Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 16: Contract Theory

Outline

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Moral Hazard

Moral Hazard

- Moral hazard (or hidden action). A scenario in which an agent cannot observe the actions taken by other agents.
- *Examples*:
  - Health insurance companies cannot observe the actions that their clients take to maintain good health.
  - Car insurance companies cannot observe how careful drivers are, but hey can design insurance policies to give incentives to clients to be as careful as possible, such as
    - Progressive's "Snapshot" device monitors the insurance individual's driving behavior;
    - "pay-per-drive" insurance which provides discounts for low mileage.

Moral Hazard

- Consider you are hired by a small firm, which pays you $\$400$ a week to work 6 hours a day.
- If the contract does not specify a target outcome of your effort, *how much effort will you exert?*
- The firm does not monitor your work, and the contract sets a flat weekly pay.
- You may not work as much as you could every minute of the day.
- Workers may exert effort because of nonmonetary incentives.

Contracts When Effort is Observable

Contracts When Effort Is Observable

- The firm could specify a salary connected to the effort you exert.
  - Effort is difficult to measure for the firm.
- Alternatively, it could write a contract specifying that your pay will increase based on the output you produce.
  - Effort does not simply materialize into a constant amount of output because of random shocks (e.g., focus, sickness, sleep patterns, distractions with other co-workers).
  - Randomness between effort and output emerges both in manual and intellectual tasks and cannot be ignored by managers at the time of drafting contracts.

Contracts When Effort Is Observable

- *Example 16.1: Finding optimal contracts when effort is observable.*
  - Consider a worker with utility function $u(w) = \sqrt{w}$, where $w \geq 0$ denotes her salary.
  - The worker experiences disutility from exerting effort, e, measured by $g(e) = e$, and her reservation utility is $\bar{u} \geq 0$.
    - $\bar{u}$ captures the utility that she would obtain in an alternative job.
  - Assume $\bar{u} = 0$ and there are two efforts levels the worker can exert:
    - Hight effort, $e_H = 5$.
    - Low effort, $e_L = 0$.

Contracts When Effort Is Observable

- *Example 16.1* (continued):

Table 16.1 Probability of sales for each effort level

	\$0 in sales	\$100 in sales	\$400 in sales
High effort	0.1	0.3	0.6
Low Effort	0.6	0.3	0.1

- The expected sales when the worker exerts high effort is

$$(0.1 \times 0) + (0.3 \times 100) + (0.6 \times 400) = \$270,$$

  while expected sales when she exerts low effort are

$$(0.6 \times 0) + (0.3 \times 100) + (0.1 \times 400) = \$70.$$

- *How can the firm induce high or low effort from the worker?*

Contracts When Effort Is Observable

- *Example 16.1* (continued):
  - The worker accepts the high-effort contract if
$$u(w_H) - g(e_H) \geq \bar{u},$$
$$\sqrt{w_H} - 5 \geq 0,$$
$$\sqrt{w_H} \geq 5,$$
$$(\sqrt{w_H})^2 = 5^2 \quad \Longrightarrow w_H = \$25.$$
  - Because the firm seeks to pay the lowest possible salary, it will reduce w_H until $w_H = \$25$.
  - Similarly, the worker accepts the low effort contract if
$$u(w_L) - g(e_L) \geq \bar{u},$$
$$\sqrt{w_L} - 0 \geq 0,$$
$$w_L = \$0.$$

Contracts When Effort Is Observable

- *Example 16.1* (continued):
  - Comparing the firm's expected profits (measuring expected sales less salary),
    - With high effort (e_H), $\$270 - \$25 = \$245$.
    - With low effort (e_L), $\$70 - \$0 = \$70$.
  - Therefore, the firm offers a contract $(w_H, w_L) = (\$25, \$0)$, inducing the agent to exert a high effort level.

Contracts When Effort Is Observable

- Role of risk aversion.
  - In example 16.1, the worker is risk averse (her utility function $u(w) = \sqrt{w}$ is concave).
  - The firm is risk neutral (its profit function is linear in money).
  - The principal offers a contract that pays a generous salary when the worker exerts high effort but a lower payoff if she exerts low effort.
  - If the worker is less risk averse (e.g., $u(w) = w^{9/10}$), wages become less generous.
  - If the worker is more risk averse (e.g., $u(w) = w^{1/10}$) she needs a more generous compensation.

Contracts When Effort is Unobservable

Contracts When Effort Is Unobservable

- When the firm cannot observe the effort of the worker, it needs to provide her incentives to exert the amount of effort that maximizes profits.
- Under observable effort, inducing e_H gives rise to 2 effects:
  - Positive effect: It increases expected profits (higher outcomes are more likely)
  - Negative effect: It is more expensive to induce than e_L as e_H requires a higher salary.
- Assume the positive effects offset the negative effect.
- When effort is not observable, expected benefits from e_H are unaffected, while its expected costs go up.

Contracts When Effort Is Unobservable

- *Example 16.2: Finding optimal contracts when effort is unobservable.*
  - Consider example 16.1, but now effort is unobservable.

Table 16.2 Probability of high and low outputs for each effort level

	High output	Low output
High effort	0.6	0.4
Low Effort	0.1	0.9

Contracts When Effort Is Unobservable

- *Example 16.2* (continued):
  - Assuming the firm prefers to induce high effort, its problem is

$$\max_{w_H, w_L} \$270 - \underbrace{[0.6w_H + 0.4w_L]}_{\text{Expected labor cost}}$$

subject to

$$\underbrace{0.6\sqrt{w_H} + 0.4\sqrt{w_L}}_{\text{Expected utility from high effort}} - 5 \geq 0 \qquad \text{(PC)}$$

$$\underbrace{0.6\sqrt{w_H} + 0.4\sqrt{w_L}}_{\text{Expected utility from high effort}} - 5 \geq \underbrace{0.1\sqrt{w_H} + 0.9\sqrt{w_L}}_{\text{Expected utility from low effort}} - 0. \qquad \text{(IC)}$$

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - The "participation constraint" (PC) states that the worker prefers to exert high effort (obtaining $0.6\sqrt{w_H} + 0.4\sqrt{w_L}$, but suffering an effort cost of 5), than rejecting the contract (receiving a payoff of 0).
  - The "incentive constraint" (IC) indicates that the worker prefers to exert high than low effort.

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - In this context, IC holds with equality.
  - If IC did not hold, the firm could still reduce the salary offered to the worker when high (low) output is observed, increasing its profits.
  - Because IC holds,

$$0.6\sqrt{w_H} + 0.4\sqrt{w_L} - 5 = 0.1\sqrt{w_H} + 0.9\sqrt{w_L},$$

$$0.5\sqrt{w_H} - 0.5\sqrt{w_L} = 5.$$

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - Solving for w_H in the IC,

$$w_H = (\sqrt{w_L} + 10)^2.$$

  - Plugging this result everywhere we had w_H in the previous maximization problem,

$$\max_{w_L} \$270 - [0.6(\sqrt{w_L} + 10)^2 + 0.4w_L],$$

subject to $$0.6(\sqrt{w_L} + 10) + 0.4\sqrt{w_L} - 5 \geq 0 \qquad \text{(PC)}$$

  - A common approach of the problem is to ignore the PC and treat it as unconstraint maximization problem. And once we solve it, we need to check that our results satisfy the PC.

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - Differentiating the firm's objective function with respect to w_L,

$$\frac{\partial \pi}{\partial w_L} = -\left[\frac{0.6}{\sqrt{w_L}}(\sqrt{w_L} + 10) + 0.4\right]$$
$$= -0.6 - \frac{6}{\sqrt{w_L}} - 0.4$$
$$= -1 - \frac{6}{\sqrt{w_L}}.$$

    which is negative for all salaries w_L.
  - Therefore, the firm reduces this salary as much as possible, to $w_L^* = 0$.

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - The firm pays $w_L^* = 0$ after observing low output, and

    $$w_H^* = (\sqrt{w_L} + 10)^2 = (\sqrt{0} + 10)^2 = \$100,$$

    after observing high output.
  - Relative to the case where effort is observable, the firm still pays $w_L^* = 0$ after observing low output.
  - However, w_H increases from $\$24$ to $\$100$ when effort is unobservable.

Contracts When Effort Is Observable

- *Example 16.2* (continued):
  - We check that the PC holds with strict inequality because

$$0.6\left(\sqrt{0} + 10\right) + 0.4\sqrt{0} - 5 = 1 > 0.$$

  - When effort is observable, PC holds with equality, leaving the worker indifferent between accepting and rejecting the contract.

Contracts When Effort Is Unobservable

- Information rent. A utility gain that an agent enjoys when moving from a symmetric to an asymmetric context.
  - The worker's utility is larger when the firm cannot observe her effort than when the firm can observe it.

Preventing Moral Hazard

Preventing Moral Hazard

- Given the inefficiencies emerging under moral hazard, firms seek to observe the worker's effort.
- The firm manager may monitor the worker's effort.
- For monitoring to be effective:

  (1) Workers must know that monitoring may occur.

  (2) They must know when their effort will be monitored.

Adverse Selection

Adverse Selection

- Adverse selection (hidden information). A context where the agent cannot observe some private information of the other agent.
- *Examples*:
  - A buyer not observing a used car's quality.
  - A manager not observing a job applicant's ability.
  - An insurance company not being able to observe the risk of an insured party (an individual's health or driving ability).
- Lack of information could lead the uninformed party to make a wrong decision.

Markets for Lemons

Markets for Lemons

- When information asymmetries exit (i.e., buyers and sellers have access to different amounts of information) markets might fail.
- Akerlof (1970):
  - Consider a used-cars market, where quality is denoted by q.
  - Quality is a random variable whose realization is observed by the seller, but not by the buyer. Assume $q \sim U\left[0, \frac{3}{2}\right]$.
  - A car of quality q is valued as much for the buyer, and at discounted value $\frac{q}{3/2} = \frac{2}{3}q$ by the seller.
  - The buyer and the seller could find prices between $\frac{2}{3}q$ for which the trade makes both parties better off.

Markets for Lemons– Symmetric Information

- When both the seller and buyer observe the car's quality, q.
- The seller needs to charge a price p that maximizes her profits subject to guaranteeing this price is accepted by the buyer,

$$\max_p p - \frac{2}{3} q$$

$$\text{subject to } q - p \geq 0. \qquad \text{(PC)}$$

- The buyer's PC must hold with equality (i.e., $q - p = 0$ or $q = p$). Otherwise, the seller could charge a higher price, still accepted by the buyer. Inserting $q = p$ into the seller's PMP

$$\max_p p - \frac{2}{3} p = \frac{1}{3} p \, .$$

Differentiating with respect to p, we obtain $\frac{1}{3}$.

Markets for Lemons– Symmetric Information

- Since $\frac{1}{3} > 0$, a corner solution exists where the seller increase price p as much as possible
$$p = q.$$
- Therefore, the seller charges a price equal to the car's quality q, which in this scenario the buyer can perfectly observe.
- All car types are traded: from those with q close to zero (poor quality, or "lemons") to those with q close to $\frac{3}{2}$ (good quality, or "peaches").
- In summary, when both parties observe the car's quality, no market failure arise.

Markets for Lemons–Asymmetric Information

- Consider a context where the buyer is not able to observe the car's true quality.
- The buyer will accept a price p if she receives a positive expected utility, $E[q] - p \geq 0$.
- The car's expected utility, $E[q]$, can be found as follows:

$$E[q] = \frac{\frac{3}{2} + 0}{2} = \frac{3}{4},$$

because $q \sim U\left[0, \frac{3}{2}\right]$.

Markets for Lemons–Asymmetric Information

- The seller's problem is

$$\max_{p} \; p - \frac{2}{3}q$$

subject to

$$\frac{3}{4} - p \geq 0 \text{ or } p \leq \frac{3}{4}. \qquad \text{(PC)}$$

- The seller can raise the price p until the PC holds with equality, $p = \frac{3}{4}$.

Markets for Lemons–Asymmetric Information

- But we have then solved this problem:
  - The seller sets the highest acceptable price by the buyer, as any higher price yields a negative *expected* utility for the buyer.
- This price leads the seller to offer cars with quality q that satisfies

$$p - \frac{2}{3}q = \frac{3}{4} - \frac{2}{3}q \geq 0,$$

$$\frac{3}{4} \geq \frac{2}{3}q,$$

$$\frac{3/4}{2/3} = \frac{9}{8} \geq q.$$

Markets for Lemons–Asymmetric Information

- Offering cars with qualities above $\frac{9}{8}$ is unprofitable for the seller.
- Under symmetric information, the seller and the buyer can trade cars of *all quality* levels.
- However, under asymmetric information, only bad cars ("lemons") exist in this market.

Markets for Lemons– Asymmetric Information

- *Lemons in other markets:* Labor market.
  - Buyers of job services (firms) have access to less information than sellers of labors (job applicants).
  - A worker privately observes her productivity, θ, but firms do not.
  - Firms only offer a wage equal to the worker's expected productivity, $w = E[\theta]$.

Markets for Lemons– Asymmetric Information

- *Lemons in other markets:* Labor market (cont.).
  - This salary $w = E[\theta]$:
    - attracts only workers whose productivity lies below such a salary (i.e., $\theta \leq E[\theta]$),
    - but it does not attract workers with high productivity (i.e., $\theta > E[\theta]$).
  - Asymmetric information prevents the existence of a market of high-skilled workers.

Markets for Lemons–Asymmetric Information

- *Overcoming the lemon problem.*
  - Sellers try to overcome this market failure by offering warranties.
  - If sellers offer warranties when selling a high-quality car (peach) but not a low-quality car (lemon), the observation of the warranty signals its true quality to the buyer.
  - In this context, as operating in the symmetric information scenario, markets for both lemons and peaches exist.
  - More recent tools are CARFAX and certified preowned vehicles.

Principal-Agent Model

Principal-Agent Model

- Consider a scenario between a principal (firm) and an agent (worker).
- The principal's profits are given by

$$\pi(e) = \log(e) - w,$$

which is increasing in e, but a decreasing rate because $\log(e)$ is concave. And profits decrease in w.

Principal-Agent Model

- The agent's utility is
$$u(w, e) = w - \theta e^2,$$
which is increasing in w. The term θe^2 (worker's "cost effort") is increasing and convex in e, and increasing in θ.
- Parameter θ is either high, θ_H, or low, θ_L, where $\theta_H > \theta_L$.

Principal-Agent Model– Symmetric Information

- When the firm observes θ, it knows the cost θe that the worker incurs from exerting effort.
- The firm sets a salary w to solve

$$\max_{w,e} \log(e) - w$$

$$\text{subject to } w - \theta e^2 \geq 0. \qquad \text{(PC)}$$

- (PC) guarantees that the worker accepts the contract.
- The firm seeks to decreases w as much as possible, while still guaranteeing workers' acceptance. That is PC holds with equality,

$$w - \theta e^2 \geq 0 \quad \Longrightarrow w = \theta e^2.$$

- Inserting this result into the firm's PMP,

$$\max_{e} \log(e) - \theta e^2.$$

Principal-Agent Model–Symmetric Information

- Differentiating and solving with respect to e,

$$\frac{1}{e} - 2\theta e = 0,$$

$$\frac{1}{e} = 2\theta e,$$

$$\frac{1}{2\theta} = e^2 \quad \Longrightarrow e^{SI} = \left(\frac{1}{2\theta}\right)^{\frac{1}{2}}.$$

- Because $\theta_H > \theta_L$, efforts satisfy

$$e_H^{SI} = \left(\frac{1}{2\theta_H}\right)^{\frac{1}{2}} < \left(\frac{1}{2\theta_L}\right)^{\frac{1}{2}} = e_L^{SI},$$

implying that the high-cost worker exerts a lower effort than the low-cost worker.

Principal-Agent Model–Symmetric Information

- We find optimal wages using $w = \theta e^2$.
- When the firm observes θ_H, it offers a wage of

$$w_H^{SI} = \theta_H (e_H^{SI})^2 = \theta_H \times \left[\left(\frac{1}{2\theta_H}\right)^{\frac{1}{2}}\right]^2 = \theta_H \frac{1}{2\theta_H} = \$\frac{1}{2}.$$

- Similarly, when it observes θ_L, the firm offers a wage of

$$w_L^{SI} = \theta_L (e_L^{SI})^2 = \theta_L \times \left[\left(\frac{1}{2\theta_L}\right)^{\frac{1}{2}}\right]^2 = \theta_L \frac{1}{2\theta_L} = \$\frac{1}{2}.$$

- The firm pays the high worker the same salary as if she were a low-type worker, but this salary induces here to exert a lower effort level than the low-type worker.

Principal-Agent Model–Symmetric Information

- *Example 16.3: Principal-agent problem under symmetric information.*
  - Consider that $\theta_H = 2$ and $\theta_L = 1$.
  - Using the previous results, when the firm observers the worker's type to be $\theta_H = 2$, it requires an effort level of

$$e_H^{SI} = \left(\frac{1}{2\theta_H}\right)^{\frac{1}{2}} = \left(\frac{1}{2 \times 2}\right)^{\frac{1}{2}} = \frac{1}{2}.$$

  - When the firm observes $\theta_L = 1$, it requires an effort of

$$e_L^{SI} = \left(\frac{1}{2\theta_L}\right)^{\frac{1}{2}} = \left(\frac{1}{2 \times 1}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

  - The firm demands more effort from the worker with the lowest cost of effort, $e_L^{SI} > e_L^{SI}$. But pays it the same salary $\$\frac{1}{2}$.

Principal-Agent Model–Asymmetric Information

- The firm does not observe the worker's type θ, but it knows a proportion γ of workers is θ_H, while a proportion $1-\gamma$ is θ_L.
- The firm maximize its expected profits.
- Like in the symmetric information scenario,
  - both types of workers must be willing to work for the firm.
- Unlike the symmetric information scenario,
  - we must require that each type of worker prefers to choose the contract meant for her.

Principal-Agent Model–Asymmetric Information

- The firm solves the following PMP:

$$\max_{w_H, e_H, w_L, e_L} \underbrace{\gamma[\log(e_H) - w_H]}_{\text{If worker is high type}} + \underbrace{(1-\gamma)[\log(e_L) - w_L]}_{\text{If worker is low type}}$$

subject to

$$w_H - \theta_H e_H^2 \geq 0, \qquad (\text{PC}_\text{H})$$

$$w_L - \theta_L e_L^2 \geq 0, \qquad (\text{PC}_\text{L})$$

$$w_H - \theta_H e_H^2 \geq w_L - \theta_H e_L^2, \qquad (\text{IC}_\text{H})$$

$$w_L - \theta_L e_L^2 \geq w_H - \theta_L e_H^2. \qquad (\text{IC}_\text{L})$$

- The firm offers a menu of two contracts: (w_H, e_H) and (w_L, e_L).

Principal-Agent Model– Asymmetric Information

- The four constraints specify:
  - PC_H , the high-type worker finds her contract acceptable.
  - PC_L, the low-type worker finds her contract acceptable.
  - IC_H, the high-type worker prefers the contract meant for her.
  - IC_L, the low-type worker prefers the contract meant for her.
- PC constrains guarantee the voluntary participation of all types of workers, while IC constraints ensure self-selection

Principal-Agent Model–Asymmetric Information

- The PC of the least efficient agent (PC_H in this context), and the IC of the most efficient agent (IC_L in this case) hold with equality,

$$w_H - \theta_H e_H^2 = 0,$$
$$w_L - \theta_L e_L^2 = w_H - \theta_L e_H^2.$$

- Inserting the binding PC_H into the binding IC_L,

$$w_L - \theta_L e_L^2 = \underbrace{\theta_H e_H^2}_{w_H = \theta_H e_H^2 \text{ from } PC_H} - \theta_L e_H^2,$$

$$w_L = \theta_H e_H^2 + \theta_L (e_L^2 - e_H^2).$$

Principal-Agent Model–Asymmetric Information

- Inserting

$$w_H = \theta_H e_H^2,$$

$$w_L = \theta_H e_H^2 + \theta_L (e_L^2 - e_H^2).$$

into the PMP, we obtain:

$$\max_{e_H, e_L} \gamma[\log(e_H) - \underbrace{\theta_H e_H^2}_{w_H}] + (1-\gamma)\left[\log(e_L) - \underbrace{[\theta_H e_H^2 + \theta_L (e_L^2 - e_H^2)]}_{w_L}\right].$$

Principal-Agent Model–Asymmetric Information

- Differentiating with respect to e_L,

$$(1-\gamma)\left(\frac{1}{e_L}-2\theta_L e_L\right)=0, \qquad (\mathrm{FOC}_{E_L})$$

Rearranging and solving for e_L, we find that

$$e_L^{AI}=\left(\frac{1}{2\theta_L}\right)^{\frac{1}{2}}.$$

Principal-Agent Model–Asymmetric Information

- Differentiating with respect to e_H,

$$\gamma\left(\frac{1}{e_H} - 2\theta_H e_H\right) - 2e_H(1-\gamma)(\theta_H - \theta_L) = 0, \qquad (\text{FOC}_{E_H})$$

Rearranging and solving for e_H, we obtain

$$e_H^{AI} = \left(\frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]}\right)^{\frac{1}{2}}.$$

Principal-Agent Model– Asymmetric Information

- Using the expression for the binding IC$_L$, $w_L = \theta_H e_H^2 + \theta_L(e_L^2 - e_H^2)$, we find the wage for the low-type worker,

$$w_L^{AI} = \theta_H \frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]} + \theta_L \left(\frac{1}{2\theta_l} - \frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]} \right)$$

$$= (\theta_H - \theta_L) \frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]} + \theta_L \frac{1}{2\theta_L}$$

$$= \frac{(1+\gamma)\theta_H - \theta_L}{2[\theta_H - (1-\gamma)\theta_L]}.$$

Principal-Agent Model–Asymmetric Information

- The wage of the high-type is found using the binding PC_{H}, $w_H = \theta_H e_H^2$,

$$w_H^{AI} = \theta_H \left(e_H^{AI}\right)^2$$
$$= \theta_H \frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]}.$$

Principal-Agent Model– Asymmetric Information

- *Example 16.4: Principal-agent problem under asymmetric information.*
  - Let us continue with example 16.3, where we had that $\theta_L = 1$ and $\theta_H = 2$.
  - And assume the probability of high-cost workers in the pool of workers is $\gamma = \frac{1}{3}$.

Principal-Agent Model–Asymmetric Information

- *Example 16.4* (continued):
  - The optimal effort levels are:

$$e_L^{AI} = \left(\frac{1}{2\theta_L}\right)^{\frac{1}{2}} = \left(\frac{1}{2 \times 1}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}},$$

$$e_H^{AI} = \left(\frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]}\right)^{\frac{1}{2}} = \left(\frac{\frac{1}{3}}{2[2 - (1 - \frac{1}{3}) \times 1]}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{8}}.$$

Principal-Agent Model– Asymmetric Information

- *Example 16.4* (continued):
  - The optimal wages are:

$$w_L^{AI} = \frac{(1+\gamma)\theta_H - \theta_L}{2[\theta_H - (1-\gamma)\theta_L]} = \frac{\left(1+\frac{1}{3}\right)\times 2 - 1}{2[2-(1-\frac{1}{3})\times 1]} = \$\frac{5}{8}.$$

$$w_H^{AI} = \theta_H \frac{\gamma}{2[\theta_H - (1-\gamma)\theta_L]} = 2\frac{\frac{1}{3}}{2[2-(1-\frac{1}{3})\times 1]} = \$\frac{1}{4}.$$

Principal-Agent Model–Comparing Information Settings

- The introduction of asymmetric information entails no changes in effort for the worker with low cost of effort,
$$e_L^{SI} = e_L^{AI} = \frac{1}{\sqrt{2}}.$$
  - This is known as "no distortion at the top".
- However, the high-cost worker exerts less effort when the firm is uninformed about her type,
$$e_H^{AI} = \frac{1}{\sqrt{8}} < \frac{1}{2} = e_H^{SI}.$$
- Salaries are, in contrast, higher for the low-cost worker (the efficient worker) but lower for the high-cost (inefficient) worker,
$$w_L^{AI} = \$\frac{5}{8} > \$\frac{1}{2} = w_L^{SI}$$
$$w_H^{AI} = \$\frac{1}{4} < \$\frac{1}{2} = w_H^{SI}.$$

Principal-Agent Model–Comparing Information Settings

- The efficient worker earns a positive information rent under asymmetric information because she obtains a larger wage exerting the same level of effort,

$$u_L^{AI} = w_L^{AI} - \theta_L \left(e_L^{AI}\right)^2 = \frac{5}{8} - \left(1 \times \frac{1}{2}\right) = \frac{1}{8},$$

$$u_L^{SI} = w_L^{SI} - \theta_L \left(e_L^{SI}\right)^2 = \frac{1}{2} - \left(1 \times \frac{1}{2}\right) = 0.$$

- The inefficient worker is left with zero utility (no information rent) in both symmetric and asymmetric information contexts.

$$u_H^{AI} = w_H^{AI} - \theta_H \left(e_H^{AI}\right)^2 = \frac{1}{4} - \left(2 \times \frac{1}{8}\right) = 0,$$

$$u_H^{SI} = w_H^{SI} - \theta_H \left(e_H^{SI}\right)^2 = \frac{1}{2} - \left(2 \times \frac{1}{4}\right) = 0.$$

Preventing Adverse Selection

Preventing Adverse Selection

- *Screening*.
  - A menu (or list) of contracts–one for individuals with high risk and another for individuals with low risk– and each type has incentives to select the contract meant for her.
  - They work as "screening devices" to identify which individuals are more or less risky. By selecting the contract each individual prefers, she is revealing her riskiness to the company.

Preventing Adverse Selection

- *Signaling*.
  - The informed party (worker) does something costly to signal her type to the uninformed party (firm).
  - In the principal-agent model, signaling can occur only if the worker has the ability to send a signal before the firm offers the contract.
  - *Example*: Role of education as signaling device.
    - Consider a worker has an undergraduate degree
    - She has two available options: earn a master's degree or not. This degree does not change her productivity.
    - An efficient worker (θ_L) suffers a cost of $\$100$ from earing the degree.
    - An inefficient worker (θ_H) suffers a cost of $\$300$.

Preventing Adverse Selection

- *Signaling*.
  - *Example*: Role of education as signaling device (cont.)
    - The cost difference reflects that the efficient worker can finish her coursework faster, reducing tuition costs and opportunity costs.
    - The firm is uninformed about the worker's efficiency. Observing a master's degree signals that the worker is more likely to be efficient.
  - Education can work as signal that the informed agent (worker) uses to convey information to the uninformed agent (firm).
  - However, it is costly, giving rise to inefficiencies relative to the complete information scenario.

Preventing Adverse Selection

- *Legal rules*.
  - Most countries provide buyers with rights that can help ameliorate adverse selection problems.
  - *Example*: Laws requiring the seller to replace the object if it breaks down during a certain period after the purchase.
    - Known as "Implied Warranties."

Appendix:
Showing that PC_H and IC_L Hold with Equality

PH_C and IC_L Hold with Equality

- *PC_L is slack.*
  - The incentive compatibility condition of the low-cost worker, IC_L, is $w_L - \theta_L e_L^2 \geq w_H - \theta_L e_H^2$. Because $\theta_H > \theta_L$ by definition,

$$w_L - \theta_L e_L^2 \geq w_H - \theta_L e_H^2 \underbrace{>}_{\text{By } \theta_H > \theta_L} w_H - \theta_L e_H^2 \geq 0$$

  - Combining the first and last elements of the inequality,

$$w_L - \theta_L e_L^2 > 0.$$

  - This result coincides with the PC_L. We just showed that PC_L holds with strict inequality ($>$) rather than with a weak inequality ($\geq$).

PH_C and IC_L Hold with Equality

- *IC_L binds.*
  - The incentive compatibility condition of the low-cost worker, IC_L, must hold with equality (i.e., it must bind).
  - Otherwise, the principal could reduce the wage that it offers to the low-cost worker, still inducing her to take the contract meant for her.
  - Therefore, IC_L holds with equality, implying

$$w_L - \theta_L e_L^2 = w_H - \theta_L e_H^2,$$

$$w_L = w_H + \theta_L (e_L^2 - e_H^2).$$

PH_C and IC_L Hold with Equality

- *IC_H is slack.*
  - The incentive compatibility condition of the high-cost worker, IC_H, says that
$$w_H - \theta_H e_H^2 \geq w_L - \theta_H e_L^2.$$
  - Using the binding IC_L, $w_L = w_H + \theta_L(e_L^2 - e_H^2)$,
$$w_H - \theta_H e_H^2 \geq [w_H + \theta_L(e_L^2 - e_H^2)] - \theta_H e_L^2,$$
$$\theta_H(e_L^2 - e_H^2) - \theta_L(e_L^2 - e_H^2) \geq 0,$$
$$(\theta_H - \theta_L)(e_L^2 - e_H^2) \geq 0,$$
which is strictly positive because $\theta_H > \theta_L$ and if $e_L > e_H$.
  - Therefore, IC_H holds with strict inequality. The higher-cost agent has no incentives to take the contract for the low-cost. Doing so would entail a loss.

PH_C and IC_L Hold with Equality

- *PC_H binds.*
  - The participation constraint of the high-cost worker, PC_H, holds with equality (i.e., it must bind).
  - Otherwise, the firm can still reduce the wage offered to the high-cost worker, while inducing her to take the contract meant for her.