Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 9:
Partial and General Equilibrium

Outline

- Features of Perfectly Competitive Markets
- Profit Maximization Problem
- Supply Curves
- Short-Run Supply Curve
- Market Equilibrium
- Producer Surplus
- General Equilibrium
- A Look at Behavioral Economics–Market Experiments
- Appendix. Efficient Allocations and MRS

Features of Perfectly Competitive Markets

Perfectly Competitive Markets

- Perfectly competitive markets are:
  - *Fragmented*: Many small firms. An increase in the production of either firm does not alter market prices.
  - *Undifferentiated products*: Products of all firms in the industry are regarded as identical by consumers.
  - *Perfect pricing information*: Consumers can easily compare sellers' prices at not cost.
  - *Free entry and exit*: In the long run, firms can enter the market if positive profits can be earned, or to exit if they incur losses.
- *Example*: Markets of agricultural commodities (e.g. wheat).

Profit Maximization Problem

Profit Maximization Problem

- In a perfectly competitive industry, all firms are price takers (they take market price p as given).
- Every firm's PMP is

$$\max_{q} \pi = TR(q) - TC(q) = \underbrace{pq}_{TR(q)} - TC(q). \qquad (9.1)$$

  - The firm chooses output level to maximize its profit, which is the difference between total revenue and its total cost (resulting after solving the cost-minimization problem).

Profit Maximization Problem

- The PMP can be understood as a 3-step procedure:
  1. Find input demands for L and K that minimize the firm's cost subject to reaching a specific production level q (i.e., input combinations that solve the CMP)
  2. Insert input demands into the firm's costs to obtain its total cost $TC(q) = wL + rK$.
  3. Insert $TC(q)$ found in step 2 into the firm's profit equation (9.1), $\pi = pq - TC(q)$.
- Completed steps 1-3, differentiate the profit function only with respect to q,

$$p - \frac{\partial TC}{\partial q} = 0 \quad \Longrightarrow p = MC(q).$$

Profit Maximization Problem

- The result $p = MC(q)$ says that, to maximize its profits, the firm increases its output q until the point where
  - the price from selling an additional unit coincides
  - with the additional cost that the firm needs to incur to produce such unit.
- If, instead, the firm chooses q for which
  - $p > MC(q)$, it could increase profits by producing *more* units.
  - $p < MC(q)$, it could increase its profits by producing *fewer* units.

Profit Maximization Problem

- We check that $p = MC(q)$ is a condition to maximize (rather than minimize) profits by finding the second-order condition.
- Differentiating $p - MC(q) = 0$ with respect to q,

$$0 - \frac{\partial MC}{\partial q} < 0 \text{ so long as } \frac{\partial MC}{\partial q} > 0.$$

  - If the firm's marginal costs are increasing in output, condition $p = MC(q)$ guarantees the firm is maximizing its profits.

Profit Maximization Problem

- *Example 9.1: PMP in the Cobb-Douglas case.*
  - Consider the firm with Cobb-Douglas production function in example 8.6, $q = L^{1/2}K^{1/2}$.
  - Total cost function is $TC(q) = 40q$.
  - Inserting $TC(q)$ into the the firm's PMP,
$$\max_{q} \pi = pq - 40q.$$
  - Differentiating with respect to q,
$$p - 40 = 0 \quad \Rightarrow p = \$40.$$

Profit Maximization Problem

- *Example 9.1* (continued):
  - This figure depicts this supply curve with linear cost function.
    - At $p \geq \$40$, the firm produces as much as possible.
    - At $p < \$40$, the firm finds optimal not to supply any units, $q = 0$.

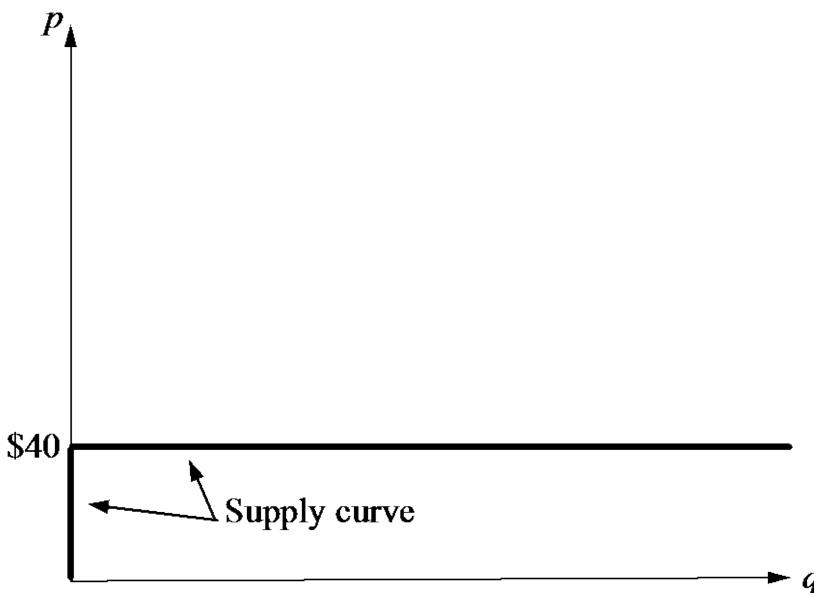


Figure 9.1a

Profit Maximization Problem

- *Example 9.1* (continued):
  - Assume now the firm's cost function was $TC(q) = 40q^2$, which is convex in q.
  - Condition $p = MC(q)$ becomes
    $$p = 80q.$$
  - Solving for q we find the firm's supply curve,
    $$q = \frac{p}{80}.$$

Profit Maximization Problem

- *Example 9.1* (continued):
  - This figure depicts this supply curve with a convex cost function.
    - $q = \frac{p}{80}$ originates at zero, and grows in p with a slope of $\frac{1}{80}$.
    - The firm supplies more units as price increases.

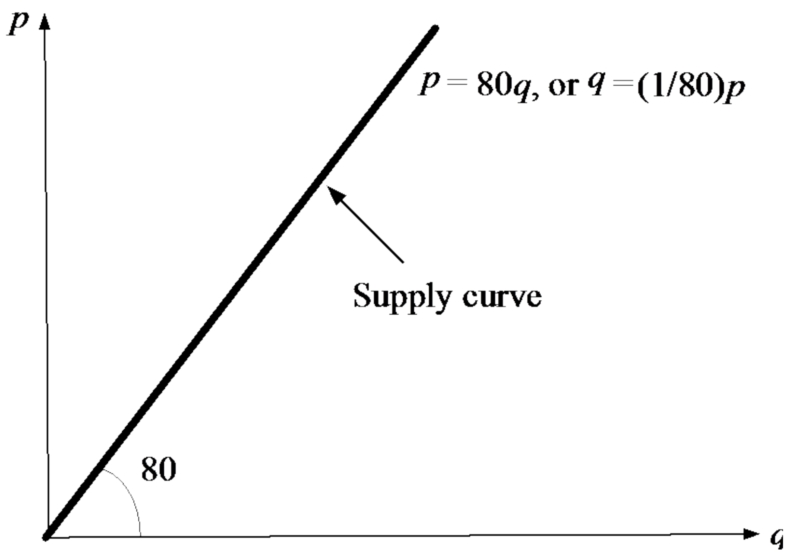


Figure 9.1b

Supply Curves

Individual Firm Supply

- We use the result from the firm's PMP, $p = MC(q)$, to obtain the firm's supply curve.
  - The mapping from the vertical to the horizontal axis says, for each price p, how many units the firm produces to maximize its profits.

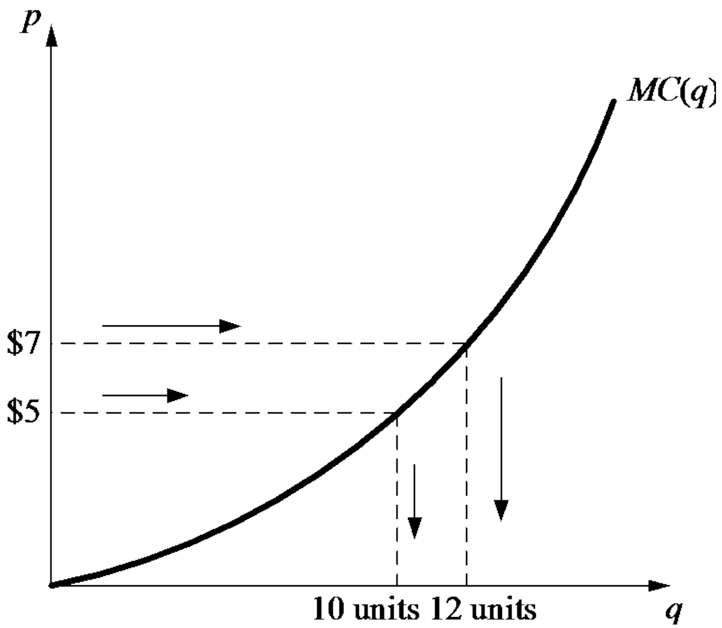


Figure 9.2

Individual Firm Supply

- In the long run, there are no fixed costs, and average cost curve, $AC(q)$, only includes variable costs.
  - The firm could supply units of output even when the market price p falls below $AC(q)$, but with losses.
  - Then, this production strategy would never be chosen.
  - The supply curve can be given by $p = MC(q)$ but only on the segment of the $MC(q)$ that lies above $AC(q)$.

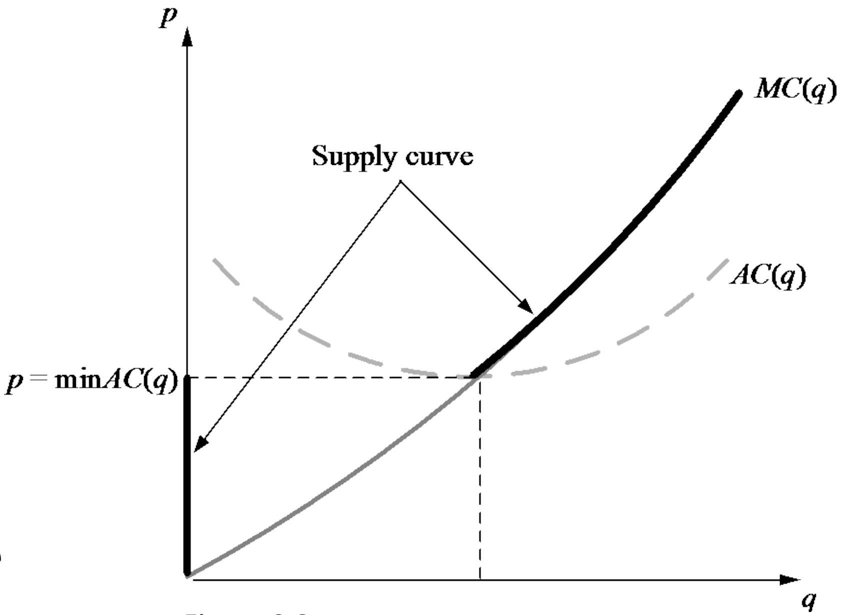


Figure 9.3

Individual Firm Supply

- *Example 9.2: Finding the long-run supply curve.*
  - Consider a firm with $TC(q) = -5q + 2q^2$.
  - Differentiating $TC(q)$ with respect to q, we find

$$MC(q) = \frac{\partial TC(q)}{\partial q} = -5 + 4q.$$

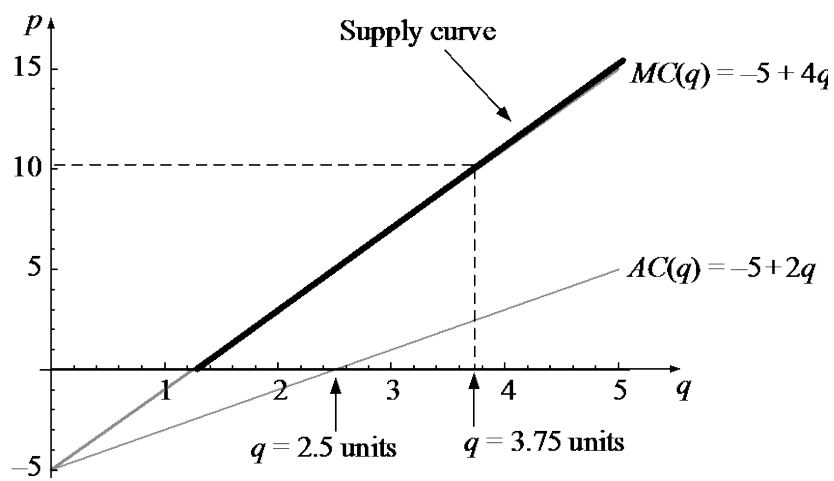


Figure 9.4

Individual Firm Supply

- *Example 9.2* (continued):
  - Setting $p = MC(q)$ and solving for q,
$$p = -5 + 4q,$$
$$q = \frac{p+5}{4} = \frac{p}{4} + \frac{5}{4}.$$
  - This curve, however, is not necessarily the firm's supply curve. To find it, we first need to find the firm's average cost curve
$$AC(q) = \frac{TC(q)}{q} = \frac{-5q+2q^2}{q} = -5 + 2q.$$
  - To obtain the point where $MC(q)$ and $AC(q)$ cross, which is the firm's "shutdown price," we set $MC(q) = AC(q)$,
$$-5 + 4q = -5 + 2q \quad \Longrightarrow q = 0.$$

Individual Firm Supply

- *Example 9.2* (continued):
  - At $q = 0$, $MC(0) = -5 + (4 \times 0) = -5$.
  - We summarize our results with the supply function:

$$q(p) = \begin{cases} \frac{p}{4} + \frac{5}{4} \text{ if } p > 0 \\ 0 \quad \text{otherwise.} \end{cases}$$

- If market price is $p = \$10$, $q = \frac{10}{4} + \frac{5}{4} = \frac{15}{4} = 3.75$ units.
- If market price is $p = \$16$, $q = \frac{16}{4} + \frac{5}{4} = \frac{21}{4} = 5.25$ units.

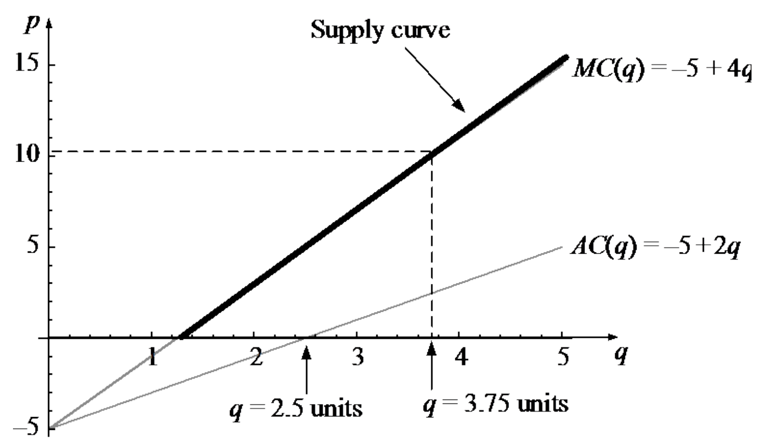


Figure 9.4

Market Supply

- We obtain the market supply by aggregating the individual supply curve of each firm.
  - Horizontally summing across all individual demands in the industry.

Market Supply

- *Example 9.3: Finding market supply.*
  - Consider N firms, each with the individual supply curve found in example 9.2:

$$q(p) = \begin{cases} \frac{p}{4} + \frac{5}{4} \text{ if } p > 0, \\ 0 \quad \text{otherwise.} \end{cases}$$

  - The market supply is $N \times q(p)$,

$$q(p) = \begin{cases} \mathrm{N}\left(\frac{p}{4} + \frac{5}{4}\right) \text{if } p > 0, \\ 0 \quad \text{otherwise.} \end{cases}$$

  - At $p = \$10$, every firm supplies $q = \frac{10}{4} + \frac{5}{4} = 3.75$ units, which entails an aggregate supply of $N \times 3.75$ units of output.

Short-Run Supply Curve

Short-Run Supply Curve

- The previous analysis considers the long-run approach (i.e., the amount of all inputs could be altered).
- We next analyze how the firm's supply curve is affected if the manager operates in a short-run scenario (i.e., the amount of at least one input is fixed).
- Consider

$$TC(q) = \underbrace{a}_{FC} + \underbrace{bq + cq^2}_{VC(q)},$$

where $a \geq 0$ captures the fixed cost, FC (i.e., the part of total costs that is unaffected by changes in output).

$bq + cq^2$, which depends on q, measure the firm's variable costs, $VC(q)$.

Short-Run Supply Curve

- In this scenario, the average cost becomes

$$AC(q) = \frac{TC(q)}{q} = \underbrace{\frac{a}{q}}_{AFC} + \underbrace{bq + cq}_{AVC(q)}.$$

  where $\frac{a}{q} \geq 0$ is the average fixed cost, $AFC(q)$.

  $bq + cq$, denotes the average variable cost, $AVC(q)$.

- Because $AC(q) = AFC + AVC(q)$,

$$AFC = AC(q) - AVC(q).$$

  - $AC(q)$ lies above $AVC(q)$.

Short-Run Supply Curve

- $AC(q)$ curve lies above $AVC(q)$ curve.

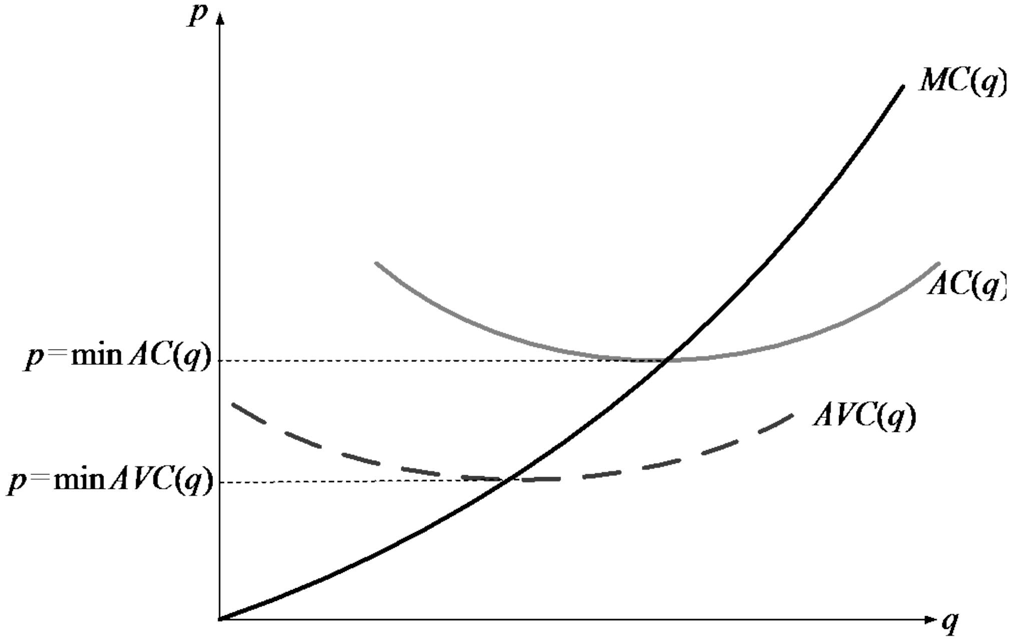


Figure 9.5

Short-Run Supply Curve

- For generality, we can allow a share of fixed cost, a, to be sunk (unrecoverable) or nonsunk (recoverable),

$$a = a_S + a_{NS}.$$

where a_S denotes sunk fixed cost;

a_{NS} represents non-sunk fixed cost.

- In this context, the firm's average fixed cost is

$$AFC(q) = \frac{a}{q} = \frac{a_S + a_{NS}}{q}.$$

- And the average non-sunk cost is $ANSC(q) = \frac{a_{NS}}{q} + b + 2q$, which is lower than the $AC(q)$, but higher than $AFC(q)$.

Short-Run Supply Curve

- $ANSC(q)$ is lower than the $AC(q)$.

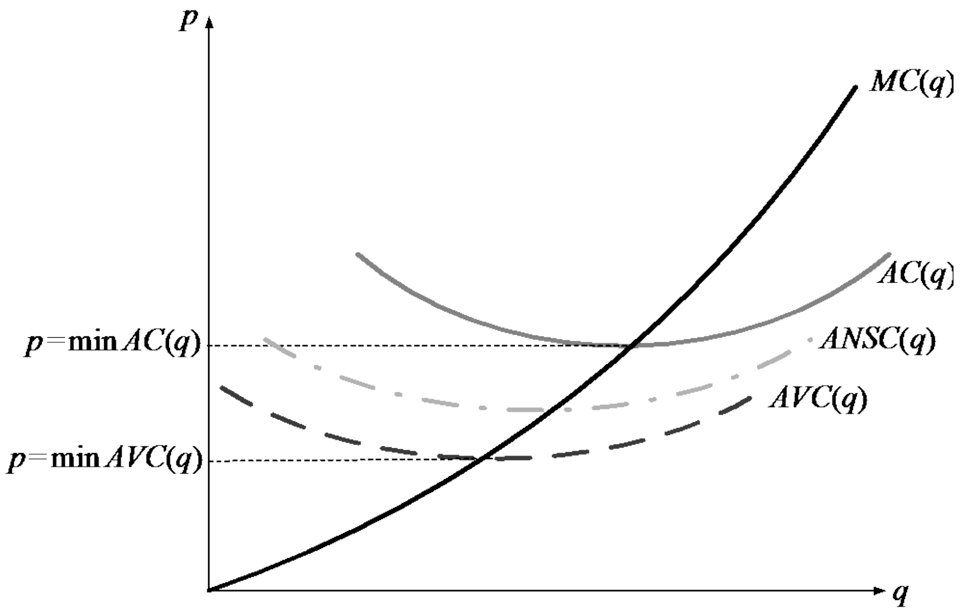


Figure 9.6

Short-Run Supply Curve

Which is the firm's supply curve in a short-run context?

- By altering its output decision, the firm can avoid $AVC(q)$ and its nonsunk costs, but it cannot recover sunk costs.
- The firm will produce positive amounts so long as the market price exceeds its $ANSC$ because its non-sunk costs aggregate all those cost categories than can be avoided or recovered.
- The shutdown price in a short-run scenario lies at exactly the minimum of the $ANSC$, at the point where $ANSC$ crosses $MC(q)$.

Short-Run Supply Curve

- *Example 9.4: Finding the short-run supply curve.*
  - Consider a firm with the same TC function as in example 9.2, but with $10 in fixed costs,
$$TC(q) = 10 - 5q + 2q^2.$$
  - Assume this fixed cost is evenly distributed into $5 sunk costs and $5 non-sunk costs.
  - In example 9.2 we found $MC(q) = 5 - 4q$.
  - The expression of the non-sunk costs is given by
$$NSC(q) = 5 - 5q + 2q^2.$$
  - Then, average non-sunk cost is
$$ANSC(q) = \frac{NSC(q)}{q} = \frac{5}{q} - 5 + 2q.$$

Short-Run Supply Curve

- *Example 9.4* (continued):
  - Setting $MC(q) = ANSC(q)$ we find their crossing point,

$$5 + 4q = \frac{5}{q} - 5 + 2q,$$

$$2q = \frac{5}{q}.$$

  - Solving for q, $q = \sqrt{2.5} \cong 1.58$ units.
  - Inserting this output level into the $MC(q)$ curve, we find the shutdown price in the short-run scenario,

$$p = -5 + (4 \times 1.58) = \$1.32.$$

Short-Run Supply Curve

- *Example 9.4* (continued):
  - In summary, the firm's short-run supply curve is

$$q(p) = \begin{cases} \frac{p}{4} + \frac{5}{4} & \text{if } p > \$1.32, \\ 0 & \text{otherwise.} \end{cases}$$

  - Comparing the short-run supply with the long-run supply in example 9.2,

$$q(p) = \begin{cases} \frac{p}{4} + \frac{5}{4} & \text{if } p > 0, \\ 0 & \text{otherwise.} \end{cases}$$

    - In the short-run, the firm needs a higher price to start producing positive amounts, $\$1.32$, than in the long-run, $\$0$.
    - In the short-run some costs are fixed and sunk (unrecoverable), inducing the firm to start producing only when it faces a high price.

Market Equilibrium

Short-Run Equilibrium

- In the short run we can assume the number of firms in the industry, N, is given.
  - The time span is short (e.g., a day or a week) to prevents other firms from entering or exiting the industry.

- In this scenario, to analyze the equilibrium output and market price, we can use
  - the market demand (aggregate demand after summing all individual demands);
  - and the market supply (aggregate supply after summing all individual supplies).

Short-Run Equilibrium

- *Example 9.5: Finding short-run equilibrium output and price.*
  - Consider a market demand $q^D(p) = 100 - 2p$.
  - And the aggregate supply curve in example 9.3,

$$q^S(p) = \begin{cases} \mathrm{N}\left(\frac{p}{4} + \frac{5}{4}\right) \text{if } p > 0 \\ 0 \quad \text{otherwise.} \end{cases}$$

  - They cross each other when $q^D(p) = q^S(p)$,

$$100 - 2p = \mathrm{N}\left(\frac{p}{4} + \frac{5}{4}\right),$$

$$8p + N(5 + p) = 400.$$

  - Solving for p,

$$p = \frac{5(80-N)}{8+N}, \text{ which is decreasing in } N.$$

Short-Run Equilibrium

- *Example 9.5* (continued):
  - Price $p = \frac{5(80-N)}{8+N}$ crosses the shutdown price of $1.32 at

$$\frac{5(80-N)}{8+N} = 1.32,$$

$$5(80-N) = 1.32(8+N) \quad \Longrightarrow N = 61.62.$$

  - When $N \leq 62$ (e.g., $N = 10$),

$$p = \frac{5(80-N)}{8+N} = \frac{5(80-10)}{8+10} = \frac{350}{18} \cong 19.44.$$

$$q = 100 - 2\frac{5(80-N)}{8+N} = \frac{110N}{8+N} = \frac{110 \times 10}{8+10} = \frac{1{,}100}{18} \cong 61.1.$$

  - When $N > 62$ (e.g., $N = 90$), every firm sets individual production at zero, $q = 0$, and aggregate output is also zero in equilibrium.

Long-Run Equilibrium

- In a perfectly competitive market:
  - Firm entry can occur if potential entrants can make more profits in this market than in other industries.
  - Firm exit can happen if incumbent firms make less profits than in other easily accessible industries.
- The industry reaches an equilibrium when no more firms have incentives to enter or exit.
- For that to occur, two conditions must hold:
  1. Profits for every firm are zero, $p = minAVC(q)$.
  2. Aggregate demand and supply cross each other, that is, $q^D(p) = q^S(p)$.

Long-Run Equilibrium

- *Example 9.6: Finding long-run equilibrium output and price.*
  - Consider the same market demand as in example 9.5,

$$q^D(p) = 100 - 2p,$$

$$MC(q) = -4 + 4q, \text{ and}$$

$$AC(q) = \frac{10}{q} - 5 + 2q.$$

Long-Run Equilibrium

- *Example 9.6* (continued):
  - Because in the long-run equilibrium, the production of every firm, q, must satisfy $p = MC(q) = AC(q)$, we must have that

$$MC(q) = AC(q),$$
$$-5 + 4q = \frac{10}{q} - 5 + 2q,$$
$$2q = \frac{10}{q},$$
$$q^2 = 5,$$
$$\sqrt{q^2} = \sqrt{5},$$
$$q = 2.24 \text{ units.}$$

Long-Run Equilibrium

- *Example 9.6* (continued):
  - $q = 2.24$ units is the output level where curve $MC(q)$ crosses $AC(q)$ at its minimum.
  - All firms produce an output of $q = 2.24$ units, at an equilibrium price of $p = MC(2.24) = -5 + (4 \times 2.24) = \$3.94$, which is the shutdown price in this scenario.
  - Next, we use the second condition (no entry or exit incentives) to obtain the number of firms operating in equilibrium, N^*.

Long-Run Equilibrium

- *Example 9.6* (continued):
  - We set $q^D(p) = q^S(p)$,

$$100 - 2p = N\left(\frac{p}{4} + \frac{5}{4}\right).$$

  - Inserting equilibrium price, $p = \$3.94$, into this expression, we find the equilibrium number of firms is

$$100 - 2(\times 3.94) = N\left(\frac{3.94}{4} + \frac{5}{4}\right),$$

$N^* = \frac{92.12}{2.23} = 41.21$ (i.e., 41 firms are active in the industry)

Long-Run Equilibrium

- *Example 9.6* (continued):
  - As demand increases, N^* grows as well.
  - For instance, if demand increases from

$$q^D(p) = 100 - 2p$$

to

$$q^D(p) = 4{,}000 - 2p,$$

the equilibrium number of firms grows to $N^* = 1{,}786$.

    - Because all firms produce the same output, $q = 2.24$ units, an increase in demand attracts more firms to the industry.

Producer Surplus

Producer Surplus

- The producer surplus (PS) is the difference between the price that the producer receives for its product, p, and its marginal cost from producing that unit.
- Graphically, PS is given by the area below the market price p and the firm's supply curve because the latter is found by setting $p = MC(q)$ and solving for q.

Producer Surplus

- Recall, that $MC(q)$ comes from
$$MC(q) = \frac{\partial TC(q)}{\partial q};$$
and $TC(q)$ was the result of minimizing firm's costs.
- PS measures the profit margin that the firm makes by comparing the price it receives from each unit against the minimal increase in costs from producing 1 extra unit.

Producer Surplus

- *Example 9.7: Finding producer surplus*.
  - Consider the supply function found in example 9.6,
$$p = -5 + 4q \quad \Longrightarrow q = \frac{p}{4} + \frac{5}{4}.$$
  - The shutdown price was $p^{ShutDown} = \$3.94$.
  - We evaluate PS when market price is $p = \$15$.

$$PS = A + B$$

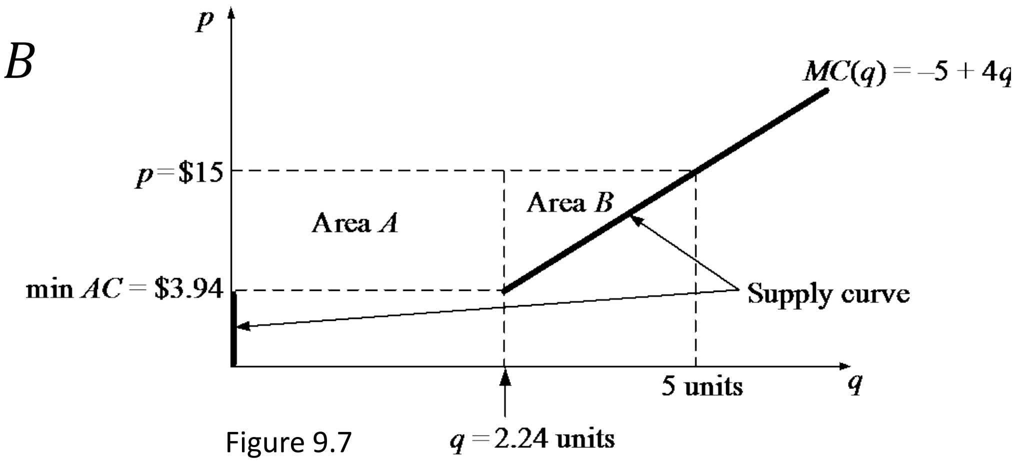


Figure 9.7

Producer Surplus

- *Example 9.7* (continued):

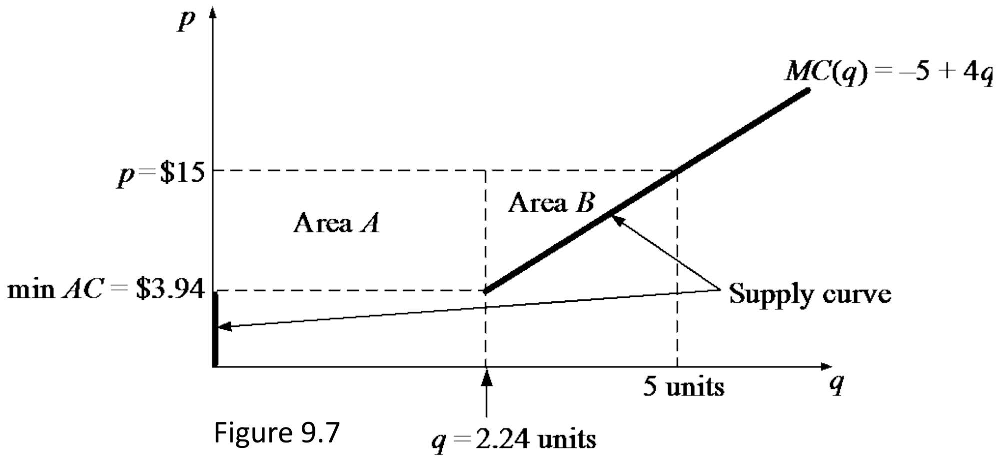


Figure 9.7

$$PS = \underbrace{\left(p - p^{ShutDown}\right)q^{ShutDown}}_{\text{Area } A} + \underbrace{\frac{1}{2}\left(p - p^{ShutDown}\right)\left(q - q^{ShutDown}\right)}_{\text{Area } B},$$

where $p = \$15$ is market price; $p^{ShutDown} = \$3.94$ is the shutdown price; $q^{ShutDown} = 2.24$ units produced at shutdown price; and $q = \frac{15}{4} + \frac{5}{4} = \frac{20}{4} = 5$ are units sold at $p = \$15$.

Producer Surplus

- *Example 9.7* (continued):

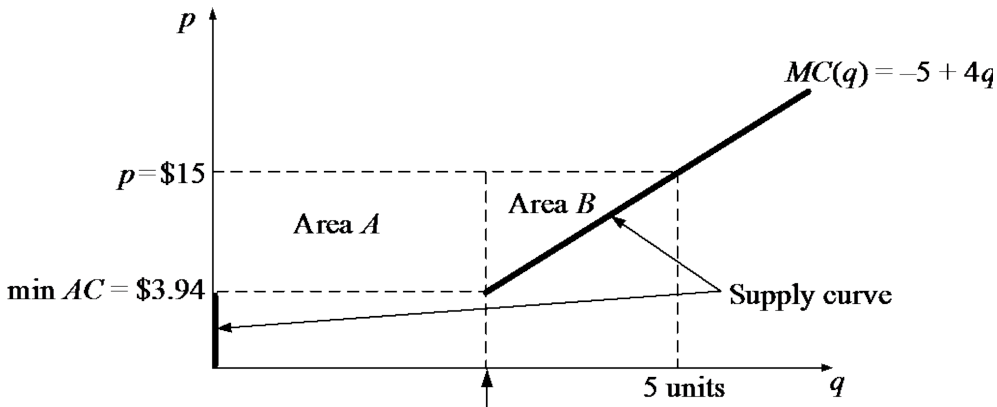


Figure 9.7

$$PS = \underbrace{(15-3.94)2.24}_{\text{Area } A} + \underbrace{\frac{1}{2}(15-3.94)(5-2.24)}_{\text{Area } B} = 24.7 + 15.26,$$

$$PS = \$40.03.$$

General Equilibrium

General Equilibrium

- We extend our analysis to markets with more than one good.
- We seek to find equilibrium prices for which the demand and supply of every good are compatible with one another.
- For simplicity, consider markets with two goods, 1 and 2.
- An endowment of

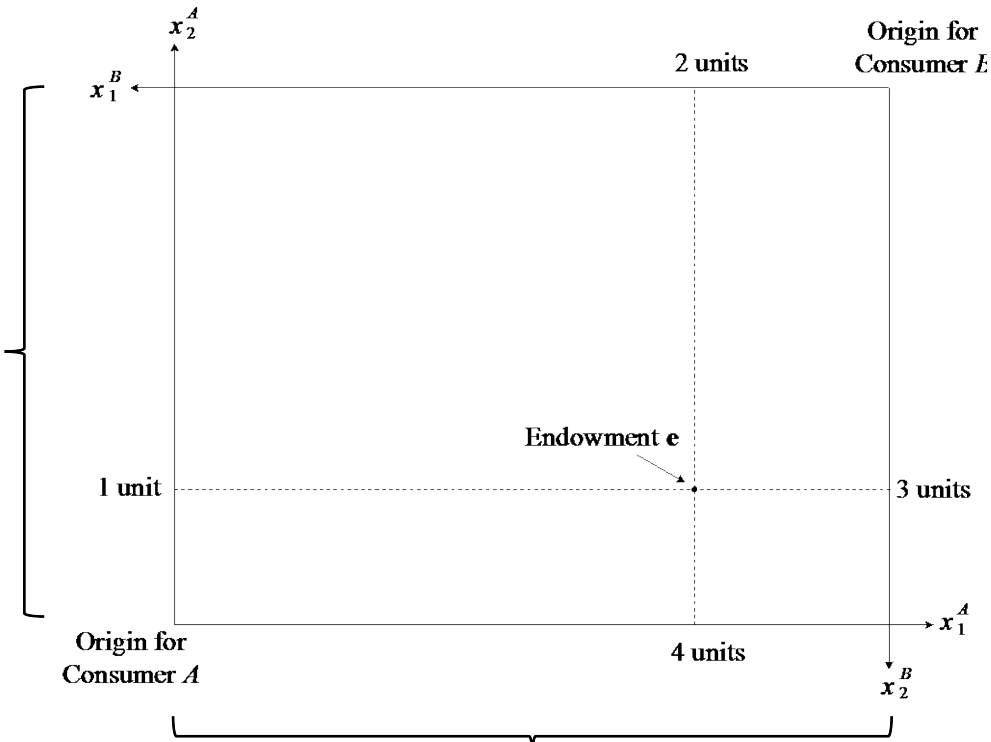
$$\boldsymbol{e} \equiv \left(e_1^A, e_2^A; e_1^B, e_2^B\right)$$

  denotes the amount of goods 1 and 2 that consumers A and B enjoy when they do not trade.
  - *Example*: $\boldsymbol{e} = (4,1;2,3)$.

General Equilibrium

- Endowment $\boldsymbol{e} = (4,1;2,3)$ using the Edgeworth box.

The length of the vertical axis represents total endowment of good 2, $e_2^A + e_2^B = 4$ units.



The length of the horizontal axis represents total endowment of good 1, $e_1^A + e_1^B = 6$ units.

Figure 9.8

General Equilibrium

- An allocation

$$\boldsymbol{x} \equiv \left(x_1^A, x_2^A; x_1^B, x_2^B\right)$$

  lists the amount of goods 1 and 2 that consumers A and B enjoy.

- Allocation $\boldsymbol{x}$ can differ from initial endowment $\boldsymbol{e}$ if individuals trade among themselves.

- An allocation $\boldsymbol{x}$ is feasible if $\boldsymbol{x} \leq \boldsymbol{e}$ → the aggregate amount that all individuals consume, $\boldsymbol{x}$, does not exceed the aggregate amount they initially owned, $\boldsymbol{e}$.

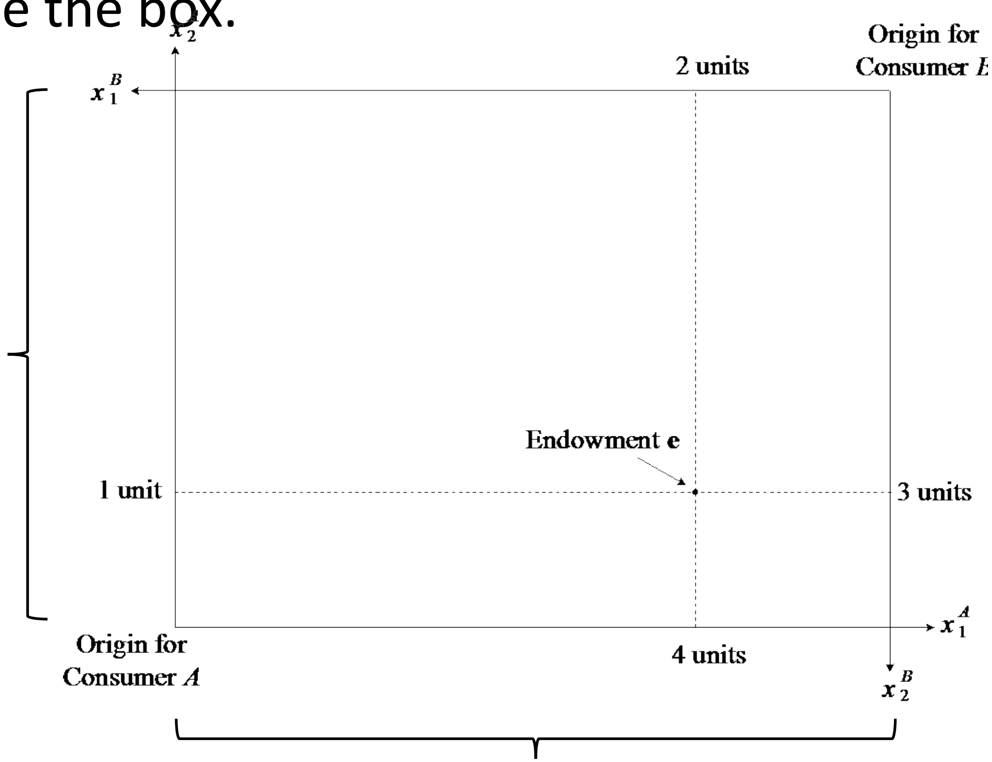
$$x_1^A + x_1^B \leq e_1^A + e_1^B \text{ for good 1, and}$$

$$x_2^A + x_2^B \leq e_2^A + e_2^B \text{ for good 2.}$$

General Equilibrium

- Feasibility says that the allocation that consumers A and B enjoy can not lie outside the box.

The length of the vertical axis represents total endowment of good 2, $e_2^A + e_2^B = 4$ units.



The length of the horizontal axis represents total endowment of good 1, $e_1^A + e_1^B = 6$ units.

Figure 9.8

Equilibrium Prices

- Equilibrium price. A price vector (p_1, p_2) is in equilibrium if it clears the markets for both good 1 and good 2.
  - The demand for every good $j = \{1,2\}$ from all individuals in the economy ("aggregate demand") coincides with the supply of that good in the economy ("supply demand"), $q_j^D(p_j) = q_j^S(p_j)$.
    - If $q_j^D(p_j) > q_j^S(p_j)$, agents could charge a higher price, implying the initial price was not in equilibrium.
    - If $q_j^D(p_j) < q_j^S(p_j)$, buyers wouldn't be willing to pay so much for the product, forcing suppliers to reduce prices.
    - When $q_j^D(p_j) = q_j^S(p_j)$, suppliers and buyers have no incentives to increase or decrease prices.

Equilibrium Prices

- For simplicity, consider a "barter economy" without production, where consumers A and B sell the endowments they do not consume.
- Supply of good $j = \{1,2\}$ is given by the total amount of good j in the endowments of consumers A and B.
- The demand from consumer A is obtained solving her UPM, which yields the tangency condition,

$$MRS_{1,2}^{A} = \frac{p_1}{p_2}.$$

- Similarly, the demand from consumer B is

$$MRS_{1,2}^{B} = \frac{p_1}{p_2}.$$

Equilibrium Prices

- The demands from these two individuals are compatible if the price ratio $\frac{p_1}{p_2}$ satisfies

$$MRS^A_{1,2} = MRS^B_{1,2} = \frac{p_1}{p_2}.$$

- This condition says that the market is in equilibrium when the indifference curves of consumers A and B are tangent.
  - Their slopes, captured by the marginal rate of substitution (MRS), coincide.

Equilibrium Prices

- *Example 9.8: Finding an equilibrium allocation and price.*
  - Consider 2 consumers with Cobb-Douglas utility function $u^i(x_1^i, x_2^i) = x_1^i x_2^i$ for every consumer i.
  - Initial endowments are:
    - $(e_1^A, e_2^A) = (100, 350)$ for consumer A;
    - $(e_1^B, e_2^B) = (100, 50)$ for consumer B.
  - Consumers have symmetric preferences, but start with asymmetric endowments.

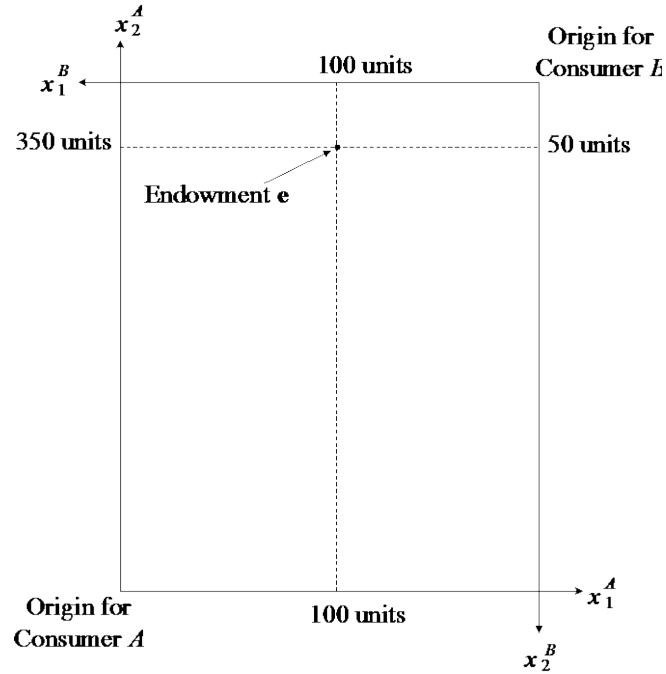


Figure 9.9

Equilibrium Prices

- *Example 9.8* (continued):
  - *Consumer A*. Using her tangency condition,

$$MRS^A_{1,2} = \frac{p_1}{p_2},$$

$$\frac{x^A_2}{x^A_1} = \frac{p_1}{p_2} \quad \Longrightarrow p_2 x^A_2 = p_1 x^A_1.$$

Inserting this result into consumer A's budget constraint,

$$p_1 x^A_1 + p_2 x^A_2 = p_1 e^A_1 + p_2 e^A_2,$$
$$p_1 x^A_1 + p_1 x^A_1 = p_1 100 + p_2 350,$$

which simplifies into consumer A's demand for good 1:

$$x^A_1 = 50 + 175\frac{p_2}{p_1}.$$

Equilibrium Prices

- *Example 9.8* (continued):
  - *Consumer A* (cont.).

    Plugging $x_1^A = 50 + 175\frac{p_2}{p_1}$ back into the tangency condition,

$$p_1 x_1^A = p_2 x_2^A,$$

$$p_1 \underbrace{\left(50 + 175\frac{p_2}{p_1}\right)}_{x_1^A} = p_2 x_2^A.$$

Solving for x_2^A, yields consumer A's demand for good 2,

$$x_2^A = 175 + 50\frac{p_1}{p_2}.$$

Equilibrium Prices

- *Example 9.8* (continued):
  - *Consumer B*. Using her tangency condition,

$$MRS_{1,2}^{B} = \frac{p_1}{p_2},$$

$$\frac{x_2^B}{x_1^B} = \frac{p_1}{p_2} \quad \Longrightarrow p_2 x_2^B = p_1 x_1^B.$$

Following the same steps as for consumer A, we obtain that consumer B's demands are:

$$x_1^B = 50 + 25\frac{p_2}{p_1}$$

and

$$x_2^B = 25 + 50\frac{p_1}{p_2}.$$

Equilibrium Prices

- *Example 9.8* (continued):
  - Lastly, we find equilibrium prices.

    Inserting the demands from good 1 from consumers A and B into the feasibility condition,

$$x_1^A + x_1^B = 100 + 100,$$

$$\underbrace{\left(50 + 175\frac{p_2}{p_1}\right)}_{x_1^A} + \underbrace{\left(50 + 25\frac{p_2}{p_1}\right)}_{x_1^B} = 200,$$

$$100 + 200\frac{p_2}{p_1} = 200.$$

Solving for $\frac{p_2}{p_1}$, we find an equilibrium price ratio of

$$\frac{p_2}{p_1} = \frac{1}{2}.$$

Equilibrium Prices

- *Example 9.8* (continued):
  - Plugging this price ratio into the consumers' demands yields an equilibrium allocation of
    - $x_1^A = 137.5$ and $x_2^A = 275$.
    - $x_1^B = 62.5$ and $x_2^B = 125$.
  - Relative to initial endowment,
    - Consumer A gives up $350 - 275 = 75$ units of good 1 to gain $137.5 - 100 = 37.5$ units of good 1.
    - Consumer B gains 75 units of good 1 and gives up 37.5 units of good 2.

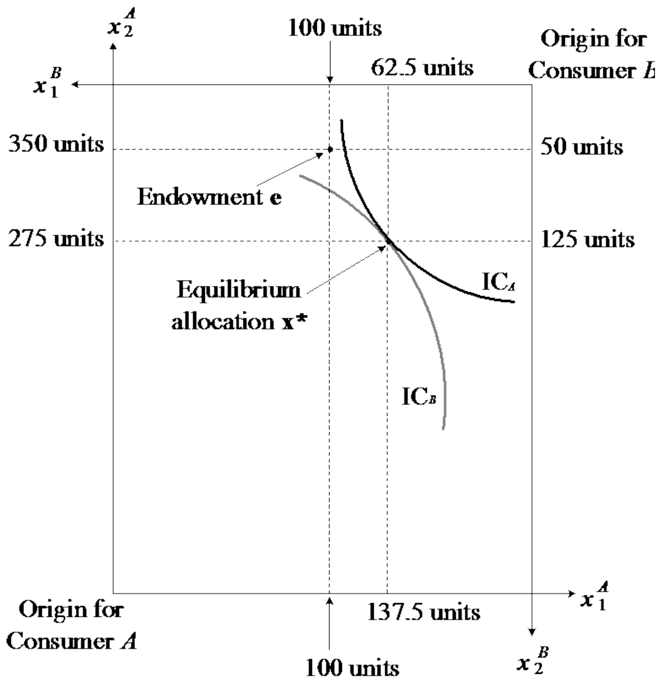


Figure 9.10

Equilibrium Prices

- *Example 9.8* (continued):
  - Finally, we show that both consumes are made better off by trading.
    - Consumer A's utility is:
      - with initial endowment: $350 \times 100 = 35{,}000$,
      - with the equilibrium allocation: $275 \times 137.5 = 37{,}812.5$.
    - Consumer B's utility is:
      - with initial endowment: $50 \times 100 = 812.5$,
      - with the equilibrium allocation: $62.5 \times 125 = 7{,}812.5$.

Efficient Allocation

- We next examine whether equilibrium allocations are efficient.
- If the equilibrium allocation when individuals exchange goods is efficient, no government intervention is needed.
- Consider two assumptions:
  1. Every consumer's utility function is strictly increasing in the good she enjoys, being unaffected by the amount of goods the other individual consumes.
  2. Markets for goods 1 and 2 exist with prices p_1 and p_2, which all consumers take as given.

Efficient Allocation

- A feasible allocation $\boldsymbol{x}$ is efficient if we cannot find another feasible allocation $\boldsymbol{y}$ that strictly increases the utility of at least one individual without reducing the utility of any other individual.
- Efficiency entails that indifference curves of consumers A and B must be tangent, having the same slope,

$$MRS_{1,2}^{A} = MRS_{1,2}^{B}.$$

Equilibrium versus Efficiency

- First Welfare Theorem. Every equilibrium allocation is efficient.
  - The equilibrium allocation emerging when individuals are allowed to trade cannot be improved by a social planner who reassigns goods between consumers.
- But markets are *not* always efficient. They are efficient as long as assumptions (1) and (2) hold.
  - Assumption (1) is violated when consumers care about the amount of goods that others enjoy (exhibiting envy or guilt).
  - Assumption (2) is violated when the market for one of the good does not exist (e.g., a bad such pollution), or if it does exist, consumers have market power, failing to take prices as given.

Efficient Allocation

- *Example 9.9: Finding efficient allocations.*
  - Consider the consumers of example 9.8. The tangency condition $MRS_{1,2}^{A} = MRS_{1,2}^{B}$ yields
$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} \quad \Longrightarrow x_2^A x_1^B = x_2^B x_1^A.$$
  - The feasibility requirements for good 1 and good 2 say
$$x_1^A + x_1^B = 100 + 100 \Longrightarrow x_1^B = 200 - x_1^A.$$
$$x_2^A + x_2^B = 350 + 50 \Longrightarrow x_2^B = 400 - x_2^A.$$
  - Inserting these feasibility equations into the tangency condition,
$$x_2^A \underbrace{\left(200 - x_1^A\right)}_{x_1^B} = \underbrace{\left(400 - x_2^A\right)}_{x_2^B} x_1^A,$$

Efficient Allocation

- *Example 9.9* (continued):

  which simplifies to

  $$x_2^A = 2x_1^A.$$

  - For an allocation to be efficient:
    - Consumer A must enjoy twice as many units of good 2 than of good 1.
    - Consumer B must enjoy the remaining $x_1^B = 200 - x_1^A$ units of good 1 and $x_2^B = 400 - x_2^A$ of good 2.

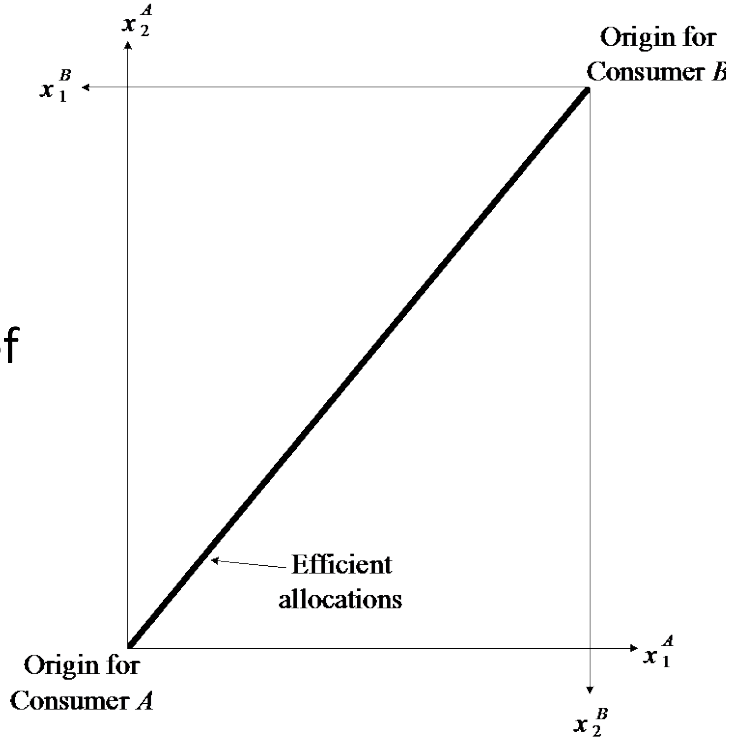


Figure 9.11

Efficient Allocation

- *Example 9.10: Testing the First Welfare Theorem.*

  Is the equilibrium allocation found in example 9.8 efficient?

  - To be efficient, the condition $x_2^A = 2x_1^A$ found in example 9.9 must hold.
  - YES!, the equilibrium allocation in example 9.8 for consumer A, $x_1^A = 137.5$ and $x_2^A = 275$, is efficient because

$$x_2^A = 2x_1^A,$$

$$x_2^A = 2 \times 137.5 = 275 \text{ units.}$$

Efficient Allocation

- *Example 9.10* (continued):
  - The equilibrium allocation lies on the line of efficient allocations.

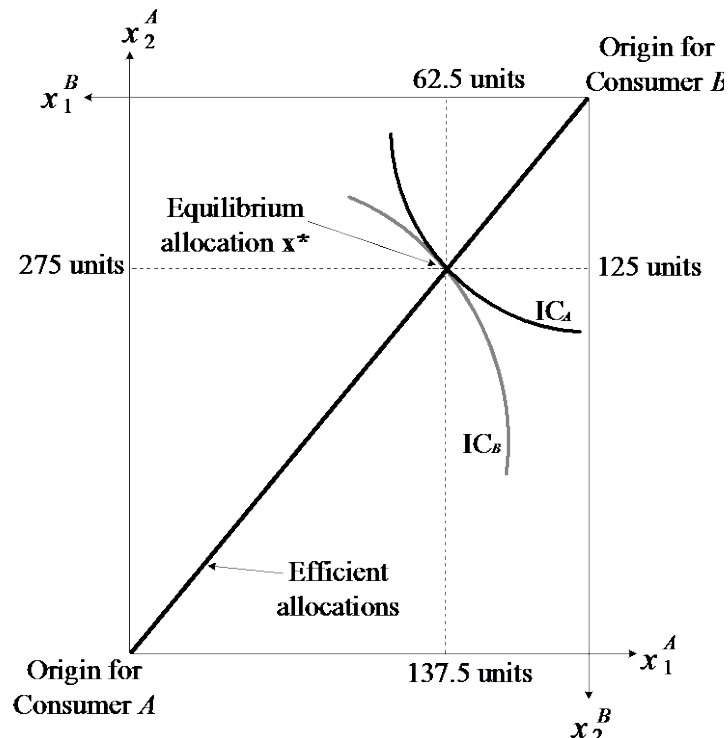


Figure 9.12

Equilibrium versus Efficiency

- The First Welfare Theorem, informally, says that if we let market forces work, a social planner won't be able to improve welfare.
- This theorem provides an argument against market intervention.
- A natural question is whether the converse relationship of that in the First Welfare Theorem also holds:

Can every efficient allocation emerge as an equilibrium outcome?

Equilibrium versus Efficiency

- Second Welfare Theorem. Consider an efficient allocation $\bar{\boldsymbol{x}}$, and a redistribution of the initial endowment, from $\boldsymbol{e}$ to $\bar{\boldsymbol{e}}$, which satisfy $\boldsymbol{p}\bar{\boldsymbol{e}}^{\boldsymbol{i}} = \boldsymbol{p}\bar{\boldsymbol{x}}^{\boldsymbol{i}}$ for every individual $i = \{A, B\}$. Then, every efficient allocation can be supported as an equilibrium allocation given the new endowment $\bar{\boldsymbol{e}}$.
- Consider a scenario where society prefers, among all efficient allocations, a specific allocation $\bar{\boldsymbol{x}}$.
  - The theorem says that this allocation can emerge in equilibrium if we redistribute the initial endowments and allow individuals to trade.
  - One way to redistribute endowments is by taxing some consumers and distributing the collected amount among other as a subsidy.

Equilibrium versus Efficiency

- *Example 9.11: Testing the Second Welfare Theorem.*
  - The efficient allocations in example 9.9 satisfy $x_2^A = 2x_1^A$, where $x_1^A \in [0{,}200]$.
  - A specific allocation satisfying this condition is
    - For consumer A,
$$x_1^A = 100 \text{ units},$$
$$x_2^A = 200 \text{ units}.$$
    - Leaving for consumer B,
$$x_1^B = 200 - x_1^A = 200 - 100 = 100 \text{ units},$$
$$x_2^B = 400 - x_2^A = 400 - 200 = 200 \text{ units}.$$

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Which redistribution of the initial endowment can lead to such an allocation emerging in equilibrium?*
  - From the equilibrium allocation in example 9.8, we know

$$MRS_{1,2}^{A} = \frac{x_2^A}{x_1^A} = \frac{p_1}{p_2} \Rightarrow p_1 x_1^A = p_2 x_2^A, \qquad (9.2)$$

$$MRS_{1,2}^{B} = \frac{x_2^B}{x_1^B} = \frac{p_1}{p_2} \Rightarrow p_1 x_1^B = p_2 x_2^B. \qquad (9.3)$$

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - This efficient allocation can be implemented if the social planner
    - sets a tax $t_B > 0$ to individual B,
    - with the amount collected going to individual A as a subsidy (technically as a "negative tax", $t_A < 0$).

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Consumer A*. Consumer A's budget constraint in terms of the subsidy t_A she receives is

$$p_1 x_1^A + p_2 x_2^A = \underbrace{p_1 e_1^A + p_2 e_2^A}_{\text{Value of initial endowment}} + \underbrace{t_A}_{\text{Tax/Subsidy}}.$$

After substituting equation (9.2), $p_1 x_1^A = p_2 x_2^A$, and her initial endowment $\left(e_1^A, e_2^A\right) = (100{,}350)$,

$$2p_1 x_1^A = 100p_1 + 350p_2 + t_A.$$

Solving for x_1^A,

$$x_1^A = 50 + 175\frac{p_2}{p_1} + \frac{t_A}{2p_1}.$$

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Consumer A* (cont.).

    We now take x_1^A and do the following:

    1. Insert the specific efficient allocation we seek to implement,

    $$\left(x_1^A, x_2^A, x_1^B, x_2^B\right) = (100,200,100,200).$$

    2. Insert the equilibrium price ratio

    $$\frac{p_2}{p_1} = \frac{1}{2}.$$

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Consumer A* (cont.).
    3. Normalize the price of good 2, so that in equilibrium
    $$p_2 = \$1,$$
    $$p_1 = \$2.$$
    Then,
    $$100 = 50 + 175\frac{1}{2} + \frac{t_A}{2 \times 2},$$
    $t_A = -\$150$ (a subsidy of $\$150$ to consumer A).

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Consumer* B. Applying a similar argument, we express her budget constraint as a function of the tax t_B she faces,

$$p_1 x_1^B + p_2 x_2^B = \underbrace{p_1 e_1^B + p_2 e_2^B}_{\text{Value of initial endowment}} + \underbrace{t_B}_{\text{Tax/Subsidy}}.$$

After substituting equation (9.3), $p_1 x_1^B = p_2 x_2^B$, and her initial endowment $(e_1^B, e_2^B) = (100{,}50)$,

$$2p_1 x_1^B = 100p_1 + 50p_2 + t_B.$$

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - *Consumer* B. (cont.).

    Solving for x_1^B,

$$x_1^B = 50 + 25\frac{p_2}{p_1} + \frac{t_B}{2p_1}.$$

After applying steps (1) to (3),

$$100 = 50 + 25\frac{1}{2} + \frac{t_B}{2 \times 2},$$

$t_B = \$150$ (a tax of $\$150$ to consumer B).

Equilibrium versus Efficiency

- *Example 9.11* (continued):
  - Finally, we confirm that the tax imposed on consumer B
  $$t_B = \$150,$$
  coincides with the subsidy provided to consumer A,
  $$t_A = -\$150.$$
  - So, the redistribution scheme is neutral.

Adding Production to the Economy

- Previously, we have considered exchange economies, with no production.
- When we allow for firms, the analysis still holds, but new elements arise.
- *Equilibrium allocations.*
  - We still need every consumer i to solve her UMP, which yields the tangency condition $MRS_{1,2}^{i} = \frac{p_1}{p_2}$.
  - But we require every firm j to solve its PMP, yielding the tangency condition $MRT_{1,2}^{j} = \frac{p_1}{p_2}$, which denotes the marginal rate of transformation of input j (such as labor).

Adding Production to the Economy

- *Equilibrium allocations* (cont.).

$$MRT_{1,2}^{j} = \frac{MP_j^1}{MP_j^2}.$$

  - The tangency condition $MRT_{1,2}^{j} = \frac{p_1}{p_2}$ says that the firm rearranges the use of every input j between the productions of good 1 and 2 until their relative productivity, $\frac{MP_j^1}{MP_j^2}$, coincides with these goods' price ratio, $\frac{p_1}{p_2}$.
  - In summary, an equilibrium allocation with production requires

$$MRS_{1,2}^{i} = MRT_{1,2}^{j} = \frac{p_1}{p_2}.$$

Adding Production to the Economy

- *Efficient allocations.*
  - The definition of efficiency still applies. An allocation is efficient is we cannot find another feasible allocation that makes at least one consumer strictly better off and no consumers worse off.
  - Mathematically, efficiency with production requires

    $$MRS_{1,2}^{i} = MRT_{1,2}^{j},$$

    the rate at which consumers are willing to trade goods coincides with the rate at which firms are capable of transforming one good into another.
  - First and Second Welfare Theorems also hold.

A Look at Behavioral Economics–Market Experiments

Market Experiments

- Several controlled experiments have tested than an equilibrium price ratio helps clear markets.
- Experiments construct a “double auction”, in which
  - every buyer is informed of her value for the object;
  - every seller is informed that her reservation price reflects the cost of producing the good.
- Then, simultaneously,
  - every seller is asked to announce a price for the good;
  - every buyer announces the price that she is willing to pay.

Market Experiments

- The experimenter
  - aggregates the WTP from all buyers to depict an approximated demand curve; and
  - aggregates the prices from all sellers to construct an approximated supply curve.
- Market price and quantity are determined finding the point where demand and supply curves cross each other.
- Experimental results converge relatively fast to theoretical predictions.

Appendix.
Efficient Allocations and MRS

Efficient Allocations and MRS

- We show that an efficient allocation in an economy with 2 consumers and 2 goods must satisfy

$$MRS^A_{1,2} = MRS^B_{1,2}.$$

- For an allocation $\boldsymbol{x}$ to be efficient, it must solve

$$\max_{\boldsymbol{x}} u^A(\boldsymbol{x})$$

subject to

$$u^B(\boldsymbol{x}) \geq \bar{u}^B$$

and

$$x^A_1 + x^B_1 \leq e^A_1 + e^B_1 \qquad \text{(Feasibility of good 1)}$$

$$x^A_2 + x^B_2 \leq e^A_2 + e^B_2 \qquad \text{(Feasibility of good 2)}$$

(Feasibility conditions say that, in aggregate, individuals do not consume more than initial endowments)

Efficient Allocations and MRS

- An allocation is efficient if it maximizes consumer A's utility without reducing the utility of consumer B below a certain cutoff level $\bar{u}^B$, while satisfying the feasibility condition for each good.
- The Lagrangian is

$$\mathcal{L} = u^A(\boldsymbol{x}) + \lambda[u^B(\boldsymbol{x}) - \bar{u}^B] + \mu_1[e_1^A + e_1^B - x_1^A - x_1^B] + \mu_2[e_2^A + e_2^B - x_2^A - x_2^B],$$

where λ denotes the Lagrange multiplier associated with the first constraint (the utility of consumer B cannot be below $\bar{u}^B$);

μ_1 represents the Lagrange multiplier associated with the feasibility constraint for good 1;

μ_2 is the Lagrange multiplier associated with the feasibility constraint for good 2.

Efficient Allocations and MRS

- Differentiating with respect to x_1^A and x_2^A,

$$\frac{\partial u^A(\boldsymbol{x})}{x_1^A} - \mu_1 = 0 \text{ and } \frac{\partial u^A(\boldsymbol{x})}{x_2^A} - \mu_2 = 0. \tag{9.5}$$

  in the case of interior solutions.

- Similarly, differentiating with respect to x_1^B and x_2^B,

$$\lambda\frac{\partial u^B(\boldsymbol{x})}{x_1^B} - \mu_1 = 0 \text{ and } \lambda\frac{\partial u^B(\boldsymbol{x})}{x_2^B} - \mu_2 = 0. \tag{9.6}$$

- Dividing the two equations that make up (9.5) yields

$$\frac{\frac{\partial u^A(\boldsymbol{x})}{x_1^A}}{\frac{\partial u^A(\boldsymbol{x})}{x_2^A}} = \frac{\mu_1}{\mu_2}, \tag{9.7}$$

Efficient Allocations and MRS

and dividing the two equations that make up (9.6) yields

$$\frac{\frac{\partial u^B(\boldsymbol{x})}{x_1^B}}{\frac{\partial u^B(\boldsymbol{x})}{x_2^B}} = \frac{\mu_1}{\mu_2}, \tag{9.8}$$

because λ cancels out.

- Because equations (9.7) and (9.8) are both equal to $\frac{\mu_1}{\mu_2}$,

$$\frac{\frac{\partial u^A(\boldsymbol{x})}{x_1^A}}{\frac{\partial u^A(\boldsymbol{x})}{x_2^A}} = \frac{\frac{\partial u^B(\boldsymbol{x})}{x_1^B}}{\frac{\partial u^B(\boldsymbol{x})}{x_2^B}} \quad \Rightarrow MRS_{1,2}^A = MRS_{1,2}^B.$$