EconS 501 - Microeconomic Theory I
Homework #1 - Due date: September 7th, in class.

1. **Exercise from FMG.** Chapter 1, exercise 16.

2. **Lexicographic preferences.** Show that lexicographic preferences are convex.

3. **Three examples of preference relations.** Describe each of the following three preference relations formally, giving a utility function that represents the preferences wherever possible, draw some representative indifference sets, and determine whether the preferences are monotone, continuous, and convex.

   (a) The consumer prefers the bundle (x_1, x_2) to the bundle (y_1, y_2) if and only if (x_1, x_2) is further from $(0, 0)$ than is (y_1, y_2), where the distance between two bundles is measured with the Euclidean distance.

   (b) The consumer prefers any balanced bundle, containing the same amount of each good, to any unbalanced bundle. Between balanced bundles, he prefers the one with the largest quantities. Between unbalanced bundles, he prefers the bundle with the largest quantity of good 2.

   (c) The consumer cares first about the sum of the amounts of the goods; if the sum is the same in two bundles, he prefers the bundle with more of good 1.

4. **Ideal bundle.** The consumer has in mind an ideal bundle x^*. He prefers bundle x to y if and only if x is closer to x^* than y is, that is, $x \succsim y$ if and only if

$$|x_1 - x_1^*| + |x_2 - x_2^*| \leq |y_1 - y_1^*| + |y_2 - y_2^*| .$$

   Show that this preference relation is continuous and convex.

5. **Rationalizable choices.** Determine whether each of the following five choice functions over a set X is rationalizable. If the answer is positive, find a preference relation that rationalizes the choice function. Otherwise, prove that the choice function is not rationalizable.

   (a) The set X consists of candidates for a job. An individual has a complete ranking of the candidates. When he has to choose from a set A, he first orders the candidates in A alphabetically, and then examines the list from the beginning. He goes down the list as long as the new candidate is better than the previous one. If the nth candidate is the first who is better than the $(n+1)$th candidate, he stops and chooses the nth candidate. If in his journey he never gets to a candidate who is inferior to the previous one, he chooses the last candidate.

   (b) The set X consists of n basketball teams, indexed 1 to n. The teams participate in a round-robin tournament (also known as all-play-all, where every team plays against every other team. An individual knows, for every pair of teams, which one wins. When he chooses a team from a set A, he chooses the one with the largest number of wins among the games between teams in A. If more than one team has the largest number of wins, he chooses the team with the lowest index among the tied teams.

(c) The set X consists of pictures. An individual has in mind L binary criteria, each of which takes the value 0 (the criterion is not met) or 1 (the criterion is met). Examples of such criteria are whether the painting is modern, whether the painter is famous, and whether the price is above \$1,000. The criteria are ordered: criterion_1, criterion_2, . . . , criterion_L. When the individual chooses a picture from a subset of X, he rejects those that do not satisfy the first criterion. Then, from those that satisfy the first criterion, he rejects those that do not satisfy the second criterion. And so on, until only one picture remains. Assume that any two alternatives have a criterion by which they differ, so that the procedure always yields a unique choice.

(d) An individual has in mind two numerical functions, u and v, on the set X. For any set $A \subseteq X$, he first looks for the u-maximal alternative in A. If its u value is at least 10, he selects it. If not, he selects the v-maximal alternative in A.

(e) An individual has in mind a preference relation on the set X. Each alternative is either red or blue. Given a set $A \subseteq X$, he chooses the best alternative among those with the color that is more common in A. In the case of a tie, he chooses among the red alternatives.