EconS 501 - Micro Theory I
Assignment #5 - Due date: October 31st, in class.

1. **Additional lottery.** An individual faces the monetary lottery L, where $(p_1, p_2, ..., p_K)$ with $p_k \geq 0$ denoting the probability of outcome k, and $(z_1, z_2, ..., z_K)$ representing the profile of monetary outcomes. Consider that we make the offer of replacing every monetary outcome, z_k, in the support of L with the lottery that yields $z_k - 1$ and $z_k + 1$ each of them with equal probability.

   (a) Describe the lottery that he faces if he accepts the offer.

   (b) Show that if he is strictly risk-averse, he will reject the offer.

2. **Casino.** An individual has wealth $w > 0$ and has to choose an amount $x > 0$, after which a lottery is conducted in which with probability α he earns $2x$ and with probability $1 - \alpha$ he loses x (his payoff is zero). Show that the amount he chooses, x, increases in probability α.

3. **IA implies monotonicity.** Consider an individual with preference relation $\succsim$ over monetary lotteries, which satisfies the IA. Show that, for every two monetary outcomes, x and y, where $x > y$, we must have that

$$\alpha x + (1 - \alpha)y \succsim \beta x + (1 - \beta)y$$

   also holds if and only if probabilities α and β satisfy $\alpha > \beta$. Interpret your results.

4. **Non-constant coefficient of absolute risk aversion**. Suppose that the utility function is given by
$$u(w) = aw - bw^2,$$
   where $a, b > 0$, and $w > 0$ denotes income.

   (a) Find the coefficient of absolute risk-aversion, $r_A(w, u)$. Does it increase or decrease in wealth? Interpret.

   (b) Let us now consider that this decision maker is deciding how much to invest in a risky asset. This risky asset is a random variable R, with mean $\overline{R} > 0$ and variance σ_R^2. Assuming that his initial wealth is W, state the decision maker's expected utility maximization problem, and find first order conditions.

   (c) What is the optimal investment in risky assets?

   (d) Show that the optimal amount of investment in risky assets is a decreasing function in wealth. Interpret.

5. **Exercises from Rubinstein:**

   (a) Lecture 7 (Expected utility): Exercise 4.