Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 14:

Imperfect Competition

Outline

- Summary of Market Structures
- Measuring Market Power
- Models of Imperfect Competition
  - Cournot Model – Simultaneous Quantity Competition
  - Bertrand Model – Simultaneous Price Competition
  - Cartels and Collusion
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Summary of Market Structures

Industry	N of firms	Type of Good	Price-takers?	Entry Barriers?
Perfect competition	Many	Homogeneous	Yes	No
Monopoly	One	No close substitutes	No	Yes
Oligopoly	Some	Homogeneous or heterogeneous	No	Yes

Measuring Market Power

Measuring Market Power

- A common measure of market power is the number of firms in an industry, $N \geq 1$.
  - It does not inform about market shares.
  - *Example*:
    - Consider two industries, A and B, with $N = 3$ firms each.
    - In industry A, one of the firms enjoys a 98% market share.
    - In industry B, market share is evenly distributed, each firm holds 33.33%.
- The Herfindahl-Hirschman index (HHI) of market concentration accounts for both the number of firms and their market shares.

Measuring Market Power

- The HHI is given by

$$HHI = (s_1)^2 + (s_2)^2 + \cdots (s_N)^2,$$

  where s_1 represents the market share of firm 1 (in %), s_2 is that of firm 2, and similarly for all remaining N firms in the industry.

- In a monopoly, in which a single firm captures the entire market share, $s_1 = 100$,

$$HHI = (100)^2 = 10{,}000.$$

- In a duopoly, with two firms evenly sharing market power,

$$HHI = 2(50)^2 = 2(2{,}500) = 5{,}000.$$

Measuring Market Power

- In an oligopoly, with $1{,}000$ firms, each capturing $\frac{1}{1{,}000}$ of the market share,

$$HHI = \left(\frac{100}{1{,}000}\right)^2 + \left(\frac{100}{1{,}000}\right)^2 + \cdots + \left(\frac{100}{1{,}000}\right)^2$$

$$= 1{,}000\left(\frac{100}{1{,}000}\right)^2 = 10.$$

- Generally, in an industry with $N \geq 1$, with $s_i = \frac{1}{N}$,

$$HHI = \left(\frac{100}{N}\right)^2 + \left(\frac{100}{N}\right)^2 + \cdots + \left(\frac{100}{N}\right)^2$$

$$= N\left(\frac{100}{N}\right)^2 = \frac{10{,}000}{N},$$

which converges to zero when N is sufficiently large.

Measuring Market Power

- The HHI ranges from $10{,}000$ to 0.
  - A high HHI arises in highly concentrated industries.
  - A low HHI emerges when market power is more evenly distributed.
- *Examples*:
  - US light bulb market, with around 57 firms,
    - $HHI = 2{,}757$. Some of these firms enjoy a large market share.
  - Glass container manufacturing, with 22 firms,
    - $HHI = 2{,}582$. Market shares are more evenly split among firms (i.e., the market is less concentrated).

Models of Imperfect Competition

Models of Imperfect Competition

- Consider a market with $N \geq 2$ firms, all of them selling a relatively homogeneous product (e.g., brands of unflavored water).
- In this scenario, we consider three models of firm competition:
  (1) Cournot model of simultaneous quantity competition.
  (2) Bertrand model of simultaneous price competition.
  (3) Stackelberg model of sequential quantity competition.

Cournot Model– Simultaneous Quantity Competition

Cournot Model

- Consider an industry with $N = 2$ firms selling a homogeneous product.
- Every firm independently and simultaneously chooses its profit maximizing output (q_1 for firm 1 and q_2 for firm 2).
- The market price is determined by inserting q_1 and q_2 into the inverse demand function $p(q_1, q_2)$. Assume this function is linear, $p(q_1, q_2) = a - b(q_1 + q_2)$, where $a, b > 0$.
- Firm 1's total cost function is $TC_1(q_1) = cq_1$, where $c > 0$.
- Firm 2's total cost function is symmetric, $TC_2(q_2) = cq_2$.

Cournot Model

Firm 1. Its PMP is to choose q_1 to solve

$$\max_{q_1} \pi_1 = TR_1 - TC_1 = \underbrace{p(q_1, q_2)q_1}_{TR_1} - \underbrace{cq_1}_{TC_1}$$

$$= [a - b(q_1 + q_2)]q_1 - cq_1,$$

where $TR_1 = p(q_1, q_2)q_1$ denotes total revenue (price per units sold), and $TC_1 = cq_1$ is its total cost.

- To maximize its profits, firm 1 differentiate this expression with respect to q_1,

$$\frac{\partial \pi_1}{q_1} = a - 2bq_1 - bq_2 - c = 0.$$

Rearranging and solving for q_1,

Cournot Model

$$a - c - bq_2 = 2bq_1,$$
$$q_1(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2, \qquad (BRF_1)$$

which is referred to as firm 1's "best response function."

- The best response function describes the profit maximizing output that firm 1 chooses as a response to each of the output levels that firm 2 selects.
  - If $a = 10$, $b = 1$, and $c = 2$, firm 1's best response function becomes
  $$q_1(q_2) = \frac{10-2}{2 \times 1} - \frac{1}{2}q_2 = 4 - \frac{1}{2}q_2.$$
  - If firm 2 produces $q_2 = 3$ units, firm 1 responds with
  $$q_1(2) = 4 - \frac{1}{2}2 = 2.5 \text{ units.}$$

Cournot Model

- Firm 1's best response function, $q_1(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_2$.
  - It originates at $\frac{a-c}{2b}$ on the vertical axis when firm 2 chooses $q_2 = 0$.
  - It decreases with a slope of $-1/2$ for every unit of q_2.
  - When $q_1\left(\frac{a-c}{b}\right) = \frac{a-c}{2b} - \frac{1}{2}\frac{a-c}{2b}$
    $= 0$ units.

    As firm 2 increases q_2, firm 1 is left with a smaller residual demand to serve.

    When $q_2 \geq \frac{a-c}{b}$, firm 1 shut down, producing $q_1 = 0$.

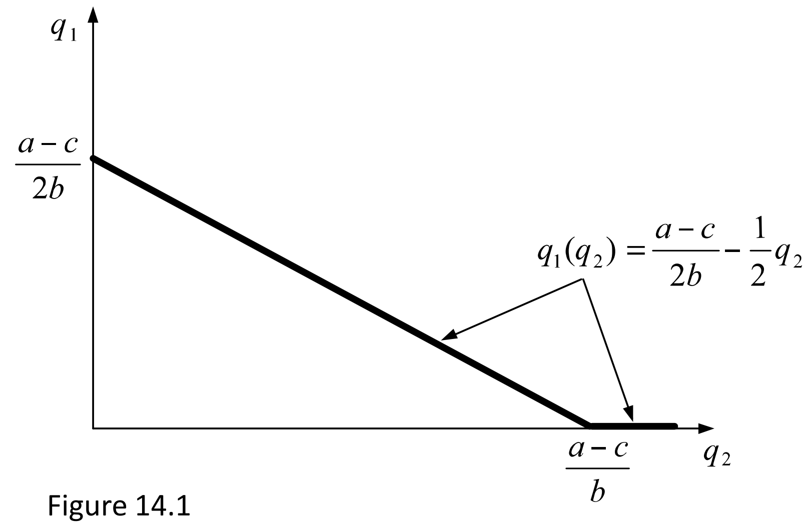

Figure 14.1

Cournot Model

Firm 2. A similar argument applies to firm 2, which solves

$$\max_{q_2} \pi_2 = TR_2 - TC_2 = \underbrace{p(q_1, q_2)q_2}_{TR_2} - \underbrace{cq_2}_{TC_2}$$

$$= [a - b(q_1 + q_2)]q_2 - cq_2.$$

- Differentiating with respect to q_2,

$$\frac{\partial \pi_2}{q_2} = a - bq_1 - 2bq_2 - c = 0.$$

  Rearranging and solving for q_2, we find firm 2's best response function,

$$a - c - bq_1 = 2bq_2,$$

$$q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1. \qquad (BRF_2)$$

Cournot Model

- Firm 2's best response function, $q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1$, is symmetric to that of firm 1 because both face the same demand and costs.
  - It originates at $\frac{a-c}{2b}$ when firm 1 is inactive but it decreases at a rate of $1/2$ as firm 1 increases its production.

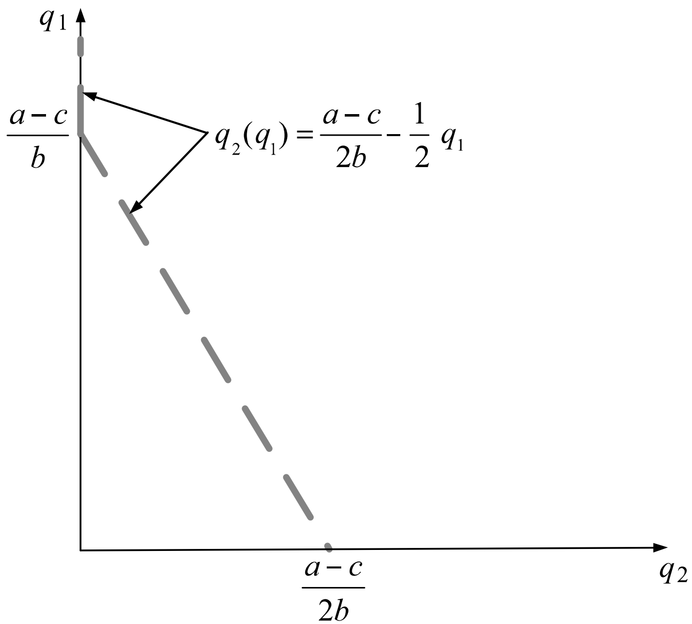

Figure 14.2

Cournot Model

- Superimposing firm 1's and firm 2's best response functions, we obtain their crossing point: Cournot Equilibrium.

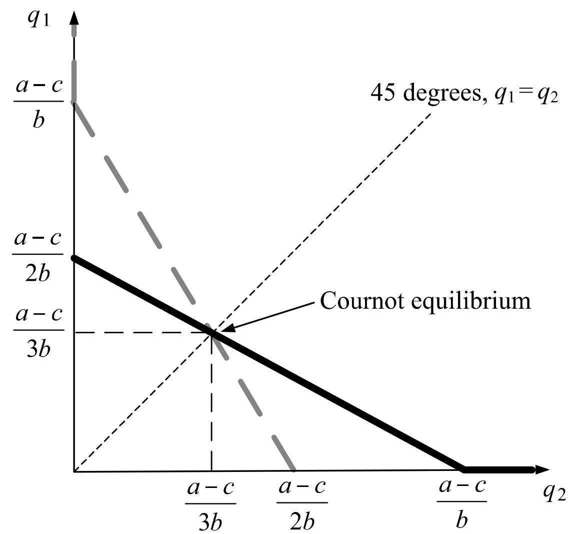

Figure 14.3

- Both firms are choosing output levels that are the best response to the output of its rival (i.e., firms are selecting *mutual* best responses, which is the Nash Equilibrium (NE) of a game).

Cournot Model

- To find the point where the best response functions cross each other, we can insert BRF_2 into BRF_1,

$$q_1 = \frac{a-c}{2b} - \frac{1}{2}\underbrace{\left(\frac{a-c}{2b} - \frac{1}{2}q_1\right)}_{q_2},$$

- Rearranging and solving for q_1, we find q_1^*,

$$\frac{3}{4}q_1 = \frac{a-c}{2b},$$

$$q_1^* = \frac{a-c}{3b}.$$

Cournot Model

- Inserting this output level into BRF_1, we find q_2^*,

$$q_2\left(\frac{a-c}{3b}\right) = \overbrace{\frac{a-c}{2b}}^{q_1^*} - \frac{1}{2}\frac{a-c}{3b}$$

$$= \frac{3(a-c)-(a-c)}{6b},$$

$$q_2^* = \frac{a-c}{3b}.$$

Cournot Model

- The output pair $(q_1^*, q_2^*) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)$ is the Nash Equilibrium of the Cournot game.

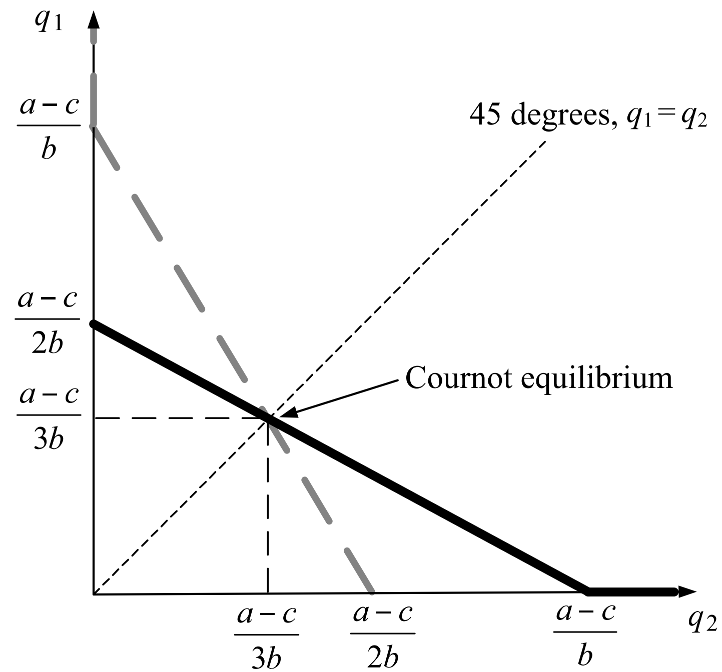

Figure 14.3

Cournot Model

- An alternative approach to solve for the equilibrium output is to invoke symmetry.
- Because firms are symmetric in their revenues and costs, we can claim that there must be a symmetric equilibrium where

$$q_1^* = q_2^* = q^*.$$

- Inserting this property into either firm's BRF,

$$q^* = \frac{a-c}{2b} - \frac{1}{2}q^*,$$
$$\frac{3}{2}q^* = \frac{a-c}{2b},$$
$$q^* = \frac{a-c}{3b}.$$

Cournot Model

- We find equilibrium price by evaluating the inverse demand function

$$p(q_1, q_2) = a - b(q_1 + q_2)$$

at $q_1^* = q_2^* = \frac{a-c}{3b}$,

$$p^* = p\left(\frac{a-c}{3b}, \frac{a-c}{3b}\right) = a - b\left(\frac{a-c}{3b} + \frac{a-c}{3b}\right)$$

$$= a - \frac{2(a-c)}{3} = \frac{a+2c}{3}.$$

Cournot Model

- Finally, equilibrium profits for every firm $i = \{1,2\}$ are

$$\pi_i^* = p^* q_i^* - c q_i^* = \left(\frac{a+2c}{3}\right)\frac{a-c}{3b} - c\,\frac{a-c}{3b}$$

$$= \frac{(a+2c)(a-c)}{9b} - \frac{3c(a-c)}{9b}$$

$$= \frac{a^2 - 2ac + c^2}{9b},$$

or, more compactly,

$$\pi_i^* = \frac{(a-c)^2}{9b}$$

because $(a-c)^2 = a^2 - 2ac - c^2$.

It can be alternatively expressed as $\pi_i^* = b(q^*)^2$.

Cournot Model

- *Example 14.1: Cournot model with symmetric costs.*
  - Consider a duopoly with $p(q_1, q_2) = 12 - q_1 - q_2$, where every firm $i = \{1,2\}$ faces a symmetric cost function $TC_i(q_i) = 4q_i$.
  - *Firm 1's best response function.* Firm 1 chooses q_1 to solve

$$\max_{q_1} \pi_1 = (12 - q_1 - q_2)q_1 - 4q_1.$$

Differentiating with respect to q_1,

$$\frac{\partial \pi_1}{q_1} = 12 - 2q_1 - q_2 - 4 = 0.$$

Rearranging and solving for q_1,

$$8 - q_2 = 2q_1,$$

$$q_1(q_2) = 4 - \frac{1}{2}q_2. \qquad (BRF_1)$$

Cournot Model

- *Example 14.1* (continued):
  - *Firm 2's best response function.* Firm 2 chooses q_2 to solve

$$\max_{q_2} \pi_2 = (12 - q_1 - q_2)q_2 - 4q_2.$$

Differentiating with respect to q_2,

$$\frac{\partial \pi_2}{q_2} = 12 - q_1 - 2q_2 - 4 = 0.$$

Rearranging and solving for q_1,

$$8 - q_1 = 2q_2,$$

$$q_2(q_1) = 4 - \frac{1}{2}q_1, \qquad (BRF_2)$$

which is symmetric to that of firm 1.

Cournot Model

- *Example 14.1* (continued):
  - *Finding equilibrium output.*

    We can invoke symmetry, and claim

    $$q_1^* = q_2^* = q^*.$$

    Inserting this property into either firm's best response function, and solving for q^*,

    $$q^* = 4 - \frac{1}{2}q^*,$$
    $$\frac{3}{2}q^* = 4 \quad \Longrightarrow q^* = \frac{8}{3}.$$

Cournot Model

- *Example 14.1* (continued):
  - *Finding equilibrium output* (cont.).

    Equilibrium price is

    $$p^*\left(\frac{8}{3},\frac{8}{3}\right) = 12 - q^* - q^* = 12 - \frac{8}{3} - \frac{8}{3} = \frac{20}{3} \cong \$6.67,$$

    producing for every firm $i = \{1,2\}$ equilibrium profits of

    $$\pi_i^* = p^*q^* - cq^* = \left(\frac{20}{3}\right)\frac{8}{3} - 4\frac{8}{3} = \frac{160}{9} - \frac{96}{9} = \frac{64}{9}.$$

Cournot Model

- *Example 14.2: Cournot model with asymmetric costs.*
  - Consider two firms competing á la Cournot, facing the same inverse demand as in example 14.1,

$$p(q_1, q_2) = 12 - q_1 - q_2,$$

but different cost functions

$$TC_1(q_1, q_2) = 4q_1,$$
$$TC_2(q_1, q_2) = 3q_2.$$

Cournot Model

- *Example 14.2* (continued):
  - *Firm 1's best response.*

    Firm 1's PMP is

$$\max_{q_1} \pi_1 = (12 - q_1 - q_2)q_1 - 4q_1.$$

This problem coincides with the one in example 14.1, yielding the same best response function,

$$q_1(q_2) = 4 - \frac{1}{2}q_2.$$

Cournot Model

- *Example 14.2* (continued):
  - *Firm 2's best response.* Firm 2's PMP is

$$\max_{q_2} \pi_2 = (12 - q_1 - q_2)q_2 - 3q_2.$$

Differentiating with respect to q_2,

$$\frac{\partial \pi_2}{q_2} = 12 - q_1 - 2q_2 - 3 = 0.$$

Rearranging and solving for q_2, yields

$$9 - q_1 = 2q_2 \Longrightarrow q_2(q_1) = \frac{9}{2} - \frac{1}{2}q_1.$$

This function has the same slope as that in example 14.1, but it originates at $9/2$ rather than at 4. This indicates that, for every output of firm 1, firm 2's output is now largest because its marginal cost is 3 rather than 4.

Cournot Model

- *Example 14.2* (continued):
  - *Finding equilibrium output*. We cannot invoke symmetry because firms face different production costs. Inserting BRF_2 into BRF_1, and solving for q_1,

$$q_1 = 4 - \frac{1}{2}\left(\frac{9}{2} - \frac{1}{2}q_1\right),$$

$$q_1 = 4 - \frac{9}{4} + \frac{1}{4}q_1,$$

$$\frac{3}{4}q_1 = \frac{7}{4} \quad \Longrightarrow q_1^* = \frac{7}{3} \cong 2.33.$$

Inserting this result into BRF_2,

$$q_2^* = \frac{9}{2} - \frac{1}{2}\frac{7}{3} = \frac{10}{3} \cong 3.33,$$

where $q_2^* > q_1^*$ because firm 2's marginal cost is lower.

Cournot Model

- *Example 14.2* (continued):

  In this scenario, equilibrium price and equilibrium profits are

$$p^* = \frac{19}{3},$$
$$\pi_1^* = \frac{49}{9},$$
$$\pi_2^* = \frac{100}{9}.$$

  Firm 2, which is benefiting from a cost advantage, earns a larger profit than firm 1 which suffers from a cost disadvantage.

Bertrand Model–
Simultaneous Price Competition

Bertrand Model

- Two firms produce a homogeneous good and face common marginal cost, $c > 0$.
- They simultaneously and indecently set prices p_1 and p_2.
  - If $p_1 < p_2$, firm 1 captures all the demand, while firm 2 captures none:
    $$x_1(p_1, p_2, I) > 0,$$
    $$x_2(p_1, p_2, I) = 0.$$
  - If $p_1 > p_2$, firm 2 captures all demand.
  - If $p_1 = p_2$, both firms equally share market demand:
    $$\frac{1}{2}x_1(p_1, p_2, I) > 0,$$
    $$\frac{1}{2}x_2(p_1, p_2, I) > 0.$$

Bertrand Model

- The Bertrand model of price competition claims that, in equilibrium:
$$p_1 = p_2 = c.$$
- To show this result, we next demonstrate that all possible price pairs (p_1, p_2) that are different from $(p_1, p_2) = (c, c)$, cannot be equilibria.

Bertrand Model

- We need to show that any price different than the marginal cost, c, is "unstable" in the sense that at least one firm has incentives to deviate to a different price.
- We examine:
  1. Asymmetric price profiles, where $p_1 \neq p_2$.
  2. Symmetric price profiles, where $p_1 = p_2$.

Bertrand Model

1. *Asymmetric price profiles*.

   (a) Consider $p_1 > p_2 > c$.

   - Firm 2 sets the lowest price and captures the entire market by making a positive margin because $p_2 > c$.
   - This profile cannot be stable because firm 1 has incentives to deviate undercutting firm2's prices by charging $p_1' = p_2 - \varepsilon$, where $\varepsilon \to 0$ indicates a small reduction in firm 2's price.

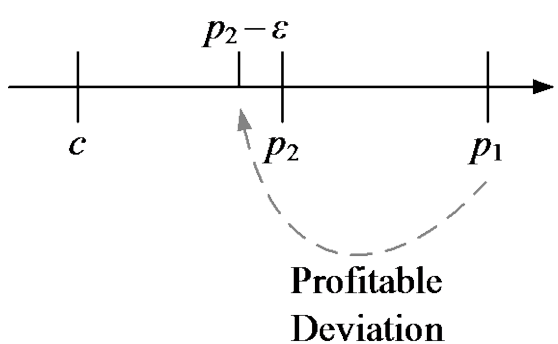

Figure 14.4

Bertrand Model

1. *Asymmetric price profiles* (cont.).

   (b) Consider $p_1 > p_2 = c$.

   - Firm 2 captures the entire market, but it makes no profit per unit.
   - Firm 1 would not have incentives to undercut firm 2's price that would entail charging a price below c, incurring in a per unit cost.
   - Instead, firm 2 would have incentives to deviate by increasing its price from $p_2 = c$ to slightly below its rival's price, $p_2' = p_1 - \varepsilon$, and make a higher profit.

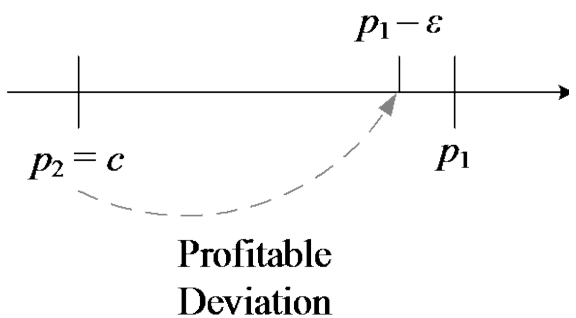

Figure 14.5

Bertrand Model

2. *Symmetric price profiles*.

   (a) Consider $p_1 = p_2 > c$.

   - Both firms evenly share the market because their prices are the same.
   - Every firm i has the incentive to deviate by undercutting its rival's price p by a small amount ε, by charging $p_i' = p - \varepsilon$, where $\varepsilon \to 0$.

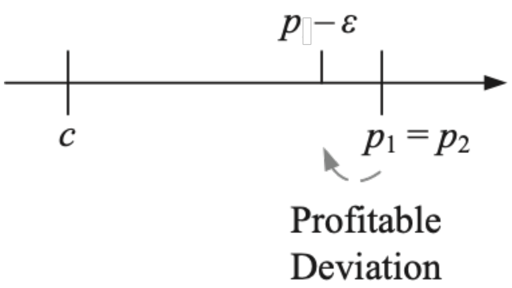

Figure 14.6

Bertrand Model

2. *Symmetric price profiles (cont.).*

   (b) Consider $p_1 = p_2 = c$.

   - Prices coincide, leading firms to evenly share the market.
   - These prices leave no positive margin per unit because $p_i = c$ for every firm i.
   - No firm can strictly increase its payoff by unilaterally deviating:
     - A lower price would attract all consumers, but at a lower per unit loss.
     - A higher price would reduce the deviating firm's sales to zero.

We can claim that setting $p_i = c$ is a weakly dominant strategy in the Bertrand model of price competition because no firm can strictly increase its profit by deviating from such a price.

Bertrand Model

- *Example 14.3: Bertrand model.*
  - Consider the inverse demand function in example 14.1, $p(q_1, q_2) = 12 - q_1 - q_2$.
  - Because $Q \equiv q_1 + q_2$ denotes the aggregate output in the industry, the inverse demand can be expressed as
$$p(Q) = 12 - Q.$$
  - In the Bertrand model of price competition, all firms in the industry lower their prices until
$$p = c \quad \Longrightarrow 12 - Q = c.$$
Solving for Q, $Q^* = 12 - c$.
    - If $c = 4$, $Q = 12 - 4 = 8$ units, each of which sold at a price of $\$4$.

Reconciling the Cournot and Bertrand models

- *Why are the results in the Cournot model and Bertrand model so dramatically different?*
  - In the Cournot model,
    - firms set a price above marginal cost, making a positive profit.
  - In the Bertrand model,
    - firms set $p = c$, earning no economic profits.

Reconciling the Cournot and Bertrand models

- These differences are driven by the absence of capacity constraints in the Bertrand model:
  - If a firm charges 1 cent less than its rival, it captures the market demand, regardless of its size.
- This assumption might be reasonable for goods such as online movie streaming
  - but difficult to justify with others (e.g., smartphones) with a world demand that cannot be served by a single firm.

Cartels and Collusion

Cartels and Collusion

- Firms competing in quantities can earn profits below those under monopoly, which is emphasized when firm compete in prices.
- *What if, rather than competing, firms were to coordinate their production decisions?*
- We analyze how collusion can help firm increase their profits, and under which condition cooperation holds.
- Cartels seek to coordinate production decisions to raise profits and profits for participants.
  - In a cartel firms seek to maximize their *joint* rather than their individual profits.
  - *Example*: OPEC.

Cartels and Collusion

- *Example 14.4: Collusion when firms compete in quantities.*
  - Consider the industry in example 14.2, where

$$p(q_1, q_2) = 12 - q_1 - q_2,$$
$$TC_i(q_i) = 4q_i \text{ for every firm } i.$$

  - If firms join a cartel, they choose q_1 and q_2 to maximize their joint profits, $\pi = \pi_1 + \pi_2$ as follows:

$$\max_{q_1, q_2} \pi = \underbrace{(12 - q_1 - q_2)q_1 - 4q_1}_{\pi_1} + \underbrace{(12 - q_1 - q_2)q_2 - 4q_2}_{\pi_2}.$$

Cartels and Collusion

- *Example 14.4* (continued):
  - The previous expression can be simplified as

$$\max_{q_1,q_2} (12 - q_1 - q_2)(q_1 + q_2) - 4(q_1 + q_2),$$

$$\max_{q_1,q_2} [12 - (q_1 + q_2)](q_1 + q_2) - 4(q_1 + q_2).$$

Cartels and Collusion

- *Example 14.4* (continued):
  - Because $Q = q_1 + q_2$ denotes aggregate output, we can rewrite the cartel's PMP as it were a single monopolist,

$$\max_{q_1,q_2} [12 - Q]Q - 4Q.$$

  - Differentiating with respect to Q,

$$12 - 2Q - 4 = 0.$$

Cartels and Collusion

- *Example 14.4* (continued):
  - Solving for Q,
$$Q^* = \frac{8}{2} = 4 \text{ units.}$$
  - Because firms are symmetric, each produces $q_i = \frac{Q^*}{2} = 2$ units.
  - In contrast, under Cournot competition, every firm produces $q = \frac{8}{3} \cong 2.66$ units.

Cartels and Collusion

- *Example 14.4* (continued):
  - Under cartel, every firm limits its own production to increase market price and profits.
  - We confirm this result by finding that the cartel price is

    $$p(2,2) = 12 - 2 - 2 = \$8,$$

    which is higher than under Cournot competition (\$6.67).
  - The cartel profits for every firm i are

$$\pi_i = (12 - q_1 - q_2)q_i - 4q_i = (12 - 2 - 2)2 - (4 \times 2) = \$8,$$

while under Cournot competition, $\pi_i^* = \frac{64}{9} \simeq \$7.11$.

Cartels and Collusion

- *Why are cartel profits larger than under Cournot competition?*
  - Under Cournot, when every firm increases its output, it considers the effect of such additional production has in its own profits, but it ignores the effect on its rival's profit.
  - Under the cartel, firms take into account each other's benefits. Firms produce less but elevate market prices and increase profits.
- We next identify the conditions to sustain collusion over time.
  - If firms interact only once, cooperation cannot be sustained in equilibrium.
  - If firms interact infinitely (there is a probability that firms will be in the industry tomorrow), cooperation can be sustained.

Cartels and Collusion

- *Example 14.5: Sustaining cooperation within the cartel.*
  - Assume that firms play an infinitely repeated Cournot game, and they seek to coordinate their production decisions through the following Grim-Trigger Strategy (GTS):
    1. In $t = 1$, every firm starts cooperating (producing 2 units).
    2. In $t > 1$,
       (a) Every firm continues cooperating, so long as all firms cooperated in all previous periods.
       (b) If, instead, it observes some past cheating (deviating this GTS), then it produces the Cournot output $q^* = \frac{8}{3}$ hereafter.

Cartels and Collusion

- *Example 14.5* (continued):
  - We only need to check if every firm has incentives to deviate from the GTS: (1) after observing a history of cooperation; and (2) after observing that some firm/s cheated.
  - *Cooperation.* If firm i continues cooperating (producing under cartel $q = 2$), it obtains profit of $\$8$. Then, its stream of discounted payoffs from cooperating is

$$8 + \delta 8 + \delta^2 8 + \cdots = 8(1 + \delta + \delta^2 + \cdots)$$
$$= \frac{8}{1-\delta},$$

where δ denotes the discount factor weighting future payoffs.

Cartels and Collusion

- *Example 14.5* (continued):
  - *Best deviation.* If firm i deviates from $q = 2$ while its rival sticks to the cartel agreement, its profits could increase.

    What is firm i's best deviation? We need to evaluate its profits when its rival produces the cartel output, $q_j = 2$,

    $$(12 - q_i - 2)q_i - 4q_i = (10 - q_i)q_i - 4q_i.$$

  - Differentiating wit respect to q_i,

    $$10 - 2q_i - 4 = 0 \quad \Longrightarrow q_i = 3 \text{ units.}$$

  - Inserting this "best deviation" into firm i's profits, deviation profits are

    $$\pi^{Dev} = (10 - 3)3 - (4 \times 3) = \$9 > \text{cartel profit of } \$8.$$

Cartels and Collusion

- *Example 14.5* (continued):
  - If firm i deviates, its stream of discounted payoffs becomes

$$\underbrace{9}_{\text{Deviation}} + \underbrace{\delta\frac{64}{9} + \delta^2\frac{64}{9} + \cdots}_{\text{Punishment}} = 9 + \frac{64}{9}(\delta + \delta^2 + \cdots)$$
$$= 9 + \frac{64}{9}\delta(1 + \delta + \cdots)$$
$$= 9 + \frac{64}{9}\,\frac{\delta}{1-\delta}.$$

    - The deviating firm increases its profits for from \$8 to \$9 one period.
    - Its defection is detected by its cartel partner, which triggers an infinite punishment in which both firms produce the Cournot output, yielding a profit of $\frac{64}{9}$ thereafter.

Cartels and Collusion

- *Example 14.5* (continued):
  - *Comparing profits*. Every firm i prefers to cooperate if

$$\frac{8}{1-\delta} \geq 9 + \frac{64}{9}\frac{\delta}{1-\delta},$$

$$(1-\delta)\frac{8}{1-\delta} \geq \left[9 + \frac{64}{9}\frac{\delta}{1-\delta}\right](1-\delta),$$

$$8 \geq 9(1-\delta)\frac{64}{9}\delta.$$

  - The cartel output can be sustained with this GTS if

$$\delta \geq \frac{9}{17} \cong 0.53.$$

That is, if firms assign sufficiently importance to their profits. If $\delta < 0.53$, the cartel agreement cannot be sustained.

Stackelberg Model– Sequential Quantity Competition

Stackelberg Model

- We modify the Cournot model by considering that firms *sequentially* compete in quantities.
- The structure of the game is:
  1. Firm 1 chooses its output q_1.
  2. Firm 2 observes q_1 and responds with its own output q_2.
- This timing may be due to industry or legal reasons that provide firm 1 with an advantage.
  - *Example*: Firm 1 is the first to develop a new product, allowing it to choose its output before firm 2.
- This is a sequential-move game in which Firm 1 is the leader and firm 2 is the follower. We solve it by applying backward induction.

Stackelberg Model

- **Firm 2 (follower).**
  - Firm 2 takes the leader's output q_1 as given, because it is already chosen by the time firm 2 gets to move. Its PMP is

$$\max_{q_2} \; [a - b(q_1 + q_2)]q_2 - cq_2.$$

  - Differentiating with respect to q_2,

$$a - bq_1 - 2q_2 - c = 0,$$

  and solving for q_2,

$$q_2(q_1) = \frac{a - c}{2b} - \frac{1}{2} q_1. \qquad (BRF_2)$$

    - This BRF coincides with that of the Cournot model. In both scenarios firm 2 treats firm 1's output q_1 as given, because firm 2 cannot alter it (Cournot) or because q_1 is already produced (Stackelberg).

Stackelberg Model

- **Firm 1 (leader).**
  - Firm 1 chooses its output q_1 to maximize its profits,

$$\max_{q_1} \; [a - b(q_1 + q_2)]q_1 - cq_1.$$

  - Firm 1 can anticipate that firm 2 will respond with

$$BRF_2 = q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1,$$

  as this maximizes the follower's profits.

Stackelberg Model

- **Firm 1 (leader)** (cont.)
  - Inserting BRF_2 into the leader's PMP,

$$\max_{q_1}[a - b(q_1 + \underbrace{q_2(q_1)}_{BRF_2})]q_1 - cq_1$$

$$\max_{q_1}\left[a - b\left(q_1 + \underbrace{\left(\frac{a-c}{2b} - \frac{1}{2}q_1\right)}_{q_2(q_1)\text{ from }BRF_2}\right)\right]q_1 - cq_1.$$

  - After simplifying,

$$\max_{q_1}\frac{1}{2}(a + c - bq_1)q_1 - cq_1.$$

Stackelberg Model

- **Firm 1 (leader)** (cont.)
  - Differentiating with respect to q_1 , and solving for q_1,

$$\frac{1}{2}(a - c - 2bq_1) = 0,$$

$$q_1^* = \frac{a - c}{2b}.$$

  - We find the follower's equilibrium output by inserting q_1^* into BRF_2,

$$q_2\left(\frac{a-c}{2b}\right) = \frac{a-c}{2b} - \frac{1}{2}\underbrace{\left(\frac{a-c}{2b}\right)}_{q_1^*} = \frac{2(a-c)}{4b} - \frac{a-c}{4b} = \frac{a-c}{4b},$$

which is half of leader output $q_2^* = \frac{1}{2}q_1^*$.

Stackelberg Model

- More generally, the subgame perfect equilibrium (SPE) of the game is described as

$$q_1^* = \frac{a-c}{2b},$$

$$q_2(q_1) = \frac{a-c}{2b} - \frac{1}{2}q_1,$$

because the follower's BRF allows firm 2 to optimally respond to the leader's output, both:

  - in equilibrium, $q_1^* = \frac{a-c}{2b}$,
  - and off the equilibrium $q_1^* \neq \frac{a-c}{2b}$.

Stackelberg Model

- If instead, the follower chooses $q_2^* = \frac{a-c}{4b}$ in the SPE of the game, we would provide no information about how the follower responds if the leader "made a mistake" by deviating from q_1^*.

- The leader produces more in the Stackelberg model than in Cournot,

$$\frac{a-c}{2b} > \frac{a-c}{4b},$$

  whereas the follower produces less,

$$\frac{a-c}{4b} < \frac{a-c}{3b}.$$

Stackelberg Model

- In this scenario, equilibrium price is

$$\begin{aligned} p^* &= a - b\left(\frac{a-c}{2b} + \frac{a-c}{4b}\right) \\ &= a - \frac{2(a-c)}{4} - \frac{a-c}{4} \\ &= \frac{3a+c}{4}. \end{aligned}$$

- The equilibrium profits for the leader are

$$\pi_1^* = \left(\frac{3a+c}{4} - c\right)\frac{a-c}{2b} = \frac{3(a-c)^2}{8b}.$$

- And equilibrium profits for the follower are

$$\pi_2^* = \left(\frac{3a+c}{4} - c\right)\frac{a-c}{4b} = \frac{3(a-c)^2}{16b},$$

that is exactly half of the leader's profits, $\pi_2^* = \frac{1}{2}\pi_1^*$.

Stackelberg Model

- *Example 14.6: Stackelberg model.*
  - Consider the same inverse demand function as in example 14.1, $p(q_1, q_2) = 12 - q_1 - q_2$, and marginal cost $c = 4$.
  - Inserting the follower's BRF found in example 14.1, $q_2(q_1) = 4 - \frac{1}{2}q_1$, into the leader's PMP,

$$\max_{q_1} \left[12 - \left(q_1 + \left(4 - \frac{1}{2}q_1\right)\right)\right] q_1 - 4q_1 ,$$

$$\max_{q_1} \frac{1}{2}(16 - q_1)q_1 - 4q_1.$$

  - Differentiating with respect to q_1,

$$8 - q_1 - 4 = 0.$$

Stackelberg Model

- *Example 14.6* (continued):
  - Solving for q_1 we find the profit-maximizing output for the leader, $q_1^* = 4$ units.
  - Then, $q_2 = 2$ units.
  - In this scenario, equilibrium price is $p^* = \$6$.
  - And equilibrium profits become

$$\pi_1^* = (6 \times 4) - (4 \times 4) = \$8,$$
$$\pi_2^* = (6 \times 2) - (4 \times 2) = \$4.$$

Product Differentiation

Product Differentiation

- Most goods are differentiated from those of their rivals:
  - Coke and Pepsi, in the soda industry.
  - Dell and Lenovo, the the computer industry.
  - iPhone and Samsung Galaxy, in the smartphone market.
- *Demand for product differentiation.*
  - Consider two firms, A and B, with inverse demand functions,
    $$p_A(q_A, q_B) = a - bq_A - dq_B,$$
    $$p_B(q_A, q_B) = a - bq_B - dq_A,$$
    where $b, d \geq 0$ and $b \geq d$
  - These demands are symmetric. Let us focus on good A.
    - An increase in q_A or q_B reduces p_A, but the effect of q_A is larger because $b > d$. The "own-price effects" dominate "cross-price" effect.

Product Differentiation

- When $d = 0$, the inverse demand function for good A collapses to
$$p_A(q_A, q_B) = a - bq_A,$$
indicating that every firm's price is unaffected by its rival's output, as in two separate monopolies.
- When $d = b$, the inverse demand function for good A becomes
$$p_A(q_A, q_B) = a - bq_A - bq_B = a - b(q_A + q_B),$$
reflecting that p_A is symmetrically affected by an increase in either q_A or q_B as in the Cournot model with homogeneous goods.

Product Differentiation

- *Best responses with product differentiation.*
  - Assume every firm $i = \{A, B\}$ faces a cost function $TC(q_i) = cq_1$, where $c > 0$.
  - The PMP of firm A is
$$\max_{q_A} [a - bq_A - dq_B]q_A - cq_A.$$
  - Differentiating with respect to q_A,
$$a - c - 2bq_A - dq_B = 0.$$
  - Rearranging and solving for q_A,
$$a - c - dq_B = 2bq_A,$$
$$q_A(q_B) = \frac{a-c}{2b} - \frac{d}{2b}q_B.$$

Product Differentiation

- Figure 14.7 depicts BRF_A, $q_A(q_B) = \frac{a-c}{2b} - \frac{d}{2b} q_B$.
  - $q_A = \frac{a-c}{2b}$ when $q_B = 0$, but it decrease at rate of $\frac{d}{2b}$.
  - $q_A = 0$ when $q_B \geq \frac{a-c}{d}$.
  - When $d = 0$, BRF_A reduces to $q_A(q_B) = \frac{a-c}{2b}$ (monopolist).

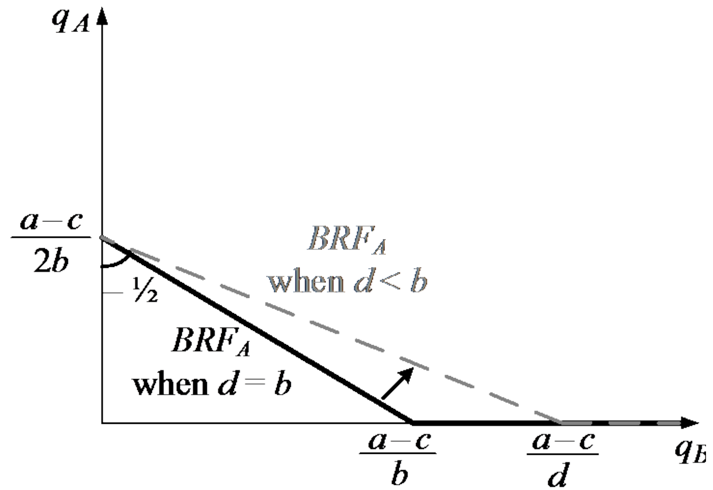

Figure 14.7

Product Differentiation

- Figure 14.7 depicts BRF_A, $q_A(q_B) = \frac{a-c}{2b} - \frac{d}{2b} q_B$.
  - When $d = b$, $q_A(q_B) = \frac{a-c}{b} - \frac{1}{2} q_B$ (Cournot), with a slope of $-1/2$.
  - When $d < b$, the slope becomes smaller than $-1/2$. Competition is ameliorated, because every firm i is induce to reduce its output when products are differentiated.

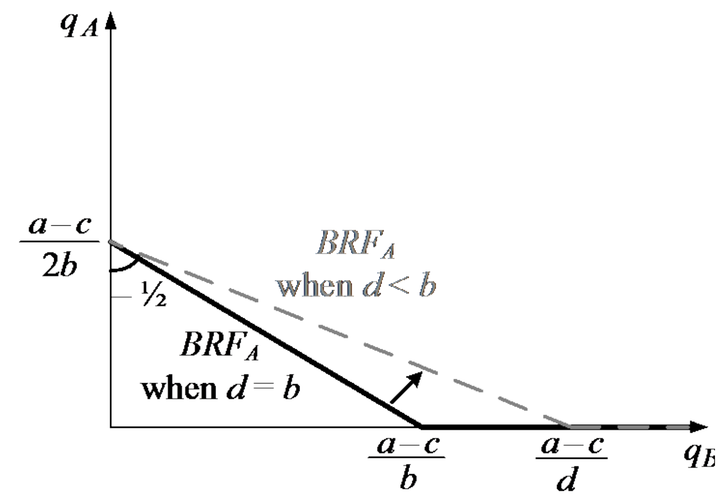

Figure 14.7

Product Differentiation

- We can invoke symmetry in equilibrium output $q_i^* = q_j^* = q^*$,

$$q = \frac{a-c}{2b} - \frac{d}{2b}q,$$
$$\frac{(2b+d)q}{2b} = \frac{a-c}{2b},$$
$$q^* = \frac{a-c}{2b+d}.$$

- When products are completely differentiated ($d = 0$), this output becomes $q^* = \frac{a-c}{2b}$, as in monopoly.
- When products are homogeneous ($d = b$), $q^* = \frac{a-c}{2b+b} = \frac{a-c}{3b}$, as in the Cournot model.

Product Differentiation

- Equilibrium price is given by

$$p_i^* = a - bq_i^* + dq_j^* = a - b\underbrace{\frac{a-c}{2b+d}}_{q_i^*} - d\underbrace{\frac{a-c}{2b+d}}_{q_j^*}$$

$$= \frac{ab + c(b+d)}{2b+d}.$$

- Equilibrium profits for every firm i are

$$\pi_i^* = (p^* - c)q^* = \left(\frac{ab + c(b+d)}{2b+d} - c\right)\frac{a-c}{2b+d}$$

$$= \frac{(a-c)^2 b}{(2b+d)^2}.$$

Product Differentiation

- When products are completely differentiated ($d = 0$),

$$\pi_i^* = \frac{(a-c)^2}{4b},$$

  as in monopoly.

- When products are homogeneous ($d = b$),

$$\pi_i^* = \frac{(a-c)^2 b}{(2b+b)^2} = \frac{(a-c)^2}{9b},$$

  as in Cournot model.

Stackelberg Model

- *Example 14.7: Output competition with product differentiation.*
  - Consider two firms, A and B, facing the demand curves
$$p_A(q_A, q_B) = 100 - 5q_A - 2q_B,$$
$$p_B(q_A, q_B) = 100 - 5q_B - 2q_A.$$
  - Parameters are $a = 100$, $b = 5$, and $d = 2$, which indicates that own-price effects are larger than cross-price effect (i.e., $b > d$).
  - Both firms have symmetric marginal cost of $c = 3$.
  - Inserting these parameters in in the previous equilibrium results, equilibrium output is
$$q^* = \frac{100-3}{(2\times5)+2} = \frac{97}{12} \cong 8.08 \text{ units.}$$

Stackelberg Model

- *Example 14.7* (continued):
  - The equilibrium price is

$$p_i^* = \frac{(100 \times 5) + 3(5 + 2)}{(2 \times 5) + 2} = \frac{521}{12} \cong \$43.41.$$

  - And profits become

$$\pi_i^* = \frac{(100 - 3)^2 5}{[(2 \times 5) + 2]^2} \cong \$326.7.$$

Appendix.
Cournot Model with *N* Firms

Cournot Model with *N* Firms

- The inverse demand function is
$$p(Q) = a - bQ.$$
- $Q = q_i + Q_{-i}$ denotes the aggregate output by all firms.
  - q_i is the output that firm i produces.
  - Q_{-i} represents the production of all the firms different than firm i,
$$Q_{-i} = q_1 + q_2 + \cdots + q_{i-1} + q_{i+1} + \cdots + q_N.$$
  - If $N = 4$, and $i = 2$, then $Q_{-2} = q_1 + q_3 + q_4$.
- We can rewrite the inverse demand function as
$$p(q_i, Q_{-i}) = a - b\underbrace{(q_i + Q_{-i})}_{Q}.$$

Cournot Model with *N* Firms

- If all N firms face the same marginal cost c, where $a > c > 0$, every firm i solves the following PMP:

$$\max_{q_i} [a - b(q_i + Q_{-i})]q_i - cq_i. \qquad (PMP_i)$$

- Differentiating with respect to firm i's output,

$$a - 2bq_i - bQ_{-i} - c = 0.$$

- Rearranging a solving for q_i, we find firm i's best response function,

$$a - c - bQ_{-i} = 2bq_i,$$

$$q_i(Q_{-i}) = \frac{a - c}{2b} - \frac{1}{2}Q_{-i}. \qquad (BRF_i)$$

Cournot Model with *N* Firms

- $BRF_i \equiv q_i(Q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}$ informs about this firm's profit maximizing output q_i, as a function of its rivals' output Q_{-i}.
  - It originates at $\frac{a-c}{2b}$ and decreases in Q_{-i} at a rate of $1/2$.
  - This function captures the Cournot model with 2 firms as a special case.
  - If we consider only two firms, i and j, then firm i has a single rival (firm j), and $Q_{-i} = q_j$.

Cournot Model with *N* Firms

- Because all firms are symmetric, they all solve a problem similar to PMP_i, obtaining $BRF_i \equiv q_i(Q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}Q_{-i}$.
- We can invoke symmetry in equilibrium output,

$$q_1 = q_2 = \cdots = q_N = q.$$

- Then,

$$Q = Nq,$$

$$Q_{-i} = (N-1)q.$$

Cournot Model with *N* Firms

- Inserting this result into BRF_i,

$$q_i(Q_{-i}) = \frac{a-c}{2b} - \frac{1}{2}\underbrace{(N-1)q}_{Q^*}.$$

- Rearranging and solving for q,

$$\frac{2q+(N-1)q}{2} = \frac{a-c}{2b},$$

$$q[2+(N-1)] = \frac{a-c}{b},$$

$$q^* = \frac{1}{N+1}\frac{a-c}{b}.$$

which is decreasing in the number of firms operating in the market, N. As more firms compete, the individual production of each firm decreases.

Cournot Model with *N* Firms

- The aggregate output becomes

$$Q^* = Nq^* = N\left(\frac{1}{N+1}\frac{a-c}{b}\right),$$

  which increases as more firms enter the industry.

- The equilibrium price is

$$\begin{aligned} p(Q^*) = a - bQ^* &= a - b\left[N\left(\frac{1}{N+1}\frac{a-c}{b}\right)\right] \\ &= \frac{a+Nc}{N+1}, \end{aligned}$$

  which is decreasing in N.

Cournot Model with *N* Firms

- *Monopoly* ($N = 1$):

$$q^* = \frac{1}{1+1}\frac{a-c}{b} = \frac{a-c}{2b},$$

$$Q^* = Nq^* = \frac{a-c}{2b},$$

$$p^* = \frac{a+c}{1+1} = \frac{a+c}{2}.$$

Cournot Model with *N* Firms

- Duopoly ($N = 2$):

$$q^* = \frac{1}{2+1}\frac{a-c}{b} = \frac{a-c}{3b},$$

$$Q^* = Nq^* = 2q^* = 2\frac{a-c}{3b},$$

$$p^* = \frac{a+2c}{2+1} = \frac{a+2c}{3}.$$

Cournot Model with N Firms

- *Perfect competition* $(N \rightarrow +\infty)$:

$$\lim_{N \to +\infty} q^* = \lim_{N \to +\infty} \frac{1}{N+1} \frac{a-c}{b} = 0\,,$$

$$\lim_{N \to +\infty} Q^* = \lim_{N \to +\infty} N\left(\frac{1}{N+1} \frac{a-c}{b}\right) = \frac{a-c}{b},$$

$$\lim_{N \to +\infty} p^* = \lim_{N \to +\infty} \frac{a+Nc}{N+1} = c.$$