Asymmetric regulators in polluting mixed oligopolies: Agency problems and second-mover advantage∗

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Abstract

We investigate privatization decisions in a mixed oligopoly market, with and without environmental regulation. We consider three agents: the manager of the public firm, the environmental agency, and the regulator choosing privatization levels; allowing them to assign different weights to pollution. When environmental policy is absent, we find that privatization decisions in equilibrium suffer from agency problems, yielding potentially inefficient privatizations. When environmental regulation is present and privatization decisions precede this regulation, privatizations have no impact on equilibrium output; while the opposite holds when environmental policy is chosen first. Our results, then, identify the presence of a second-mover advantage when asymmetric government agencies act sequentially.

Keywords: Mixed Oligopoly, Privatization, Emission fees, Asymmetric regulators.

JEL classification: D43, H23, L33, Q58

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1 Introduction

Mixed oligopolies, where private and public firms coexist, are common in several industries, such as banking services, university education, and airlines. They are also frequently observed in polluting industries. Examples include railways in the US and Canada, electricity and steel industry in China, car automakers in France, biochemical companies in the UK, and the energy sector in New Zealand. In this paper, we investigate privatization decisions in this type of market, how these decisions are affected by the presence or absence of environmental regulation, and the potential conflict arising between different government agencies.

Our model considers a mixed oligopoly where a public firm competes with several private companies. As a benchmark, we first examine a context in which firms do not face environmental policy. In this setting, the regulator chooses the degree of privatization of the public firm or, alternatively, the relative weight that this firm assigns on profits and welfare; and, given this privatization level, firms compete à la Cournot in the second stage. For generality, we allow for the regulator and the manager of the public firm to exhibit different environmental preferences (i.e., marginal environmental damage from additional pollution), such an asymmetry has not yet been studied in this context. We show that the firm remains public when the regulator assigns a weakly larger weight to pollution than the manager, which embodies the setting of symmetric preferences as a special case; as in Matsumura (1988). However, if the regulator assigns less importance to environmental damages than the manager, a partial privatization is socially optimal to the regulator, which increases as the preference divergence between the regulator and manager becomes more intense.

As expected, the preference divergence generates an agency problem. In particular, the regulator choosing the privatization level is the principal and the manager of the public firm is the agent. In this context, the principal (regulator) anticipates the agent’s (manager’s) incentives in the subsequent stage, and adjusts privatization decisions, giving rise to a socially excessive privatization of the public firm. This outcome occurs when the manager assigns a higher weight on environmental damages than the regulator; otherwise, privatization decisions are viewed as socially efficient to the regulator.

We then examine how results are affected when firms are subject to environmental policy. In that setting, the regulator still chooses the privatization level in the first stage, as those decisions are often difficult to adjust. Observing the regulator’s choice, the environmental protection agency (EPA) responds with an emission fee seeking to curb firms’ pollution; and, finally, firms compete in the last stage. We demonstrate that, when the public firm is not fully public (completely or partially privatized), the EPA responds setting a positive emission fee to curb pollution. In addition, fees become more (less) stringent when the EPA (public firm’s manager, respectively) assigns a higher weight on environmental damages. Intuitively, the EPA anticipates that a more environmentally concerned manager will reduce output and, thus, pollution, making stringent fees less necessary.

\[1\text{For more examples, see, for instance, Pal and Saha (2015), and references within.}\]

\[2\text{When the public firm remains fully public, however, the EPA anticipates that this firm will produce the manager’s}\]
In the first stage, the regulator anticipates that the EPA will induce its socially optimal output in the next period. In this context, changes in privatization decisions do not affect aggregate output in the last stage, nor welfare in equilibrium. That is, the specific privatization level becomes irrelevant when the EPA is present.

Comparing our results with and without environmental policy, we first show that, as expected, aggregate output is higher when there is no environmental regulation, which the EPA views as socially excessive pollution. When firms face environmental policy, this inefficiency is ameliorated, especially when the EPA and the regulator choosing privatization levels assign similar environmental weights on pollution. When their weights differ, however, inefficiencies still arise due to socially insufficient (excessive) output from the regulator’s (EPA’s) perspective.

For comparison purposes, we extend our model, allowing for the EPA to become the first mover, followed by the regulator responding with privatization decisions. This setting helps to examine contexts in which the EPA cannot easily revise environmental regulation. We show that the regulator making privatization decisions enjoys a second-mover advantage. Indeed, for every emission fee set by the EPA, the regulator responds with a privatization decision that induces the same aggregate output, also yielding the same welfare level. This result is analogous to that when the EPA acts in second place, and where this agency benefits from a second-mover advantage inducing its most preferred aggregate output. Therefore, both agencies prefer to act second, as that lets them adjust their choice variable, ultimately inducing their most preferred output and welfare according to how they perceive the severity of environmental damage. We measure this second-mover advantage, finding that it is nil when agencies’ preferences over pollution are symmetric, but increases when their preference divergence expands. The latter is relatively common in most countries, where the EPA mainly cares about environmental damages, assigning a small weight to consumer surplus and profits, whereas privatization policy generally overlooks the negative effects of pollution, yielding a large preference asymmetry between government bodies.

Our findings provide several policy implications. First, governments should avoid implementing environmental regulations in mixed oligopoly markets if this policy is relatively difficult to adjust. Indeed, our results suggest that, when the EPA acts first, privatization decisions dictate aggregate output in equilibrium, which are not affected by emission fees and align more closely with the regulator’s environmental concerns. In contrast, when privatization decisions are more difficult to revise than environmental regulation, emission fees induce the EPA’s preferred aggregate output according to their environmental concerns. In this context, privatization policy becomes ineffective at changing output levels. As a consequence, privatization decisions could be ignored when governments anticipate that they are difficult to adjust or, alternatively, when they are followed by environmental regulation.

socially optimal output (according to her weight on environmental damages) which is, importantly, unaffected by the stringency of emission fees.
1.1 Related literature

The literature on partial privatization in mixed oligopolies stems mainly from Matsumura (1998). Subsequent papers extended the study of mixed markets by focusing on several dimensions, such as policy impacts, product differentiation, foreign competition, and innovation. However, environmental policy is not considered in these works. Matsumura and Tomaru (2013), for instance, study the optimal tax-subsidy policies in mixed oligopolies finding that, if the burden of the tax is considered, privatization of the public firm impacts welfare. For models that investigate asymmetric costs where the public firm competes with foreign firms, see Pal and White (1998), Chang (2005), and Wang et al. (2009); for R&D investment and its effects in output competition, see Ishibashi and Matsumura (2006); for merger decisions in mixed oligopolies, see Kamijo and Nakamura (2009); for the effects of product differentiation, see Barcena-Ruiz and Garzon (2003), Matsumura and Matsushima (2003, 2004), and Li (2006); for the effects of allowing entry, see Fujiwara (2007); and for the effects of cross-ownership on privatization decisions, see Jain and Pal (2011). Finally, Amir and De Feo (2014) allow for endogenous timing in mixed oligopolies, showing that Stackelberg competition can be sustained in the subgame perfect equilibrium of the game, as supposed to games where all firms are private.

While some papers in this literature consider environmental policy (emissions fees and abatement subsidies) they assume that all agents assign the same weight on environmental damages. Mujumdar and Pal (1998) investigate a mixed duopoly which faces an exogenous fee finding that total output is unaffected by the fee and instead shifts output to the public firm. Barcena-Ruiz and Garzon (2006) allow for endogenous environmental regulation and firms investing in pollution abatement, focusing on whether the public firm should be fully public or private; but still assuming that all agents assign the same weight on pollution. This setting is extended to show that partial privatization is optimal by Pal and Saha (2015). Haruna and Goel (2018) also consider a polluting and abating mixed oligopoly but focus instead on whether optimal environmental policy is a tax or a subsidy, and not on the privatization of the public firm. In a similar context, Wang and Wang (2009) allow for product differentiation in a mixed duopoly, showing that abatement is higher in the mixed oligopoly than the private market. Elnaboulsi et al. (2022) evaluate environmental policy in a context where the regulator and private firms face cost uncertainties, showing that emission fees are affected by the regulator’s information precision.

Using a setting similar to that of Barcena-Ruiz and Garzon (2006) and Pal and Saha (2015), we study partial privatization in a mixed oligopoly with endogenous emission fees. However, we allow for each agent to assign a different weight on pollution, we compare privatization decisions with and without environmental regulation, and evaluate the output effect of changing the timing of the privatization decision and the emission fee.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 (4)
examines equilibrium behavior without (with) environmental policy, comparing equilibrium output with and without regulation. Section 5 studies how our results are affected if, instead, the EPA sets emission fees in the first stage, identifying each agency’s second-mover advantage; and section 6 concludes.

2 Model

Consider a mixed oligopoly with one publicly owned firm (denoted as firm 0) and \( N \geq 1 \) private firms. In this context, aggregate output is \( Q \equiv q_0 + \sum_{i=1}^{N} q_i \), where \( q_0 \) and \( q_i \) denote the public and private firms’ output, respectively, and the inverse demand function is \( p(Q) = 1 - Q \). For convenience, we also denote the aggregate output of private firms as \( Q_{-0} = \sum_{i=1}^{N} q_i \). Every firm faces a marginal cost \( c \), where \( 0 \leq c < 1 \). Every unit of output produces a unit of emissions \( e_j \), where \( j \in \{0, i\} \), and firms face a per-unit emissions fee \( t \). Each private firm \( i \) maximizes its profits

\[
\pi_i = (1 - Q)q_i - cq_i - tq_i
\]

while the public firm maximizes a combination of social welfare and profits

\[
V_0 = \alpha WP + (1 - \alpha) \pi_0
\]

where \( \pi_0 = p(Q)q_0 - cq_0 - tq_0 \) denotes the public firm’s profits; social welfare is given by \( WP = CS + PS + T - ED \), and

\[
CS + PS = \int_0^Q p(y)dy - cQ - tQ
\]

represents consumer and producer surplus, respectively; \( T = tQ \) is total tax revenue from the emission fee, thus making tax revenue welfare neutral; and \( ED = d_P Q^2 \) is the environmental damage from emissions, where \( d_P \) denotes the weight that the manager of the public firm \( (P) \) assigns to pollution.

For generality, we allow the manager of the public firm to exhibit a different concern for pollution, \( d_P \), than the regulator, \( d_R \), and the EPA, \( d_E \). The asymmetry between \( d_R \) and \( d_P \) could indicate, for instance, that the manager of the public firm was appointed during a previous administration, thus not sharing the same environmental preferences as the current regulator, \( d_R \). A similar argument applies to the potential asymmetry between \( d_R \) and \( d_E \).

Our setting, however, allows for symmetry across all three players as a special case, \( d_R = d_P = d_E = d \); or between two of the three agents, \( d_R = d_P \neq d_E \), or \( d_R \neq d_P = d_E \).

Our model allows for different interpretations. According to a “technocrat” view, the EPA would observe the true environmental damage based on scientific reports, \( d_E \), while regulator and manager

\[ \text{footnote}{^5} \]There are typically recorded conflicts between the head of EPA and other government officials. During the G.W. Bush administration, for instance, Vice President Cheney, whose Energy Task Force recommended reducing regulations to promote coal, oil, and gas industries, frequently undermined Christine Whitman (head of EPA during 2001-2003). For other examples, see Fredrickson et al. (2018).
of the public firm would hold biased estimates of these reports based on their own preferences, \( d_R \) and \( d_P \). In an alternative approach, the regulator would represent citizens’ environmental concerns (through their vote in democratic elections), entailing that \( d_R \) would capture the true environmental damage of a population, whereas \( d_E \) and \( d_P \) could differ, potentially being higher or lower than \( d_R \).

We consider a three-stage game as follows:

1. The regulator chooses the privatization level, \( \alpha \).
2. Observing \( \alpha \), the EPA sets the emission fee \( t \) per unit of emissions.
3. Observing \( \alpha \) and \( t \), public and private firms compete à la Cournot.

### 2.1 Third stage

Operating by backward induction, we first identify equilibrium output in the third stage.

**Lemma 1.** Every private firm \( i \)'s best response function is \( q_i(q_0) = \frac{1 - c - t}{N + 1} - \frac{1}{N + 1}q_0 \), while that of the public company is

\[
q_0(q_i) = \frac{1 - c - t(1 - \alpha)}{2 + \alpha(2d_P - 1)} - \frac{1 + 2d_P\alpha}{2 + \alpha(2d_P - 1)}Nq_i.
\]

The private firm’s best response function is decreasing in \( t \) and \( c \), while unaffected by \( \alpha \). The public firm’s best response function is decreasing in \( c \), \( d_P \), \( t \), and \( N \). The slope of the public firm’s best response function decreases in \( \alpha \), while the intercept increases in \( \alpha \) if and only if \( d_P < \overline{d}_P \) where \( \overline{d}_P \equiv \frac{1 - c - t}{1 - c - t - \frac{1}{2}} \).

As expected, every firm’s best response function shifts downwards when the firm becomes less efficient (higher \( c \)). The intercept of the public firm’s best response function decreases as the environmental damage from emissions \( d_P \) increases, and increases in \( \alpha \) when the environmental damage from emissions \( d_P \) is relatively small. Both types of firms’ best response function decrease as the emission fee \( t \) becomes more stringent, thus indicating lower output levels.

Overall, the above results entail that, when pollution is severe (not severe) an increase in \( \alpha \) shifts the public firm’s best response function downward (upward), but produces a flattening effect in both cases. Intuitively, while the public firm produces fewer (more) units when \( \alpha \) increases, it responds less significantly to a given one-unit increase in \( q_i \).

The next lemma uses the above best response functions to identify equilibrium output.

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6In addition, the public firm’s best response function is flatter as emissions become more severe (higher \( d_P \)) and as the public firm assigns a larger weight on welfare (higher \( \alpha \)).
Lemma 2. In the third stage, equilibrium output levels are

\[ q_i(t, \alpha) = \frac{(1 - c - t)(1 + 2\alpha d_P) - \alpha(1 - c)}{2 + \alpha(2d_P - 1) + (1 - \alpha)N}, \quad \text{and} \]

\[ q_0(t, \alpha) = \frac{(1 - c - t)(1 - 2\alpha d_P N) + (N + 1)\alpha}{2 + \alpha(2d_P - 1) + (1 - \alpha)N}, \]

which are positive if \( t < \frac{(1 - c)[1 + \alpha(2d_P - 1)]}{1 + 2\alpha d_P} \equiv t_i \) and \( t < \frac{(1 - c)(2\alpha d_P N - 1)}{\alpha(2d_P N + N + 1) - 1} \equiv t_0 \), respectively, where \( t_i < t_0 \). In addition, both output levels are decreasing in the number of firms \( N \), if \( t < \frac{(1 - c)[1 + \alpha(2d_P - 1)]}{1 + 2\alpha d_P} \).

The ranking of cutoffs \( t_i \) and \( t_0 \) give rise to three regions according to the stringency of the emission fee \( t \): (i) when \( t < t_i \), both types of firms are active; (ii) when \( t_i \leq t \leq t_0 \), only the public firm is active; and (iii) otherwise, no type of firm is active. The next corollary evaluates aggregate output and its comparative static results.

Corollary 1. Aggregate output is,

\[ Q(t, \alpha) = Nq_i(t, \alpha) + q_0(t, \alpha) = \frac{(1 - c - t)(1 + (1 - \alpha)N) + \alpha t}{2 + \alpha(2d_P - 1) + (1 - \alpha)N}, \]

which is decreasing in the emission fee, \( t \), environmental damage, \( d_P \), and marginal cost, \( c \). Further, aggregate output increases in privatization \( \alpha \) if and only if \( d_P < \frac{1 - c + (N + 1)t}{2(N + 1)(1 - c - t)} \), and increases in the number of firms \( N \) if and only if \( d_P > \frac{1}{2} \left( \frac{1 - c}{1 - c - t} - \frac{1}{\alpha} \right) \).

The final condition in which aggregate output increases in the number of firms \( N \) is satisfied if \( d_P > \frac{1}{2}d_P \). To understand the above result, the following corollary evaluates aggregate output \( Q(t, \alpha) \) at different special cases.

Corollary 2. Aggregate output simplifies to the following expressions:

1. When all firms are private, \( \alpha = 0 \), aggregate output is \( Q(t, 0) = \frac{(1 - c - t)(N + 1)}{N + 2} \), which increases in \( N \).

2. When there is a fully public firm, \( \alpha = 1 \), aggregate output is \( Q(t, 1) = \frac{1 - c}{2d_P + 1} \).

3. When the manager of the public firm assigns no weight to environmental damage, \( d_P = 0 \), aggregate output is \( Q(t, \alpha) = \frac{(1 - c - t)[1 + (1 - \alpha)N] + \alpha t}{2 - \alpha + (1 - \alpha)N} \), which increases in \( N \) if and only if \( t < (1 - c)(1 - \alpha) \).

4. Without the EPA, \( t = 0 \), aggregate output becomes \( Q(0, \alpha) = \frac{(1 - c)[1 + (1 - \alpha)N]}{2 + \alpha(2d_P - 1) + (1 - \alpha)N} \), which unambiguously increases in \( N \), and decreases in \( \alpha \) if and only if \( d_P > \frac{1}{2(N + 1)} \).
When all firms are private, \( \alpha = 0 \), and environmental regulation is absent, \( t = 0 \), we obtain the same results as in the standard Cournot setting, where aggregate output becomes \( Q(0,0) = \frac{(1-c)(N+1)}{N+2} \). When firms face emission fees, however, this aggregate output decreases to \( Q(t,0) = \frac{1-c-t}{N+2} \). In contrast, aggregate output changes to \( Q(t,1) = 1 - c \frac{dP}{2dP+1} \) when, still facing emission fee \( t \), there is a fully public firm, \( \alpha = 1 \). In this context, if the public firm and EPA place the same weight on environmental damages, \( dP = dE \), aggregate output coincides with the socially optimal output, i.e., \( Q(t,1) = Q^{SO} \), where \( Q^{SO} \) is decreasing in the weight that the manager of the public firm assigns to the environmental damage from emissions, \( dP \).

The results from the third stage are used in the following two sections with and without the EPA present.

3 Equilibrium Analysis without EPA

As a baseline, this section analyzes the case in which environmental regulation is absent. Hence, using the results in Lemma 2 and considering \( t = 0 \), we focus on the first stage.

3.1 First stage - EPA is absent

In the first stage, the regulator chooses the privatization level \( \alpha \) by solving

\[
\max_{\alpha \in [0,1]} W = \int_0^{Q(0,\alpha)} p(y)dy - cQ(0,\alpha) - dR [Q(0,\alpha)]^2
\]

which is evaluated at the equilibrium aggregate output \( Q(0,\alpha) \); as identified in Corollary 2 (point 4). Recall that the regulator considers a weight \( dR \) on pollution.

**Proposition 1.** When the EPA is absent, the regulator’s optimal level of privatization is:

1. If \( dR < \frac{1}{2(N+1)} \), then \( \alpha^*(dR,dP) = 0 \) and the public firm is completely privatized.

2. If \( \frac{1}{2(N+1)} \leq dR < dP \), then \( \alpha^*(dR,dP) = \frac{2dR(N+1)-1}{2(NdR+dP)-1} \), which satisfies \( 0 < \alpha^*(dR,dP) < 1 \), is increasing in \( dR \) and \( N \), but decreasing in \( dP \).

3. If \( dR \geq dP \), then \( \alpha^*(dR,dP) = 1 \) and a fully public firm is socially optimal to the regulator.

Proposition 1 identifies privatization decisions according to different values of environmental damage \( dR \) and \( dP \). When the regulator assigns a low value to environmental damages, \( dR < \frac{1}{2(N+1)} \), she completely privatizes the public firm regardless of the weight that the manager of the public firm assigns to pollution. Figures 1a and 1b depict this case for values of \( dR \) close to the origin (\( dR < 1/4 \) when \( N = 1 \) in figure 1a and \( dR < 1/10 \) when \( N = 4 \) in figure 1b). When the regulator assigns an intermediate weight, she privatizes a share of the public firm (interior solution, as illustrated by the curves in figures 1a and 1b). If, instead, \( dP \) increases, \( \alpha^* \) shifts downward, indicating that the
regulator privatizes a larger share of the firm. Finally, when the regulator’s weight on pollution is relatively high, \( d_R \geq d_P \), she does not privatize the firm; which includes the case in which regulator and the manager of the public firm are symmetric in their preferences, \( d_R = d_P = d \).

Figure 1: Optimal privatization levels, \( \alpha^* \).

Figure 1b illustrates that, as more private firms operate, the regulator has less incentives to privatize: she completely privatizes the public firm under a more restrictive range of \( d_R \) (left side of the figures), and when \( \alpha^* \) is interior it shifts upwards relative to figure 1a. Intuitively, the presence of more private firms produces more pollution, and the regulator uses a larger \( \alpha^* \) (less privatization) as a “pollution-curbing” tool when she is concerned about pollution.

To better understand the role of the players’ preferences in the results, figure 2 solves for \( d_R \) in \( \alpha^*(d_R, d_P) \), yielding \( d_R = \frac{1 - \alpha(1 - 2d_P)}{2[1 + N(1 - \alpha)]} \). This expression identifies, for a given privatization level \( \alpha^* \), the \((d_R, d_P)\)-pairs for which \( \alpha^* \) is optimal, i.e., level sets or “iso-privatization” curves. First, note that these level sets have a weakly positive slope. Intuitively, this means that, when the manager of the public firm assigns a larger weight on environmental damages, the regulator must also increase the importance that she assigns to these damages to keep the privatization level unchanged. Otherwise, an increase in \( d_P \) without an accompanying increase in \( d_R \) would induce the regulator to choose a lower value of \( \alpha^* \) (i.e., privatizing a larger share of the public firm). Second, the level sets associated to more intense privatizations (lower \( \alpha^* \)) graphically lie further away from the 45-degree line, indicating more asymmetric preferences between regulator and manager.

When the regulator assigns more weight on environmental damages than the public firm does, \( d_R \geq d_P \) (in the southeast region in figure 2), the regulator anticipates excessive output (and pollution), and keeps this firm fully public. This includes the case of symmetric preferences between the regulator and the manager of the public firm, \( d_R = d_P = d \), as in Matsumura (1998), in the 45-degree line of the figure. If, instead, \( d_R < d_P \), the regulator anticipates output to be insufficient
and chooses to privatize a share of it. In addition, an increase in the number of private firms (as depicted in figure 2b) hinders privatization decisions, intuitively requiring a larger asymmetry between the regulator and the manager of the public firm. We next evaluate equilibrium welfare in this setting.

Corollary 3. Equilibrium welfare when the EPA is absent is

\[
W_{NoEPA} = \left\{ \begin{array}{ll}
(1-c)^2(N+1)(N+3-2d_R(N+1)) & \text{if } d_R < \frac{1}{2(N+1)}, \ (\alpha^* = 0), \\
\frac{(1-c)^2}{2(2d_R+1)} & \text{if } \frac{1}{2(N+1)} \leq d_R < d_P, \ (\alpha^* \in (0,1)) \\
\frac{(1-c)^2[2(2d_P-d_R)+1]}{2(2d_P+1)^2} & \text{if } d_P \leq d_R, \ (\alpha^* = 1),
\end{array} \right.
\]

which decreases in \(d_R\) and \(c\). In addition, it is increasing in \(N\) in the first case, and in \(d_P\) in the third case.

Intuitively, as the regulator assigns a higher weight on environmental damages, welfare decreases. Welfare in this case does not depend on the weight that the manager of the public firm places on welfare (though that induces more privatization), as choosing \(\alpha\) in this stage aligns the incentives of the public firm with the regulator.

3.2 Socially excessive privatizations

The difference \(SEP = \alpha^*(d_R,d_R) - \alpha^*(d_P,d_R)\) measures how the regulator’s privatization decisions are affected by the agency problem between her and the manager of the public firm. Since \(\alpha^*(d_R,d_R) = 1\) when agents are symmetric, the above expression simplifies to \(SEP = 1 - \alpha^*(d_P,d_R)\), which satisfies \(SEP \in [0,1]\), and reflects “socially excessive privatizations.” Given that \(SEP \geq 0\) under all parameter values, the preference asymmetry induces the regulator to choose a lower \(\alpha^*\) than she would otherwise, privatizing too much of the public firm. The next corollary identifies \(SEP\) and its comparative statics.
Corollary 4. Inefficient privatizations are:

1. If \( d_R < \frac{1}{2(N+1)} \), then \( SEP = 1 \).

2. If \( \frac{1}{2(N+1)} \leq d_R < d_P \), then \( SEP = \frac{2(d_P - d_R)}{2(Nd_R + d_P) - 1} \), which is unambiguously increasing in \( d_P \), but decreasing in \( d_R \) and \( N \).

3. If \( d_R \geq d_P \), then \( SEP = 0 \).

Figure 3: SEP given different levels of the regulator’s evaluation of environmental damage \( d_R \).

Figure 3 illustrates \( SEP \), as a function of \( d_R \) on the horizontal axis, and evaluated at different values of \( d_P \). For a given \( d_P \), an increase in \( d_R \) (rightward movement along the curve) decreases \( SEP \), approaching it to zero where agents are symmetric in their environmental concerns, \( d_P = d_R \). Overall, when the manager of the public firm assigns more weight on pollution than the regulator, \( d_R < d_P \), \( SEP \) becomes positive, and privatizations are excessive in equilibrium, moving rightward in the figure. Otherwise, when \( d_R \geq d_P \), the firm remains fully public, \( \alpha^*(d_P, d_R) = 1 \), as shown in figure 3, privatizations coincide with those when agents are symmetric, \( \alpha^*(d_R, d_R) = 1 \), and \( SEP = 0 \) (see right side of the figure).

A natural question is whether the regulator’s strategic privatization decisions drives private firms to shut down under larger or more restrictive conditions than when being unregulated. To address this point, consider first a setting where the public firm is completely nationalized, \( \alpha = 1 \). In this context, if the manager of the public firm does not assign any weight to environmental damages, \( d_P = 0 \), as in Matsumura (1998), she seeks to produce the perfectly competitive output \( Q = 1 - c \), leaving no room for private firms to produce positive units. Otherwise, they would induce a negative price per unit, \( p < c \), earning negative profits. In contrast, if the manager of the public firm assigns a positive weight on environmental damages, \( d_P > 0 \), she seeks to produce less than the perfectly competitive output, \( Q < 1 - c \), to ameliorate aggregate pollution, leaving room for private firms to produce a positive output at a profit.

The above results are attenuated when the public firm is partially privatized, \( \alpha < 1 \), since this firm produces fewer units, both when \( d_P = 0 \) and otherwise. Therefore, private firms face positive
profit margins, and remain active, under most parameter conditions. Section 4 investigates if these findings are facilitated or hindered by the presence of environmental policy.

4 Equilibrium Analysis with EPA

4.1 Second stage

In the second stage the EPA first finds its aggregate socially optimal output, \(Q_{SO}\), then identifies the fee \(t\) that solves \(Q(t, \alpha) = Q_{SO}\) where privatization, \(\alpha\), is taken as given from the first stage. In this context, the EPA finds the socially optimal output by solving,

\[
\max_{\mathcal{Q}} W = \int_0^\mathcal{Q} (1 - y) \, dy - c\mathcal{Q} - d_E\mathcal{Q}^2,
\]

Differentiating with respect to \(\mathcal{Q}\) and solving, yields the socially optimal output \(\mathcal{Q}_{SO} = \frac{1 - c}{2d_E + 1}\), which coincides with the aggregate output when the regulator assigns full weight on social welfare, \(Q(t, 1) = \frac{1 - c}{2d_R + 1}\), if \(d_E = d_R\); but lies below \(Q(t, 1)\) if \(d_E > d_R\), and above otherwise. In addition, \(Q_{SO}\) is decreasing in firms’ costs (\(c\)), but decreasing in the EPA’s environmental damage, \(d_E\).

The next lemma characterizes the EPA decision in this stage.

**Lemma 3.** When \(\alpha < 1\), the EPA sets the emissions fee,

\[
t^* = \frac{(1 - c)[2d_E(1 + (1 - \alpha)N) - 1 - \alpha(2d_P - 1)]}{(1 - \alpha)(2d_E + 1)(N + 1)},
\]

which increases in \(d_E\) and \(\alpha\) if \(d_E > d_P\), while it is decreasing in \(d_P\), and increasing in \(N\) if and only if \(\alpha < \frac{1}{1 + d_E - d_P}\); and is positive if and only if \(d_E \geq \tilde{d}_E \equiv \frac{1 + \alpha(2d_P - 1)}{2(1 + (1 - \alpha)N)}\), where cutoff \(\tilde{d}_E\) is decreasing in \(N\), increasing in \(d_P\), and increasing in \(\alpha\) if and only if \(d_P > \frac{1}{2(1 + N)}\).

Therefore, when the regulator chooses a lower privatization level in the first stage (higher \(\alpha\)), the EPA responds setting a more stringent emission fee in the second stage. This indicates that government agencies treat each other’s choice variables (\(t\) and \(\alpha\)) as strategic complements, suggesting the potential for the EPA to benefit from a second-mover advantage, as we show in subsequent sections.

In addition, when the public firm is at least partially privatized, \(\alpha < 1\), the EPA sets a positive emission fee to curb aggregate pollution, which becomes more stringent as more firms join the industry if the public firm is sufficiently privatized, \(\alpha < \frac{1}{1 + 2(d_E - d_P)}\). In addition, this emission fee is increasing in the value that the EPA assigns to environmental damages, \(d_E\), but decreasing in the value that the manager of the public firm assigns to these damages \(d_P\). However, when the public firm is fully public (\(\alpha = 1\)), the EPA can anticipate that this firm will choose an optimal output according to its environmental damage \(d_P\), i.e., \(Q(t, 1) = \frac{1 - c}{2d_P + 1}\), as shown in Corollary 2, regardless of the EPA’s evaluation of environmental damages, \(d_E\). This makes the emission fee ineffective from
the EPA’s perspective, since, regardless of the fee, the public firm chooses an output which induces its own socially optimal output.

Furthermore, Lemma 3 identifies that the emission fee becomes more stringent when the market is more competitive (higher $N$); but less stringent when the public firm assigns a larger importance to environmental damages (higher $d_P$), since in this case it produces less output and pollution. In addition, the fee is more (less) stringent when a larger share of the public firm is privatized (lower $\alpha$) and the public firm’s environmental concern is sufficiently high (low, respectively).

Evaluating the equilibrium output of every private firm $i$ at the equilibrium fee $t^*$, yields

$$q_i(t^*, \alpha) = \frac{(1 - c) \left[ 1 - \alpha(1 + 2d_E - 2d_P) \right]}{(1 - \alpha)(N + 1)(2d_E + 1)}$$

which is positive for all $d_E < \bar{d}_E \equiv \frac{1 - \alpha(1 - 2d_P)}{2\alpha}$. Cutoff $\bar{d}_E$ is unambiguously decreasing in $\alpha$, increasing in $d_P$, and satisfies $\bar{d}_E > \tilde{d}_E$ for all parameter values. Therefore, when a larger share of the public firm is privatized (lower $\alpha$), private firms are active under larger conditions. Similarly, when the manager of the public firm assigns more weight to environmental damage (higher $d_P$), this firm produces fewer units, helping private firms stay active under larger conditions.

As a consequence, our results identify three output regions depending on $d_E$: (i) when $d_E < \bar{d}_E$, the EPA provides a per-unit subsidy to firms, which are all active; (ii) when $\tilde{d}_E \leq d_E < \bar{d}_E$, the EPA sets a positive but lax emission fee, inducing all private firms to remain active; and (iii) otherwise, the EPA sets a stringent emission fee inducing private firms to shut down. For simplicity, we focus our analysis on cases (i) and (ii), considering that $d_E < \bar{d}_E$ throughout the remainder of the paper.

4.2 First stage - EPA is present

If the EPA is present, the regulator anticipates that the EPA sets emission fee $t^*$ in the subsequent stage, inducing firms to produce a socially optimal output $Q^{SO}$. In this context, the regulator chooses the weight $\alpha$ that solves

$$W_{EPA} = \int_0^{Q(t^*, \alpha)} p(y) dy - cQ(t^*, \alpha) - d_R \left[ Q(t^*, \alpha) \right]^2$$

where individual and aggregate output levels are now evaluated at the equilibrium fee $t^*$ identified in Lemma 3; as opposed to the regulator’s problem in the absence of EPA, which was evaluated at $t = 0$. Once equilibrium emission fee and output are substituted in, the problem simplifies to

$$W_{EPA} = \max_{\alpha \in [0,1]} \frac{(1 - c)^2 [2(2d_E - d_R) + 1]}{2(2d_E + 1)^2},$$

which is not a function of $\alpha$. This implies that a continuum of privatization levels, $\alpha \in [0, 1]$, are socially optimal. Intuitively, the regulator anticipates that the EPA’s presence in the next stage will induce socially optimal output $Q^{SO}$ regardless of the privatization level she selects in the first stage. Therefore, when the EPA is present, every value of $\alpha$ produces the same welfare level $W_{EPA}$. 

13
The next corollary examines the comparative static of this welfare.

**Corollary 5.** *Equilibrium welfare when the EPA is present, \( W_{EPA} \), is decreasing in \( c \) if \( d_E > \frac{2d_R - 1}{d_R} \), \( d_R \), and in \( d_E \) if \( d_E > d_R \).

### 4.3 Output comparisons

In this section, we compare our equilibrium outcomes when the EPA is present (sections 4.1-4.2) and absent (section 3.1), to identify the effects of environmental regulation and privatization policy.

**Corollary 6.** *When the EPA is present, aggregate output is \( Q^{E}_{SO} = \frac{1 - c}{2d_E + 1} \), and when the EPA is absent aggregate output becomes \( Q^{R}_{SO} = \begin{cases} \frac{(1-c)(N+1)}{N+2} & \text{if } d_R < \frac{1}{2(N+1)}, \ (\alpha^* = 0) \\ \frac{1-c}{2d_R + 1} & \text{if } \frac{1}{2(N+1)} \leq d_R < d_P, \ (\alpha^* \in (0, 1)) \\ \frac{1-c}{2d_P + 1} & \text{if } d_P \leq d_R, \ (\alpha^* = 1) \end{cases} \)

which satisfies \( Q^{E}_{SO} \leq Q^{R}_{SO} \) for all \( d_E > \frac{1}{2(N+1)} \) in the first case, for all \( d_E \geq d_R \) in the second case, and for all \( d_E > d_P \) in the last case.

Intuitively, when the EPA assigns a larger weight to environmental damages than the regulator, \( d_E > d_R \), it sets stringent emission fees, curbing production (and pollution) below the aggregate output that the regulator induces when the EPA is absent. Alternatively, we can interpret \( Q^{E}_{SO} \) as the EPA’s socially optimal output, and \( Q^{R}_{SO} \) as the regulator’s socially optimal output. Therefore, from the perspective of the EPA, \( Q^{E}_{SO} \) is socially excessive, while the regulator finds it welfare maximizing. Similarly, from the regulator’s viewpoint, \( Q^{E}_{SO} \) is socially insufficient, while the EPA deems it ideal given its preferences. However, when agencies are symmetric, \( d_E = d_R \), they both induce the same aggregate output in equilibrium, \( Q^{E}_{SO} = Q^{R}_{SO} \).

### 5 Allowing for the EPA to move first

Consider now an alternative time structure where the EPA sets emission fee \( t \) in the first stage, the regulator responds choosing a privatization level \( \alpha \) in the second stage, and firms compete in the last stage. This setting corresponds to policy regimes where the EPA cannot easily revise environmental regulation, while privatization decisions, \( \alpha \), can be more rapidly adjusted. Our previous setting would describe, instead, countries where industrial policy is relatively rigid, or long lasting, while the EPA can easily adjust emission fees to market conditions.

**Third stage.** In this stage, aggregate output is still \( Q(t, \alpha) = \frac{2(1-c-t)-\alpha(1-c-2t)}{2\alpha(d_p-1)+3} \), as shown in Lemma 2.
**Second stage.** In the second stage, the regulator anticipates $Q(t, \alpha)$ in the subsequent stage, and chooses $\alpha$ to solve

$$\max_{\alpha \in [0,1]} W = \int_0^{Q(t,\alpha)} p(y)dy - cQ(t, \alpha) - d_R[Q(t, \alpha)]^2$$

The next proposition identifies the regulator’s socially optimal privatization level.

**Proposition 2.** When the EPA chooses emission fees in the first stage, the regulator’s socially optimal level of privatization is:

1. If $d_R < \frac{1-c+(N+1)t}{2(N+1)(1-c-t)}$, then $\alpha^*(d_R, d_P, t) = 0$ and the public firm is completely privatized.
2. If $d_R < \frac{1-c+(N+1)t}{2(N+1)(1-c-t)} < d_P$ or $d_R < \frac{(1-c)(2d_P-1)-(N+1)t}{2[2(N+1)(1-c-t)]}$, then
   $$\alpha^*(d_R, d_P, t) = \frac{(2d_R+1)(N+1)t-(1-c)(2d_P(N+1)-1)}{2d_R+1(N+1)(1-c-t)},$$
   which satisfies $0 < \alpha^*(d_R, d_P, t) < 1$, is increasing in $d_R$ if $d_P > \frac{1-c+(N+1)t}{2(N+1)(1-c-t)}$, decreasing in $d_P$ if $d_R > \frac{1-c+(N+1)t}{2(N+1)(1-c-t)}$, and decreasing in $t$ if $d_P > d_R$.
3. If $d_R \geq d_P$, then $\alpha^*(d_R, d_P, t) = 1$ and a fully public firm is optimal.

Therefore, the regulator now chooses a privatization level which is not only a function of $d_R$ and $d_P$, but also a function of the fee $t$ set by the EPA in the previous stage, $\alpha^*(d_R, d_P, t)$. In addition, this privatization level coincides with that in Proposition 1, $\alpha^*(d_R, d_P)$, when fees are absent, $t = 0$, that is, $\alpha^*(d_R, d_P, 0) = \alpha^*(d_R, d_P)$ in all cases. The comparative statics of $\alpha^*(d_R, d_P, t)$ are similar to those in Proposition 1, except for $\alpha^*(d_R, d_P, t)$ being decreasing in fee $t$ if and only if $d_P > d_R$. Intuitively, when the manager of the public firm cares more about pollution than the regulator, a more stringent fee (higher $t$) induces the regulator to respond privatizing a larger portion of the public firm (lower $\alpha^*$), since the regulator anticipates that the public manager will reduce its production, and thus pollution, in the last stage of the game. In contrast, when the regulator assigns more importance to pollution than the manager of the public firm, $d_P < d_R$, a more stringent fee drives the regulator to privatize a smaller share of the public firm. In this case, the regulator anticipates the public firm producing too much pollution, so keeping a larger portion in public hands is welfare improving.

**First stage.** At the beginning of the game, the EPA anticipates the regulator setting $\alpha^*(d_R, d_P, t)$, which entails an aggregate output $Q(\alpha^*(d_R, d_P, t), t) = \frac{1-c}{2d_R+1}$. Therefore, the EPA inserts $\frac{1-c}{2d_R+1}$ into its welfare function, yielding the following maximization problem:

$$\max_{t \geq 0} W = \frac{(1-c)^2[1-2(d_E+2d_R)]}{2(2d_R+1)^2}$$

which is not a function of emission fee $t$. This yields a continuum of emission fees in equilibrium or, alternatively, every emission fee $t$ produces the same welfare level. Intuitively, the EPA can
anticipate that the regulator responds with privatization level \( \alpha^*(d_R, d_P, t) \) in the next stage which, regardless of the specific fee \( t \) chosen by the EPA in the first stage, will induce the same aggregate output \( \frac{1-c}{2d_R+1} \).

5.1 Second-mover advantage

The regulator’s ability to respond to the EPA’s fee helps her strategically induce her socially optimal output \( \frac{1-c}{2d_R+1} \), instead of the EPA’s, \( \frac{1-c}{2d_E+1} \). This result is analogous to that in the previous section, where the regulator chooses \( \alpha \) and the EPA responds setting the emission fee \( t \). Therefore, the government agency that sets its policy last happens to enjoys a “second-mover advantage.” Indeed, this agency can decrease (increase) aggregate output if its environmental concerns are more (less) severe.

The next corollary evaluates the regulator’s second-mover advantage, understood as the welfare gain that this agent enjoys from moving second instead of first, \( SMA^R = W(d_R, Q_{SO}^R) - W(d_R, Q_{SO}^{EPA}) \); and, similarly, the EPA’s second-mover advantage, \( SMA^{EPA} = W(d_E, Q_{SO}^{EPA}) - Q(d_E, Q_{SO}^R) \).

**Corollary 7.** The regulator’s second-mover advantage is \( SMA^R = \frac{2(1-c)^2(d_R-d_E)^2}{(2d_E+1)^2(2d_R+1)^2} \), which is positive for all admissible parameters, increasing in \( d_E \), and decreasing in \( d_R \). Similarly, the EPA’s second-mover advantage is \( SMA^{EPA} = \frac{2(1-c)^2(d_E-d_R)^2}{(2d_E+1)(2d_R+1)^2} \), which is unambiguously positive, increasing in \( d_R \), and decreasing in \( d_E \).

Both \( SMA^R \) and \( SMA^{EPA} \) are positive for all \( d_E > d_R \) and zero when \( d_E = d_R \). In addition, the second-mover advantage for each agent decreases in its own evaluation of environmental damage and increases in the other agent’s evaluation. As the gap in the two agents’ preferences increases, \( d_E - d_R \), the benefit of moving second increases as the target output from each policy widens.

If \( d_E = d_R \), both agencies’ preferences are symmetric, entailing that they would like to induce the same aggregate output, and welfare. In this setting, equilibrium results with the EPA acting before or after the regulator coincide. However, when \( d_E > d_R \), the EPA prefers to choose emission fees after the regulator sets privatization levels, to induce its output, \( \frac{1-c}{2d_R+1} \), i.e., enjoying a second-mover advantage (\( SMA^{EPA} > 0 \)). The same argument applies to the regulator choosing \( \alpha \), who would prefer being the second mover when \( d_E > d_R \) (\( SMA^R > 0 \)).

6 Discussion

_Agency problem._ In the absence of environmental policy, we show that socially inefficient privatizations may arise in equilibrium. This inefficiency stems from the fact that the regulator choosing the public firm’s privatization level takes into account the divergence in environmental concerns between herself and the manager, and responds to this preference asymmetry setting a higher privatization level than she would have selected if their preferences were aligned, giving rise to excessive
privatization of the public firm. This is analogous to agency problems in principal-agent settings, where the regulator acts as a principal in our model (choosing $\alpha$) and the manager of the public firm is the agent (responds to $\alpha$ selecting the public firm’s output level).

**Ineffective EPA.** In the model where the EPA acts first, environmental policy is relatively difficult to adjust (e.g., requires majority in Congress). In that scenario, our results suggest that environmental regulation becomes, essentially, ineffective. This happens because the EPA’s decisions in the first stage are, subsequently, canceled out by privatization decisions. The latter induces the output preferred by the regulator making privatization decisions, instead of the output (and pollution) level that the EPA sought. This finding entails that governments should avoid implementing environmental regulation in mixed oligopoly markets if, importantly, such regulation is particularly rigid, i.e., more difficult to adjust than the public firm’s privatization decisions.

**Ineffective privatizations.** In contrast, the model where the EPA acts after privatization decisions reflects settings where emission fees can be easily adjusted, or, privatization decisions are relatively bureaucratic. In that setting, environmental policy induces its optimal output, canceling the privatization decision’s effect. In other words, regardless of whether the public firm is mostly public or largely privatized at the beginning of the game, the EPA induces its most preferred outcome. In this scenario, it is privatization policy which becomes ineffective; and could be abandoned when governments anticipate that it is difficult to adjust, and it is followed by a relatively flexible environmental regulation.

**Representability problems.** Our results also highlight that conflict between privatization decisions and environmental regulation originates in the preference asymmetry between these government agencies, with the EPA generally assigning a larger weight on pollution than the agency determining the public firm’s privatization level. If both agencies assign the same weight on pollution, no inefficiencies arise. However, that is quite uncommon in most countries where, often, the EPA (or respective regulator agency) is directed to mainly care about environmental damages and assign a small weight to other considerations, such as consumer surplus or profits (high $d_E$). In contrast, privatization decisions generally overlook environmental issues (low $d_R$), yielding a large preference asymmetry between government bodies, $d_E - d_R$, which ultimately aggravates the above conflict. In this policy scenario, the output level that the agency playing last induces will be extremely different from the one that the first mover sought. If the EPA acts first and privatization decisions follow, aggregate output in equilibrium will be substantially higher than what the EPA intended. If the EPA acts last, the opposite argument applies, with output being significantly lower than what the privatizing agency intended.

This result also suggests that, once society agrees on which environmental weight best represents their preferences, $d_E$ or $d_R$, a single government agency should be used to control pollution. Otherwise, it is only the agency playing last the one who dictates its preferences; canceling out the role of the agency acting in a previous stage. Informally, “one is better than two,” especially if the administrative cost of these agencies is particularly large.

**Privatization policy as an environmental tool?** When the regulator chooses the public firm’s pri-
vatization level, $\alpha$, she takes environmental damages into account, thus helping curb pollution. As a result, privatization (or nationalization) policies have environmental consequences which should not be overlooked. Nonetheless, our results suggest that, if the EPA acts after privatization decisions are implemented, this agency sets emission fees to induce its socially optimal output, implying that privatizations do not have ultimately any welfare effects. Alternatively, the role of privatization policy as a tool to control pollution only matters when the industry is not subject to environmental regulation or, if it is, that regulation happened before the privatization decision and will not be easily revised in subsequent periods.

7 Appendix

7.1 Proof of Lemma 1

The public firm solves

$$V_0 = \max_{q_0 \geq 0} \alpha W_P + (1 - \alpha) \pi_0$$

$$= \alpha \left[ \int_0^{q_0 + \sum_{i=1}^{N} q_i} p(y) dy - c \left( \sum_{i=1}^{N} q_i \right) - c q_0 - d_P \left( q_0 + \sum_{i=1}^{N} q_i \right)^2 \right]$$

$$+ (1 - \alpha) \left[ \left( 1 - (q_0 + \sum_{i=1}^{N} q_i) \right) q_0 - c q_0 - t q_0 \right]$$

Differentiating and solving for $q_0$, the public firm’s best response function is

$$q_0(q_i) = \frac{1 - c - t(1 - \alpha)}{2 + \alpha(2d_P - 1)} - \frac{1 + 2d_P \alpha}{2 + \alpha(2d_P - 1)} N q_i.$$ 

This best response function is decreasing in $d_P$:

$$\frac{\partial q_0(q_i)}{\partial d_P} = -\frac{2\alpha[1 - c - t(1 - \alpha) + (1 - \alpha)Nq_i]}{[2 + \alpha(2d_P - 1)]^2} < 0.$$ 

The slope of the public firm’s best response function decreases in $\alpha$:

$$\frac{\partial}{\partial \alpha} \frac{1 + 2d_P \alpha}{2 + \alpha(2d_P - 1)} = -\frac{1 + 2d_P}{[2 + \alpha(2d_P - 1)]^2} < 0.$$ 

The intercept of the public firm’s best response function decreases in $d_P$:

$$\frac{\partial}{\partial d_P} \frac{1 - c - t(1 - \alpha)}{2 + \alpha(2d_P - 1)} = \frac{1 - c - t(1 - \alpha)}{[2 + \alpha(2d_P - 1)]^2} < 0.$$
The intercept of the public firm’s best response function changes in $\alpha$ as follows:

$$\frac{\partial}{\partial \alpha} \left( \frac{1 - c - t(1 - \alpha)}{2 + \alpha(2d_P - 1)} \right) = \frac{1 + c(2d_P - 1) + 2d_P(t - 1) + t}{[2 + \alpha(2d_P - 1)]^2},$$

which increases if and only if $d_P < \overline{d_p}$ where $\overline{d_p} \equiv \frac{1-c}{1-c-t} - \frac{1}{2}$.

In the third stage, every private firm $i$ solves

$$\max_{q_i \geq 0} \pi_i = \left[ 1 - (q_0 + q_i + \sum_{j \neq i} q_j) \right] q_i - (c + t)q_i$$

Differentiating with respect to $q_i$, yields

$$1 - q_0 - 2q_i - \sum_{j \neq i} q_j - (c + t) = 0$$

In a symmetric equilibrium, every private firm produces the same output, $q_i = q_j$, implying that the above first-order condition simplifies to $1 - q_0 - (N + 1)q_i - c - t = 0$, and, solving for $q_i$, we find that every private firm $i$’s best response function is

$$q_i(q_0) = \frac{1 - c - t}{N + 1} - \frac{1}{N + 1}q_0.$$

### 7.2 Proof of Lemma 2

Solving the two best response functions for $q_i$ and $q_0$ gives the equilibrium output for each firm. Private firm output $q_i(t, \alpha) > 0$ if and only if $t < \frac{(1-c)[\alpha(2d_P-1) + 1]}{2ad_P + 1} \equiv t_i$, and public firm output $q_0(t, \alpha) > 0$ if and only if $t < \frac{(1-c)[2ad_PN-1]}{\alpha(2d_PN+N+1)-1} \equiv t_0$. If $d_P > \frac{\alpha^{-2}N(1-\alpha)}{\alpha}$, then $t_i < t_0$, which always holds as $\frac{\alpha-2-N(1-\alpha)}{\alpha} < 0$.

The comparative statics for public and private firm outputs with respect to the number of firms is

$$\frac{\partial q_i(t, \alpha)}{\partial N} = -\frac{(1 - \alpha)[\alpha(1 - c)(2d_P - 1) + 1 - c - t - 2ad_pl]}{[2 + N(1 - \alpha) + \alpha(2d_P - 1)]^2} < 0$$

$$\frac{\partial q_0(t, \alpha)}{\partial N} = -\frac{(2ad_P + 1)[\alpha(1 - c)(2d_P - 1) + 1 - c - t - 2ad_pl]}{[2 + N(1 - \alpha) + \alpha(2d_P - 1)]^2} < 0$$

Each comparative static is negative if $t < \frac{(1-c)[1+\alpha(2d_P-1)]}{1+2d_P\alpha}$, which is the condition needed for $q_i(t, \alpha) > 0$. 

19
7.3 Proof of Corollary 1

The comparative statics for aggregate output $Q(t, \alpha)$ are as follows:

\[
\begin{align*}
\frac{\partial Q(t, \alpha)}{\partial t} &= -\frac{(1-\alpha)(N+1)}{2 + N(1-\alpha) + \alpha(2d_P - 1)} < 0 \\
\frac{\partial Q(t, \alpha)}{\partial d_P} &= -\frac{2\alpha[(1-c-t)((1-\alpha)N+1) + \alpha]}{(2 + N(1-\alpha) + \alpha(2d_P - 1))^2} < 0 \\
\frac{\partial Q(t, \alpha)}{\partial c} &= -\frac{1 + N(1-\alpha)}{2 + N(1-\alpha) + \alpha(2d_P - 1)} < 0 \\
\frac{\partial Q(t, \alpha)}{\partial \alpha} &= -\frac{1}{2 + N(1-\alpha) + \alpha(2d_P - 1)} < 0 \\
\frac{\partial Q(t, \alpha)}{\partial N} &= \frac{(1-\alpha)[\alpha(1-c)(2d_P - 1) + 1 - t(2\alpha d_P + 1)]}{(2 + N(1-\alpha) + \alpha(2d_P - 1))^2}
\end{align*}
\]

where $\frac{\partial Q(t, \alpha)}{\partial \alpha} > 0$ if and only if $d_P < \frac{1-c+(N+1)t}{2(N+1)(1-c-t)}$, and $\frac{\partial Q(0, \alpha)}{\partial N} > 0$ if and only if $d_P > \frac{1}{2}\left(\frac{1-c}{1-c-t} - \frac{1}{\alpha}\right)$.

7.4 Proof of Corollary 2

When $\alpha = 0$, $Q(t, 0)$ satisfies $\frac{\partial Q(t, 0)}{\partial N} = \frac{1-c-t}{N+2} < 0$. When $d_P = 0$, $Q(t, \alpha)$ satisfies $\frac{\partial Q(t, \alpha)}{\partial N} = \frac{(1-\alpha)(1-c)(1-\alpha) - t}{2 + N(1-\alpha) - \alpha^2}$, which increases in $N$ if and only if $t < (1-c)(1-\alpha)$.

Without the EPA, $t = 0$, $Q(0, \alpha)$ satisfies $\frac{\partial Q(0, \alpha)}{\partial N} = \frac{(1-\alpha)[\alpha(2d_P - 1) + 1 - t(2\alpha d_P + 1)]}{2 + N(1-\alpha) + \alpha(2d_P - 1)} > 0$, and $\frac{\partial Q(0, \alpha)}{\partial \alpha} = \frac{-(1-c)[2d_P(N+1)-1]}{2 + N(1-\alpha) + \alpha(2d_P - 1)} < 0$ if and only if $d_P > \frac{1}{2(N+1)}$.

7.5 Proof of Proposition 1

Differentiating the regulator's problem in the first stage with respect to $\alpha$, we find that

\[
\frac{\partial W}{\partial \alpha} = \frac{(1-c)^2[2d_P(N+1) - 1][\alpha(2d_P - 1) + 2d_R((\alpha - 1)N - 1) + 1]}{[\alpha - 2ad_P + (\alpha - 1)N - 2]^3}
\]

Setting it equal to zero and solving for $\alpha$, we obtain the optimal weight on social welfare as

\[
\alpha^*(d_R, d_P) = \frac{2d_R(N+1) - 1}{2(Nd_R + d_P) - 1}
\]

where the numerator is positive if $d_R > \frac{1}{2(N+1)}$, while the denominator is positive if $d_R > \frac{1-2d_P}{2N}$. Since $\frac{1}{2(N+1)} > \frac{1-2d_P}{2N}$, condition $d_R > \frac{1}{2(N+1)}$ guarantees that $\alpha^*(d_R, d_P) > 0$. In addition,
\[ \alpha^*(d_R, d_P) < 1 \] for all \( d_R < d_P \); otherwise, \( \alpha^*(d_R, d_P) = 1 \). The comparative statics are

\[
\frac{\partial \alpha^*(d_R, d_P)}{\partial d_R} = \frac{2[2d_P(N + 1) - 1]}{(2d_P + 2d_RN - 1)^2} > 0,
\]

\[
\frac{\partial \alpha^*(d_R, d_P)}{\partial d_P} = \frac{2[1 - 2d_R(N + 1)]}{(2d_P + 2d_RN - 1)^2} < 0,
\]

\[
\frac{\partial \alpha^*(d_R, d_P)}{\partial N} = \frac{4d_R(d_P - d_R)}{(2d_P + 2d_RN - 1)^2}.
\]

The last result means that \( \alpha^* \) decreases in \( N \) if and only if \( d_R > d_P \). However, from the above findings, \( \alpha^* = 1 \) when \( d_R > d_P \), meaning that when \( d_R > d_P \), \( \alpha^* \) is unaffected by \( N \). For \( \alpha^* \) to be affected by \( N \), it must be that \( \alpha^* \) lies between 0 and 1, which happens when \( \frac{1}{2(N+1)} < d_R < d_P \), making \( \alpha^* \) increasing in \( N \) since \( d_P > d_R \).

### 7.6 Proof of Corollary 3

When the EPA is absent, aggregate output simplifies to \( Q(0, \alpha^*) = \frac{1-c}{2d_R+1} \), which helps us simplify welfare \( W_{\text{NoEPA}} \) to the following

\[ W_{\text{NoEPA}} = Q(0, \alpha) - \frac{Q(0, \alpha)^2}{2} - cQ(0, \alpha) - d_R (Q(0, \alpha))^2. \]

which simplifies first to

\[ W_{\text{NoEPA}} = (1 - c)Q(0, \alpha) - \left( d_R + \frac{1}{2} \right) Q(0, \alpha)^2 \]

Plugging \( \alpha^* \) into \( Q(0, \alpha) \) yields

\[ W_{\text{NoEPA}} = \frac{(1 - c)^2}{2d_R + 1} - \left( d_R + \frac{1}{2} \right) \left( \frac{1 - c}{2d_R + 1} \right)^2, \]

which can be rewritten as

\[ W_{\text{NoEPA}} = \frac{(1 - c)^2}{2(2d_R + 1)} \]

and satisfies \( \frac{\partial W_{\text{NoEPA}}}{\partial d_R} = -\frac{(1-c)^2}{[2(2d_R+1)]^2} < 0 \).

If \( d_R < \frac{1}{2(N+1)} \), then \( \alpha^* = 0 \), aggregate output is \( Q(0, 0) = \frac{(1-c)(N+1)}{N+2} \), and welfare is

\[ W_{\text{NoEPA}} = \frac{(1 - c)^2(N + 1)(N + 3 - 2d_R(N + 1))}{2(N + 2)^2} \]

and satisfies \( \frac{\partial W_{\text{NoEPA}}}{\partial N} = \frac{(1-c)^2[1-2d_R(N+1)]}{(N+2)^2} > 0 \) since, in this case, \( d_R < \frac{1}{2(N+1)} \).

If \( d_R > d_P \), then \( \alpha^* = 1 \), aggregate output is \( Q(0, 1) = \frac{1-c}{2d_P+1} \), and welfare is

\[ W_{\text{NoEPA}} = \frac{(1 - c)^2(4d_P - 2d_R + 1)}{2(2d_P + 1)^2}, \]
and satisfies $\frac{\partial W_{\text{NoEPA}}}{\partial d_P} = -\frac{4(1-c)^2(d_R-d_P)}{(2d_P+1)^2} > 0$ since, in this case, $d_R > d_P$.

### 7.7 Proof of Corollary 4

When $\alpha^*$ is interior (case 2 of Proposition 1), $SEP$ is given by

$$SEP = \frac{2d_R(N + 1) - 1}{2(Nd_R + d_P) - 1} - \frac{2d_R(N + 1) - 1}{2(Nd_R + d_R) - 1} = \frac{2(d_P - d_R)}{2(d_P + Nd_R) - 1},$$

which satisfies

$$\frac{\partial SEP}{\partial d_P} = \frac{4(N + 1)d_R - 2}{[2(d_P + d_R) - 1]^2} > 0$$

which holds if $d_R \geq \frac{1}{2(N+1)}$, which is satisfied by definition. In addition,

$$\frac{\partial SEP}{\partial d_R} = -\frac{4(N + 1)d_P - 2}{[2(d_P + d_R) - 1]^2} < 0$$

which holds if $d_P \geq \frac{1}{2(N+1)}$, which is satisfied in this case;

$$\frac{\partial SEP}{\partial N} = \frac{4d_R(d_R - d_P)}{[2(d_P + d_R) - 1]^2}$$

which is positive for $d_R > d_P$.

### 7.8 Proof of Lemma 3

The regulator sets the fee $t$ which solves $Q_{SO} = Q(t, \alpha)$:

$$\frac{(1 - c - t)[1 + (1 - \alpha)N] + \alpha t}{2 + \alpha(2d_P - 1) + (1 - \alpha)N} = \frac{1 - c}{2d_E + 1}$$

which, solving for $t$, yields $t^* = \frac{(1-c)[2d_E(1+1-\alpha)N-1-\alpha(2d_P-1)]}{(1-\alpha)(2d_E+1)(N+1)}$.

The comparative statics are

$$\frac{\partial t^*}{\partial d_E} = \frac{2(1-c)[2 - \alpha + 2\alpha d_P + (1 - \alpha)N]}{(1 - \alpha)(2d_E + 1)^2(N + 1)} > 0$$

$$\frac{\partial t^*}{\partial d_P} = \frac{2\alpha(1-c)}{(1-\alpha)(2d_E+1)(N+1)} < 0$$

$$\frac{\partial t^*}{\partial c} = -\frac{\alpha + 2d_E[1 + N(1 - \alpha)] - 2\alpha d_P - 1}{(1 - \alpha)(2d_E + 1)(N + 1)}$$

$$\frac{\partial t^*}{\partial \alpha} = \frac{2(1-c)(d_E - d_P)}{(1-\alpha)^2(2d_E+1)(N+1)} > 0$$

$$\frac{\partial t^*}{\partial N} = -\frac{(1-c)[\alpha + 2\alpha(d_E - d_P) - 1]}{(1-\alpha)(2d_E+1)(N+1)^2}$$
The comparative static \( \frac{\partial t^*}{\partial c} \) is positive if and only if \( d_P > \frac{1}{2(N+1)} \) and \( d_E > \frac{1}{2(N+1)} \). The final comparative static, \( \frac{\partial t^*}{\partial N} \), is positive if and only if \( \alpha < \frac{1}{1+2(d_E-d_P)} \).

Emission fee \( t^* \) is unambiguously increasing in \( \alpha \). In addition, evaluating \( t^* \) at \( \alpha = 0 \) yields \( t^* = \frac{(1-c)2d_E N}{(2d_E+1)(N+1)} \), which is unambiguously positive. Solving the equilibrium emission fee \( t^* \) for \( d_E \) yields a lower bound on \( d_E \) which guarantees that the emission fee \( t^* \) is positive, \( d_E \geq \hat{d}_E = \frac{1+\alpha(2d_P-1)}{2[1+(1-\alpha)N]} \).

The comparative statics of the cutoff \( \hat{d}_E \), are

\[
\frac{\partial \hat{d}_E}{\partial N} = \frac{2(1-\alpha)(\alpha - 2\alpha d_P - 1)}{[2 + 2N(1 - \alpha)]^2} < 0
\]

\[
\frac{\partial \hat{d}_E}{\partial d_P} = \frac{\alpha}{1 + N(1 - \alpha)} > 0
\]

\[
\frac{\partial \hat{d}_E}{\partial \alpha} = \frac{2d_P(N + 1) - 1}{2[1 + N(1 - \alpha)]^2}
\]

showing that \( \hat{d}_E \) is unambiguously decreasing in \( N \), increasing in \( d_P \), and increasing in \( \alpha \) if and only if \( d_P > \frac{1}{2(1+N)} \).

### 7.9 Proof of Corollary 5

Welfare when the EPA is active is given as

\[
W_{EPA} = (1-c)Q(t^*, \alpha) - \left( d_R + \frac{1}{2} \right) Q(t^*, \alpha)^2,
\]

which, after plugging in \( Q(t^*, \alpha) \), simplifies to

\[
W_{EPA} = (1-c) \left( \frac{1-c}{2d_E + 1} \right) - \left( d_R + \frac{1}{2} \right) \left( \frac{1-c}{2d_E + 1} \right)^2.
\]

Further rearranging yields

\[
W_{EPA} = \frac{(1-c)^2(4d_E - 2d_R + 1)}{2(2d_E + 1)^2}.
\]

The comparative static results are

\[
\frac{\partial W_{EPA}}{\partial c} = -\frac{(1-c)(4d_E - 2d_R + 1)}{(2d_E + 1)^2}
\]

which is negative if \( d_E > \frac{2d_R - 1}{4} \); and

\[
\frac{\partial W_{EPA}}{\partial d_R} = -\frac{(1-c)^2}{(2d_E + 1)^2} < 0
\]

\[
\frac{\partial W_{EPA}}{\partial d_E} = -\frac{4(1-c)^2(d_E - d_R)}{(2d_E + 1)^3}
\]
which is negative if \( d_E > d_R \).

### 7.10 Proof of Corollary 6

The values of \( Q_{SO}^R \) come directly from Corollary 3:

\[
Q_{SO}^R = \begin{cases} 
\frac{(1-c)(N+1)}{N+2} & \text{if } d_R < \frac{1}{2(N+1)}, (\alpha^* = 0) \\
\frac{1-c}{2d_R+1} & \text{if } \frac{1}{2(N+1)} \leq d_R < d_P, (\alpha^* \in (0,1)) \\
\frac{1-c}{2d_P+1} & \text{if } d_P \leq d_R, (\alpha^* = 1)
\end{cases}
\]

In the first case, \( Q_{SO}^R > Q_{SO}^E \) if

\[
\frac{(1-c)(N+1)}{N+2} > \frac{1-c}{2d_E+1},
\]

which, solving for \( d_E \) yields that the expression holds if and only if \( d_E > \frac{1}{2(N+1)} \).

In the second case, \( Q_{SO}^R > Q_{SO}^E \) if

\[
\frac{1-c}{2d_R+1} > \frac{1-c}{2d_E+1},
\]

which holds if and only if \( d_E > d_R \).

In the third case, \( Q_{SO}^R > Q_{SO}^E \) if

\[
\frac{1-c}{2d_P+1} > \frac{1-c}{2d_E+1},
\]

which holds if and only if \( d_E > d_P \).

### 7.11 Proof of Proposition 2

Differentiating with respect to \( \alpha \), yields

\[
-\frac{1}{(\alpha - 2\alpha d_P + (\alpha - 1)n - 2)^3} \left[ ((2d_P + 1)(N + 1)t - (1-c)(2d_P(N + 1) - 1)) \cdot (\alpha + c(2\alpha d_P - \alpha + 2d_R((\alpha - 1)N - 1) + 1)) - 2\alpha d_P + 2\alpha d_R(N(t - 1) + t) - 2d_R(N + 1)(t - 1) + (\alpha - 1)(N + 1)t - 1 \right] = 0
\]

and solving for \( \alpha \), we obtain

\[
\alpha^*(d_R, d_P, t) = \frac{(2d_R + 1)(N + 1)t - (1-c)[2d_R(N + 1) - 1]}{(2d_R + 1)(N + 1)t - (1-c)(2d_P + 2d_RN - 1)}
\]

The numerator is positive if \( d_R > \frac{1-c+(N+1)t}{2(N+1)(1-c-t)} \) while the numerator is positive if \( d_R > \frac{(1-c)(2d_R-1)-(N+1)t}{2(N+1)(1-c-t)} \).

For values of \( d_R \) that satisfy these two conditions, \( \alpha^*(d_R, d_P, t) > 0 \). In addition, \( \alpha^*(d_R, d_P, t) < 1 \)
for values of $d_R < d_P$; otherwise $\alpha^*(d_R, d_P, t) = 1$. The comparative statics are:

$$\frac{\partial \alpha^*(d_R, d_P, t)}{\partial d_R} = -\frac{2(1-c)[(2d_P+1)(N+1)t - (1-c)(2d_P(N+1)-1)]}{[(2d_R+1)(N+1)t - (1-c)(2d_P+2d_RN-1)]^2} < 0 \text{ if } d_P < \frac{1-c + (N+1)t}{2(N+1)(1-c-t)}$$

$$\frac{\partial \alpha^*(d_R, d_P, t)}{\partial d_P} = \frac{2(1-c)[(2d_R+1)(N+1)t - (1-c)(2d_P+2d_RN-1)]}{[(2d_R+1)(N+1)t - (1-c)(2d_P+2d_RN-1)]^2} < 0 \text{ if } d_R > \frac{1-c + (N+1)t}{2(N+1)(1-c-t)}$$

$$\frac{\partial \alpha^*(d_R, d_P, t)}{\partial t} = \frac{2(1-c)(2d_R+1)(N+1)(d_R-d_P)}{[(2d_R+1)(N+1)t - (1-c)(2d_P+2d_RN-1)]^2} > 0 \text{ if } d_R > d_P$$

### 7.12 Proof of Corollary 7

Social welfare to each respective agent is $W(d, Q) = \frac{1}{2}Q^2 + (1-c)Q - dQ^2$, where $d$ is replaced with $d_R$ in the regulator’s welfare and $d_E$ in the EPA’s welfare, and $Q$ is replaced with aggregate output induced by the respective policy setter, $Q_R = \frac{1-c}{2d_R+1}$ when the regulator acts second, or $Q_{EPA} = \frac{1-c}{2d_E+1}$ when the EPA acts second. In the case of $SMA^R$, the regulator’s second-mover advantage is the difference in welfare she perceives when she moves second and first:

$$SMA^R = W(d_R, Q_R) - W(d_R, Q_{EPA}) = \frac{(1-c)^2}{2(2d_R+1)} - \frac{(1-c)^2(1+4d_E-2d_R)}{2(2d_E+1)^2}$$

$$= \frac{2(1-c)^2(d_E-d_R)^2}{(2d_E+1)^2(2d_R+1)} > 0$$

which is unambiguously positive. The comparative statics are

$$\frac{\partial SMA^R}{\partial c} = \frac{4(1-c)(d_E-d_R)^2}{(2d_E+1)^2(2d_R+1)} < 0$$

$$\frac{\partial SMA^R}{\partial d_E} = \frac{4(1-c)^2(d_E-d_R)}{(2d_E+1)^3} > 0 \text{ if } d_E > d_R$$

$$\frac{\partial SMA^R}{\partial d_R} = -\frac{4(1-c)^2(d_E-d_R)(d_E+d_R+1)}{(2d_E+1)^2(2d_E+1)^2} < 0 \text{ if } d_E > d_R$$

Similarly, the $SMA^{EPA}$ is

$$SMA^{EPA} = W_{EPA}(d_E, Q_{EPA}) - W_{NoEPA}(d_E, Q_R) = \frac{(1-c)^2}{2(2d_E+1)} - \frac{(1-c)^2(1+4d_R-2d_E)}{2(2d_R+1)^2}$$

$$= \frac{2(1-c)^2(d_E-d_R)^2}{(2d_E+1)(2d_R+1)^2} > 0$$

which is unambiguously positive and exhibits similar comparative statics to those of $SMA^R$, namely, decreasing in $c$ and $d_R$, but increasing in $d_E$.

### References


