Advanced Microeconomic Theory

Chapter 9: Externalities and Public Goods

Outline

- Externalities
- The Coase Theorem
- Pigouvian Taxation
- Tragedy of the Commons
- Pollution Abatement
- Public Goods
- Lindahl Equilibria
- Asymmetric Information

Externalities

Externalities

- ***Externality*** emerges when the well-being of a consumer or the production possibilities of a firm is directly affected by the actions of another agent in the economy.
  - Example: the production possibilities of a fishery are affected by the pollutants that a refinery dumps into a lake.
  - The effects from one agent to another are not captured by the price system.
- The effects transmitted through the price system are referred to as "***pecuniary externalities***."

Externalities

- Consider a polluting firm (agent 1) and an individual affected by such pollution (agent 2).
- The firm's profit function is

$$\pi(p, x)$$

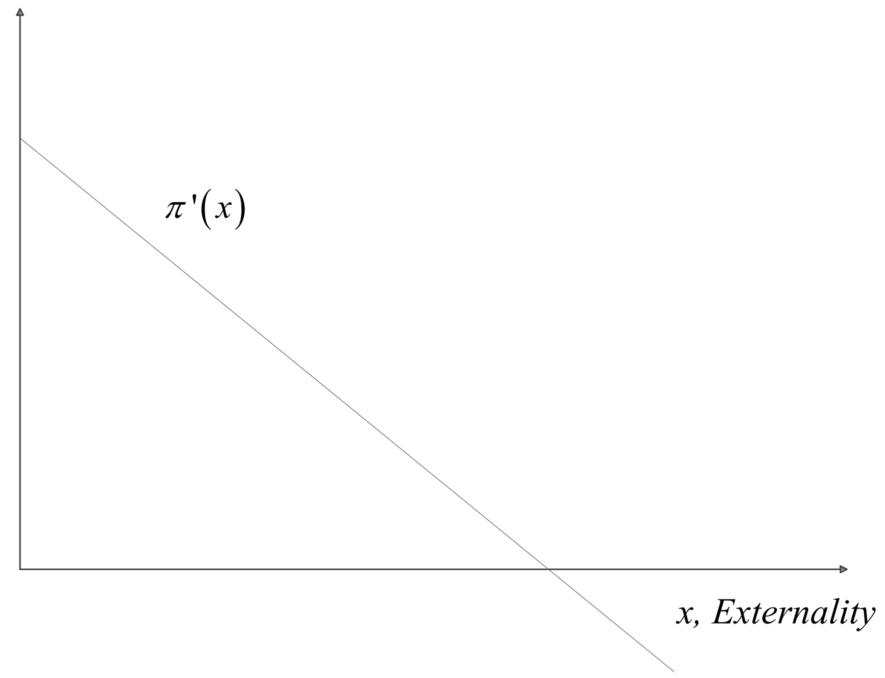
where p is the price vector and x is the amount of externality generated.
- Assume that p is given (i.e., p is parameter). Then, the profit function becomes

$$\pi(x)$$

where $\pi'(x) > 0$ and $\pi''(x) < 0$.

Externalities

- The firm obtains a positive and significant benefit from the first unit of the externality-generating activity.
- But the additional benefit from further units is decreasing.



Externalities

- The individual's (i.e., agent 2's) utility is given by
$$u(q, x)$$
where $q \in \mathbb{R}^N$ is a vector of N-tradable goods and $x \in \mathbb{R}_+$ is the negative externality, with $\frac{\partial u}{\partial x} < 0$ and $\frac{\partial u}{\partial q_k} \geq 0$ in every good k.
- Let $q^*(p, w, x)$ denote the individual's Walrasian demand. Then,
$$v(x) = u(q^*(p, w, x), x)$$
is the indirect utility function with $v'(x) < 0$ for all $x > 0$.

Externalities

- ***Example***:
  - Consider the firm's profit function is given by $\pi = py - cy^2$, where $y \in \mathbb{R}_+^L$ is output and $p > c > 0$.
  - If every unit of output generates a unit of pollution, i.e., $x = y$, the profit function becomes $\pi(x) = px - cx^2$.
  - FOC wrt x yields $\pi'(x^*) = p - 2cx^* = 0$, producing $p = 2cx^*$ or $x^* = \frac{p}{2c}$.

Externalities

- ***Example*** (continued):
  - If every unit of output y generates $\frac{1}{\alpha}$ units of pollution, i.e., $y = \frac{1}{\alpha}x$, where $\alpha > 0$, the profit function becomes

$$\pi(x) = p\frac{x}{\alpha} - c\left(\frac{x}{\alpha}\right)^2.$$

  - Taking FOC with respect to x yields

$$\pi'(x^*) = \frac{p}{\alpha} - 2c\frac{x^*}{\alpha}\frac{1}{\alpha} = 0,$$

with a competitive equilibrium level of pollution of

$$x^* = \alpha\frac{p}{2c}.$$

Externalities

- ***Competitive equilibrium***: All agents independently and simultaneously solve their PMP (for firms) or UMP (for consumers).
  - The firm independently chooses the level of the externality-generating activity, x, that solves its PMP

$$\max_{x \geq 0} \ \pi(x)$$

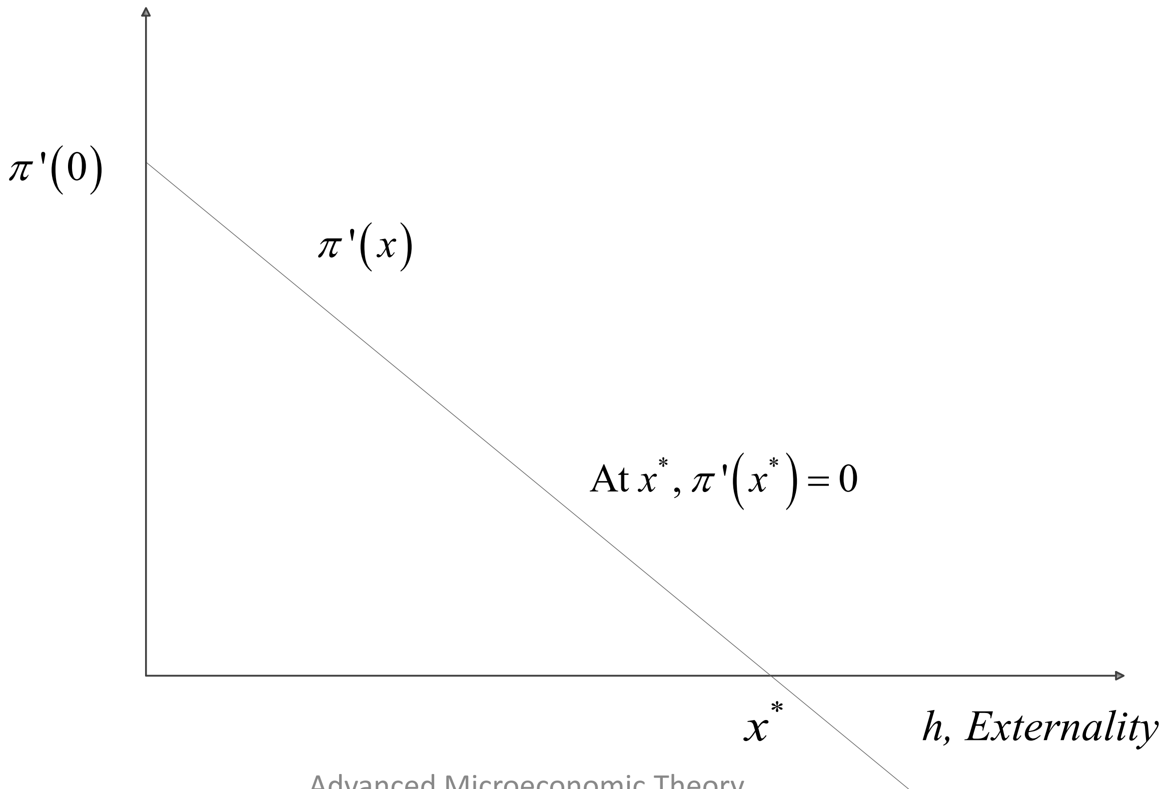
  - Taking FOC with respect to x yields

$$\pi'(x^*) \leq 0$$

    with equality if $x > 0$ (interior solution).

Externalities

– Firm increases the externality-generating activity until the point where the marginal benefit from an additional unit is exactly zero, i.e., $\pi'(x^*) = 0$.



Externalities

– The UMP of the individual affected by pollution is

$$\max_{q \geq 0} \ u(q, x) \quad \text{s.t.} \quad pq \leq w$$

where $p \in \mathbb{R}_+^N$ is the given price vector.

– Notice that $q \in \mathbb{R}^N$ does not include pollution as one of the N-tradable goods.

– Hence the individual cannot affect the level of the externality generating activity x.

- Uninteresting case
- This assumption is later relaxed

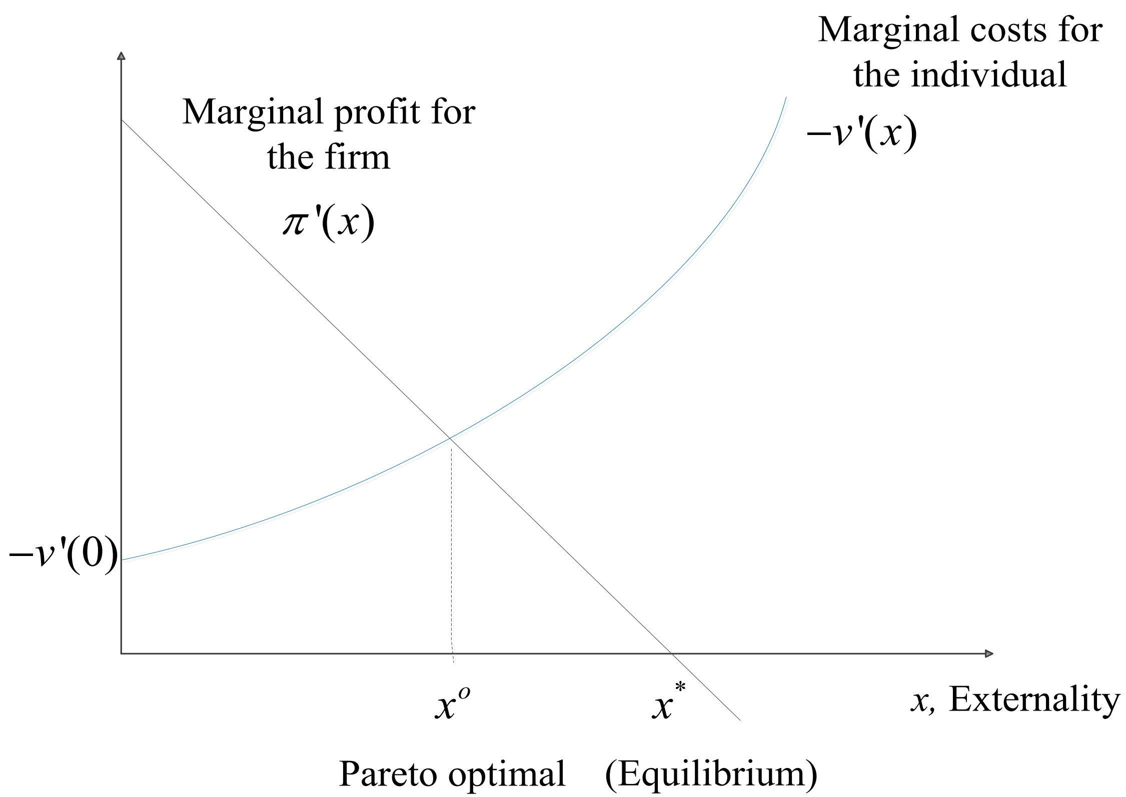
Externalities

- ***Pareto optimum***:
  - The social planner selects the level of x that maximizes social welfare
$$\max_{x \geq 0} \; \pi(x) + v(x)$$
  - Taking FOC with respect to x yields
$$\pi'(x^0) \leq -v'(x^0) \text{ with equality if } x^0 > 0$$
    where x^0 is the Pareto optimal amount of the externality.
  - Intuitively, at a Pareto optimal (and interior) solution, the marginal benefit of the externality-generating activity, $\pi'(x^0)$, is equal to its marginal cost, $-v'(x^0)$.

Externalities

- Pareto optimal and equilibrium externality level (negative externality).

- Too much externality x is produced in the competitive equilibrium relative to the Pareto optimum, i.e., $x^* > x^0$.



Externalities

- ***Example***:
  - Consider a firm with marginal profits of
  $$\pi'(x) = a - bx, \text{ where } a, b > 0$$
  which is decreasing in x.
  - Assume a consumer with marginal damage function of
  $$v'(x) = -c - dx, \text{ where } c, d > 0$$
  which is increasing in x.

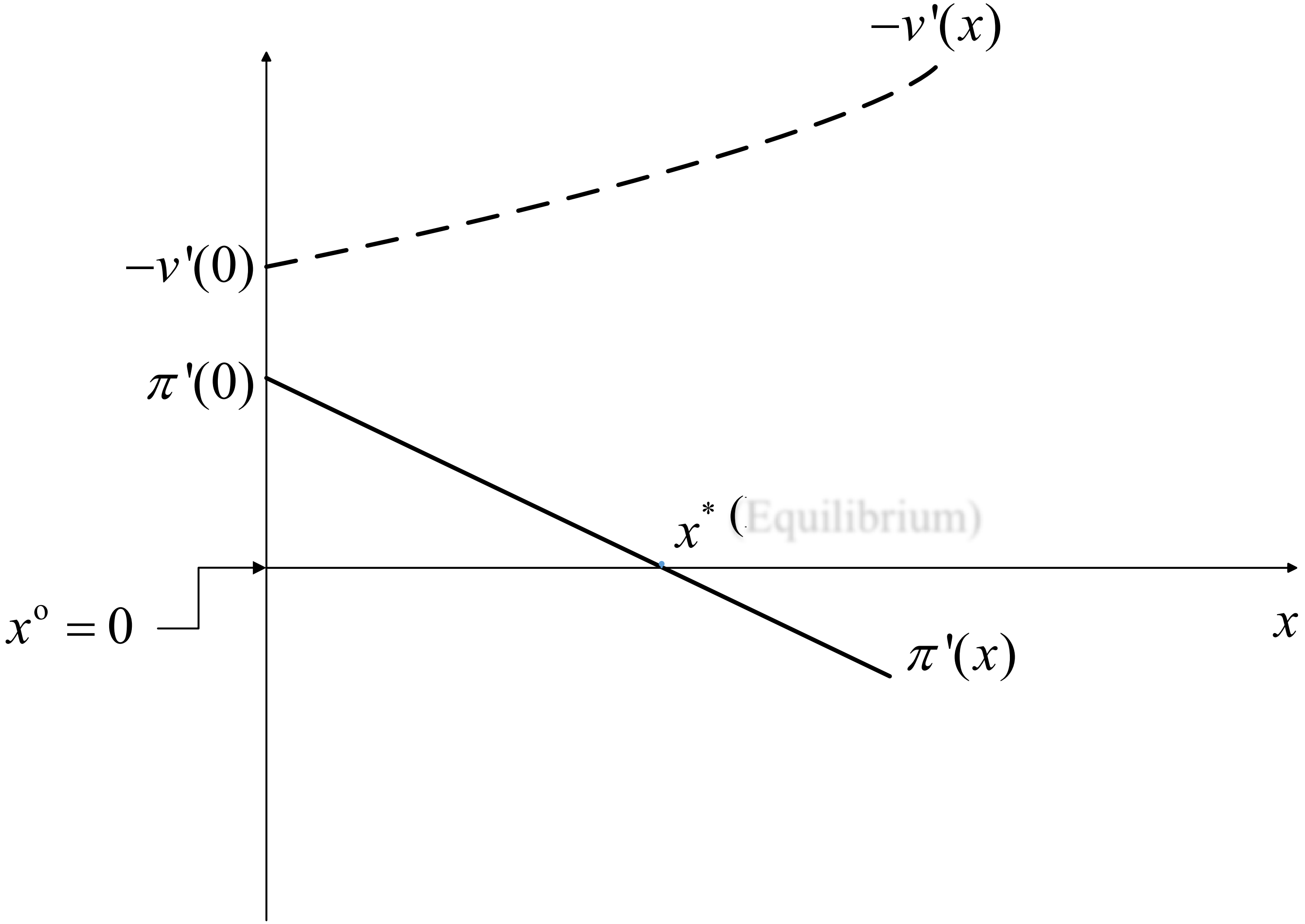
Externalities

- ***Example*** (continued):
  - The competitive equilibrium amount of externality x^* solves $\pi'(x^*) = 0$, i.e., $a - bx^* = 0$. Hence,
  $$x^* = \frac{a}{b}$$
  - The socially optimal level of the externality x^0 solves $\pi'(x^0) = -v'(x^0)$, i.e., $a - bx^0 = c + dx^0$. Thus,
  $$x^0 = \frac{a-c}{b+d}$$
  which is positive if $\pi'(0) > -v'(0)$, i.e., $a > c$.

Externalities

- Negative externalities are not necessarily eliminated at the Pareto optimal solution.
- This would only occur at the extreme case when $-v'(0) > \pi'(0)$.
- In this setting, curve $\pi'(x)$ and $-v'(x)$ do not cross, and the Pareto optimal solution only occurs at the corner where $x^0 = 0$.

Externalities



Externalities

- If firm's production activities produce a ***positive externality*** in the individual's wellbeing, then

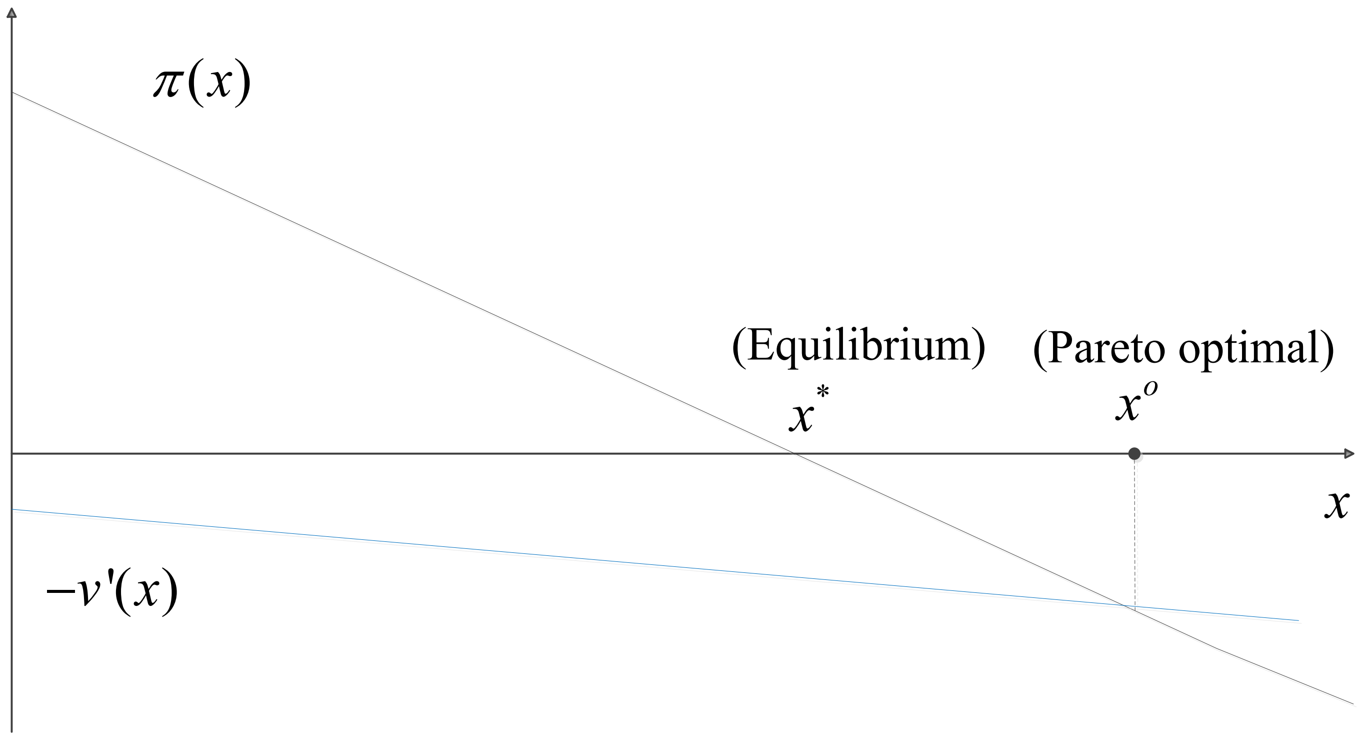
$$v'(x) > 0 \text{ and } -v'(x) < 0$$

  - That is, $-v'(x) < 0$ lies in the negative quadrant.
  - $\pi'(x)$ remains unaffected.

- In this setting, there is an underproduction of the externality-generating activity relative to the Pareto optimum, i.e., $x^* < x^0$.

Externalities

- Pareto optimal and equilibrium externality level (positive externality).



Externalities

- ***Example*** (Positive externalities):
  - Consider two neighboring countries, $i = \{1,2\}$, simultaneously choosing how many resources (in hours) to spend on recycling activities, r_i.
  - The net benefit from recycling is:

$$\pi_i(r_i, r_j) = \left(a - r_i + \frac{r_j}{2}\right) r_i - b r_i$$

    where $a, b > 0$, and b denotes the marginal cost of recycling.
  - Country i's average benefit, $\left(a - r_i + \frac{r_j}{2}\right)$, is increasing in r_j because a clean environment produces positive external effects on the other country.

Externalities

- ***Example*** (continued):
  - Let us first find the competitive equilibrium allocation.
  - Taking FOC with respect to r_i yields country i's BRF:

$$r_i(r_j) = \frac{a-b}{2} + \frac{1}{4} r_j$$

  - Symmetrically, country j's BRF is

$$r_j(r_i) = \frac{a-b}{2} + \frac{1}{4} r_i$$

    - The positive slope of the BRFs indicates that countries' recycling activities are strategic complements.

Externalities

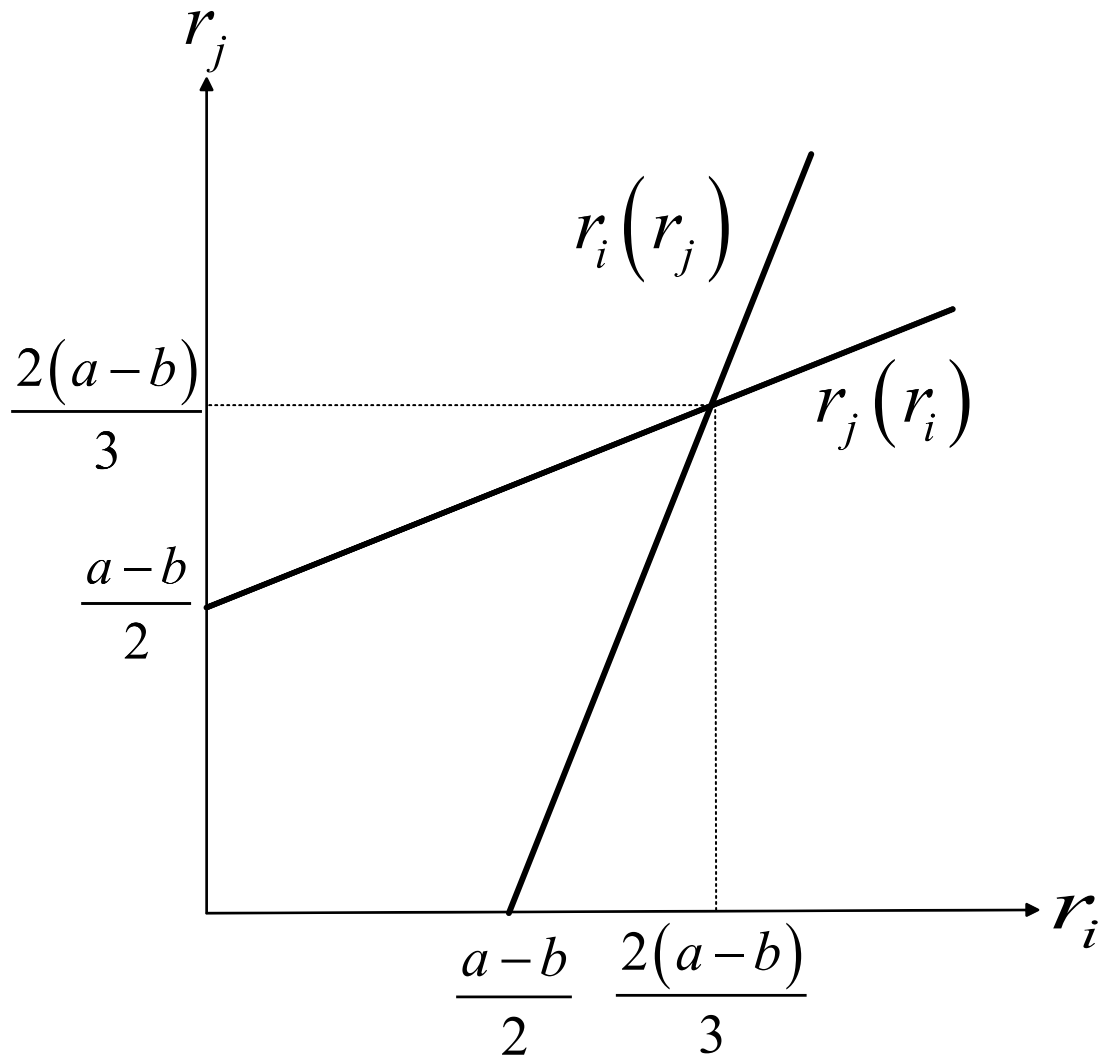
- ***Example*** (continued):
  – Simultaneously solving the two BRFs yields

$$r_i = \frac{\frac{r_i}{4} + \frac{a-b}{2}}{4} + \frac{a-b}{2}$$

  – And rearranging, we obtain an equilibrium level of recycling

$$r_i^* = \frac{2}{3}(a - b) \text{ for } i = \{1,2\}$$

Externalities



Externalities

- ***Example*** (continued):
  - A social planner simultaneously selects r_i and r_j in order to maximize social welfare

$$\max_{r_i, r_j} \left(a - r_i + \frac{r_j}{2}\right) r_i - b r_i + \left(a - r_j + \frac{r_i}{2}\right) r_j - b r_j$$

  - FOCs:

$$a - 2r_i + \frac{r_j}{2} - b + \frac{r_j}{2} = 0$$

$$a - 2r_j + \frac{r_i}{2} - b + \frac{r_i}{2} = 0$$

Externalities

- ***Example*** (continued):
  - Simultaneously solving the two FOCs yields the socially optimal levels of recycling

$$r_i^0 = a - b \text{ for every } i = \{1,2\}$$

  - Note that

$$r_i^0 = a - b > \frac{2}{3}(a - b) = r_i^*$$

Solutions to the Externality Problem:

Property Rights

Property Rights

- This is a less intrusive approach:
  - let the parties bargain over the externality
  - no government intervention
- Key assumptions:
  - The property rights over the externality-generating activity must be:
    - Easy to identify, and
    - Enforceable.
  - No bargaining costs.
- As long as property rights are clearly assigned, the two parties will negotiate in such a way that the optimal level of the externality-producing activity is implemented (***Coase Theorem***)

Property Rights

- ***Property rights assigned to consumer 2***:
  - Let us assign property rights to the individual suffering the negative externality
    - "externality-free" environment: at the initial state no externality is generated, i.e., $x = 0$
  - The firm must then pay the affected individual if it wants to increase the externality over zero.
  - In particular, let us assume that affected individual makes a take-it-or-leave-it-offer where the firm must pay $\$T$ in exchange of x units of pollution.

Property Rights

– The firm agrees to pay $\$T$ to the affected individual iff

$$\pi(x) + w_1 - T \geq \underbrace{\pi(0)}_{\text{current state}} + w_1$$

or $\pi(x) - T \geq \pi(0)$

– Hence, the affected individual's UMP becomes that of choosing (x, T) that solves

$$\max_{x \geq 0, T} \; v(x) + w_2 + T$$
$$\text{s.t.} \;\; \pi(x) - T \geq \pi(0)$$

– The constraint is binding, since the affected individual will raise $\$T$ until the point where the firm is exactly indifferent between accepting and rejecting the offer.

Property Rights

– Hence, $\pi(x) - T = \pi(0)$ or $\pi(x) - \pi(0) = T$.
– Plugging this result into the affected individual's UMP, we obtain

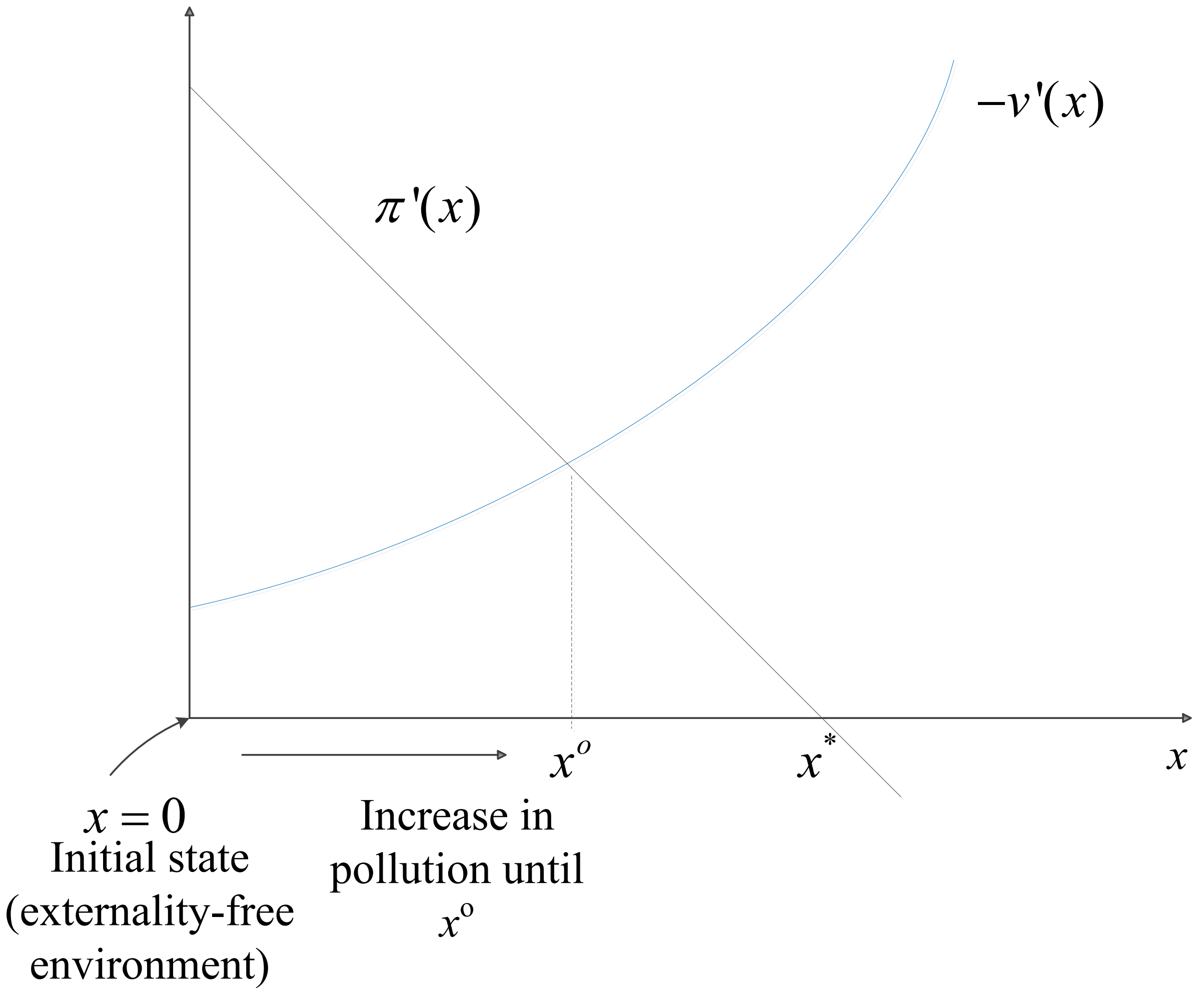
$$\max_{x \geq 0} v(x) + w_2 + \underbrace{\pi(x) - \pi(0)}_{T}$$

– FOCs with respect to x yields:

$$v'(x) + \pi'(x) \leq 0 \quad \Leftrightarrow \quad \pi'(x) \leq -v'(x)$$

– This coincides with the FOCs to the social planner's problem.
– Therefore, bargaining allows for the level of the externality x to reach the optimal level, i.e., $x = x^0$.

Property Rights



Property Rights

- ***Property rights assigned to the firm***:
  - What if the property rights were assigned to the firm (i.e., polluter)?
  - If there is no bargaining between the firm and the affected individual, the firm would pollute until $x = x^*$, where $\pi'(x^*) = 0$.
  - However, the affected individual can pay $\$T$ to the firm in exchange of a lower level of pollution, x, where $x < x^*$.

Property Rights

– The polluter is willing to accept this offer iff

$$\pi(x) + w_1 + T \geq \underbrace{\pi(x^*)}_{\text{current state}} + w_1$$

$$\text{or} \quad \pi(x) + T \geq \pi(x^*)$$

– Thus, the affected individual's UMP becomes that of choosing (x, T) that solves

$$\max_{x \geq 0, T} \; v(x) + w_2 - T$$

$$\text{s.t.} \; \pi(x) + T \geq \pi(x^*)$$

– Note that $\$T$ now enters negatively into the affected individual's utility, but positively into the firm's.

Property Rights

– The constraint is binding, since the affected individual reduces the $\$T$ until the point where the firm is indifferent between accepting and rejecting the offer T.

– Hence, $\pi(x) + T = \pi(x^*)$ or $T = \pi(x^*) - \pi(x)$.

– Inserting this result into the affected individual's UMP, we obtain

$$\max_{x \geq 0} v(x) + w_2 \underbrace{-\pi(x^*) + \pi(x)}_{-T}$$

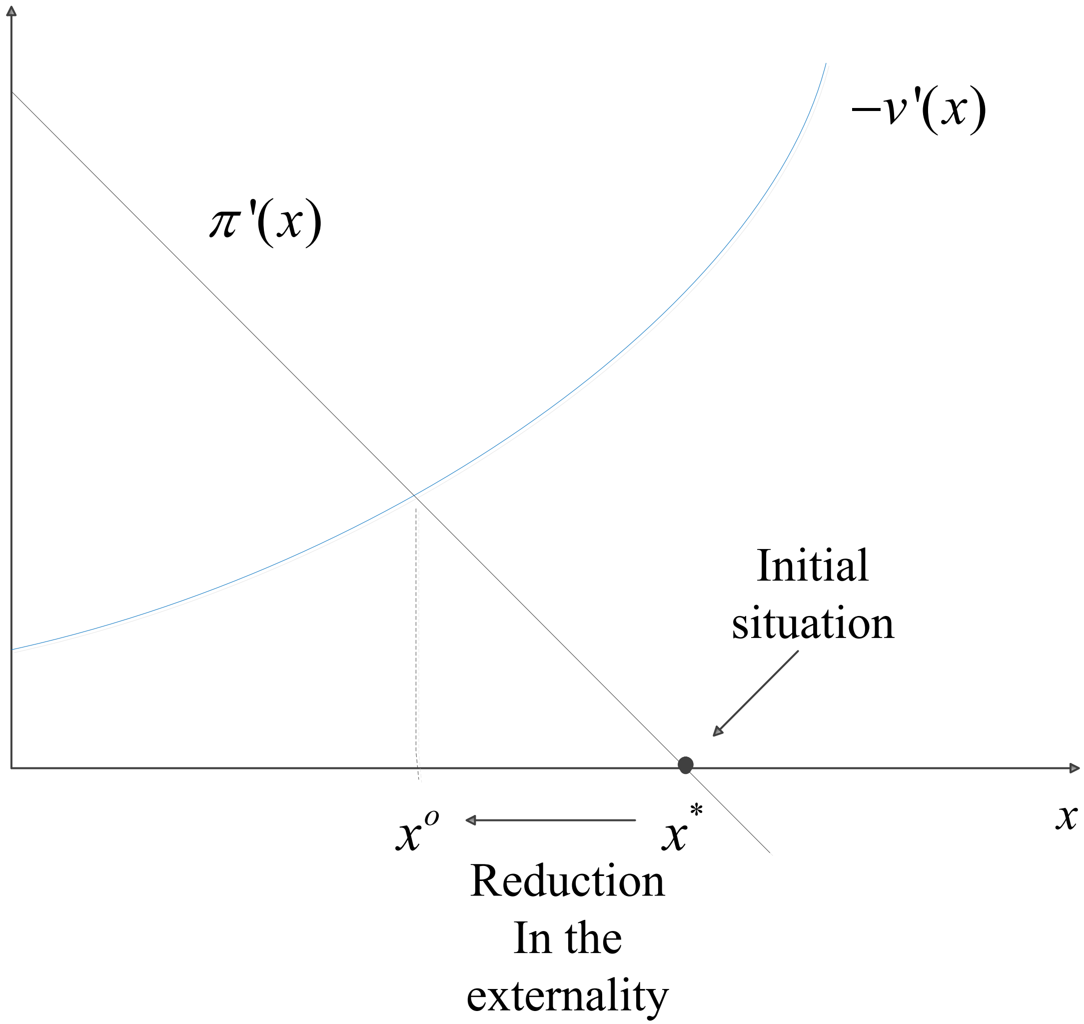
– FOCs with respect to x yields:

$$v'(x) + \pi'(x) \leq 0 \iff \pi'(x) \leq -v'(x)$$

Property Rights

- Again, the above coincides with the FOCs at the optimal level of the externality (i.e., social planner's problem), where $x = x^0$.

Property Rights



Property Rights

- In summary, regardless of the initial assignment of property rights over the externality-generating activity…
  - agents can negotiate to increase or decrease the externality, x, until reaching the Pareto optimal level x^0.

Property Rights

- ***Coase Theorem***: If bargaining between the agents generating and affected by the externality is possible and costless, then
  - the initial allocation of property rights does not affect the level of the externality.
  - That is, bargaining helps set the level of the externality at the optimal level $x = x^0$.
- The allocation of property rights, however, affects the final wealth of the two agents!

Property Rights

- If property rights are assigned to the individual affected by the externality:
  - the firm must pay $T = \pi(x^0) - \pi(0)$ to the affected individual in order to increase the externality from $x = 0$ to $x = x^0$
  - the affected individual's utility is then

$$v(x^0) + T \text{ or } v(x^0) + \pi(x^0) - \pi(0)$$

    while that of the firm is

$$\pi(x^0) - T \text{ or } \pi(x^0) - \pi(x^0) + \pi(0) = \pi(0)$$

  - the affected individual's utility is higher than that of the firm iff

$$v(x^0) + \pi(x^0) - \pi(0) > \pi(0) \iff \pi(x^0) + v(x^0) > 2\pi(0)$$

Property Rights

- If property rights are assigned to the firm generating the externality:
  - the affected individual must pay $T = \pi(x^*) - \pi(x^0)$ to the firm in order to reduce the externality from $x = x^*$ to $x = x^0$
  - the firm's utility is then
    $$\pi(x^0) + T \text{ or } \pi(x^0) + \pi(x^*) - \pi(x^0) = \pi(x^*)$$
    while that of the affected individual is
    $$v(x^0) - T \text{ or } v(x^0) - \pi(x^*) + \pi(x^0)$$
  - the firm's utility is higher than that of the affected individual iff
    $$\pi(x^*) > v(x^0) - \pi(x^*) + \pi(x^0) \Leftrightarrow 2\pi(x^*) > \pi(x^0) + v(x^0)$$

Property Rights

- Combining the two inequalities, we can conclude that the agent with the bargaining power (e.g., the property right over the lake) has a total utility higher than the agent without the bargaining power iff

$$2\pi(x^*) > \pi(x^0) + v(x^0) > 2\pi(0)$$

where $\pi(x^0) + v(x^0)$ measures the aggregate welfare at the social optimum, i.e., x^0.

Property Rights

- ***Disadvantages of the Coase Theorem***:
  - Property rights must be perfectly defined
    - Who should I bargain with?
  - Bargaining must be costless.
    - Not true if many agents are involved.
  - Property rights must be perfectly enforced:
    - The level of x must be perfectly observable and measurable by both parties
    - If a party does not comply with the agreement, it can be brought to court at no cost.
  - These assumptions are not satisfied in many cases, which limit the possibility of using negotiations.

Property Rights

- ***Advantages of the Coase Theorem***:
  - Only the parties involved must know the marginal benefits and costs associated with the externality
  - The regulator does not need to know anything!

Property Rights

- ***Remark***:
  - If the two parties are firms (e.g., fishery and refinery), a form of bargaining could be the sale of one firm to the other, i.e., a merger.
  - This is efficient as now the merged firm would internalize the effects that pollution imposes on the production process of the fishery.

Solutions to the Externality Problem:
More Intrusive Approaches

Quota

- Setting a quota (emission standard) that bans production levels higher than the Pareto optimal level x^0.
- The social planner must be perfectly informed about the benefits and damages of the externality for all consumers.

Pigouvian Taxation

- This policy sets a tax t_x per unit of the externality-generating activity x.
- What is the level of tax t_x that restores efficiency?
- Let us start by re-writing the firm's PMP

$$\max_{x \geq 0} \; \pi(x) - t_x \cdot x$$

- FOC with respect to x:

$$\pi'(x) - t_x \leq 0 \;\; \Longrightarrow \;\; \pi'(x) \leq t_x$$

or $\pi'(x) = t_x$ for interior solutions.

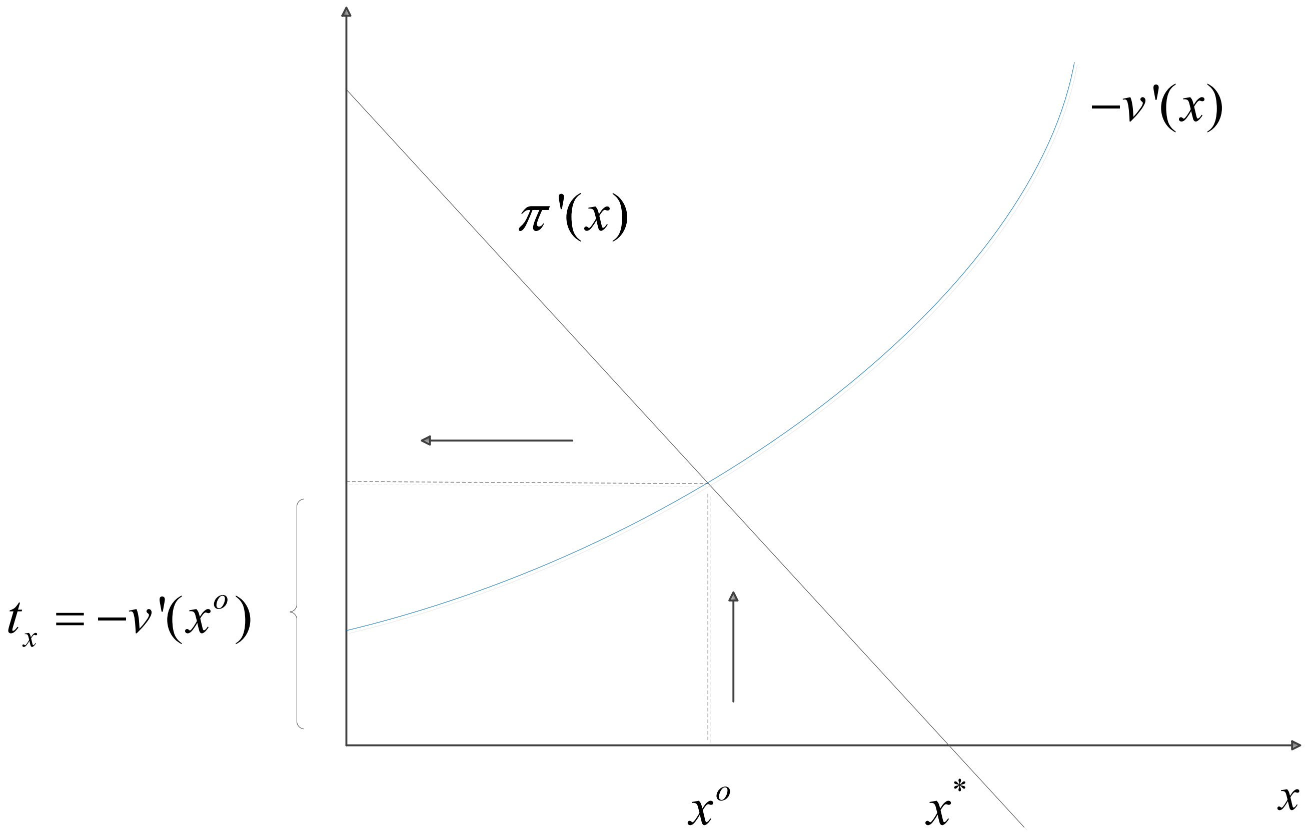
- *Intuition*: the firm increases x until the point where the marginal benefit from an additional unit of x coincides with the per-unit tax t_x.

Pigouvian Taxation

- We know that at the social optimum (i.e., x^0)
$$\pi'(x^0) = -v'(x^0)$$
- Hence, the tax t_x needs to be set at
$$t_x = -v'(x^0)$$
- This forces the firm to internalize the negative externality that its production generates on the consumer's wellbeing at x^0.

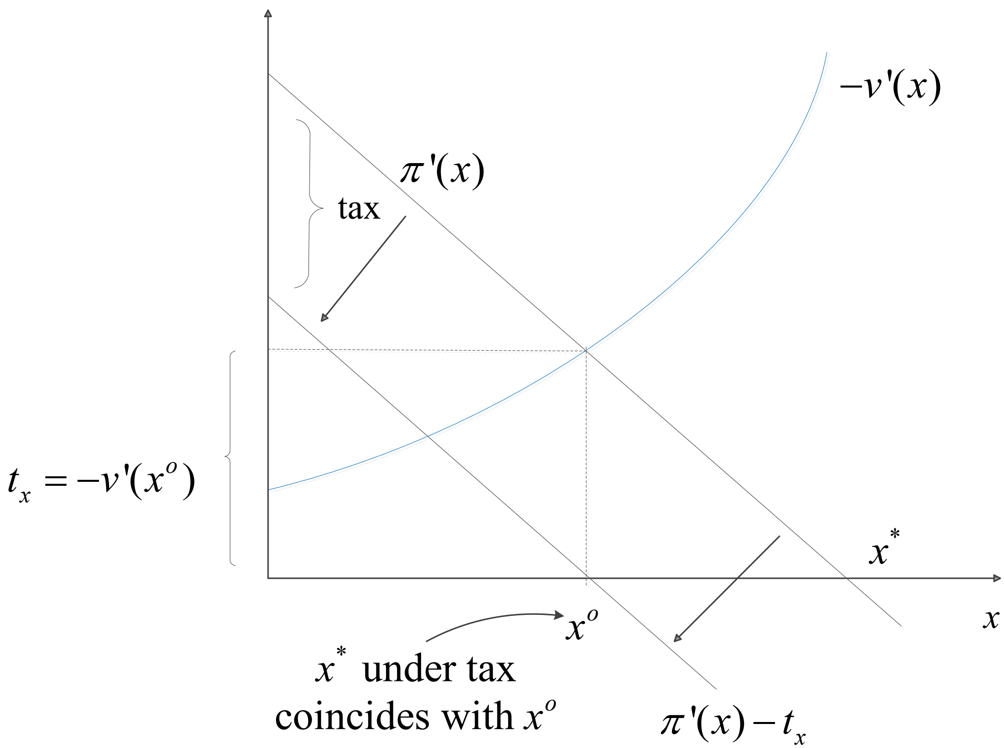
Pigouvian Taxation

- The tax t_x leads the firm to choose a level of x equal to x^0



Pigouvian Taxation

- The tax produces a downward shift in $\pi'(x)$.
- The new marginal benefit curve $\pi'(x) - t_x$ crosses the horizontal axis exactly at x^0.

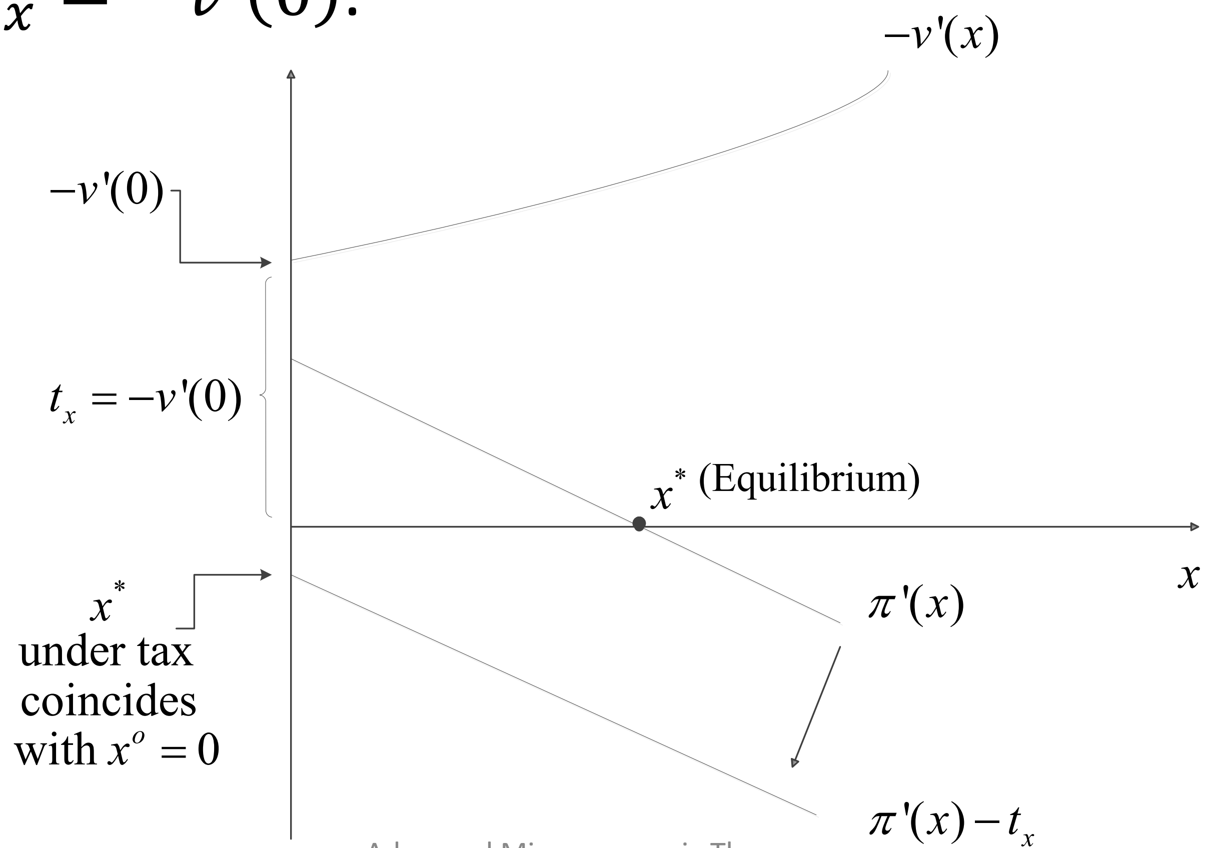


Pigouvian Taxation

- The optimality-restoring tax t_x is equal to the marginal externality at the optimal level x^0.
  - That is, it is equal to the amount of money that the affected individual would be willing to pay in order to reduce x slightly from its optimal level x^0.
- The tax t_x induces the firm to internalize the externality that it causes on the individual.

Pigouvian Taxation

- If the negative externality is very substantial (and the socially optimum is at $x^0 = 0$), the optimal Pigouvian tax is $t_x = -v'(0)$.

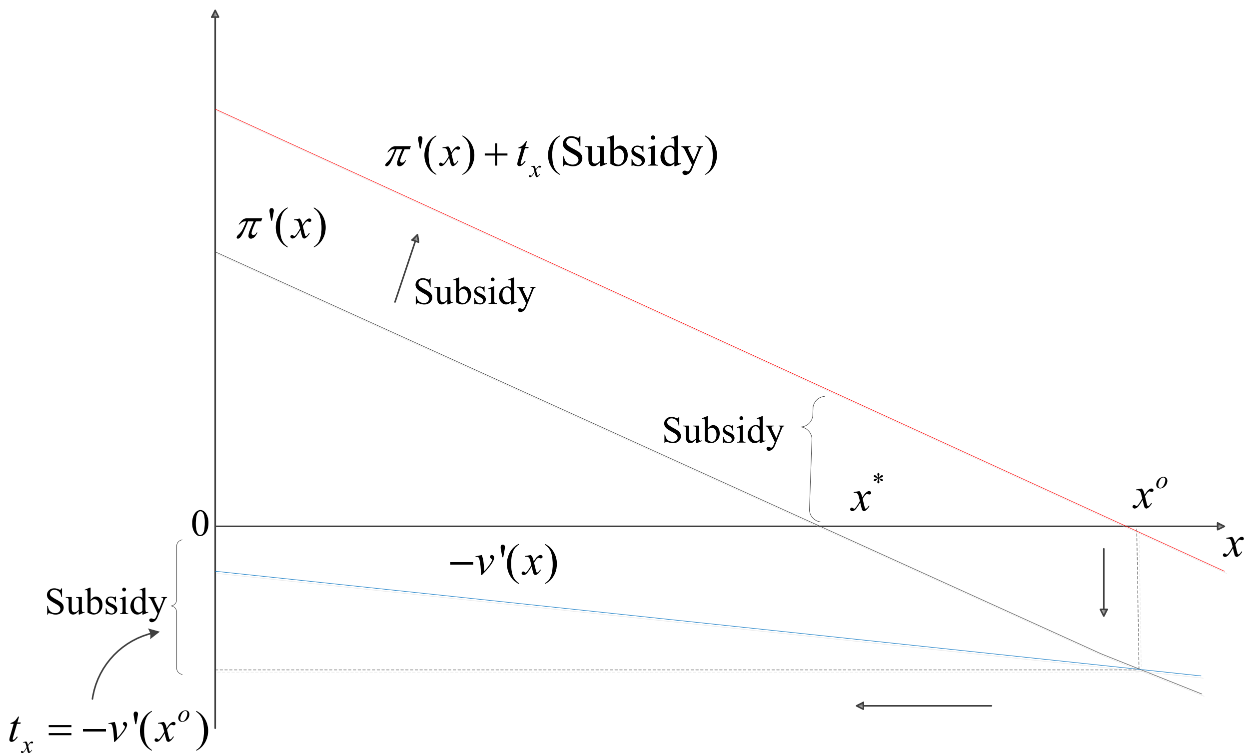


Pigouvian Subsidy

- Previous discussions can also be extended to *positive externalities*.
- Since now $v'(x^0) > 0$ (i.e., x increases individual's welfare), the optimality-correcting tax is
$$t_x = -v'(x^0) < 0$$
- We thus set "negative taxes" on the externality: a per-unit subsidy (s_x).
- The firm receives a payment of t_x for each unit of the positive externality it generates.

Pigouvian Subsidy

- The per-unit subsidy produces an upward shift in the marginal benefits of the firm.
- The firm has incentives to increase x beyond the competitive equilibrium level x^* until reaching the Pareto optimal level x^0.



Pigouvian Policy: Important Points

a) A tax on the negative externality is equivalent to a subsidy inducing agents to reduce the externality.

– Consider a subsidy $s_x = -v'(x^0) > 0$ for every unit that the firm's choice of x is below the equilibrium level of x^*.

– The firm's PMP becomes:

$$\max_{x \geq 0} \pi(x) + s_x(x^* - x) = \pi(x) + \underbrace{s_x x^*}_{\text{subsidy}} - \underbrace{s_x x}_{\text{per unit tax}}$$

– FOC with respect to x yields

$$\pi'(x^0) - s_x \leq 0 \text{ or } \pi'(x^0) \leq s_x$$

Pigouvian Policy: Important Points

- This FOC coincides with that under the Pigouvian taxation (taxing the negative externality at t_x), plus a lump-sum tax of $t_x x^*$.

- Hence, a subsidy for the reduction of the externality (combined with a lump-sum tax $t_x x^*$) can exactly replicate the outcome of the Pigouvian tax.

Pigouvian Policy: Important Points

b) *The Pigouvian tax levies a tax on the externality-generating activity (e.g., pollution) but not on the output that generated such pollution.*
   - Taxing output might lead the firm to reduce output, but it does not necessarily guarantee a reduction in pollutant emissions.
   - A tax on output can induce the firm to reduce emissions if emissions bear a constant relationship with output.

Pigouvian Policy: Important Points

c) *The quota and the Pigouvian tax are equally effective under complete information.*

   – They might not be equivalent when regulators face incomplete information about the benefits and costs of the externality for consumers and firms.

Solutions to the Externality Problem:
Tradable Externality Permits

Tradable Externality Permits

- Every permit grants the right to generate one unit of the externality.
- Suppose that $x^0 = \sum_j x_j^0$ total permits are given to the firms, with every firm receiving $\bar{x}_j$ of them.
- Let p_x^* denote the equilibrium price of these permits.
- Firm j's PMP is

$$\max_{x \geq 0} \; \pi_j(x_j) + p_x^*(\bar{x}_j - x_j)$$

where firm j must pay a price p_x^* for every permit it needs to buy in excess of its initial endowment $\bar{x}_j$.

Tradable Externality Permits

- FOC wrt x_j yields
$$\pi'_j(x_j) - p_x^* \leq 0$$
with equality if $x_j > 0$ (interior solution).

- If all J firms carry out this PMP, we need the market clearing condition
$$x^0 = \sum_j x_j$$

- The efficiency can be restored by setting
$$p_x^* = -\sum_{i=1}^{I} v_i'(x^0)$$

Tradable Externality Permits

- Thus, firm j's FOCs become

$$\pi_j'(x_j^0) + \sum_{i=1}^{I} v_i'(x^0) \le 0$$

with equality if $x_j^0 > 0$ (interior solution).

- This condition coincides with the FOC that solves the social planner's problem.

- Therefore, every firm j is induced to voluntarily choose $x_j = x_j^0$.

Tradable Externality Permits

- The advantage of tradable externality permits relative to quotas or taxes:
  - Government officials do not need so much information.
  - They only need data about the optimal level of pollution x^0.
    - Specifically, data on aggregate industry profits and damages from the externality
    - But not on individual firms and consumers

Regulating a Polluting Monopolist

Regulating a Polluting Monopolist

- Consider a monopolist facing a linear inverse demand curve $p(x) = 1 - x$ and constant marginal production costs $c < 1$.
- Assume that the monopolist's production generates an environmental damage measured by $ED(x) = d(x)^2$, where $d > 0$.
  – pollution is convex in output
- The social planner seeks x_{SO} that maximizes

$$W(x) = CS(x) + PS(x) + T - ED(x)$$

where $CS(x)$ and $PS(x)$ denote consumer and producer surplus, respectively, and T represents tax revenue from the emission fee.

Regulating a Polluting Monopolist

- Let us first find the monopolist's profit-maximizing level of x for a given emission fee t.
- The PMP of the monopolist is

$$\max_{x \geq 0} \quad (1-x)x - (c+t)x$$

- FOC:

$$1 - 2x - (c+t) = 0$$

- Solving for x yields an output function

$$x(t) = \frac{1-(c+t)}{2}, \quad x'(t) < 0$$

Regulating a Polluting Monopolist

- The social planner solves
$$\max_{x\geq 0} W(x) = CS(x) + PS(x) + T - ED(x)$$
- where $CS(x) \equiv \frac{1}{2}(x)^2$, $PS(x) \equiv (1-x)x - (c+t)x$, $T \equiv tx$, and $ED(x) \equiv d(x)^2$.
- FOC:
$$2\frac{1}{2}x + [1 - 2x - (c+t)] + t - 2dx = 0$$
or re-arranging, $1 - c = x(1+2d)$
- Solving for x yields $x_{SO} = \frac{1-c}{1+2d}$.

Regulating a Polluting Monopolist

- The emission fee t that induces the monopolist to produce x_{SO} is found by solving
$$\frac{1-(c+t)}{2} = \frac{1-c}{1+2d}$$
which yields
$$t = (2d-1)\frac{1-c}{1+2d} \quad \text{or} \quad t = (2d-1)x_{SO}$$
- Notice that
  - $t > 0$ (a taxation) iff $d > 1/2$
  - $t < 0$ (a subsidy) iff $d < 1/2$

Regulating a Polluting Monopolist

- *Intuition*:
  - If the market failure arising from the environmental externality is sufficiently large, i.e., $d > 1/2$, the regulator imposes a tax policy in order to reduce the production from the polluting monopolist.
  - If, in contrast, the market failure from the externality is less damaging, i.e., $d < 1/2$, the regulator subsidizes the monopolist's production.

Regulating a Polluting Oligopoly

Regulating a Polluting Oligopoly

- Consider a Cournot oligopoly facing a linear inverse demand curve $p(X) = 1 - X$, where X is aggregate output.
- Assume that an incumbent firm can
  - enjoy a cost advantage relative to the entrant, i.e., $c_{inc} < c_{ent} < 1$, or
  - face the same production costs, i.e., $c_{inc} = c_{ent} < 1$
- The oligopoly generates an environmental damage measured by $ED(X) = d(X)^2$, where $d > 0$.
  - pollution is convex in output
  - for simplicity, we assume that $d > 1/2$
- The social planner seeks X_{SO} that maximizes

$$W(X) = CS(X) + PS(X) + T - ED(X)$$

Regulating a Polluting Oligopoly

- Let us first find the oligopolists' profit-maximizing output levels for a given emission fee t.

- The PMP of the incumbent is

$$\max_{x_{inc}} \ (1 - x_{inc} - x_{ent})x_{inc} - (c_{inc} + t)x_{inc}$$

while that of the entrant's

$$\max_{x_{ent}} \ (1 - x_{ent} - x_{inc})x_{ent} - (c_{ent} + t)x_{ent}$$

- FOCs wrt x_i, where $i = \{inc, \ ent\}$, yields

$$1 - 2x_i - x_j - (c_i + t) = 0 \ \text{ for } \ j \neq i$$

Regulating a Polluting Oligopoly

- Solving for x_i yields firm i's BRF

$$x_i(x_j) = \frac{1-(c_i+t)}{2} - \frac{1}{2}x_j$$

- Plugging firm j's BRF into firm i's, we obtain

$$x_i = \frac{1-(c_i+t)}{2} - \frac{1}{2} \cdot \left(\frac{1-(c_j+t)}{2} - \frac{1}{2}x_i\right)$$

- Solving for x_i yields an equilibrium output function

$$x_i(t) = \frac{1-2c_i+c_j-t}{3}$$

which is decreasing in c_i and t, but increasing in c_j.

Regulating a Polluting Oligopoly

- The social planner solves

$$\max_{X \geq 0} W(X) = CS(X) + PS(X) + T - ED(X)$$

where $X \equiv x_{inc} + x_{ent}$, $CS(X) \equiv \frac{1}{2}(X)^2$, $PS(X) \equiv (1 - X)X - (c_{inc} + t)X$, $T \equiv tX$, and $ED(X) \equiv d(X)^2$.

  – Note that $PS(X)$ considers the incumbent's marginal costs, since it is the most efficient firm, i.e., $c_{inc} \leq c_{ent}$

- FOC:

$$X_{SO} = \frac{1 - c_{inc}}{1 + 2d}$$

Regulating a Polluting Oligopoly

- The emission fee t that induces the incumbent and entrant to produce $x_{inc,SO}$ and $x_{ent,SO}$, respectively, is found by simultaneously solving

$$x_{inc,SO} + x_{ent,SO} = \frac{1-c_{inc}}{1+2d}$$

$$x_{inc}(t) = \frac{1-2c_{inc}+c_{ent}-t}{3}$$

$$x_{ent}(t) = \frac{1-2c_{ent}+c_{inc}-t}{3}$$

- Two cases:
  1) Cost symmetry, $c_{inc} = c_{ent}$
  2) Cost asymmetry, $c_{inc} < c_{ent}$

Regulating a Polluting Oligopoly

- ***Case 1: Cost symmetry,*** $c_{inc} = c_{ent}$
  - Simultaneously solving the three equations yields

$$t = \frac{4d-1}{2}\frac{1-c_{inc}}{1+2d} \quad \text{or} \quad t = (4d-1)\frac{X_{SO}}{2}$$

where $t > 0$ iff $d > 1/4$.

    - The regulator imposes emission fees on the oligopoly even in settings in which he would not impose a fee to a monopoly, i.e., $d \in \left[\frac{1}{4}, \frac{1}{2}\right]$.
  - Substituting t into the output function $x_i(t)$ yields

$$x_{inc,SO} = x_{ent,SO} = \frac{1}{2} \cdot \frac{1-c_{inc}}{1+2d} = \frac{X_{SO}}{2}$$

Regulating a Polluting Oligopoly

- ***Case 2: Cost asymmetry,*** $c_{inc} < c_{ent}$
  - Simultaneously solving the three equations yields
  $$t = \frac{A(1-c_{ent})-B(1-c_{inc})}{2A}$$
  where $A \equiv 1 + 2d$ and $B \equiv 2(1-d)$, and $t > 0$ iff $c_{ent} < \frac{4d-1+Bc_{inc}}{A}$.
  - Substituting t into the output function $x_i(t)$ yields
  $$x_{inc,SO} = \frac{-1 + 4d + 3Ac_{ent} - 2(1+5d)c_{inc}}{6A}$$
  $$x_{ent,SO} = \frac{1 - Ac_{ent} + 2dc_{inc}}{2A}$$
  which are positive iff $c_{ent} > \frac{2(1+5d)c_{inc}+1-4d}{3A}$ and $c_{ent} < \frac{1+2dc_{inc}}{A}$, respectively.

Regulating a Polluting Oligopoly

– Let us check if the three conditions for c_{ent} are binding.

1) The condition that guarantees $x_{inc,SO} > 0$, i.e., $c_{ent} > \frac{2(1+5d)c_{inc}+1-4d}{3A}$, holds for all $c_{inc} < c_{ent}$.
   - The cutoff originates in the negative quadrant ($-\frac{4d-1}{3A}$) and lies below the 45^{o}-line for all c_{inc}.
   - Hence, the cutoff is not binding.

Regulating a Polluting Oligopoly

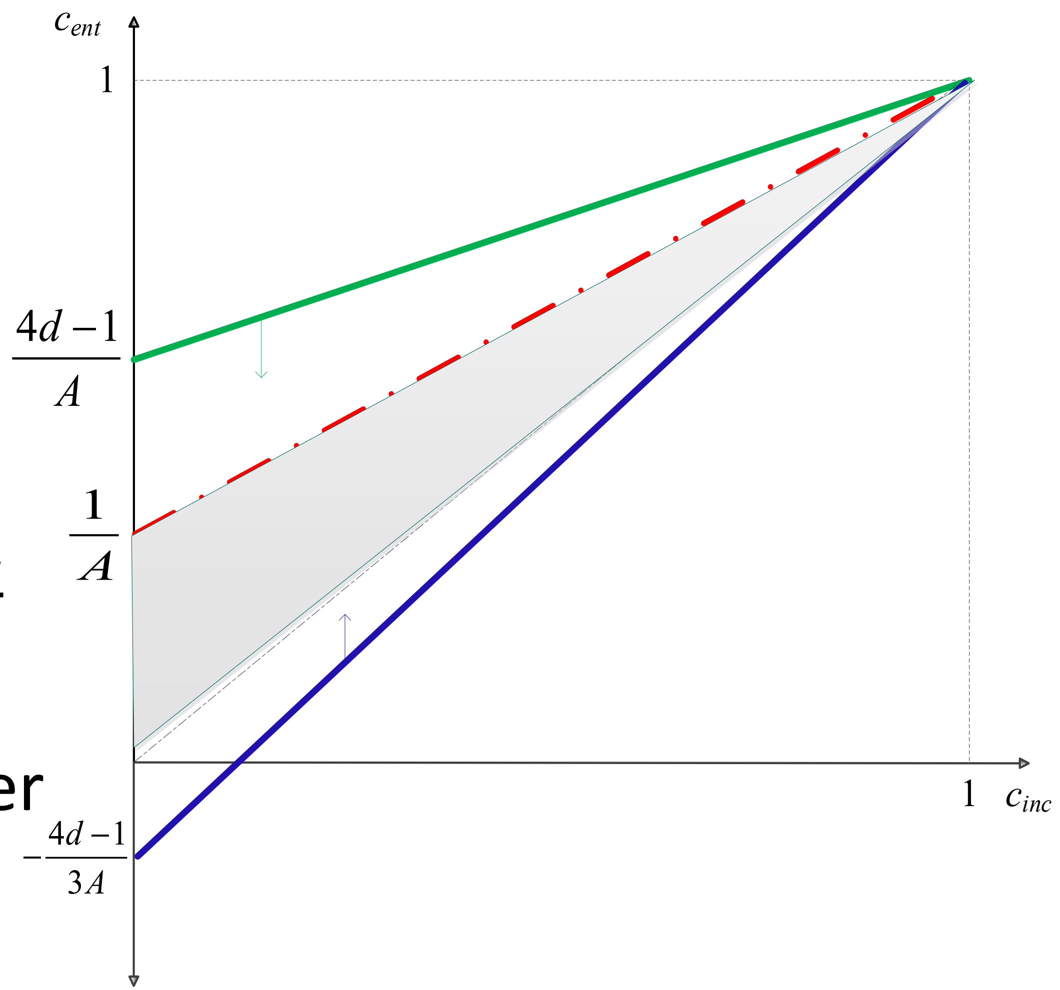
2) The condition that guarantees $x_{ent,SO} > 0$, i.e., $c_{ent} < \frac{1+2dc_{inc}}{A}$, is more restrictive than the condition that guarantees $t > 0$, i.e., $c_{ent} < \frac{4d-1+Bc_{inc}}{A}$.
   - Cutoff $\frac{1+2dc_{inc}}{A}$ originates at $1/A$, while the cutoff $\frac{4d-1+Bc_{inc}}{A}$ originates at a higher vertical intercept, $\frac{4d-1}{A}$ since $4d-1>1$ but with a less steep slope because $B \equiv 2(1-d) < 2d$, both of which hold when pollution is relatively severe, i.e., $d > 1/2$.
   - Hence, the cutoff $c_{ent} < \frac{4d-1+Bc_{inc}}{A}$ is not binding.
   - Only the cutoff $c_{ent} < \frac{1+2dc_{inc}}{A}$ is binding.

Regulating a Polluting Oligopoly

- In order to have a positive emission fee that induces positive output levels from both firms, we need

$$c_{inc} < c_{ent} < \frac{1+2dc_{inc}}{A}$$

where the entrant suffers a slightly higher but not too high cost than the incumbent.



Regulating a Polluting Oligopoly

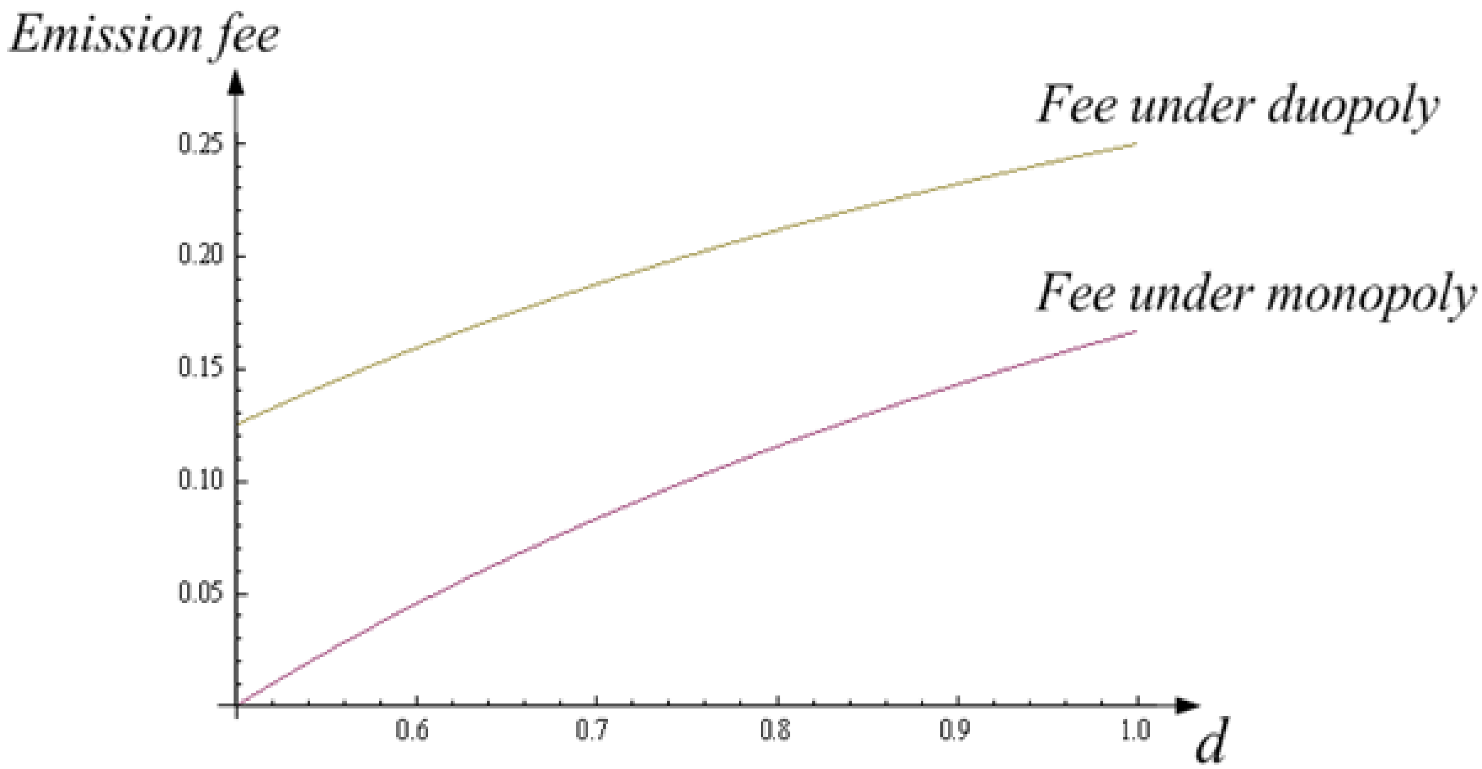
- Fee comparison:
  - The regulator sets more stringent fees to the duopolists than to the monopolist
  - For the case of cost symmetry $c_{inc} = c_{ent}$,

$$(4d-1)\frac{X_{SO}}{2} > (2d-1)X_{SO}$$
$$\Leftrightarrow \ 4d-1 > 4d-2$$

  - Similar results arise for a different marginal cost.
  - *Intuition*:
    - The unregulated duopoly generates a larger amount of pollution than unregulated monopoly.
    - Hence, the regulator sets a more stringent fee on the former.

Regulating a Polluting Oligopoly

- Fee comparison:



Presence of Asymmetric Information in Externality Problems

Presence of Asymmetric Information in Externality Problems

- Consider a setting in which firms generate the externality whereas consumers are affected by that externality.
- Let $v(x, \eta)$ be the derived utility of a consumer of type $\eta \in \mathbb{R}$ from x amount of externality.
- Let $\pi(x, \theta)$ be the derived profit function of a firm of type $\theta \in \mathbb{R}$ which generates x amount of externality.
- Consider that parameters η and θ are privately observed by the consumer and firm, respectively.
  - Agents do not observe each other's types, but know the ex-ante likelihoods of η and θ.
  - For simplicity, we consider that parameters η and θ are independently distributed.
- Functions $v(x, \eta)$ and $\pi(x, \theta)$ are strictly concave in the externality x for any value of η and θ.

Presence of Asymmetric Information in Externality Problems

- Let us first consider the decentralized bargaining procedure.
- Bargaining in the presence of asymmetric information does not necessarily lead to an efficient level of the externality x^0.
- Suppose that the consumer has the right to an externality-free environment, and he makes a take-it-or-leave-it offer to the firm.
- Assume that there are two levels of the negative externality: $x = 0$ and $x = \bar{x}$
  - the consumer prefers $x = 0$ to $x = \bar{x}$, whereas the firm prefers $x = \bar{x}$ to $x = 0$.

Presence of Asymmetric Information in Externality Problems

- The benefits that a firm of type θ obtains from having an externality level $x = \bar{x}$ is

$$b(\theta) = \pi(\bar{x}, \theta) - \pi(0, \theta) > 0$$

- The costs that a consumer of type η bears from having an externality level $x = \bar{x}$ is

$$c(\eta) = v(0, \eta) - v(\bar{x}, \eta) > 0$$

- What matters in the negotiation between the consumer and the firm are the precise values of $b(\theta)$ and $c(\eta)$.
  - The CDF of $b(\theta)$ and $c(\eta)$ are $G(b)$ and $F(c)$, respectively.
  - The PDF of $b(\theta)$ and $c(\eta)$ are $g(b) > 0$ for all $b > 0$ and $f(c) > 0$ for all $c > 0$, respectively.

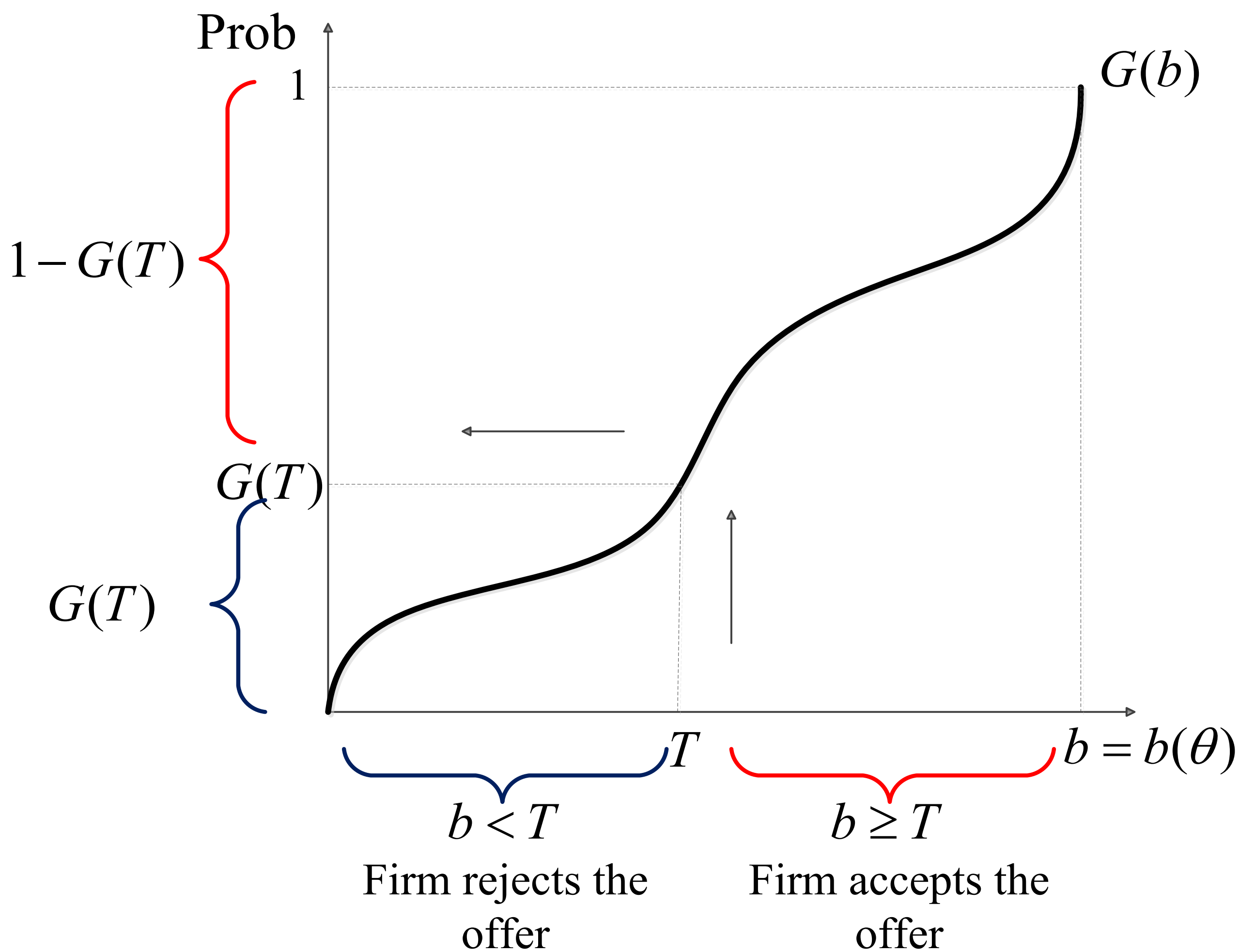
Presence of Asymmetric Information in Externality Problems

- In the absence of an agreement, the level of the externality remains at $x = 0$.
  - Consumer has the right to the resource
- *Pareto optimal outcome*: the firm should be allowed to set a level of the externality $x = \bar{x}$ whenever $b > c$.
  - Intuitively, the firm is willing to pay the consumer more than the damage that the consumer suffers from the externality.
  - Hence, $x = \bar{x}$ would be agreed by a firm and consumer if they were perfectly informed about each other's marginal benefits and costs.

Presence of Asymmetric Information in Externality Problems

- Let us now start analyzing equilibrium strategies in this context.
- What amount should the consumer demand from the firm (a take-it-or-leave-it-offer) when his cost of the externality is exactly $c(\eta) = c$?
- A θ-type firm will agree to pay T iff its benefits, $b(\theta) = b$, satisfy $b \geq T$.
- Hence, the consumer knows the probability of the firm accepting the payment of T is equal to the probability that $b \geq T$, i.e., $1 - G(T)$.

Presence of Asymmetric Information in Externality Problems



Presence of Asymmetric Information in Externality Problems

- Hence, the consumer chooses the value of the offer T that maximizes his expected utility
$$\max_{T \geq 0} \; [1 - G(T)](T - c)$$
where
  - $1 - G(T)$ is the probability that an offer of T is accepted by the firm
  - $T - c$ is the net gain that a consumer (with cost c) obtains if the offer is accepted

- FOC wrt T yields
$$[1 - G(T_c^*)] - g(T_c^*)(T_c^* - c) \leq 0$$
and in interior solution, $[1 - G(T_c^*)] = g(T_c^*)(T_c^* - c)$.

Presence of Asymmetric Information in Externality Problems

- Re-arranging,

$$\frac{1-G(T_c^*)}{g(T_c^*)} + c = T_c^*$$

- Since the ratio $\frac{1-G(T_c^*)}{g(T_c^*)} \neq 0$, we have that $T_c^* > c$.
- This implies the firm rejects the consumer's offer when b satisfies $T_c^* > b > c$.
  - However, since $b > c$, Pareto optimality requires that the externality is increased until $x = \bar{x}$.
  - But in this setting the consumer's offer is rejected with positive probability for $T_c^* > b > c$.

Presence of Asymmetric Information in Externality Problems

- Complete information:
  - The firm and consumers are willing to bargain and have the externality produced when they are perfectly informed about their benefits and costs.
  - A welfare improvement for both parties.
- Asymmetric information:
  - The lack of information hinders the success of the mutually beneficial agreements.
  - Decentralized bargaining does not necessarily yield efficient outcomes.

Quotas under Incomplete Information

Quotas under Incomplete Information

- Unlike complete information settings:
  - Government intervention (quotas or taxes) does not necessarily achieve efficient outcomes when the agents are asymmetrically informed.
  - In addition, quotas or taxes are not perfectly substitutable between one another.

- For given η and θ, the aggregate surplus resulting from externality level x is

$$v(x,\eta) + \pi(x,\theta)$$

Quotas under Incomplete Information

- The Pareto optimal level of the externality $x(\eta, \theta)$ solves

$$\max_{x \geq 0} \; v(x, \eta) + \pi(x, \theta)$$

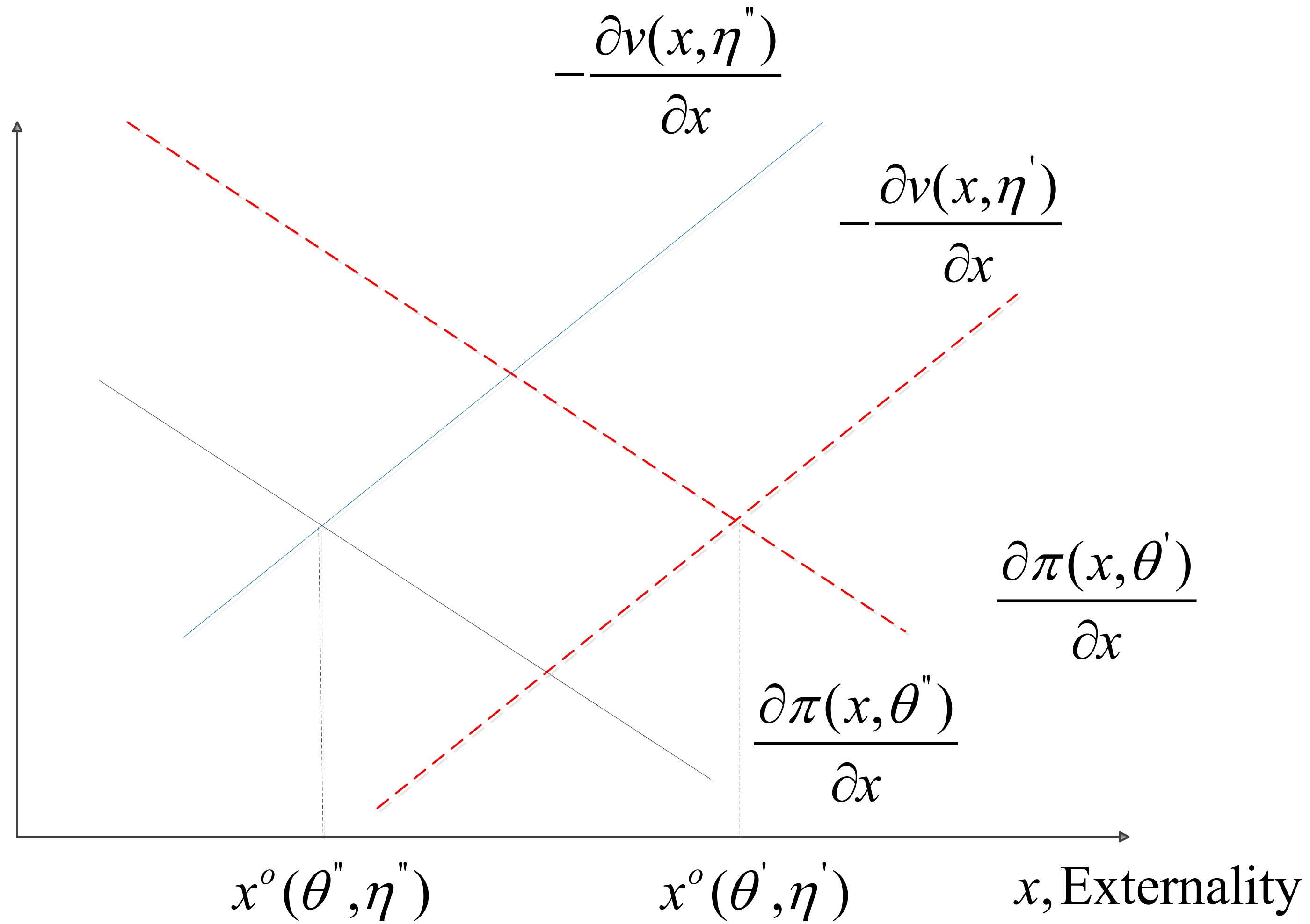
- FOC wrt x yields

$$\frac{\partial v(x,\eta)}{\partial x} + \frac{\partial \pi(x,\theta)}{\partial x} \leq 0$$

or, at an interior optimum,

$$\frac{\partial v(x,\eta)}{\partial x} + \frac{\partial \pi(x,\theta)}{\partial x} = 0$$

Quotas under Incomplete Information



Quotas under Incomplete Information

- Suppose that a quota is fixed at the level of the externality $x = \hat{x}$.
- The firm's PMP becomes

$$\max_{x \geq 0} \ \pi(x, \theta) \quad \text{s.t. } x \leq \hat{x}$$

- Let $x^q(\hat{x}, \theta)$ be the externality level that solves this PMP.
  - Since the PMP does not depend on η, $x^q(\hat{x}, \theta)$ is completely insensitive to η.
  - Thus, $x^q(\hat{x}, \theta)$ cannot be efficient.
  - The efficient level of externality is $x^0(\theta, \eta)$.

Quotas under Incomplete Information

- The level of the quota $\hat{x}$ is such that $\frac{\partial \pi(\hat{x},\theta)}{\partial x} > 0$ for all $\theta > 0$.
- Thus, the profit-maximizing level of the externality is $x^q(\hat{x}, \theta) = \hat{x}$.
- That is, the firm would like to increase the externality x beyond $\hat{x}$, but it cannot since it already reached the quota.

Quotas under Incomplete Information

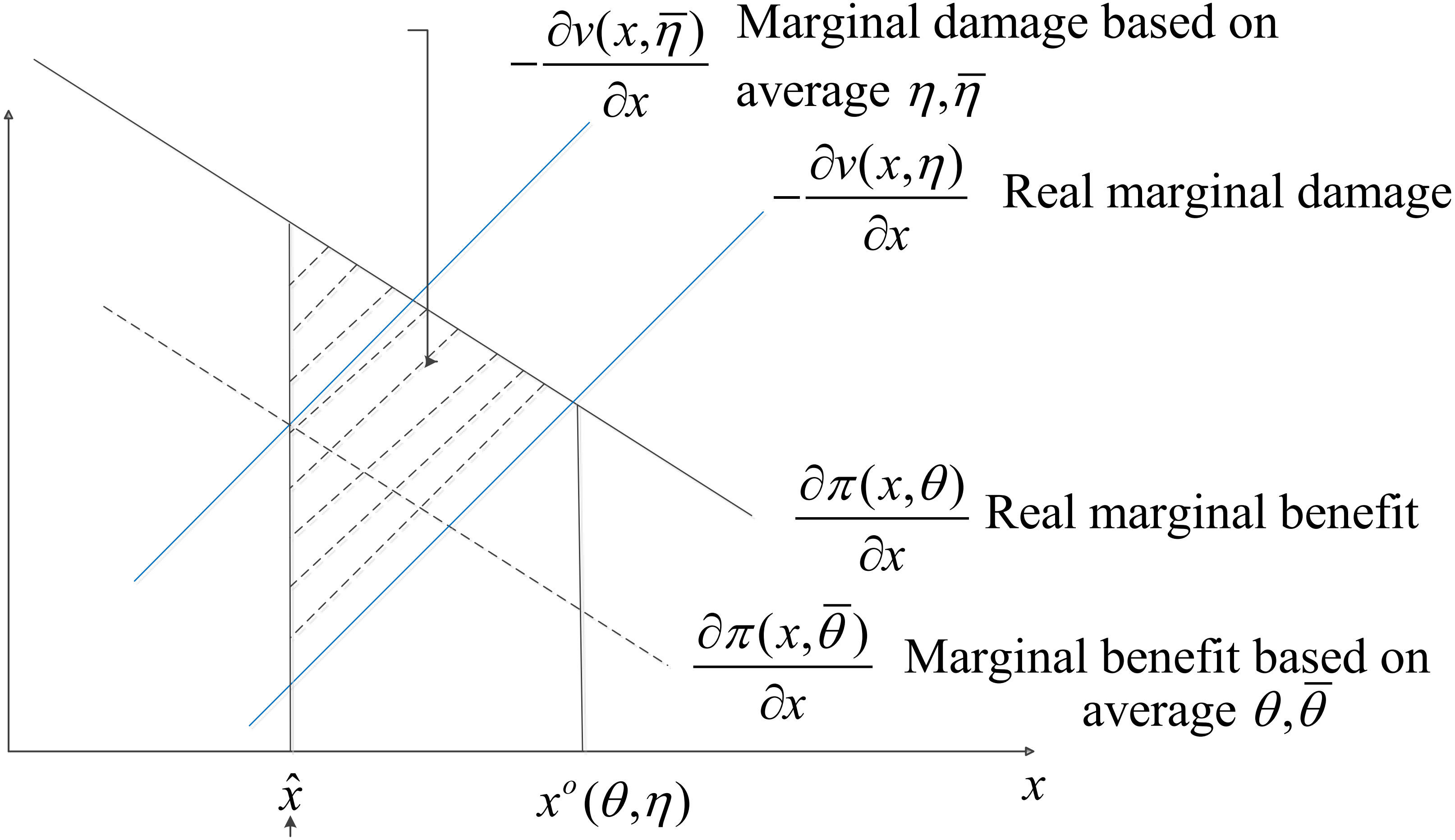
- The welfare loss of quota $\hat{x}$ relative to the socially optimal level of externality $x^0(\theta,\eta)$ is

$$\overbrace{[v(x^q(\hat{x},\theta),\eta)+\pi(x^q(\hat{x},\theta),\theta)]}^{\text{Aggregate surplus with the quota } \hat{x}} - \underbrace{v(x^0(\theta,\eta),\eta)+\pi(x^0(\theta,\eta),\theta)}_{\text{Aggregate surplus at the PO}}$$

or, more compactly,

$$\int_{x^o(\theta,\eta)}^{x^q(\hat{x},\theta)}\left(\frac{\partial\pi(x,\theta)}{\partial x}+\frac{\partial v(x,\eta)}{\partial x}\right)dx$$

Quotas under Incomplete Information



Taxes under Incomplete Information

Taxes under Incomplete Information

- Suppose that the regulator imposes a tax t per unit of the externality.
- The firm's PMP becomes

$$\max_{x \geq 0} \; \pi(x,\theta) - tx$$

- FOC yields $\frac{\partial \pi(x,\theta)}{\partial x} \leq t$ or, in an interior solution, $\frac{\partial \pi(x,\theta)}{\partial x} = t$.
- Let $x^t(t,\theta)$ denote the amount of the externality that solves the FOC (interior solution).
  - $x^t(t,\theta)$ is completely insensitive to η.
  - Thus, $x^t(t,\theta)$ cannot be efficient.

Taxes under Incomplete Information

- The welfare loss caused by the imposition of a tax relative to the socially optimal level of externality $x^0(\theta,\eta)$ is

$$\overbrace{[v(x^t(t,\theta),\eta)+\pi(x^t(t,\theta),\theta)]}^{\text{Aggregate surplus with tax } t}$$
$$-\underbrace{v(x^0(\theta,\eta),\eta)+\pi(x^0(\theta,\eta),\theta)}_{\text{Aggregate surplus at the PO}}$$

or, more compactly,

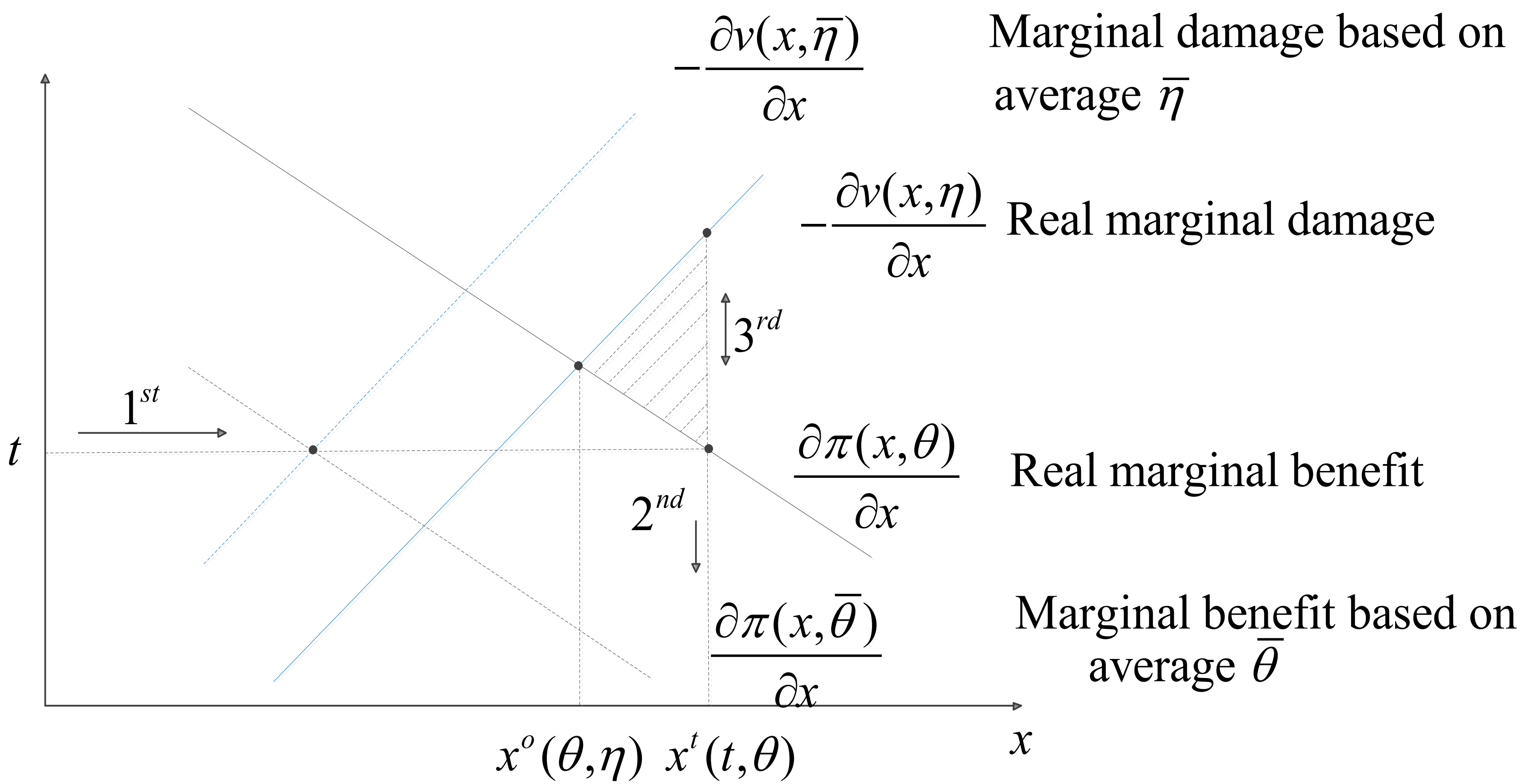
$$=\int_{x^o(\theta,\eta)}^{x^t(t,\theta)}\left(\frac{\partial\pi(x,\theta)}{\partial x}+\frac{\partial v(x,\eta)}{\partial x}\right)dx$$

Taxes under Incomplete Information

- The tax must be set at the point that maximizes aggregate surplus, evaluated at the average value of θ and η, $(\bar{\theta}, \bar{\eta})$, that is

$$t = -\frac{\partial v\left(x^{o}\left(\bar{\theta}, \bar{\eta}\right), \bar{\eta}\right)}{\partial x}$$

Taxes under Incomplete Information



Comparing Policy Instruments under Incomplete Information

Policy Comparison

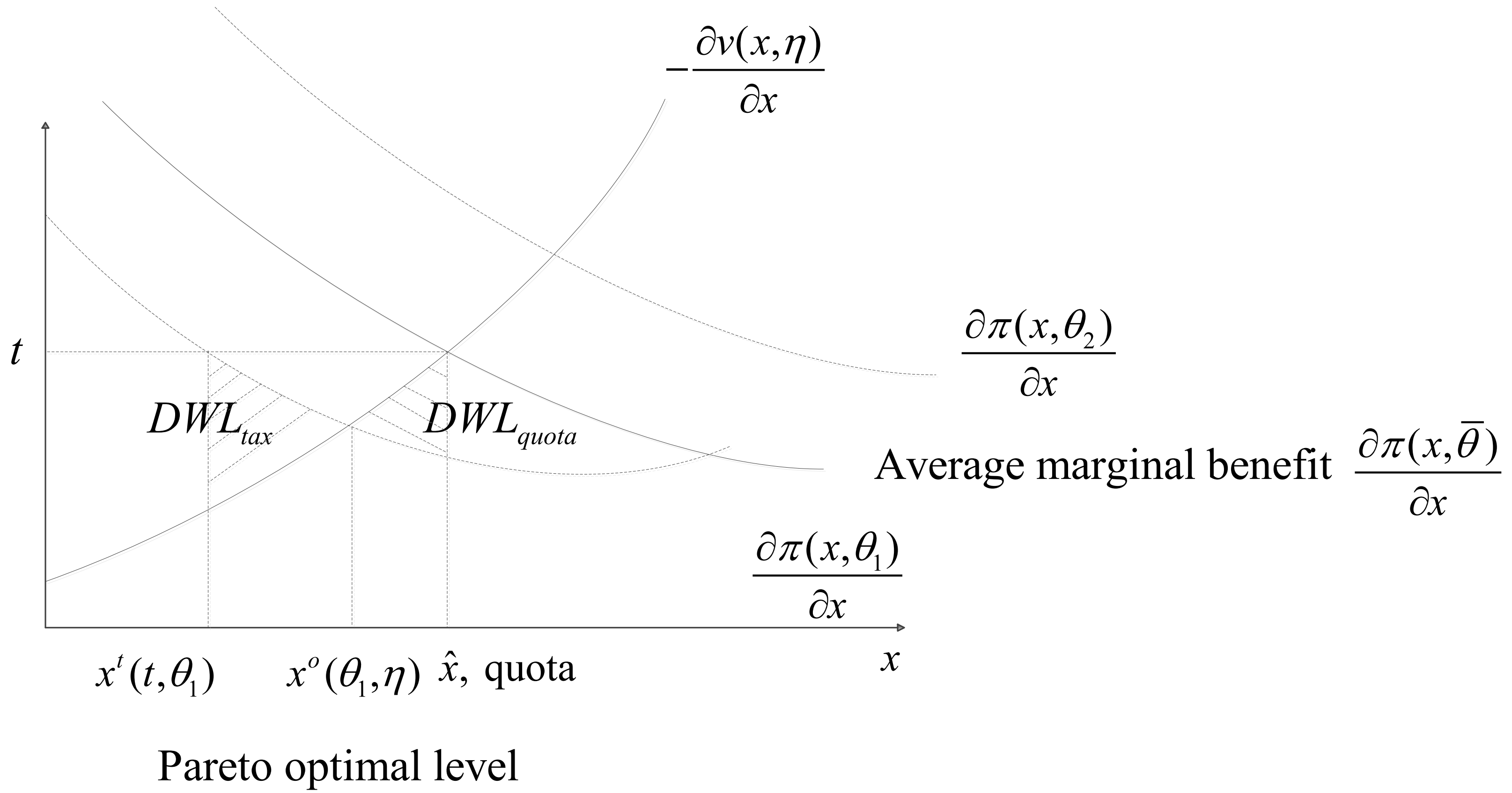
- Both quotas and emission fees create inefficiencies under incomplete information.
- Which instrument, despite being imperfect, performs better?
  - "second-best" policy
- It depends on the elasticity of the marginal damage and marginal benefit functions.
- Consider a setting where:
  - the realization of parameter θ is $\theta = \theta_1$
  - the regulator has relatively precise information about the marginal damage function, but he is uncertain about the firm's marginal benefit function.

Policy Comparison

- The regulator sets:
  - a quota $\hat{x}$ at the point where the observed marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, crosses the average marginal benefit function, i.e., $\frac{\partial \pi(x,\overline{\theta})}{\partial x}$.
  - an emission fee t at the height at which the observed marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, crosses the average marginal benefit function, i.e., $\frac{\partial \pi(x,\overline{\theta})}{\partial x}$.

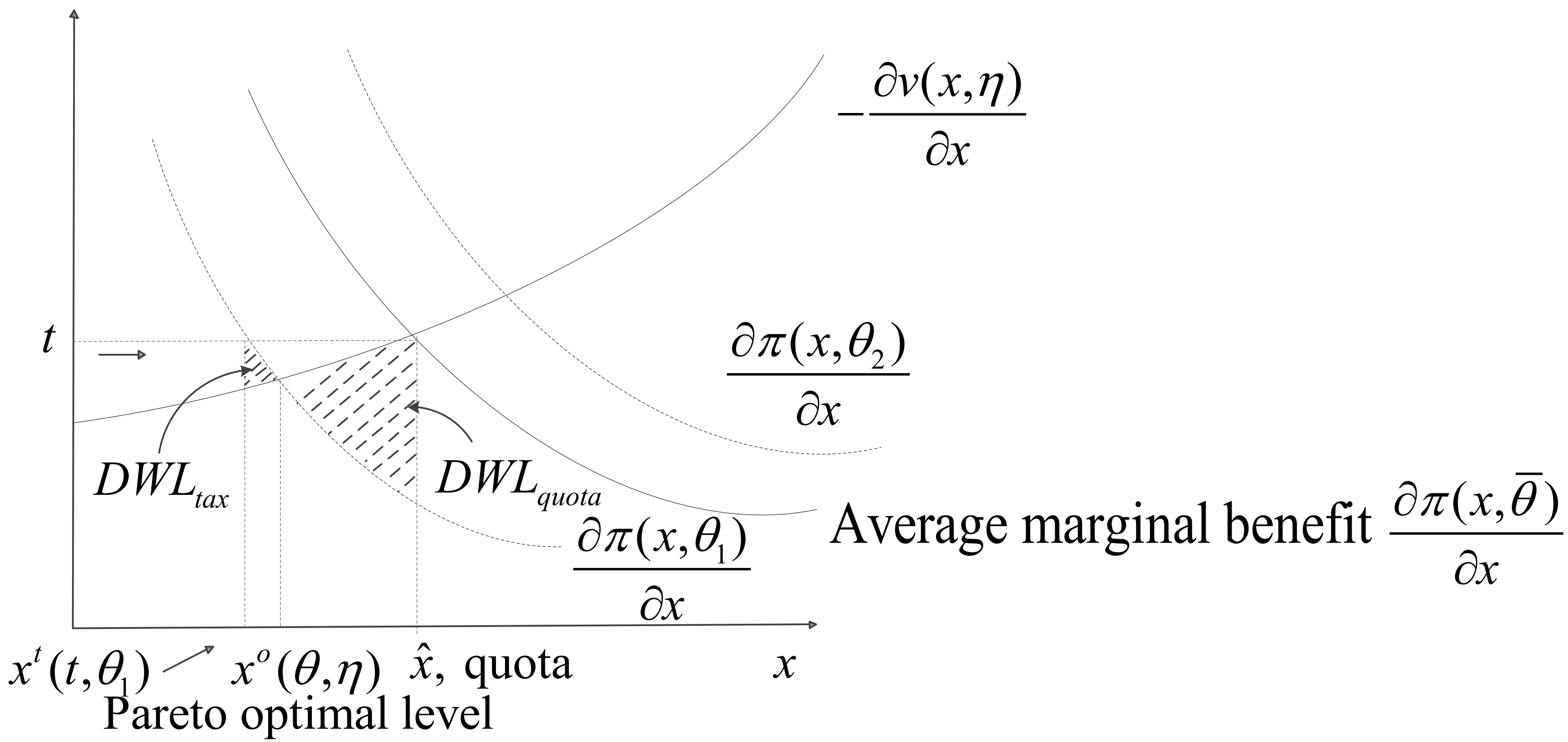
Policy Comparison

- The marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, is relatively sensitive to x.



Policy Comparison

- The marginal damage function, i.e., $-\frac{\partial v(x,\eta)}{\partial x}$, is not very sensitive to x.



Policy Comparison

- For a given elasticity of the marginal profit function, at the socially optimal level of the externality:
  - quota performs better than emission fee when the marginal damage function is relatively inelastic
  - emission fee performs better than quota when the marginal damage function is relatively elastic

Tragedy of the Commons

Tragedy of the Commons

- "Tragedy of the Commons" considers a common pool resource (CPR, e.g., a fishing ground or an aquifer) exploited by $n \geq 2$ firms, and shows that firms' exploitation is socially excessive.
  - That is, the sum of the individual exploitation levels that each firm independently and simultaneously chooses exceeds the aggregate exploitation level that a social planner would select.

Tragedy of the Commons

- Consider a setting in which each firm exploiting the CPR chooses an individual appropriation level $x_i \geq 0$.
- In particular, every firm *i*'s profit function is

$$\pi_i(x_i, X_{-i}) = p(X)x_i - c(x_i, X_{-i})$$

where $p(X)$ denotes the inverse demand function, $p'(X) < 0, p''(X) \geq 0$, $X \equiv \sum_{i=1}^{N} x_i$ represents aggregate output, and $c(x_i, X_{-i})$ represents firm *i*'s cost of appropriating x_i units of the resource when its rivals extract $X_{-i} \equiv \sum_{j \neq i} x_j$, $\frac{\partial c(\cdot)}{\partial x_i} > 0$, $\frac{\partial^2 c(\cdot)}{\partial x_i^2} \geq 0$, and $\frac{\partial^2 c(\cdot)}{\partial x_i \partial x_j} > 0$ implying that the CPR becomes more scarce and more difficult to exploit by firm *i*.

Tragedy of the Commons

- The equilibrium is solved by every firm *i* simultaneously and independently chooses its appropriation level x_i to maximize its profit $\pi_i(x_i, X_{-i})$. Taking FOC wrt x_i, and assuming an interior solution, we obtain

$$p(X^*) + p'(X^*)x_i^* = \frac{\partial c(x_i^*, X_{-i}^*)}{\partial x_i}$$

where $X^* = x_i^* + X_{-i}^*$

- Intuitively, the marginal revenues and marginal costs from a large individual approximation x_i must offset each other in equilibrium. Solving for x_i in the FOC above, we find firm *i*'s BRF, $x_i^*(X_{-i}^*)$, describing firm *i*'s appropriation as a function of its rivals' X_{-i}.

Tragedy of the Commons

- Consider a commons with:
  - linear inverse demand $p(X) = a - bX$, where $a, b > 0$; and
  - cost function $c(x_i, X_{-i}) = cx_i(1 + \alpha X_{-i})$, where $c > 0$, and $\alpha \geq 0$ represents how other firms' appropriation, X_{-i}, affect firm *i*'s costs.
- The FOC for equilibrium appropriation becomes

$$a - b(x_i + X_{-i}) + (-b)x_i = c(1 + \alpha X_{-i})$$

- Solving for firm *i*'s appropriation x_i yields a BRF of

$$x_i^*(X_{-i}) = \frac{a - c}{2b} - \frac{b + c\alpha}{2b} X_{-i}$$

which is decreasing in other firms' appropriation X_{-i}, thus reflecting that appropriation of firm *i* and *j* are strategic substitutes; that is, as firm *j* increases its appropriation, firm *i* responds by decreasing his own.

Tragedy of the Commons

- In a symmetric equilibrium where $x_i^* = x_j^*$ for every firm $i \neq j$, the preceding BRF becomes
$$x_i^* = \frac{a-c}{2b} - \frac{b+c\alpha}{2b}(N-1)x_i^*$$
since $X_{-i} = (N-1)x_i^*$. Solving for x_i^*,
$$x_i^* = \frac{a-c}{(N+1)b + c\alpha(N-1)}$$
which is decreasing in the number of firms *N*, and in the external effect α.
- For instance, when $N = 2$ this equilibrium appropriation becomes $x_i^* = \frac{a-c}{3b+c\alpha}$.
- Whereas, when firms are price takers, that is, $b = 0$ entailing that $p(x) = a$, equilibrium appropriation becomes $x_i^* = \frac{a-c}{c\alpha(N-1)}$.

Tragedy of the Commons

- To more generally identify the conditions where the BRF of firm i, $x_i^*(X_{-i})$, decreases in X_{-i}, we differentiate its FOC wrt x_j,

$$\underbrace{\frac{\partial p(X^*)}{\partial x_j}}_{-} + \underbrace{\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j}}_{+} \cdot x_i - \underbrace{\frac{\partial^2 c(x_i^*, X_{-i}^*)}{\partial x_i \partial x_j}}_{+}$$

which is negative if

$$\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j} \cdot x_i < -\frac{\partial p(X^*)}{\partial x_j} + \frac{\partial^2 c(x_i^*, X_{-i}^*)}{\partial x_i \partial x_j}$$

- In the special case of linear demand, $p(X) = a - bX$, $\frac{\partial p(X^*)}{\partial x_i} = -b$, implying that the cross-partial derivative is zero, $\frac{\partial^2 p(X^*)}{\partial x_i \partial x_j} = 0$, such that the inequality above unambiguously holds.

Tragedy of the Commons

- We next show that the equilibrium appropriation levels are socially excessive.
- In particular, if all firms maximize their joint profits, their maximization problem becomes

$$\max_{x_1, x_2, \ldots, x_N} p(X) \cdot X - \sum_{i=1}^{N} c(x_i, X_{-i})$$

since $\sum_{i=1}^{N} p(X) X_i = p(X) \cdot X$. Taking FOC wrt x_i,

$$p(X^{SO}) + p'(X^{SO}) \cdot x_i^{SO} + p'(X^{SO}) \cdot \sum_{j \neq i} x_j^{SO} = \frac{\partial c\left(x_i^{SO}, X_{-i}^{SO}\right)}{\partial x_i} + \sum_{j \neq i} \frac{\partial c\left(x_j^{SO}, X_{-j}^{SO}\right)}{\partial x_i}$$

for every firm *i* and $j \neq i$. This FOC differs from the equilibrium behavior in 2 ways:

- It considers aggregate (rather than individual) marginal revenue on the left-hand side because firms now internalize the effect that selling more units has on the revenues of all other firms (see the third term on the left-hand side) rather than on their own revenues alone.
- It includes the increase in marginal costs that other firms experience as a result of a larger approximation by firm *i* (i.e., a negative externality in costs).

Tragedy of the Commons

- Writing down the difference between firm *i*'s FOC and the FOC from the joint maximization problem of all firms,

$$\left[p(X) + p'(X)x_i - \frac{\partial c(x_i, X_{-i})}{\partial x_i}\right]$$

$$-\left[p(X) + p'(X)x_i + p'(X)\sum_{j\neq i} x_j - \frac{\partial c(x_i, X_{-i})}{\partial x_i} - \sum_{j\neq i}\frac{\partial c(x_j, X_{-j})}{\partial x_i}\right]$$

  which simplifies to

$$-p'(X)\sum_{j\neq i} x_j + \sum_{j\neq i}\frac{\partial c(x_j, X_{-j})}{\partial x_i}$$

- Since $p'(X) \leq 0$ by definition (i.e., by the law of demand), both terms in this expression are positive, entailing that the equilibrium appropriation is social excessive, that is, $x_i^* \geq x_i^{SO}$ for every firm *i*.
- Such a result is often referred to as the "tragedy of the commons".

Tragedy of the Commons

- Consider the following two extreme cases:

1. When $\frac{\partial c(x_j, X_{-j})}{\partial x_i} = 0$, firm *i*'s appropriation does not increase its rivals' costs, but demand is negatively sloped, $p'(X) < 0$.
   - Then, the above problem coincides with that for the standard cartel, where firms collude to reduce their production and increase profits.
2. When $p'(X) = 0$, firms take prices as given, but every firm *i*'s appropriation increases its rivals' costs, that is, $\frac{\partial c(x_j, X_{-j})}{\partial x_i} > 0$.
   - This may occur when many firms are selling the same product in the international market. In this case, the inverse demand function collapses to a (exogenous) price *p*, and the result $x_i^* \geq x_i^{SO}$ now indicates that the social optimum internalizes the external effect that each firm's appropriation generates on its rivals' costs.

Tragedy of the Commons

- Let us return to our example with:
  - linear demand $p(X) = a - bX$, and
  - cost function $c(x_i, X_{-i}) = cx_i(1 + \alpha X_{-i})$.
  - For simplicity, consider $N = 2$ firms.
- In this context, the joint profit maximization problem becomes

$$\max_{x_i, x_j} \left[a - b(x_i + x_j)\right] \cdot (x_i + x_j) - cx_i(1 + \alpha x_j) - cx_j(1 + \alpha x_i)$$

- Taking FOC with respect to every x_i yields
  $a - 2b(x_i + x_j) - c(1 + 2\alpha x_j) \leq 0$ for all $i \in \{1,2\}$ and $j \neq i$

- Assuming an interior solution and solving for x_i, we obtain

$$x_i = \frac{a - c}{2b} - \frac{2(b + c\alpha)}{2b} x_j$$

Tragedy of the Commons

- By symmetry, we can simultaneously solve for x_i and x_j to obtain the socially optimal appropriation levels
$$x_i^{SO} = x_j^{SO} = \frac{a-c}{2(2b+c\alpha)}$$
which are lower than the equilibrium appropriation x_i^*, since
$$\frac{a-c}{2(2b+c\alpha)} < \frac{a-c}{3b+c\alpha}$$
simplifies to $-c\alpha < b$, which holds for $c, b > 0$ and $\alpha \geq 0$ by definition.
- For example, consider $a = b = 1$, $c = 1/2$, and $\alpha = 1/4$.
  - In this setting, $x_i^* = 0.16 > 0.12 = x_i^{SO}$.

Tragedy of the Commons

- Interestingly, $x_i^{SO} < x_i^*$ holds in the two extreme cases described above:
  1. When $\alpha = 0$ but $b > 0$, since
  $$x_i^{SO} = \frac{a-c}{4b} < \frac{a-c}{3b} = x_i^*$$
  1. When $\alpha > 0$ but $b = 0$, which yields
     - $p = a$, and
     - $x_i^{SO} = \frac{a-c}{2c\alpha} < \frac{a-c}{c\alpha} = x_i^*$

Tragedy of the Commons

- Consider a setting with $n \geq 2$ symmetric firms, facing a market demand function $p(Q) = 1 - Q$, where Q is aggregate output. Each firm j has a convex cost function, $c(q_j) = \theta q_j^2$, where $\theta \geq 1$.
- The PMP of every firm j is

$$\max_{q_j \geq 0} \ (1 - q_j - q_{-j})q_j - \theta q_j^2$$

where $q_{-j} = \sum_{k \neq j} q_k$ represents the aggregate output of all firms but j.

Tragedy of the Commons

- Taking FOC with respect to q_j :

$$1 - 2(1+\theta)q_j - q_{-j} = 0$$

- Solving for q_j yields firm j's BRF

$$q_j(q_{-j}) = \frac{1}{2(1+\theta)} - \frac{1}{2(1+\theta)} q_{-j}$$

- Note:
  – If firm j was alone in the commons, $q_{-j} = 0$, it would produce

$$q_j(0) = \frac{1}{2(1+\theta)}$$

  – Firm j's output decreases as the aggregate output of other firms, q_{-j}, increases.

Tragedy of the Commons

- Since all firms are symmetric, in equilibrium

$$q_j = q_i = q^*$$

- Hence, the BRF of firm j can be written as

$$q^* = \frac{1}{2(1+\theta)} - \frac{n-1}{2(1+\theta)} q^*$$

which, solving for q^*, entails an equilibrium output of

$$q^* = \frac{1}{n+1+2\theta}$$

with equilibrium profits of

$$\pi^* = [1 - q^* - (n-1)q^*]q^* - \theta(q^*)^2 = \frac{1+\theta}{(n+1+2\theta)^2}$$

Tragedy of the Commons

- Aggregate output is

$$Q^* = nq^* = \frac{n}{n+1+2\theta}$$

- Aggregate profits are

$$\Pi^* = n\pi^* = \frac{n(1+\theta)}{(n+1+2\theta)^2}$$

which reach their maximum when

$$\frac{\partial \Pi^*}{\partial n} = \frac{(1+\theta)(1-n+2\theta)}{(1+n+2\theta)^3} = 0$$

where solving for n yields

$$n^* = 1 + 2\theta$$

Tragedy of the Commons

- Since the inverse demand function is linear, consumer surplus is

$$CS^* = \frac{(Q^*)^2}{2} = \frac{n^2}{2(n+1+2\theta)^2}$$

- The exploitation of the commons entails negative environmental externality (i.e., reduces biodiversity).
  - We consider a convex environmental damage function

$$ED = dQ^2,\ d > 0$$

  - Thus, the equilibrium aggregate environmental damage is

$$ED^* = d(Q^*)^2 = \frac{dn^2}{(n+1+2\theta)^2}$$

Tragedy of the Commons

- The resulting social welfare is

$$SW^* = CS^* + \Pi^* - ED^*$$
$$= \frac{n^2}{2(n+1+2\theta)^2} + \frac{n(1+\theta)}{(n+1+2\theta)^2} - \frac{dn^2}{(n+1+2\theta)^2}$$

- Consumer surplus increases in the number of firms n, i.e.,

$$\frac{\partial CS^*}{\partial n} = \frac{n(1+2\theta)}{(n+1+2\theta)^3} > 0$$

- The difference $\Pi^* - ED^*$ decreases in n, i.e.,

$$\frac{\partial[\Pi^* - ED^*]}{\partial n} = -\frac{(1+\theta)(n-1-2\theta)+2dn(1+2\theta)}{(n+1+2\theta)^3} < 0$$

Tragedy of the Commons

- *Intuition*: increasing the number of firms has two opposing effects on welfare:
  1) *Positive effect*: it increases consumer surplus (as a larger output entails lower prices);
  2) *Negative effects*: it decreases industry profits AND generates more environmental damage.

- The positive effect coincides with the negative effects when $\frac{\partial SW}{\partial n} = 0$, which occurs when

$$\frac{n(1+2\theta)}{(n+1+2\theta)^3} = \frac{(1+\theta)(n-1-2\theta)+2dn(1+2\theta)}{(n+1+2\theta)^3}$$

Tragedy of the Commons

- Solving for n,

$$\bar{n} = \frac{(1+\theta)(1+2\theta)}{2d(1+2\theta) - \theta}$$

- Comparing, $\bar{n} < n^*$, and solving for d, yields $d > 1/2$.
- *Intuition:*
  - If the environmental damage from exploitation is sufficiently large, i.e., $d > 1/2$, the number of firms in the industry is socially excessive, and firm concentration would be welfare-improving;
  - The opposite applies where $d \leq 1/2$.

Pollution Abatement

Pollution Abatement

- Let us analyze emission fees that induce firms to reduce their emissions in the least costly method.
  – *Example*: using end-of-pipe technologies, redesigning their production process, or just reducing their output.

- We examine settings where environmental damages are:
  a) uniformly distributed
  b) non-uniformly distributed

Pollution Abatement: Uniform Pollutants

- Consider a regulator seeking to limit total pollution to a maximum level x^0, so
$$x^0 \geq \sum_j x_j$$
- Firms' production functions are given by
$$y_j(p, w)$$
where p is the price of output while w is input prices.
- Firm j invests in abatement technology to limit its emissions to a given level x_j. The abatement function is
$$a_j(y_j, V_j) = x_j$$
where V_j denotes the use of abatement inputs, each with a price p_V.

Pollution Abatement: Uniform Pollutants

- Firm j can reach a target emission level x_j by
  - producing few units of output and using few abatement inputs (i.e., low y_j and V_j), or
  - producing a large amount of output but using large amounts of abatement inputs (i.e., high y_j and V_j).

Pollution Abatement: Uniform Pollutants

- If a social planner had the ability to choose the use of production and abatement inputs across firms, i.e., $(z_1, \dots, z_N)$ and $(V_1, \dots, V_N)$, he would solve

$$\min_{z_j, V_j} \sum_j (w z_j + p_V V_j)$$

s.t. $y_j(p, w) = \bar{y}_j$,

$\sum_j a_j(\bar{y}_j, V_j) \leq x^0$, and

$x_j \geq 0$ for every firm j.

Pollution Abatement: Uniform Pollutants

- The Lagrangian of this constrained maximization problem is

$$\mathcal{L} = \sum_j \left(wz_j + p_V V_j\right) + \sum_j \lambda_j \left[\bar{y}_j - y_j(p,w)\right] + \mu\left[\sum_j a_j(\bar{y}_j, V_j) - x^0\right]$$

- FOC with respect to z_j yields

$$w = \lambda_j \frac{\partial y_j(p,w)}{\partial z_j} \text{ for every firm } j$$

- FOC with respect to V_j yields

$$p_V = -\mu \frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j} \text{ for every firm } j$$

Pollution Abatement: Uniform Pollutants

- That is,
  - production input z_j should be increased until the cost of an additional unit, w, coincides with its marginal benefit in terms of additional production.
  - abatement input V_j should be increased until its costs, p_V, coincide with its marginal benefit
- For a profit maximizing firm to voluntarily select V_j at the socially optimal level, we need to set an emission fee on pollution, t_j^*, that coincides with μ.

Pollution Abatement: Uniform Pollutants

- Consider the cost minimization problem of firm j

$$\min_{z_j, v_j} \; wz_j + p_V V_j + t_j x_j$$

$$\text{s. t.} \quad y_j(p, w) = \bar{y}_j, \text{ and}$$

$$a_j(\bar{y}_j, V_j) = x_j$$

which can be re-written as

$$\min_{z_j, v_j} \; wz_j + p_V V_j + t_j a_j(\bar{y}_j, V_j)$$

$$\text{s. t.} \quad y_j(p, w) = \bar{y}_j$$

- Hence, the Lagrangian for firm j is

$$\mathcal{L} = wz_j + p_V V_j + t_j a_j(\bar{y}_j, V_j) + \theta_j \left[\bar{y}_j - y_j(p, w) \right]$$

Pollution Abatement: Uniform Pollutants

- FOC with respect to z_j yields
$$w = \theta_j \frac{\partial y_j(p,w)}{\partial z_j}$$
which is similar to the FOC we found for the social planner.

- FOC with respect to V_j yields
$$p_V = -t_j \frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j}$$
which coincides with the FOC we found for the social planner if $t_j = \mu$.

Pollution Abatement: Uniform Pollutants

- Solving for t_j in the latter equation yields

$$t_j = -\frac{p_V}{\frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j}}$$

- For optimality, we need $t_j = \mu$. Thus, for every two firms j and k, $k \neq j$, we must have

$$-\frac{p_V}{\frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j}} = -\frac{p_V}{\frac{\partial a_k(\bar{y}_k, V_k)}{\partial V_k}}$$

or, rearranging, $\frac{\partial a_k(\bar{y}_k, V_k)}{\partial V_k} = \frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j}$.

  – That is, the marginal benefits from dedicating more inputs to abatement coincide at the social optimum.

Pollution Abatement: Non-Uniform Pollutants

- Consider now non-uniform pollution sources
  - *Example*: rivers.
- The amount of pollution measured at a particular measuring station, m_k, depends on:
  a) total pollution, x, and
  b) how pollution from a firm j transfers to the monitoring station located nearby firm k, as captured by d_{kj}.
- The measurement at station m_k is given by

$$m_k = \sum_j d_{kj} x_j$$

Pollution Abatement: Non-Uniform Pollutants

- The regulator seeks to limit the measurement in each station k so it does not exceed a cutoff $\bar{m}_k$

$$m_k = \sum_j d_{kj} x_j \leq \bar{m}_k$$

- The social planner problem is

$$\min_{z_j, v_j} \sum_j (wz_j + p_v V_j)$$

$$\text{s. t.} \quad y_j(p, w) = \bar{y}_j, \text{ and}$$

$$\sum_j d_{kj} \left[a_j(\bar{y}_j, V_j) \right] \leq \bar{m}_k$$

where $a_j(\bar{y}_j, V_j) = x_j$.

Pollution Abatement: Non-Uniform Pollutants

- The Lagrangian to this program is

$$\mathcal{L} = \sum_j \left(wz_j + p_V V_j\right) + \sum_j \lambda_j \left[\bar{y}_j - y_j(p, w)\right] + \sum_k \mu_k \left[\sum_j d_{kj} a_j\left(\bar{y}_j, V_j\right) - \bar{m}_k\right]$$

- FOC with respect to z_j yields

$$w = \lambda_j \frac{\partial y_j(p,w)}{\partial z_j} \quad \text{for every firm } j$$

- FOC with respect to V_j yields

$$p_V = -\sum_k \mu_k d_{kj} \frac{\partial a_j(\bar{y}_j, V_j)}{\partial V_j}$$

Pollution Abatement: Non-Uniform Pollutants

- In the case of two firms,

$$p_V = -\mu_1 d_{12} \frac{\partial a_2(\bar{y}_2, V_2)}{\partial V_2} - \mu_2 d_{22} \frac{\partial a_2(\bar{y}_2, V_2)}{\partial V_2}$$

  which differs from the solution under uniform pollution.

- In order to set an emission fee to firm 2,

$$-t_2 \frac{\partial a_2(\bar{y}_2, V_2)}{\partial V_2} = -\mu_1 d_{12} \frac{\partial a_2(\bar{y}_2, V_2)}{\partial V_2} - \mu_2 d_{22} \frac{\partial a_2(\bar{y}_2, V_2)}{\partial V_2}$$

- Solving for t_2 yields

$$t_2 = \mu_1 d_{12} + \mu_2 d_{22}$$

Pollution Abatement: Non-Uniform Pollutants

- The regulator can set taxes based on the pollution recorded at each monitoring point, m_j, rather than on emission fees.
- That is, at every measurement point k, firm j pays a tax $d_{kj}\mu_j$ per unit of emissions.

Public Goods

Public Goods

- Before defining public goods, let us define two properties:
  - ***Non-excludability***: If the good is provided, no consumer can be excluded from consuming it.
  - ***Non-rivalry***: Consumption of the good by one consumer does not reduce the quantity available to other consumers.

	Rivalrous	**Non-rivalrous**
Excludable	Private Good	Club Good
Non-excludable	Common property resource	Public good

Public Goods

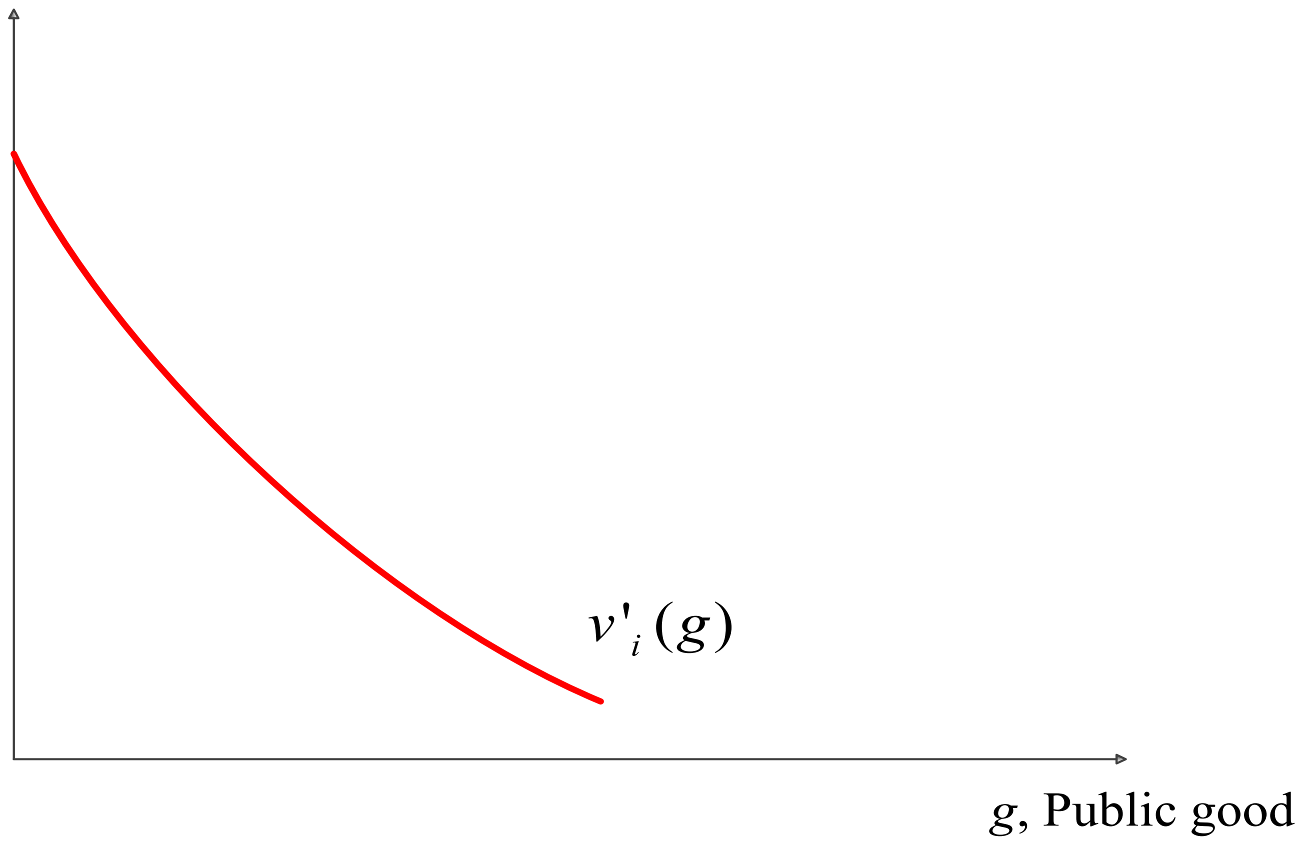
- ***Private goods***, e.g., an apple. These goods are rival and excludable in consumption.
- ***Club goods***, e.g., golf course. These goods are non-rival but excludable in consumption.
- ***Common property resources***, e.g., fishing grounds. These goods are rival but non-excludable in consumption.
- ***Public goods***, e.g., national defense. These goods are non-rival and non-excludable in consumption.

Public Goods

- Consider I consumers, one public good g and L traded private goods.
- Every consumer i's marginal utility from the consumption of g units of a public good is $v_i'(g)$
  - Note that g does not have a subscript because of non-rivalry (every individual can enjoy g units of the public good)
- We consider the case of a public good, where $v_i'(g) > 0$ for every individual i
  - A "public bad" would imply $v_i'(g) < 0$ for every i
- We assume that $v_i''(g) < 0$, which represents a positive but decreasing marginal utility from additional units of the public good.

Public Goods

- Marginal benefit from the public good



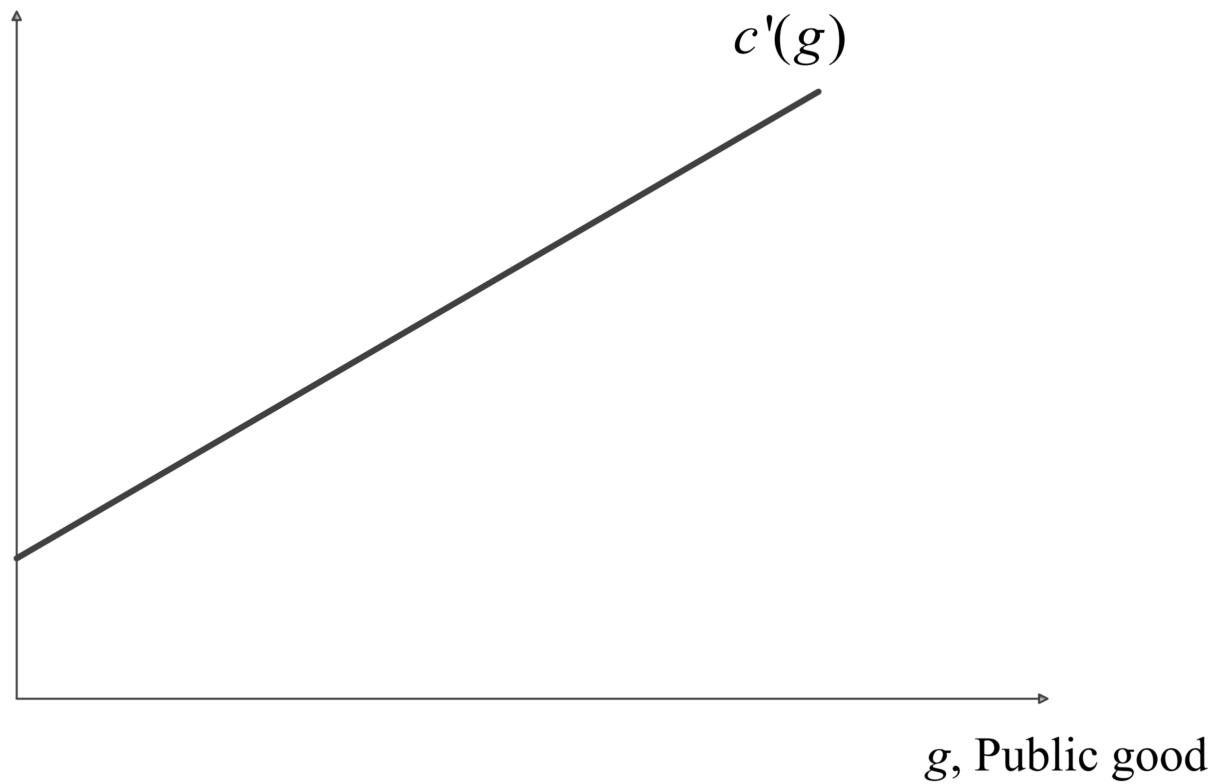
Public Goods

- We assume that the marginal utility from the public good, $v_i'(g)$, is independent on the private good.

- The cost of supplying x units of the public good is $c(g)$, where $c'(g) > 0$ and $c''(g) > 0$ for all g.
  - That is, the costs of providing the public good are increasing and convex in g.

Public Goods

- Marginal costs from providing the public good



Public Goods

- Let us first find the Pareto optimal allocation

$$\max_{g\geq 0} \sum_{i=1}^{I} v_i(g) - c(g)$$

- FOC with respect to g yields

$$\sum_{i=1}^{I} v_i'(g^o) - c'(g^o) \leq 0$$

with equality if $g^o > 0$.

- SOCs are satisfied since

$$\sum_{i=1}^{I} v_i''(g^o) - c''(g^o) \leq 0$$

Public Goods

- In case of an interior solution, the optimal level of public good is achieved for the level of g^o that solves

$$\sum_{i=1}^{I} v_i'(g^o) = c'(g^o)$$

- That is, the sum of the consumers' marginal benefit from an additional unit of the public good is equal to its marginal cost (***Samuelson rule***).

- The Pareto optimal level of public goods does not coincide with that of private goods, where, for interior solutions,

$$v_i'(g_i^*) = c_i'(g_i^*)$$

- That is, every individual i's private marginal benefit from the private good is equal to its marginal cost.

Public Goods

- ***Example*** (Discrete public good):
  - Consider a public good with $g = \{0,1\}$, i.e., it is either produced or not.
  - Every individual i has a valuation $v_i(g) = \alpha_i g$ for the good, where $\alpha_i \geq 0$ is individual i's value for this good.
  - Total cost of producing the public good is cg, where $c > 0$.
  - The Pareto optimal condition requires

$$\sum_{i=1}^{I} v_i'(g) = c$$

Public Goods

- ***Example*** (continued):
  - In the discrete setting, the public good is produced if

$$\sum_{i=1}^{I} v_i'(g) \geq c$$

  - That is, if the aggregate marginal valuation for the public good is weakly higher than its marginal cost.

Inefficiency of the Private Provision of Public Goods

Inefficiency of the Private Provision of Public Goods

- Let us consider the case in which a market exists for the public good and that each consumer i chooses how much of the public good to buy, denoted as $g_i \geq 0$, taking as given a market price of p.
- The total amount of the public good purchased by all I individuals is hence $g = \sum_{i=1}^{I} g_i$.
- Consider a single producer of the public good with a cost function $c(g)$.

Inefficiency of the Private Provision of Public Goods

- Formally, at a competitive equilibrium price p^*, each consumer i's purchase of the public good, g_i^*, must solve

$$\max_{g_i \geq 0} \; v_i\left(g_i + \sum_{k \neq i} g_k^*\right) + w_i - p^* g_i$$

  - The first term reflects that individual i benefits from both the g_i units of the public good he purchases and $\sum_{k \neq i} g_k^*$ units of the public good that all other individuals acquire;
  - In determining his purchases of the public good, individual i takes the purchases of all the other individuals as given;
  - Consumer i pays $p^* g_i$ when acquiring g_i units of the public good.

Inefficiency of the Private Provision of Public Goods

- FOC with respect to g_i yields
$$v_i'(g_i^* + \sum_{k\neq i} g_k^*) - p^* \leq 0$$
with equality if $g_i^* > 0$ (interior solution).
- For compactness, let g^* denote the total purchases of the public good, that is,
$$g^* = g_i^* + \sum_{k\neq i} g_k^*.$$
- Hence, the above FOC can be expressed as
$$v_i'(g^*) - p^* \leq 0$$
with equality if $g_i^* > 0$ (interior solution)

Inefficiency of the Private Provision of Public Goods

- On the other hand, the firm's PMP is

$$\max_{g \geq 0} \quad p^* g - c(g)$$

- FOC with respect to g yields

$$p^* - c'(g^*) \leq 0$$

  with equality if $g > 0$ (interior solution).

- Finally, the market clearing condition implies that the total amount of the public goods produced coincides with the amount consumed by all individuals.

Inefficiency of the Private Provision of Public Goods

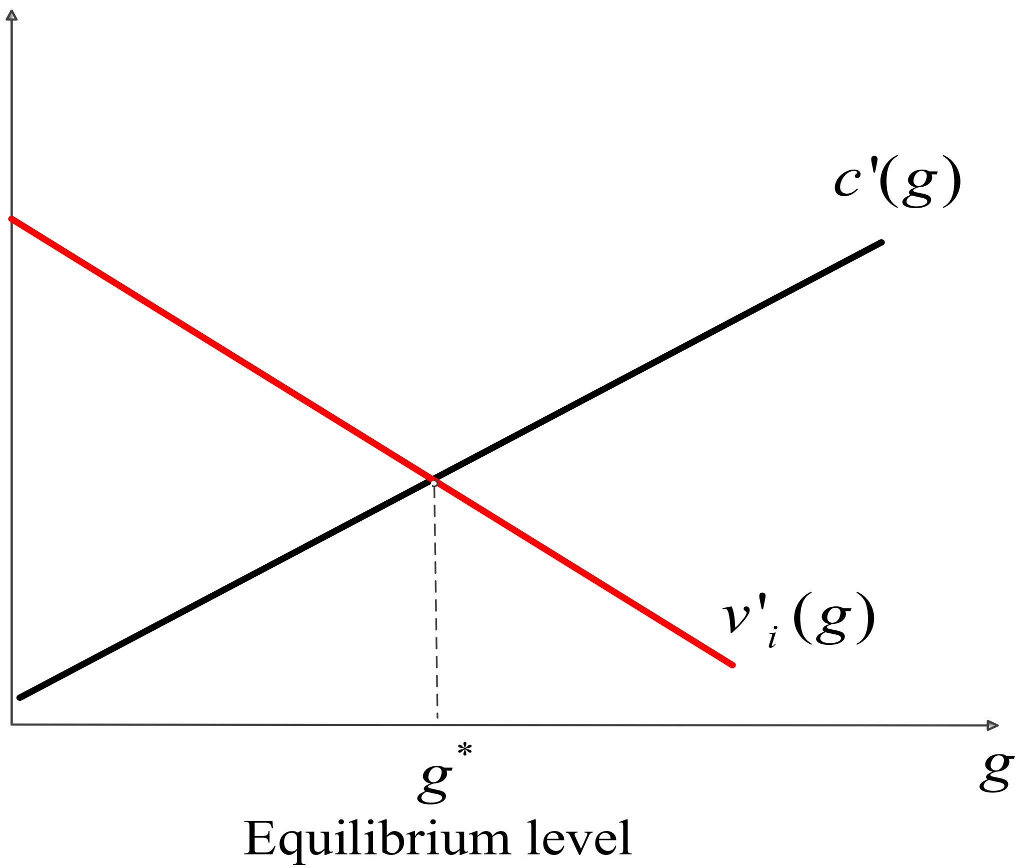
- Combining the FOCs for consumers and the firm, we obtain

$$v_i'(g^*) = c'(g^*) \text{ if } g^* > 0,$$
$$v_i'(g^*) < c'(g^*) \text{ if } g^* = 0$$

- Intuitively, individual i increases his consumption of the public good until the point in which his marginal benefit from the public good equals the marginal cost.

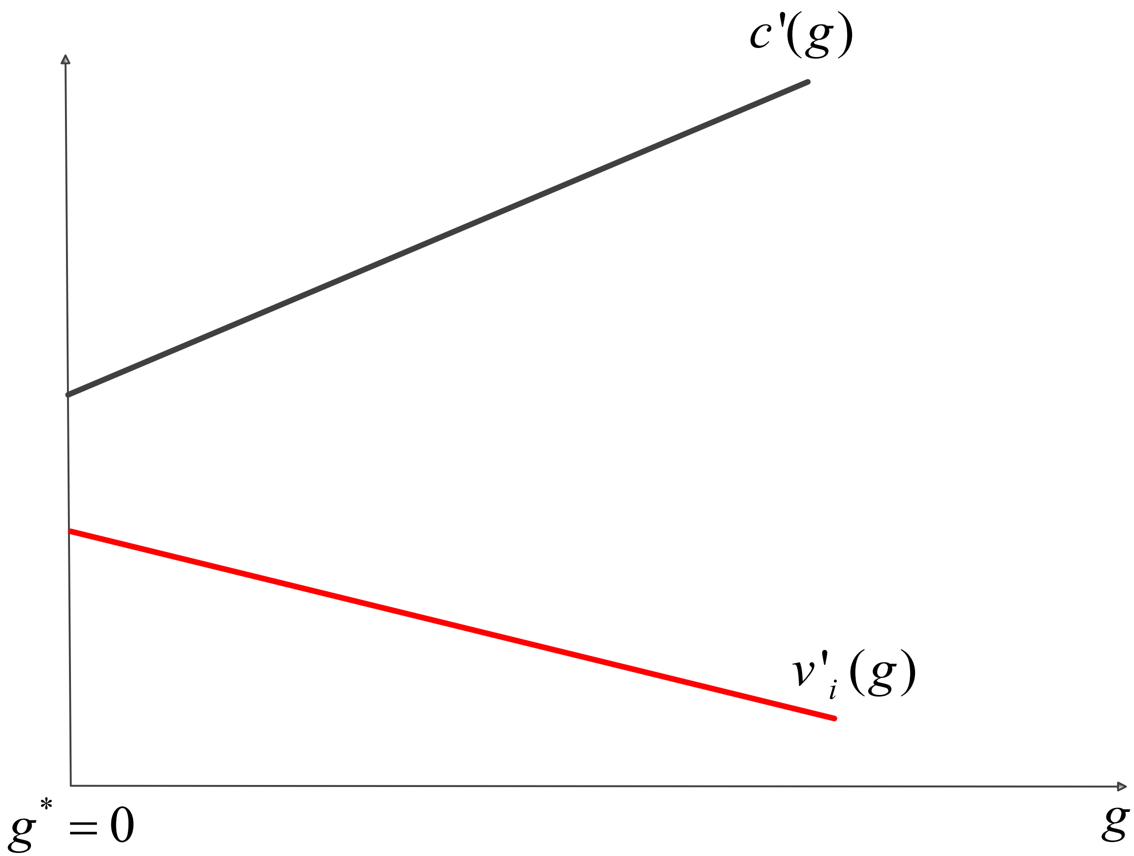
Inefficiency of the Private Provision of Public Goods

- Equilibrium level of public good (interior solution).



Inefficiency of the Private Provision of Public Goods

- Equilibrium level of public good (corner solution).



Inefficiency of the Private Provision of Public Goods

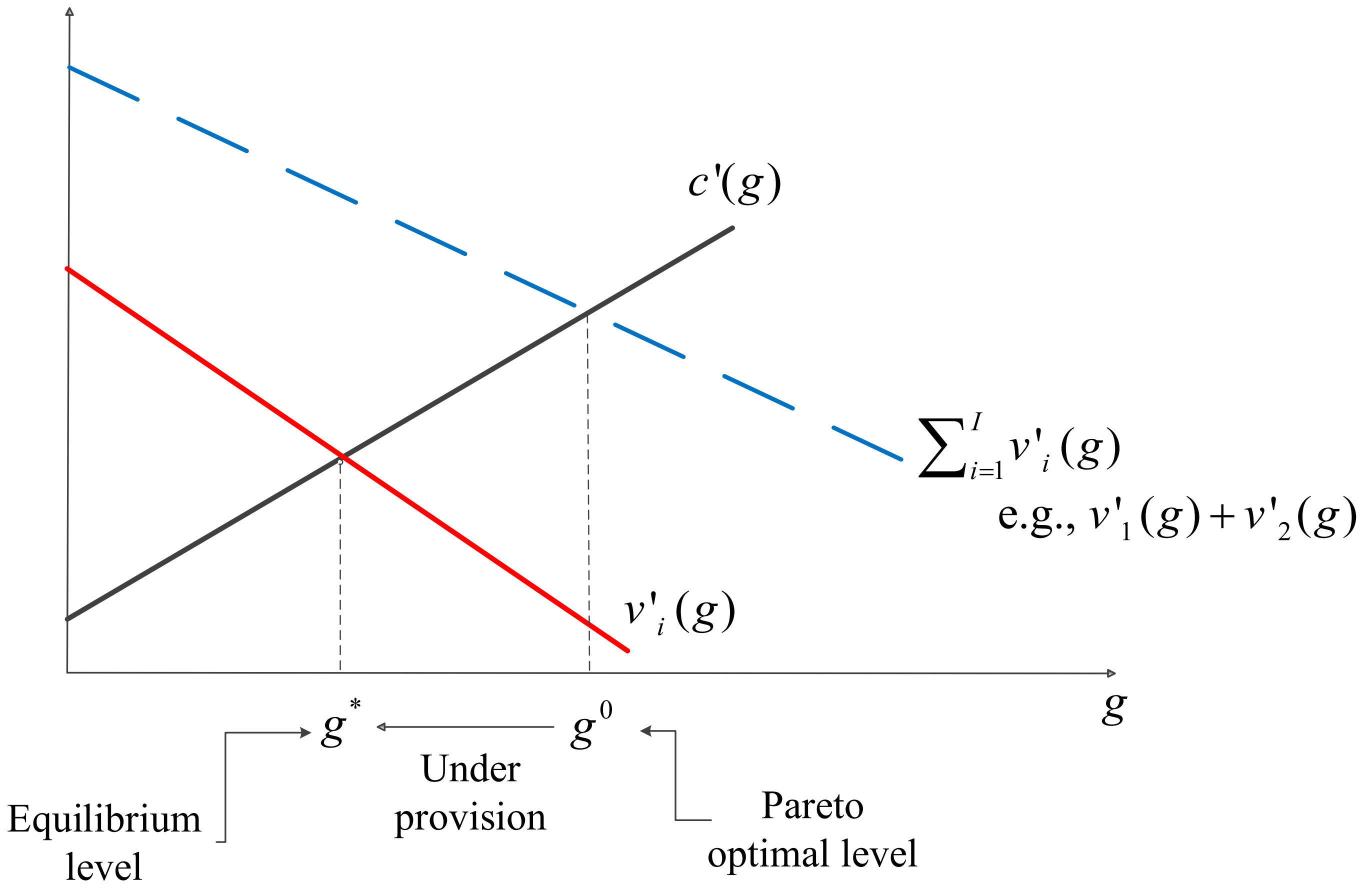
- However, at the Pareto optimality, we must have

$$\sum_{i=1}^{I} v_i'(g^o) = c'(g^o)$$

- That is, the summation of the marginal benefit that all individuals obtain from the public good must equal the marginal cost.
- Hence, there is an *underprovision* of the public good, $g^* < g^o$.
  - *Exception*: when the marginal cost curve is vertical, i.e., $c'(g) = +\infty$. In this case, $g^* = g^o$.

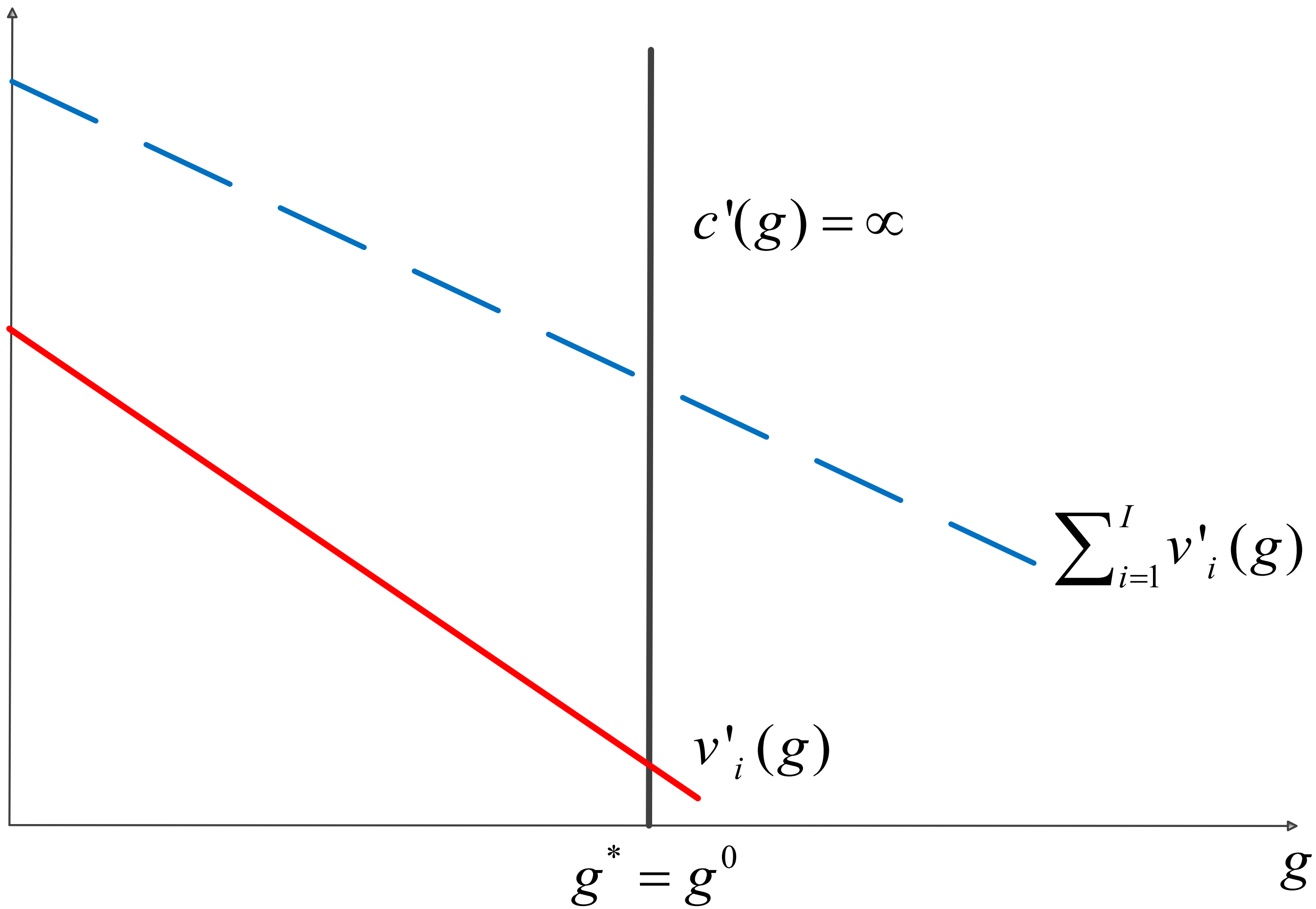
Inefficiency of the Private Provision of Public Goods

- Pareto optimal and equilibrium level of public good



Inefficiency of the Private Provision of Public Goods

- Pareto optimal coincides with equilibrium level of public good



Inefficiency of the Private Provision of Public Goods

- *Intuition*:
  - Each individual's purchase of the public good benefits not only him, but also all other individuals in the economy.
  - Each individual does not internalize the positive externalities that his individual purchase of the public good generates on other individuals.
  - Hence, each individual does not have enough incentives to purchase sufficient amounts of the public good.
  - This leads to the *free-rider problem*, whereby the public good is underprovided.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (Private contributions to a public good):
  - Consider an economy with two individuals $i = \{1,2\}$, with quasilinear utility function
$$u_i(G, y_i) = y_i + m_i \log(G)$$
where
    - $m_i > 0$ denotes the value that individual i assigns to total contributions to the public good, $G = g_i + g_j$
    - y_i is a composite private good commodity
    - Assume that $m_1 > m_2$
  - For simplicity, the price of both private and public good is 1, thus entailing a budget constraint $g_i + y_i = w$ for every individual i.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Using the budget constraint $g_i + y_i = w$, or $y_i = w - g_i$, and the fact that $G = g_i + g_j$, we can rewrite the above UMP as the following unconstrained program

$$\max_{g_i \geq 0} \quad w - g_i + m_i \log(g_i + g_j)$$

  - Taking FOC with respect to g_i yields

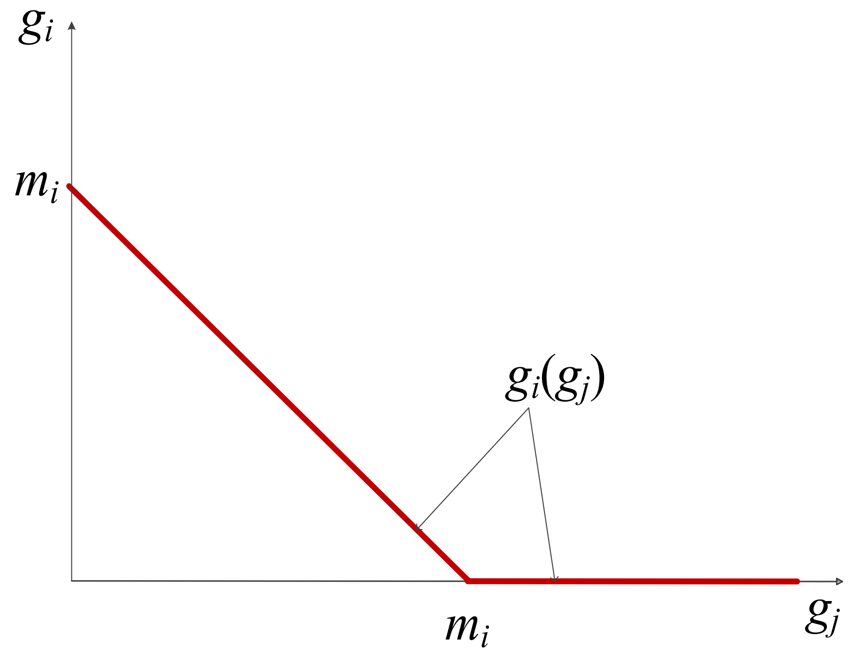
$$-1 + \frac{m_i}{g_i + g_j} = 0$$

  - Solving for g_i produces BRF $g_i(g_j)$

$$g_i(g_j) = \begin{cases} m_i - g_j & \text{if } g_j < m_i \\ 0 & \text{otherwise} \end{cases}$$

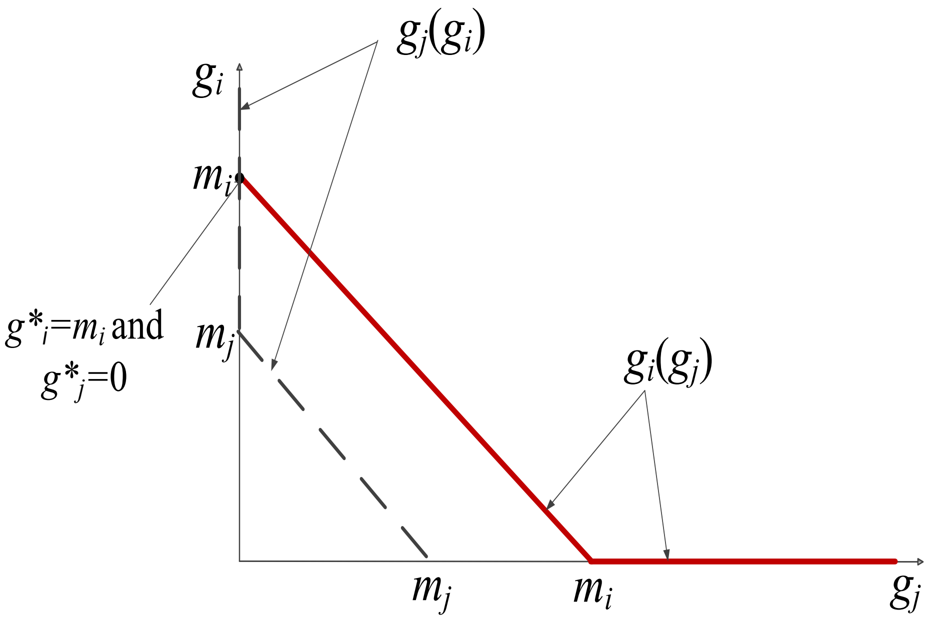
Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Individual i's BRF $g_i(g_j)$.
  - Individual j's BRF $g_j(g_i)$ is analogous.



Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - The equilibrium level of $\left(g_i^*, g_j^*\right)$ is obtained by simultaneously solving the two BRFs, $g_i\left(g_j\right)$ and $g_j\left(g_i\right)$.
  - Hence, $g_1^* = m_1 > 0$ and $g_2^* = 0$, since $m_1 > m_2$.



Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - In contrast, a social planner would maximize total welfare by solving

$$\max_{g_i, g_j} \quad w - g_i + m_i \log(g_i + g_j)$$
$$+w - g_j + m_j \log(g_j + g_i)$$

  - FOC:

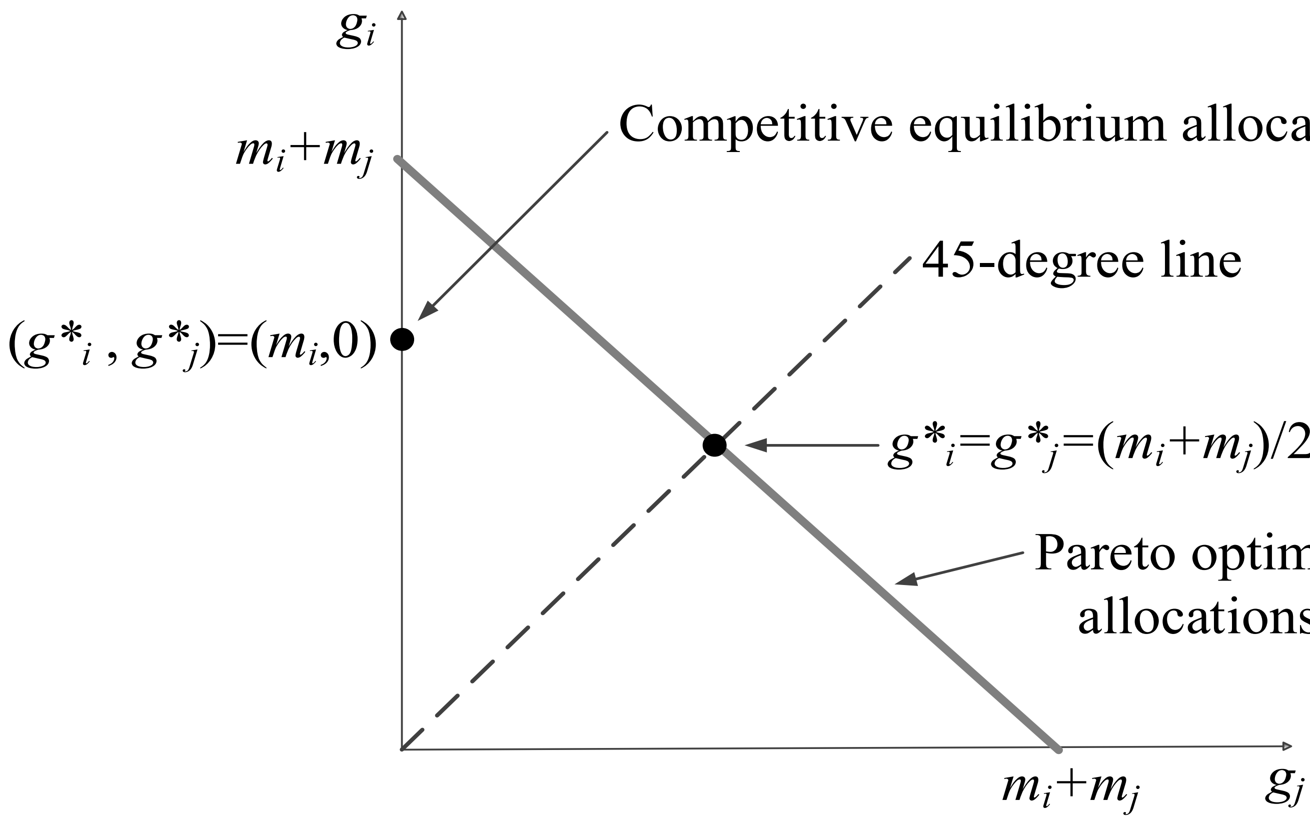
$$-1 + \frac{m_i + m_j}{g_i + g_j} = 0$$

  - Solving for g_i, we obtain a continuum of Pareto optimal allocations

$$g_i^{SO} = m_i + m_j - g_j^{SO}$$

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):



Neutrality and Crowding out

Neutrality and Crowding out

- ***Revenue neutral policies***: an income tax to one individual which is entirely allocated as a transfer to another individual.
- Consider a Cobb-Douglas utility function
$$u_i(x_i, G) = x_i^{\alpha} G^{1-\alpha}$$
where
  - x_i denotes the private good
  - G represents total contributions to the public good.
- Assume for simplicity that the price of the private and public good is 1, and that individual i's income is w_i.

Neutrality and Crowding out

- Every individual $i = \{1,2\}$ solves the utility maximization problem

$$\max_{x_i, g_i} \; x_i^{\alpha}\left(g_i + g_j\right)^{1-\alpha}$$
$$\text{s.t.} \quad x_i + g_i = w_i$$

  where $G = g_i + g_j$.

- Since $x_i = w_i - g_i$, the above maximization problem can be reduced to an unconstrained program

$$\max_{g_i \geq 0} \; (w_i - g_i)^{\alpha}(g_i + g_j)^{1-\alpha}$$

Neutrality and Crowding out

- FOC wrt g_i yields

$$-\alpha(w_i - g_i)^{\alpha-1}(g_i + g_j)^{1-\alpha}$$
$$+(1-\alpha)(w_i - g_i)^{\alpha}(g_i + g_j)^{-\alpha} \leq 0$$

- In the case of interior solutions, solving for g_i produces individual i's BRF

$$g_i(g_j) = (1-\alpha)w_i - \alpha g_j$$

- Simultaneously solving the two BRFs yields the equilibrium condition

$$g_i^* = \frac{w_i - \alpha w_j}{1+\alpha}$$

Neutrality and Crowding out

- Equilibrium aggregate donation is

$$G^* = g_i^* + g_j^* = \frac{(1-\alpha)(w_1+w_2)}{1+\alpha}$$

which lies below the socially optimal donation that a benevolent planner would select, i.e., $G^* < G^{SO}$, where

$$G^{SO} = (1-\alpha)(w_1 + w_2)$$

Neutrality and Crowding out

- Now, consider transfer $dw_1 > 0$ to individual 1 and a tax $dw_2 < 0$ so that

$$dw_1 + dw_2 = 0 \text{ or } dw_1 = -dw_2$$

- Individual i's equilibrium contribution g_i^* is affected as follows:

$$dg_i^* = \frac{dw_i - \alpha dw_j}{1+\alpha}$$

- Since $dw_i = -dw_j$, the above expression can be re-written as

$$dg_i^* = \frac{dw_i + \alpha dw_i}{1+\alpha} = \frac{(1+\alpha)dw_i}{1+\alpha} = dw_i$$

Neutrality and Crowding out

- Hence, individual i's contribution is exactly:
  - increased by the amount of the transfer he receives (if $dw_i > 0$), or
  - reduced by the tax he bears (if $dw_i < 0$).
- However, his contribution change dg_i^* is unaffected by the initial income distribution (i.e., w_i and w_j).
- As a consequence, aggregate donations are unaffected by income redistributions, since

$$dG^* = dg_1^* + dg_2^* = dw_1 + dw_2$$

which is zero by definition (i.e., $dw_1 = -dw_2$).

Neutrality and Crowding out

- This condition shows the crowding out effect of levying taxes to fund the production of the public good.

- That is, a \$1 tax, i.e., $dw_i = -1 < 0$, reduces every individual i's private contributions to the public good by exactly \$1, since $dg_i^* = dw_i = -1$.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (Increasing the number of contributors):
  - Let us extend the previous setting of two individuals with Cobb-Douglas preferences to a context with N individuals.
  - Assume all donors have the same income

$$w_1 = w_2 = \cdots = w_n = w$$

  - In this setting, every individual i chooses g_i to solve the following utility maximization problem

$$\max_{g_i \geq 0} \ \alpha \log(w - g_i) + (1 - \alpha) \log\left(g_i + \sum_{j \neq i} g_j \right)$$

Inefficiency of the Private Provision of Public Goods

– Differentiating with respect to g_i, we obtain

$$\frac{\alpha}{w - g_i} - \frac{1 - \alpha}{g_i + \sum_{j \neq i} g_j} = 0$$

which we rearrange to yield

$$\alpha g_i + \alpha G_{-i} = (1 - \alpha)w - (1 - \alpha)g_i$$

where $G_{-i} \equiv \sum_{j \neq i} g_j$ stands for the aggregate donations of all individuals other than i.

– Solving for g_i, every individual i's best response function becomes

$$g_i(G_{-i}) = (1 - \alpha)w - G_{-i}$$

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - Invoking symmetry in equilibrium, i.e., $g_1 = g_2 = \cdots = g_n = g$, we have $G_{-i} = (N-1)g$, yielding
  $$g = (1-\alpha)w - \alpha(N-1)g$$
  - Solving for g, we obtain an equilibrium donation
  $$g^* = \frac{(1-\alpha)w}{1+\alpha(N-1)}$$
  - An aggregate contribution is
  $$G^* = Ng^* = \frac{N(1-\alpha)w}{1+\alpha(N-1)}$$

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - The effect of increasing the number of contributors N on the equilibrium results:

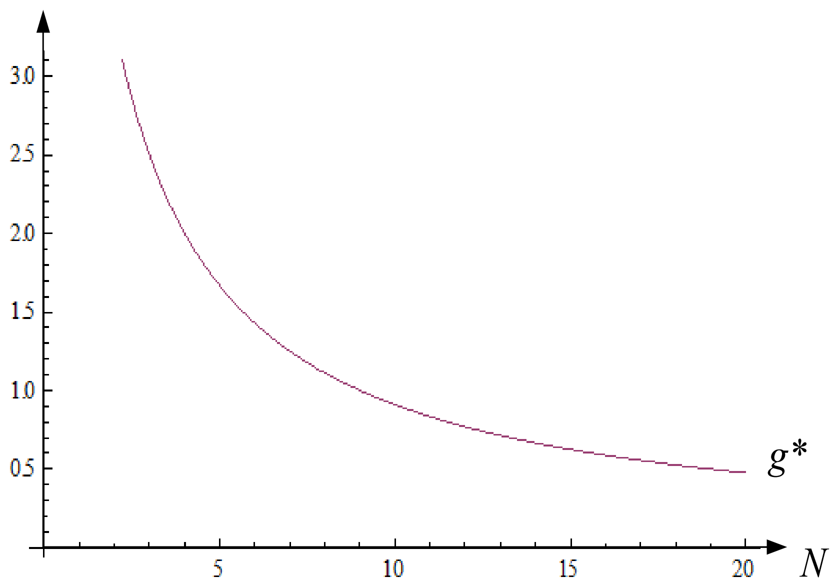
$$\frac{\partial g^*}{\partial N} = \frac{-\alpha(1-\alpha)w}{[1+\alpha(N-1)]^2} < 0, \text{ and}$$

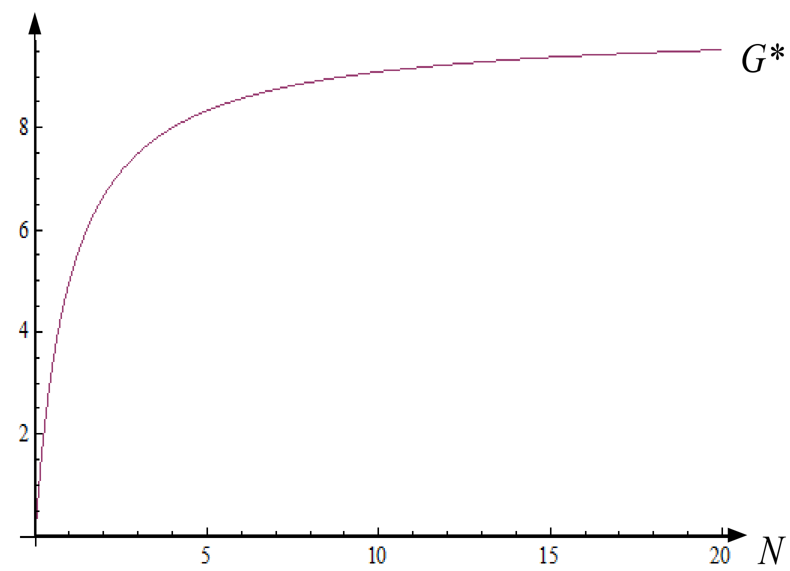
$$\frac{\partial G^*}{\partial N} = \frac{(1-\alpha)^2 w}{[1+\alpha(N-1)]^2} > 0$$

  - That is, while individual contributions decrease as a result of more donors potentially contributing to the public good, the overall effect of adding more donors is still positive.

Inefficiency of the Private Provision of Public Goods

- ***Example*** (continued):
  - We assume $\alpha = \frac{1}{2}$ and $w = 10$. Hence, the above expressions become $g^* = \frac{10}{1+N}$ and $G^* = N\frac{10}{1+N}$.





Remedies to the Under-Provision of Public Goods

Remedies to the Under-Provision

- ***Quantity-based intervention***: a direct governmental provision of the public good
- ***Price-based intervention***: taxes or subsidies
  - Assume two consumers with benefit functions $v_1(x_1 + x_2)$ and $v_2(x_1 + x_2)$, respectively, where x_i denotes the amount of the public good purchased by consumer i.
  - Similarly to our analysis of externalities, we can design a subsidy s_i per unit of the public good purchased by every consumer i that induces him to take into account the positive external effect of his purchases of the public good on the other individual's welfare.

Remedies to the Under-Provision

– Hence, the subsidy must be $s_i = v'_{-i}(x^o)$, where $v'_{-i}(x^o)$ reflects the marginal benefit that all other individuals obtain from enjoying x^o units of the public good.

– Note that this analysis is equivalent to that of imposing a tax $t_i = -v'_{-i}(x^o)$ per unit of the public good when the overall amount of public good falls below x^o, as we next describe.

Remedies to the Under-Provision

- Every consumer i's UMP becomes that of selecting $\tilde{x}_i$ for a given level of $\tilde{x}_j$

$$\max_{x_i \geq 0} \underbrace{v_i\left(x_i + \tilde{x}_j\right) + \underbrace{s_i x_i}_{\text{subsidy}}}_{\text{Total utility from } x_i} - \underbrace{\tilde{p} x_i}_{\text{cost}}$$

- Taking FOC with respect to x_i yields

$$v_i'\left(\tilde{x}_i + \tilde{x}_j\right) + s_i - \tilde{p} \leq 0$$

with equality if $\tilde{x}_i > 0$ (interior solution).

Remedies to the Under-Provision

- Using the market clearing condition $\tilde{x} = \tilde{x}_i + \tilde{x}_j$, and the fact that in a competitive equilibrium the PMP implies $\tilde{p} = \tilde{c}'(\tilde{x})$, the above FOC becomes
$$v_i'(\tilde{x}) + s_i \leq c'(\tilde{x})$$
- Finally, note that for a subsidy s_i to be optimal, we need
$$s_i = v_{-i}'(x^o) = v_j'(x^o)$$
which allows us to rewrite the above FOC as
$$v_i'(x^o) + v_j'(x^o) \leq c'(x^o)$$

Remedies to the Under-Provision

- Hence, we need a subsidy $s_i = v'_{-i}(x^o)$ which, for the case of only two consumers i and j, implies $s_i = v'_j(x^o)$.
- In the case of N individuals, the subsidy to consumer i would be

$$s_i = v'_j(x^o) + v'_k(x^o) + \cdots = \sum_{j \neq i} v'_j(x^o)$$

- The introduction of a subsidy might seem an effective and easy solution to the under-provision problem in public goods.
- However, the regulator might not have access to information about the marginal benefits of the public good for every consumer.

Lindahl Equilibria

Lindahl Equilibria

- Private provision of a public good results in inefficiencies, i.e., $x^* < x^0$.
  - This can be solved by the use of quantity-based or price-based regulation.
- There is, however, a market solution that *in principle* can achieve optimality.
- Consider a market where every individual's consumption of the public good is a distinct commodity with its own market.
- Denote the price of this personalized good by p_i, which can differ across consumers.

Lindahl Equilibria

- If consumer i faces a price p_i^{**}, his UMP is

$$\max_{x_i \geq 0} \; v_i(x_i) + w_i - p_i^{**} x_i$$

- FOC wrt x_i yields

$$v_i'(x_i^{**}) - p_i^{**} \leq 0$$

with equality if $x_i^{**} > 0$.

- Hence, at the aggregate level,

$$\sum_{i=1}^{I} v_i'(x_i^{**}) \leq \sum_{i=1}^{I} p_i^{**}$$

Lindahl Equilibria

- On the other hand, the firm produces a bundle of I goods (one for each consumer), with PMP

$$\max_{x \geq 0} \quad \underbrace{\textstyle\sum_{i=1}^{I} (p_i^{**} x)}_{\text{Total revenue}} - c(x)$$

- FOC wrt x yields

$$\textstyle\sum_{i=1}^{I} p_i^{**} - c'(x^{**}) \leq 0, \text{ or}$$

$$\textstyle\sum_{i=1}^{I} p_i^{**} \leq c'(x^{**})$$

with equality if $x^{**} > 0$ (interior solution).

Lindahl Equilibria

- Using the condition we found for consumers, i.e., $\sum_{i=1}^{I} v_i'(x_i^{**}) \leq \sum_{i=1}^{I} p_i^{**}$, with the above condition, we have

$$\sum_{i=1}^{I} v_i'(x_i^{**}) \leq \sum_{i=1}^{I} p_i^{**} \leq c'(x^{**})$$
$$\Longrightarrow \sum_{i=1}^{I} v_i'(x_i^{**}) \leq c'(x^{**})$$

  which implies that the equilibrium level of the public good that every consumer purchases is exactly the efficient level, i.e., $x^{**} = x^{o}$.
- This type of equilibrium in personalized markets for the public good is usually known as the ***Lindahl equilibrium***.

Lindahl Equilibria

- *Why do we obtain efficiency?*
  - First, we define personalized markets for the public good.
  - Second, each consumer, taking the price of his personalized good as given, fully determines his own level of consumption of the public good.
  - Positive externalities are eliminated.

Lindahl Equilibria

- *Are these personalized markets for the public good realistic?*
  - We need excludability between the different personalized public goods, which might only be applicable to very specific public goods
    - e.g., some forms of health care, college education, etc.
  - Even if excludability was possible, personalized markets would be monopsonistic (there is only one buyer on the demand side)
    - Thus, the price-taking assumption is difficult to support.

Lindahl Equilibria

- ***Example*** (Calculating a Lindahl equilibrium):
  - Consider three first-year economics graduate students, Eric (E), Chris (C), and Matt (M) deciding to purchase a microwave (a public good) for their office.
  - The utility function of each student as

    $$u_i(x_i, y) = \ln x_i + \alpha_i \ln y$$

    where
    - x_i denotes the utility gained by student $i = \{E, C, M\}$ from private purchases (i.e., all other goods);
    - y denotes the utility gained by the total amount spent on a new microwave by the three students;
    - α_i denotes the benefit student i obtains from the microwave.

Lindahl Equilibria

- ***Example*** (continued):
  - For simplicity, assume both the price of the private good, x_i, and the wealth of each student is 1. In addition, assume that $p_E + p_C + p_M = 1$.
  - The UMP for student i is

$$\max_{x_i} \ \ln x_i + \alpha_i \ln y$$
$$\text{s.t.} \ \ x_i + p_i y \leq 1$$

  where p_i represents the Lindahl price each student pays.

Lindahl Equilibria

- ***Example*** (continued):
  - Since the budget constraint holds with equality, the UMP becomes
$$\max_{y} \ \ln(1 - p_i y) + \alpha_i \ln y$$
  - FOC wrt y yields
$$\frac{p_i}{1 - p_i y} = \frac{\alpha_i}{y}$$
  - Rearranging, we obtain
$$p_i y = \frac{\alpha_i}{1 + \alpha_i}$$

Lindahl Equilibria

- ***Example*** (continued):
  - Summing across all three students yields
  $$(p_E + p_C + p_M)y = y = \frac{\alpha_E}{1+\alpha_E} + \frac{\alpha_C}{1+\alpha_C} + \frac{\alpha_M}{1+\alpha_M}$$
  - Substituting this value for y in equation $p_i y = \frac{\alpha_i}{1+\alpha_i}$ and solving for the Lindahl prices produces
  $$p_i^* = \frac{\frac{\alpha_i}{1+\alpha_i}}{\frac{\alpha_E}{1+\alpha_E}+\frac{\alpha_C}{1+\alpha_C}+\frac{\alpha_M}{1+\alpha_M}}$$
  - For instance, if $\alpha_E = 1, \alpha_C = \alpha_M = 0.6$, then $y = 1.25$ with Lindahl prices of $p_E = 0.4$ and $p_C = p_M = 0.3$.

Public Goods that Experience Congestion

Public Goods that Experience Congestion

- Consider that the number of individuals consuming the public good *reduces* the benefit that each user i enjoys from the good.
- Hence, the utility function becomes

$$v_i(x_i, x_{-i}) + w_i$$

  where
  - x_i is the contribution of individual i
  - x_{-i} is those of all other individuals, which enters negatively in $v_i(\cdot)$, thus capturing the ***congestion effect*** (as in CPR goods, see slide 149).
- Utility increases in x_i, but decreases in the amount of the public good consumed by other individuals x_{-i}.

Public Goods that Experience Congestion

- The social planner's maximization problem is

$$\max_{x_1,\ldots,x_n} \sum_{i=1}^{N}[v_i(x_i, x_{-i}) + w_i] - C(x)$$

- FOC wrt x_i yields

$$\frac{\partial v_i(x_i,x_{-i})}{\partial x_i} + \sum_{j\neq i}\frac{\partial v_j(x_i,x_{-i})}{\partial x_i} - \frac{\partial C(x)}{\partial x_i} \leq 0 \text{ for all } i$$

which in the case of interior solutions becomes

$$\frac{\partial v_i(x_i,x_{-i})}{\partial x_i} + \sum_{j\neq i}\frac{\partial v_j(x_i,x_{-i})}{\partial x_i} = \frac{\partial C(x)}{\partial x_i}$$

Public Goods that Experience Congestion

- Summing over all N individuals, we obtain

  $$\sum_{i=1}^{N} \frac{\partial v_i(x_i, x_{-i})}{\partial x_i} + \sum_{i=1}^{N} \sum_{j \neq i} \frac{\partial v_j(x_i, x_{-i})}{\partial x_i} = \sum_{i=1}^{N} \frac{\partial c(x)}{\partial x_i}$$

  which coincides with the standard ***Samuelson rule*** for the optimal provision of public goods, except for the second term.
- Intuitively, this term reflects the negative externality that individual i suffers from a larger consumption of the public good by all other individuals.
- As a consequence, the socially optimal amount of public good will tend to be smaller than in the absence of congestion effects.

Behavioral Motives in Public Good Games

Warm-Glow Benefits

- Individuals obtain a "warm glow" benefit from their donations to the public good
  - See Andreoni (1990)
- Individual i's utility function $u_i(x_i, G, g_i)$ increases in his consumption of:
  - private good, x_i
  - total contributions to the public good, G
  - warm glow of donating dollars to the public good, g_i
- The presence of warm-glow in the donors' utility function prevents the "crowding-out".

Warm-Glow Benefits

- Donor i's UMP is

$$\max_{x_i, g_i, G} \; u_i(x_i, G, g_i)$$
$$\text{s.t.} \quad x_i + g_i = w_i$$
$$g_i + G_{-i} = G$$

  where $G_{-i} = \sum_{j \neq i} g_j$ is total donations from all other individuals.
- Since $g_i = G - G_{-i}$ and $x_i = w_i - g_i = w_i - G + G_{-i}$, the UMP becomes

$$\max_{G \geq 0} \; u_i(w_i - G + G_{-i}, G, G - G_{-i})$$

Warm-Glow Benefits

- FOC wrt G yields a total donations function of
$$G = f_i(w_i + G_{-i}, G_{-i})$$
which only depends on the elements of $u_i(\cdot)$ that are different from G.
- The individual donation function of player i, $g_i = G - G_{-i}$, is
$$g_i = f_i(w_i + G_{-i}, G_{-i}) - G_{-i}$$
  – The first term in $f_i(\cdot)$ is common to public good games in which donors do not benefit from warm glow.
  – In contrast, the second term of $f_i(\cdot)$ arises because of the warm glow benefits that donors obtain.

Warm-Glow Benefits

- *Intuition*:
  - the first term captures *altruistic* motivations in public goods; whereas
  - the second component measures *egoistic* motivations (because the warm glow benefit is private).

Warm-Glow Benefits

- Let f_{ia} (f_{ie}) denote the first order derivative of $f_i(\cdot)$ wrt the first argument (altruism) and the second argument (egoism), where

$$f_{ia} \in (0,1),\ f_{ie} > 0 \text{ and } 0 < f_{ia} + f_{ie} \leq 1$$

- This notation helps us obtain the following "altruism coefficient"

$$\alpha_i = \frac{\frac{\partial f_i}{\partial w_i}}{\frac{\partial f_i}{\partial G_{-i}}} = \frac{f_{ia}}{f_{ia}+f_{ie}}$$

  - $\alpha_i = 1$ for pure altruists, where $f_{ie} = 0$
  - $\alpha_i = f_{ia}$ for pure egoists since $f_{ia} + f_{ie} = 1$

Warm-Glow Benefits

- Let us now consider a transfer from individual 2 to 1, i.e., $dw_1 > 0$ and $dw_2 < 0$ where $dw_1 = -dw_2$.
- Let us examine how total donations G and individual contributions are affected by the transfer.
- Totally differentiating g_i yields

$$dg_i = f_{ia}(dw_i + dG_{-i}) + f_{ie}dG_{-i} - dG_{-i}$$

- Factoring out dG_{-i} yields

$$dg_i = f_{ia}\, dw_i + (f_{ia} + f_{ie} - 1)dG_{-i}$$

Warm-Glow Benefits

- Since $G_{-i} = G - g_i$ (by definition), substituting $dG_{-i} = dG - dg_i$ in the above expression yields

$$dg_i = f_{ia}dw_i + (f_{ia} + f_{ie} - 1)dG - (f_{ia} + f_{ie} - 1)dg_i$$

- Rearranging,

$$(f_{ia} + f_{ie})dg_i = (f_{ia} + f_{ie} - 1)dG + f_{ia}dw_i$$

- Dividing both sides by $(f_{ia} + f_{ie})$ and using the definition of α_i yields

$$dg_i = \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}dG + \alpha_i dw_i$$

Warm-Glow Benefits

- This captures how the donation of individual i is affected by a change in his wealth dw_i.
- Aggregating across donors

$$\sum_{i=1}^{I} dg_i = \sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}} dG + \sum_{i=1}^{I} \alpha_i \, dw_i$$

- Since $\sum_{i=1}^{I} dg_i = dG$, then the change in aggregate contributions, dG, is

$$dG = \sum_{i=1}^{I} \frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}} dG + \sum_{i=1}^{I} \alpha_i \, dw_i$$

Warm-Glow Benefits

- We can now rearrange and solve for dG to obtain

$$dG = \frac{1}{1-\sum_{i=1}^{I}\frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}} \sum_{i=1}^{I} \alpha_i \, dw_i$$

- For compactness, let

$$c \equiv \frac{1}{1-\sum_{i=1}^{I}\frac{f_{ia}+f_{ie}-1}{f_{ia}+f_{ie}}}$$

- Thus, the expression for dG can be expressed as

$$dG = c \sum_{i=1}^{I} \alpha_i \, dw_i$$

Warm-Glow Benefits

- Finally, since $dw_1 = -dw_2$ and $dw_j = 0$ for all other individuals $j \neq 1 \neq 2$, the above expression becomes

$$dG = c\,[\alpha_1 dw_1 + \alpha_2(-dw_1)] = c(\alpha_1 - \alpha_2)dw_1$$

- Thus $dG \geq 0$ only holds if $\alpha_1 \geq \alpha_2$, but dG is negative otherwise.
- Intuitively, the transfer from individual 2 to 1 is not necessarily neutral: it can increase total donations if and only if individual 1 is more altruistic than individual 2 ($\alpha_1 \geq \alpha_2$).

Warm-Glow Benefits

- ***Example*** (Warm glow in public goods):
  - Consider a public good game with two individuals $i = \{1,2\}$, each with Cobb-Douglas utility function

$$u_i(x_i, G, g_i) = a \log x_i + b \log G + c_i \log g_i$$

where $c_1 > c_2$ represent the warm-glow benefit that individuals obtain from their contributions to the public good.

Warm-Glow Benefits

- ***Example*** (continued):
  - Since $x_i = w_i - g_i$ and $G = g_i + g_j$, we can rewrite the above expression as

$$u_i(w_i - g_i, g_i + g_j, g_i)$$
$$= a\log(w_i - g_i) + b\log(g_i + g_j) + c_i \log g_i$$

  - FOC wrt g_i yields

$$-\frac{a}{w_i - g_i} + \frac{b}{g_i + g_j} + \frac{c_i}{g_i} = 0$$

Warm-Glow Benefits

- ***Example*** (continued):
  - For simplicity, consider $w_i = 10$ for both individuals, $a = 1, b = \frac{1}{3}$, and warm-glow parameters $c_1 = \frac{1}{4}$ and $c_2 = \frac{1}{5}$.
  - Solving for g_1 yields player 1's best response function, $g_1(g_2)$.
  - Operating similarly for player 2, we find his best response function $g_2(g_1)$.
  - Simultaneously solving for g_1 and g_2 yields equilibrium contribution levels of

    $$g_1^* = 2.995 \text{ and } g_2^* = 2.623$$

    with aggregate donations of $G^* = 5.61$.

Warm-Glow Benefits

- ***Example*** (continued):
  - We can implement a \$2 transfer from the individual with high warm-glow parameter to that with low warm-glow parameter.
  - After implementing the transfer from individual 1 to 2, wealth levels become
  $$w_1 = \$8 \text{ and } w_2 = \$12$$
  which modifies individual donations to
  $$g_1^* = 2.2 \text{ and } g_2^* = 3.45$$
  with aggregate donations of $G^* = 5.65$.

Social Preferences

- Consider the public good game where two players simultaneously and independently choose between contributing (C) or not contributing (NC) to the public good.

		Player 2	
		C	NC
Player 1	C	a, a	c, b
	NC	b, c	d, d

- Both players' payoffs satisfy $b > a > d > c$, thus indicating that both players have incentives to free ride.

Social Preferences

- Every player's best response is NC, both when his opponent contributes (since $b > a$), and when he does not (since $d > c$).
- In fact, both players find C to be strictly dominated by NC, ultimately implying that the strategy profile (NC,NC) is the unique equilibrium of the stage game.

Social Preferences

- Consider that players exhibit Fehr and Schmidt (1999)-type social preferences:

  $$U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$$

  where
  - x_i and x_j are player i's and j's payoffs, respectively
  - α_i is player i's disutility from *envy*, which occurs when $x_i < x_j$ implying $U_i(x_i, x_j) = x_i - \alpha_i(x_j - x_i)$
  - β_i is player i's disutility from *guilt* which occurs when $x_i > x_j$ implying $U_i(x_i, x_j) = x_i - \beta_i(x_i - x_j)$
- Fehr and Schmidt (1999) assume that players' envy is always stronger than their guilt, i.e., $\alpha_i \geq \beta_i$ and $0 \leq \beta_i < 1$.

Social Preferences

- Hence, the above payoff matrix can be reformulated as follows

Player 2

		C	NC
Player 1	C	a, a	$c - \alpha_1(b-c), b - \beta_2(b-c)$
	NC	$b - \beta_1(b-c), c - \alpha_2(b-c)$	d, d

- Every player's utility level decreases when he is either:
  - the player with the highest payoff in the group (due to guilt), e.g., player 1 under outcome (NC,C), or
  - the player with the lowest payoff in the group (due to envy), e.g., player 1 under outcome (C,NC).

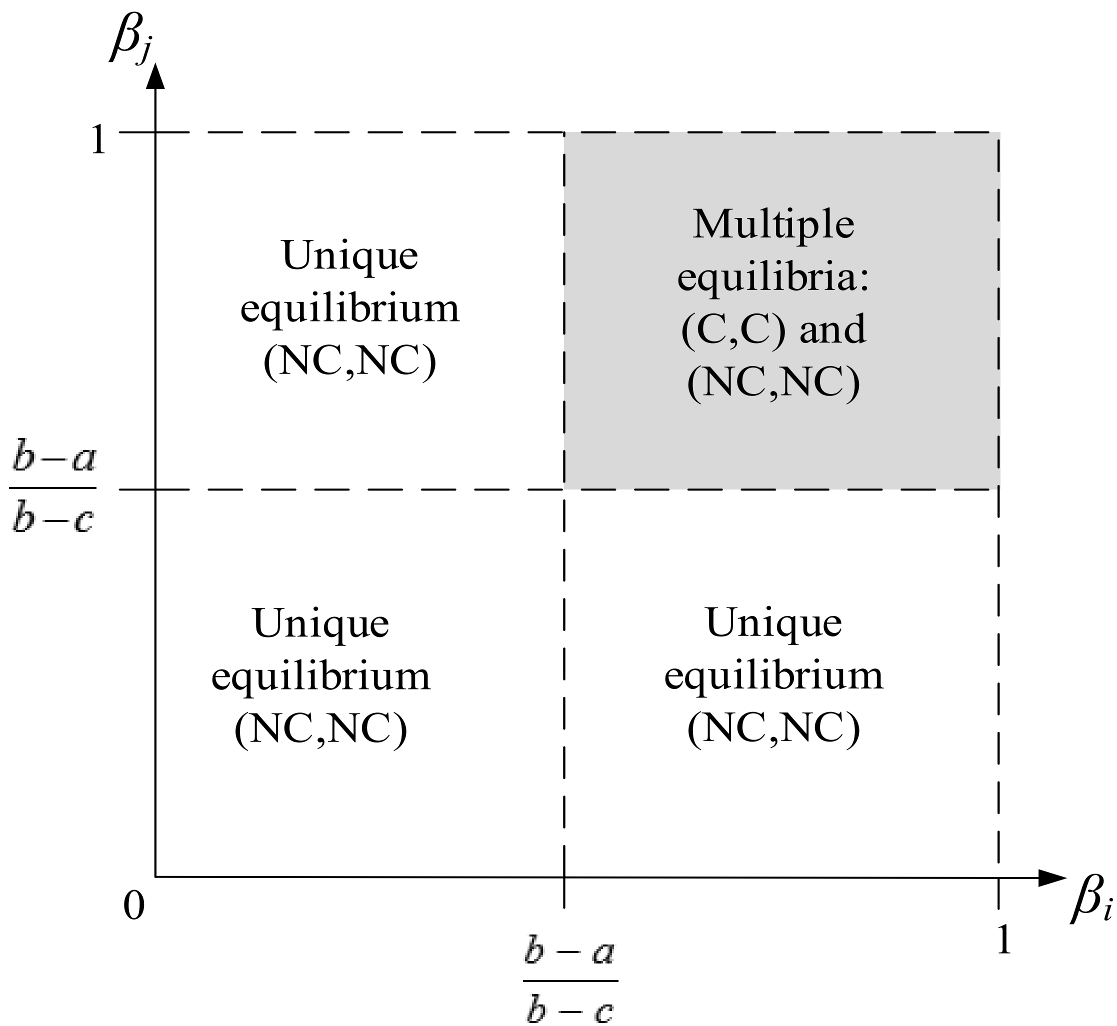
Social Preferences

- Let us first analyze player i's BRF:
  - When player j chooses C, player i prefers NC if $a \leq b - \beta_i(b-c)$, or, more compactly, $\beta_i \leq \frac{b-a}{b-c}$.
  - When player j chooses NC, player i prefers NC if $c - \alpha_i(b-c) \leq d$, which is always true given that $\frac{c-d}{b-c} < 0 \leq \alpha_i$.
  - Therefore, if $\beta_i \leq \frac{b-a}{b-c}$, NC becomes a strictly dominant strategy for player i.
  - If, instead, $\beta_i > \frac{b-a}{b-c}$, then player i's best response to C is C, but his best response to NC is still NC.

Social Preferences

- Hence, Nash equilibria of the game in pure strategies are:
  - (NC,NC) if either $\beta_i \leq \frac{b-a}{b-c}$ or $\beta_j \leq \frac{b-a}{b-c}$
  - (C,C) and (NC,NC) if $\beta_i, \beta_j \geq \frac{b-a}{b-c}$
- Intuitively,
  - if at least one player has relatively low concerns about guilt, the unique Nash equilibrium of the game coincides with that where players have no concerns about the fairness of the payoff distribution (standard preferences).
  - if both individuals are sufficiently concerned about fairness, we can identify two different Nash equilibria. That is, every player's best response is to select the same action as his opponent.

Social Preferences



Competition for Status Acquisition

Competition for Status Acquisition

- Consider a public good game with two agents privately contributing to its provision.
- Let g_i denote individual i's voluntary contributions to the public good, and $x_i \geq 0$ represent his consumption of private goods.
- Every player obtains the return $m \in [0, +\infty)$ from total contributions to the public good, i.e., $G = g_i + g_j$.
- In addition, each player benefits from *status acquisition* if his donation is larger than his rival's.
- Assume that every player is endowed with w monetary units that can be distributed between private and public good consumption.
- The marginal utility individual i derives from his consumption of the private good is one.

Competition for Status Acquisition

- The representative contributor's utility function is
$$u_i(x_i, G) = x_i + \ln[mG + \alpha_i(g_i - g_j)]$$
- In this setting, the *status* subject i acquires by contributing g_i is given by $g_i - g_j$.
  - That is, subject i enhances his relative status if his contribution is greater than individual j's; otherwise, subject i perceives himself as an individual with lower status than subject j.
- The difference is scaled by $\alpha_i \in [0, +\infty)$, indicating the importance of relative status for subject i.
- Complete information game

Competition for Status Acquisition

- The UMP of every player i is

$$\max_{x_i, g_i} \; x_i + \ln\left[mG + \alpha_i\left(g_i - g_j\right)\right]$$

$$\text{s.t.} \;\; x_i + g_i = w,$$

$$g_i + g_j = G, \text{ and}$$

$$g_i, g_j \geq 0$$

- Using $x_i = w - g_i \geq 0$ and $g_i + g_j = G$, we obtain the following unconstrained program

$$\max_{g_i \geq 0} \; w - g_i + \ln\left[m\left(g_i + g_j\right) + \alpha_i\left(g_i - g_j\right)\right]$$

Competition for Status Acquisition

- Note that an increase in player j's contribution, g_j, imposes two types of externalities on player i's utility level:
  - ***Positive externality***: arising from the public good nature of player j's contributions.
  - ***Negative externality***: player j's donations reduce the status perception of player i.

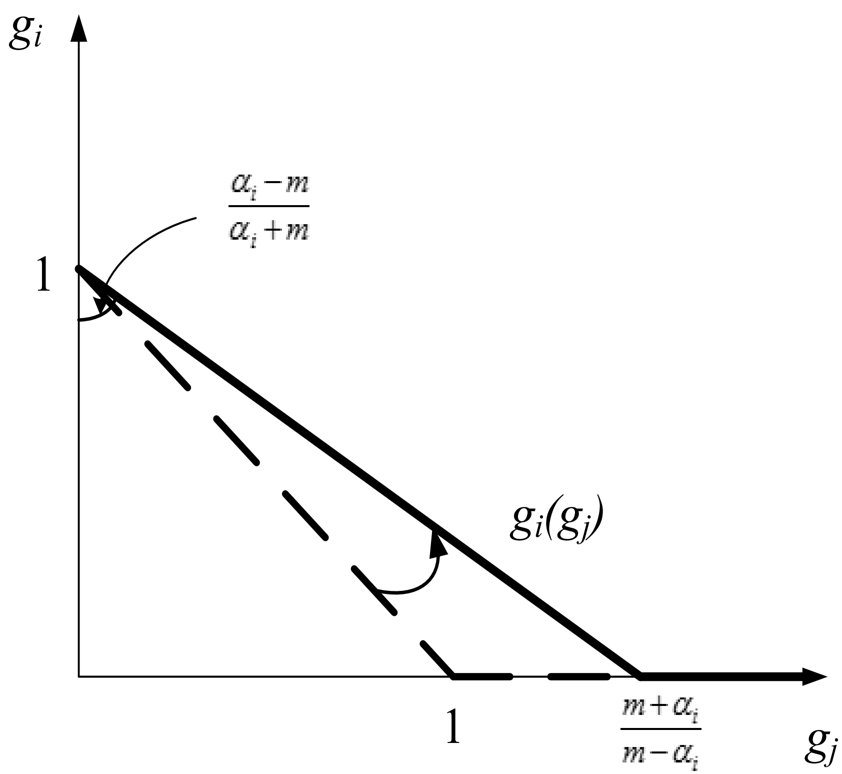
Competition for Status Acquisition

- FOC wrt g_i yields player i's BRF

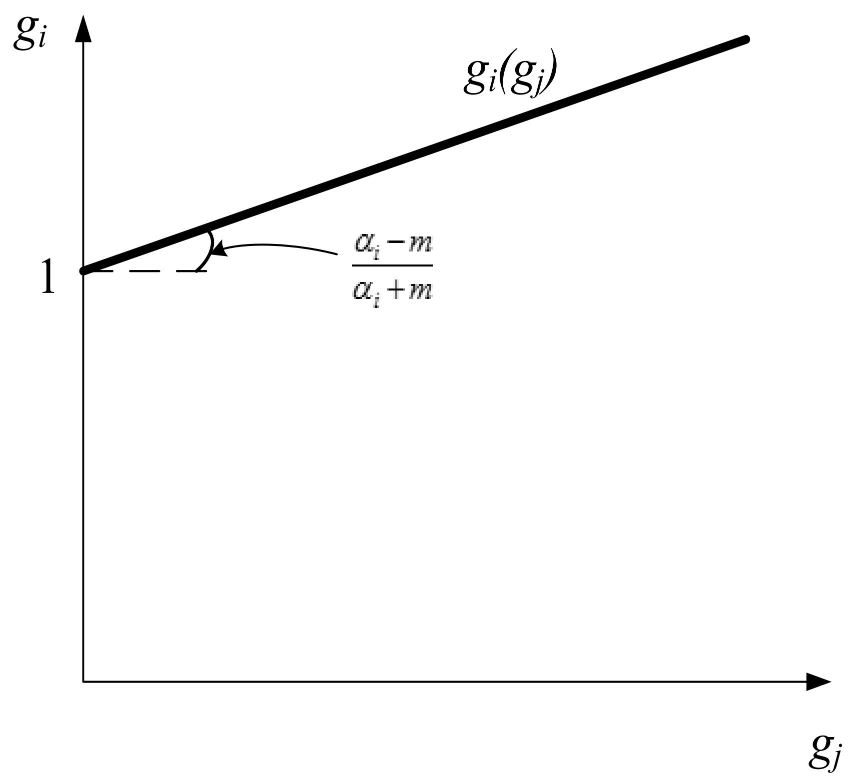
$$g_i(g_j) = \begin{cases} 1 + \frac{\alpha_i - m}{\alpha_i + m} g_j & \text{if } g_j \in \left[0, \frac{m+\alpha_i}{m-\alpha_i}\right] \\ 0 & \text{if } g_j > \frac{m+\alpha_i}{m-\alpha_i} \end{cases}$$

- Note that player i's BRF is
  - decreasing in g_j if $\alpha_i < m$
  - increasing in g_j if $\alpha_i > m$
  - increasing in α_i

Competition for Status Acquisition



$g_i(g_j)$ when $\alpha_i < m$



$g_i(g_j)$ when $\alpha_i > m$

Competition for Status Acquisition

- *Remarks*:
  - When $\alpha_i < m$, the positive externality that player j's contributions impose on player i's utility dominates the negative externality.
    - Player i considers player j's contributions as strategic *substitutes* of his own.
  - When $\alpha_i > m$, the negative externality that player j's contributions impose on player i's utility dominates the positive externality.
    - Player i considers player j's donations as strategic *complements* to his own.

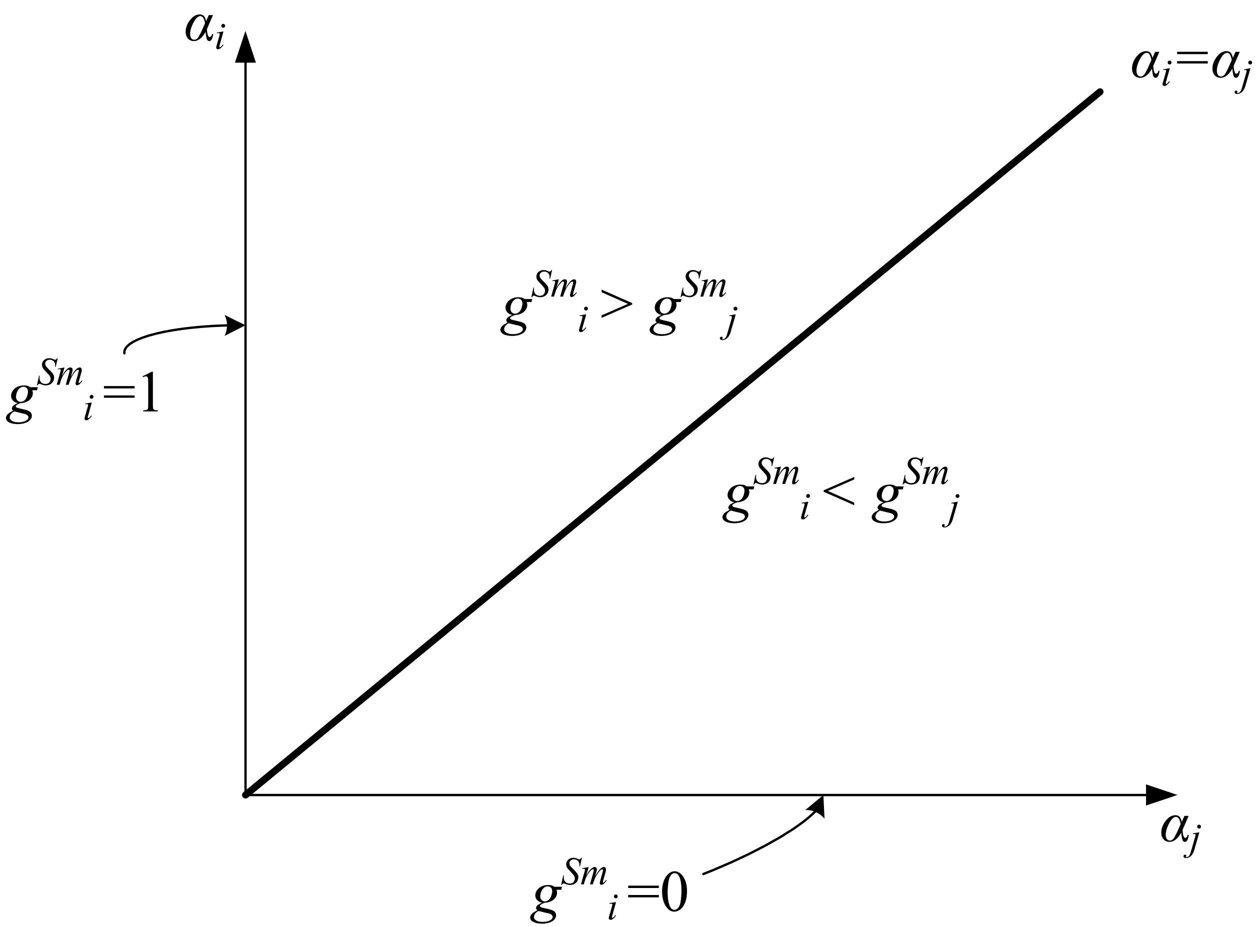
Competition for Status Acquisition

- Nash equilibrium contribution of player i:

$$g_i^{Sm} = \begin{cases} 1 \ \text{ if } \ \alpha_i > 0 \text{ and } \alpha_j = 0 \\ \frac{\alpha_i(\alpha_j + m)}{(\alpha_i + \alpha_j)m} \ \text{ if } \ \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 0 \ \text{ if } \ \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

where $g_i^{Sm} + g_j^{Sm} = 1$ if $\alpha_i = \alpha_j = 0$.

Competition for Status Acquisition



Competition for Status Acquisition

- Total contributions to the public good are

$$G^{Sm} = \begin{cases} 1 \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j = 0 \\ 1 + \frac{2\alpha_i \alpha_j}{(\alpha_i + \alpha_j)m} \quad \text{if} \quad \alpha_i > 0 \text{ and } \alpha_j > 0 \\ 1 \quad \text{if} \quad \alpha_i = 0 \text{ and } \alpha_j > 0 \end{cases}$$

where G^{Sm} is weakly increasing in both α_i and α_j, and maximized when $\alpha_i = \alpha_j = \alpha$.

Competition for Status Acquisition

- *Intuition*:
  - Total contributions when *either* player does not value status coincide with total contributions when *none* of them does, $G^{Sm} = 1$.
  - An increase in the status concerns of only one individual (α_i or α_j) raises total contributions.
  - Finally, G^{Sm} is higher when players' value of status acquisition are homogeneous, i.e., $\alpha_i = \alpha_j = \alpha$.

Appendix 1:
More General Policy Mechanisms

More General Policy Mechanisms

- In the presence of incomplete information, standard policy tools (e.g., quotas and emission fees) entail welfare losses.
- Let us examine more general policy mechanisms that try to maximize social surplus in the context of incomplete information.
- We consider mechanisms in which we ask agents to self-report their types.

More General Policy Mechanisms

- We ask the firm:
  - What is your benefit from increasing the externality level from $x = 0$ to $x = \bar{x}$, i.e., $b = b(\theta)$, given your private observation of θ?
- We ask the consumer:
  - What is your damage from the externality, i.e., $c = c(\eta)$, given your private observation of η?

More General Policy Mechanisms

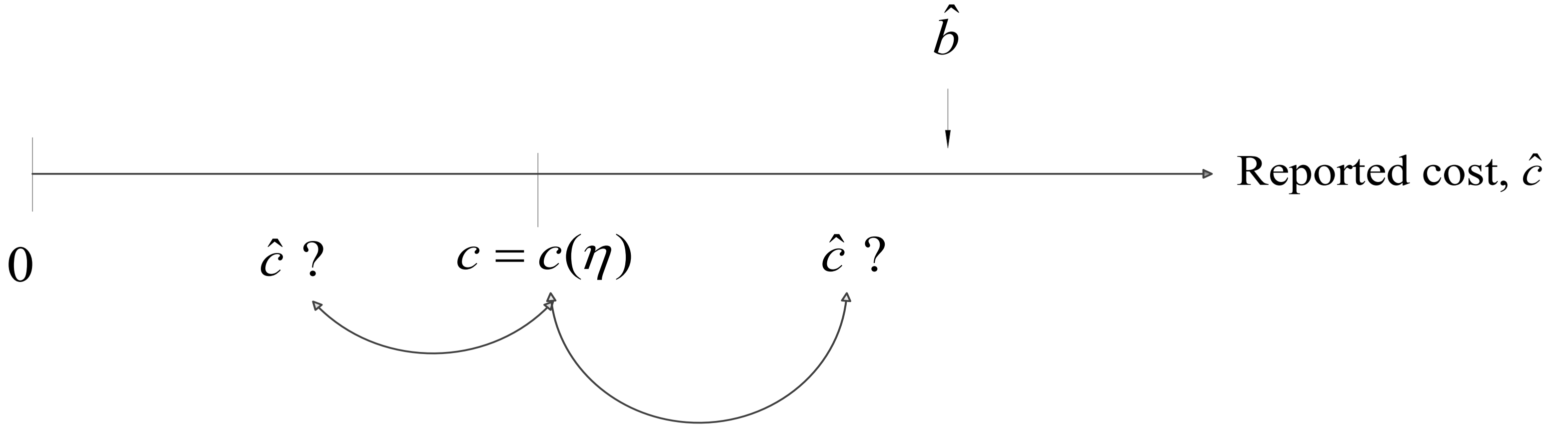
- The mechanism we are interested in focuses on providing incentives to all parties to guarantee that a truth-telling equilibrium emerges.
- ***Groves-Clark-Vickrey (GCV) mechanism***:
  - The regulator declares that it will set the level of the externality at $x = \bar{x}$ if $\hat{b} > \hat{c}$.
    - If this is the case, the government pays $\hat{b}$ to the consumer and charges $\hat{c}$ to the firm.
    - Not a typo!
  - Otherwise, the regulator keeps the level of the externality at $x = 0$.

More General Policy Mechanisms

- Wouldn't this type of mechanism induce firms to *underreport* their benefits?
  - That is, stating a benefit $\hat{b} < b$, in order to reduce the compensation that they have to provide to those consumers affected by the externality.
- Also, wouldn't this type of mechanism induce consumers to *overestimate* their damages?
  - That is, stating a cost $\hat{c} > c$, in order to guarantee that the externality is not allowed or, if allowed, they are substantially compensated for the cost they suffer.

More General Policy Mechanisms

- ***Consumer***:
  - Consider a consumer with a real cost $c = c(\eta)$.
  - Let us examine the consumer's optimal announcement, $\hat{c}$, given a firm's report of a benefit $\hat{b} > c$.



More General Policy Mechanisms

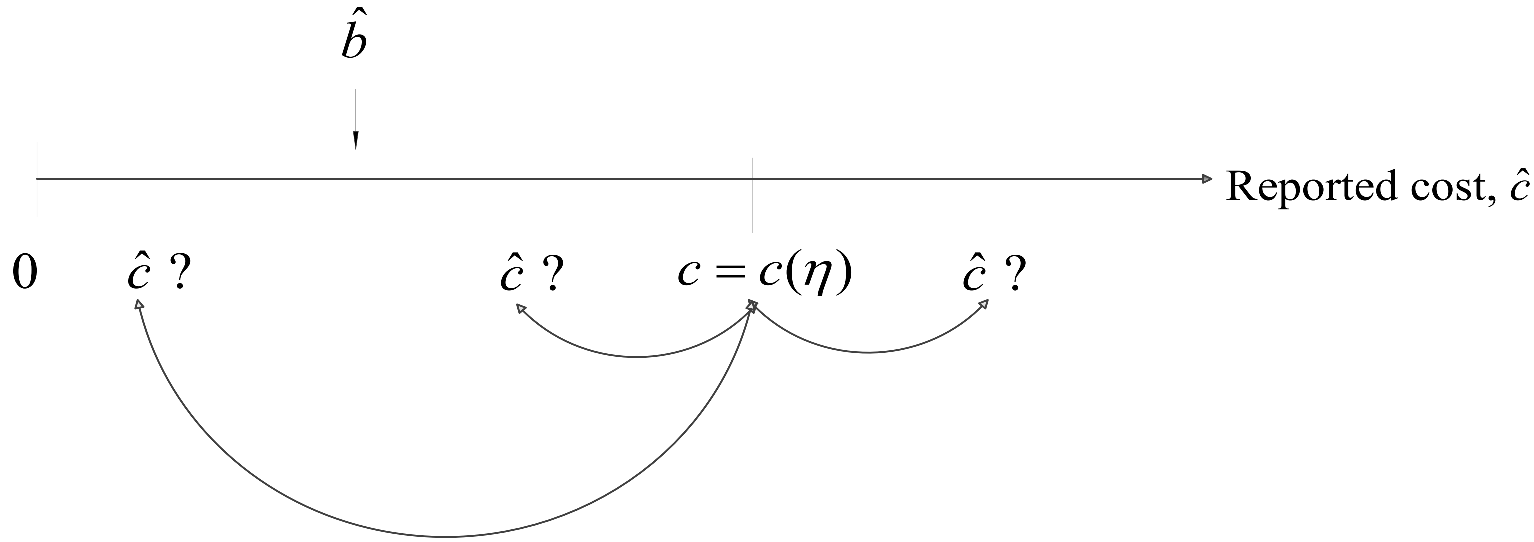
- The consumer does not have incentives to slightly over-report her cost, i.e., $c < \hat{c} < \hat{b}$, or underreport it, i.e., $\hat{c} < c$, since in both cases the compensation she receives is $\hat{b}$.
  - The compensation that the consumer receives is unaffected by her report, inducing the consumer to truthfully reveal her cost $c = c(\eta)$.

More General Policy Mechanisms

- If the consumer over-reports her costs, i.e., $\hat{c} > \hat{b}$, the regulator would decide to not allow the externality, i.e., $x = 0$.
  - Such outcome yields a lower payoff for the consumer than the above outcomes, whereby a report $\hat{c} = c$ yields a compensation of $\hat{b}$ from the firm.

More General Policy Mechanisms

– Let us now examine the consumer's optimal announcement, $\hat{c}$, given a firm's announcement of a benefit $\hat{b} < c$.



More General Policy Mechanisms

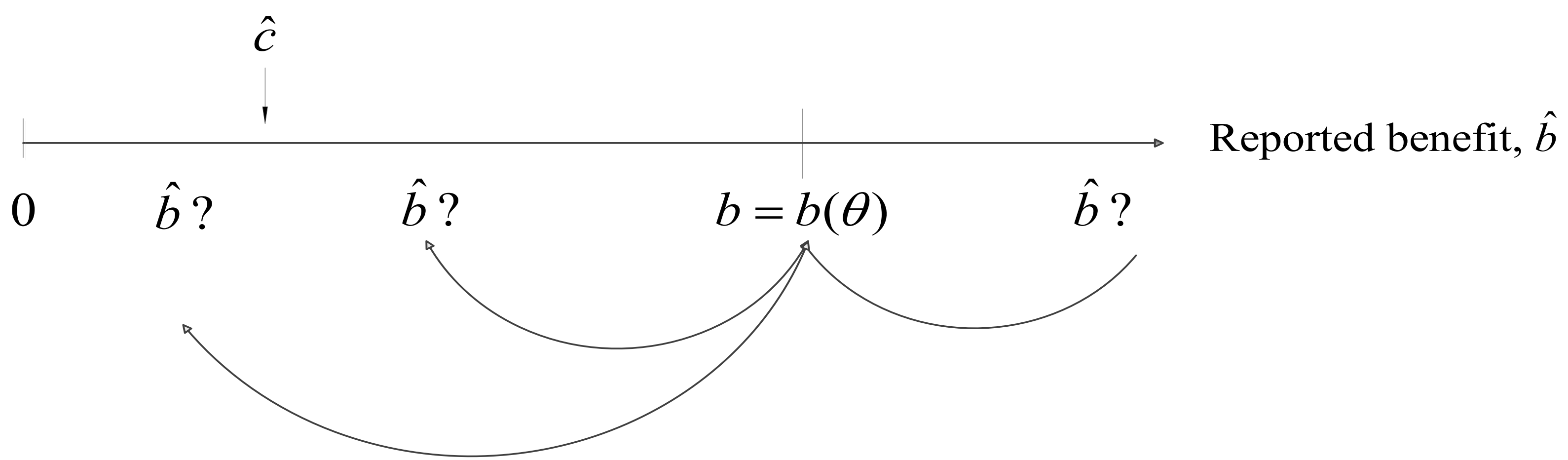
- If the consumer over-reports her costs, i.e., $\hat{c} > \hat{b}$, the regulator would decide to not allow the externality, i.e., $x = 0$.
- If the consumer slightly underreports her cost $\hat{c}$, i.e., $\hat{b} < \hat{c} < c$, the externality is still not allowed by the regulator, given that reports satisfy $\hat{c} > \hat{b}$.
- Finally, an extreme underreport of her costs, i.e., $\hat{c} < \hat{b}$, is not sensible either:
  - While the externality is now allowed (since $\hat{b} > \hat{c}$), the consumer receives a subsidy $\hat{b}$ below her true cost c, i.e., $c > \hat{b}$.

More General Policy Mechanisms

- Hence, the consumer has incentives to truthfully reveal the damage she suffers from the externality, $\hat{c} = c(\eta)$, regardless of the precise report $\hat{b}$ that the firm makes.
  - That is, truthfully reporting her cost is a weakly dominant strategy for the consumer.

More General Policy Mechanisms

- ***Firm***:
  - Consider a firm with a real benefit $b = b(\theta)$.
  - Let us first examine the firm's optimal announcement, $\hat{b}$, given a consumer's report of a cost $\hat{c} < b$.



More General Policy Mechanisms

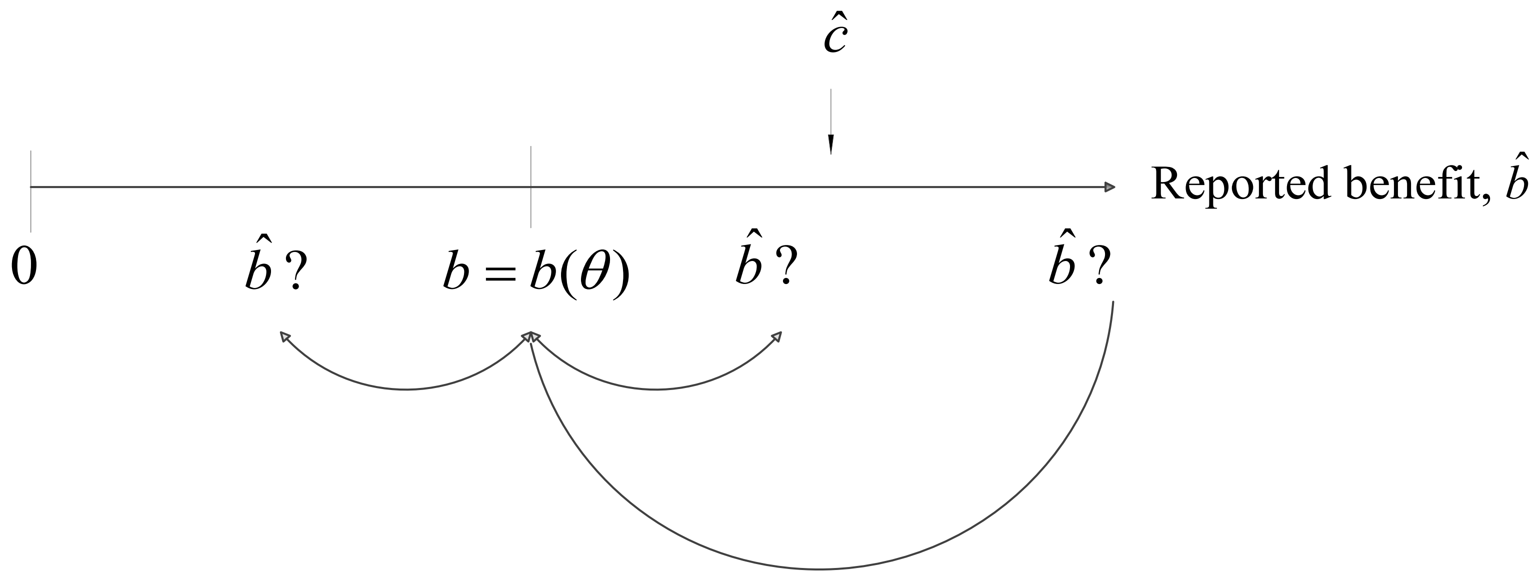
- The firm has no incentives to over-report its true benefit b, i.e., $\hat{b} > b$.
  - The firm would have to pay the same compensation to the consumer, $\hat{c}$, and the externality would still be allowed since reports satisfy $\hat{b} > \hat{c}$.
- The firm has no incentives to slightly underreport its true benefit, i.e., $\hat{c} < \hat{b} < b$.
  - The compensation that the firm has to pay is still $\hat{c}$ and the externality is allowed, since reports still satisfy $\hat{b} > \hat{c}$.

More General Policy Mechanisms

- Finally, the firm has no incentives to extremely underreport its true benefit, i.e., $\hat{b} < \hat{c}$.
  - In this case, the externality would not be allowed by the government given that reports satisfy $\hat{b} < \hat{c}$.

More General Policy Mechanisms

– Let us now consider the case where consumer's report $\hat{c}$ lies above the firm's true benefit b, i.e., $\hat{c} > b$.



More General Policy Mechanisms

- If the firm over-reports its benefit, i.e., $b < \hat{c} < \hat{b}$, the externality would be allowed (since $\hat{b} > \hat{c}$).
  - However, the firm has to pay a compensation $\hat{c}$ to the consumer which is higher than the real benefit that the firm obtains from the externality, i.e., $b < \hat{c}$.

- If the firm slightly over-reports its benefits, i.e., $b < \hat{b} < \hat{c}$, or underreports it, i.e., $\hat{b} < b < \hat{c}$, the externality will not be allowed given that reports would now satisfy $\hat{b} < \hat{c}$.

More General Policy Mechanisms

– Hence, the firm prefers no externality whatsoever, i.e., $x = 0$.
  - The true benefit that the firm obtains from the externality b lies below the cost $\hat{c}$ that the consumer declared to experience.

– Hence, truthfully reporting its benefit from the externality is a weakly dominant strategy for the firm.