Information Transmission during the Trial:  
The Role of Punitive Damages and Legal Costs*

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January 16, 2017

Abstract

This paper studies an incomplete information model in which a preventable accident occurred. The judge determining punitive damages observes the firm’s (defendant) investment decisions, but is uninformed about the firm’s experience adopting safety measures. Our model allows firms to file an appeal if the judge’s verdict is incorrect, which the judge may accept or reject. We identify under which conditions a separating equilibrium exists where the firm’s investment decisions signal its type to the judge, who responds with a correct verdict, thus avoiding future appeals. Our paper also finds conditions under which a pooling equilibrium exists whereby the firm’s investment in precaution conceals its type from the judge, who can respond with an incorrect verdict thus giving rise to appeals. Furthermore, we show that the separating equilibrium is more likely to arise if the percentage of revenue that defendants are required to pay in punitive damages decreases, if the punitive-to-compensatory ratio increases, and if the legal cost of filing an appeal increases.

KEYWORDS: Signaling game, punitive damages, legal cost, appeal.

JEL CLASSIFICATION: C72, D82, K4.

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*We would like to thank James J. Jozefowicz, Moon Y. Parks and participants at the 90th Annual Western Economic Association International conference for providing valuable comments and feedback on the paper.
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1 Introduction

The 1989 Exxon Oil spill released 11 million gallons of crude oil into Prince William Sound in Alaska and created vast economic and environmental damages. In September 1994, a jury awarded $286.8 million for compensatory damages to commercial fishermen and $5 billion in punitive damages to be paid by Exxon (Duffield, 1997).\(^1\) After several appeals, punitive damages were reduced to $507.5 million. A similar pattern of reduced punitive damages after appeals occurred in the hot coffee lawsuit in Liebeck v. McDonald’s (1994).\(^2\) We show that unreasonably high punitive damages can induce firms to file unnecessary appeals and demonstrate that the judge’s choice of punitive damage affects firms’ decision to invest in precaution.

Our model examines a setting of incomplete information in which the judge, upon observing a preventable accident (e.g., environmental disaster), cannot infer whether the firm (defendant) is reckless or cautious. A reckless type of firm has a poorer managerial ability at adopting safety measures and employee training than the cautious type of firm. The structure of the game is the following: in the first period, the firm decides its precaution level, either high or low; and the judge, after observing such investment, chooses punitive damages based on either a share of the firm’s revenue (wealth approach) or as a proportion of compensatory damages (ratio approach).\(^3\) In the second period, the firm can appeal the judge’s verdict, as in the case of the Exxon Oil spill, which the judge can accept or reject.

We identify a separating equilibrium where the cautious firm chooses a high precaution level while the reckless type invests in low, which emerges when investment costs are intermediate. In this setting, the judge can infer the firm’s type upon observing its investment decision, leading him to a correct verdict, that is, the ratio approach after a high precaution but the wealth approach otherwise. As a consequence, in this equilibrium outcome no appeals are filed saving time and resources to both parties. We also find a pooling equilibrium where both types of firm invest in high precaution, which emerges when investment costs are relatively low. In this context, the firm’s investment in precaution conceals its type from the judge, who responds with the wealth or ratio approach depending on his prior beliefs (i.e., frequency of cautious types). Hence, the cautious type needs to appeal in the second period when the judge responds with the wealth approach; an appeal that is only due to the judge’s lack of information in the first period.\(^4\)

\(^1\)Punitive damages is the monetary compensation awarded to injured parties. This amount goes beyond what is necessary to compensate individuals for losses (compensatory damage) and it is intended to punish and encourage the defendant to be more cautious; see Kemezy v. Peters, 79 F. 3d 33 (7th Cir. 1996). In the Exxon Oil spill, the amount of compensatory damages was evaluated using the contingent evaluation method; for more details, see Carson et al., (1992).

\(^2\)In this case, the jury awarded Liebeck, a 79-year-old woman who accidentally spilled hot coffee in her pelvic region and suffered third-degree burns, $2.7 million in punitive damages, and $160,000 in compensatory damages. This amount was approximately two days of McDonald’s coffee sales revenue at that time. However, punitive damages were eventually reduced to $480,000 (Shapiro, 1995).

\(^3\)The U.S. Supreme Court suggests that a punitive-to-compensatory ratio below 4:1 is reasonable. Eisenberg et al. (2001) and Segalla (2005) discuss the factors affecting the selection of punitive damages.

\(^4\)We also find the existence of a pooling equilibrium in which both types of firm choose to invest in low precaution. However, we show that such equilibrium does not survive the Cho and Kreps’ (1987) Intuitive Criterion as it is based on insensible off-the-equilibrium beliefs.
Our results show that the emergence of these equilibria depends on: (1) the burden of the wealth approach (as a percentage of revenue); (2) the size of the punitive-to-compensatory ratio; and (3) the legal cost that the firm incurs when appealing the judge’s decision. First, more severe punitive damages based on the wealth approach shrink the range of parameters under which the (informative) separating equilibrium arises but expand those for which the (uninformative) pooling equilibrium exists. Intuitively, in the separating equilibrium the reckless type has more incentives to deviate towards a high investment as the wealth approach it receives in equilibrium becomes more costly. The opposite argument applies in the pooling equilibrium where both types of firm receive the ratio approach. In this case, a deviation towards low precaution would be followed by more costly punitive damages based on the wealth approach. Hence, both types of firm have less incentives to deviate.

Second, if the punitive-to-compensatory ratio increases, the separating (pooling) equilibrium is sustained under larger (more restrictive) parameter values. In the separating equilibrium, the reckless type has less incentives to deviate towards high precaution since the gap in punitive damages between the ratio and wealth approach diminishes. By contrast, in the pooling equilibrium the judge responds with the ratio approach. Therefore, an increase in the punitive-to-compensatory ratio provides more incentives for firms to deviate towards a low investment. Hence, judicial systems that recommend as punitive damages a small share of the defendant’s revenue using the wealth approach and/or large punitive-to-compensatory ratios promote the emergence of the separating equilibrium in which information flows from the defendant’s actions to the judge, ultimately inducing correct verdicts and no further appeals. In contrast, a legal system requiring defendants to pay a large percentage of their revenue in punitive damages and low punitive-to-compensatory ratios would reduce firms’ incentives to behave as under the separating equilibrium. As a consequence, these punitive damages would prevent information transmission, leading to more likely incorrect verdicts followed by appeals.

Finally, if the legal cost of appeals increases, the separating equilibrium is sustained under larger parameter values, while the pooling equilibrium is unaffected. Specifically, the cautious type has less incentives to deviate to low precaution, as it would be responded by the judge with the wealth approach, requiring the firm to appeal at a higher legal cost. While high legal costs are often blamed for hindering judicial processes, our results suggest that they can actually facilitate the existence of the separating equilibrium, and thus the transmission of information to the judge.

Related literature. Many papers propose different approaches to determining punitive damages. In the context of the wealth approach, Abraham et al. (1989) and Polinsky and Shavell (1998) argue that punitive damages based on a defendant’s wealth leads to over-deterrence and, similarly, Rhee (2012) supports that the wealth approach creates unpredictability in the outcome decided by the jury, providing incentives for defendants to overinvest in precaution. Our equilibrium results, hence, rationalize such behavior since a more severe wealth approach expands the range of parameters for which the pooling equilibrium exists. In addition, our findings also highlight that such overinvestment hinders the judge’s ability to infer the firm’s type.
Other papers examine how changes in legal costs can induce selection thus producing information revelation, see Brown and Ayres (1994) and Shavell (1995). Hence, this paper also connects to the literature on signaling and information transmission, such as Spence (1974) analyzing labor market signaling; Milgrom and Roberts (1982), who consider an entrant uncertain about the incumbent’s unit costs; Matthews and Mirman (1983), Ridley (2008) and Espinola-Arredondo et al. (2011) who examine settings in which the entrant does not observe market demand. In addition, our paper relates with the law and economics literature analyzing settings of incomplete information. Farmer and Pecorino (1994), Swanson and Mason (1998) and Heyes et al. (2004) examine a setting in which the defendant is uninformed about the plaintiff’s degree of risk aversion.\footnote{Similarly, Schweizer (1989), Spier (1992), Daughety and Reinganum (1994), and Friedman and Wittman (2006) consider a model in which both plaintiff and defendant are privately informed, and bargain a settlement in the first stage in order to avoid trial. If the bargaining process is unsuccessful, these studies assume that the winner is exogenously determined with a given probability.}

In contrast, we focus on the trial process where the judge can strategically respond to the firm’s investment in order to infer the firm’s type and determine punitive damages.\footnote{For a detailed survey on the modeling of pretrial settlement bargaining, both under complete and incomplete information, see Daughety and Reinganum (2012).}

In a different setting, Bebchuck and Guzman (1996) examine the role of legal costs on the bargaining process between the defendant and the plaintiff, showing that hourly fees induce the defendant to accept less favorable settlement offers than contingent fees. Heyes et al. (2004) analyze the impact of legal expenses insurance on plaintiffs’ bargaining strategies in post-accident negotiations. They show that legal insurance not only increases plaintiffs’ negotiating power, but also encourages the defendant to invest in precaution.\footnote{From the perspective of injured parties, Naysnerski and Tietenberg (1992) find that factors such as penalty remedies and attorney fee reimbursements have an impact on litigation activities.} Our paper, however, finds that a decrease in legal costs induces firms to reduce their precaution since appeals becomes cheaper and, in addition, hinders the judge’s ability to infer the firm’s type (since the separating equilibrium is less likely to arise).

Section 2 discusses the model, and section 3 examines the complete information context. Section 4 analyzes the incomplete information game, and section 5 discusses our results and policy implications.

2 Model

We examine a sequential-move game in which the judge is uninformed about the firm’s true type: either cautious \((C, \text{ with probability } \theta)\) or reckless \((R, \text{ with probability } 1 – \theta)\), where \(\theta \in (0, 1)\). After observing its type, the firm invests in precaution, such as safety measures, which can be either high \((H)\) or low \((L)\), at a cost \(c_j\) where \(j \in \{H, L\}\). For simplicity, we normalize \(c_L = 0\), and assume that \(c_H > 0\). The probability that an accident occurs is \(p^i_j\) and, thus, \(1 – p^i_j\) represents the probability that an accident does not occur, where \(i \in \{C, R\}\). We assume that, for a given investment level \(j\), it is more likely that an accident occurs when the firm is a reckless than a cautious type, \(p^C_j < p^R_j\). Even if both types of firms invest the same amount in safety measures, the reckless type exhibits a
poorer managerial ability than the cautious type at reducing the risk of an accident. In addition, for a given firm’s type $i$, the probability of an accident increases when moving from a high to a low precaution, $p_H^i < p_L^i$.

The firm’s profit when an accident does not occur is $r - c_j$, where $r$ denotes the firm’s revenue while $c_j$ is the cost of precaution. However, if there is an accident, the firm’s payoff is also affected by the compensatory and punitive damages. The compensatory damage is a function of the harm caused by the accident which, for simplicity, we assume to only depend on the investment in precaution. The firm’s type only affects the probability of an accident but, once the accident occurs, its harm is only a function of the investment level. Compensatory damages are denoted by $d_j$, where $d_L > d_H$. That is, if an accident occurs, the firm must pay $d_L$ if it invested in low precaution and $d_H$ if it did in high precaution. In addition, the judge decides whether to assign punitive damages based on the ratio approach or the wealth of the defendant. If the judge chooses the ratio approach, the firm must pay $\beta d_j$, where $\beta > 1$ indicates the punitive-to-compensatory ratio. Otherwise, the firm pays punitive damages based on wealth, $\alpha r$, where $\alpha \in (0,1)$ represents a percentage of revenue. In addition, we assume that punitive damages based on wealth exceed those based on the ratio approach, i.e., $\alpha r > \beta d_j$; as reported in Thomson and Scolnick (2008). The judge sets punitive damages as a tool to encourage the firm to be cautious. Hence, under complete information, the judge selects the wealth approach for a reckless firm, and the ratio approach for a cautious firm, which we refer to as a correct verdict. This response yields a payoff of $V$ for the judge, while an incorrect verdict, such as choosing the ratio approach after a reckless firm invests in low precaution, entails a loss of $\bar{V}$ to the judge, where $\bar{V} > 0 > V$.\footnote{Federal judges for the Supreme Court and district courts are appointed by the President for terms of good behavior, which means that they cannot be released, but can be impeached for improper behavior (United States Courts). We consider that an incorrect verdict leads to a detriment of the judge’s reputation.}

The time structure of the game is the following:

1. In the first period,

   (a) The firm decides the investment in precaution level, $H$ or $L$.

   (b) Nature then determines whether an accident happens with probability $p_j^i$ where $i \in \{C, R\}$ and $j \in \{H, L\}$.

   (c) If no accident occurs, the game ends. If, instead, an accident happens, the judge, upon observing firm’s investment, responds by selecting punitive damages (wealth or ratio).

2. In the second period,

   (a) The firm chooses whether to appeal the judge’s decision.\footnote{In reality, the firm can appeal multiple times to higher courts if it is not satisfied with the judge’s decision. However, for simplicity, we focus on a two-period game.}

   (b) If no appeal is filed, the game ends. If an appeal is requested at a legal cost $e > 0$, the judge responds by accepting or rejecting it.\footnote{In this case, we assume that the judge observes the firm’s type, since he has collected more evidence about the accident.}
If the judge ruled a correct verdict in the first period, his decision does not change in the second period. However, if the judge’s verdict was incorrect, his decision changes to a correct verdict in the second period. In this case, his reputation value increases from \( V \) to \( V' \), where \( V' > V > V \). Finally, the legal cost of appealing is assumed to be sufficiently low, in order to guarantee that firms have incentives to appeal when they face an incorrect decision from the judge, i.e., \( e < \alpha r - \beta d_j \) for all \( j \). (Otherwise, no type of firm would appeal under any circumstances.) We next identify under which conditions the single-crossing property holds in this context.

**Lemma 1.** The single-crossing property is satisfied if and only if

\[
\beta (p^C_L d_L - p^C_H d_H) + p^C_L e \geq p^R_L \alpha r - p^R_H \beta d_H
\]

In the binary setting that our model considers, the single-crossing property implies that the increase in payoffs that the cautious type obtains from choosing a high rather than a low investment is larger than that of the reckless type. Such a property is satisfied if the increase in expected punitive damages that the cautious firm experiences when choosing a low precaution (which includes the lower probability of an accident and the expected legal cost of appealing an incorrect verdict) is larger than that of the reckless firm, thus providing the former with more incentives to invest in high precaution than the latter. We assume that this condition holds throughout the paper.

**Socially optimal investment.** We next identify the socially optimal investment level that the social planner would choose. For a type \( i \) firm, a high investment yields a larger welfare than a low investment if

\[
r - c_H - p^i_H d_H \geq r - c_L - p^i_L d_L
\]

where \( r - c_j \) represents profits for an investment level \( j \), whereas \( p^i_j d_j \) denotes the expected damage from the accident. Solving for \( c_H \), and recalling that \( c_L = 0 \), we obtain \( c_H \leq p^i_L d_L - p^i_H d_H \). Intuitively, the increase in cost from investing in high precaution is smaller than the increase in expected damages from keeping investment at low precaution (as the accident is more likely to occur). When this condition holds for both types of firm, it is socially optimal that both invest in high precaution; when it is satisfied for only a type \( i \) firm, it is optimal that it invests in high while type \( j \neq i \) invests in low; and when the condition does not hold for either type of firm, it is optimal that both types of firm invest in low precaution.

In the following sections, we first examine the complete information setting as a benchmark and then focus on the incomplete information context.

### 3 Complete information

In this setting, the judge is able to observe the firm’s type. As a consequence, the firm’s investment in precaution cannot be used as a signal to convey or conceal its type from the judge. We solve the game by backward induction and the next proposition summarizes our findings.
Proposition 1. In a context of complete information, there exists a unique subgame perfect equilibrium in which:

1. Cautious firm. In the first period, it invests in a high precaution level if and only if $c_H \leq (1 + \beta) \left[ pc_L d_L - p_H d_H \right]$. In the second period, this firm only appeals after receiving an incorrect verdict (the wealth approach);

2. Reckless firm. In the first period, it invests in a low precaution level if and only if $c_H \geq \left[ pr_L d_L - pr_H d_H \right] + \alpha r (pr_L - pr_H)$. In the second period, this firm never appeals;

3. Judge. In the first period, he chooses the ratio (wealth) approach when facing the cautious (reckless, respectively) firm. In the second period, he accepts an appeal in all contingencies;

where the equilibrium exists if $\beta \left[ p^C_L d_L - p^C_H d_H \right] > \alpha r \left[ p^R_L - p^R_H \right]$.

Hence, along the equilibrium path, the cautious firm invests in high precaution if its costs are sufficiently low. This condition is more likely to hold if the punitive-to-compensatory ratio, $\beta$, is relatively high, and the additional expected compensatory damage that the firm pays when investing in low precaution, $p^C_L d_L - p^C_H d_H$, is also high. Observing the firm’s type, the judge responds with the ratio approach, which is not appealed by the firm. Intuitively, the appeal would not change the judge’s verdict, yet would entail a legal cost. In contrast, the reckless firm chooses a low precaution level if high precaution is relatively expensive. This condition depends on the increase in expected compensatory damages when the firm chooses a low precaution, $p^R_L d_L - p^R_H d_H$; and on the increase in expected punitive damages from the wealth approach, $\alpha r \left( pr_L - pr_H \right)$, since an accident is more likely to occur when the firm invests in low precaution.\(^{11}\) Finally, given that the single crossing property holds, this equilibrium exists if $\beta \left[ p^C_L d_L - p^C_H d_H \right] > \alpha r \left[ p^R_L - p^R_H \right]$. That is, the expected increase in punitive damages from choosing a low investment is more severe for the cautious than the reckless type of firm.

We next compare our equilibrium results against the socially optimal investment levels found in section 2. For the cautious type, its high investment in equilibrium is socially optimal if $c_H \leq p^C_L d_L - p^C_H d_H$ and its low investment (off-the-equilibrium) is optimal if $c_H > (1 + \beta) \left[ p^C_L d_L - p^C_H d_H \right]$. However, its high investment becomes socially excessive otherwise. That is, the firm and social planner choose a high (low) investment in precaution when its costs are extremely high (low, respectively). In contrast, when investment costs are intermediate, the firm selects a high investment in equilibrium in order to avoid potential penalties while the regulator would choose a low investment. Similarly, the low investment that the reckless type selects in equilibrium is socially optimal if $c_H > \left[ p^R_L d_L - p^R_H d_H \right] + \alpha r \left( p^C_L - p^C_H \right)$ and its high investment (off-the-equilibrium) is optimal if $c_H \leq p^R_L d_L - p^R_H d_H$. However, its high investment is socially excessive otherwise, thus following a similar pattern as for the cautious type of firm.

\(^{11}\)Hence, this firm does not appeal, both in- and out-the-equilibrium path, since a wrong verdict (ratio approach) is beneficial for the firm.
4 Incomplete information

In this information context, the firm knows its type, but the judge cannot observe it. The only information that the judge observes is the firm’s investment in precaution. The judge’s beliefs about facing a cautious firm, upon observing a high or low investment in precaution, are $\mu$ and $\gamma$ respectively, where $\mu, \gamma \in [0, 1]$. Next, we discuss the Perfect Bayesian Equilibrium (PBE) results of the incomplete information game. We focus on the first period, since second-period behavior coincides with that identified in the subgame perfect equilibrium of Proposition 1. Figure 1 in the appendix provides a graphical representation of the game.

4.1 Separating equilibria

Proposition 2. A separating PBE can be sustained in which: (1) the cautious (reckless) firm invests in high (low) precaution as long as $C_2 \leq c_H \leq C_1$ (where $C_2 \leq C_1$ by the single-crossing property); and (2) upon observing high (low) investment, the judge chooses the ratio (wealth, respectively) approach, where

$$
C_1 \equiv (1 + \beta) \left[ p_L^c d_L - p_H^c d_H \right] + p_L^c e, \quad \text{and}
$$

$$
C_2 \equiv \left[ p_L^R d_L - p_H^R d_H \right] + (p_L^R \alpha r - p_H^R \beta d_H).
$$

Therefore, firms behave as under complete information, but the conditions on $c_H$ differ. In particular, the judge’s inability to observe the firm’s type leads the cautious firm to choose a high precaution under a wider range of $c_H$ than in the case of complete information, i.e., this firm is more willing to invest in high precaution. Intuitively, if this firm deviates to a low investment, the judge identifies it as a reckless firm, responding with the wealth approach, thus forcing the cautious firm to appeal, which entails an additional legal cost $e$.

In contrast, the reckless firm is now more attracted to invest in high precaution than under complete information, since such investment in high precaution would conceal its type from the judge, receiving a lower punitive damage (ratio approach). Hence, the minimal cost $c_H$ that supports the reckless firm choosing low investment becomes more demanding.

Furthermore, cutoff $C_1$ is constant in $\alpha$ while $C_2$ increases, thus shrinking the range of $c_H$ for which the separating equilibrium exists. For values of $\alpha$ sufficiently high, $C_2$ lies above $C_1$, implying that this equilibrium cannot be supported. That is, when the burden of the wealth approach increases, the reckless firm is less willing to select a low investment, preferring to deviate towards high precaution. In contrast, cutoff $C_1$ increases in $\beta$ while $C_2$ decreases, thus expanding the range of $c_H$ for which the equilibrium exists. In this case, the reckless firm has less incentives to deviate towards high precaution since the punitive damage gap between the wealth and ratio approach, $p_L^R \alpha r - p_H^R \beta d_H$, narrowed. A similar argument applies when the legal cost, $e$, increases, since cutoff $C_1$ increases while $C_2$ remains unaffected. In particular, the cautious firm has less incentives to invest in low precaution, as this deviation would be responded by the judge with the
wealth approach, requiring the firm to appeal at a higher cost.\textsuperscript{12} We next examine the opposite strategy profile.

**Lemma 2.** The separating strategy profile in which the cautious (reckless) firm invests in low (high, respectively) precaution cannot be sustained as a PBE.

In this setting, the cautious (reckless) firm would choose a low (high) investment, which the judge infers to originate from the cautious (reckless) type, and responds with the ratio (wealth) approach. The reckless firm, however, would invest in high precaution and yet suffer the wealth approach, thus providing it with strong incentives to deviate, ultimately implying that this equilibrium does not exist.

**Example 1.** Consider probabilities $p_L^R = 0.9$, $p_L^C = 0.7$, $p_H^R = 0.6$ and $p_H^C = 0.1$. In addition, assume revenue $r = 150$, compensatory damages $d_L = 7$ and $d_H = 5$, a legal cost of $e = 12.3$, and judge’s payoffs of $\bar{V} = 14$, $V = 12$ and $\bar{V} = -2$. Finally, consider $\alpha = 0.2$ and $\beta = 2.5$. Under complete information, the cautious firm invests in high precaution if $c_H \leq (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right] = 15.4$, while the reckless firm invests in low if $c_H \geq \left[ p_L^R d_L - p_H^R d_H \right] + \alpha r \left( p_L^R - p_H^R \right) = 12.3$. Therefore, the subgame perfect equilibrium exists if $c_H \in [12.3, 15.4]$. Under incomplete information, the cutoffs identified in Proposition 2 become $C_1 = 24.01$ and $C_2 = 22.8$, thus allowing the separating equilibrium to exist for all $c_H \in [22.8, 24.01]$. Hence, in the separating equilibrium the cautious type of firm invests in high precaution under larger conditions on $c_H$ than under complete information, but the reckless firm invests in low precaution under more restrictive conditions. Finally, the social planner would choose a high investment for a firm with type $i$ if $c_H \leq p_L^i d_L - p_H^i d_H$, which entails cutoffs of $c_H \leq 4.4$ for the cautious type and $c_H \leq 6$ for the reckless type. Therefore, the separating equilibrium yields a socially optimal investment level for the reckless type but a socially excessive investment for the cautious type, since the regulator would choose a low investment for this firm.

### 4.2 Pooling equilibria

This subsection analyzes strategy profiles in which both types of firm choose the same investment level. We seek to guarantee that the judge’s verdict affects the firm’s incentives when selecting its precaution level. Hence, we assume that the cost of investing in high precaution, $c_H$, is: (1) not extremely low, i.e., $c_H > (1 + \beta) \left[ p_L^R d_L - p_H^R d_H \right]$; and (2) not prohibitively expensive, i.e., $c_H < (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right].\textsuperscript{13}

**Proposition 3.** A pooling PBE can be sustained in which the cautious and reckless types of firm invest in high precaution if and only if

\textsuperscript{12}Similarly as in the case of complete information, the equilibrium investment level of both types of firm is socially optimal if the investment cost $c_H$ is sufficiently high or sufficiently low, but becomes socially excessive otherwise.

\textsuperscript{13}As shown in the proof of Lemma 1, the single-crossing condition entails that $p_L^R d_L - p_H^R d_H > p_L^C d_L - p_H^C d_H$, thus implying that our analysis focuses on $c_H \in \left[ (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right], (1 + \beta) \left[ p_L^R d_L - p_H^R d_H \right] \right]$. 

9
i. Case 1: investment costs satisfy $c_H < C_2$; and the judge, upon observing high (low) investment, responds with the ratio (wealth, respectively) approach given beliefs $\mu = \theta > \bar{\theta}_H$ and $\gamma < \bar{\theta}_L$;

ii. Case 2: investment costs satisfy $c_H < C_4$ and if $\frac{p_R}{p_H} > \frac{\alpha r - \beta d_H}{\alpha r - \beta d_L}$, and the judge responds with the wealth approach regardless of the investment level given beliefs $\mu = \theta < \bar{\theta}_H$ and $\gamma < \bar{\theta}_L$;

where $\bar{\theta}_j \equiv \frac{p_R^j (\bar{V} - V)}{p_C^j (\bar{V} - V) + p_R^j (\bar{V} - V)}$, and cutoff $C_4 \equiv [p_L^R d_L - p_H^R d_H] + \alpha r (p_L^R - p_H^R)$;

Both types of firms choose to invest in high precaution if they face a sufficiently low cost $c_H$. However, the precise cutoff that induces firms to invest in high precaution varies depending on the judge’s prior belief that, upon observing a high investment (in equilibrium), he faces a cautious type, i.e., whether $\theta$ lies above or below $\bar{\theta}_H$. In particular, when cautious types are frequent, $\theta > \bar{\theta}_H$, the judge responds in equilibrium with the ratio approach, while he chooses the wealth approach otherwise.\footnote{Case 2 is sustained if the reckless type experiences a large increase in the probability of an accident should it deviate to a low investment, thus providing this firm with incentives to choose high precaution.}

In addition, since $C_2 > C_4$ both types of firm have more incentives to invest in high precaution when the judge responds with the ratio approach (Case 1) than with the wealth approach (Case 2). We next analyze the comparative statics of cutoff $\bar{\theta}_j$.

**Corollary 1.** Cutoff $\bar{\theta}_j$ decreases in $p_C^j$, $\bar{V}$ and $V_j$; but increases in $p_R^j$ and $V$.

Cutoff $\bar{\theta}_H$ represents the conditional probability that, upon observing an accident after the firm invested in high precaution, such an accident originates from a reckless firm.\footnote{Cutoff $\bar{\theta}_j$ is positive since $\bar{V} > V > V_j$ by definition, and smaller than one if $\frac{\bar{V} - V}{V - V_j} = \frac{p_R^j}{p_C^j}$.} Hence, this cutoff decreases in $p_C^j$, as an accident is more likely to stem from the cautious type, inducing the judge to respond with the ratio approach under larger conditions. The opposite argument applies to $p_R^j$, since an accident is more likely to originate from the reckless type, which increases cutoff $\bar{\theta}_H$. In addition, $\bar{\theta}_H$ decreases in the payoff that the judge obtains from a correct verdict, $\bar{V}$, and a wrong verdict that is not corrected after an appeal (i.e., the penalty $V_j$ is less severe). Therefore, the range of priors satisfying $\theta > \bar{\theta}_H$ expands, implying that the judge becomes less hesitant to apply the ratio approach as his rewards are more generous. In contrast, $\bar{\theta}_H$ increases in the judge’s payoff from changing his verdict, $V$. Intuitively, being wrong in the first stage is not so costly, thus leading the judge to respond with the wealth approach under larger parameter values.

When the percentage of revenue that firms pay in the wealth approach, $\alpha$, increases, the range of $c_H$ for which the pooling PBE exists expands in both Cases 1 and 2 since cutoffs $C_2$ and $C_4$ increase in $\alpha$. Specifically, firms have more incentives to choose a high precaution, since deviating to low precaution would result in a more costly wealth approach. If $\beta$ increases, cutoff $C_2$ decreases, $C_4$ remains constant, but the condition supporting case 2 is less likely to hold. Therefore, an increase in the punitive-to-compensatory ratio makes the existence of this equilibrium more difficult. Finally, an increase in the legal cost does not modify the conditions under which the equilibrium is supported.
since cutoffs $C_2$ and $C_4$ are unaffected by $e$. This is due to the fact that all cutoffs that support the pooling PBEs are those corresponding to the reckless firm, which does not appeal regardless of the judge’s verdict.

**Proposition 4.** A pooling PBE can be sustained in which the cautious and reckless types of firm invest in low precaution if and only if investment costs satisfy
\[
c_H \geq (1 + \beta) \left[ p^C_L d_L - p^H_H d_H \right] - e p^C_H;
\]
and the judge, upon observing low (high) investment, responds with the ratio (wealth, respectively) approach given beliefs $\gamma = \theta \geq \overline{\theta}_L$ and $\mu < \overline{\theta}_H$.

In contrast to the pooling PBE of Proposition 3, both types of firm must now face an expensive cost of high precaution in order to choose a low investment. Similarly to our above discussion, cutoff $\overline{\theta}_L$ reflects the probability of facing a reckless type, conditional on an accident occurring after a low investment. Hence, in equilibrium, the judge responds with the ratio approach if the probability of facing a cautious type $\theta$ is sufficiently high, $\theta \geq \overline{\theta}_L$. In this case, cutoff $\overline{\theta}_L$ decreases (increases) in $p^C_L$ ($p^R_L$), thus expanding (shrinking, respectively) the set of priors $\theta$ for which the judge responds with the ratio approach. Intuitively, when $p^C_L$ increases, an accident is more likely to originate from a cautious firm, inducing the judge to respond with the ratio approach under larger conditions. (Since the next subsection shows that this pooling PBE violates the Cho and Kreps’ (1987) Intuitive Criterion, we do not elaborate on its comparative statics.)

**Example 2.** Considering the parameter values in Example 1, the pooling equilibrium in Case 1 of Proposition 3 is sustained when $c_H < 22.8$ if the judge’s beliefs satisfy $\theta \geq 0.97$ and $\gamma < 0.91$. Since the set of admissible costs is $c_H \in [12.3, 15.4]$, this PBE exists for all $c_H$. Case 2, however, is only supported if $c_H < 12.3$ and $\theta < 0.97$ and $\gamma \geq 0.91$. Hence, this PBE cannot be sustained in the set of admissible costs. In this range, the regulator would choose low investment for both types of firm, whereas both types of firm invest in high precaution in the pooling equilibrium, thus yielding a socially excessive investment. Following a similar parametric example in the pooling equilibrium of Proposition 4, we obtain that the PBE exists when $c_H > 14.17$ and the judge’s beliefs satisfy $\theta \geq 0.91$ and $\mu < 0.97$.

### 4.3 Equilibrium Refinement

In this section, we apply Cho and Kreps’ (1987) Intuitive Criterion to the set of pooling PBEs identified in Propositions 3 and 4 in order to eliminate those based on insensible off-the-equilibrium beliefs.

**Proposition 5.** The pooling PBEs in which both types of firm choose a high investment in precaution survive the Cho and Kreps’ (1987) Intuitive Criterion. However, the pooling equilibrium in which both types of firm choose a low investment in precaution does not survive the Intuitive Criterion.
In Case 1 of Proposition 3, the judge responds with the ratio approach in equilibrium, which eliminates any incentives for the cautious type to deviate. The reckless type, in contrast, could have incentives to deviate since investing in high precaution in equilibrium does not reduce the probability of an accident as significantly as for the cautious type. However, deviating would signal its type to the judge, who would respond with the wealth approach. As a consequence, no type of firm deviates and this case survives the Intuitive Criterion (IC). Similarly, in Case 2 the reckless firm could have incentives to deviate if the judge responded with the ratio approach, while the cautious type may have incentives only if \( c_H \) is sufficiently high. Therefore, either only the reckless or both types of firm deviate, allowing the judge to infer that a low precaution originates from either the reckless type or from both (thus keeping his beliefs unaffected). In both cases, he maintains his response choosing the wealth approach. In summary, all pooling PBEs of Proposition 3 survive the IC.

In the pooling PBE of Proposition 4 where both types of firm choose low precaution, the cautious type has incentives to deviate while the reckless does not. In particular, this occurs because the decrease in expected punitive damages that the cautious type experiences when choosing a high precaution is significantly larger than that of the reckless firm, thus inducing only the former to deviate. Hence, the judge updates his beliefs upon observing a high investment, and responds with the ratio approach, implying that this pooling PBE violates the IC.

5 Discussion and Conclusions

Informative investments. In the separating equilibrium, the judge uses the investment decision as a signal to infer the firm’s type, which leads to correct verdicts and saves time and resources in future appeals. Our results show that the emergence of such an equilibrium critically depends on parameters \( \alpha, \beta \) and \( e \). First, an increase in the burden of the wealth approach (i.e., the percentage of revenue that firms pay if found liable) shrinks the set of parameter values for which this equilibrium exists, and for sufficiently high burdens the equilibrium cannot be sustained. In this context, only the pooling PBE in which both types of firm invest in high precaution arises in equilibrium, hindering the judge’s ability to infer the firm’s type upon observing its investment in precaution. A similar argument applies if firms face a low punitive-to-compensatory ratio, \( \beta \), or inexpensive legal costs, \( e \), which also shrink the set of parameters for which the separating equilibrium exists. In contrast, setting a low wealth burden combined with severe punitive-to-compensatory ratio and legal costs expands the range of \( c_H \) for which the separating PBE emerges, thus facilitating the judge’s ability to infer the firm’s type. For instance, judges with a history of setting low punitive damage using the wealth approach, such as in Liebeck v. McDonald’s (1994), or a high punitive-to-compensatory ratio, as in TXO Production Corp. v. Alliance Resource Corp. (1993), could induce future firms to behave as predicted by the separating equilibrium under larger conditions.

Investment in precaution. In addition, our results show that the pooling PBE arises when the
cost of investing in high precaution is relatively low, \( c_H < C_2 \), while the separating equilibrium is sustained for intermediate values, \( C_2 \leq c_H \leq C_1 \). This indicates that judges should pay close attention to investment costs in the industry where firms operate, since a relatively high cost would lead firms to behave as under the separating equilibrium, thus allowing the judge to infer whether the firm is cautious or reckless by only observing its investment decision. The opposite argument applies when investment costs are low, whereby the judge cannot infer the firm’s type, relying on his priors to choose punitive damages.

Judge’s rewards. We find that the judge’s response in the pooling equilibrium is sensible to his reward and penalty from choosing a correct or incorrect verdict, respectively. In particular, an increase in the reward from a correct verdict, makes the judge more willing to respond with the ratio approach. Therefore, the firm is benefited by more generous rewards to correct verdicts since it will face the ratio approach under larger conditions. In contrast, more severe punishments from incorrect verdicts are more likely to lead the judge to respond with the wealth approach. Hence, the cautious firm is harmed by a legal system in which judges are severely penalized when ruling incorrect verdicts, since it receives the wealth approach and incurs legal costs from the appealing process.

Further research. Our model considers that the judge becomes informed during the appealing process. However, if such a process is relatively short, the judge could remain uninformed about the firm’s type, thus affecting his response about accepting or rejecting the firm’s appeal, and the punitive damage decision in the first period. In addition, the judge’s reward or penalty from a verdict could be modified in the second period. This setting implies that the firm also faces uncertainty since, in the case of an appeal, it would not know whether the judge’s payoff is radically different than that in the first period, thus, affecting his investment decision.

6 Appendix

6.1 Proof of Lemma 1

When the judge responds with the ratio approach, the payoff increase that the cautious firm obtains from choosing high investment is larger than that of the reckless firm if

\[
[p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]] - [p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C)r] \geq [p_H^R [r - c_H - (d_H + \beta d_H)] + (1 - p_H^R) [r - c_H]] - [p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R)r]
\]

which simplifies to \( p_L^C d_L - p_H^C d_H \geq p_L^R d_L - p_H^R d_H \).

When the judge responds, instead, with the wealth approach, the payoff increase that the cautious firm obtains from choosing high investment is larger than that of the reckless firm if

\[
[p_H^C [r - c_H - e - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]] - [p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C)r] \geq [p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H]] - [p_L^R [r - (d_L + \alpha r)] + (1 - p_L^R)r]
\]
which simplifies to $C_1 > \hat{C}$, where $C_1 \equiv (1+\beta) \left[ p_L^C d_L - p_H^C d_H \right] + p_L^C e$ and $\hat{C} \equiv p_L^H e + \alpha \left( p_L^R - p_H^R \right) + (p_L^R d_L - p_H^R d_H)$. In addition, consider a cutoff $C_2 \equiv \left[ p_L^R d_L - p_H^R d_H \right] + (p_L^R \alpha - p_H^R \beta d_H)$, which lies above $\hat{C}$ for all $e < \frac{p_L^R}{p_H^R} (\alpha - \beta d_H)$, a condition that holds since $e < \alpha \beta - \beta d_H$ and $p_H^R > p_L^C$ by definition. Furthermore, $C_1 \geq C_2$ if

\[ (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right] + p_L^C e > \left[ p_L^R d_L - p_H^R d_H \right] + (p_L^R \alpha - p_H^R \beta d_H) \]

or,

\[ \beta \left[ p_L^C d_L - p_H^C d_H \right] + p_L^C e - \left[ p_L^R \alpha - p_H^R \beta d_H \right] \geq \left[ p_L^R d_L - p_H^R d_H \right] - \left[ p_L^C d_L - p_H^C d_H \right] . \]

Since the right-hand side is negative, a sufficient condition for the above inequality to hold is

\[ \beta \left[ p_L^C d_L - p_H^C d_H \right] + p_L^C e - \left[ p_L^R \alpha - p_H^R \beta d_H \right] \geq 0 \]

or

\[ p_L^C d_L - p_H^C d_H \geq \frac{\left[ p_L^R \alpha - p_H^R \beta d_H \right] - p_L^C e}{\beta} \] \hspace{1cm} (A)

Finally, the term on the right-hand side satisfies

\[ \frac{\left[ p_L^R \alpha - p_H^R \beta d_H \right] - p_L^C e}{\beta} \geq p_L^R d_L - p_H^R d_H \]

since, solving for $e$, we obtain $\frac{p_L^R}{p_H^R} (\alpha - \beta d_L) > e$, which holds given that $e < \alpha \beta - \beta d_L$ and $p_L^R > p_L^C$ by definition. Therefore, if condition (A) holds, $p_L^C d_L - p_H^C d_H \geq p_L^R d_L - p_H^R d_H$ is also satisfied. In summary, we have shown that $C_1 \geq C_2$ if condition (A) holds, and that $C_2 > \hat{C}$ is supported under all parameter conditions. Hence, when condition (A) holds, $C_1 > \hat{C}$.

### 6.2 Proof of Proposition 1

**Second Period.**

*Judge.* When facing a cautious firm, in the second period the judge is indifferent between accepting and not accepting its appeal when the firm invested in high precaution level and was penalized with the ratio approach, since the judge’s payoffs are the same, i.e., $\nabla$ in both cases. Similarly, the judge accepts the appeal of a cautious firm that invested in high precaution and received the wealth approach (wrong verdict) since $V > \nabla$, which holds by definition.

In addition, the judge accepts the appeal of a cautious firm that invested in low precaution and received the ratio approach since he is indifferent between accepting and rejecting the appeal (he obtains $\nabla$ in both cases). The judge accepts the appeal of a cautious firm that invested in low precaution and received the wealth approach (wrong verdict) since $V > \nabla$.

When facing a reckless firm, the judge is indifferent between accepting and not accepting its appeal when this firm invested in low precaution level and was penalized with the wealth approach since the judge’s payoffs are the same, i.e., $\nabla$ in both cases. A similar argument applies when a
reckless firm invests in low precaution and receives the ratio approach (wrong verdict). In this case, the judge accepts the appeal since $V > V'$.

In addition, the judge accepts an appeal of a reckless firm that invested in high precaution level and received the wealth approach since he obtains a payoff of $V$ in both cases. Finally, the judge accepts the appeal of a reckless firm that invested in high precaution level and received the ratio approach (wrong verdict) since $V > V'$.

_Cautious firm._ The cautious firm appeals in the second period after investing in high precaution level and receiving the wealth approach, since the firm anticipates the judge accepting the appeal if
\[
  r - c_H - \epsilon - [d_H + \beta d_H] > r - c_H - [d_H + \alpha r]
\]
which, solving for $\epsilon$, yields $\epsilon < \alpha r - \beta d_H$, which holds by definition.

This firm does not appeal after investing in high precaution if it received the ratio approach, since its appeal would not be accepted yielding a payoff $r - c_H - \epsilon - [d_H + \beta d_H]$, which is lower than its payoff from not appealing, $r - c_H - [d_H + \beta d_H]$, i.e., the firm saves the legal cost $\epsilon$.

In addition, it does not appeal if it invested in low precaution and it received the ratio approach, since its payoff from not appealing is $r - [d_L + \beta d_L]$, while that of appealing is $r - \epsilon - [d_L + \beta d_L]$.

Finally, if this firm invested in low precaution and receiving the wealth approach, it appeals if its payoff from appealing, $r - \epsilon - [d_L + \beta d_L]$ (since its appeal is subsequently accepted), exceeds that from not appealing, $r - [d_L + \alpha r]$, that is, if $\epsilon < \alpha r - \beta d_L$, which holds by definition.

_Reckless firm._ The reckless firm does not appeal in the second period after investing in low precaution and receiving the wealth approach, since its payoff from not appealing is $r - [d_L + \alpha r]$, which exceeds that from appealing, $r - \epsilon - [d_L + \alpha r]$. Similarly, this firm does not appeal after investing in low precaution but receiving the ratio approach, since its payoff from not appealing, $r - [d_L + \beta d_L]$, is higher than that from appealing, $r - \epsilon - [d_L + \alpha r]$, if $\epsilon > \beta d_L - \alpha r$, which holds for all $\epsilon$ since $\beta d_H < \alpha r$ by definition.

A similar argument applies to this firm after investing in high precaution and receiving the ratio approach, since its payoff from not appealing, $r - c_H - [d_H + \beta d_H]$, exceeds that from appealing, $r - c_H - \epsilon - [d_H + \alpha r]$, since $\epsilon > \beta d_H - \alpha r$ holds by definition.

Finally, this firm does not appeal after investing in high precaution and facing the wealth approach since
\[
  r - c_H - \epsilon - [d_H + \alpha r] < r - c_H - [d_H + \alpha r]
\]
holds for all $\epsilon$.

**First Period**

_Judge._ The judge chooses the ratio (wealth) approach when the firm is cautious (reckless, respectively) since $V > V'$, which holds regardless of the firm’s investment.
Cautious firm. Anticipating the ratio approach, this firm invests in a high precaution level if
\[
p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \\
\geq p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r
\]
which, solving for \(c_H\), yields
\[
c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H]
\]

Reckless firm. Anticipating the wealth approach, this firm invests in a low precaution level if
\[
p_L^R [r - (d_L + \alpha r)] + (1 - p_L^R) r \\
\geq p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H]
\]
which, solving for \(c_H\), yields
\[
c_H \geq [p_L^R d_L - p_H^R d_H] + \alpha r (p_L^R - p_H^R)
\]

Both conditions on \(c_H\) simultaneously hold as long as
\[
(1 + \beta) [p_L^C d_L - p_H^C d_H] > [p_L^R d_L - p_H^R d_H] + \alpha r (p_L^R - p_H^R)
\]
rearranging,
\[
\beta [p_L^C d_L - p_H^C d_H] - \alpha r [p_L^R - p_H^R] > [p_L^R d_L - p_H^R d_H] - [p_L^C d_L - p_H^C d_H] \equiv \Gamma
\]
where \(\Gamma < 0\) by the single-crossing property. Therefore, \(\beta [p_L^C d_L - p_H^C d_H] - \alpha r [p_L^R - p_H^R] > 0\) if
\[
\beta [p_L^C d_L - p_H^C d_H] > \alpha r [p_L^R - p_H^R].
\]

6.3 Proof of Proposition 2

In this strategy profile, the judge updates his beliefs by Bayes’ rule, yielding \(\mu = 1\) and \(\gamma = 0\). Hence, upon observing a high investment in precaution, the judge response is the ratio approach since \(\overline{V} > V\); and after observing a low investment, he responds with the wealth approach given that \(\overline{V} > V\).

Anticipating these responses by the judge, the cautious firm chooses a high investment if and only if
\[
p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \\
\geq p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C) r
\]
which, solving for $c_H$, yields

$$c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H] + p_L^C e \equiv C_1.$$ 

In contrast, the reckless firm chooses a low investment if and only if

$$p^R_L [r - (d_L + \alpha r)] + (1 - p^R_L) r \geq p^R_H [r - c_H - (d_H + \beta d_H)] + (1 - p^R_H) [r - c_H]$$

which, solving for $c_H$, yields

$$c_H \geq [p^R_L d_L - p^R_H d_H] + (p^R_L \alpha r - p^R_H \beta d_H) \equiv C_2.$$ 

By the single-crossing property (see proof of Lemma 1), $C_1 \geq C_2$, thus guaranteeing the existence of this separating PBE.

6.4 Proof of Lemma 2

In this strategy profile, the judge updates his beliefs by Bayes’ rule, yielding $\mu = 0$ and $\gamma = 1$. Therefore, upon observing a low investment, the judge responds with the ratio approach since $V > V$; and after observing a high investment, he responds with the wealth approach given that $V > V$.

Anticipating the judge’s response, the cautious firm chooses a low investment if and only if

$$p^C_L [r - (d_L + \beta d_L)] + (1 - p^C_L) r \geq p^C_H [r - c_H - e - (d_H + \beta d_H)] + (1 - p^C_H) [r - c_H]$$

which, solving for $c_H$, yields

$$c_H \geq (1 + \beta) [p^C_L d_L - p^C_H d_H] - p^C_H e \equiv C_1 - e(p^C_L + p^C_H).$$

The reckless firm chooses a high investment if and only if

$$p^R_H [r - c_H - (d_H + \alpha r)] + (1 - p^R_H) [r - c_H] \geq p^R_L [r - (d_L + \beta d_L)] + (1 - p^R_L) r$$

which, solving for $c_H$, yields

$$c_H \leq [p^R_L d_L - p^R_H d_H] - (p^R_H \alpha r - p^R_L \beta d_L) \equiv C_2 - (p^R_H \alpha r - p^R_L \beta d_L) - (p^R_L \alpha r - p^R_H \beta d_H).$$
In addition, the equilibrium would exist if
\[ C_2 - (p_H^R \alpha r - p_L^R \beta d_L) - (p_L^R \alpha r - p_H^R \beta d_H) \geq C_1 - e(p_L^C + p_H^C). \]
simplifying, we obtain
\[ C_2 + e(p_L^C + p_H^C) \geq C_1 + p_L^R (\alpha r - \beta d_L) + p_H^R (\alpha r - \beta d_H) \]
which cannot hold since \( C_1 \geq C_2 \) by the single-crossing property, legal cost satisfy \( e < \alpha r - \beta d_j \) for all \( j \in \{H, L\} \), and \( p_j^R > p_j^C \) for all \( j \). Therefore, this separating strategy profile cannot be supported as a PBE.

6.5 Proof of Proposition 3

In this strategy profile, the judge cannot update his beliefs, yielding \( \mu = \theta \) (in equilibrium) and \( \gamma \in [0, 1] \) (off-the-equilibrium). Therefore, upon observing a high investment, the judge responds with the ratio approach if
\[ \theta p_H^C \gamma + (1 - \theta)p_H^R \gamma \geq \theta p_H^C \nu + (1 - \theta)p_H^R \nu \]
which, solving for \( \theta \), yields
\[ \theta \geq \frac{p_H^R (\nu - \gamma)}{p_H^C (\nu - \gamma) + p_H^R (\nu - \nu)} \equiv \bar{\theta}_H \]
and, upon observing low investment in precaution, the judge responds with the wealth approach if
\[ \gamma p_L^C \nu + (1 - \gamma)p_L^R \nu \geq \gamma p_L^C \nu + (1 - \gamma)p_L^R \nu \]
which, solving for \( \gamma \), yields
\[ \gamma \leq \frac{p_L^R (\nu - \gamma)}{p_L^C (\nu - \gamma) + p_L^R (\nu - \nu)} \equiv \bar{\theta}_L \]

We next analyze four different cases depending on the judge’s beliefs:

Case 1. The judge’s beliefs are \( \theta \geq \bar{\theta}_H \) and \( \gamma < \bar{\theta}_L \), that is, the judge chooses ratio approach after high investment and wealth approach after low investment. Anticipating the judge’s response, the cautious firm chooses a high investment if and only if
\[ p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \geq p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C) r \]
which, solving for \( c_H \), yields
\[ c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H] + p_L^C e \equiv C_1. \]
The reckless firm chooses a high investment if and only if
\[
p_H^R [r - c_H - (d_H + \beta d_H)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \alpha r)] + (1 - p_L^R) r
\]
which, solving for \(c_H\), yields
\[
c_H \leq [p_L^R d_L - p_H^R d_H] + (p_L^R \alpha r - p_H^R \beta d_H) \equiv C_2.
\]

Hence, since \(C_1 \geq C_2\) by the single-crossing property, and \(C_2 > (1 + \beta) [p_L^R d_L - p_H^R d_H]\) since \(\alpha r > \beta d_L\), the pooling equilibrium exists in Case 1 if \(c_H \leq C_2\).

**Case 2.** The judge’s beliefs are \(\theta < \theta_H\) and \(\gamma < \theta_L\), that is, the judge chooses wealth approach regardless of the investment decision. Anticipating the judge’s response, the cautious firm chooses a high investment if and only if
\[
p_H^C [r - c_H - e - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \geq p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C) r
\]
which, solving for \(c_H\), yields
\[
c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H] + e (p_L^C - p_H^C) \equiv C_3.
\]

The reckless firm chooses a high investment if and only if
\[
p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \alpha r)] + (1 - p_L^R) r
\]
which, solving for \(c_H\), yields
\[
c_H \leq [p_L^R d_L - p_H^R d_H] + \alpha r (p_L^R - p_H^R) \equiv C_4.
\]

In addition, \(C_3 > C_4\) since \(C_1 \geq C_2\) by the single-crossing property, \(e < \alpha r - \beta d_H\), and \(p_H^R > p_H^C\). Therefore, for the pooling equilibrium to exist in Case 2 we need \(c_H \leq C_4\). Hence, the equilibrium exists if and only if \(C_4 \geq C_B\) which entails \(\frac{p_H^R}{p_H^C} > \frac{\alpha r - \beta d_H}{\alpha r - \beta d_L}\).

**Case 3.** The judge’s beliefs are \(\theta \geq \theta_H\) and \(\gamma \geq \theta_L\), that is, the judge chooses ratio approach regardless of the investment decision. Anticipating the judge’s response, the cautious firm chooses a high investment if and only if
\[
p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \geq p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r
\]
which, solving for \(c_H\), yields
\[
c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H].
\]
The reckless firm chooses a high investment if and only if

\[ p_H^R [r - c_H - (d_H + \beta d_H)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R) r \]

which, solving for \( c_H \), yields

\[ c_H \leq (1 + \beta) [p_L^R d_L - p_H^R d_H] \equiv C_B. \]

Therefore, since \( c_H \leq C_B \) by assumption, the pooling equilibrium in Case 3 cannot be supported.

**Case 4.** The judge’s beliefs are \( \theta < \bar{\theta}_H \) and \( \gamma \geq \bar{\theta}_L \), that is, the judge chooses wealth approach after observing a high investment, but the ratio approach otherwise. Anticipating the judge’s response, the cautious firm chooses a high investment if and only if

\[ p_C^H [r - c_H - e - (d_H + \beta d_H)] + (1 - p_C^H) [r - c_H] \geq p_C^L [r - (d_L + \beta d_L)] + (1 - p_C^L) r \]

which, solving for \( c_H \), yields

\[ c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H] - e p_H^C = C_3 - p_L^C e. \]

The reckless firm chooses a high investment if and only if

\[ p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R) r \]

which, solving for \( c_H \), yields

\[ c_H \leq (p_L^R d_L - p_H^R d_H) - [p_H^R \alpha r - p_L^R \beta d_L] \equiv C_5. \]

In addition, \( C_3 - p_L^C e > C_5 \) since \( C_1 \geq C_2 \) by the single-crossing property, legal cost satisfy \( e < \alpha r - \beta d_j \) for all \( j \in \{H, L\} \), and \( p_j^R > p_j^C \) for all \( j \). Therefore, for the pooling equilibrium to exist in Case 4 we would need \( c_H \leq C_5 \). However, \( C_5 < C_B \) since \( \alpha r > \beta d_H \), implying that the pooling PBE of Case 4 cannot be sustained.

### 6.6 Proof of Corollary 1

Differentiating cutoff \( \bar{\theta}_j \) with respect to \( p_j^C \) yields

\[
\frac{p_j^R (V - \bar{V}) (\bar{V} - \bar{V})}{\left[ p_j^C V + p_j^R \bar{V} - \left( p_j^C + p_j^R \right) \bar{V} \right]^2} < 0
\]

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and with respect to $p_j^R$ yields
\[
- \frac{p_j^C (V - \overline{V}) (\overline{V} - \overline{V})}{[p_j^C V + p_j^R \overline{V} - (p_j^C + p_j^R) \overline{V}]^2} > 0
\]

Differentiating cutoff $\bar{\theta}_j$ with respect to $\overline{V}$ yields
\[
- \frac{p_j^C p_j^R (V - \overline{V})}{[p_j^C V + p_j^R \overline{V} - (p_j^C + p_j^R) \overline{V}]^2} < 0
\]

and with respect to $\overline{V}$ yields
\[
\frac{p_j^C p_j^R (V - \overline{V})}{[p_j^C V + p_j^R \overline{V} - (p_j^C + p_j^R) \overline{V}]^2} < 0
\]

and with respect to $V$ yields
\[
\frac{p_j^C p_j^R (\overline{V} - \overline{V})}{[p_j^C V + p_j^R \overline{V} - (p_j^C + p_j^R) \overline{V}]^2} > 0
\]

6.7 Proof of Proposition 4

In this strategy profile, the judge cannot update his beliefs, yielding $\gamma = \theta$ (in equilibrium) and $\mu \in [0, 1]$ (off-the-equilibrium). Therefore, upon observing a low investment, the judge responds with the wealth approach if
\[
\theta p_L^C V + (1 - \theta) p_L^R \overline{V} \geq \theta p_L^C \overline{V} + (1 - \theta) p_L^R \overline{V}
\]

which, solving for $\theta$, yields
\[
\theta < \frac{p_L^R (\overline{V} - \overline{V})}{p_L^C (\overline{V} - \overline{V}) + p_L^R (\overline{V} - \overline{V})} \equiv \bar{\theta}_L
\]

and, upon observing low investment in precaution, the judge responds with the wealth approach if
\[
\mu p_H^C V + (1 - \mu) p_H^R \overline{V} \geq \mu p_H^C \overline{V} + (1 - \mu) p_H^R \overline{V}
\]

which, solving for $\gamma$, yields
\[
\mu < \frac{p_H^R (\overline{V} - \overline{V})}{p_H^C (\overline{V} - \overline{V}) + p_H^R (\overline{V} - \overline{V})} \equiv \bar{\theta}_H.
\]

We next analyze four different cases depending on the judge’s beliefs:
Case 1. The judge’s beliefs are $\theta < \bar{\theta}_L$ and $\mu < \bar{\theta}_H$, that is, the judge chooses the wealth approach regardless of the firm’s investment decision. Anticipating the judge’s response, the cautious firm chooses a low investment if and only if
\[
p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C) r \geq p_H^C [r - c_H - e - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]
\]
which, solving for $c_H$, yields $c_H \geq C_3$, which cannot hold since $C_3 > (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right]$. Hence, the pooling equilibrium in Case 1 cannot be sustained.

Case 2. The judge’s beliefs are $\theta \geq \bar{\theta}_L$ and $\mu < \bar{\theta}_H$, that is, the judge chooses ratio (wealth) approach after observing a low (high) investment. Anticipating the judge’s response, the cautious firm chooses a low investment if and only if
\[
p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r \geq p_H^C [r - c_H - e - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]
\]
which, solving for $c_H$, yields $c_H \geq C_3 - p_L^C e$. The reckless firm chooses a high investment if and only if
\[
p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R) r \geq p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H]
\]
which, solving for $c_H$, yields $c_H \geq C_5$. Since $C_3 - p_L^C e > C_5$ as shown in the proof of Proposition 4, the pooling equilibrium exists in Case 2 if $c_H \geq C_3 - p_L^C e$, where $C_3 - p_L^C e < (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right]$ for all parameter values.

Case 3. The judge’s beliefs are $\theta < \bar{\theta}_L$ and $\mu \geq \bar{\theta}_H$, that is, the judge chooses ratio (wealth) approach after observing a high (low) investment. Anticipating the judge’s response, the cautious firm chooses a low investment if and only if
\[
p_L^C [r - e - (d_L + \beta d_L)] + (1 - p_L^C) r \geq p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]
\]
which, solving for $c_H$, yields $c_H \geq C_1$, which cannot hold since $C_1 > (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right]$. Hence, the pooling equilibrium in Case 3 cannot be sustained.

Case 4. The judge’s beliefs are $\theta \geq \bar{\theta}_L$ and $\mu \geq \bar{\theta}_H$, that is, the judge chooses ratio approach regardless of the firm’s investment decision. Anticipating the judge’s response, the cautious firm chooses a low investment if and only if
\[
p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r \geq p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H]
\]
which, solving for $c_H$, yields $c_H \geq (1 + \beta) \left[ p_L^C d_L - p_H^C d_H \right]$, which cannot hold by definition.

6.8 Proof of Proposition 5

We first apply the Intuitive Criterion (IC) to the PBEs in Proposition 3.

Case 1. Deviation to low investment is equilibrium dominated for the cautious type if and only
if its equilibrium payoff is greater than the highest payoff it could obtain from deviating to low (which occurs when the judge responds with the ratio approach), that is,

\[ p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \geq p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r \]

or, if \( c_H \leq (1 + \beta) [p_L^C d_L - p_H^C d_H] \equiv C_A \), which is satisfied for all admissible values of \( c_H \). Deviation to low investment is equilibrium dominated for the reckless type if and only if

\[ p_H^R [r - c_H - (d_H + \beta d_H)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R) r \]

or, if \( c_H \leq (1 + \beta) [p_L^R d_L - p_H^R d_H] \equiv C_B \), which cannot hold since \( c_H \) must satisfy \( c_H > C_B \) by definition. Therefore, only the reckless type has incentives to deviate. The judge can update his belief as a low investment only originates from a reckless firm, thus, responding with a wealth approach, eliminating the incentives of this type of firm to deviate. Hence, the pooling PBE survives the IC in Case 1.

**Case 2.** Deviation to low investment is equilibrium dominated for the cautious type if and only if

\[ p_H^C [r - c_H - e - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \geq p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r \]

or, if \( c_H \leq C_1 - e (p_L^C + p_H^C) = C_3 - p_L^C e \). Deviation to low investment is equilibrium dominated for the reckless type if and only if

\[ p_H^R [r - c_H - (d_H + \alpha r)] + (1 - p_H^R) [r - c_H] \geq p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R) r \]

or, if \( c_H \leq C_5 \), which does not hold since \( C_5 < C_B \). Therefore, the reckless type has incentives to deviate. We next check if these conditions hold under the range of \( c_H \) for which this pooling equilibrium is supported, i.e., \( c_H \leq C_A \). If \( C_3 - p_L^C e < C_A \) then two regions emerge: (1) if \( C_B < c_H < C_3 - p_L^C e \), only the reckless type deviates; and (2) if \( C_3 - p_L^C e < c_H < C_A \), both types deviate. In region (1), the judge can update his belief since a low investment only originates from a reckless type, thus, responding with the wealth approach, eliminating the incentives of this type of firm to deviate. In region (2) the judge cannot update his beliefs upon observing a low investment. Hence, the pooling PBE survives the IC in all regions. A similar argument to that in region (2) applies when \( C_3 - p_L^C e > C_A \).

We next analyze if the PBE in Proposition 4 survives the IC. Deviation to high investment is equilibrium dominated for the cautious type if and only if

\[ p_L^C [r - (d_L + \beta d_L)] + (1 - p_L^C) r \geq p_H^C [r - c_H - (d_H + \beta d_H)] + (1 - p_H^C) [r - c_H] \]

or, if \( c_H \geq C_1 - p_L^C e \), where \( C_1 - p_L^C e = C_A \). Hence, the cautious type has incentives to deviate to high precaution for all admissible \( c_H \), i.e., \( c_H \in [C_B, C_A] \). Deviation to high investment is
equilibrium dominated for the reckless type if and only if
\[
p_L^R [r - (d_L + \beta d_L)] + (1 - p_L^R)r \geq p_H^R [r - c_H - (d_H + \beta d_H)] + (1 - p_H^R)[r - c_H]
\]
or, if \( c_H \geq (1 + \beta) [p_L^R d_L - p_H^R d_H] \equiv C_B \), which holds for all admissible values of \( c_H \). Hence, the reckless firm does not deviate for any \( c_H \) in this PBE. Therefore, only the cautious firm deviates, the judge updates his belief since a high investment only originates from a cautious type, thus responding with the ratio approach. In summary, the pooling PBE of Proposition 4 violates the IC.

References


Figure 1: Two-period game tree.