Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 4:
Substitution and Income Effects

Outline

- Income Changes
- Price Changes
- Income and Substitution Effects
- Putting Income and Substitution Effects Together
- Appendix A. Not All Goods Can Be Inferior
- Appendix B. Alternative Representation of Income and Substitution Effects

Income Changes

Income Changes

- We analyze how the demand for a good (optimal consumption bundle) changes as the consumer's income increases.
- *Four ways* to measure a change in demand:
    1. Using the Derivative of Demand.
    2. Using Income Elasticity.
    3. Using the Income-Consumption Curve.
    4. Using the Engel Curve.

Income Changes

1. Using the Derivative of Demand.
   - Formally, $x(p_x, p_y, I)$ represents consumer demand for good x.
   - Normal goods. A consumer's demand for good x is normal if

$$\frac{\partial x(p_x, p_y, I)}{\partial I} > 0.$$

     - She demands more units of good x as her income increases. *Example*: holiday packages.
   - Inferior goods. A consumer's demand for good x is inferior if

$$\frac{\partial x(p_x, p_y, I)}{\partial I} < 0.$$

     - She cut her consumption as soon as she can afford to do so. *Example*: food staples.

Income Changes

- *Example 4.1: Increasing income in a Cobb-Douglas utility function*.
  - Consider an individual with $u(x, y) = xy$, who faces prices p_x, p_y, and income I.
  - Her optimal consumption for good x (i.e., her demand) is

$$x = \frac{I}{2p_x}.$$

  - This demand is increasing in income because $\frac{\partial x}{\partial I} = \frac{1}{2p_x} > 0.$
  - Hence good x is normal in consumption.
  - Similarly, the demand of good y, $y = \frac{I}{2p_y}$, is also increasing in income.

Income Changes

2. Using Income Elasticity.

- We can represent the relationship between income and demand by using the formula of income elasticity,

$$\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \underbrace{\frac{I}{x(p_x, p_y, I)}}_{>0},$$

which measures the % change in quantity demanded per 1% change in income.

  - $\varepsilon_{x,I} > 0$ when the good is *normal*, $\frac{\partial x(p_x,p_y,I)}{\partial I} > 0$.
  - $\varepsilon_{x,I} < 0$ when the good is *inferior*, $\frac{\partial x(p_x,p_y,I)}{\partial I} < 0$.

Income Changes

2. Using Income Elasticity (cont.).
    - A good with $\varepsilon_{x,I} > 1$, is regarded as *luxury*.
        - An 1% increase in income produces a more-than-proportional increase in the quantity demanded of the good.
        - *Example*: electronic gadgets, yachts.
    - A good with $0 < \varepsilon_{x,I} < 1$, is regarded as *necessity*.
        - A 1% increase in income yields a less-than-proportional increase in demand.
        - *Example*: water, electricity.
    - When $\varepsilon_{x,I} = 0$, the consumer purchases the same amount of the good regardless of her income.

Income Changes

2. Using Income Elasticity (cont.).

- *Summary*. Types of goods according to their income elasticity.

Income Elasticity, $\varepsilon_{x,I}$	Type of Good	Example
$\varepsilon_{x,I} < 0$	Inferior	Canned food
$0 < \varepsilon_{x,I} < 1$	Necessity	Water
$\varepsilon_{x,I} > 1$	Luxury	Yachts

Table 4.1

Income Changes

- *Example 4.2: Finding income elasticity in the Cobb-Douglas scenario*.
  - From example 4.1, the demand for good x is $x = \frac{I}{2p_x}$, and $\frac{\partial x}{\partial I} = \frac{1}{2p_x}$.
  - We can evaluate the income elasticity of good x as

$$\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)}$$

$$= \frac{1}{2p_x} \frac{I}{\frac{I}{2p_x}} = \frac{1}{2p_x} 2p_x = 1.$$

  - The good is normal ($\varepsilon_{x,I} > 0$), but it is neither a luxury (which requires $\varepsilon_{x,I} > 1$) nor a necessity (which needs $\varepsilon_{x,I} < 1$).

Income Changes

3. Using the Income-Consumption Curve.

- Depict the optimal consumption bundle at initial income I_1, Bundle A (where IC_1 is tangent to BL_1)
- When income increases, budget line shifts to BL_2, Bundle B is optimal.
- Income increases again producing BL_3, Bundle C is optimal.
- The "income-consumption curve" yields after connecting optimal consumption bundles.

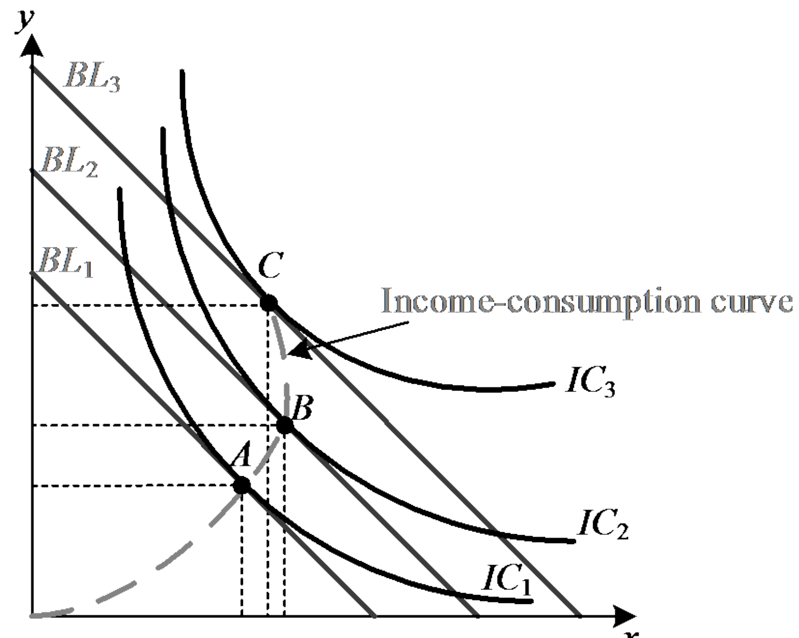

Figure 4.1

Income Changes

3. Using the Income-Consumption Curve (cont.).

- When the slope of the income-consumption curve is:
  - *Positive* (segment $A - B$), the consumer increases her purchases of both x and y → normal goods.
  - *Negative* (segment *B-C*), she decreases her purchases of x but increases purchases of y → one of the goods must be inferior.

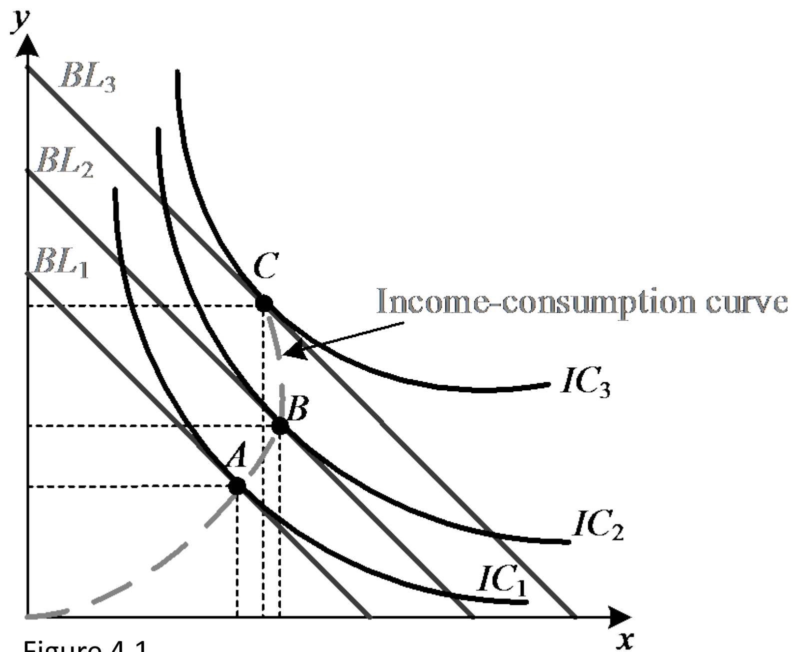

Figure 4.1

Income Changes

- *Example 4.3: Finding income-consumption curves*.
  - From example 4.1, the demand for good x is $x = \frac{I}{2p_x}$, and the demand of good y is $y = \frac{I}{2p_y}$.
  - The ratio of these demands is

$$\frac{y}{x} = \frac{\frac{1}{2p_y}}{\frac{1}{2p_x}} = \frac{p_x}{p_y}.$$

  which is the slope of the ray connecting the origin $(0,0)$ with any optimal consumption bundle.

  - *Example:* When $p_x = \$4$ and $p_y = \$2$, this ratio is $\frac{y}{x} = \frac{4}{2} = 2$.
    - The optimal consumption of goods y and x maintain a two-to-one relationship. Graphically, the income-consumption curve is a straight line.

Income Changes

4. Using the Engel Curve.

- The Engel curve represents how income affects the demand of a good by plotting the demand for the good on the vertical axis, and the income on the horizontal axis.

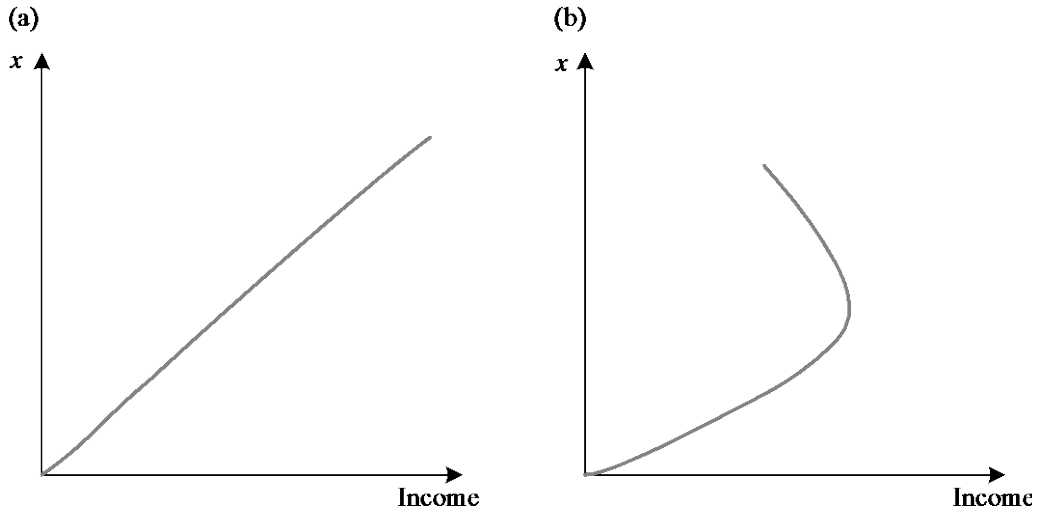

Figure 4.2

Income Changes

4. Using the Engel Curve (cont.).
   - Figure 4.2a depicts a positively sloped Engel curve, which implies the good is normal.
     - The number of units purchased increases with income.
     - *Example*: Products such as real state.

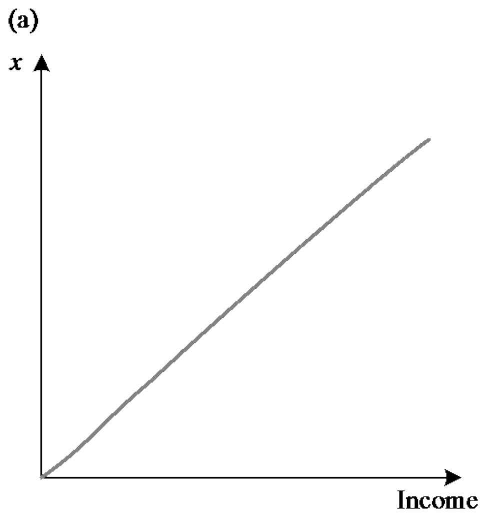

Figure 4.2a

Income Changes

4. Using the Engel Curve (cont.).
   - Figure 4.2b depicts an Engel curve that has a positive slope for low-income levels, but eventually becomes negatively sloped.
     - The good is normal when the individual is not very rich.
     - She starts regarding the good as inferior once she is sufficiently rich.
     - *Example*: Canned food or public transportation.

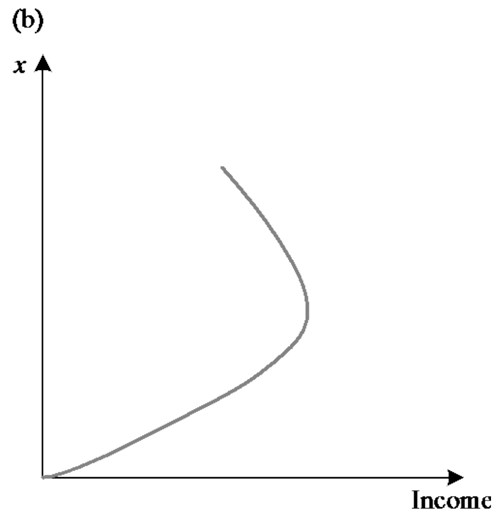

Figure 4.2b

Income Changes

- *Example 4.3: Finding Engel curves.*
  - From example 4.1, the demand for good x is $x = \frac{I}{2p_x}$.
  - Solving for I, we obtain an Engel curve of

$$I = (2p_x)x.$$

  - This Engel curve originates at zero, and has a slope of $2p_x$ (e.g., a slope of 6 if $p_x = \$3$).
  - This slope is positive and constant in x, indicating the consumer regards good x as normal (demand increases in income) for all income levels.

Income Changes

- Remark – Not All Goods Can be Inferior.
  - Figure 4.3 depicts an individual facing income I_1 at budget line BL_1.
  - When her income increases to I_2, budget line shifts to BL_2.

    Which bundle B does the consumer selects?

    - If B lies in region a, she increases consumption of x and y.
    - If B lies in region b, she purchase more of y (normal) but few of x (inferior).
    - If B lies in region c, she buys few of y (inferior) but more of x (normal).

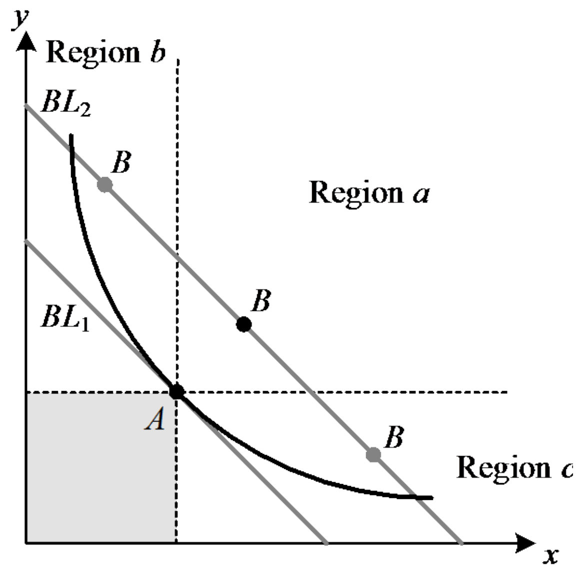

Figure 4.3

  - Hence, either both goods are normal, or only one of them is inferior.

Prices Changes

Price Changes

- We analyze how demand changes as the price of one good increases.
- *Three ways* to measure a change in price:
  1. Using the Derivative of Demand.
  2. Using Income Elasticity.
  3. Using the Income-Consumption Curve.

Price Changes

1. Using the Derivative of Demand.
   - Formally, $x(p_x, p_y, I)$ represent a consumer demand for good x.
   - Her demand curve for the good is negatively sloped if

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} < 0.$$

     - She purchase fewer units as the good becomes more expensive, keeping her income and price of all other goods constant.
   - Her demand curve is positively sloped if

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} > 0.$$

     - Quantity demanded and price go in the same direction. This types of goods are referred as "Giffen goods."

Price Changes

- *Example 4.5: Demand and price changes*.
  - From example 4.1, the demand for good x is $x = \frac{I}{2p_x}$.
  - If the price of good x increases by a small amount, the consumer's purchases respond as follows:

$$\frac{\partial x(p_x, p_y, I)}{\partial p_x} = -\frac{I}{2p_x^2} < 0,$$

    given that $p_x, I > 0$.
  - The demand function $x = \frac{I}{2p_x}$ decreases in price .
  - Graphically, this demand function has a negative slope.

Price Changes

2. Using the Price-Elasticity of Demand.

- We can represent the relationship between price of good x and its demand by using the formula of price-elasticity,

$$\varepsilon_{x,p_x} = \frac{\partial x(p_x, p_y, I)}{\partial p_x} \underbrace{\frac{p_x}{x(p_x, p_y, I)}}_{>0}.$$

- If we increase price p_x by 1%, quantity demanded changes by $\varepsilon_{x,p_x}\%$.

- For most goods, the demand function has a negative slope, entailing that the price-elasticity must be also negative.

Price Changes

- *Example 4.6: Price elasticity and demand*.
  - From example 4.1, the demand for good x is $x = \frac{I}{2p_x}$.
  - Using the formula of price elasticity,

$$\varepsilon_{x,p_x} = \frac{\partial x(p_x, p_y, I)}{\partial p_x} \frac{p_x}{x(p_x, p_y, I)}$$

$$= -\frac{I}{2p_x^2} \frac{p_x}{\frac{I}{2p_x}} = -1.$$

  - A 1% increase in price p_x produces a proportional reduction in its own demand (i.e., purchases of good x decrease by exactly 1%).

Price Changes

2. Using the Price-Elasticity of Demand (cont).

- The "cross-price elasticity" of demand for good y to p_x is,

$$\varepsilon_{y,p_x} = \frac{\partial y(p_x, p_y, I)}{\partial p_x} \frac{p_x}{y(p_x, p_y, I)}.$$

  - If we increase the price of good x by 1%, the quantity demanded of good y changes by changes by ε_{y,p_x}%.
  - In example 4.1, the demand for good y is $y = \frac{I}{2p_y}$, implying the demand of good y is independent of p_x.
  - Therefore, $\frac{\partial y(p_x,p_y,I)}{\partial p_x} = 0$, entailing $\varepsilon_{y,p_x} = 0$. A 1% increase in the price of good x does not affect the demand of good y.

Price Changes

3. Using the Price-Consumption Curve.

- Figure 4.4a illustrates a decrease in the price of good x, from $p_x = \$2$ in BL_1 to $p_x = \$1$ in BL_2, and to $p_x = \$0.5$ in BL_3.
- This figure also depicts the optimal consumption bundles the consumer selects at each price.
- The graph connects optimal bundles $A - C$ with a curve, which is referred as "price-consumption curve," which has a positive slope at all levels of price p_x.
  - *Example*: Good x is housing.

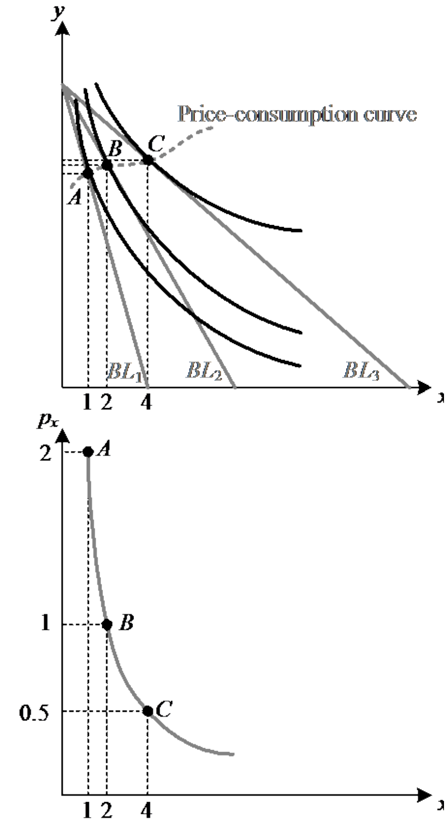

Figure 4.4a

Price Changes

3. Using the Price-Consumption Curve (cont.).

- Figure 4.4b illustrates a situation in which the individual also increases her consumption of x, the good that becomes cheaper.
- For good y,
  - she decreases her purchases when moving from bundle A to B (when p_x decreases from \$2 to \$1);
  - she increases her purchases afterwards (when p_x further decrease to \$0.5).

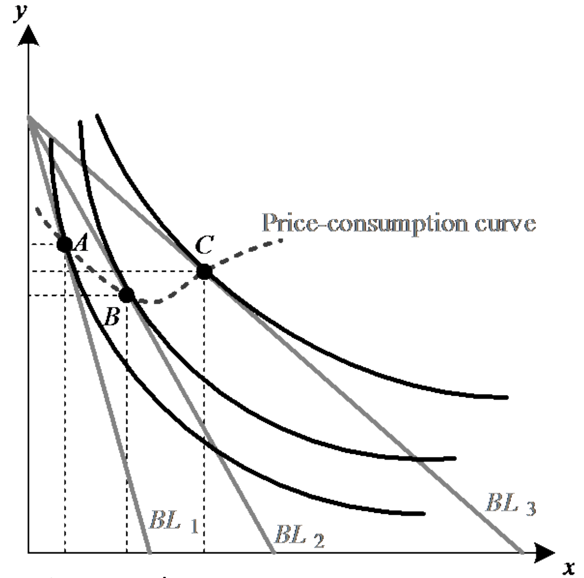

Figure 4.4b

Price Changes

- *Example 4.7: Finding price-consumption curves*.
  - From example 4.3, the demand for good x is $x = \frac{I}{2p_x}$, and for good y is $y = \frac{I}{2p_y}$.
  - The ratio of these demands is

$$\frac{y}{x} = \frac{\frac{I}{2p_y}}{\frac{I}{2p_x}} = \frac{p_x}{p_y},$$

  which gives the slope of the ray connecting the origin $(0,0)$ with any optimal consumption bundle.
    - An increase in p_x, increases the value of the ratio. The consumer moves to optimal bundles with more y and less x.
    - An increase in p_y, decreases the value of the ratio. The consumer purchases more of x but less of y.

Income and Substitution Effects

Income and Substitution Effects

- An increase in the quantity demanded after a price decrease produces 2 simultaneous effects:
  - Income effect.
  - Substitution effect.

Income Effect

- Income effect (IE). The change in the quantity demanded due to an *increase in purchasing power*, with the price of the item held constant.
  - A cheaper good x allows the individual to afford more units of all goods (i.e., larger purchasing power).
- An increase in income can induce the consumer to increase (decrease) her purchases of a good if she regards it as normal (inferior), allowing for positive (negative) income effects.

Substitution Effect

- Substitution effect (SE). The change in the quantity demanded due to a change in its price, holding the utility level constant.
  - The quantity change is due to a change in the relative price for the two goods, and not due an increase in the consumer's purchasing power.
- After a price decrease (increase), the substitution effect always lead to an increase (decrease) in the quantity demanded of the good that became relatively less (more) expensive.

Putting Income and Substitution Effects Together

Putting IE and SE Together

- IE and SE of a price decrease for normal goods.
  - When facing BL_1, the consumer selects A, where she reaches IC_1, purchasing x_A.
  - When the price of $x \downarrow$, the budget line pivots upward to BL_2, and she chooses bundle C with x_C.
  - The difference $x_C - x_A$, measures the total effect (TE).

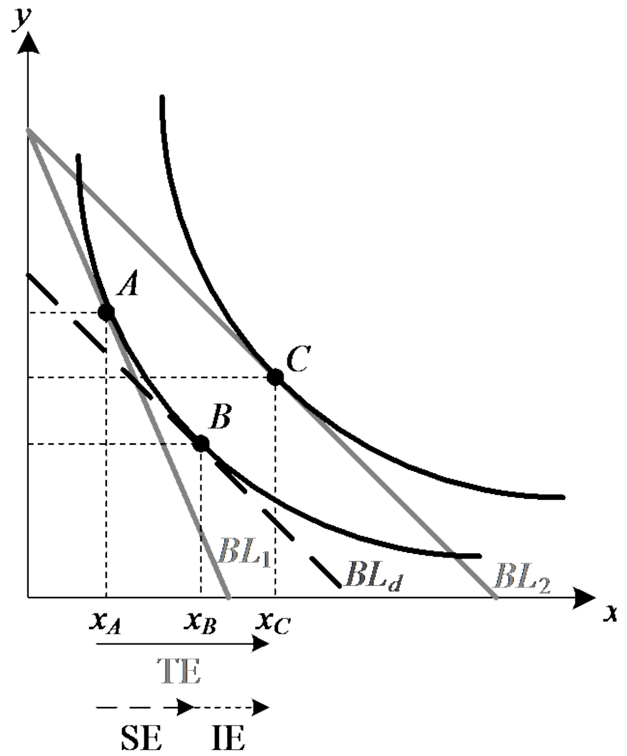

Figure 4.5

Putting IE and SE Together

- IE and SE of a price decrease for normal goods (cont.).
  - To separate TE into substitution effects (SE) and income effects (IE), we need to shift BL_2 downward (reducing her income) to make her reach the same utility level as before the price change.
  - The resulting BL_d is parallel to BL_2, having the final price ratio. And it is tangent to IC_1 at bundle B, where she purchases x_B.

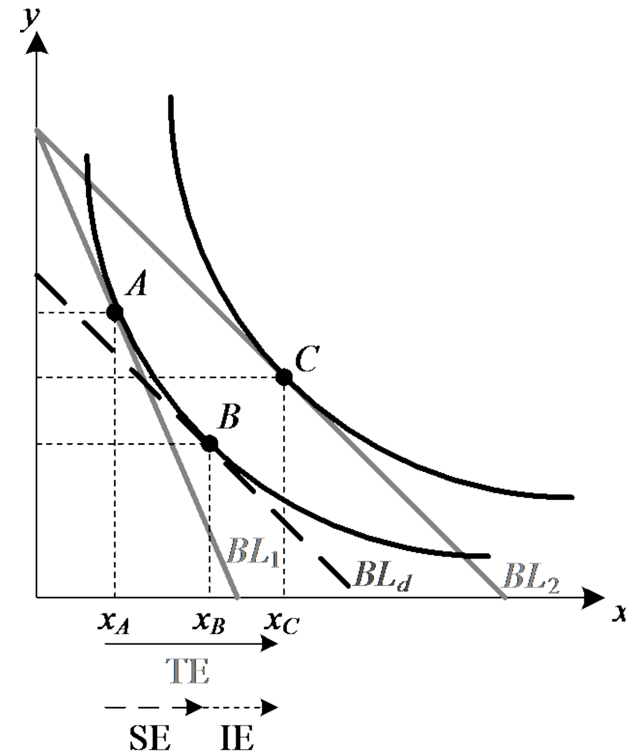

Figure 4.5

Putting IE and SE Together

- IE and SE of a price decrease for <u>normal</u> goods (cont.).

  1. The increase in consumption from x_A to x_B reflects the *substitution effect* (SE).
  2. The increase in consumption from x_B to x_C reflects the *income effect* (IE).
  3. The sum of SE and IE is the *total effect* (TE).

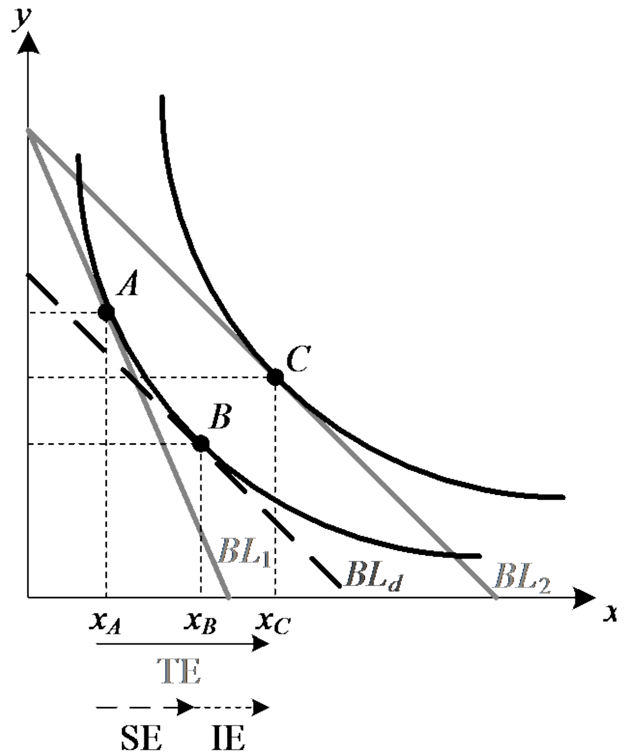

Figure 4.5

Putting IE and SE Together

- IE and SE when good x is regarded as inferior.
  - The income effect (IE) is negative, but it only partially offsets the substitution effect (SE), producing a positive total effect (TE).

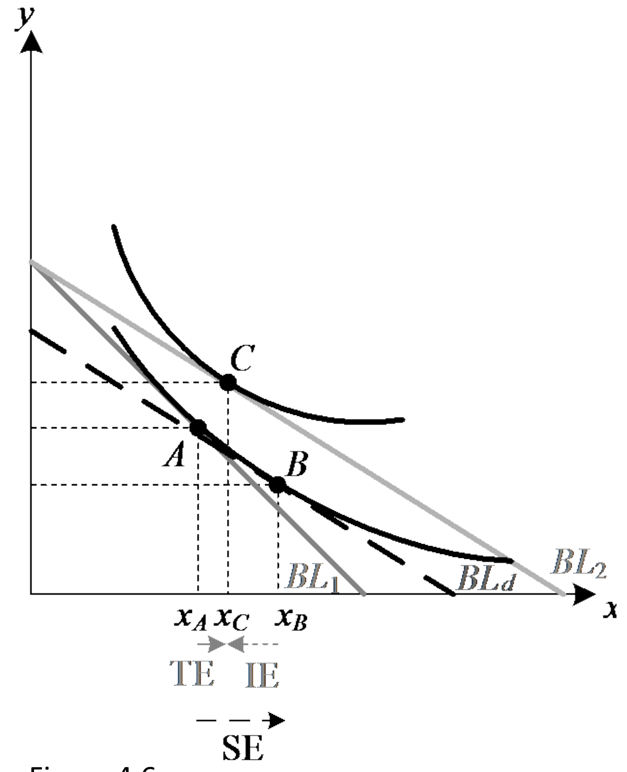

Figure 4.6a

Putting IE and SE Together

- IE and SE with a Giffen good.
  - The income effect (IE) is negative, but large enough to offset the substitution effect (SE) and produce a negative total effect (TE).

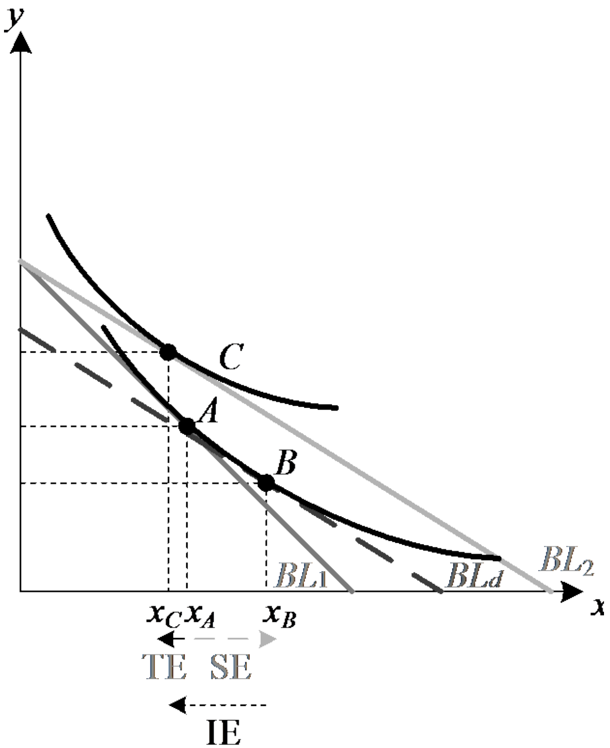

Figure 4.6b

Putting IE and SE Together

- Summary:

Table 4.2

Price Decrease				Price Increase			
Type of good	SE	IE	TE	Type of Good	SE	IE	TE
Normal	$+$	$+$	$+$	Normal	$-$	$-$	$-$
Inferior	$+$	$-$	$-$	Inferior	$-$	$+$	$+$
Giffen	$+$	$-$	$-$	Giffen	$-$	$+$	$+$

Income and Substitution Effects

- *Example 4.8*: *Finding IE and SE with a Cobb-Douglas utility function*.
  - Consider $u(x, y) = xy$, $I = \$100$, and $p_y = \$1$. And assume $p_x = \$3$ decreases to $p_x' = \$2$.
  - *Finding initial bundle A* (at $p_x = \$3$):
    - The tangency condition is
    $$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Longrightarrow \frac{y}{x} = \frac{3}{1} \quad \Longrightarrow y = 3x.$$
    - Inserting this result into the budget line, $3x + y = 100$,
    $$3x + 3x = 100 \Longrightarrow x = \frac{100}{6} \text{ units},$$
    $$y = 3\frac{100}{6} = 50 \text{ units}.$$
    - At $p_x = \$3$, optimal bundle is $A = \left(\frac{100}{6}, 50\right)$.

Income and Substitution Effects

- *Example 4.8* (continued):
  - *Finding final bundle C* (at $p_x' = \$2$):
    - The tangency condition is

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Longrightarrow \frac{y}{x} = \frac{2}{1} \quad \Longrightarrow y = 2x.$$

    - Inserting this result into the new budget line, $2x + y = 100$,

$$2x + 2x = 100 \Longrightarrow x = \frac{100}{4} = 25 \text{ units},$$

$$y = 2 \times 25 = 50 \text{ units}.$$

    - At $p_x' = \$2$, optimal bundle is $C = (25{,}50)$.
  - The total effect (TE) of the decrease in p_x is an increase of

$$TE = x_C - x_A = 25 - \frac{100}{6} \cong 8.3 \text{ units}.$$

Income and Substitution Effects

- *Example 4.8* (continued):
  - *Finding decomposition bundle B*:
    - Bundle B satisfies 2 conditions:

      1. At B the consumer reaches the same utility level as at A

         $$u\left(\frac{100}{6}, 50\right) = xy = \frac{100}{6} \times 50 \cong 833.3.$$

      2. The decomposition budget line BL_d, has the same slope as BL_2, and is tangent to the indifference curve. That is,

         $$\frac{MU_x}{MU_y} = \frac{p_x'}{p_y},$$

         $$\frac{y}{x} = \frac{2}{1} \quad \Longrightarrow y = 2x.$$

Income and Substitution Effects

- *Example 4.8* (continued):
  - *Finding decomposition bundle B* (cont.):
    - In summary, previous conditions state that,

$$xy = 833.3 \text{ and } y = 2x.$$

    - Inserting one equation into the other,

$$x(2x) = 833.3,$$
$$x^2 = 416.6,$$
$$\sqrt{x^2} = \sqrt{416.6},$$

$$y = 2 \times 20.4 = 40.8 \text{ units}.$$

    - Then, bundle B is $B = (20.4{,}40.8)$.

Income and Substitution Effects

- *Example 4.8* (continued):
  - The substitution effect of the decrease in p_x is

$$SE = x_A - x_B = 20.4 - \frac{100}{6} \cong 3.74 \text{ units.}$$

The consumer increases purchases of good x by 3.74 units only due to the lower price of this good, but still reaches the same utility level as before the price change.

  - The income effect of this price decrease is

$$IE = x_C - x_B = 25 - 20.4 = 4.6 \text{ units}$$

For a given price ratio, the consumer increases her consumption of good x by 4.6 units because a cheaper good x increases her purchasing power.

Income and Substitution Effects

- *Example 4.9*: *Finding IE and SE with quasilinear utility.*
  - Consider $u(x,y) = 2x^{1/2} + y$, $I = \$100$, and $p_y = \$1$. And assume $p_x = \$3$ decreases to $p_x' = \$2$ as in example 4.8.
  - *Finding initial bundle A* (at $p_x = \$3$):
    - The tangency condition $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ becomes

$$\frac{\frac{1}{x^{\frac{1}{2}}}}{1} = \frac{3}{1} \Longrightarrow \frac{1}{x^{\frac{1}{2}}} = \frac{3}{1},$$

$$\frac{1}{3} = x^{\frac{1}{2}} \Longrightarrow \left(\frac{1}{3}\right)^2 = \left(x^{1/2}\right)^2,$$

$$x = \frac{1}{9} \cong 0.11 \text{ units.}$$

Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding initial bundle A* (at $p_x = \$3$) (cont.):
    - Inserting this result into the budget line,

$$3x + y = 100,$$

$$(3 \times 0.11) + y = 100,$$

$$y = 100 - 0.33 \cong 99.67 \text{ units.}$$

    - At $p_x = \$3$, optimal bundle is $A = (0.11, 99.67)$.

Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding final bundle C* (at $p_x' = \$2$):
    - The tangency $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$ condition yields

$$\frac{\frac{1}{x^{\frac{1}{2}}}}{1} = \frac{2}{1} \quad \Longrightarrow \frac{1}{x^{\frac{1}{2}}} = 2$$

$$\frac{1}{2} = x^{1/2} \Longrightarrow \left(\frac{1}{2}\right)^2 = \left(x^{1/2}\right)^2,$$

$$x = \tfrac{1}{4} \cong 0.25 \text{ units.}$$

Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding final bundle C* (at $p_x' = \$2$) (cont.):
    - Inserting this result into the new budget line,

$$2x + y = 100,$$

$$(2 \times 0.25) + y = 100$$

$$y = 100 - 0.5 = 95.5 \text{ units.}$$

    - At $p_x' = \$2$, optimal bundle is $C = (0.25, 95.5)$.
  - The total effect of the decrease in p_x is an increase of

$$TE = x_C - x_A = 0.25 - 0.11 = 0.14 \text{ units.}$$

Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding decomposition bundle B*:
    1. At B the consumer reaches the same utility level as at A
       $u(0.11,99.67) = \left(2 \times 0.11^{1/2}\right) + 99.67 \cong 100.33.$
    2. The decomposition budget line BL_d, has the same slope as BL_2, and is tangent to the indifference curve. That is,

$$\frac{MU_x}{MU_y} = \frac{p'_x}{p_y},$$

$$\frac{\frac{1}{x^{1/2}}}{1} = \frac{2}{1} \Longrightarrow \frac{1}{x^{\frac{1}{2}}} = 2 \Longrightarrow \frac{1}{2} = x^{1/2},$$

$$\left(\frac{1}{2}\right)^2 = \left(x^{1/2}\right)^2 \Longrightarrow x = \frac{1}{4} \cong 0.25 \text{ units.}$$

Income and Substitution Effects

- *Example 4.9* (continued):
  - *Finding decomposition bundle B* (cont.):
    - In summary, previous conditions state that,

      $2x^{1/2} + y = 100.33$ and $x = 0.25$.

    - Inserting one equation into the other,

      $$\left(2 \times 0.25^{1/2}\right) + y = 100.33,$$
      $$(2 \times 0.5) + y = 100.33,$$

      $y = 100.33 - 1 = 99.33$ units.

    - Then, bundle B is $B = (0.25, 99.33)$.

Income and Substitution Effects

- *Example 4.9* (continued):
  - The substitution effect of the decrease in p_x is

    $$SE = x_A - x_B = 0.22 - 0.11 = 0.14 \text{ units.}$$

  - The income effect of this price decrease is

    $$IE = x_C - x_B = 0.25 - 0.25 = 0 \text{ units.}$$

  - $SE = TE$:
    - The consumer, after experiencing a cheaper good x, uses her increased purchasing power to buy more units of good y alone, rather than increasing purchases of good x.

IE and SE on the Labor Market

- We apply the analysis of IE and SE to the case of hours of leisure an individual enjoys, L.
- Because the day has only 24, the analysis of leisure choices allows us to examine it counterpart, working hours, H.

$$L + H = 24,$$
$$H = 24 - L.$$

IE and SE on the Labor Market

- Figure 4.7a represents an individual facing a salary of w per working hour.
  - Bl_1 originates at the horizontal intercept at $L = 4$h/day ($H = 0$).
  - At this point, $I = 0$. She cannot buy any unit of the composite good y, in the vertical axis.
  - If $H = 1$, I increases to w. If we works all day, her income is $24w$. She does not enjoy leisure but can purchase the largest amount of y.

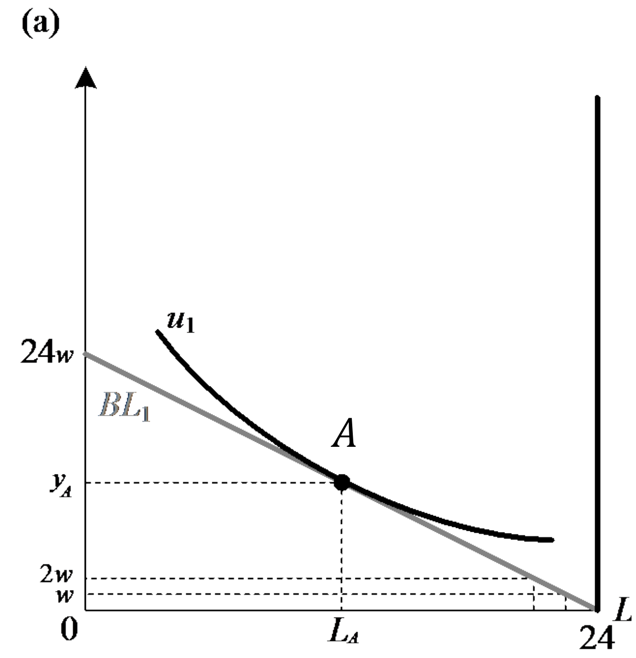

Figure 4.7a

IE and SE on the Labor Market

- Figure 4.7a represents an individual facing a salary of w per working hour.
  - The indifference curve moves northeast. Her utility increases as she enjoy more L and y.
  - At hourly wage of w, she chooses the optimal bundle A, in which BL_1 is tangent to her indifference curve u_1. She enjoys L_A hours of leisure and y_A goods.

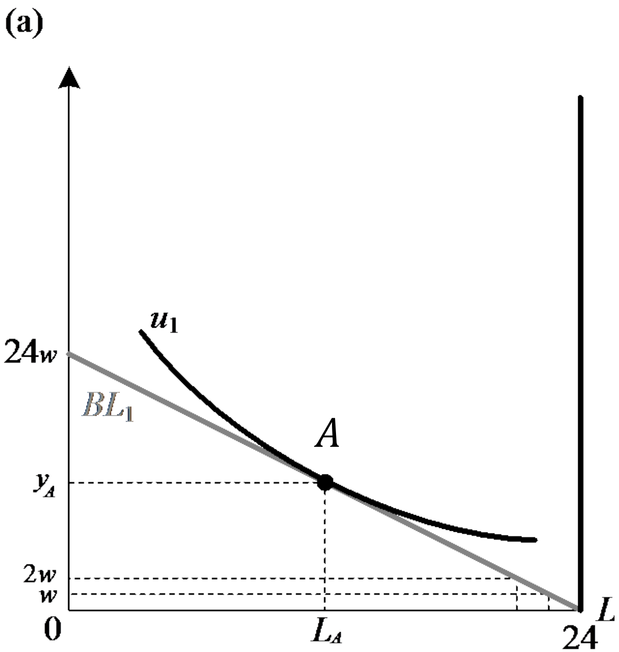

Figure 4.7a

IE and SE on the Labor Market

- Figure 4.7b depicts an increase in the worker's hourly salary from w to w'.
  - The new budget line BL_2 becomes steeper than BL_1.
  - Working 24h her income becomes $24w'$, which lies above BL_1 because $24w' > w$.
  - With this salary, she chooses a new optimal bundle C, where she enjoys L_C.
  - The total effect from the salary increase is $TE = L_C - L_A$.

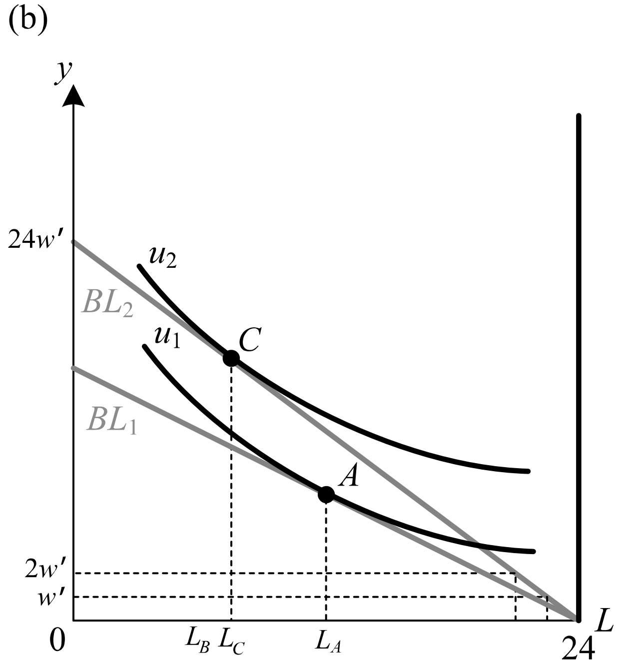

Figure 4.7b

IE and SE on the Labor Market

- Figure 4.8 decomposes total effect into substitution and income effects.
    - To examine SE and IE we find the decomposition budget line BL_D, which is tangent to initial indifference curve, u_1, at bundle B, where she enjoys L_B hours of leisure.

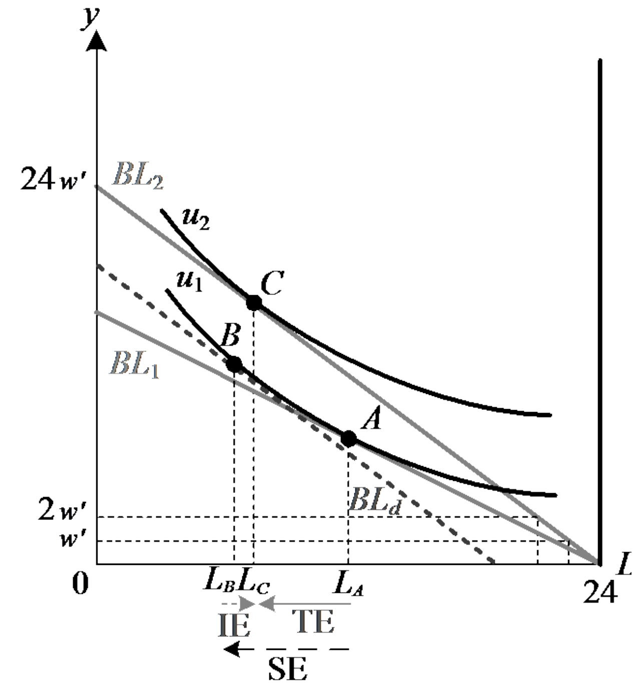

Figure 4.8

IE and SE on the Labor Market

- Figure 4.8 decomposes total effect into substitution and income effects.
  - The substitution effect is

    $$SE = L_A - L_B.$$

    A higher salary/h induces her to work more hours.
  - The income effect is

    $$IE = L_C - L_B.$$

    As she becomes richer, she can afford to work less and enjoy more leisure.

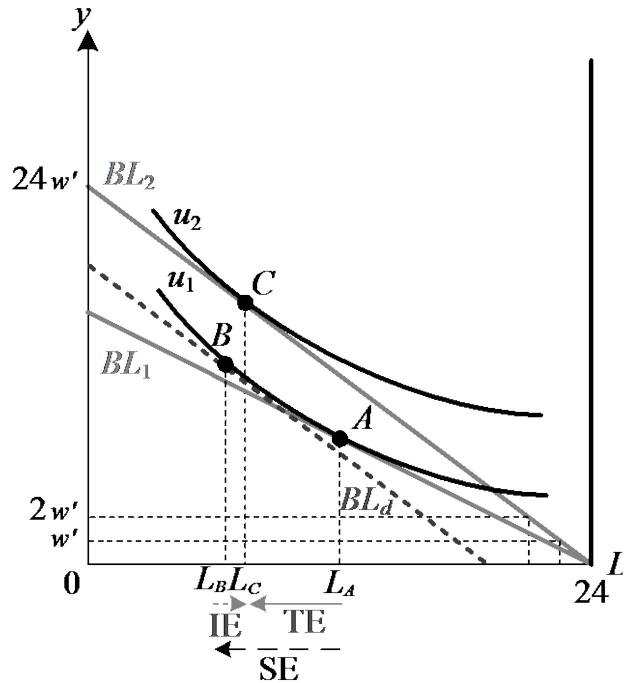

Figure 4.8

Appendix A.
Not All Goods Can be Inferior

Not All Goods Can Be Inferior

- We can use income elasticities to prove that NOT all goods can be inferior.
- When the consumer chooses her optimal bundle, this bundle must lie on the budget line.
- Formally,

$$p_x x(p_x, p_y, I) + p_y y(p_x, p_y, I) = I.$$

- We want to analyze the effect of an income change. We differentiate the budget line with respect to I:

$$p_x \frac{\partial x(p_x, p_y, I)}{\partial I} + p_y \frac{\partial y(p_x, p_y, I)}{\partial I} = 1.$$

Not All Goods Can Be Inferior

- We obtain the expression of income elasticity,
  - We multiply the first term by $\frac{x(p_x,p_y,I)\times I}{x(p_x,p_y,I)\times I}$;
  - We multiply the second term by $\frac{y(p_x,p_y,I)\times I}{y(p_x,p_y,I)\times I}$,

$$p_x \frac{\partial x(p_x,p_y,I)}{\partial I} \frac{x(p_x,p_y,I)}{x(p_x,p_y,I)} \frac{I}{I} + p_y \frac{\partial y(p_x,p_y,I)}{\partial I} \frac{y(p_x,p_y,I)}{y(p_x,p_y,I)} \frac{I}{I} = 1.$$

- Rearranging,

$$\underbrace{\frac{p_x x(p_x,p_y,I)}{I}}_{\theta_x} \underbrace{\frac{\partial x(p_x,p_y,I)}{\partial I} \frac{I}{x(p_x,p_y,I)}}_{\varepsilon_x, I} + \underbrace{\frac{p_y y(p_x,p_y,I)}{I}}_{\theta_y} \underbrace{\frac{\partial y(p_x,p_y,I)}{\partial I} \frac{I}{y(p_x,p_y,I)}}_{\varepsilon_y, I} = 1.$$

Not All Goods Can Be Inferior

- More compactly,

$$\theta_x \varepsilon_{x,I} + \theta_y \varepsilon_{y,I} = 1.$$

where $\theta_x = \frac{p_x x(p_x,p_y,I)}{I}$ is the budget share spent on good x (θ_x is a percentage, $0 \leq \theta_x \leq 1$)

$\varepsilon_{x,I} = \frac{\partial x(p_x,p_y,I)}{\partial I} \frac{I}{x(p_x,p_y,I)}$ is the definition of income elasticity.

- Can *all* goods be inferior? If it would be the case, $\varepsilon_{x,I} < 0$ and $\varepsilon_{y,I} < 0$,

$$\underbrace{\overbrace{\theta_x}^{+}\overbrace{\varepsilon_{x,I}}^{-}}_{-} + \underbrace{\overbrace{\theta_y}^{+}\overbrace{\varepsilon_{y,I}}^{-}}_{-} \neq 1.$$

NO! One of the goods must be normal.

Appendix B. Alternative Representation of IE and SE

Alternative IE and SE

- From the discussion of the utility maximization problem (UMP), and the expenditure minimization problem (EMP),
  - the demand function from the UMP is $x^U(p_x, p_Y, I)$,
  - which is evaluated at income level $I = e(p_x, p_y, \bar{u})$. This is the necessary income to purchase optimal bundle solving EMP.

Alternative IE and SE

- Therefore,

$$x^U\left(p_x, p_y, e(p_x, p_y, \bar{u})\right) = x^E(p_x, p_y, \bar{u}), \qquad (4.1)$$

where $e(p_x, p_y, \bar{u}) = p_x x^E(p_x, p_y, \bar{u}) + p_y y^E(p_x, p_y, \bar{u})$.

The optimal bundle solving the UMP (left side) coincides with the bundling solving the EMP (right side).

Alternative IE and SE

- Because the IE and SE measure how purchases of good x are affected by a change in its price, p_x, we differentiate equation (4.1) with respect to p_x,

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial e}\frac{\partial e}{\partial p_x} = \frac{\partial x^E}{\partial p_x}. \qquad (4.2)$$

Alternative IE and SE

- To understand the left side of equation (4.2), recall that price p_x shows up:
  - in the first and second argument of $x^U\left(p_x, p_y, e\left(p_x, p_y, \bar{u}\right)\right)$, meaning we need to differentiate separately each of them.
  - in $e\left(p_x, p_y, \bar{u}\right) = p_x x^E\left(p_x, p_y, \bar{u}\right) + p_y y^E\left(p_x, p_y, \bar{u}\right)$, meaning we need to apply the chain rule.

Alternative IE and SE

- Differentiating $e(p_x, p_y, \bar{u}) = p_x x^E(p_x, p_y, \bar{u}) + p_y y^E(p_x, p_y, \bar{u})$, with respect to with respect to p_x

$$\frac{\partial e}{\partial p_x} = x^E(p_x, p_y, \bar{u})$$

and $x^U\left(p_x, p_y, e(p_x, p_y, \bar{u})\right) = x^E(p_x, p_y, \bar{u})$.

Alternative IE and SE

- We can insert this result at the end of the left side of equation (4.2),

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial e} x^U = \frac{\partial x^E}{\partial p_x}. \tag{4.3}$$

- Finally, because $e\left(p_x, p_y, \bar{u}\right) = I$, we express equation (4.3) as

$$\frac{\partial x^U}{\partial p_x} + \frac{\partial x^U}{\partial I} x^U = \frac{\partial x^E}{\partial p_x}.$$

Alternative IE and SE

- Rearranging yields the so-called Slutsky equation:

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE},$$

The total effect of a decrease in p_x (measured by the effect on the demand function from solving UMP) is given by:

  - the substitution effect (as captured by the change in demand found solving the EMP)
  - and the income effect.

Alternative IE and SE

- The substitution effect is measured by $\frac{\partial x^E}{\partial p_x}$.
- A decrease in price p_x implies $\frac{\partial x^E}{\partial p_x} < 0$.
- This result does not rely on goods being normal or inferior.

Alternative IE and SE

- For the income effect, however, the sign depends on the goods being normal or inferior.

- When goods are normal $\frac{\partial x^U}{\partial I} > 0$), then

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE \text{ is } -} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE \text{ is } -} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE \text{ is } -}.$$

- When goods are inferior $\frac{\partial x^U}{\partial I} < 0$), then

$$\underbrace{\frac{\partial x^U}{\partial p_x}}_{TE \text{ is } ?} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE \text{ is } -} - \underbrace{\frac{\partial x^U}{\partial I} x^U}_{IE \text{ is } +}$$

Income and Substitution Effects

- *Example 4.10: Applying the Slutsky equation to the Cobb-Douglas case.*
  - Consider the Cobb-Douglas utility function from example 4.1.
  - After solving the UMP, we found $x^U(p_x, p_y, I) = \frac{I}{2p_x}$.
  - In that situation,

$$\frac{\partial x^U}{\partial p_x} = -\frac{I}{2(p_x)^2} \quad \text{and} \quad \frac{\partial x^U}{\partial I} = -\frac{1}{2p_x}$$

  - Applying the Slutsky equation,

$$\underbrace{-\frac{I}{2(p_x)^2}}_{TE} = \underbrace{\frac{\partial x^E}{\partial p_x}}_{SE} - \underbrace{\frac{1}{2p_x}\frac{I}{2p_x}}_{IE},$$

implying that SE is $\frac{\partial x^E}{\partial p_x} = -\frac{I}{4(p_x)^2}$.

Income and Substitution Effects

- *Example 4.10* (continued):
  - For instance, if $p_x = \$3$ and $I = \$100$:
    - SE is $-\frac{I}{4(p_x)^2} = -\frac{100}{4(3)^2} = -\frac{25}{9}$.
    - IE is $-\frac{1}{2p_x}\frac{I}{2p_x} = -\frac{1}{2\times3}\frac{100}{2\times3} = -\frac{25}{9}$.
  - The two effects reinforce each other, producing a TE of

$$-\frac{I}{2(p_x)^2} = -\frac{100}{2(3)^2} = -\frac{50}{9} \simeq 5.55 \text{ units.}$$

Income and Substitution Effects

- *Example 4.10* (continued):
  - This results implies that:
    - A marginal increase of the price of good x decreases the quantity demanded by 5.55 units,
    - where half of this decrease can be attributed to SE alone (change in price ratio).
    - The remaining half is explained by the IE (smaller purchasing power).

Using Elasticities to Represent the Slutsky Equation

- First, multiply the left and right sides by $\frac{p_x}{x^U}$,

$$\frac{\partial x^U}{\partial p_x}\frac{p_x}{x^U} = \frac{\partial x^E}{\partial p_x}\frac{p_x}{x^U} - \frac{\partial x^U}{\partial I}x^U\frac{p_x}{x^U}. \tag{4.4}$$

- Next, multiply the second term in right side of equation (4.4) by $\frac{I}{I} = 1.$
- And note that $x^U\left(p_x, p_y, e(p_x, p_y, \bar{u})\right) = x^E(p_x, p_y, \bar{u})$ in the first term of the right side.

Using Elasticities to Represent the Slutsky Equation

- Equation (4.4) becomes

$$\frac{\partial x^U}{\partial p_x}\frac{p_x}{x^U} = \frac{\partial x^E}{\partial p_x}\frac{p_x}{x^E} - \frac{\partial x^U}{\partial I}x^U\frac{p_x}{x^U}\frac{I}{I}, \tag{4.5}$$

where the left side corresponds to the definition of price elasticity, $\varepsilon_{x,p_x} = \frac{\partial x^U}{\partial p_x}\frac{p_x}{x^U}$; and the first term in the right side is $\varepsilon^E_{x,p_x} = \frac{\partial x^E}{\partial p_x}\frac{p_x}{x^E}$.

Using Elasticities to Represent the Slutsky Equation

- Furthermore, the second term on the right side can be arranged as:

$$\frac{\partial x^U}{\partial I} x^U \frac{p_x}{x^U} \frac{I}{I} = \underbrace{\frac{\partial x^U}{\partial I} \frac{I}{x^U}}_{\varepsilon_{x,I}} \underbrace{\frac{p_x x^U}{I}}_{\theta_x},$$

where $\varepsilon_{x,I} = \frac{\partial x(p_x, p_y, I)}{\partial I} \frac{I}{x(p_x, p_y, I)}$ represents the income-elasticity of demand, and $\theta_x = \frac{p_x x^U}{I}$ denotes the budget share that the individual spends on good x.

Using Elasticities to Represent the Slutsky Equation

- As a consequence, equation (4.5)

$$\underbrace{\frac{\partial x^U}{\partial p_x}\frac{p_x}{x^U}}_{\varepsilon_{x,p_x}} = \underbrace{\frac{\partial x^E}{\partial p_x}\frac{p_x}{x^E}}_{\varepsilon^E_{x,p_x}} - \underbrace{\frac{\partial x^U}{\partial I}\frac{I}{x^U}}_{\varepsilon_{x,I}}\underbrace{\frac{p_x x^U}{I}}_{\theta_x}, \qquad (4.6)$$

can be rewritten as

$$\varepsilon_{x,p_x} = \varepsilon^E_{x,p_x} - \theta_x \varepsilon_{x,I}.$$

Using Elasticities to Represent the Slutsky Equation

- Consider two extreme examples:
  1. *Demand for garlic*.

     The budget share of most consumers is negligible (i.e., $\theta_x \cong 0$), implying that equation (4.6) reduces to

     $$\varepsilon_{x,p_x} \cong \varepsilon^E_{x,p_x}.$$

     - The IE is close to zero, and hence, the SE coincides with TE.

Using Elasticities to Represent the Slutsky Equation

- Consider two extreme examples:

  2. *Demand for housing.*

     The budget share is much larger (i.e., $\theta_x = 0.3$). If we have estimates of $\varepsilon_{x,p_x} = -0.6$, and $\varepsilon_{I,p_x} = 1.3$, then we can find ε^E_{x,p_x} by using equation (4.6):

$$-0.6 = \varepsilon^E_{x,p_x} - (0.3 \times 1.3),$$
$$\varepsilon^E_{x,p_x} = -0.21.$$

  - A 1% increase in the price reduces demand by 0.6% if wealth is left unaffected.
  - However, if the consumer receives additional wealth (to maintain utility before the price change), demand reduces to 0.21%.