Intermediate Microeconomic Theory

Tools and Step-by-Step Examples

Chapter 11:
Price Discrimination and Bundling

Outline

- Price Discrimination
  - First-Degree Price Discrimination
  - Second-Degree Price Discrimination
  - Third-Degree Price Discrimination
- Bundling

Price Discrimination

Price Discrimination

Can the monopolist do even better? YES!

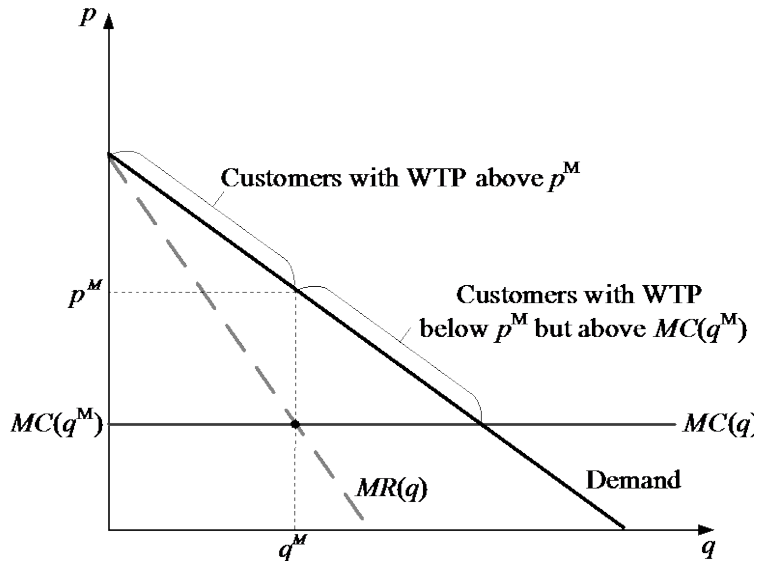

Figure 11.1

- The monopolist could increase its profits if it could charge different prices to specific customer ("price discriminate").

Price Discrimination

- Three types of price discrimination:
  - *First-degree:*
    - The monopolist sets a *different price for each customer* coinciding with her willingness-to-pay (WTP).
  - *Second-degree:*
    - The monopolist offers a *quantity discount* to buyers purchasing a large amount of the product.
  - *Third-degree:*
    - *T*he monopolist charges *different prices to different groups of customers*, each with a different demand curve.

Conditions for Price Discrimination

- The monopolist can price discriminate under the following conditions:
  - *No arbitrage*. The good cannot be resold from a consumer to another.
    - Otherwise, individuals with a low WTP would purchase the good at a low price and resell to individuals with a high WTP.
  - *Information about WTP*. The monopolist needs some information about customers' WTP for its good.
    - While detailed information about WTP is rarely observed, firms at least can gather information for various groups of customers.

First-Degree Price Discrimination

First-Degree Price Discrimination

- The monopolist charges to every consumer i a price that coincides with her maximum WTP.
- *Personalized price*:
  - If the monopolist faces inverse demand $p(q) = a - bq$, it charges:
    - A price $p = a$ to the individual with higher WTP;
    - A price $p = a - \$0.01$ to the individual with the 2^{nd}-highest WTP;
    - etc.
- The monopolist stops this pricing strategy when $p = MC(q)$ because customers with WTP below $MC(q)$ would entail a per-unit loss.

First-Degree Price Discrimination

- The firm extracts all the surplus from every consumer (the area below the demand curve and above the marginal cost function).
- The output produced under first-degree price discrimination, q^{FD}, coincides with that under perfectly competitive market, q^{PC}, because at q^{PC}, the demand curve crosses the firms' marginal cost, $p(q) = MC(q)$.

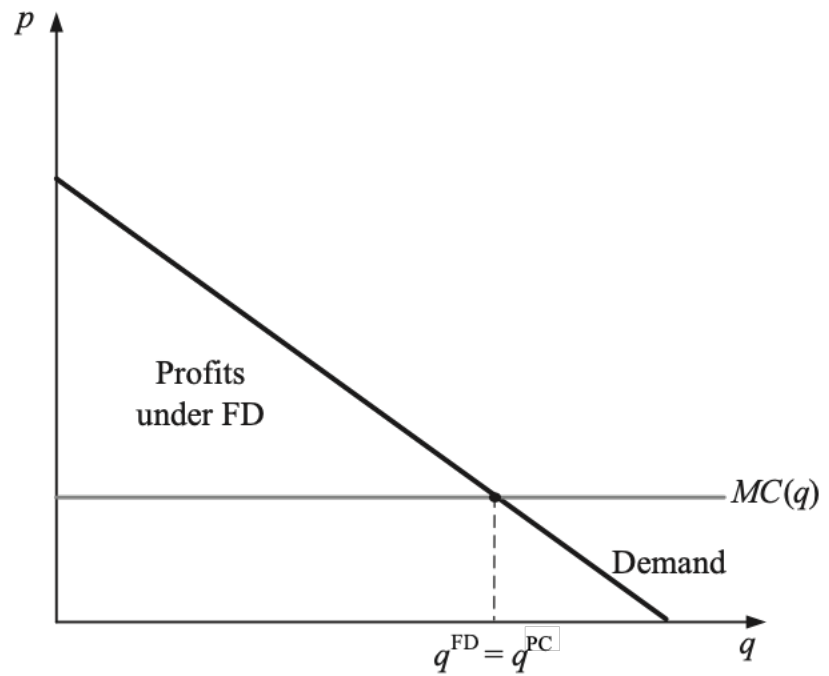

Figure 11.2

First-Degree Price Discrimination

- *Example 11.1: First-degree price discrimination.*
  - Consider a monopolist facing inverse demand curve $p(q) = a - bq$, where $a, b > 0$, and total cost function is $TC(q) = cq$, where $c > 0$.
  - *Uniform price*. The monopolist would produce
$$MR(q) = MC(q),$$
$$a - 2bq = c \Longrightarrow q^M = \frac{a-c}{2b},$$
which entails a monopoly price of
$$p^M = a - b\frac{a-c}{2b} = \frac{a+c}{2},$$
with profits $\pi^M = \frac{(a-c)^2}{4b}$.

First-Degree Price Discrimination

- *Example 11.1* (continued):
  - *First-degree price discrimination*. The monopolist produces an output level where $p(q) = MC(q)$,

$$a - bq = c \Longrightarrow q^{FD} = \frac{a-c}{b}.$$

  - Profits coincides with the area of the triangle below the demand curve $p(q) = a - bq$, and above marginal cost c,

$$\pi^{FD} = \frac{1}{2}\underbrace{(a-c)}_{\text{Height}}\underbrace{\left(\frac{a-c}{b} - 0\right)}_{\text{Base}} = \frac{(a-c)^2}{2b}.$$

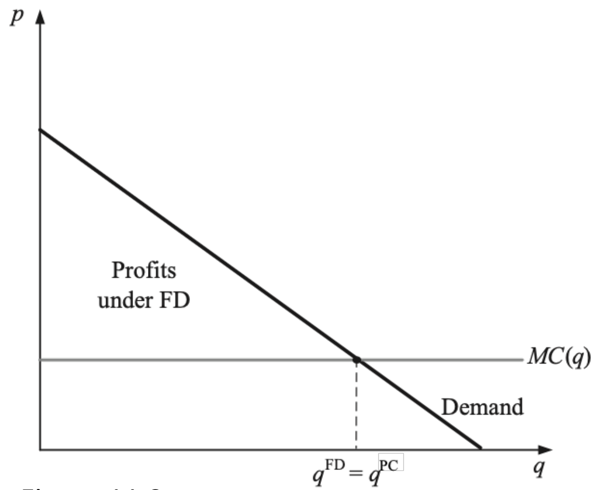

Figure 11.2

First-Degree Price Discrimination

- *Example 11.1* (continued):
  - Profits under first-degree price discrimination exceeds those under uniform (unique) price, $\pi^{FD} > \pi^{M}$

$$\frac{(a-c)^2}{2b} > \frac{(a-c)^2}{4b},$$
$$\frac{1}{2b} > \frac{1}{4} \Longrightarrow 4b > 2b.$$

  - If the monopolist faces $p(q) = 10 - q$ (i.e., $a = 10$, $b = 1$) and $c = 2$,

$$\pi^{M} = \frac{(a-c)^2}{4b} = \frac{(10-2)^2}{4} = \$16,$$
$$\pi^{FD} = \frac{(a-c)^2}{2b} = \frac{(10-2)^2}{2} = \$32.$$

First-Degree Price Discrimination

- Summary:
  - First-degree price discrimination extracts all possible surplus from consumers.
  - However, the monopolist needs a massive amount of information. It needs to know the maximum WTP for every buyer.
  - First-degree discrimination is is relatively uncommon.
    - *Example*: Free Application for Federal Student Aid (FAFSA)
      - The form that the students submit includes relatively detailed information about the student and her family’s income, which is highly correlated with their WTP for education.

Second-Degree Price Discrimination

Second-Degree Price Discrimination

- The monopolist offers a quantity discount to individuals willing to purchase several units, such as discounts in bulk.
- The monopolist charges at least two prices:
  - One for each of the first q_1 units,
    - E.g., $p_1 = \$4$ for the first 3 units.
  - Another for each unit beyond q_1 units,
    - E.g., $p_2 = \$2$ for all units after 3.
- *Example*: Utilities, such as electricity and water, and in mass transit systems.

Second-Degree Price Discrimination

- There are three unknowns that the firm needs to determine:
  - Where should the monopolist set the boundary, q_1, where customers can start benefiting from quantity discount?
  - Which price should the monopolist set for each unit in the first block, p_1?
  - Which price should it set for each unit in the second block, p_2?

Second-Degree Price Discrimination

- To find these three unknowns, we set up the following monopolist problem:

$$\max_{q_1,q_2} \underbrace{p_1 q_1}_{TR_1} + \underbrace{p_2(q_2 - q_1)}_{TR_2} - TC(q_2),$$

where $TR_1 = p_1 q_1$ denotes total revenue from units in the first block, from $q = 0$ to $q = q_1$;

$TR_2 = p_2(q_2 - q_1)$ is total revenue from units in the second block, from q_1 to q_2;

$TC(q_2)$ is total cost evaluated at q_2 because the firm produces a total of q_2 units.

- This problem ask: Choose the number of units in the first block, q_1, and in the second block, $q_2 - q_1$, to maximize profits from both blocks.

Second-Degree Price Discrimination

- *Example 11.3: Second-degree price discrimination.*
  - Consider a monopolist facing inverse demand function $p(q) = 10 - q$.
  - The firm total cost function is $TC(q) = cq$, where $c > 0$.
  - The monopolist's PMP is

$$\max_{q_1,q_2} \pi = \overbrace{\underbrace{(10 - q_1)}_{p_1} q_1}^{TR_1} + \overbrace{\underbrace{(10 - q_2)}_{p_2}(q_2 - q_1)}^{TR_2} - \overbrace{cq_2}^{TC(q_2)}.$$

  - Differentiating with respect to q_1,

$$\frac{\partial \pi}{\partial q_1} = 10 - 2q_1 - (10 - q_2) = 0,$$

$$-2q_1 + q_2 = 0 \Longrightarrow q_1 = \frac{q_2}{2}.$$

Second-Degree Price Discrimination

- *Example 11.3* (continued):
  - Differentiating now with respect to q_2,

$$\frac{\partial \pi}{\partial q_2} = 10 - 2q_2 + q_1 - c = 0,$$

$$q_2 = \frac{10 + q_1 - c}{2}.$$

  - Inserting the expression for q_1 into the expression for q_2,

$$q_2 = \frac{10 + \overbrace{\frac{q_2}{2}}^{q_1} - c}{2},$$

$$3q_2 + 2c = 20 \Rightarrow q_2 = \frac{2(10 - c)}{3}.$$

  - Inserting this result into q_1, $q_1 = \frac{10-c}{3}$.

Second-Degree Price Discrimination

- *Example 11.3* (continued):
  - We find the optimal prices for each block by plugging q_1, and q_2 into the inverse demand function,

$$p(q_1) = 10 - \frac{10-c}{3} = \frac{20+c}{3},$$
$$p(q_2) = 10 - \frac{2(10-c)}{3} = \frac{2(5+c)}{3}.$$

  - *Numerical example*. If marginal cost is $c = \$4$,
    - $q_1 = \frac{10-4}{3} = 2$ units at $p_1 = \frac{20+4}{3} = \$8$/unit in the 1st block.
    - $q_2 = \frac{2(10-4)}{3} = 4$ units, implying $q_2 - q_1 = 4 - 2 = 2$ units in the 2nd block at $p_2 = \frac{2(5+4)}{3} = \$6$/unit.

Second-Degree Price Discrimination

- *Example 11.3* (continued):
  - These prices an output levels generate profits of

$$\pi = (8 \times 2) + (6 \times 2) - (4 \times 4) = \$12.$$

  - If instead, the monopolist charged a uniform price for all its customers,
    - Output q^M would solve to $10 - 2q = 4 \Rightarrow q^M = 3$ units.
    - At price of $p^M = 10 - 3 = \$7$.
    - Profits would be only $\pi^M = (7 \times 3) - (4 \times 3) = \$9$.
  - As expected, the monopolist increases its profits by price discriminating.

Non-linear pricing

- Uniform pricing is known as “linear pricing.”
  - Price per unit is the same, regardless of how many units the consumer purchases.
- Second-price discrimination is known as “non-linear pricing.”
  - Price per unit is not constant in output.

Third-Degree Price Discrimination

Third-Degree Price Discrimination

- The monopolist charges different prices to group of customers with different demands.
  - Its needs to identify which group the customer belongs to.
- Mathematically, the monopolist treats each group of customers as a separate monopoly.
  - Customers in one group cannot resell the good to customers in another group (i.e., there is no arbitrage condition).
- The monopolist finds the marginal revenue curve for each demand function, and it sets each of them equal to the firm's marginal cost.

Third-Degree Price Discrimination

- *Example 11.4: Third-degree price discrimination.*
  - Consider a small town with only one movie-theater.
  - As a monopolist, the movie theater faces 2 groups of customers, which it can easily distinguish by checking if they have student ID:
    - *Students*, who have a lower WTP, captured by $p_1(q) = 10 - q$.
    - *Non-students*, who have a higher WTP, measured by $p_2(q) = 25 - q$.
  - The marginal cost of a ticket is the same for both types of customers, $MC = \$3$.

Third-Degree Price Discrimination

- *Example 11.4* (continued):
  - The monopolist seeks to maximize its profits from both groups,

$$\max_{q_1,q_2} \pi = \pi_1 + \pi_2 = \underbrace{(10 - q_1)q_1 - 3q_1}_{\pi_1} + \underbrace{(25 - q_2)q_2 - 3q_2}_{\pi_2}.$$

  - Differentiating with respect to q_1,

$$10 - 2q_1 = 3 \Rightarrow q_1 = 3.5 \text{ tickets.}$$

  - Differentiating with respect to q_2,

$$25 - 2q_2 = 3 \Rightarrow q_2 = 11 \text{ tickets.}$$

Third-Degree Price Discrimination

- *Example 11.4* (continued):
  - Since profits from each group only depends on the number of tickets sold to that group, the PMP can alternative written as two separate problems:

$$\max_{q_1} \pi_1 = (10 - q_1)q_1 - 3q_1, \qquad (\textit{Students})$$

$$\max_{q_2} \pi_2 = (25 - q_2)q_2 - 3q_2. \qquad (\textit{Non-students})$$

  - The firm treats each group as a separate monopoly, setting the monopoly rule $MR = MC$:
    - *Students*: $MR_1 = MC$,

$$10 - 2q_1 = 3 \Longrightarrow q_1 = 3.5 \text{ units.}$$

$$p_1 = 10 - 3.5 = \$6.5.$$

Third-Degree Price Discrimination

- *Example 11.4* (continued):
  - *Non-students*: $MR_2 = MC$,

$$25 - 2q_2 = 3 \Longrightarrow q_2 = 11 \text{ units.}$$
$$p_2 = 25 - 11 = \$14.$$

- As a result, total profits become

$$\pi = \pi_1 + \pi_2 = [(6.5 \times 3.5) - (3 \times 3.5)] + [(14 \times 11) - (3 \times 11)]$$
$$= 12.25 + 121$$
$$= \$133.25.$$

Screening

- In example 11.3, students pay much less than non-students at movies ($\$6.50$ vs. $\$14$).
- Customers might try to pose as part of the low-demand group to buy at a lower price:

What can the monopolist do to avoid such a strategy?

Screening

- The firm can use screening to infer the customer's unobserved demand. Screening must satisfy key properties to work:
    1) It must be perfectly observable.
    2) It must be strongly correlated with the customer's WTP.

- *Example*:

A student ID can be observable by an employee of the movie theater, and it is negatively correlated with the customer's WTP.

Bundling

Bundling

- *Example*: You can buy a desktop computer as a whole (monitor + CPU + keyboard + mouse) or buy each unit separately.
- Three forms of bundling:
  - *No bundling*, the firm does not bundle any good, e.g., the buyer can purchase each part of the computer separately.
  - *Pure bundling*, the firm allows the buyer to purchase either the bundle, e.g., the whole computer, or no good at all.
  - Mixed bundling, the firm sets prices for each individual item and for the bundle, the buyer can choose whether to buy an item or the bundle.
- The monopolist can increase profits by offering pure bundling as long as the customer's demand for the different items is negatively correlated.

Bundling

- *Example 11.5:* ***Bundling.***
  - Consider a monopolist selling computers.

Table 11.1

	CPU	Monitor	Both items (Computer)
Consumer 1 WTP	$\$500$	$\$100\beta$	$\$500 + \$100\beta$
Consumer 2 WTP	$\$500\alpha$	$\$100$	$\$500\alpha + \$100$
Average cost (cost/unit)	$\$400$	$\$80$	$\$400 + \$80$

$\alpha, \beta \in (0,1)$

- Consumer 1 has the higher WTP for the CPU, but the lower for monitor.
- Consumer 2 has the higher WTP for the monitor, but the lower for CPU.
- Assume, consumer 1 has a higher WTP for the bundling,

$$\$500 + \$100\beta > \$500\alpha + \$100.$$

Bundling

- *Example 11.5* (continued):
  - *No bundling*. The firm sells the CPU either at $\$500$ or $\$500\alpha$, where $\$500 > \$500\alpha$.

CPU	$\$500\alpha$	$\$500$
Which consumer buy?	1 and 2	1
Profits	$= (2 \times 500\alpha)$ $- (2 \times 400)$ $= 1{,}000\alpha - 800$	$= 500 - 400$ $= 100$

- The firm will choose to entice both consumers only if

$$1{,}000\alpha - 800 > 100 \Longrightarrow \alpha > 0.9.$$

- The firm entices both types of consumers when consumer 2's WTP for the CPU is relatively close to that of consumer 1 (i.e., α closer to 1).

Bundling

- *Example 11.5* (continued):
  - *No bundling*. The firm sells the monitor either at $\$100$ or $\$100\beta$.

Monitor	$\$100\beta$	$\$100$
Which consumer buy?	1 and 2	2
Profits	$= (2 \times 100\beta) - (2 \times 80)$ $= 200\beta - 160$	$= 100 - 80$ $= 20$

- The firm will choose to entice both consumers only if
$$200\beta - 160 > 20 \Longrightarrow \beta > 0.9.$$
- The firm entices both types of consumers as long as consumer 1's WTP for the monitor is relatively close to that of consumer 2 (i.e., β closer to 1).

Bundling

- *Example 11.5* (continued):
  - *Bundling*. With pure bundling, the firm has 2 pricing options to sells the whole computer.

Bundle	$\$500\alpha + \$100$	$\$500 + \$100\beta$
Which consumer buy?	1 and 2	1
Profits	$= 2 \times (500\alpha + 100) - (2 \times 480)$ $= 1{,}000\alpha - 760$	$= (500 + 100\beta) - 480$ $= 20 + 100\beta$

    - The firm will choose to entice both consumers only if

$$1{,}000\alpha - 760 > 20 + 100\beta,$$
$$\alpha > 0.78 + 0.1\beta \equiv \bar{\alpha}.$$

    - We analyze what happens in six regions.

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

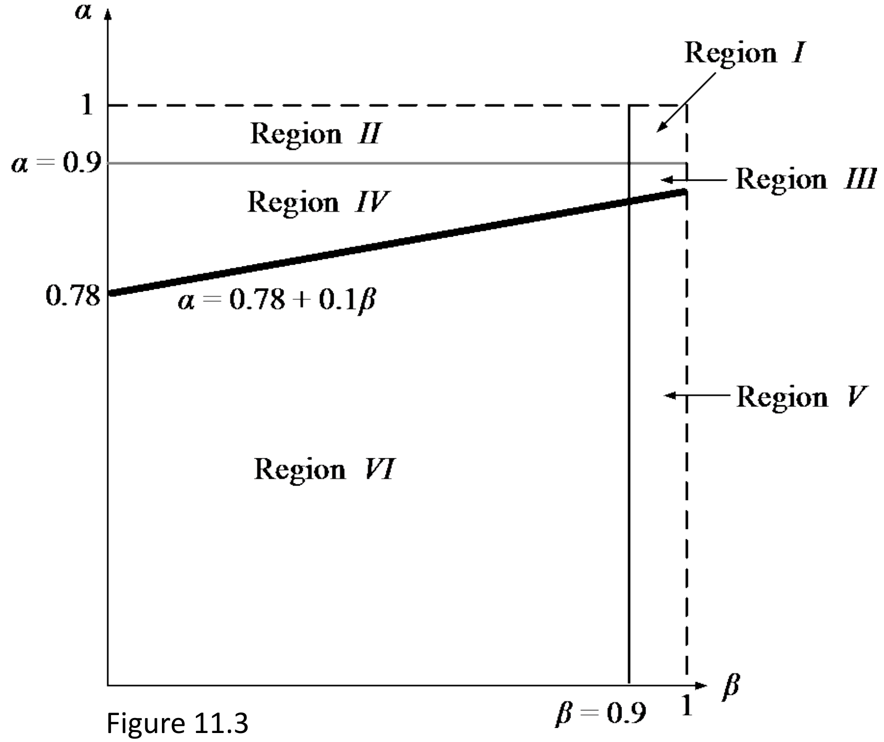

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

  Region I. If $\alpha > 0.9$ and $\beta > 0.9$, condition $\alpha > \bar{\alpha}$ holds.

The firm prefers to sell the CPU, the monitor, and the bundle to both customers.

It prefers to sell the bundle rather the separated items because

$$\underbrace{1{,}000\alpha - 760}_{\text{Profits from bundle}} > \underbrace{1{,}000\alpha - 800}_{\text{Profits from CPU}} + \underbrace{200\beta - 160}_{\text{Profits from monitor}},$$

$$-760 > 200\beta - 960,$$
$$\beta < 1,$$

which holds by assumption (negative correlated demands)

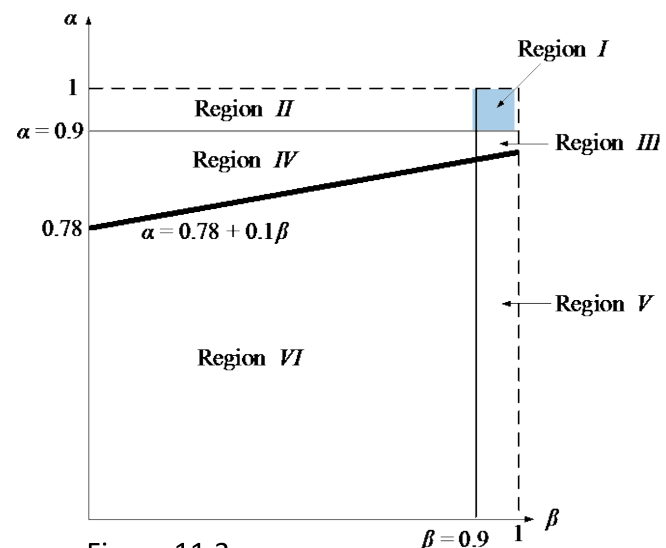

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

Region II. If $\alpha > 0.9$ but $\beta < 0.9$, condition $\alpha > \bar{\alpha}$ still holds.

The firm sells the CPU and the bundle to both customers, and the monitor to customer 2 alone.

The firm offers bundling given that

$$\underbrace{1{,}000\alpha - 760}_{\text{Profits from bundle}} > \underbrace{1{,}000\alpha - 800}_{\text{Profits from CPU}} + \underbrace{20}_{\text{Profits from monitor}},$$

$$780 > 760.$$

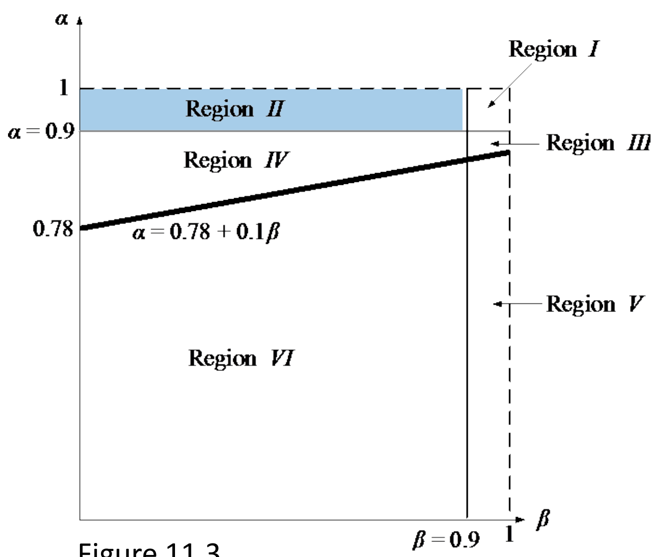

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

Region III. If $\alpha < 0.9$, $\beta > 0.9$, and $\alpha > \bar{\alpha}$.

The firm sells the monitor and the bundle to both consumers, but CPU to customer 1 alone.

The firm offers bundling because

$$\underbrace{1{,}000\alpha - 760}_{\text{Profits from bundle}} > \underbrace{100}_{\text{Profits from CPU}} + \underbrace{200\beta - 160}_{\text{Profits from monitor}},$$

$$1{,}000\alpha > 700 + 200\beta,$$
$$\alpha > 0.7 + 0.2\beta.$$

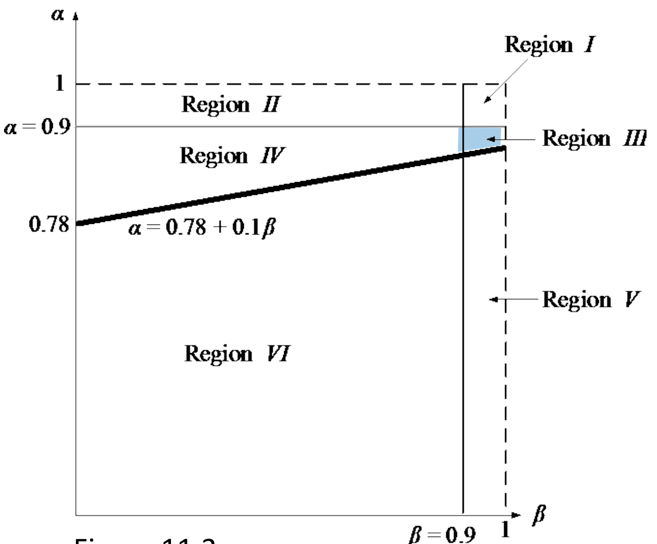

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

Region III. If $\alpha < 0.9$, $\beta > 0.9$ and $\alpha > \bar{\alpha}$ (cont.).

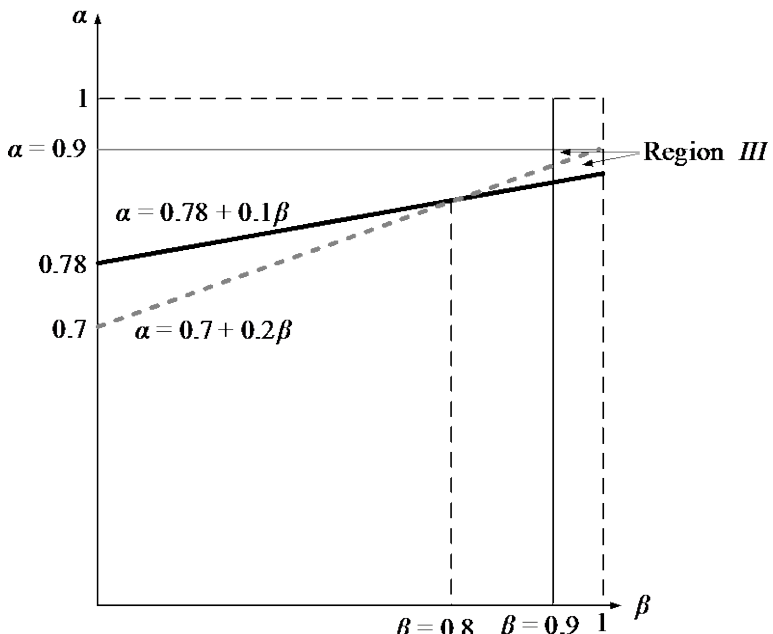

Figure 11.4

If we plot the line $\alpha = 0.7 + 0.2\beta$, Region III is divided in two areas:

- In the area above the dashed line, $\alpha > 0.7 + 0.2\beta$ holds, and the firm prefers to bundle.
- In the area below the dashed line, this condition is violated, and the firm sells each item separately.

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

  Region IV. If $\alpha < 0.9$, $\beta > 0.9$, and $\alpha > \bar{\alpha}$.

  The firm sells the bundle to both customers, the CPU to customer 1 alone, and the monitor to customer 2 alone.

  The firm offers bundling because

$$\underbrace{1{,}000\alpha - 760}_{\text{Profits from bundle}} > \underbrace{100}_{\text{Profits from CPU}} + \underbrace{20}_{\text{Profits from monitor}},$$

$$1{,}000\alpha > 880,$$
$$\alpha > 0.88.$$

Because condition $\alpha > \bar{\alpha}$ is satisfied, and cutoff $\bar{\alpha}$ reaches its highest point at 0.88, $\alpha > 0.88$ holds.

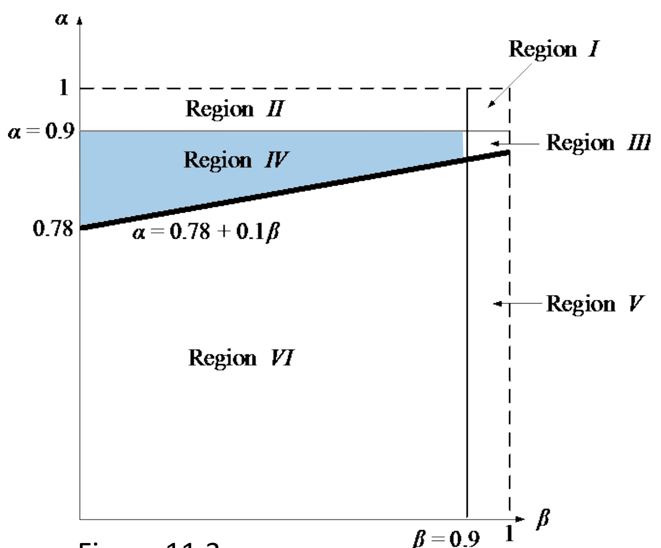

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

Region V. If $\alpha < 0.9$, $\beta > 0.9$, and $\alpha < \bar{\alpha}$.

The firm sells the monitor to both customers, the CPU to customer 1 alone, and the bundle to customer 1 alone.

The firm does not offer bundling because

$$\underbrace{20 + 100\beta}_{\text{Profits from bundle}} < \underbrace{100}_{\text{Profits from CPU}} + \underbrace{200\beta - 160}_{\text{Profits from monitor}},$$

$$80 < 100\beta,$$
$$0.8 < \beta,$$

which holds because $\beta > 0.9$ is satisfied by all points in this region.

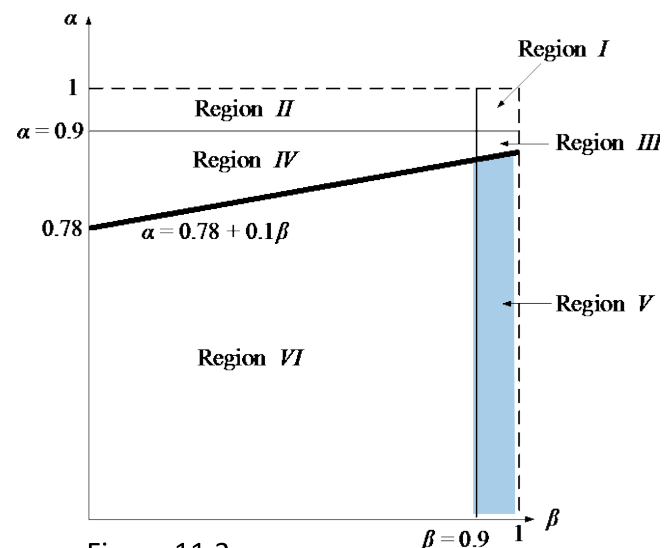

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - *Bundling* (cont.).

  Region VI. If $\alpha < 0.9$, $\beta < 0.9$, and $\alpha < \bar{\alpha}$.

  The firm sells the CPU to customer 1 alone, the monitor to customer 2 alone, and the bundle to customer 1 alone.

  Offering bundling is unprofitable because

$$\underbrace{20 + 100\beta}_{\text{Profits from bundle}} < \underbrace{100}_{\text{Profits from CPU}} + \underbrace{20}_{\text{Profits from monitor}},$$

$$100\beta < 100,$$
$$\beta < 1,$$

which holds by assumption (negative correlated demands)

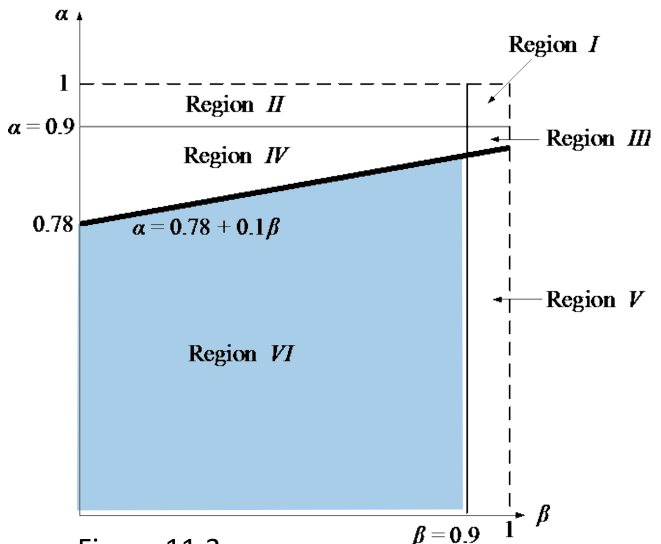

Figure 11.3

Bundling

- *Example 11.5* (continued):
  - In summary:
    - The firm finds bundle profitable in Regions I, II, and IV, which can be defined by condition $\alpha > \bar{\alpha}$, and in the top area of Region III, defined by $\alpha > 0.7 + 0.2\beta$.

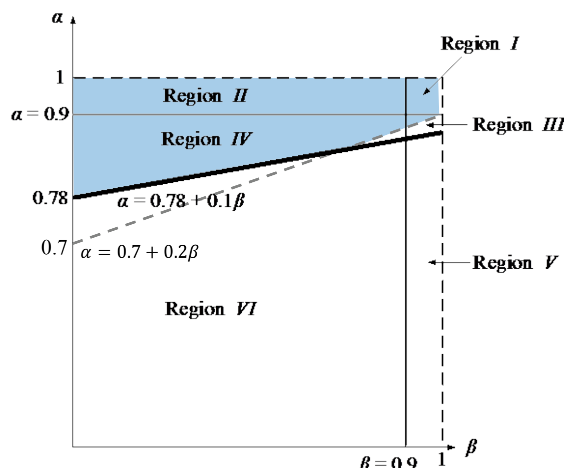

Figure 11.3

    - Otherwise, the firm sells each item separately.