_i)^2 - 2D (1 - c_i) (1 - c_j) + C_j (1 - c_j)^2}{8 (1 - \beta^2)^2}$$

where $C_i \equiv 4(1 - \beta^2) + 2\beta r - s_i - \beta^2 s_j$ and $D \equiv (1 + \beta^2) r - \beta [s_i + s_j - 4(1 - \beta^2)]$.

Therefore, the AA approves the merger if and only if $SW^M \geq SW^{NM}$ for $\beta \leq m_i < \frac{1}{\beta}$, solving

$$[(4 - \beta^2)^2 C_i - 4(1 - \beta^2)^2 E_i] m_i^2 - 2[(4 - \beta^2)^2 D - 4(1 - \beta^2)^2 F] m_i^2 + [(4 - \beta^2)^2 C_j - 4(1 - \beta^2)^2 E_j] \geq 0$$

Otherwise, if firm $i$ shuts down the rival firm $j$ after the merger, social welfare becomes

$$SW_i^M = \frac{(3 - 2\alpha_i^2 d) (1 - c_i)^2}{8}$$

Then, the AA approves the merger if and only if $SW_i^M \geq SW^{NM}$ for $\frac{1}{\beta} \leq m_i < \frac{2}{\beta}$, solving

$$[(4 - \beta^2)^2 (3 - 2\alpha_i^2 d) - 4E_i] m_i^2 + 8Fm_i - 4E_j \geq 0$$

In summary, mergers are welfare-improving if and only if $m_i \leq m_i^{SW}$ or $m_i \geq m_i^{SW}$, where

$$m_i^{SW} = \begin{cases} \frac{(4 - \beta^2)^2 (3 - 2\alpha_i^2 d) - 4E_i}{2 \sqrt{4F^2 + E_i (4 - \beta^2)^2 (3 - 2\alpha_i^2 d) - 4E_i} - 2F} & \text{if } \beta \leq m_i < \frac{1}{\beta}, \\
\frac{(4 - \beta^2)^2 D - 4(1 - \beta^2)^2 F - \sqrt{[(4 - \beta^2)^2 D - 4(1 - \beta^2)^2 F]^2 - [(4 - \beta^2)^2 C_i - 4(1 - \beta^2)^2 E_i] [(4 - \beta^2)^2 C_j - 4(1 - \beta^2)^2 E_j]}}{(4 - \beta^2)^2 C_i - 4(1 - \beta^2)^2 E_i} & \text{if } \frac{1}{\beta} \leq m_i < \frac{2}{\beta}. \end{cases}$$

$$m_i^{SW} = \begin{cases} \frac{(4 - \beta^2)^2 D - 4(1 - \beta^2)^2 F + \sqrt{[(4 - \beta^2)^2 D - 4(1 - \beta^2)^2 F]^2 - [(4 - \beta^2)^2 C_i - 4(1 - \beta^2)^2 E_i] [(4 - \beta^2)^2 C_j - 4(1 - \beta^2)^2 E_j]}}{(4 - \beta^2)^2 C_i - 4(1 - \beta^2)^2 E_i} & \text{if } \beta \leq m_i < \frac{1}{\beta}, \\
\frac{2 \sqrt{4F^2 + E_i (4 - \beta^2)^2 (3 - 2\alpha_i^2 d) - 4E_i} - 2F}{(4 - \beta^2)^2 (3 - 2\alpha_i^2 d) - 4E_i} & \text{if } \frac{1}{\beta} \leq m_i < \frac{2}{\beta}. \end{cases}$$

7.2.8 Proof of Lemma 4

When both firms remain active after the merger, which happens when $\beta \leq m_i < \frac{1}{\beta}$, firm $i$ has incentives to merge with firm $j$ if and only if $\pi_i^M + \pi_j^M \geq \pi_i^{NM} + \pi_j^{NM}$, solving

$$(4 + 5\beta^2) m_i^2 - 2\beta (8 + \beta^2) m_i + 4 + 5\beta^2 \geq 0$$
which is positive for all values of $0 \leq \beta \leq 1$ and $m_i > 0$.

When the merger shuts down the relatively inefficient firm $j$, which happens when $\frac{1}{\beta} \leq m_i < \frac{2}{\beta}$, firm $i$ still has incentives to merge with firm $j$ if and only if $\pi_i^M \geq \pi_i^{NM} + \pi_j^{NM}$, yielding

\[-\beta^2 (12 - \beta^2) m_i^2 + 32 \beta m_i - 4 (4 + \beta^2) \geq 0\]

that reduces to $\frac{2(4 + \beta^2)}{\beta(12 - \beta^2)} < m_i < \frac{2}{\beta}$. Since $\frac{2(4 + \beta^2)}{\beta(12 - \beta^2)} < \frac{1}{\beta}$, the above inequality holds for all $\frac{1}{\beta} \leq m_i < \frac{2}{\beta}$.

7.2.9 Proof of Lemma 5

Substituting $c_i + t_i$ for $c_i$ in Lemma 1, we find firm $i$’s output as a function of the emission fees. In addition, $\frac{\partial q_i^{NM,R}}{\partial t_i} = -\frac{2}{4-\beta^2} > -\frac{1}{2(1-\beta^2)} = \frac{\partial q_i^{M,R}}{\partial t_i}$ when both firms remain active after the merger.

7.2.10 Proof of Lemma 6

Taking the first order condition of (A1) with respect to $q_i$, we obtain

\[1 - c_i - (1 + 2\alpha_i^2 d) q_i - (\beta + 2\alpha_i\alpha_j d) q_j = 0\]

Rearranging and solving for $q_i$, socially optimal output becomes

\[q_i^{SO} = \frac{\left(1 + 2\alpha_i^2 d\right) \left(1 - c_i\right) - (\beta + 2\alpha_i\alpha_j d) \left(1 - c_j\right)}{\left(1 + 2\alpha_i^2 d\right) \left(1 + 2\alpha_j^2 d\right) - (\beta + 2\alpha_i\alpha_j d)^2}\]

which is positive when $\frac{\beta + 2\alpha_i\alpha_j d}{1 + 2\alpha_j^2 d} \leq m_i < \overline{m}_i \equiv \frac{1 + 2\alpha_i^2 d}{\beta + 2\alpha_i\alpha_j d}$.

When $m_i < \overline{m}_i$, the EPA sets $q_i^{SO} = 0$ but when $m_i \geq \overline{m}_i$, it solves $\max_{q_i > 0} SW (q_i, 0)$, yielding

\[q_i^{SO} = \frac{1 - c_i}{1 + 2\alpha_i^2 d}\]

7.2.11 Proof of Corollary 5

Differentiating $q_i^{SO}$ with respect to $\beta$, we obtain

\[\frac{\partial q_i^{SO}}{\partial \beta} = \frac{2 (\beta + 2\alpha_i\alpha_j d) \left(1 + 2\alpha_i^2 d\right) \left(1 - c_i\right) - \left[\left(1 + 2\alpha_i^2 d\right) \left(1 + 2\alpha_i^2 d\right) + (\beta + 2\alpha_i\alpha_j d)^2\right] (1 - c_j)}{\left[\left(1 + 2\alpha_i^2 d\right) \left(1 + 2\alpha_j^2 d\right) - (\beta + 2\alpha_i\alpha_j d)^2\right]^2}\]

which is positive if and only if $m_i > \frac{(1+2\alpha_i^2 d)(1+2\alpha_j^2 d)+(\beta+2\alpha_i\alpha_j d)^2}{2(\beta+2\alpha_i\alpha_j d)(1+2\alpha_j^2 d)}$.
7.2.12 Proof of Corollary 6

Differentiating $q_i^{SO}$ with respect to $d$, we find

$$\frac{\partial q_i^{SO}}{\partial d} = -\frac{2(\alpha_i - \alpha_j \beta)^2(1 - c_i) + 2[\alpha_i \alpha_j (1 - \beta)^2 - \beta (\alpha_i - \alpha_j)^2]}{(1 + 2\alpha_i^2 d)(1 + 2\alpha_j^2 d) - (\beta + 2\alpha_i \alpha_j d)^2} (1 - c_j)$$

which is negative if and only if $m_i > \bar{m}_i^d \equiv \frac{\beta(\alpha_i - \alpha_j)^2 - \alpha_i \alpha_j (1 - \beta)^2}{(\alpha_i - \alpha_j)^2}$. Since $\frac{\partial m_i^d}{\partial \alpha_j} = -\frac{\alpha_i (1 - \beta)^2}{(\alpha_i - \alpha_j)^2} < 0$, $\bar{m}_i^d$ attains a maximum at $\alpha_j = 0$, which coincides with $\bar{m}_i^R = \beta$, so $\frac{\partial q_i^{SO}}{\partial d} < 0$ for all parameter values.

7.2.13 Proof of Corollary 7

Differentiating $q_i^{SO}$ with respect to $\alpha_i$, we have

$$\frac{\partial q_i^{SO}}{\partial \alpha_i} = 2d \frac{2(\alpha_j \beta - \alpha_i) \left( 1 + 2\alpha_i^2 d \right)(1 - c_i) - \left[ (1 + 2\alpha_i^2 d) \left( \alpha_j - \alpha_i \beta \right) + \left( \beta + 2\alpha_i \alpha_j d \right) \left( \alpha_j - \alpha_i \right) \right] (1 - c_j)}{\left[ (1 + 2\alpha_i^2 d)(1 + 2\alpha_j^2 d) - (\beta + 2\alpha_i \alpha_j d)^2 \right]^2}$$

which is positive if $m_i > \bar{m}_i^q \equiv \frac{\left( 1 + 2\alpha_i^2 d \right)(\alpha_j - \alpha_i \beta) + \left( \beta + 2\alpha_i \alpha_j d \right) \left( \alpha_j - \alpha_i \right)}{2(\alpha_i - \alpha_j)(1 + 2\alpha_i^2 d)}$ when $\alpha_j \beta > \alpha_i$, yielding $\frac{\partial m_i^q}{\partial \alpha_i} = \frac{\alpha_i (1 - \beta)^2 \left( 1 + 2\alpha_i^2 d \right) + 2\alpha_i d (\alpha_j - \alpha_i \beta)^2}{2(\alpha_i - \alpha_j)^2 (1 + 2\alpha_i^2 d)} > 0$.

Differentiating $q_i^{SO}$ with respect to $\alpha_j$, we have

$$\frac{\partial q_i^{SO}}{\partial \alpha_j} = 2d \frac{2(\alpha_i - \alpha_j \beta) \left( \beta + 2\alpha_i \alpha_j d \right)(1 - c_i) - \left[ (1 + 2\alpha_j^2 d) \left( \alpha_i - \alpha_j \beta \right) + \left( \beta + 2\alpha_i \alpha_j d \right) \left( \alpha_i - \alpha_j \right) \right] (1 - c_j)}{\left[ (1 + 2\alpha_i^2 d)(1 + 2\alpha_j^2 d) - (\beta + 2\alpha_i \alpha_j d)^2 \right]^2}$$

which is positive if $m_i > \bar{m}_i^q \equiv \frac{\left( 1 + 2\alpha_j^2 d \right)(\alpha_i - \alpha_j \beta) + \left( \beta + 2\alpha_i \alpha_j d \right) \left( \alpha_i - \alpha_j \right)}{2(\alpha_i - \alpha_j)(\beta + 2\alpha_i \alpha_j d)}$ when $\alpha_i > \alpha_j \beta$, yielding $\frac{\partial m_i^q}{\partial \alpha_j} = -\frac{\alpha_i (1 - \beta)^2 \left( \beta + 2\alpha_i \alpha_j d \right)^2 + 2\alpha_i d (1 + 2\alpha_j^2 d) (\alpha_i - \alpha_j \beta)^2}{2(\alpha_i - \alpha_j)^2 (\beta + 2\alpha_i \alpha_j d)^5} < 0$.

7.2.14 Proof of Proposition 1

When $m_i^R \leq m_i < \bar{m}_i^R$, the EPA chooses $t_i^{NM}$ to solve $q_i^{NM,R} = q_i^{SO}$ for every unmerged firm $i$ and $t_i^M$ to solve $q_i^{M,R} = q_i^{SO}$ for every merged firm $i$, yielding

$$t_i^{NM} = \frac{\left[ \beta + 2\alpha_i \left( 2\alpha_j - \alpha_j \beta \right) d \right](1 - c_j) - \left[ 1 + 2(\alpha_j^2 - \alpha_i^2 + \alpha_i \alpha_j \beta) d \right](1 - c_i)}{(1 + 2\alpha_i^2 d)(1 + 2\alpha_j^2 d) - (\beta + 2\alpha_i \alpha_j d)^2}$$

$$t_i^M = \frac{4\alpha_i d \left( \alpha_j - \alpha_i \beta \right)(1 - c_j) - \left[ 1 - \beta^2 + 2 \left( \alpha_j^2 - \alpha_i^2 \right) d \right](1 - c_i)}{(1 + 2\alpha_i^2 d)(1 + 2\alpha_j^2 d) - (\beta + 2\alpha_i \alpha_j d)^2}$$
Otherwise, when \( m_i \geq \overline{m}_i^R \), the EPA chooses \( t_i^M \) to solve \( q_i^{M,R} = q_i^{SO} \) for firm \( i \), yielding

\[
t_i^M = \frac{(2\alpha_i^2 d - 1)(1 - c_i)}{1 + 2\alpha_i^2 d}
\]

### 7.2.15 Proof of Corollary 8

When \( m_i^R \leq m_i < \overline{m}_i^R \), \( t_i^{NM} > t_i^M \) if and only if \( m_i < \frac{1 + 2\alpha_i^2 d}{\beta + 2\alpha_i \alpha_j d} \), which coincides with \( \overline{m}_i^R \).

### 7.2.16 Proof of Corollary 9

When \( m_i^R \leq m_i < \overline{m}_i^R \),
\[
\frac{\partial m_i}{\partial d} = -\frac{4(\beta - \alpha)(1 - \beta^2)}{(1 - \beta^2 + 2\alpha^2 d - 2\beta^2)} < 0 \text{ if } \beta > \alpha \text{ and } \frac{\partial \overline{m}_i}{\partial d} = -\frac{1 - \beta^2}{4\alpha d(1 - \alpha \beta)} < 0.
\]

### 7.2.17 Proof of Lemma 8

When \( m_i^R \leq m_i < \overline{m}_i^R \), every firm \( i \) earns a profit of \( \pi_i^{NM,R} = (q_i^{SO})^2 \) before the merger.

After the merger, the merged firm earns a profit of
\[
\pi_i^{M,R} = \pi_i^{M,R} + \pi_j^{M,R} = (1 - q_i^{SO} - \beta q_j^{SO} - c_i - t_i^M) q_i^{SO} + (1 - q_j^{SO} - \beta q_i^{SO} - c_j - t_j^M) q_j^{SO} = (q_i^{SO} + \beta q_j^{SO}) q_i^{SO} + (q_j^{SO} + \beta q_i^{SO}) q_j^{SO} = (q_i^{SO})^2 + 2\beta q_i^{SO} q_j^{SO} + (q_j^{SO})^2
\]

Since \( \pi_i^{M,R} \geq \pi_i^{NM,R} + \pi_j^{NM,R} \), every firm \( i \) has incentives to merge for all values of \( \beta \in [0, 1] \).

When \( m_i \geq \overline{m}_i^R \), firm \( i \) (j) earns \( \pi_i^{M,R} = (q_i^{SO})^2 \) (zero profits) with and without the merger.

### 7.2.18 Proof of Lemma A1

No merger. Let \( N_i \) denote the set of all type-\( i \) firms. Every type-\( i \) firm chooses \( q_i \) to solve
\[
\max_{q_i \geq 0} \pi(q_i) = \left(1 - q_i - \sum_{k \in N_i, k \neq i} q_k - \beta \sum_{j \in N_j} q_j - c_i\right) q_i
\]

where \( i, j \in \{B, G\} \) denotes the type of firm \( i \) and \( j \), respectively.

Taking the first order condition with respect to \( q_i \), and assuming interior solutions,
\[
1 - 2q_i - \sum_{k \in N_i, k \neq i} q_k - \beta \sum_{j \in N_j} q_j - c_i = 0
\]
Since all type-\(i\) firms are symmetric, in equilibrium we must have \(q_i = q_k\), yielding
\[
q_i(q_j) = \frac{1 - \beta \sum_{j \in N_j} q_j - c_i}{n_i + 1}
\]
Intersecting the best response functions \(q_i(q_j)\) and \(q_j(q_i)\), equilibrium output becomes
\[
q_{iNM} = \frac{(n_j + 1)(1 - c_i) - \beta n_j (1 - c_j)}{(n_i + 1)(n_j + 1) - \beta^2 n_in_j}
\]
In this context, every firm \(i\) produces positive units before the merger if and only if
\[
m_i > \frac{\beta n_j}{n_i + 1}
\]
**Merger.** Let \(K_i\) (\(K_j\)) denote the subset of merged firms within the set of all type-\(i\) (-\(j\)) firms, \(N_i\) (\(N_j\)). When \(k_i \leq n_i\) type-\(i\) and \(k_j \leq n_j\) type-\(j\) firms merge, the merged firm solves
\[
\max_{q_i^M, q_j^M \geq 0} \pi(q_i^M, q_j^M) = \left(1 - q_i^M - \sum_{i \in \{N_i \setminus K_i\}} q_i^U - \beta q_j^M - \beta \sum_{j \in \{N_j \setminus K_j\}} q_j^U - c_i\right)q_i^M
\]
\[
+ \left(1 - q_j^M - \sum_{j \in \{N_j \setminus K_j\}} q_j^U - \beta q_i^M - \beta \sum_{i \in \{N_i \setminus K_i\}} q_i^U - c_j\right)q_j^M
\]
Taking the first order condition with respect to \(q_i^M\), and assuming interior solutions,
\[
1 - 2q_i^M - \sum_{i \in \{N_i \setminus K_i\}} q_i^U - 2\beta q_j^M - \beta \sum_{j \in \{N_j \setminus K_j\}} q_j^U - c_i = 0
\]
Since \(\sum_{j \in \{N_j \setminus K_j\}} q_j^U = (n_j - k_j)q_j^UM\) and \(\sum_{i \in \{N_i \setminus K_i\}} q_i^U = (n_i - k_i)q_i^UM\), we obtain that
\[
q_i^M(q_i^UM) = \frac{1 - c_i - \beta (1 - c_j) - (1 - \beta^2)(n_i - k_i)q_i^UM}{2(1 - \beta^2)}
\]
On the other hand, the unmerged firm \(i\) chooses \(q_i^UM\) to solve
\[
\max_{q_i^UM \geq 0} \pi(q_i^UM) = \left(1 - q_i^UM - \sum_{k \in \{N_i \setminus K_i\}, k \neq i} q_k^UM - q_i^M - \beta \sum_{j \in \{N_j \setminus K_j\}} q_j^U - \beta q_j^M - c_i\right)q_i
\]
Taking the first order condition with respect to \(q_i^UM\), and assuming interior solutions,
\[
1 - 2q_i^UM - \sum_{k \in \{N_i \setminus K_i\}, k \neq i} q_k^UM - q_i^M - \beta \sum_{j \in \{N_j \setminus K_j\}} q_j^U - \beta q_j^M - c_i = 0
\]
Since \( \sum_{k \in (N_i \setminus K_i), k \neq i} q_k^U = (n_i - k_i - 1) q_i^U \) and \( \sum_{j \in (N_i \setminus K_j)} q_j^U = (n_j - k_j) q_j^U \), we have that
\[
1 - (n_i - k_i + 1) q_i^U - q_i^M - \beta (n_j - k_j) q_j^U - \beta q_j^M - c_i = 0
\]

Solving for \( q_i^U \), we obtain the best response function of the unmerged firm \( i \), as follows
\[
q_i^U (q_i^M, q_j^M) = \frac{(n_j - k_j + 1) (1 - c_i) - \beta (n_j - k_j) (1 - c_j) - [(1 - \beta^2) (n_j - k_j) + 1] q_i^M - \beta q_j^M}{(n_i - k_i + 1) (n_j - k_j + 1) - \beta^2 (n_i - k_i) (n_j - k_j)}
\]

Simultaneously solving for the equilibrium output of merged and unmerged firms, we find
\[
q_i^M = \frac{[\beta^2 (n_i - k_i) + n_j - k_j + 2] (1 - c_i) - \beta (n_i - k_i + n_j - k_j + 2) (1 - c_j)}{(1 - \beta^2) (n_i - k_i + 2) (n_j - k_j + 2) - \beta^2 (n_i - k_i) (n_j - k_j)}
\]
\[
q_i^U = \frac{(n_j - k_j + 2) (1 - c_i) - \beta (n_j - k_j) (1 - c_j)}{(n_i - k_i + 2) (n_j - k_j + 2) - \beta^2 (n_i - k_i) (n_j - k_j)}
\]

After the merger, every merged firm \( i \) produce positive units if and only if
\[
\frac{n_i - k_i + \beta^2 (n_j - k_j) + 2}{\beta (n_i - k_i + n_j - k_j + 2)} > m_i \geq \frac{\beta (n_i - k_i + n_j - k_j + 2)}{\beta^2 (n_i - k_i) + n_j - k_j + 2}
\]

while every unmerged firm \( i \) produces positive units if and only if
\[
m_i \geq \frac{\beta (n_j - k_j)}{n_j - k_j + 2}
\]

When \( \frac{n_i - k_i + 2}{\beta (n_i - k_i + 1)} > m_i \geq \frac{n_i - k_i + \beta^2 (n_j - k_j) + 2}{\beta (n_i - k_i + n_j - k_j + 2)} \), the merged firm \( i \) (or \( j \)) produces positive (zero) units, that is, \( q_j^M = 0 \). Setting \( n_i - k_i + 1 \) \( (n_j - k_j) \) for the number of type-i (-j) firms, every firm \( i \) produces
\[
q_i = \frac{(n_j - k_j + 1) (1 - c_i) - \beta (n_j - k_j) (1 - c_j)}{(n_i - k_i + 2) (n_j - k_j + 1) - \beta^2 (n_i - k_i + 1) (n_j - k_j)}
\]
which is positive since \( \frac{n_i - k_i + \beta^2 (n_j - k_j) + 2}{\beta (n_i - k_i + n_j - k_j + 2)} > \frac{\beta (n_i - k_i)}{n_j - k_j + 1} \). In this context, every unmerged firm \( j \) produces
\[
q_j^U = \frac{(n_i - k_i + 2) (1 - c_j) - \beta (n_i - k_i + 1) (1 - c_i)}{(n_i - k_i + 2) (n_j - k_j + 1) - \beta^2 (n_i - k_i + 1) (n_j - k_j)}
\]

7.2.19 Proof of Corollary A2

When \( \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \), the inequality \( q_i^M + q_j^M < k_i q_i^N + k_j q_j^N \) reduces to
\[
m_i < \frac{[n_i - k_i - \beta (n_j - k_j) + 2] y - (1 + \beta) (k j n_i - \beta k_i n_j + k_j) x}{(1 + \beta) (k j n_i - \beta k_i n_j + k_j) x + [\beta (n_i - k_i) - (n_j - k_j) - 2] y},
\]

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and when \( \frac{w_i}{\beta z} \leq m_i < \frac{n_i-k_i+2}{\beta(n_i-k_i+1)} \), the inequality \( q_i^M < k_i q_i^{NM} + k_j q_j^{NM} \) reduces to

\[
m_i < \frac{\beta (n_j - k_j) y + \left[ (n_i - k_i + 2) (n_j - k_j + 1) - \beta^2 (n_i - k_i + 1) (n_j - k_j) \right] (k_j n_i - \beta k_i n_j + k_j)}{(n_j - k_j + 1) y - \left[ (n_i - k_i + 2) (n_j - k_j + 1) - \beta^2 (n_i - k_i + 1) (n_j - k_j) \right] (k_i n_j - \beta k_j n_i + k_i)},
\]

both of which are satisfied by the assumption that \( \frac{\beta (n_j - k_j + 1)}{n_j - k_j + 2} \leq m_i < \frac{n_i-k_i+2}{\beta(n_i-k_i+1)} \).

### 7.2.20 Proof of Lemma A3

Before the merger, aggregate output of type-\( i \) all firms is

\[
Q_i^{NM} = n_i q_i^{NM} = \frac{n_i (n_j + 1) (1 - c_i) - \beta n_i n_j (1 - c_j)}{y}
\]

After the merger, aggregate output of all type-\( i \) firms is

\[
Q_i^M = q_i^M + (n_i - k_i) q_i^{UM} = \begin{cases} 
\frac{(n_i-k_i)[(n_j-k_j+2)(1-c_i)-\beta(n_j-k_j+1)(1-c_j)]}{v_j} & \text{if } \frac{\beta(n_j-k_j+1)}{n_j-k_j+2} \leq m_i < \frac{w_i}{\beta z} \\
\frac{v_j(1-c_i)-\beta(x-z)(1-c_j)}{(1-\beta^2)x} & \text{if } \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \\
\frac{(n_i-k_i+1)[(n_j-k_j+1)(1-c_i)-\beta(n_j-k_j)(1-c_j)]}{v_i} & \text{if } \frac{w_j}{\beta z} \leq m_i < \frac{n_i-k_i+2}{\beta(n_i-k_i+1)}
\end{cases}
\]

Consumer surplus from consuming \( Q_i \) units of good \( i \) and \( Q_j \) units of good \( j \) is

\[
CS(Q_i, Q_j) = \frac{1}{2} \left( Q_i^2 + 2\beta Q_i Q_j + Q_j^2 \right) - (1 - Q_i - \beta Q_j) Q_i - (1 - Q_j - \beta Q_i) Q_j
\]

\[
= \frac{1}{2} \left( Q_i^2 + 2\beta Q_i Q_j + Q_j^2 \right) + \left( p_i - c_i \right) Q_i + \left( p_j - c_j \right) Q_j - d \left( \alpha_i Q_i + \alpha_j Q_j \right)^2
\]

In this context, social welfare becomes

\[
SW(Q_i, Q_j) = \frac{1}{2} \left( Q_i^2 + 2\beta Q_i Q_j + Q_j^2 \right) + \left( p_i - c_i \right) Q_i + \left( p_j - c_j \right) Q_j - d \left( \alpha_i Q_i + \alpha_j Q_j \right)^2
\]

\[
= \left( 1 - \frac{s_i}{2} Q_i - c_i \right) Q_i + \left( 1 - \frac{s_j}{2} Q_j - c_j \right) Q_j - r Q_i Q_j
\]

Specifically, social welfare before the merger is

\[
SW^{NM} = \frac{E_i (1 - c_i)^2 - 2 F (1 - c_i) (1 - c_j) + E_j (1 - c_j)^2}{2 y^2}
\]

Whereas, social welfare after the merger becomes

\[
SW^M = \begin{cases} 
\frac{I_i (1-c_i)^2 - 2 H_i (1-c_i) (1-c_j) + G_j (1-c_j)^2}{2 v_j^2} & \text{if } \frac{\beta(n_j-k_j+1)}{n_j-k_j+2} \leq m_i < \frac{w_i}{\beta z} \\
\frac{J_i (1-c_i)^2 - 2 L_i (1-c_i) (1-c_j) + J_j (1-c_j)^2}{2 (1-\beta^2) x^2} & \text{if } \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \\
\frac{G_i (1-c_i)^2 - 2 H_i (1-c_i) (1-c_j) + I_i (1-c_j)^2}{2 v_i^2} & \text{if } \frac{w_j}{\beta z} \leq m_i < \frac{n_i-k_i+2}{\beta(n_i-k_i+1)}
\end{cases}
\]
Therefore, $SW^M \geq SW^{NM}$ if and only if $m_i \leq \underline{m}_i^{SW}$ and $m_i \geq \overline{m}_i^{SW}$, where $m_i$ solves

\[
\begin{align*}
\left( y^2I_j - v_j^2 E_i \right) m_i^2 &= 2(y^2 H_j - v_j^2 F) m_i + (y^2 G_j - v_j^2 E_j) \geq 0 \quad \text{if } \frac{\beta(n_i-k_j+1)}{n_j-k_j+2} \leq m_i < \frac{\beta z}{w_i} \\
\left[ y^2I_j - (1 - \beta^2) x^2 E_i \right] m_i^2 &\geq 2(1 - \beta^2) x^2 F m_i + (y^2 J_j - (1 - \beta^2) x^2 E_j) \geq 0 \quad \text{if } \frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z} \\
\left( y^2 G_j - v_j^2 E_i \right) m_i^2 &\geq 2(y^2 H_j - v_j^2 F) m_i + (y^2 I_j - v_j^2 E_j) \geq 0 \quad \text{if } \frac{w_j}{\beta z} \leq m_i < \frac{n_i-k_i+2}{\beta(n_i-k_i+1)}
\end{align*}
\]

where $E_i \equiv n_i \left[ (2y - s_i n_i) (n_i + 1) + 2\beta r n_i n_j \right] n_i (n_i + 1) - \beta^2 s_i n_i n_j^2]$, $F \equiv n_i n_j \left( r \left[ (n_i + 1) (n_j + 1) + \beta^2 n_i n_j \right] - \beta [s_i n_i (n_i + 1) + s_j n_j (n_i + 1) - 2y] \right]$, $G_i \equiv (n_i - k_i + 1) \left[ 2v_i - s_i (n_i - k_i + 1) (n_j - k_j + 1) + 2\beta r (n_i - k_i + 1) (n_i - k_j + 1) \beta^2 s_i (n_i - k_i + 1) (n_j - k_j + 1) \beta^2 s_j (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] r \left[ (n_i - k_i + 2) (n_j - k_j + 1) + \beta^2 (n_i - k_i + 1) (n_j - k_j) \right] \beta [s_i (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] \beta [s_j (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] \beta [s_j (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right]$, $H_i \equiv (n_i - k_i + 1) (n_j - k_j) \beta [s_i (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] \beta [s_j (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] \beta [s_j (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right]$, $I_i \equiv (n_j - k_j) \left( 2v_i - s_i (n_i - k_i + 2) (n_j - k_j) + 2\beta r (n_i - k_i + 1) (n_j - k_j) \right] \beta s_j (n_i - k_i + 1) (n_j - k_i + 1) + s_j (n_i - k_i + 2) (n_j - k_j + 2) (n_i - k_i + 1) (n_j - k_j + 1) + s_j (n_i - k_i + 1) (n_j - k_j) \right] \beta \left( x - z \right] [s_i v_j + s_j v_i - 2(1 - \beta^2)x \right]$. In the setting of $n_i = n_j = k_i = k_j$, we obtain that $E_i \equiv 4(4 - \beta^2 + \beta r - s_i) - \beta^2 s_j$, $F \equiv r \left( 4(1 - \beta^2) - 2\beta (s_i + s_j - 4 + \beta^2) \right] \beta [s_i + s_j - 4(1 - \beta^2) \right] = 4D$, which are consistent with our results in Lemma 3.

7.2.21 Proof of Lemma A4

Substituting $p_j = 1 - Q_i - \beta Q_j$ into $\pi_i$, firm $i$ earns a profit of $\pi_i^{NM} = (q_i^{NM})^2$ before the merger; and after the merger, firm $i$ obtains $\pi_i^M = q_i^{UM} q_i^M$ if merged and $\pi_i^{UM} = (q_i^{UM})^2$ if not merged.

Aggregate profits of all the firms that merge, $\pi^{NM} \equiv k_i q_i^{NM} + k_j q_j^{NM}$, before the merger are

\[
\left[ k_i (n_j + 1)^2 + \beta^2 k_j n_j^2 \right] (1-c_i)^2 - 2(1-c_i)^2 \beta [k_i n_j (n_j + 1) + k_j n_i (n_i + 1)] (1-c_i) (1-c_j) + \beta^2 k_i n_j^2 (1-c_j)^2
\]

When $\frac{\beta z}{w_i} \leq m_i < \frac{w_j}{\beta z}$, total profits of the merged firm, $\pi^M \equiv \pi_i^M + \pi_j^M$, after the merger are

\[
\left[ (n_j - k_j + 2) w_i + \beta z (n_i-k_i) z \right] (1-c_i)^2 - \beta (n_i-k_i) w_j + (n_i-k_i) w_j ] (1-c_i) (1-c_j) + \left[ (n_i-k_i+2) w_j + \beta z (n_i-k_i) z \right] (1-c_j)^2 (1-\beta^2 z)^2
\]

When $\frac{w_j}{\beta z} \leq m_i < \frac{n_i-k_i+2}{\beta (n_i-k_i+1)}$, profits of the merged firm, $\pi^M \equiv \pi_i^M$, after the merger become

\[
\frac{(n_j - k_j + 1)^2 (1-c_i)^2 - 2(1-c_i)^2 \beta (n_j - k_j) (n_j - k_j + 1) (1-c_i) (1-c_j) + \beta^2 (n_j - k_j)^2 (1-c_j)^2}{\left[ (n_i-k_i+2) (n_j - k_j + 1) - \beta^2 (n_i-k_i+1) (n_j - k_j) \right]^2}
\]

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Therefore, firm $i$ has incentives to merge if and only if $\pi^M \geq \pi^{NM}$, where $m_i$ solves

$$
\begin{align*}
\left\{ \begin{array}{ll}
\beta R_i - \sqrt{\beta^2 R_i^2 - O_i S_i} & \equiv m_i^{\text{NR}} \leq m_i \leq \overline{m}_i^{\text{NR}} \equiv \beta R_i + \sqrt{\beta^2 R_i^2 - O_i S_i} \\
\beta P_i - \sqrt{\beta^2 P_i^2 - 4 M_i M_i} & \equiv \beta \left( \frac{m_i^{\text{NR}}}{S_i} \right) \leq \beta \left( \frac{m_i}{S_i} \right) \leq \beta \left( \frac{\overline{m}_i^{\text{NR}}}{S_i} \right) \\
\beta R_i - \sqrt{\beta^2 R_i^2 - O_i S_i} & \equiv \beta \left( \frac{m_i}{O_i} \right) - \sqrt{\beta^2 R_i^2 - O_i S_i} \\
\end{array} \right.
\end{align*}
$$

if $\beta(n_j - k_j + 1) \leq m_i < \beta \left( \frac{m_i}{w_i} \right)$

$$
\begin{align*}
\text{if } \frac{m_i}{w_i} \leq m_i < \frac{m_i}{\beta(n_j - k_j + 1)}
\end{align*}
$$

For compactness, we define $M_i \equiv (1 - \beta^2) x^2 [k_i(n_j + 1) + \beta^2 k_j n_j^2] - [w_i(n_j - k_j + 2) + \beta^2 (n_j - k_j) z] y^2$, $O_i \equiv v_i^2 [k_i(n_j + 1) + \beta^2 k_j n_j^2] - (n_j - k_j + 1)^2 y^2$, $P \equiv 2(1 - \beta^2)x^2[k_i n_j(n_j + 1) + k_j n_i(n_i + 1)] - [(n_i - k_i + n_j - k_j + 4) z + (n_i - k_i) w_j + (n_j - k_j) w_i] y^2$, $R_i \equiv v_i^2 [k_i n_j(n_j + 1) + k_j n_i(n_i + 1)] - (n_j - k_j + 1)(n_j - k_j) y^2$, and $S_i \equiv v_i^2 [k_i(n_j + 1) + \beta^2 k_j n_j^2] - \beta^2 (n_j - k_j)^2 y^2$. In the setting of $n_i = n_j = k_i = k_j = 1$, we obtain that $M_i = -4 \beta^2(4 + 5 \beta^2)$, $O_i = \beta^2(12 - \beta^2)$, $P = -8 \beta^2(8 + \beta^2)$, $R_i = 16$, and $S_i = 4(4 + \beta^2)$, which are consistent with the results in Lemma 4.

7.2.22 Proof of Lemma A5

Substituting $c_i + t_i$ for $c_i$ in Lemma A1, we find firm $i$’s output as a function of emission fees.

7.2.23 Proof of Lemma A6

Differentiating $SW( Q_i, Q_j )$ with respect to $Q_i$, socially optimal output solves

$$
Q_i^{SO} = \frac{s_j}{s_i} \left( \frac{1 - c_i}{1 - c_j} - r \left( \frac{1 - c_j}{1 - c_j} \right) \right)
$$

which is positive when $\frac{r}{s_i} \equiv m_i^{R_i} \leq m_i \leq \overline{m}_i^{R_i} \equiv \frac{s_i}{r}$.

When $m_i < \overline{m}_i^{R_i}$, the EPA sets $Q_i^{SO} = 0$ but when $m_i \geq \overline{m}_i^{R_i}$, it solves $\max_{Q_i > 0} SW( Q_i, 0 )$, yielding

$$
Q_i^{SO} = \frac{1 - c_i}{s_i}
$$

Next, substituting $Q_i^{SO}$ and $Q_j^{SO}$ into the social welfare function, social welfare becomes

$$
SW^{SO} = \frac{(1 - c_i)^2 - 2 \beta (1 - c_i)(1 - c_j) + (1 - c_j)^2 + 2d \left[ \alpha_i (1 - c_j) - \alpha_j (1 - c_i) \right]^2}{2(s_i s_j - r^2)}
$$

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Finally, social welfare is concave in aggregate output, given the Hessian matrix of 

\[
H = \begin{bmatrix}
\frac{\partial^2 SW(Q_i, Q_j)}{\partial Q_i^2} & \frac{\partial^2 SW(Q_i, Q_j)}{\partial Q_i \partial Q_j} \\
\frac{\partial^2 SW(Q_i, Q_j)}{\partial Q_j \partial Q_i} & \frac{\partial^2 SW(Q_i, Q_j)}{\partial Q_j^2}
\end{bmatrix}
\]

\[= - \begin{bmatrix}
s_i & r \\
-1 & s_j
\end{bmatrix}
\]

\[-(s_i s_j - r^2) < 0\]

7.2.24 Proof of Proposition A1

To find the optimal emission fees \(t_i^{NM}\) and \(t_i^M\) for every firm \(i\), we align the aggregate output with socially optimal output, solving \(Q_i^{NM} = \frac{n_i(n_i+1)(1-c_i-t_i) - \beta n_i n_j(1-c_j-t_j)}{y} = \frac{s_i(1-c_i) - r(1-c_j)}{s_i s_j - r^2} = Q_i^{SO}\) before the merger and \(Q_i^M = \frac{v_i j(1-c_i-t_i) - \beta (x-z)(1-c_j-t_j)}{1-\beta^2 x} = \frac{s_i(1-c_i) - r(1-c_j)}{s_i s_j - r^2} = Q_i^{SO}\) after the merger, respectively, when \(\tilde{m}_i^R \leq m_i < \bar{m}_i^R\). Otherwise, when \(m_i \geq \bar{m}_i^R\), the EPA shuts down all type-\(j\) firms, setting an emission fee \(t_i^M\) for every firm \(i\) that solves \(Q_i^M = \frac{(n_i-k_i+1)(1-c_i-t_i)}{n_i-k_i+2} = \frac{1-c_i}{s_i} = Q_i^{SO}\). 

7.2.25 Proof of Lemma A8

Substituting \(t_i^{NM}\) into the profit function before the merger, every firm \(i\) generates a profit of 

\[
\pi_i^{NM,R} = \begin{cases} 
0 & \text{if } m_i < \tilde{m}_i^R \\
\left(\frac{s_i(1-c_i) - r(1-c_j)}{n_i s_i s_j - r^2}\right)^2 & \text{if } \tilde{m}_i^R \leq m_i < \bar{m}_i^R \\
\left(\frac{1-c_i}{s_i(n_i+1)}\right)^2 & \text{if } m_i \geq \bar{m}_i^R 
\end{cases}
\]

Substituting \(t_i^M\) into the profit function after the merger, every merged firm \(i\) earns a profit of 

\[
\pi_i^{M,R} = \begin{cases} 
0 & \text{if } m_i < \tilde{m}_i^R \\
\frac{(1-\beta^2)(T v_i (1-c_i)^2 - (T W_i + U_i)(1-c_i)(1-c_j) + U_i W_i (1-c_j)^2)}{[v_i v_j - \beta (x-z)^2] s_i s_j - r^2} & \text{if } \tilde{m}_i^R \leq m_i < \bar{m}_i^R \\
\left(\frac{1-c_i}{s_i(n_i-k_i+2)}\right)^2 & \text{if } m_i \geq \bar{m}_i^R 
\end{cases}
\]

In this context, firm \(i\) has incentives to merge if and only if \(\pi_i^{M,R} + \pi_j^{M,R} \geq k_i \pi_i^{NM,R} + k_j \pi_j^{NM,R}\), where \(m_i\) solving \(X_i m_i^2 - Y m_i + Z_i \geq 0\) satisfies \(m_i \leq \tilde{m}_i^\pi = \frac{Y + \sqrt{Y^2 - 4X_i Z_i}}{2X_i}\) or \(m_i \geq \bar{m}_i^\pi = \frac{Y + \sqrt{Y^2 - 4X_i Z_i}}{2X_i}\). For compactness, we define \(T_i \equiv (n_j - k_j + 2)(s_j v_i - \beta r(x-z)) + \beta(n_j - k_j)(r v_j - \beta s_j (x-z)),\)
\( U_i \equiv (n_j - k_j + 2)(rv_i - \beta s_i(x - z)) + \beta(n_j - k_j)(s_iv_j - \beta r(x - z)), V_i \equiv (s_j v_i - \beta r(x - z))w_i + \beta(rv_j - \beta s_j(x - z))z, W_i \equiv (rv_i - \beta s_i(x - z))w_i + \beta(s_iv_j - \beta r(x - z))z, X_i \equiv (1 - \beta^2)n_i^2 n_j^2(T_i V_i + U_j W_j) - [v_i v_j - \beta^2(x - z)^2](k_i n_j^2 s_j^2 + k_j n_i^2 r^2), Y \equiv (1 - \beta^2)n_i^2 n_j^2(T_i W_i + T_j W_j + U_i V_j + U_j V_i) - 2r[v_i v_j - \beta^2(x - z)^2](k_i n_j^2 s_j + k_j n_i^2 s_i), Z_i \equiv (1 - \beta^2)n_i^2 n_j^2(T_j V_j + U_i W_j) - [v_i v_j - \beta^2(x - z)^2](k_i n_j^2 s_j^2 + k_j n_i^2 r^2) \).

In the setting of \( n_j = n_j = k = k_j = 1 \), we obtain that \( T_i = 4(s_j - \beta r), U_i = 4(r - \beta s_i), V_i = 4(1 - \beta^2)s_j, W_i = 4(1 - \beta^2)r, X_i = 32\beta (1 - \beta^2)^2 r s_j, Y = 32\beta (1 - \beta^2)^2 (r^2 + s_i s_j), Z_i = 32\beta (1 - \beta^2)^2 r s_i, \) such that the above inequality is rearranged to yield \( \frac{l}{s_j} \leq m_i \leq \frac{u}{r}, \) which holds by assumption. Otherwise, firm \( i \) has incentives to merge if and only if \( k_i \geq \frac{2n_i + 3 - \sqrt{4n_i + 5}}{2}, \) as in Salant et al. (1983).

8. References


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