# EconS 501 - Micro Theory I
# Assignment #6 - Due date: November 28th, in class.

1. **Monopolist serving two interdependent markets.** Consider a monopolist producing two goods, 1 and 2, at a marginal cost $c > 0$. The demand function for product $i$ is

$$q_i(p_i, p_j) = a - bp_i + gp_j \quad \text{where } i = \{1, 2\}$$

and parameters satisfy $a > c\,(b - g)$, $b, c > 0$, and $|b| > |g|$, entailing that own-price effects dominate cross-price effects. For generality, we allow for $g > 0$ and $g < 0$.

   (a) Assume that the monopolist sets the price of good $i$ separate to that of good $j$. Find the equilibrium price pair $(p_i, p_j)$.

   (b) Suppose now that the monopolist sets the prices of goods $i$ and $j$ simultaneously. Find the equilibrium price pair $(p_i, p_j)$.

   (c) Under what condition will the distinct monopolist charge a higher (lower) price than a multi-product monopolist?

2. **Multiproduct monopoly with economies of scope.** Consider Ferdinand's food company, a monopolist producing two goods, ice cream (good 1) and cheese (good 2), which are regarded as substitutes for consumers. The inverse demand function of good $i$ is

$$p_i\,(q_i, q_j) = a - bq_i - gq_j$$

where $a, b, g > 0$ and $|b| > |g|$, entailing that own-price effects dominate cross-price effects.

In addition, Ferdinand's cost function is

$$C\,(q_1, q_2) = \frac{c}{2}\left(q_1^2 + q_2^2\right) - \beta q_1 q_2$$

where $c > 0$, and $\beta > 0$ indicates that the marginal cost of producing one good decreases in the output of another good, i.e., there are cost complementarities in production often referred as "economies of scope." When $\beta = 0$, the cost of one output is independent of the other.

   (a) Find the profit-maximizing output and associated profits of Ferdinand's food company.

   (b) How does equilibrium output change in parameters $\beta$ and $g$? Interpret your results.

   (c) *Numerical example.* Evaluate your results in parts (a) and (b) assuming parameter values $a = 1$, $\beta = 1/2$, $c = 1/3$, and $g = 1/4$. How do they change with $b$? Interpret.

3. **Persuasive advertising in monopoly.** Consider a monopolist facing the demand function

$$Q(p, A) = a - pA^{-\frac{1}{2}}$$

where $a$ is the market size, and the firm spends advertising dollars $A$ in promoting its products. For simplicity, assume that production cost is zero, and we consider $a, A > 0$.

   (a) Find the price elasticity of demand $\varepsilon_{Q,P}$. Does advertising make demand more inelastic?

   (b) Consider that the firm simultaneously chooses its price $p$ and advertising expenditure, $A$. Setup the firm's profit maximization problem and solve for the firm's equilibrium price $p^*$, advertising $A^*$, and the resulting equilibrium profits $\pi^*$. Then, evaluate your results assuming $a = 2$.

4. **Regulating a natural monopoly.** A water supply company provides water to Pullman. The demand for water in Pullman is $p(q) = 10 - q$, and this company's costs are $c(q) = 1 + 2q$.

   (a) Depict the following in a figure: the demand curve $p(q)$, the associated marginal revenue $MR(q)$, the marginal cost of production $MC(q)$ and the average cost of production $AC(q)$. Discuss why this situation illustrates a "natural monopoly."

   (b) *Marginal cost pricing.* Determine the amount of water that this firm will produce if a regulatory agency in Pullman forces the firm to price according to marginal cost (i.e., to produce an amount of output $q^*$ that solves $p(q^*) = MC(q^*)$). Find the corresponding prices and profits for the firm.

   (c) *Price discrimination.* Consider now that the regulatory agency allows the monopoly to charge two different prices: a high price $p_1$ for the first $q_1$ units, and a low price $p(q^*)$ for the remaining $(q^* - q_1)$ (i.e., the units from $q_1$ up to the output level you found in part (c), $q^*$). In addition, the regulatory agency imposes the condition that the firm cannot make any profits, $\pi = 0$, when charging these two prices.

      1. Find the value of $q_1$ and the associated price $p(q_1)$.
      2. Depict these two prices and quantities in a figure and shade the areas of benefits and losses for the firm.

5. **Monopsony, a general approach.** Consider a firm with production technology $f(x)$, where $x \in [0, 1]$ stands for the input (e.g, gas supply), and $f(x)$ represents the output that is sold in a competitive market at a price $p$. The firm is subject to an input cost function $g(x)$ that is increasing and convex in input $x$. This happens, for example, when there is only one company supplying natural gas.

   (a) Define $\varepsilon_g \equiv \frac{\partial x}{\partial g(x)} \frac{g(x)}{x}$ to be the price elasticity of gas supply, measuring the percentage change of gas supply given one percent change in gas price. Setup the firm's profit-maximization problem to maximize $\pi(x) = pf(x) - g(x)x$, and show that

$$p' f(x) = g(x) \left[1 + \frac{1}{\varepsilon_g}\right].$$

(b) Let $f(x) = x$ and $g(x) = x^{\beta}$, where $\beta > 1$. Use the expression found in part (a) to identify the optimal gas supply $x^*$. For simplicity, you may assume that $p = 1$ in the remainder of this exercise.

(c) *Comparative statics.* How does $x^*$ change with $\beta$? Explain.

(d) *Numerical example.* Evaluate the firm's optimal gas supply $x^*$ when $\beta = 1$, $\beta = 2$, $\beta = 4$, and $\beta \to +\infty$. Interpret.