# EconS 501 - Micro Theory I
# Assignment #7 - Due date: December 8th, 2022

1. **Production and Externalities.** A firm's production of bacon outside Pullman generates a smelly gas as an unpleasant side product. Let $c(y, m; \mathbf{w})$ denote the (minimum) input cost of producing $y$ tons of bacon and $m$ cubic meters of gas when input prices are given by the vector $\mathbf{w} >> \mathbf{0}$. Let $p > 0$ denote the market price of bacon. Assume that $\frac{\partial c}{\partial y} > 0$, $\frac{\partial c}{\partial m} < 0$ and that $c(y, m; \mathbf{w})$ is strictly convex in output $y$ and gas emissions $m$. Let stars $^*$ denote solutions and assume throughout that $y^* > 0$.

   (a) Show that the cost function $c(y, m; \mathbf{w})$ is concave in input prices, $\mathbf{w}$.

   (b) *Setting a quota.* Suppose that the government imposes a ceiling on gas emissions such that $m \leq \overline{m}$, i.e., a quota. Assuming that this constraint binds, write down the firm's profit maximization problem with respect to $y$, and find necessary and sufficient conditions for the firm's cost-minimizing production, $y^*$,

   (c) *Comparative statics.* Under which condition on the cost function $c(y, m; \mathbf{w})$ can we guarantee that an increase in the ceiling on gas emissions, $\overline{m}$, produces a raise in the firm's cost-minimizing production, $y^*$, i.e., $\frac{\partial y^*}{\partial \overline{m}} > 0$?

   (d) *Emission fee.* Suppose now that the government abandons its emissions ceiling and replaces it with a tax $t > 0$ on gas emissions. Thus, the new cost of producing $(y; m)$ is given by $c(y; m; \mathbf{w}) + tm$. Show that maximized profits are convex in $t$, and that the firm's choice of pollution decreases in the pollution tax, i.e., $\frac{\partial m^*}{\partial t} \leq 0$.

2. **Regulating externalities under incomplete information.** Consider a setting where a regulator does not observes the marginal profits that a polluting firm obtains from emitting additional pollution, but observes the damage that such additional pollution causes on consumers. In particular, suppose that the firm's marginal benefit from an additional unit of pollution, $h$, is

$$\frac{\partial \pi(h, \theta)}{\partial h} = \beta - bh + \theta,$$

   and that the marginal utility from an additional unit of pollution for the consumer is

$$\frac{\partial \phi(h, \eta)}{\partial h} = \gamma - ch + \eta,$$

   where $\theta$ is a random variable with expectation $E[\theta] = 0$, and strictly positive realizations, i.e., $\theta > 0$. Parameters $b$, $c$ and $\gamma$ are also strictly positive by definition, i.e., $b, c, \gamma > 0$. In this exercise, we will first determine which is the best quota and emission fee that the regulator can design given that he operates under incomplete information. Afterwards, we will evaluate the welfare that arises under each of these policy instruments, to determine which is better from a social point of view.

   (a) *Setting a quota.* In this incomplete information setting, determine which is the best quota $\widehat{h}^*$ that a social planner can select in order to maximize the expected value of aggregate surplus.

(b) *Setting an emission fee.* Find the best tax $t^*$ that this social planner can set under the context of incomplete information described above.

(c) *Policy comparison.* Compare the emission fee and the quota in terms of their associated deadweight loss. Under which conditions an uninformed regulator prefers to choose the emission fee?