Risk Aversion in Auctions with Asymmetrically Informed Bidders:

a "Desensitizer" from Uncertainty

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Abstract

In the context of first-price auctions with asymmetrically informed bidders, we show that risk aversion not only increases a player's bid, but also makes him less sensitive to the probability that other bidders are informed about his private valuation.

Keywords: First-price auctions; Asymmetrically informed bidders; Risk aversion; Unknown valuations.

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1 Introduction

Many advances on auction theory focus on situations in which *every* bidder knows each other's valuations, or on cases where *no* bidder observes each other's valuations for the object on sale. Notwithstanding the contributions of these two approaches, several real-life contexts exhibit a more intermediate informational structure, where a bidder might know the value other bidders assign to the object, while another bidders might not, i.e., bidders are asymmetrically informed. This informational environment describes situations in which some players have been present in a market for a relatively long period, while others are just entering that market.

The potential applications of auctions with *asymmetrically* informed bidders have recently generated important contributions, both theoretical —as Che and Kim (2004) and Fang and Morris (2006)— and experimental —see Andreoni *et al.* (2007). One of the main results emerging from this literature for the case of first-price auctions is that an uninformed bidder (who does not observe other bidders' valuations) tends to reduce his bid as other bidders become informed about his valuation, whereas an informed bidder (who observes other bidders' valuations) increases his bid.

In this paper, we investigate how this bidding behavior is modified in the context of risk-averse players. Intuitively, one would expect the usual results from private-value settings to extend to this environment, i.e., an increase in bids as players become more risk-averse; see Maskin and Riley (1985). As we show, this result extends to this setting, but we identify an additional effect from risk aversion, which is specific to auctions with asymmetrically informed bidders, and that to our knowledge is novel in the literature. In particular, we find that a player's bidding behavior becomes *less sensitive* to a change in the probability that other bidders are informed about his private valuation. In the extreme, when a bidder is sufficiently risk-averse, he becomes totally unaffected by the probability that his competitors are knowledgeable of his valuation. That is, his bidding strategy becomes "desensitized" from his opponents' information.

Hence, the effects of changes in the information structure on bidding behavior, present in contexts of risk-neutral bidders, become attenuated in environments with risk-averse bidders, and might be totally absent when bidders are extremely risk-averse. Interestingly, our results suggest that expected revenue is essentially unaffected by the auction's informational structure when bidders are extremely risk averse, while revenue is affected otherwise.[1] Our theoretical predictions are additionally supported by recent experimental results on auctions with asymmetrically informed bidders. Indeed, as Andreoni *et al.* (2007) show, experimental data on bidders' behavior in this type of auctions closely approximates theoretical predictions if players are assumed to be risk-averse.

The following section introduces the model and equilibrium bidding function. We provide comparative statics and compare our results with those from second-price auctions.

[1] The auctioneer's control over what information is publicly disclosed can be observed in different examples. For instance, in corporate takeovers, the seller chooses what particular information to provide about the target firm's financial status. Similarly, in the case of real estate, realtors choose whether to distribute the specific location, its orientation, etc. Bergemann and Pesendorfer (2007) and Eso and Szentes (2007) theoretically examine the seller's incentives to provide this information among bidders.

2 Model

Let us consider a first-price auction with $N = 2$ bidders. Every bidder has either a high (v_H) or low (v_L) valuation for the object, with probabilities p and $1 - p$, respectively, and $v_H > v_L \geq 0$. We assume that bidders independently and privately learn each other's valuation with probability $\alpha \in (0, 1)$. Since bidders are symmetric, each uses a symmetric bidding function $\beta(v_K, \alpha)$ for all $K = \{H, L\}$. In the case of tied bids between a high and a low-value bidder, we consider that the bidder with the highest valuation wins the object.[2] Bidders are risk averse. Let $u(\cdot)$ be an increasing and strictly concave utility function for every player, normalized so that $u(0) = 0$. The following proposition describes an equilibrium bidding function.

Proposition 1. *In a first price auction with two asymmetrically informed risk-averse bidders, an equilibrium bidding function prescribes that:*

1. *All low-value bidders, either informed or uninformed, bid v_L using degenerated (pure) strategies.*

2. *The high-value bidder who is informed that his opponent is a low-value bidder bids v_L using degenerated (pure) strategies.*

3. *The high-value bidder who is uninformed randomizes over the interval $[v_L, b']$, according to a cumulative distribution function*

$$F(b) = \frac{p\left[u(v_H - v_L) - u(v_H - b)\right]}{(1-p)(1-\alpha)u(v_H - b)} \quad \textit{where } b' \textit{ solves } u(v_H - b') = \frac{pu(v_H - v_L)}{1 - \alpha(1-p)}$$

4. *The high value bidder who is informed that his competitor is a high-value bidder randomizes over the interval $[b', b^*]$, with a cumulative distribution function*

$$G(b) = \frac{(1-\alpha)\left[u(v_H - b') - u(v_H - b)\right]}{\alpha u(v_H - b)} \quad \textit{where } b^* \textit{ solves } u(v_H - b^*) = (1-\alpha)u(v_H - b')$$

Therefore, the uninformed high-value risk-averse bidder randomizes according to a cumulative distribution function $F_B(b)$ that first-order stochastically dominates that of a risk-neutral bidder, $F_A(b)$, i.e., $F_B(b) \leq F_A(b)$, or alternatively, $1 - F_B(b) \geq 1 - F_A(b)$. Intuitively, an increase in his risk aversion leads this bidder to bid more aggressively, assigning a larger probability weight on higher bids. Note that when the bidder is extremely risk averse, his randomization is mostly centered at the upper bound b'. In this case, a change in the probability with which his opponent observes his valuation, α, does not affect his randomization. Thus, this bidder becomes less sensitive to α as his risk aversion increases. This strong preference for winning the auction, at the cost of reducing

[2] This assumption is similar to that in Che and Kim (2004). In particular, in case of a tie, the high-valuation bidder could slightly increase his bid in order to outbid the low-valuation bidder in a run-off auction.

the expected payoff if winning, resembles the standard result in auctions with risk-averse bidders; see Maskin and Riley (1985) and Krishna (2002). Our results, hence, suggest that when players are extremely risk averse, equilibrium bids as described in Maskin and Riley (1985) would not be substantially affected by changes in the informational structure of the auction, i.e., changes in α.[3] Similarly, risk aversion induces the informed high-value bidder to randomize using a distribution function $G_B(b)$ that first-order stochastically dominates that of a risk-neutral bidder $G_A(b)$, i.e., $G_B(b) \leq G_A(b)$, producing more aggressive bids.[4]

Revenue comparison. We next compare our results with those from second-price auctions. First, note that Che and Kim (2004) and Andreoni et al. (2007) show that the equilibrium bidding strategy in second-price auctions is unaffected by the informational structure, since bidding one's valuation still constitutes a dominant strategy. In addition, equilibrium bids in the second-price auction are independent on players' risk preferences; see Milgrom (2004). Expected revenue in second-price auctions, R_{SPA}, is therefore influenced by neither the informational structure (α) nor the degree of risk aversion. Introducing risk-aversion in first-price auctions, in contrast, raises the seller's expected revenue, since players become more aggressive in their bidding strategy. This is illustrated in figure 1 below, where $R_{FPA}(\alpha = 0)$ denotes expected revenue in first-price auctions in which players cannot observe each others' valuations (as in independent private values auctions). In this context, both auction formats yield the same expected revenue when players are risk-neutral —as depicted in the vertical axis of the figure, where $R_{FPA}(\alpha = 0)$ and R_{SPA} coincide— but the first-price auction generates a larger revenue as bidders become risk averse; see Maskin and Riley (1985). In contrast, when bidders can observe each others' valuations with positive probability ($\alpha > 0$) and players are risk-neutral, Che and Kim (2004) demonstrate that the first-price auction generates a lower revenue, i.e., $R_{FPA}(\alpha > 0) < R_{SPA}$. Risk aversion, however, increases expected revenue, potentially reverting Che and Kim's (2004) result when the degree of risk aversion is sufficiently strong, i.e., yielding $R_{FPA}(\alpha > 0) > R_{SPA}$. Therefore, the introduction of asymmetric information ($\alpha > 0$) produces a downward shift in the expected revenue of the first-price auction.[5] Such decrease is, nonetheless, more substantial when bidders are risk-neutral than when they are risk-averse. As described above, risk-averse players are less sensitive to the auction's information structure.

[3] Another implication of this result is related to the seller's expected revenue. Specifically, a seller running an auction in which bidders are extremely risk-averse, would neither modify equilibrium bids nor her expected revenue from the auction by distributing information about other bidders' private valuations, i.e., by modifying α. This result is, in contrast, not applicable to the case of bidders with relatively risk-neutral preferences, where changes in α can increase the seller's expected revenue.

[4] Note that an increase in risk aversion produces not only a rightward shift in the probability distributions with which players randomize, but also an increase in the upper bound of these distributions; see appendix.

[5] Our results relate to those in Bennouri and Falconieri (2006), who analyze optimal auction mechanisms with asymmetrically informed risk-averse bidders. Specifically, the authors demonstrate that the seller's expected revenue in the optimal auction decreases with the share of informed bidders. We show that such result can be extended to first-price auctions, whereby an increase in the probability that players are informed weakly decreases expected revenue.

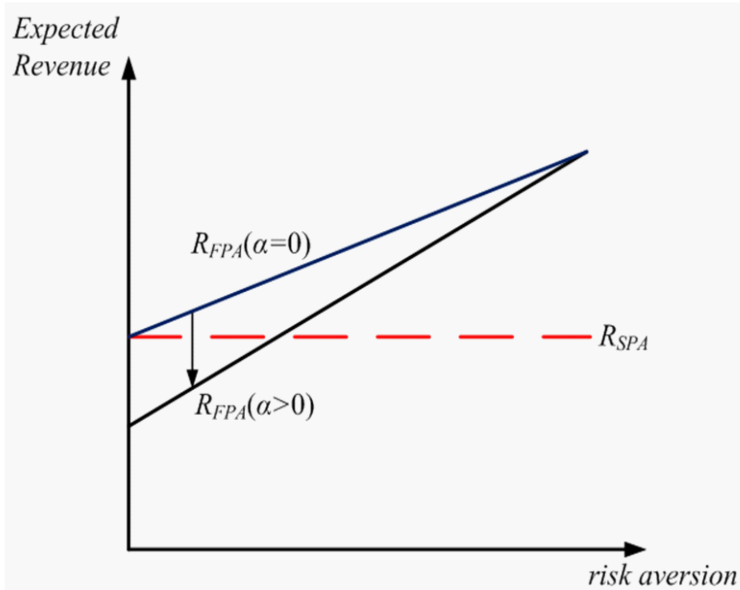

Figure 1. Revenue comparison.

3 Conclusions

We analyze a first price auction with asymmetrically informed risk-averse bidders. As in standard independent private-value auctions, we find that risk aversion increases players' bids. However, we identify an additional effect. Specifically, a risk-averse bidder becomes *less sensitive* to a change in the probability that other bidders are informed about his private valuation, suggesting that risk aversion would "desensitize" this bidder from the uncertainty surrounding the auction. A natural extension of this paper would examine, under different functional forms and parameter values, which degree of risk aversion is necessary in order to guarantee that the seller's expected revenue in the first-price is larger than in the second-price auction.

4 Appendix

First, note that there are three types of low-value bidders: uninformed, informed about his opponent's valuation being high and informed about his opponent's valuation being low. All three types clearly bid v_L. Submitting a bid above v_L would imply bidding above their own valuation, which is a strictly dominated strategy. On the other hand, bidding below v_L is never a best response, since in equilibrium no other bidder is bidding below v_L. Furthermore, the high-value bidder who is informed that his opponent is a low-value bidder bids v_L, since he knows that in equilibrium his opponent will never bid above v_L.

Let us now analyze the *uninformed* high-value bidder, who randomizes over $[v_L, b']$, according to $F(b)$. One necessary condition for equilibrium is that he is indifferent among all the elements of the support $[v_L, b']$. Hence, there must be a constant K such that

$$K = p \times u(v_H - b) + (1-p) \times [(1-\alpha)F(b)\, u(v_H - b)] \Longleftrightarrow F(b) = \frac{K - pu(v_H - b)}{(1-p)\,(1-\alpha)u(v_H - b)} \quad \text{(A.1)}$$

(note that this expression is identical to equation (3) in Maskin and Riley (1985), expect for the term $(1-\alpha)$). If we conjecture that there is no mass point at the lower bound $b = v_L$, $F(v_L) = 0$, then we can solve for K, i.e., $K = p \times u(v_H - v_L)$. Additionally, the upper bound b' must satisfy $F(b') = 1$. Using this condition and $K = p \times u(v_H - v_L)$,

$$1 = \frac{pu(v_H - v_L) - pu_H(v_H - b')}{(1-p)\,(1-\alpha)u(v_H - b')} \Longleftrightarrow u(v_H - b') = \frac{pu(v_H - v_L)}{1 - \alpha(1-p)}$$

which yields b'. Note that increasing the concavity of the utility function, from a risk-neutral bidder A, with utility function $v_H - b$ to a risk-averse bidder B, with strictly concave utility function $u(v_H - b)$, where $u(v_H - b) \geq v_H - b$ for all b, implies that the cdf associated to the risk-averse bidder, $F_B(b)$, and that of the risk-neutral bidder, $F_A(b)$, satisfy

$$F_A(b) \equiv \frac{p\,[(v_H - v_L) - (v_H - b)]}{(1-p)\,(1-\alpha)(v_H - b)} \geq \frac{p\,[u(v_H - v_L) - u(v_H - b)]}{(1-p)\,(1-\alpha)u(v_H - b)} \equiv F_B(b)$$

or rearranging, $\frac{v_H - v_L}{v_H - b} \geq \frac{u(v_H - v_L)}{u(v_H - b)}$, which holds by concavity. Hence, $F_A(b) \geq F_B(b)$ for all b, and $F_B(b)$ first-order stochastically dominates $F_A(b)$.

Let us now analyze the *informed* high-value bidder who faces a high-value bidder. This bidder randomizes over $[b', b^*]$ according to $G(b)$. A necessary condition for equilibrium is that he is indifferent among all the elements in the support. Hence, there must exist a constant M such that

$$M = (1-\alpha)\, u(v_H - b) + \alpha G(b)\, u(v_H - b) \Longleftrightarrow G(b) = \frac{M - (1-\alpha)u(v_H - b)}{\alpha u(v_H - b)} \quad \text{(A.2)}$$

If we conjecture that there is no mass point at $b = b'$, $G(b') = 0$, then we can solve for M, i.e., $M = (1-\alpha)u(v_H - b')$. Additionally, the upper bound b^* must satisfy $G(b^*) = 1$. Using this condition and the value of M found above,

$$1 = \frac{(1-\alpha)u(v_H - b') - (1-\alpha)u(v_H - b^*)}{\alpha u(v_H - b^*)} \Longleftrightarrow u(v_H - b^*) = (1-\alpha)u(v_H - b')$$

which yields b^*. Similarly as for the uninformed high-value bidder, if we increase the concavity of the utility function, we obtain that $G(b)$ satisfies

$$G_A(b) = \frac{(1-\alpha)\,[(v_H - b'_A) - (v_H - b)]}{\alpha(v_H - b)} \geq \frac{(1-\alpha)\,[u(v_H - b'_B) - u(v_H - b)]}{\alpha u(v_H - b)} \equiv G_B(b)$$

and rearranging, $\frac{v_H - b'_A}{v_H - b} \geq \frac{u_B(v_H - b'_B)}{u_B(v_H - b)}$. Since by concavity $\frac{v_H - b'_B}{v_H - b} \geq \frac{u(v_H - b'_B)}{u(v_H - b)}$ holds for a given b'_B, and $b'_B > b'_A$ (as described below), we have that $v_H - b'_A > v_H - b'_B$, implying that $\frac{v_H - b'_A}{v_H - b} \geq \frac{u(v_H - b'_B)}{u(v_H - b)}$ also holds by concavity. Thus, $G_A(b) \geq G_B(b)$ for all b, and $G_B(b)$ first-order stochastically dominates $G_A(b)$.

In order to check that the above necessary conditions for an equilibrium bidding function are also sufficient, we still need to check the following points. First, an informed high-value bidder does not want to submit any bid below b'. To see this, note that by construction an uninformed high-value bidder is indifferent among all bids in $[v_L, b']$. But an uninformed high-value bidder puts positive probability on bidding against a low-value bidder, against whom he always gains from lowering his bid. An informed high-value bidder facing another high-value bidder lacks this benefit and hence strictly loses from lowering his bid below b'. Furthermore, an uninformed high-value bidder has no incentive to increase his bid above b'. To see this, suppose he did. Then he would get a payoff which is strictly decreasing in b. A similar analysis for b^* shows sufficiency for the above distribution functions.

Note that the upper bound b' increases in the degree of risk aversion. Using the implicit equation $u(v_H - b'_B) = \frac{pu(v_H - v_L)}{1-\alpha(1-p)}$ of a risk-averse bidder, and its equivalent for a risk-neutral bidder, $v_H - b'_A = \frac{p(v_H - v_L)}{1-\alpha(1-p)}$, we obtain b'_B and b'_A, respectively. We need to show that $b'_B > b'_A$, i.e., $v_H - b'_A > v_H - b'_B$. This is satisfied when $u(v_H - b'_B) - (v_H - b'_B)$ is larger than $u(v_H - b'_B) - (v_H - b'_A)$; where the latter is alternatively represented by $\frac{pu(v_H - v_L)}{1-\alpha(1-p)} - \frac{p(v_H - v_L)}{1-\alpha(1-p)}$. Note that this condition holds if $u(v_H - b'_B) - (v_H - b'_B) > u(v_H - v_L) - (v_H - v_L)$. Since this last inequality holds by concavity, then $u(v_H - b'_B) - (v_H - b'_B)$ is larger than $\frac{pu(v_H - v_L)}{1-\alpha(1-p)} - \frac{p(v_H - v_L)}{1-\alpha(1-p)}$, implying that $b'_B > b'_A$.

A similar argument is applicable to upper bound b^*, which solves $u(v_H - b^*_B) = (1-\alpha)u(v_H - b'_B)$ for a risk-averse bidder, and $v_H - b^*_A = (1-\alpha)(v_H - b'_A)$ for a risk-neutral bidder. Given that $b'_B > b'_A$, we might have two cases. First, if $u(v_H - b'_B) < v_H - b'_A$ holds, then $(1-\alpha)u(v_H - b'_B) < (1-\alpha)(v_H - b'_A)$, implying that $v_H - b^*_B < v_H - b^*_A$, and therefore $b^*_B > b^*_A$ is satisfied. If, in contrast, $u(v_H - b'_B) \geq v_H - b'_A$ holds, then $(1-\alpha)u(v_H - b'_B) \geq (1-\alpha)(v_H - b'_A)$ must be satisfied. In this case, note that $(1-\alpha)(v_H - b'_A) > (1-\alpha)(v_H - b'_B)$ must hold. (Otherwise, $v_H - b'_A < v_H - b'_B$ would be satisfied, implying $b'_A > b'_B$, which cannot hold as described above). Therefore,

$$(1-\alpha)u(v_H - b'_B) \geq (1-\alpha)(v_H - b'_A) > (1-\alpha)\left(v_H - b'_B\right)$$

which implies that $u(v_H - b^*_B) \geq v_H - b^*_A > (1-\alpha)(v_H - b'_B)$. Hence $v_H - b^*_B < v_H - b^*_A$, showing that $b^*_B > b^*_A$.

The expected revenue, R, for the risk-neutral seller in the first-price auction is given by

$$\begin{aligned} R &= p\alpha[p\alpha E_G + p(1-\alpha)E_G] + p\alpha[(1-p)\alpha v_L + (1-p)(1-\alpha)v_L] + \\ &\quad + p(1-\alpha)[p(1-\alpha)E_F + p\alpha E_G + (1-p)\alpha E_F + (1-p)(1-\alpha)E_F] + \\ &\quad + (1-p)\alpha[p\alpha v_L + p(1-\alpha)E_F] + (1-p)\alpha[(1-p)\alpha v_L + (1-p)(1-\alpha)v_L] + \\ &\quad + (1-p)(1-\alpha)[p\alpha v_L + p(1-\alpha)E_F + (1-p)\alpha v_L + (1-p)(1-\alpha)v_L] \end{aligned}$$

where E_F denotes the expected bid from the uninformed high-value bidder (who randomizes according to the $F(b)$ distribution), i.e., $E_F = \int_{v_L}^{b'} [1 - F(b)]\, db$, since $b \geq 0$. Similarly, E_G represents the expected bid from the high-value bidder who is informed that his opponent's valuation is high (who randomizes according to $G(b)$), i.e., $E_G = \int_{b'}^{b^*} [1 - G(b)]\, db$. Note that both E_F and E_G are increasing in bidders' risk aversion, and that the expected revenue R is increasing in both E_F and E_G.

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